22.314J – Spring 2004
Problem Set II

Due 2/19/04

Problem I

This problem illustrates some important stress tensor concepts and some calculation methods for treating a stress tensor.

Consider three stress tensors $(\sigma a, \sigma b, \sigma c)$ Defined as follows:

$$\sigma a = \begin{bmatrix} -55 & -5 & 30 \\ -5 & 55 & 30 \\ 30 & 30 & 20 \end{bmatrix};$$

$$\sigma b = \begin{bmatrix} -10 & 0 & 0 \\ 0 & -10 & 0 \\ 0 & 0 & -10 \end{bmatrix}; \text{ and}$$

$$\sigma c = \sigma a + \sigma b;$$

where the stresses are given in MPa units.

1) What are the three principal stresses that characterize each of these stress tensors?

2) Draw a 3-D Mohr's circle to represent the three stress states.

3) Also place points on your Mohr's circles that give the stresses on planes normal to each o the original coordinate axes (x, y, z).

4) What are the unit vectors (direction cosines that define principal directions for the three stress tensors?

5) What is the Tresca Stress for each stress tensor? What is the von Mises Stress for each stress tensor?

Problem II

Consider a cylindrical vessel of inner radius *R* and wall thickness t with flat ends. The pressure inside the vessel is *P*i and the surrounding pressure *P*□. What is the relative error in estimating the maximum value of the stress intensity in the cylinder based on the thin shell approximation for values of:

t/R = 0.03
0.10
0.15
0.30

Consider two cases:

*P*i = 2*P*o
*P*i = 20*P*o