22.314J – Spring 2004

Problem Set III

Due 3/2/04

This problem illustrates stress calculations for a simplified pressure vessel.

1. Vessel Description The vessel is constructed of a cylinder with a hemispherical head at each end. There is no external restraint to either axial or radial displacement. The inside radius of both cylinder and hemispheres is R. The wall thickness is uniform at a value t (for both cylinder and hemispheres). The length of the cylinder is L. No flaws or stress concentrations are present.
   Dimensions are:

   R = 3.0 m;
   t = 0.21 m; and
   L = 8.6 m

2. Material Condition Material properties may be considered constant. The operating temperature is low enough so that creep may be neglected but high enough so that brittle fracture is not of concern.
   Properties include:

   Young's modulus = 190 GPa;
   Poisson's ratio = 0.29; and
   Coefficient of thermal expansion = 12 $\mu m/m$.K

3. Operating Condition. The vessel is pressurized to a design pressure p. In addition, the vessel is subjected to a cooldown transient. At the time of interest, the inside wall temperature is uniform at a value T_a, the outside is uniform at a value, $T_b = T_a + \Delta T$. Interior temperatures are distributed parabolically:

$$T = T_b - \frac{1}{4}\Delta T\left(1 - \frac{2z}{t}\right)^2$$

   where z denotes position measured radially outward from the shell center surface. The temperature distribution is the same in both cylinder and hemispheres.
   Design parameters are:

   p = 12 MPa; and
   $\Delta T = 50K$.

4. Question:
   1) What is the total (peak) stress as a function of radial position (z) at a junction between cylinder and hemisphere?

   2) What is the maximum radial displacement of the vessel cylinder and sphere?