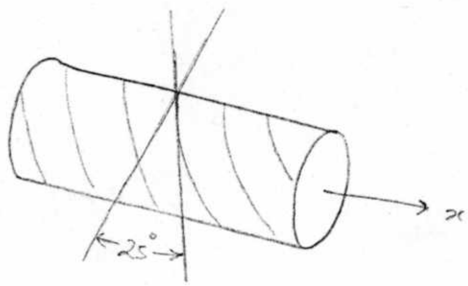
2.001 : Mechanics and Materials I

Spring 2003.

Quiz No. 2.

Problem 1



$p = 1\ MPa \quad ; \quad t = 10\ mm$

$D = 0.75\ m = 750\ mm$

$$\sigma_{xx} = \frac{p \cdot \frac{\pi}{4} D^2}{\pi D t} = \frac{pD}{4t} = \frac{1 \times 750}{4 \times 10} = 18.75\ MPa$$

$$\sigma_{\theta\theta} = \frac{pD}{2t} = \frac{1 \times 750}{2 \times 10} = 37.5\ MPa$$

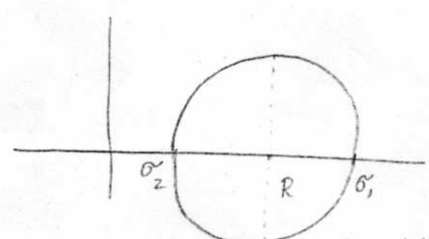
(a)

Since no torsion is applied

$\rightarrow \quad \sigma_{\theta x} = 0 \quad \Rightarrow \quad \sigma_1 = 37.5\ MPa$

$\sigma_2 = 18.75\ MPa$

$$\sigma_{shear(max)} = \frac{\sigma_1 - \sigma_2}{2} = \frac{37.5 - 18.75}{2} = 9.375\ MPa \Leftarrow$$



R = Max Shear.

(b)

$$\sigma_n' = \frac{\sigma_{xx} + \sigma_{\theta\theta}}{2} + \frac{\sigma_{xx} - \sigma_{\theta\theta}}{2}\cos 2\theta + \sigma_{x\theta}\sin 2\theta$$

$$\theta = 25^\circ$$

$$\sigma_n = \frac{18.75 + 37.5}{2} + \frac{12.75 - 37.5}{2}\cos 50$$

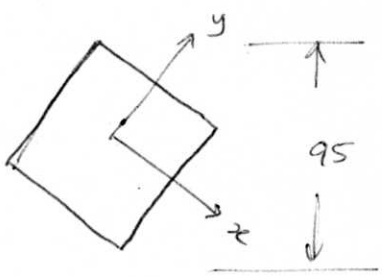
$$= 22.09 \text{ MPa}.$$

$$\sigma_{shear} = -\frac{\sigma_{xx} - \sigma_{\theta\theta}}{2}\sin 2\theta$$

$$= -\frac{18.75 - 37.5}{2}\sin 50$$

$$= 7.182 \text{ MPa}.$$

Problem 2.



$E = 100$ MPa.

$\nu = 0.4$

$G = 35.71$

Initially the length of each side of the square is

$$L_o = \sqrt{2 \times 47.5^2} = 67.175$$

Step 1

The new length of each side is

$$L = \sqrt{\left(\frac{110}{2}\right)^2 + \left(\frac{100}{2}\right)^2} = 74.33.$$

From symmetry

$$\sigma_{xx} = \sigma_{yy} = \sigma$$

$$\varepsilon_{xx} = \varepsilon_{yy} = \varepsilon$$

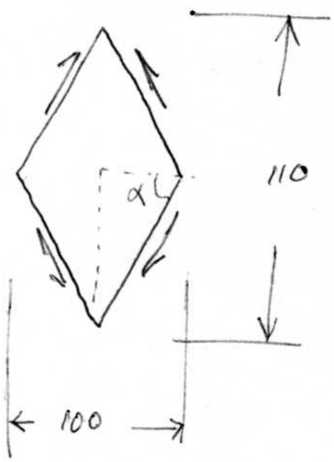
$$\varepsilon = \frac{L - L_o}{L_o} = \frac{74.33 - 67.175}{67.175}$$

