Simulation of the Dynamic Response of a Damped Taut String

By

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ABSTRACT

Marine pipeline are facing new issues involved by the increase of the depth of exploited oil and gas reservoirs. This thesis discusses the changes in the dynamic behavior of marine pipelines and proposes a simple simulation based on a taut string. The dynamic response of the taut string is modeled using two techniques, the Green's function and the modal superposition. This study demonstrates that the modal superposition technique, commonly used to assess the dynamic behavior of marine pipelines, is still valid under certain conditions.

Thesis Supervisor: J.Kim Vandiver
Title: Dean for Undergraduate Research

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1 INTRODUCTION

1.1 MARINE RISERS

The extensive consummation of oil and gas has brought the development of subsea reservoir in the second half of the 20^{th} century. The exploitation of subsea reservoirs requires heavy structures located offshore. These structures are either resting on the seabed or floating. They are facing hazards caused by exposure of submerged components to underwater currents. Critical among these structures are the marine risers. Marine risers, as shown on Figure 1, consist of a series of steel pipes connecting the surface platform to the sea bed. Risers are used to carry the oil or gas from the subsea reservoir to the platform. The structural integrity of the marine riser is crucial for the oil and gas production. The slenderness of risers makes them flexible and subjected to vibrations. When these risers are exposed to flow, vortices are shed alternatively from the sides of the cylinder. Vibration caused due to drag and lift forces resulting from the vortex shedding is called Vortex Induced Vibration (VIV).

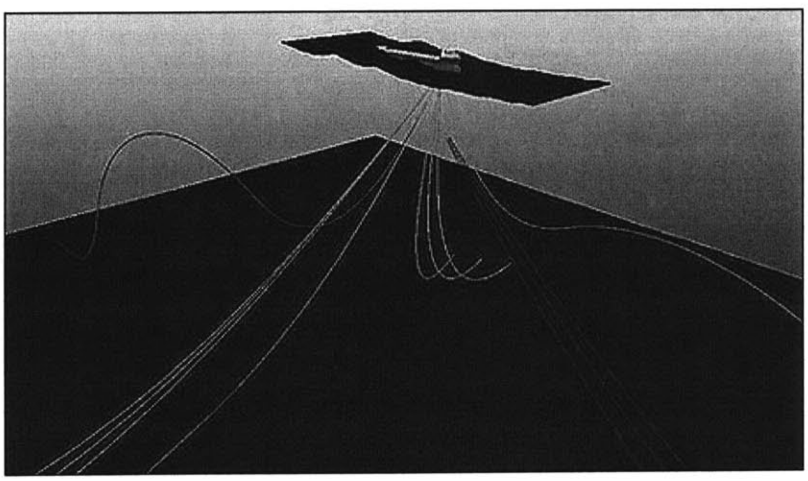

Figure 1 : Riser pipeline

For many years the oil and gas production has remained in the field of shallow water, in the range of 100ft to 1000ft. However, the increasing need for oil and gas has pushed the exploration up to 7000ft or more. Dynamic loadings become all the more an issue that exploited reservoirs are getting deeper and deeper increasing riser's length. In addition, dynamic issues are also enhanced by the fact that floating platforms move one end of the riser. At a design stage, the dynamic response of riser pipelines, modeled as strings subjected to harmonic loading is evaluated using the modal superposition technique. This technique models exactly the physics of the string for standing waves. Standing waves are the superposition of two waves having the same amplitude, wave length and frequency traveling in opposite direction. Software programs such as Shear 7 use this technique. However, when the string becomes too long, the waves decay becomes important and the wave may die before it is reflected at the end of the riser. The response is no longer a stationary wave behavior but is dominated by a traveling wave behavior. This thesis proposes to determine whether the modal superposition technique is still valid in the case of traveling wave response, i.e. in the case of long risers.

1.2 Vortex Induced Vibrations

When a fluid flows around a cylinder, there is a flow separation. The flow separation results in shed vortices. The vortex shedding creates the pressure distribution around the cylindrical pipeline to vary. The alternate shedding results in a lift force on the riser. This force is dependent on many factors such as the pipeline diameter or the current velocity. The force generated fluctuates in time and along the pipeline length.

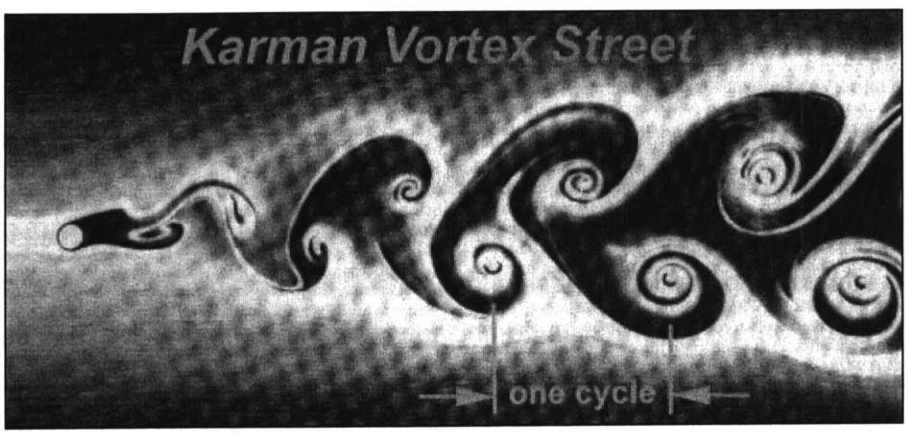

Figure 2 : Visualization of vortex induced vibration

Turbulences are generated alternatively on the top and the bottom of the cylinder as shown in Figure 2. The current flows from the left to the right of the figure. These turbulences create a force in the direction perpendicular to the current flow. The intensity of the force can be approximated to vary harmonically in time.

2 PROBLEM STATEMENT

Vortex Induced Vibration creates drag and lift forces on risers. These forces generated alternatively on either sides of the cylinder excite the riser. The riser response can damage the structure by either generating significant displacements or fatigue damage. In order to assess the dynamic response of risers, the modal superposition technique was used. This method perfectly describes the dynamic response of a string when it is subjected to standing waves. However for various reasons described hereafter in this section, the dynamic response of longer risers is not anymore made of standing waves but rather made of traveling waves. At the first glance, the use of the modal superposition technique does not seem to be adequate to predict the dynamic response of a string in case of traveling waves. However, by introducing non-resonant modes, it is possible to model traveling wave behavior. An exact representation of traveling wave would require an infinite number of non-resonant modes to be included in the solution. In practice however a good approximation may be achieved by including only a finite number of modes. This thesis presents a comparison study between the modal superposition technique and the Green's function technique. The response given by the Green's function is assumed to be the exact solution. Comparing results obtained by the modal superposition technique to the Green's function result on a wide range of conditions will tell us whether the modal superposition technique is accurate and what its limitations are.

This study is performed on a simplified model of a riser, a taut string fixed at its extremities as shown in Figure 3. The excitation has been considered as harmonic for the study.

As mentioned in [3], the spatial attenuation $n.\xi$ plays an important role in the type of response. The study gives the result of the comparison for values of $n.\xi$ ranging from 0.01 to 7.

2.1 THE SPATIAL ATTENUATION

2.1.1 Standing wave and traveling wave behavior

The spatial attenuation parameter, n.ξ governs the dynamic response behavior of the string, n being the mode number and ξ the damping ratio. For low values of n.ξ, the decay is low and the dynamic response has a standing wave behavior. For higher values of n.ξ, the decay becomes important. The decay can become high enough so that waves die before reaching the end of the string. In that case, there is no reflected wave and the dynamic response is made of traveling wave. The string has the behavior of an infinite string.

This may be explained by considering that the wave's amplitude decays over one length of the cable can be expressed as $\exp\left(\frac{-2\cdot\pi\cdot\xi\cdot L}{\lambda}\right)$. Noting that the wave length is: $\lambda = \frac{2\cdot L}{n}$, n being the mode number, it comes that, the decay is finally, $\exp(-\pi\cdot\xi\cdot n)$. Hence, the decay increases with increasing n.ξ.

When the decay is low enough, generated waves travel from the excitation location to the end of the string where they are reflected and travel back in the opposite direction.

Let's consider a wavelet generated around the middle of the string. The string displacement generated by this wavelet can be expressed as: $y_1 = A_1\cdot\sin(\omega\cdot t + k\cdot x)$. This is the general solution of the wave equation, Equation 5. The string displacement generated by the reflected wave would be written as: $y_2 = A_2\cdot\sin(\omega\cdot t - k\cdot x)$. On one extreme, when the damping is small and negligible, the decay exponent approaches 1.0 and the amplitude of the reflected wave is the same as the one which gave it birth and $A_2 = A_1$. Then, the superposition of these two waves is:

$$y_T = y_1 + y_2 = A_1\cdot[\sin(\omega\cdot t + k\cdot x) + \sin(\omega\cdot t - k\cdot x)]$$

$$y_T = A_1\cdot[\sin(\omega\cdot t)\cdot\cos(k\cdot x) - \cos(\omega\cdot t)\cdot\sin(k\cdot x) + \sin(\omega\cdot t)\cdot\cos(k\cdot x) + \cos(\omega\cdot t)\cdot\sin(k\cdot x)]$$

$$\boxed{y_T = 2\cdot A_1\cdot\sin(\omega\cdot t)\cdot\cos(k\cdot x)} \qquad \textbf{Equation 1}$$

This is the expression of a pure standing wave.

If the system is damped, the amplitude of the reflected wave A_2 becomes less than A_1; the string does not have anymore a perfect standing wave behavior. Its behavior is between the standing wave behavior and the traveling wave behavior.

On the other extreme, when the parameter n.ξ is large, the dynamic behavior of the string gets closer to an infinite string behavior.

2.1.2 The Strouhal number

The Strouhal number is a dimensionless parameter defined as follow:

$$\boxed{St = \frac{D * f_{vs}}{U}} \quad \textbf{Equation 2}$$

Where:

D: Cylinder diameter

f_{vs}: Vortex shedding frequency of a stationary body

U: Current flow velocity

It has been shown in [1] that for a Reynolds number Re varying from 100 to 100,000, the Strouhal number is roughly constant and equals 0.2.

Assuming that the vortex shedding frequency for the vibrating cylinder is the same as the stationary cylinder and that the cylinder response has the same frequency as the vortex shedding frequency, we can write the response frequency as: $f_R = \frac{U * St}{D}$.

The excited mode number n is defined as: $n = \frac{f_R}{f_1}$, f_1 being the fundamental frequency of the pipeline, $f_1 = \frac{C}{2 * L}$, C is the wave traveling velocity, $C = \sqrt{\frac{T}{\rho}}$, T being the tension of the string and ρ its linear mass.

It comes: $n = \frac{St * U}{D * f_1}$.

The wave length λ is: $\lambda = \frac{2 * L}{n} = \frac{D * C}{St * U}$.

Hence, $n = \left(\frac{2 * St * U}{C}\right) * \frac{L}{D}$, $\frac{2 * St * U}{C} = cst$.

The responsive mode is dominated by the ratio L/D.

Program such as Shear7 were developed to account responsive modes up to 10.

Let's substitute some typical values to get a sense of the ratio L/D required to get the 10^{th} mode.

Strouhal number: St=0.2

Current velocity: U=1 $m.s^{-1}$

Wave velocity: $C = \sqrt{\frac{3225}{1.258}} = 50.6 m.s^{-1}$

Hence,

$$\boxed{\frac{L}{D} = \frac{C}{2 * St * U} * n = 1265.}$$ **Equation 3**

Considering a riser with outside diameter of 25 cm, the associated length to get this ratio would be 316m.

The damping ratio can be evaluated by the following formula: $\xi = \frac{0.11}{n_{1/2}}$

$n_{1/2}$ is the number of wave length a wave travels before its amplitude decays by one-half. A typical damping ratio for a cylinder in water is 7 to 8%.

Considering the pipeline characteristics described above, $n_{1/2} = 2$. The wave amplitude would have decayed by half after traveling the equivalent distance of 2 wave length.

The wave length of the 10^{th} mode is: 63 m. In the worst case a wave traveling from the middle of the string has to travel 316m to meet the out going wave, that is to say, 5 wave lengths. This case shows a component of stationary wave behavior and the modal superposition technique is expected to give a good assessment of the physics of the string.

