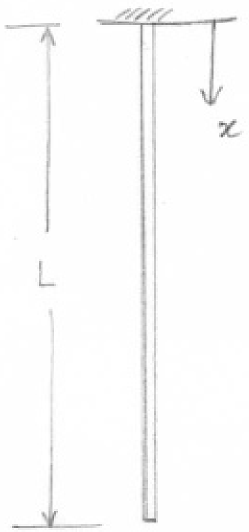
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Mechanical Engineering

2.001 - Mechanics of Materials I
Spring, 2003

Solutions for Problem Set 6

PROBLEM 1



a) Maximum length of wire that can be reeled out:

A = area of cross-section

Max. tensile stress occurs at $x = 0$ since the weight of the whole cable acts at this section.

$F = AL\rho g$ $\qquad g = 9.81 \ m/sec^2$

$\qquad\qquad\qquad \rho = 2800 \ Kg/m^3$

$$\sigma = \frac{F}{A}$$

$$\sigma = L\rho g$$

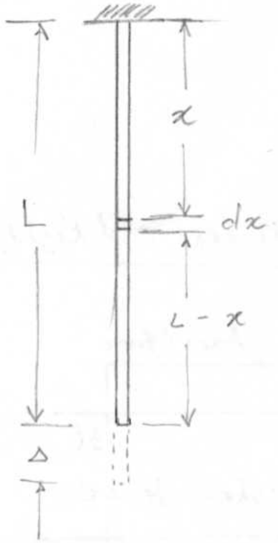
$$\sigma_{max} = 200 \ MPa$$
$$= 200 \times 10^6 \ N/m^2$$

$$\Rightarrow \quad 200 \times 10^6 = L \rho g$$

$$L = \frac{200 \times 10^6}{2800 \times 9.81}$$

$$L = 7281.2 \ m$$

(b) What is the total deformation of wire (total elongation) at the threshold of failure?



$$u = \int \varepsilon \, dx$$

$$\Delta = \int_0^L \varepsilon \, dx \qquad \varepsilon = \frac{\sigma}{E}$$

$$\sigma = \frac{\rho (L - x) A g}{A} = \rho (L - x) g$$

$$= \int_0^L \frac{\rho (L - x) g}{E} \, dx$$

$$= \int_0^L \left(\frac{\rho L g}{E} - \frac{\rho x g}{E} \right) dx$$

$$\Delta = \frac{\rho L^2 g}{E} - \frac{\rho}{2E} L^2 g$$

$$\Delta = \frac{1}{2} \frac{\rho L^2}{E} g$$

$$= \frac{1}{2} \times \frac{2800 \times (7281.2)^2 \times 9.81}{69 \times 10^9}$$

$$= 10.55 \text{ m}$$

$E = 69 \times 10^6 \text{ KN/m}^2$

$= 69 \times 10^9 \text{ N/m}^2$

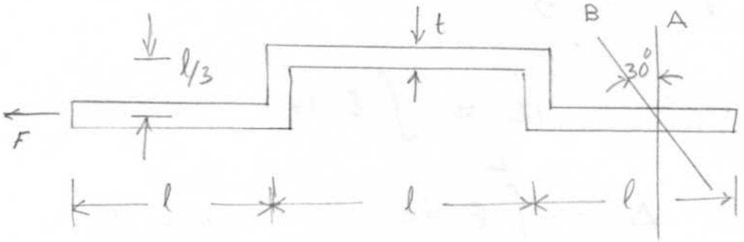
(from book CDL page. 83)

(c) The wire will become skinnier as you go up. The stresses are higher up in the wire since more weight is being supported by sections which are higher. Higher stresses mean, higher strains (Longitudinal strains).

$$\text{Lateral strains} = -\underbrace{\nu}_{\text{poisson's Ratio}} \times \text{Longitudinal strain}$$

Hence higher the stretching, the skinnier will be the wire

Problem 2



a)