$$= 0.1065$$

$$\varepsilon_{xx} = \frac{1}{E}\left[\sigma_{xx} - \nu\sigma_{yy}\right] \Rightarrow \varepsilon = \frac{1}{E}\left[\sigma - \nu\sigma\right]$$

$$\sigma = \frac{\varepsilon E}{1-\nu} = 17.75 \text{ MPa}.$$

Step 2 — The square has expanded to its new size. To make it a rhombus, a shear stress needs to be applied. The major angle of the rhombus is $> \frac{\pi}{2}$



$$\alpha = \tan^{-1} \frac{55}{50} = 0.8329812.$$

$$\gamma_{xy} = \left(\frac{\pi}{2} - 2\alpha\right) = -0.09586.$$

$$\sigma_{xy} = G\,\gamma_{xy}$$

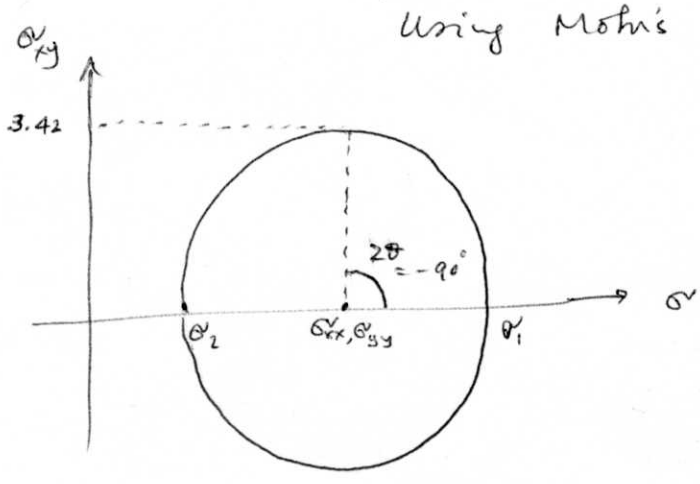
$$= 35.71 \times 0.09586$$

$$= -3.42 \text{ MPa}.$$

Step 3

Since axial loads are to applied we need to find the principal stress directions, and their orientation.

Using Mohr's circle.



$$\theta = -45^\circ.$$

$$\sigma_1 = \sigma_{xx} + |\sigma_{xy}|$$

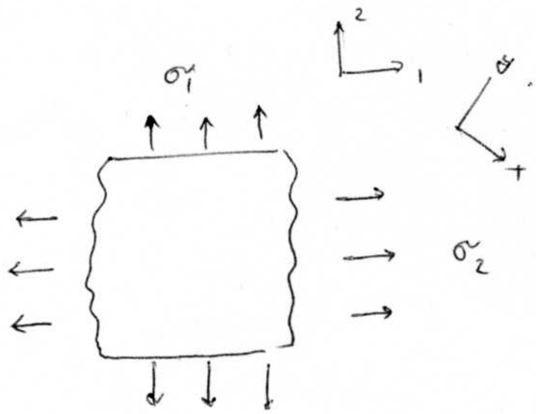
$$= 17.75 + 3.42$$

$$= 21.17 \text{ MPa}.$$

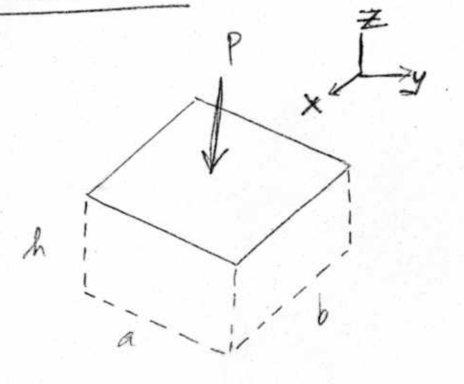
$$\sigma_2 = 17.75 - |\sigma_{xy}|$$

$$= 17.75 - 3.42$$

$$= 14.33 \text{ MPa}.$$



Problem 3.