Considering higher values of riser's length, i.e.: 6000 ft, as it exits today, the wave would die before being reflected on the string's end. The string dynamic has now an infinite string behavior.

In addition, as the length of the string increases with respect to its diameter, the responsive mode number increases, making the n.ξ parameter larger which increases the decay and tend to make the string dynamic response closer to the infinite string behavior.

3 METHOD STATEMENT

A simple taut, described in section 3.1, is studied in the following sections. As seen in the previous section, the dynamic behavior of the string is dependent on the spatial attenuation parameter $n.\xi$. It seems clear that for small values of $n.\xi$, the modal superposition technique will give an accurate result. However, this statement seems to be less accurate for higher values of $n.\xi$. In the following section, the result given by the modal superposition is compared to the result given by the Green's function technique for values of $n.\xi$ ranging from 0.01 to 7. Then, the difference between the two techniques is measured to draw conclusion on the validity of the use of the modal superposition technique.

3.1 DEFINITION OF THE STUDIED MODEL

A simple taut string system is considered for the analyses described in the following sections. These analyses focus on the transverse vibrations. However, the same conclusions would apply to other one dimensional systems. The string considered has no bending rigidity and has a constant tension along its length.

The system under study is shown in Figure 3:

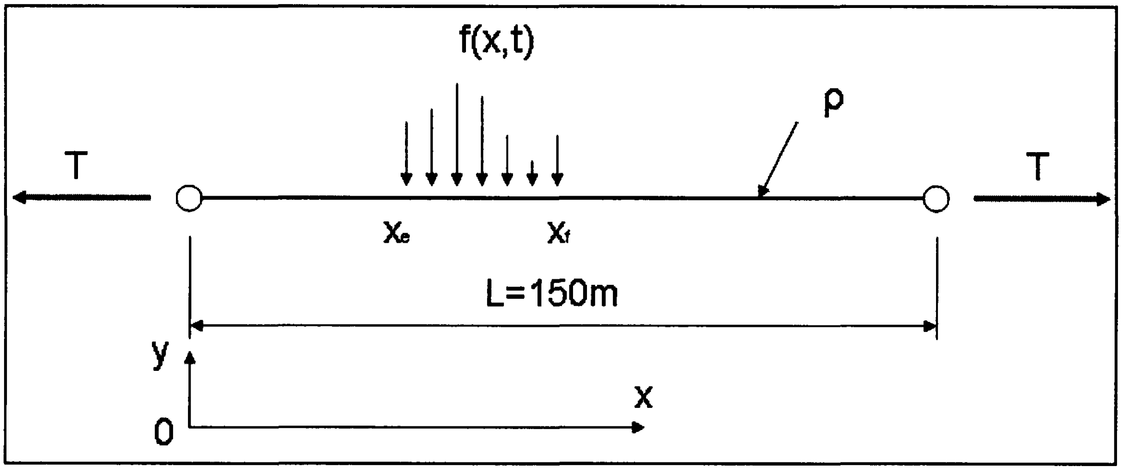

Figure 3 : String subject to distributed loading

The string considered has the following characteristics:

- Length: L = 150 m
- Linear mass: ρ = 1.2585 kg/m
- Tension: T = 3225 N
- Diameter: Φ = 0.03622 m

<u>Note:</u> the string diameter is only given to express the transverse displacement of the string in terms of number of diameters.

3.2 LOADING

The loading applied on the string simulates the forces that vortices would generate on a riser. A distributed loading acting over a finite length of the string is approximated by point loads applied every 0.5m on the string. The total force applied is not centered on the string.

It has arbitrarily been chosen that the excitation frequency would be around the frequency of the 30th mode of the string. Two frequencies were considered, the first one being exactly the frequency of the 30th mode, f = 5.062Hz and the second one being between the 30th and the 31st mode, f = 5.147Hz. It has also been decided to apply the loading on a three wave length distance. The envelope of the point loads magnitude has the shape $f(x) = \sin(k_{30} \cdot x)$ of the 30th mode, where k_{30} is the wave number of the excitation.

- Natural frequencies of the string are given by the following formula:

$$\boxed{\omega_n = \frac{n \cdot \pi}{L} \cdot \sqrt{\frac{T}{\rho}}}$$ **Equation 4**

- The wave length is given by:

$$\boxed{\lambda_n = \frac{2 \cdot L}{n}}$$ **Equation 5**

- The mode number is given by:

$$\boxed{k_n = \frac{\omega_n}{c}}$$ **Equation 6**

Mode	ω_n (rad.s^{-1})	f_n (Hz)	Λ_n (m)	k_n
30	31.81	5.062	10.00	0.63

Table 1 : Mode 30 characteristics

The table below summarizes the various value of the spatial attenuation n.ξ for various values of ξ.

ξ	n. ξ
0.00033	0.01
0.0033	0.1
0.033	1.0
0.1	3.0
0.2333	7.0

Table 2 : n.ξ for the 30th mode

The graph below shows the forces magnitude for an excitation frequency corresponding to 30th mode, f = 5.062Hz.

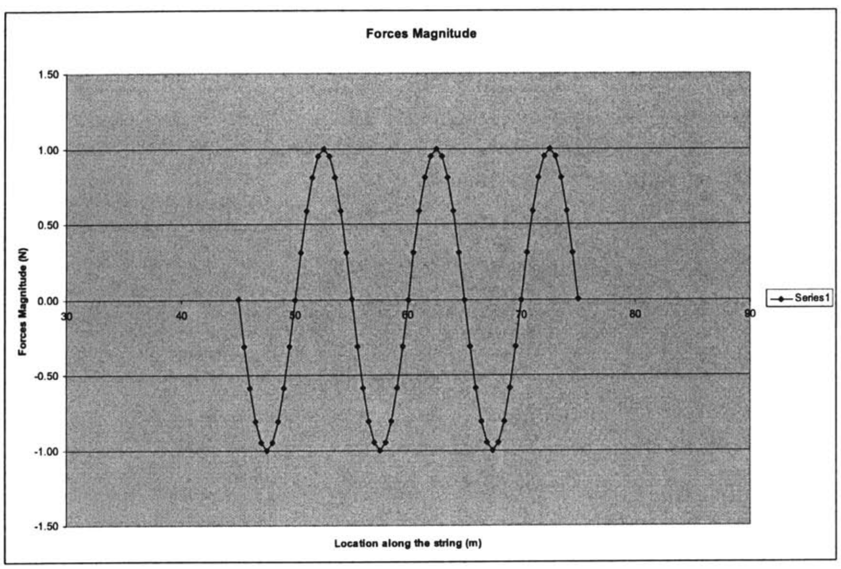

Figure 4 : Forces magnitudes, excitation frequency f = 5.062Hz

The loading corresponding to f = 5.147Hz, between the mode 30 and 31 has a different envelop profile since its wave length is slightly shorter.

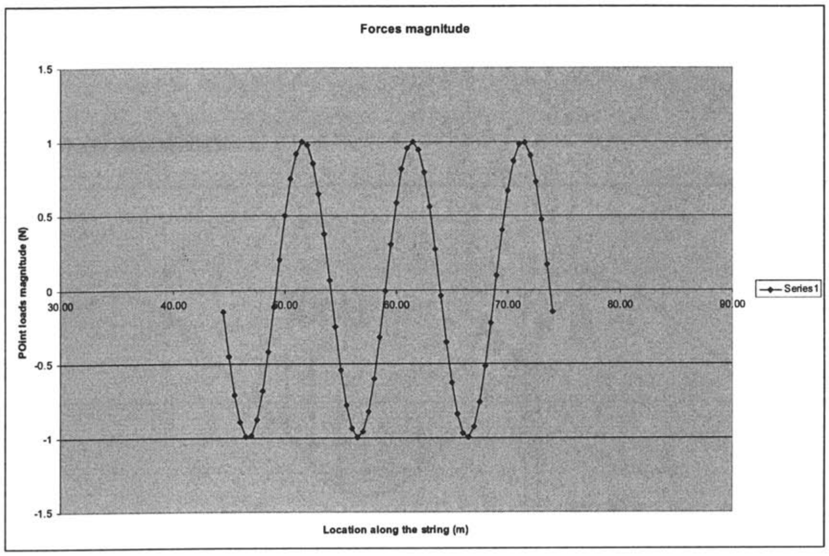

Figure 5 : Forces magnitudes, excitation frequency f = 5.147Hz

3.3 THEORY

3.3.1 Equation of motion

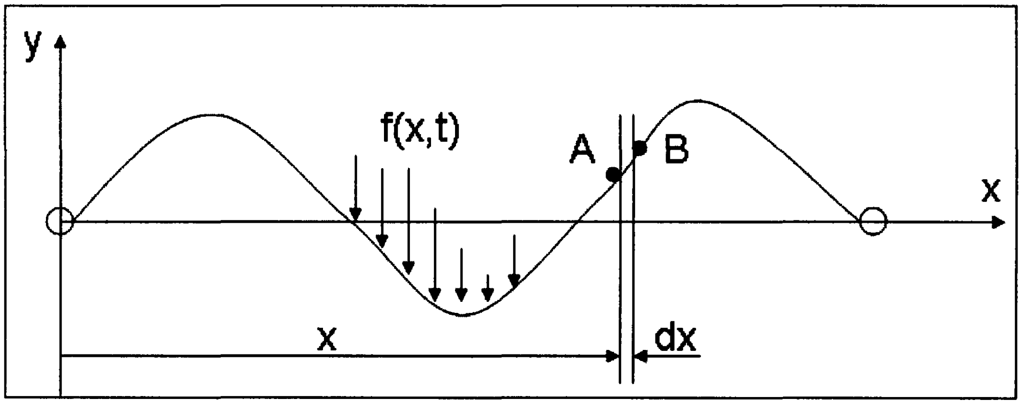

Figure 6 : Vibrating string

f(x,t) is the excitation force applied per unit length.

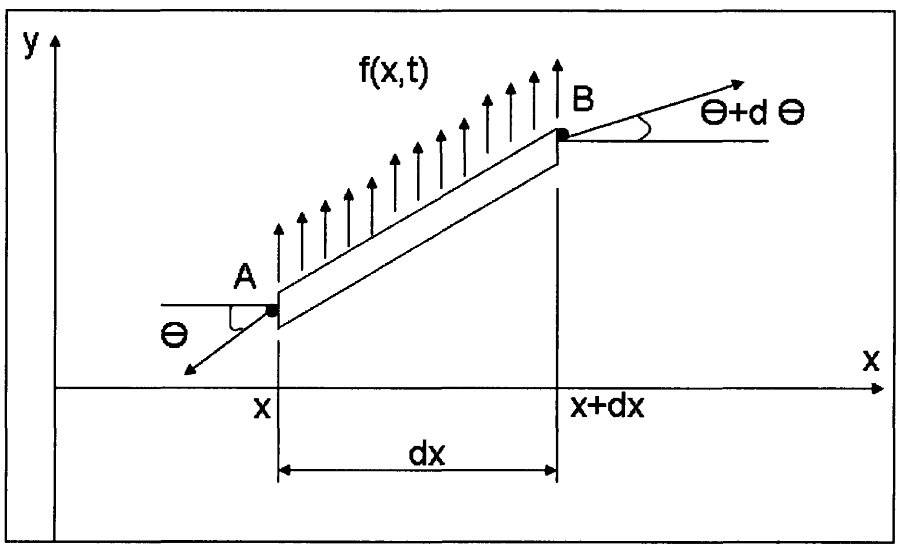

Figure 7 : Vibrating string, elementary length dx of string

The transverse displacement of the string y(x,t) are assumed to be small. The governing equation for the string is given by:

$$\boxed{\rho \cdot \frac{\partial^2 y}{\partial t^2} + c \cdot \frac{\partial y}{\partial y} - T \cdot \frac{\partial^2 y}{\partial x^2} = f(x,t)} \qquad \textbf{Equation 7}$$

With:

c: structural and environmental damping

3.3.2 The Green's function theory

The Green's function is an exact solution to the differential equation describing the motion of the string when the excitation is a point load. The idea of the Green's function technique is to use the principle of superposition to obtain the response due to a continuous load based on the solution due to a concentrated load. Since the loading accounted for in the model previously described is constituted of point loads, the total Green's function solution is the superposition of the solutions found for each point load.