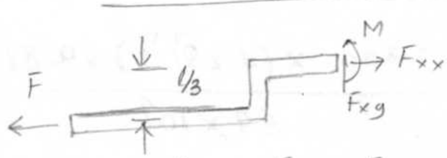
For left segment

ΣF_x: $F_{xx} = F$

ΣF_y: $F_{xy} = 0$

ΣM: $M = 0$

For Middle Segment



ΣF_x : $F_{xx} = F$

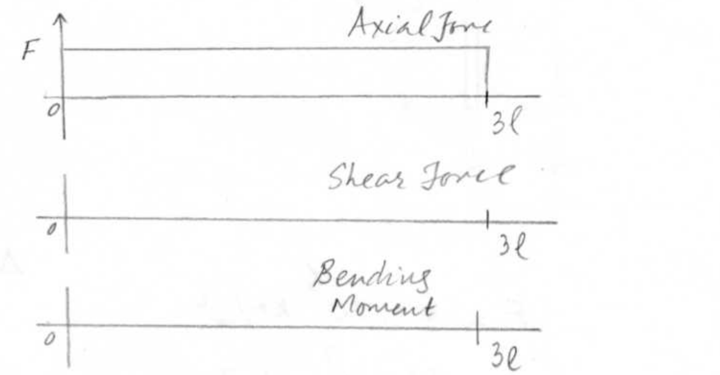
ΣF_y : $F_{xy} = 0$

ΣM : $M = F\, l/3$

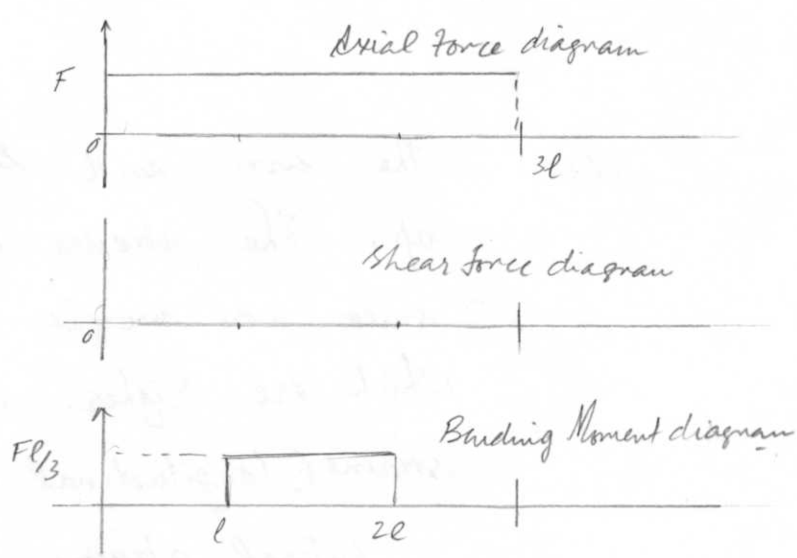
For Right Segment

Same as for left segment by symmetry

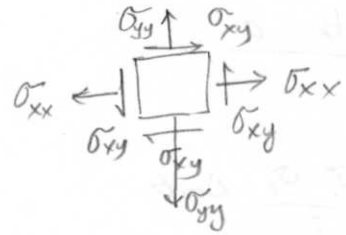
For Left and Right Segments



For Middle Segment



(b) On section (A)



Assume the dimension $\perp$ to paper is unity

$$\sigma_{xx} = \frac{F}{t} \Longleftarrow$$

$$\sigma_{xy} = 0 \Leftarrow$$ – Since no shear force acts on this face

$$\sigma_y = 0$$

Also this means that $\sigma_x = F/t$ and $\sigma_y = 0$ are principal stresses

(c) On section (B)

$\theta = 30^\circ$ $\qquad$ $\sigma_{xx} = F/t$; $\sigma_{yy} = 0$; $\sigma_{xy} = 0$

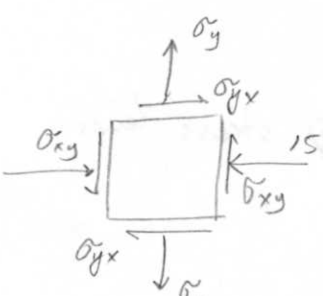
$\sigma_x' = ?$ $\quad$ $\sigma_{xy}' = ?$.

$$\sigma_{x'x'} = \frac{\sigma_{xx} + \sigma_{yy}}{2} + \frac{\sigma_{xx} - \sigma_{yy}}{2}\cos 2\theta + \sigma_{xy}\sin 2\theta$$

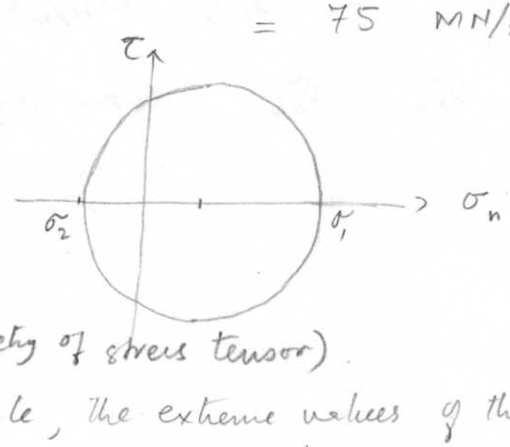
$$= \frac{F}{2t} + \frac{F}{2t}\cos 60^\circ + 0$$

$$= \frac{F}{2t} + \frac{F}{4t} = \frac{3}{4}\frac{F}{t}$$

PROBLEM 3 $\quad$ (4.15)