Let $A = ab$

a) $\sigma_{zz} = \dfrac{P}{A} = \dfrac{P}{ab}$

$u_z = \varepsilon_{zz} h$

$K = \dfrac{P}{u_z} = ?$

$\boxed{\varepsilon_{xx} = \varepsilon_{yy} = 0.}$ geometric constraint

$\varepsilon_{xx} = \frac{1}{E}\left(\sigma_{xx} - \nu(\sigma_{yy} + \sigma_{zz})\right) = 0$ — I

$\varepsilon_{yy} = \frac{1}{E}\left(\sigma_{yy} - \nu(\sigma_{xx} + \sigma_{zz})\right) = 0$ — II

$\varepsilon_{zz} = \frac{1}{E}\left(\sigma_{zz} - \nu(\sigma_{yy} + \sigma_{xx})\right)$ — III

From I and II

$\sigma_{xx} = \nu(\sigma_{yy} + \sigma_{zz})$

$\sigma_{yy} = \nu(\sigma_{xx} + \sigma_{zz})$

$= \nu\left[\nu(\sigma_{yy} + \sigma_{zz}) + \sigma_{zz}\right]$

$= \nu^2 \sigma_{yy} + \nu^2 \sigma_{zz} + \nu\sigma_{zz}$

$\sigma_{yy}(1 - \nu^2) = \nu\sigma_{zz}(1 + \nu)$

$\sigma_{yy}(1 - \nu)\cancel{(1+\nu)} = \nu\sigma_{zz}\cancel{(1+\nu)}$

$\sigma_{yy} = \dfrac{\nu}{(1-\nu)}\sigma_{zz}$

Similarly.

$\sigma_{xx} = \dfrac{\nu}{(1-\nu)}\sigma_{zz}$

substitute σ_{xx} in III

$\varepsilon_{zz} = \frac{1}{E}\left[\sigma_{zz} - \nu\left(\frac{\nu}{(1-\nu)}\sigma_{zz} + \frac{\nu}{(1-\nu)}\sigma_{zz}\right)\right]$

$\varepsilon_{zz} = \frac{1}{E}\left[\sigma_{zz} - \frac{2\nu^2}{1-\nu}\sigma_{zz}\right]$

$\varepsilon_{zz} = \dfrac{\sigma_{zz}}{E}\left(1 - \dfrac{2\nu^2}{1-\nu}\right)$

$= \dfrac{\sigma_{zz}}{E}\left(\dfrac{1-\nu-2\nu^2}{1-\nu}\right)$

$\dfrac{u_z}{h} = \dfrac{P}{abE}\left(\dfrac{1-\nu-2\nu^2}{1-\nu}\right)$

$\Rightarrow P = \dfrac{ab}{h}\dfrac{E(1-\nu)}{(1-\nu-2\nu^2)} u_z$

$\Rightarrow \boxed{K = \dfrac{ab}{h}\dfrac{E(1-\nu)}{(1-\nu-2\nu^2)}}$

↗

(b) $\nu = 0.5$

$$k = \frac{ab}{h} \frac{E\,(1-0.5)}{(1-0.5-2\times 0.5^2)}$$

$$= \frac{ab\,E \times 0.5}{h \times 0} \longrightarrow \text{infinite}.$$

Hence for $\nu = 0.5$, it is impossible to have any downward movement, no matter what force is applied. Such materials are "INCOMPRESSIBLE", like rubber. The walls of the cavity prevent any lateral expansion so the dimensions 'a' and 'b' are unchanged. They conserve the volume of the block, The third dimension 'h' must remain the same. Hence no movement in z-direction is possible. This translates mathematically in $K \to \infty$.

(c) For $\nu = 0.5$ $k = \infty$ as in part B.

For 2E and $\nu = 0.25$

$$k = 2.182 \frac{abE}{h} \lll \infty$$

Hence the material is infinitely stiff for $\nu = 0.5$. Explanation same as in part (B). For 2E and $\nu = 0.25$, the material is not incompressible so vertical deflection is possible.