3.3.2.1 Displacement

The Green's function solution to the system described in **Figure 6** is:

$$\boxed{\bar{y}(x,\omega_j) = \int_{x_e}^{x_f} g(x,s;\omega_j)\cdot f(s)ds} \qquad \textbf{Equation 8}$$

For a point load,

$$\boxed{\bar{y}(x,\omega_j) = g(x,x_p;\omega_j)\cdot P(x_p)} \qquad \textbf{Equation 9}$$

With x_p location of the p^{th} point load.

Where the Green's function g is given by:

$$g(x,s;\omega_j) = \frac{\sin\lfloor k_j \cdot (L-s)\rfloor \cdot \sin(k_j \cdot x)}{k_j \cdot T \cdot \sin(k_j \cdot L)}, \qquad x<s \qquad \textbf{Equation 10}$$

$$g(x,s;\omega_j) = \frac{\sin\lfloor k_j \cdot (L-x)\rfloor \cdot \sin(k_j \cdot s)}{k_j \cdot T \cdot \sin(k_j \cdot L)}, \qquad x>s \qquad \textbf{Equation 11}$$

Where k_j is the complex wave number for the mode j:

$$k_j = \sqrt{\frac{\rho\cdot\omega_j^{\ 2} + i\cdot c\cdot\omega_j}{T}} \qquad \textbf{Equation 12}$$

x represents the response location and s indicates the excitation location.

Finally, the total Green's function response is the superposition of the response obtained for each point load.

$$\bar{y}(x,\omega_j) = \sum_p \bar{y}_p(x,\omega_j)$$ **Equation 13**

The following magnitude of displacements is shown on graphs in section 4:

$$y(x;\omega_j) = \left|\bar{y}(x;\omega_j)\right|$$ **Equation 14**

3.3.2.2 Strain

The curvature is calculated as following:

$$y_{xxm}(x) = -K^2 \cdot y_m(x)$$ **Equation 15**

And the strain shown on graphs in section 4 is given by:

$$\boxed{RMSstrain = \frac{1 \cdot 10^6 * \left|y_{xxm}(x)\right| \cdot \frac{D}{2}}{\sqrt{2}}}$$ **Equation 16**

3.3.3 The modal superposition theory

3.3.3.1 Displacement

Displacement can be written as:

$$\boxed{y(x,t)=\sum_n Y_n(x)\cdot q_n(t)} \qquad \textbf{Equation 17}$$

With:

$Y_n(x)$: n^{th} mode shape of the system, $Y_n(x)=\sin\left(\frac{n\cdot\pi\cdot x}{L}\right)$

$q_n(t)$: n^{th} modal displacement

Substituting the formulation from Equation 17 in Equation 7 leads to modal equations of motion:

$$\boxed{M_n\cdot\ddot{q}_n(t)+R_n\cdot\dot{q}_n(t)+K_n\cdot q_n(t)=F_n(t)} \qquad \textbf{Equation 18}$$

With:

Mn: Modal mass given by $M_n=\int_0^L Y_n^2(x)\cdot\rho\cdot dx=\frac{\rho\cdot L}{2}$

C_n: Modal damping given by $C_n=\int_0^L Y_n^2(x)\cdot c\cdot dx$

K_n: Modal stiffness given by $K_n=-\int_0^L T\cdot\frac{d^2Y_n(x)}{dx^2}\cdot Y_n(x)\cdot dx=T\cdot\left(\frac{n\cdot\pi}{L}\right)^2\cdot\frac{L}{2}$

F_n: Modal force given by

$F_n(t)=\int_0^L Y_n(x)\cdot f(x,t)\cdot dx=\int_{x_e}^{x_f} Y_n(x)\cdot f(x,t)\cdot dx=\sum_m P_m\cdot\sin\left(\frac{n\cdot\pi\cdot\psi_m}{L}\right)$ with ψ_m being the location of the point load P_m.

Then, it comes that the magnitude of displacements is found as following:

$$y(x) = \sum_n \frac{\frac{F_n}{K_n}}{\sqrt{(1-r^2)+2\cdot i\cdot\xi\cdot r}} \cdot \sin\left(\frac{n\cdot\pi\cdot x}{L}\right)$$ **Equation 19**

The absolute value for each point along the string is shown in section 4 graphs.

3.3.3.2 Strain

The strain is derived from Equation 17, the curvature is given by:

$$y_{xx}(x) = \sum_n -\left(\frac{n\cdot\pi}{L}\right)^2 \cdot \frac{\frac{F_n}{K_n}}{\sqrt{(1-r^2)+2\cdot i\cdot\xi\cdot r}} \cdot \sin\left(\frac{n\cdot\pi\cdot x}{L}\right)$$ **Equation 20**

And the strain calculated as following is shown on the graphs in section 4.

$$RMSstrain = \frac{1\cdot 10^6 \cdot |y(x)| \cdot \frac{D}{2}}{\sqrt{2}}$$ **Equation 21**

4 RESULTS

In this section, results given by a string excited by the distributed load case described in section 3.2 for a range of n.ξ- are presented and compared. The Green's function is assumed to give the exact dynamic behavior of the string. Results given by the modal superposition are then compared to results given by the Green's function. The error between the two techniques is then quantified. The error found would tell us whether or not the modal superposition technique is suitable for the simulation of the dynamic behavior of the string in case of spatial attenuation.

In the case the modal superposition gives us a good result, the minimum number of modes to describe properly the dynamic response of the string will be given.

4.1 SIMULATION

4.1.1 Excitation on the 30th mode f = 5.062 Hz

In this section, calculations are performed for an excitation frequency of f = 5.062Hz corresponding to the 30th mode of the string. Various values of the spatial attenuation n.ξ listed in Table 2 are tested. Then, for a given n.ξ, the accuracy of the response is quantified for various numbers of superposed modes. Results obtained are presented in the following sections. First the Green's function response is shown and is taken as a reference, then graphs corresponding to variables values of n (number of modes) are shown.

The mode number that has the frequency closest to the excitation frequency corresponds to the minimum number of modes to be accounted for.

- **n. ξ=0.01**

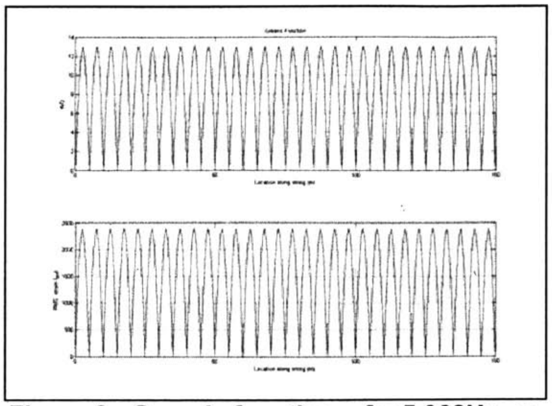

Figure 8 : Green's function – f = 5.062Hz - n. ξ = 0.01

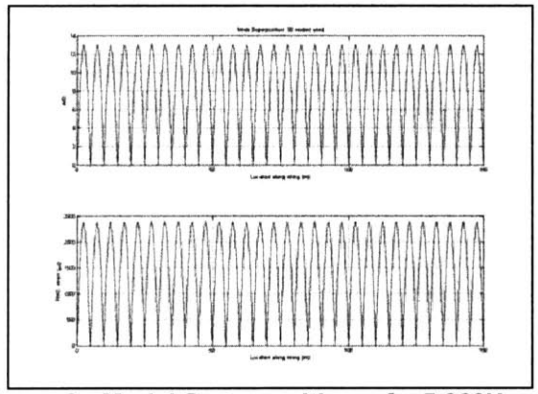

Figure 9 : Modal Superposition – f = 5.062Hz - n. ξ = 0.01 – mode 30 only

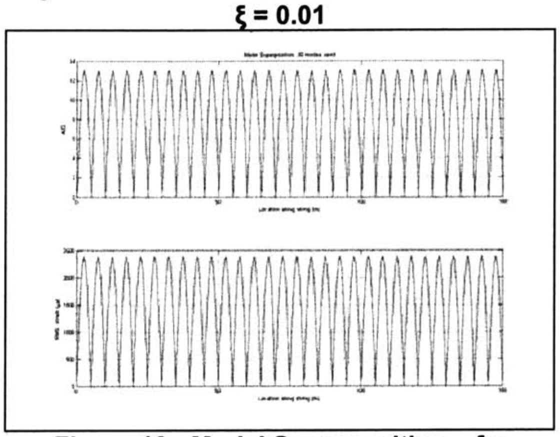

Figure 10 : Modal Superposition – f = 5.062Hz - n. ξ = 0.01 – modes 1 to 30

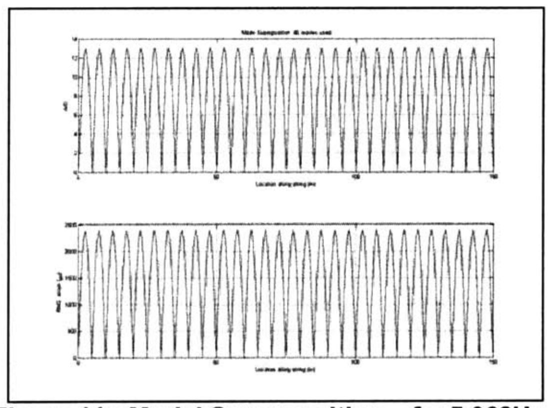

Figure 11 : Modal Superposition – f = 5.062Hz - n. ξ = 0.01 – modes 1 to 40

The string has a standing wave behavior. Accounting for the 30th mode only gives an accurate result; the participation of the other modes is probably negligible.

- **n. ξ=0.1**

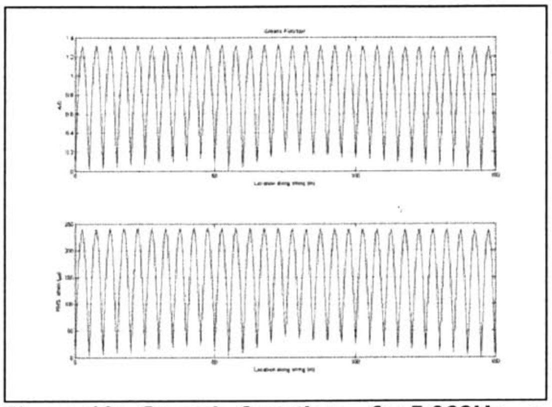

Figure 12 : Green's function – f = 5.062Hz - n. ξ = 0.1

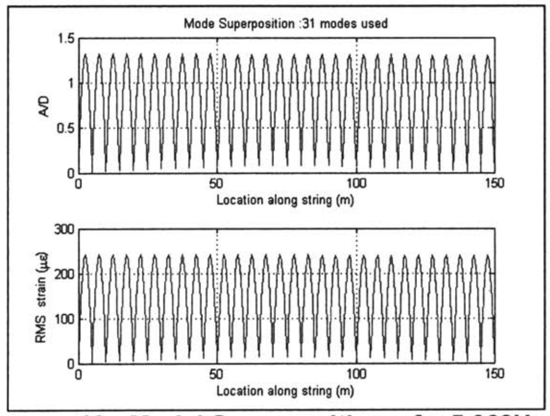

Figure 13 : Modal Superposition – f = 5.062Hz - n. ξ = 0.1 – mode 30 only

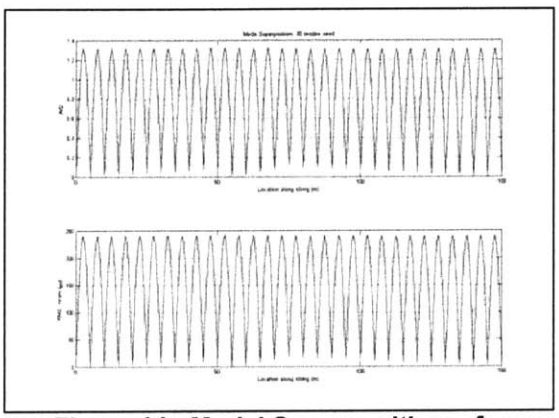

Figure 14 : Modal Superposition – f = 5.062Hz0 - n. ξ = 0.1 – modes 1 to 30

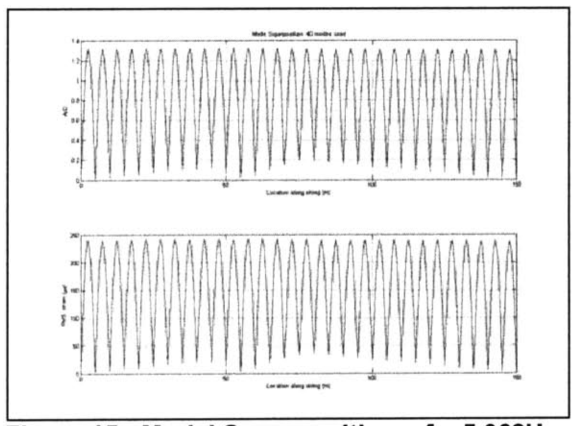

Figure 15 : Modal Superposition – f = 5.062Hz - n. ξ = 0.1 – modes 1 to 40

In this case, accounting for the mode 30 only gives a response close to the Green's function. It can be seen that by increasing the number of modes, first by considering the modes from 1 to 29 and then by increasing the number of superposed modes to 40, the response is evolving closer to the Green's function response.

- **n. ξ=1.0**

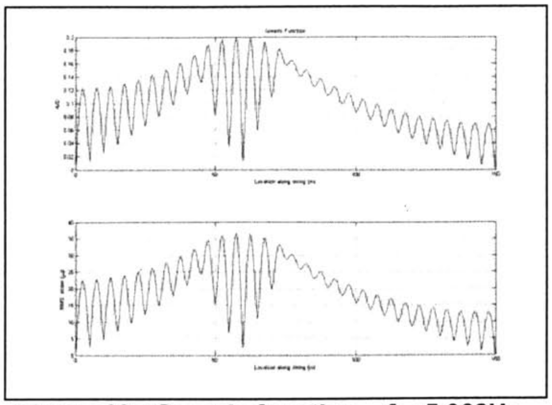

Figure 16 : Green's function – f = 5.062Hz - n. ξ = 1

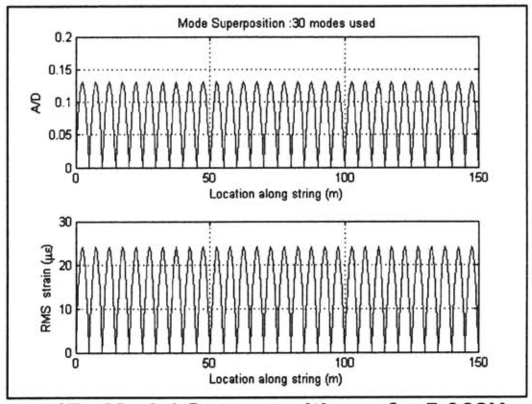

Figure 17 : Modal Superposition – f = 5.062Hz n. ξ = 1 – mode 30 only

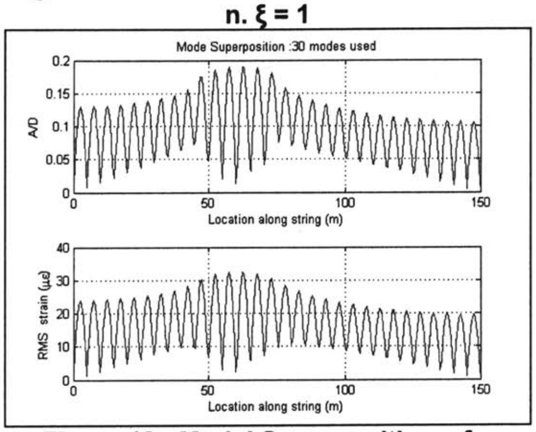

Figure 18 : Modal Superposition – f = 5.062Hz - n. ξ = 1 – modes 1 to 30

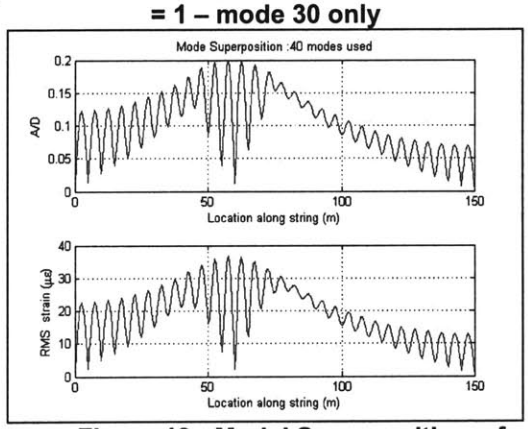

Figure 19 : Modal Superposition – f = 5.062Hz – modes 1 to 40

It is to be noticed that for the same amount of modes considered, the accuracy is not as good as for lower n.ξ values. The result given by mode 30 only is far from the Green's function result.

- **n. ξ=3.0**

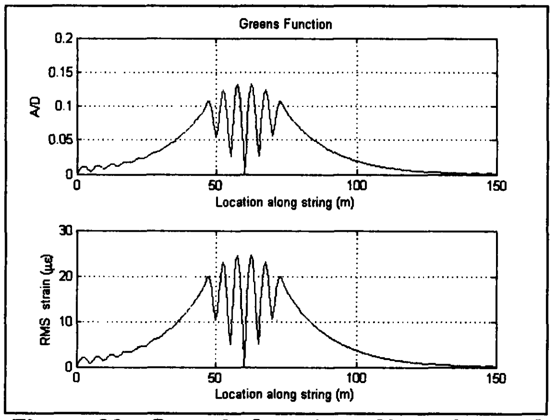

Figure 20 : Green's function - Mode 30 - n. ξ = 3

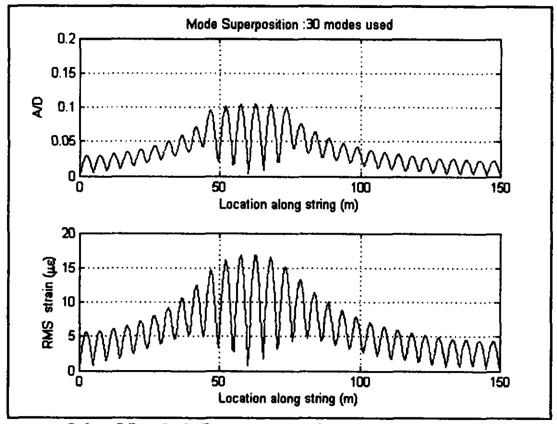

Figure 21 : Modal Superposition - Mode 30 - n. ξ = 3 – 30 modes

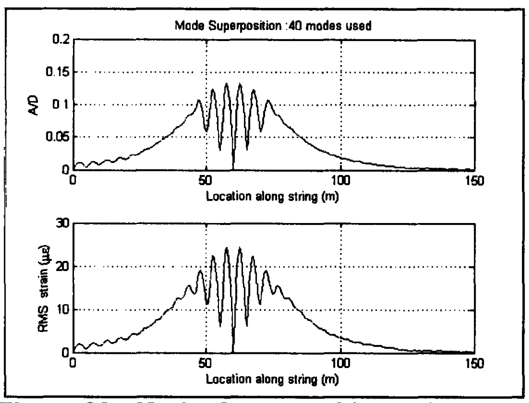

Figure 22 : Modal Superposition - Mode 30 - n. ξ = 3 – 40 modes

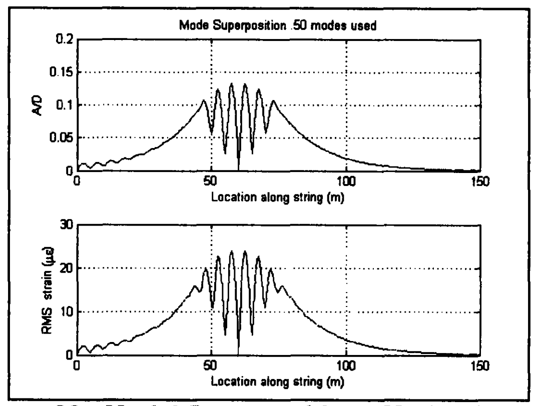

Figure 23 : Modal Superposition - Mode 30 - n. ξ = 3 – 50 modes

It is seen that the response of the string still has a standing wave behavior on the left side of the string. On the contrary, the profile of the response on the right side of the string shows an infinite string behavior. Mode 30 only gives a poor result, only the result given by the superposition of 30 modes gives a result close to the Green's function result. It is also to be noted that it seems that it takes a higher number of modes for the strain to converge.

4.1.2 Excitation between mode 30 and 31 – f = 5.147 Hz

In this section, calculations are performed for an excitation frequency of f = 5.147Hz and for the various values of spatial attenuation n. ξ listed in the Table 2. The excitation frequency f = 5.147Hz is 30.5 times the fundamental frequency of the string. Then, for a given n.ξ, the accuracy of the response is quantified for various numbers of superposed modes. Results obtained are presented in the following sections. First the Green's function response is shown and is taken as a reference, then graphs corresponding to variables values of n (number of modes) are shown.

- **n.ksi=0.01**

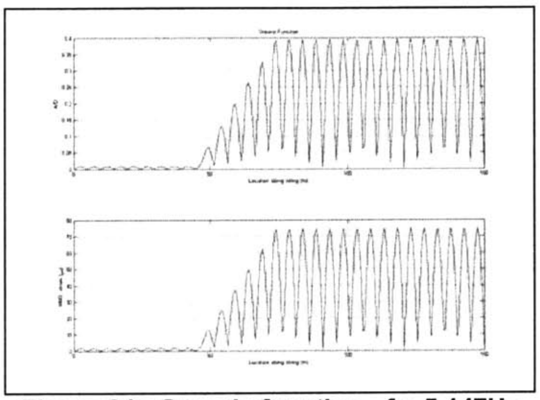

Figure 24 : Green's function - f = 5.147Hz

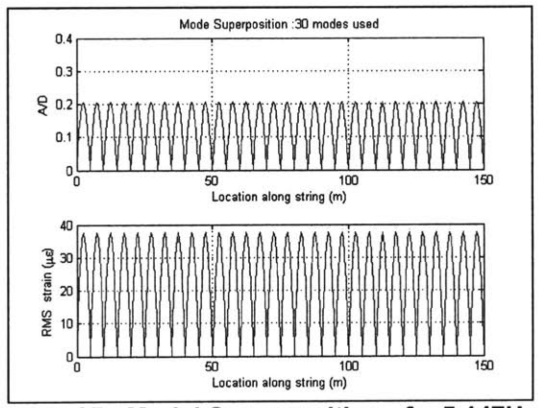

Figure 25 : Modal Superposition - f = 5.147Hz - Mode 30 only

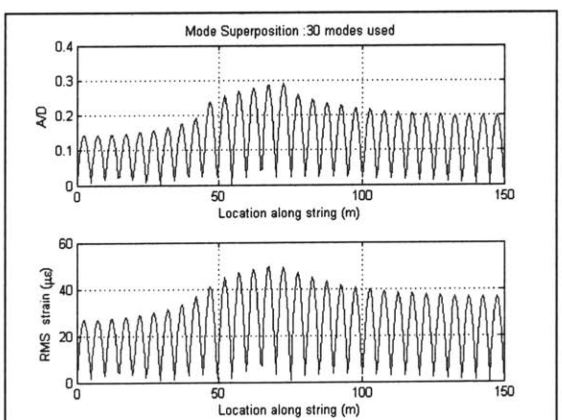

Figure 26 : Modal Superposition - f = 5.147Hz - 30 modes

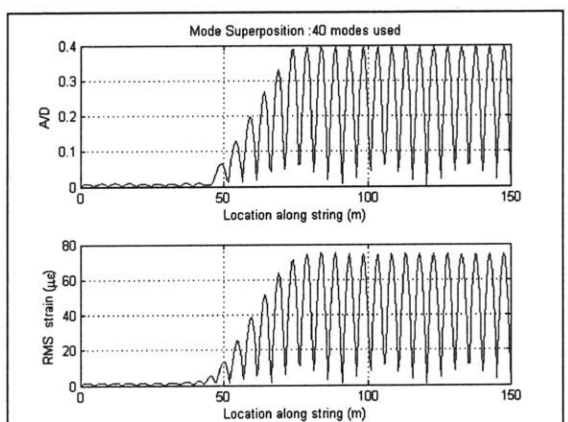

Figure 27 : Modal Superposition - f=5.147Hz - 40 modes

Mode 30 only does not give a proper result. Only the superposition of 40 modes gives a result close to the Green's function solution. It can be stated that more modes are required to obtain a good approximation of the Green's function solution when the excitation frequency does not correspond to a system natural frequency. More modes are needed to simulate a non-resonant mode response.

- **n.ksi=0.1**

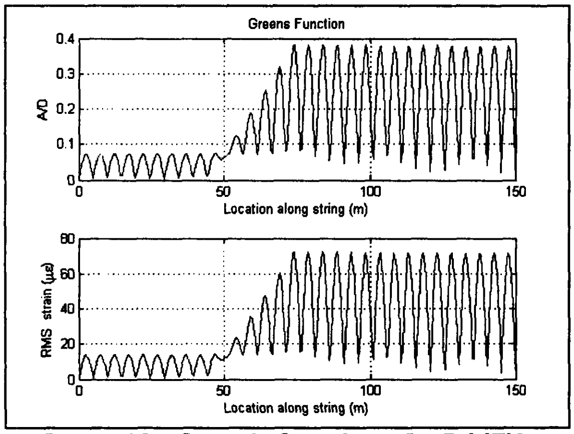

Figure 28 : Green's function - f = 5.147Hz

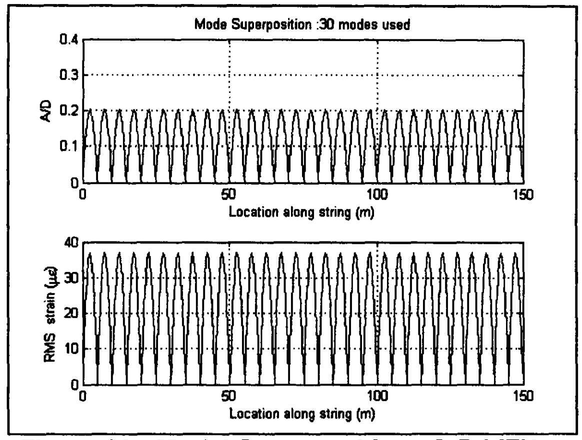

Figure 29 : Modal Superposition - f=5.147Hz - Mode 30 only

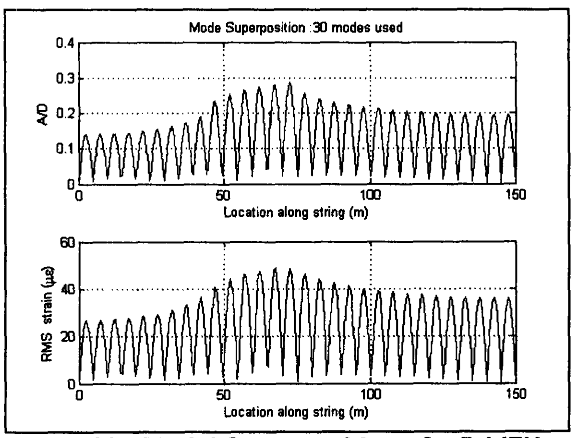

Figure 30 : Modal Superposition - f = 5.147Hz - 30modes

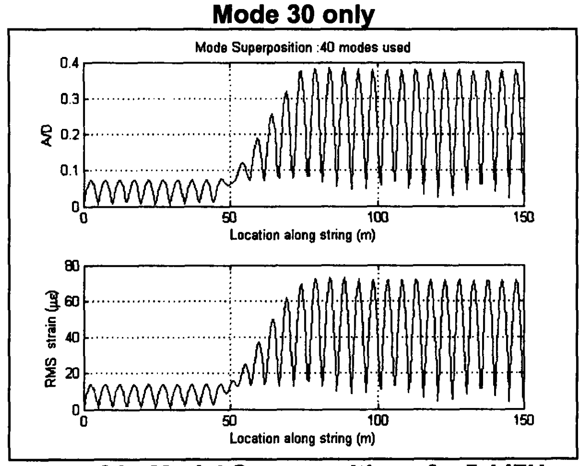

Figure 31 : Modal Superposition - f = 5.147Hz - 40 modes

The same statement as for the previous case can be made here. In addition, it is seen that for the same number of superposed modes, the accuracy of the modal superposition decreases when the spatial attenuation increases.

- **$n.\xi = 1$**

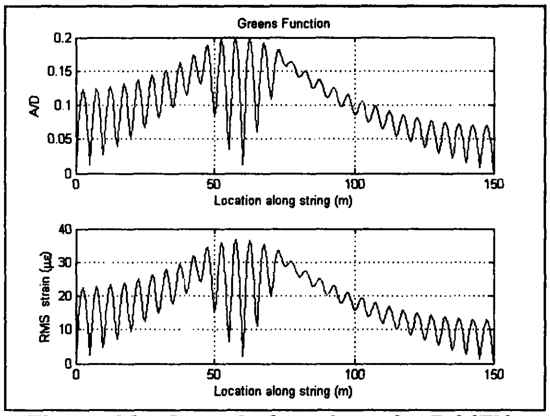

Figure 32 : Green's function - f = 5.147Hz

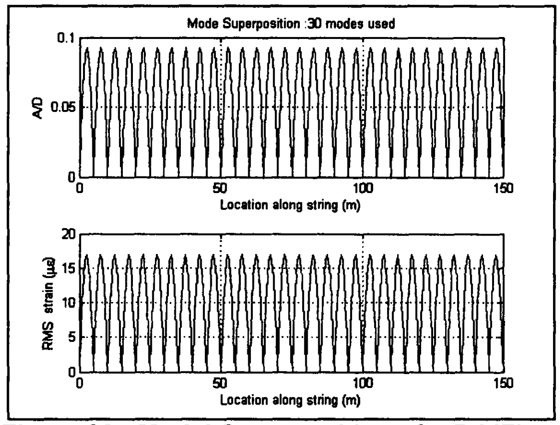

Figure 33 : Modal Superposition - f = 5.147Hz - Mode 30 only

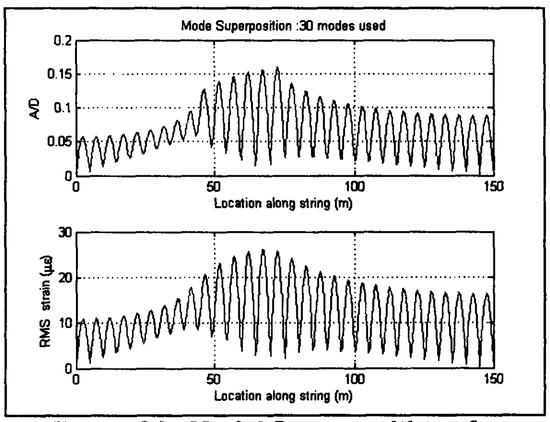

Figure 34 : Modal Superposition - f = 5.147Hz - 40 modes

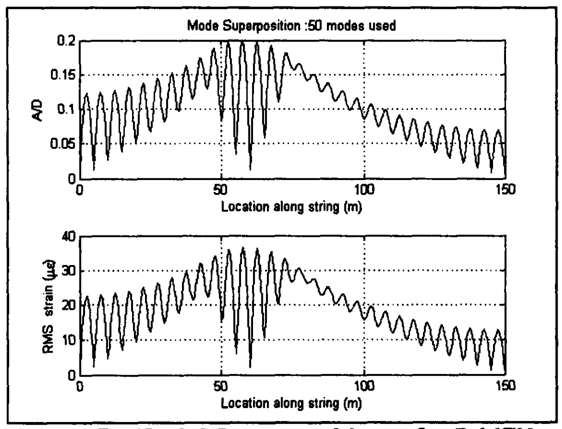

Figure 35 : Modal Superposition – f = 5.147Hz – 50 modes

The same tendency noted above is confirmed in this case.

- **$n.\xi = 3$**

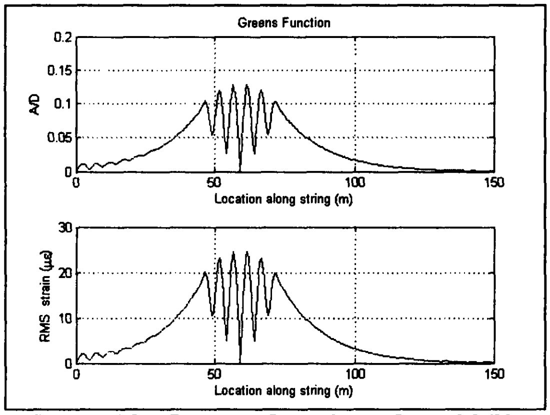

Figure 36 : Green's function - f = 5.147Hz

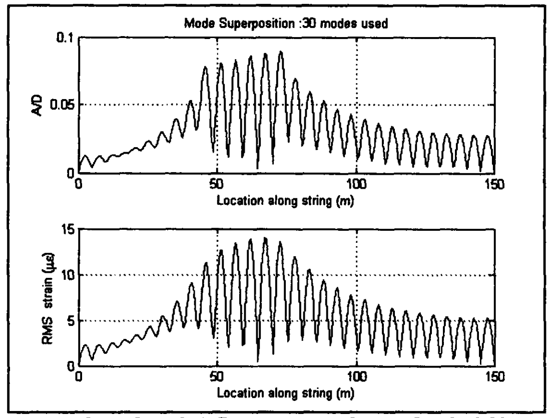

Figure 37 : Modal Superposition - f = 5.147Hz - 30 Modes

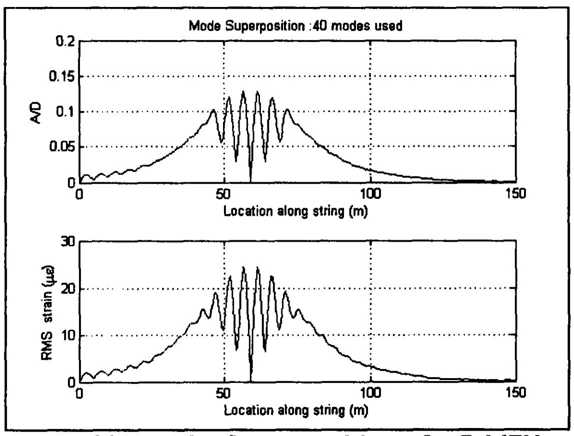

Figure 38 : Modal Superposition - f = 5.147Hz - 40 Modes

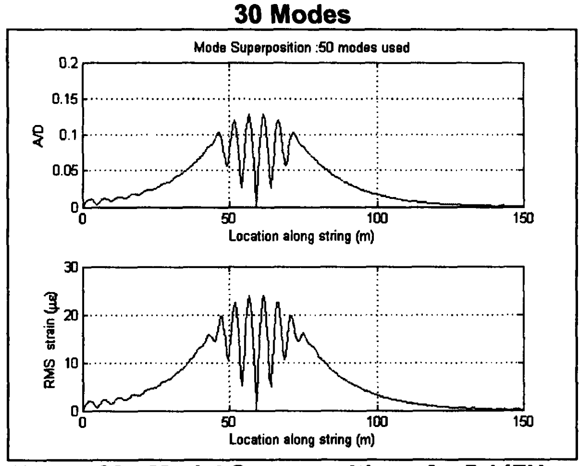

Figure 39 : Modal Superposition - f = 5.147Hz - 50 Modes

Generally, it is seen that the modal superposition solution tends to overestimate the response magnitude where the response is dominated by a traveling wave behavior and underestimate the magnitude where the response is dominated by standing wave behavior. Moreover, it appears that the strain result requires a higher number of superposed modes to give a good approximation.

4.2 ERROR QUANTIFICATION

In this section, the difference between the result given by the modal superposition and the Green's function is measured and called error since the Green's function result is assumed to be the true response. The error is quantified using the following formulae:

$$\boxed{e(\%) = \sqrt{\frac{\sum_{i=1}^{301} d^2{}_i}{\sum_{i=1}^{301} y_i^{Green^2}}} * 100} \qquad \textbf{Equation 22}$$

With:

$d_i = y_i^{Green} - y_i^{MS}$: Difference between the amplitude calculated by the Green's function and the modal superposition techniques.

y_i^{Green} : Green's function amplitude at the i^{th} point along the string

y_i^{MS} : Modal superposition amplitude at the i^{th} point along the string

Note: The amplitude is calculated every 0.5m on along the string which corresponds to 301 points.

The closer to the Green's function the modal superposition is, the better the modal superposition simulation is assumed to be. An error of less than 5% would be considered as acceptable.

The range of the spatial attenuation n.ξ has been increased up to 7 to compare results given by two cases dominated by the traveling wave behavior.

4.2.1 Excitation on mode 30 – f = 5.062Hz

Error on displacement

n.ξ	Number of modes					
	30 only	30	40	50	60	100
0.01	0.4	0.2	0.0	(1)	(1)	(1)
0.1	3.6	1.8	0.1	(1)	(1)	(1)
1	42.7	21.9	1.0	0.6	(1)	(1)
3	73.4	33.8	2.1	1.3	1.2	1.2
7	83.8	41.5	5.45	4.7	4.7	4.7

Table 3 : Error on displacement – f = 5.602Hz

Error on strain

n.ξ	Number of modes					
	30 only	30	40	50	60	100
0.01	0.4	0.2	0.0	(1)	(1)	(1)
0.1	3.6	2.0	0.2	(1)	(1)	(1)
1	42.7	25.3	4.2	3.9	(1)	(1)
3	75.5	41.4	8.7	7.9	7.6	7.2
7	84.2	51.4	16.2	14.4	13.9	13.3

Table 4 : Error on strain – f = 5.602Hz

4.2.2 Excitation between mode 30 & 31 – f = 5.147Hz

Error on displacement

n.ξ	Number of modes					
	30 only	30	40	50	60	100
0.01	68.8	64.0	0.7	0.2	(1)	(1)
0.1	68.9	56.5	0.6	0.2	(1)	(1)
1	54.5	40.2	1.1	0.7	(1)	(1)
3	77.9	44.1	2.1	1.3	1.2	1.2
7	87.1	49.3	53.5	4.7	4.6	4.6

Table 5 : Error on displacement – f=5.147Hz

Error on strain

n.ξ	Number of modes					
	30 only	30	40	50	60	100
0.01	68.9	64.5	5.5	5.5	(1)	(1)
0.1	61.6	57.3	4.3	4.2	(1)	(1)
1	55.6	44.0	4.3	4.0	(1)	(1)
3	78.5	52.3	8.6	7.8	7.6	7.2
7	87.8	59.5	16.1	14.3	13.8	13.2

Table 6 : Error on strain – f=5.147Hz

Note: (1) Since the analysis performed with a lower number of modes gives satisfactory results, the analysis was not conducted.

It appears that for low values of n.ξ the modal superposition technique gives very accurate answers. When the system is exactly excited at mode 30, i.e. when the excitation frequency is f = 5.062Hz, the mode 30 only is sufficient to obtain less than 0.4% error. As the value of n.ξ increases, the accuracy decreases for the same number of modes considered. The accuracy of results is still good when n.ξ remains less than 1.0. Over 1.0, the number of modes to be accounted for is higher. It is also confirmed that the strain takes a higher number of modes to converge.

However, the accuracy of results reaches a value that can not be improved even by increasing the considered number of modes. Moreover, it appears that as the n.ξ value increases the accuracy of the response decreases.

4.3 MODAL AMPLITUDE

According to the previous section, the accuracy of the result obtained is highly dependent on the spatial attenuation **n.ξ** and on the excitation frequency, whether the latter corresponds exactly to a natural frequency of the system or is in between two natural frequencies.

In this section, the relative influence of the various modes is investigated. For each of the above cases, the modal amplitude is plotted.

The modal amplitude formulation is as following:

$$\boxed{a_n = \frac{\frac{P_n}{K_n}}{(1-r^2)+2\cdot i\cdot\xi\cdot r}} \qquad \textbf{Equation 23}$$

With:

P_n: Modal Force – $P_n = \sum_m P_m \cdot \sin\left(\frac{n\cdot\pi\cdot\psi_m}{L}\right)$, P_m being harmonic excitation point loads and ψ_m their location along the string

K_n: Modal Stiffness – $K_n = T\cdot\left(\frac{n\cdot\pi}{L}\right)^2\cdot\frac{L}{2}$

r: Tuning ratio - $r = \frac{\omega}{\omega_n}$

ξ: Damping ratio

To compare the relative influences of the modal amplitudes, we focus on their absolute values. The modal amplitude is divided by the string diameter to get a dimensionless parameter.

4.3.1 Modal amplitudes – f = 5.062Hz (30^{th} mode)

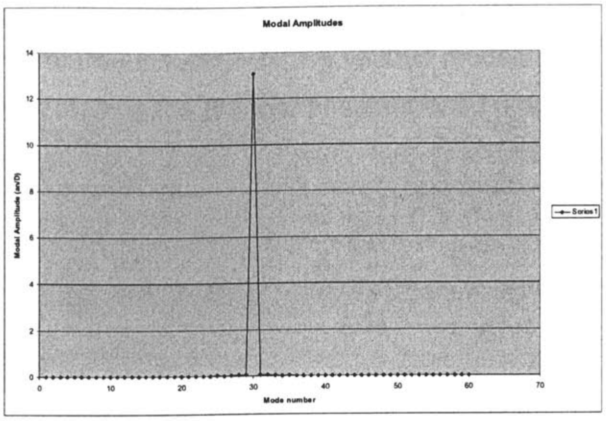

Figure 40 : Modal Amplitude - n.ksi = 0.01

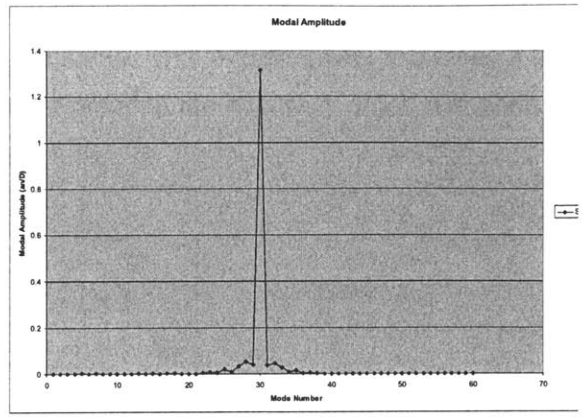

Figure 41 : Modal Amplitude - n.ksi = 0.1

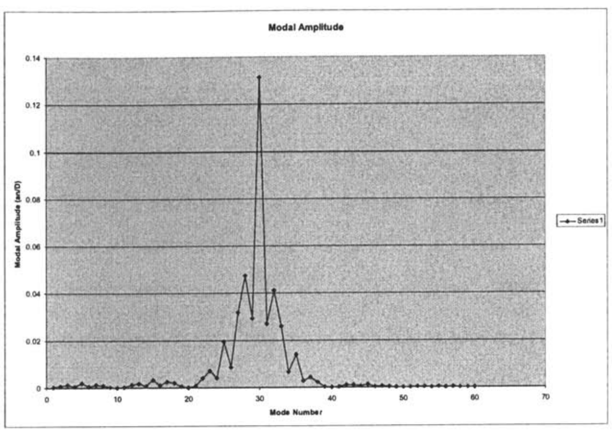

Figure 42 : Modal Amplitude - n.ksi = 1

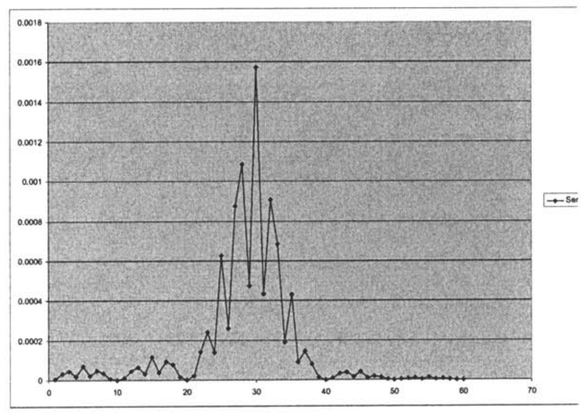

Figure 43 : Modal Amplitude - n.ksi = 3

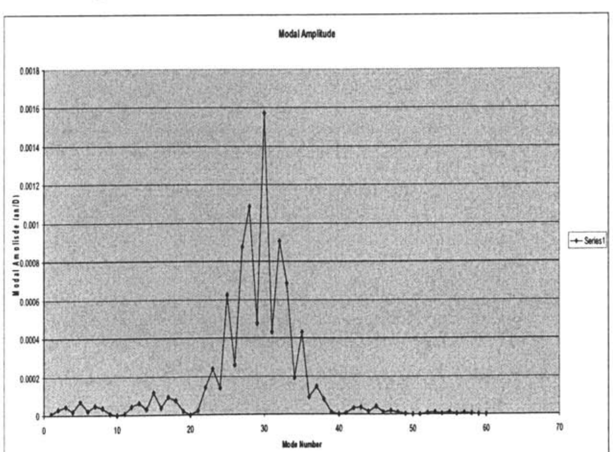

Figure 44 : Modal Amplitude - n.ksi=7

Statements made in sections 4.1 and 4.2 are confirmed by the above graphs. It can be seen that only one mode would give a very good approximation when the response is dominated by standing waves, when the excitation corresponds to a resonant mode, i.e. in the case $n.\xi = 0.01$ and $f = 5.062Hz$, Figure 40. As the spatial attenuation increases the influence of the modes around the excitation mode increases too. It can be noted that for $n.\xi$ equals 3 and 7, the relative influence of the various modes remains the same. This explains why in Tables 3 to 6, increasing the number of superposed modes does not improve the accuracy of the response.

4.3.2 Modal amplitudes – f = 5.147Hz (between the 30th and the 31st modes)

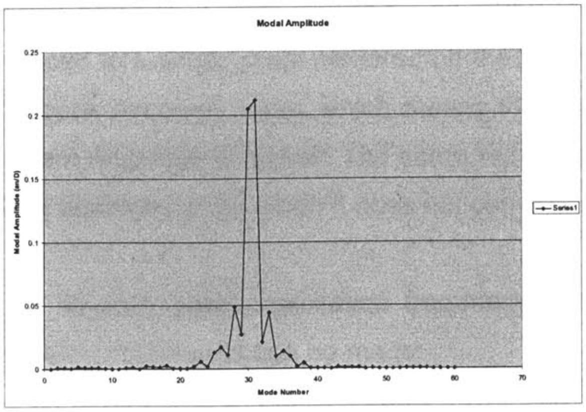

Figure 45 : Modal Amplitude - n.ksi = 0.01

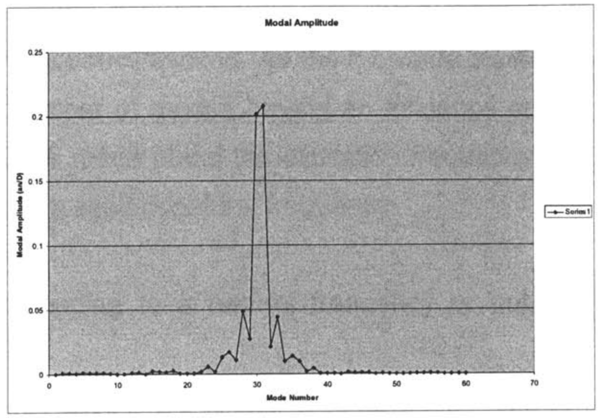

Figure 46 : Modal Amplitude - n.ksi = 0.1

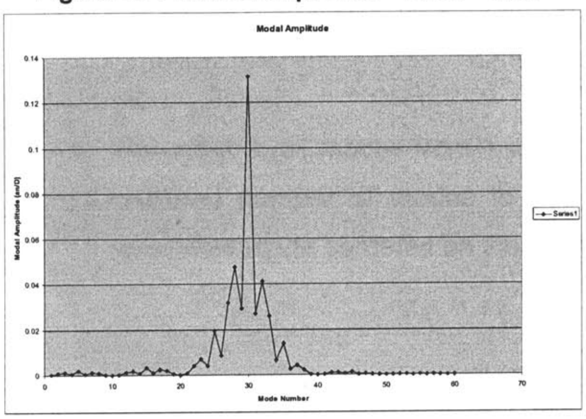

Figure 47 : Modal Amplitude - n.ksi = 1

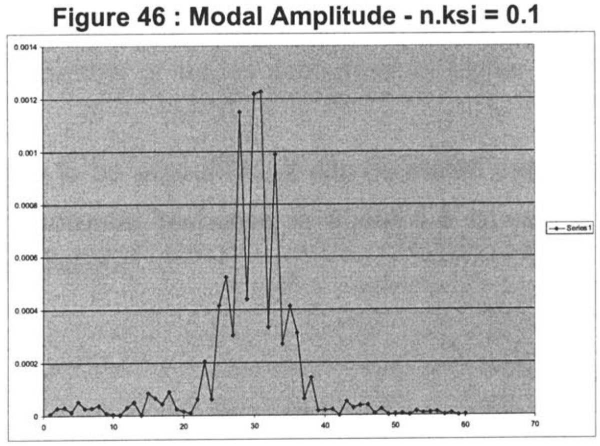

Figure 48 : Modal Amplitude - n.ksi = 3

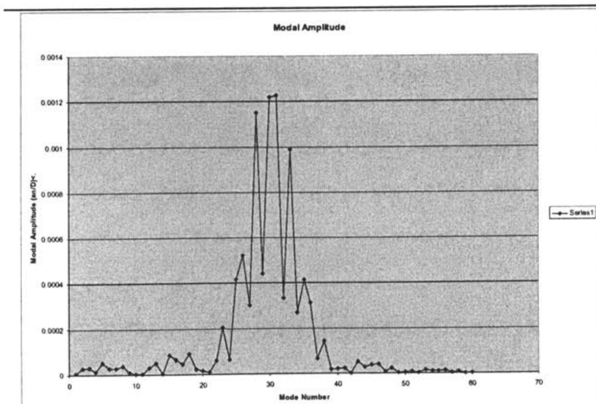

Figure 49 : Modal Amplitude - n.ksi = 7

The same remarks as the one made in section 4.3.1 apply here. It can be added that the number of modes to be considered is higher for the same value of n.ξ. This correlates the fact that the error is higher for when the excitation does not correspond to a resonant mode.

The above graphs correlate what was stated in the previous sections. When the n.ξ value is low and the excitation exactly on a system natural frequency, the amplitude influence is reduced to a single peak centered on the natural frequency excited. As the n.ξ value increases the peak becomes wider which means that the number of modes having an influence on the system response is higher. The same remark can be made about the excitation frequency; the peak becomes wider when it does not correspond to a system natural frequency.

In both cases, excitation frequency corresponding to a natural frequency or not, the following statements can be made:

- For n.ξ < 1, the number of modes to be accounted is 20, 10 below the excitation frequency, 10 above. This is confirmed by the fact that the result's accuracy for that range of n.ξ values does not change for number of modes accounted for higher than 40.
- For n.ξ > 3, all modes from 1 to 60 have to be accounted. It can be stated that the required number of modes to get an accurate response is about 1.8 times the excitation mode centered on the excitation mode.

4.3.3 Modal amplitudes around the 40th mode

In this section, the modal amplitude has been calculated for an excitation of higher frequency.

Figure 50 shows the modal amplitudes for an excitation frequency corresponding to a natural frequency. Figure 51 shows the modal amplitudes for an excitation between two natural frequencies.

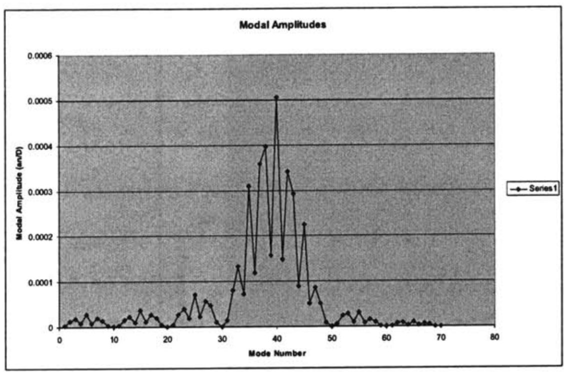

Figure 50 : Modal Amplitude – f = 6.750Hz, n.ksi = 7

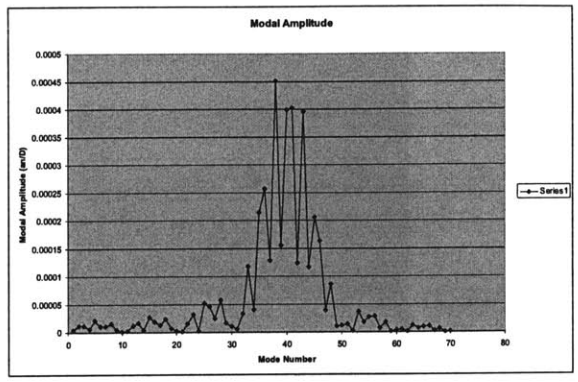

Figure 51 : Modal Amplitude – f = 6.834 Hz, n.ksi = 7

In both cases, a symmetrical peak centered on the excitation mode is obtained. Considering the superposition of modes up to 70 would give a good response. However, it was expected that the first modes would have a negligible influence on the response and the peak centered on the 30th mode in the previous section would switch to the 40th mode. It is seen that the lower modes are not negligible and that they do not have a lower influence on the response than considering an excitation around mode 30.

5 LIMITATIONS OF THE MODEL

In the simulation process, the excitation corresponding to mode 10 was considered. The frequency of the 10th mode is f = 1.687Hz. Since the wave length is three times as much as the wave length of the 30th mode, forces are applied on one wave length distance. The spatial attenuation $n.\xi = 3$ is presented here. The forces magnitude profile is shown in Figure 52:

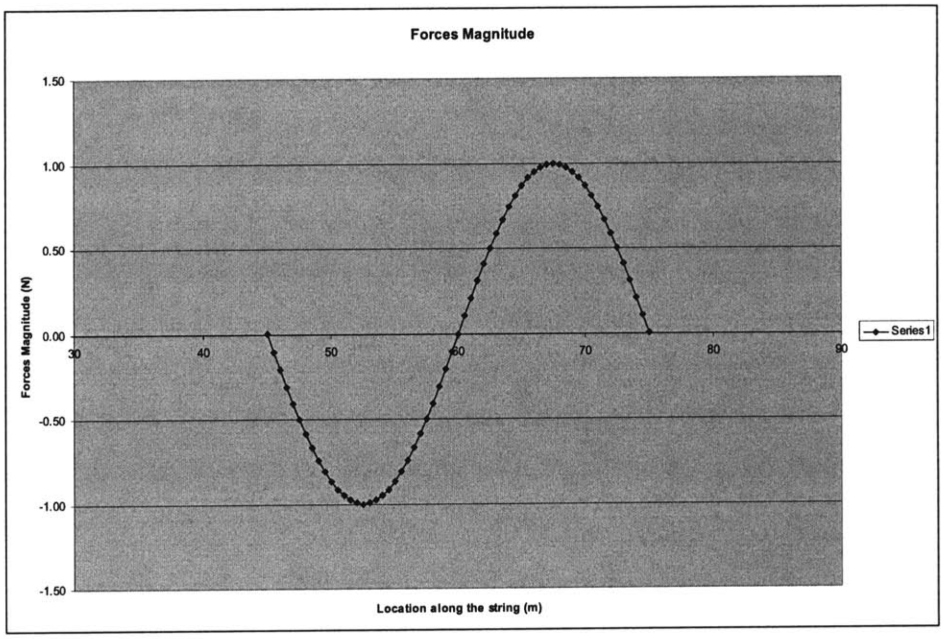

Figure 52 : Forces magnitude, excitation frequency f = 1.687Hz

The Green's function and modal superposition solutions are shown in Figures 53 and 54.

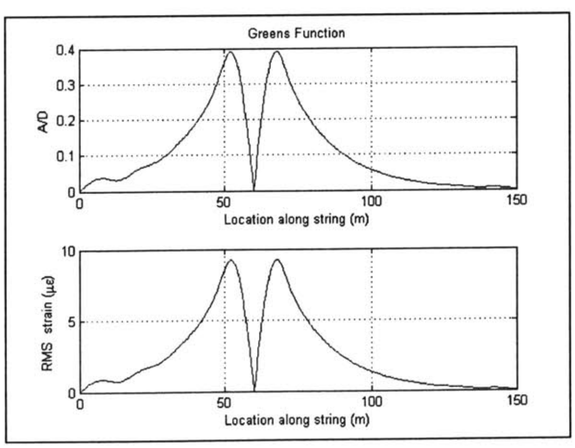

Figure 53 : Green's function - f = 1.687HzHz

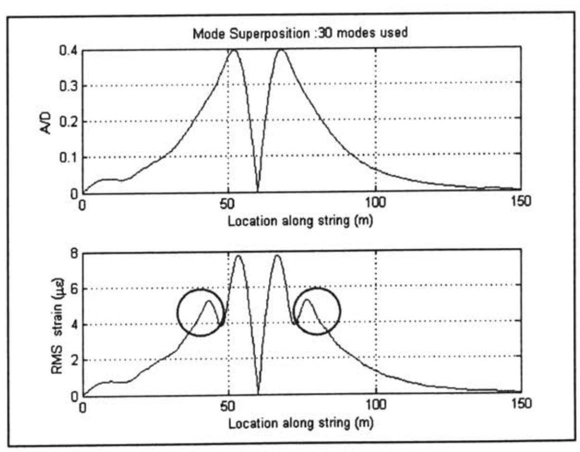

Figure 54 : Modal Superposition – f = 1.687Hz – 30 modes

It is seen that the modal superposition gives a good approximation for the displacement. However, the strain simulation gives a bad result. More specifically, the strain magnitude is underestimated and two peaks, circled in red on Figure 54, appear which do not exist on the Green's function result. These peaks remain even when increasing the number of superposed modes. Peaks are located where the first and the last point loads are applied, respectively, at 45 and 75m along the string. This phenomenon becomes visible when the response is dominated by traveling waves behavior, i.e. values of $n.\xi$ higher than 1.0.

It is recommended to avoid these issues that point loads have to be applied on more than one wave length.

6 CONCLUSION / RECOMMENDATIONS

From the analyses performed, it can be concluded that:

- The modal superposition solution gives an accurate approximation of the dynamic response of the string for values of $n.\xi$ limited to 3. The response is even exact when the excitation corresponds to a system resonant mode.
- In the worst cases, when the excitation frequency does not correspond to a natural period of the system, the number of modes to be accounted for is approximately 1.8 times the mode number whose frequency is closest to the excitation frequency.

However, the modal superposition technique has limitations:

- If it is stated that the maximum allowable error has to be limited to 5%. The modal superposition simulation gives accurate results when the spatial attenuation $n.\xi$ is less than approximately 7 for the displacement and less than approximately 3 for the strain.
- Point loads have to be applied on more than one wave length to avoid the singularities in the strain solution described in section 5.

7 REFERENCES

[1] Charles Dalton *Fundamentals of vortex-induced vibration*

[2] Singiresu S. Rao, *Mechanical Vibrations,* Fourth edition

[3] Li Li, Decmber, 1993, *A Comparison study of the Green's function and mode superposition techniques and their application to the lock-in response prediction of cylinders in currents*

[4] J.K. Vandiver, *Dimensionless parameters important to the prediction of vortex-induced vibration of long, flexible cylinders in ocean current*, Journal of Fluids and Structures (1993 7 423-455

[5] K. Vikestad, C. M. Larsen, J. K. Vandiver, *Norwegian deepwater program: Damping of vortex-induced vibrations*, OTC 11998

[6] Derivation of the Green's function solution

APPENDIX A: GREEN'S FUNCTION RESPONSE (MATLAB FILE)

```
L = 150;                      % length of string in meter;
dia = 0.03622;                % diameter in meter
x = [0:0.5:L];                %vector of points at which response is to be
computed
T = 3225;                     % Tension in Newton
zeta = 0.00033;               % Damping ratio
m = 1.2585;                   % mass per unit length in kg/m
f = 5.062;                    % frequency of the load in Hz
psi = [ 45  45.5    46  46.5    47  47.5    48  48.5    49  49.5    50  50.5
51  51.5    52  52.5    53  53.5    54  54.5    55  55.5    56  56.5    57
57.5    58  58.5    59  59.5    60  60.5    61  61.5    62  62.5    63  63.5
64  64.5    65  65.5    66  66.5    67  67.5    68  68.5    69  69.5    70
70.5    71  71.5    72  72.5    73  73.5    74  74.5    75  ]; %location of
the load, should be between 0 and L
P = [   0.00    -0.31   -0.59   -0.81   -0.95   -1.00   -0.95   -0.81   -0.59
-0.31   0.00    0.31    0.59    0.81    0.95    1.00    0.95    0.81    0.59
0.31    0.00    -0.31   -0.59   -0.81   -0.95   -1.00   -0.95   -0.81   -0.59
-0.31   0.00    0.31    0.59    0.81    0.95    1.00    0.95    0.81    0.59
0.31    0.00    -0.31   -0.59   -0.81   -0.95   -1.00   -0.95   -0.81   -0.59
-0.31   0.00    0.31    0.59    0.81    0.95    1.00    0.95    0.81    0.59
0.31    0.00    ]; % magnitude of the load in Newton


%============ End of Inputs ==========================================


w = 2*pi*f;                   % Frequency in rad/s
c = 2*m*w*zeta;               % Damping coefficient
K = sqrt((m*w^2 + i*c*w)/T); % complex wavenumber for the string;

response =[]; curv =[]; total=[];
for looppsi = 1:length(psi)

n1 =  K*L; n2 = K*psi(looppsi); n3 = K*(psi(looppsi)-L);
SN1 = sin(n1); SN2 = sin(n2); SN3 = sin(n3);
Ps = P(looppsi)/(T*K);        % Unit point load P= 1;

Coeff = [ -Ps*SN3/SN1;
          -Ps*SN2/SN1];
A1L = Coeff(1);
A1R = Coeff(2);



for loopX = 1:length(x); % Loop for section
    switch (x(loopX) <= psi(looppsi))
        case{1}
            y = A1L*sin(K*x(loopX));
            yxx = -A1L*(K^2)*sin(K*x(loopX)); % Curvature
        case{0}
            y = A1R*sin(K*(x(loopX)-L));
            yxx = -A1R*(K^2)*sin(K*(x(loopX)-L)); % Curvature

    end;% end of switch
    response(loopX,looppsi) = y;
```

```
    curv(loopX,looppsi) =  yxx;
end;% end of loopX


end;% end of looppsi


B=sum(response,2); % sums up all the columns
C=sum(curv,2);     % sums up all the columns

AbyD = abs(B)/dia  % response amplitude /diameter
strain = abs((1e6)*C*dia/2); % strain in micro strain. 1e6 converts strain to
micro strain
RMSstrain = strain/sqrt(2)  % Root mean square strain

figure;
subplot 211
plot(x,AbyD)
xlabel('Location along string (m)'); ylabel('A/D'); grid on;
title('Greens Function')

subplot 212
plot(x,RMSstrain)
xlabel('Location along string (m)'); ylabel('RMS  strain (\mu\epsilon)');
grid on;
```

APPENDIX B: MODAL SUPERPOSITION (MATLAB FILE)

```
% ============================ INPUTS ================================

L = 150;                        % length of string in meter;
dia = 0.03622;                  % diameter in meter
x = [0:0.5:L];                  % vector of points at which response is to be
computed
T = 3225;                       % Tension in Newton
zeta = 0.00033;                 % Damping ratio
m = 1.2585;                     % mass per unit length in kg/m
f = 5.062;                      % frequency of the load in Hz
psi = [ 45  45.5    46  46.5    47  47.5    48  48.5    49  49.5    50  50.5
51  51.5    52  52.5    53  53.5    54  54.5    55  55.5    56  56.5    57
57.5    58  58.5    59  59.5    60  60.5    61  61.5    62  62.5    63  63.5
64  64.5    65  65.5    66  66.5    67  67.5    68  68.5    69  69.5    70
70.5    71  71.5    72  72.5    73  73.5    74  74.5    75  ]; %location of
the load, should be between 0 and L
P = [   0.00    -0.31   -0.59   -0.81   -0.95   -1.00   -0.95   -0.81   -0.59
-0.31   0.00    0.31    0.59    0.81    0.95    1.00    0.95    0.81    0.59
0.31    0.00    -0.31   -0.59   -0.81   -0.95   -1.00   -0.95   -0.81   -0.59
-0.31   0.00    0.31    0.59    0.81    0.95    1.00    0.95    0.81    0.59
0.31    0.00    -0.31   -0.59   -0.81   -0.95   -1.00   -0.95   -0.81   -0.59
-0.31   0.00    0.31    0.59    0.81    0.95    1.00    0.95    0.81    0.59
0.31    0.00    ]; % magnitude of the load in Newton
no = 30;
n = [1:1:no];

%============= End of Inputs ========================================


w = 2*pi*f;                     % Frequency in rad/s

 responsey = []; responseyxx = [];
for loopX = 1:length(x); % Loop for section
   responseq =[]; responseS =[]; responseSxx = [] ;
    for loopN = 1:length(n);
       M = m*L/2;
       K = T*(loopN*pi/L)^2*L/2;
       wn = (K/M)^0.5;
       r = w/wn;
       for loopP = 1:length(P);
           modalForce = sum(P.*(sin(loopN*pi*psi/L)));
       end;
       q = modalForce*(1/K)/((1-r^2)+(2*zeta*r*i));
       S = sin(loopN*pi*x(loopX)/L); % changed loopX to x(loopX)
       Sxx = -((loopN*pi/L)^2)*sin(loopN*pi*x(loopX)/L); % changed loopX to
x(loopX); included ^2 with n*pi/L
       responseq = [responseq;q];
       responseS = [responseS;S];
       responseSxx = [responseSxx;Sxx];

   end;
   y = responseq' * responseS; % moved out of loopN
   yxx = responseq'*responseSxx; % moved out of loopN
   responsey = [responsey;y];
   responseyxx = [responseyxx;yxx];
end;
```

```
AbyD = abs(responsey)/dia % response amplitude /diameter
strain = (1e6)*abs(responseyxx)*dia/2; % strain in micro strain. 1e6 converts
strain to micro strain
RMSstrain = strain/sqrt(2)  % Root mean square strain

figure;
subplot 211
plot(x,AbyD)
xlabel('Location along string (m)'); ylabel('A/D'); grid
on;title(strcat('Mode Superposition : ',num2str(no),' modes used'));

subplot 212
plot(x,RMSstrain)
xlabel('Location along string (m)'); ylabel('RMS  strain (\mu\epsilon)');
grid on;
```

APPENDIX C: ERROR QUANTIFICATION (MATLAB FILE)

```
% ============================ INPUTS ===============================

L = 150;                    % length of string in meter;
dia = 0.03622;              % diameter in meter
x = [0:0.5:L];              % vector of points at which response is to be
computed
T = 3225;                   % Tension in Newton
zeta = 0.00033;             % Damping ratio
m = 1.2585;                 % mass per unit length in kg/m
f = 5.062;                  % frequency of the load in Hz
psi = [ 45  45.5    46  46.5    47  47.5    48  48.5    49  49.5    50  50.5
51  51.5    52  52.5    53  53.5    54  54.5    55  55.5    56  56.5    57
57.5    58  58.5    59  59.5    60  60.5    61  61.5    62  62.5    63  63.5
64  64.5    65  65.5    66  66.5    67  67.5    68  68.5    69  69.5    70
70.5    71  71.5    72  72.5    73  73.5    74  74.5    75  ]; %location of
the load, should be between 0 and L
P = [   0.00    -0.31   -0.59   -0.81   -0.95   -1.00   -0.95   -0.81   -0.59
-0.31   0.00    0.31    0.59    0.81    0.95    1.00    0.95    0.81    0.59
0.31    0.00    -0.31   -0.59   -0.81   -0.95   -1.00   -0.95   -0.81   -0.59
-0.31   0.00    0.31    0.59    0.81    0.95    1.00    0.95    0.81    0.59
0.31    0.00    -0.31   -0.59   -0.81   -0.95   -1.00   -0.95   -0.81   -0.59
-0.31   0.00    0.31    0.59    0.81    0.95    1.00    0.95    0.81    0.59
0.31    0.00    ]; % magnitude of the load in Newton


%============= End of Inputs ========================================


w = 2*pi*f;                   % Frequency in rad/s
c = 2*m*w*zeta;               % Damping coefficient
K = sqrt((m*w^2 + i*c*w)/T); % complex wavenumber for the string;

response =[]; curv =[]; total=[];
for looppsi = 1:length(psi)

n1 =  K*L; n2 = K*psi(looppsi); n3 = K*(psi(looppsi)-L);
SN1 = sin(n1); SN2 = sin(n2); SN3 = sin(n3);
Ps = P(looppsi)/(T*K);        % Unit point load P= 1;

Coeff = [ -Ps*SN3/SN1;
          -Ps*SN2/SN1];
A1L = Coeff(1);
A1R = Coeff(2);



for loopX = 1:length(x); % Loop for section
    switch (x(loopX) <= psi(looppsi))
        case{1}
            y = A1L*sin(K*x(loopX));
            yxx = -A1L*(K^2)*sin(K*x(loopX)); % Curvature
        case{0}
            y = A1R*sin(K*(x(loopX)-L));
            yxx = -A1R*(K^2)*sin(K*(x(loopX)-L)); % Curvature
```

```
    end;% end of switch
    response(loopX,looppsi) = y;
    curv(loopX,looppsi) =  yxx;
end;% end of loopX


end;% end of looppsi


B=sum(response,2); % sums up all the columns
C=sum(curv,2);     % sums up all the columns

AbyD1 = abs(B)/dia;             % response amplitude /diameter
strain1 = abs((1e6)*C*dia/2);   % strain in micro strain. 1e6 converts strain
to micro strain
RMSstrain1 = strain1/sqrt(2);   % Root mean square strain





% ================== Modal superposition ==============

no = 30;
n = [1:1:no];
%============== End of Inputs ==========================================


 responsez = []; responsezxx = [];
for loopX = 1:length(x); % Loop for section
   responseq =[]; responseS =[]; responseSxx = [] ;
    for loopN = 1:length(n);
       Mn = m*L/2;
       Kn = T*(loopN*pi/L)^2*L/2;
       wn = (Kn/Mn)^0.5;
       r = w/wn;

       for loopP = 1:length(P);
           modalForce = sum(P.*(sin(loopN*pi*psi/L)));
       end;

       q = modalForce*(1/Kn)/((1-r^2)+(2*zeta*r*i));
       S = sin(loopN*pi*x(loopX)/L);% changed loopX to x(loopX)
       Sxx = -((loopN*pi/L)^2)*sin(loopN*pi*x(loopX)/L);% changed loopX to
x(loopX); included ^2 with n*pi/L
       responseq = [responseq;q];
       responseS = [responseS;S];
       responseSxx = [responseSxx;Sxx];

   end;
   z = responseq' * responseS; % moved out of loopN
   zxx = responseq'*responseSxx; % moved out of loopN
```

```
   responsez = [responsez;z];
   responsezxx = [responsezxx;zxx];
end;
AbyD2 = abs(responsez)/dia; % response amplitude /diameter
strain2 = (1e6)*abs(responsezxx)*dia/2;% strain in micro strain. 1e6 converts
strain to micro strain
RMSstrain2 = strain2/sqrt(2);  % Root mean square strain

figure;
subplot 221
plot(x,AbyD1)
xlabel('Location along string (m)'); ylabel('A/D'); grid on;
title('Greens Function')

subplot 222
plot(x,RMSstrain1)
xlabel('Location along string (m)'); ylabel('RMS  strain (\mu\epsilon)');
grid on;

subplot 223
plot(x,AbyD2)
xlabel('Location along string (m)'); ylabel('A/D'); grid
on;title(strcat('Mode Superposition : ',num2str(no),' modes used'));

subplot 224
plot(x,RMSstrain2)
xlabel('Location along string (m)'); ylabel('RMS  strain (\mu\epsilon)');
grid on;

% =================== Error ===============

EA = AbyD1 - AbyD2;
ES = RMSstrain1 - RMSstrain2;

eA = sqrt(( EA' * EA )/( AbyD1' * AbyD1))*100
eS = sqrt(( ES' * ES )/(RMSstrain1' * RMSstrain1))*100
```