Maximum tensile stress $= 75$ MN/m^2



$\sigma_{xy} = 0$

$\Rightarrow \sigma_{yx} = 0$

(by symmetry of stress tensor)

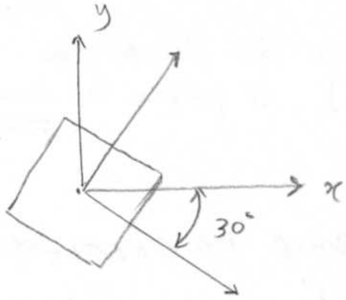
From Mohr's circle, the extreme values of the normal stresses are the principal stresses, and the corresponding shear stress is zero.

Therefore

$$\sigma_1 = 75 \text{ MN/m}^2$$

$$\sigma_2 = -15 \text{ MN/m}^2$$

Also solution of part B of this problem



For face $\perp$ to a

$\theta = -30^\circ$

$$\sigma_{aa} = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2} \cos\theta$$

$$= \frac{-15+75}{2} + \frac{-15-75}{2} \cos(-60)$$

$$\sigma_{aa} = 30 - 22.5 = 7.5 \ MN/m^2$$

$$\sigma_{ab} = -\frac{\sigma_1 - \sigma_2}{2} \sin 2\theta$$

$$= -\frac{-15-75}{2} \sin(-60) = -38.97 \ MN/m^2$$

For face $\perp$ to b

$$\theta = -30 + 90 = 60^\circ$$

$$\sigma_{bb} = \frac{-15+75}{2} + \frac{-15-75}{2} \cos(120)$$

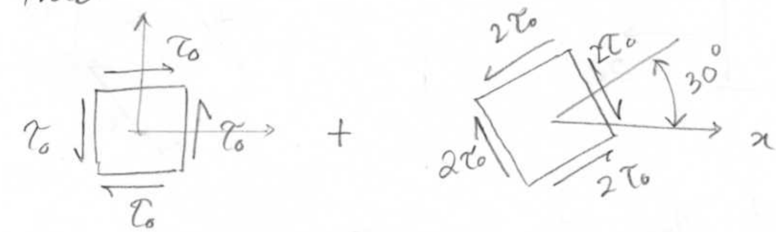
$$= 30 - 45 \cos(120)$$

$$= 52.5 \ MN/m^2$$

$\sigma_{ba} = \sigma_{ab}$. from symmetry of stress tensor.

PROBLEM 4 . 4.26

Case (a)



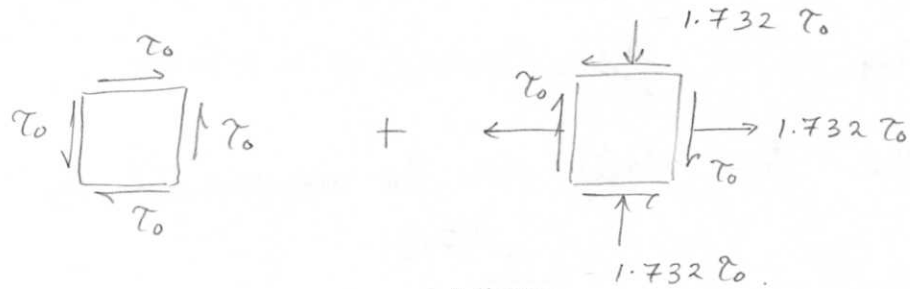
For the state on the right

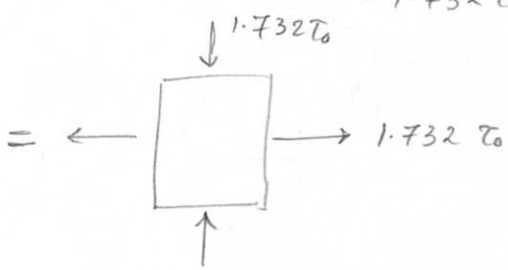
$$\sigma_x = -2\tau_o \sin(-2\times 30) = 1.732\,\tau_o$$

$$\sigma_y = -(-2\tau_o)\sin(-2\times 30) = -1.732\,\tau_o$$

$$\tau_{xy} = -2\tau_o \cos(-2\times 30) = -\tau_o$$

Therefore the two states can be represented as





Therefore the principal stresses are

$$\sigma_1 = 1.732\,\tau_o$$

$$\sigma_2 = -1.732\,\tau_o$$

(b)

we find σ_{xx} σ_{yy} and τ_{xy} for the stress stat on the right

$$\sigma_{xx} = \frac{-3\sigma_0}{2} + \frac{-3\sigma_0}{2}\cos(2\times 45)$$

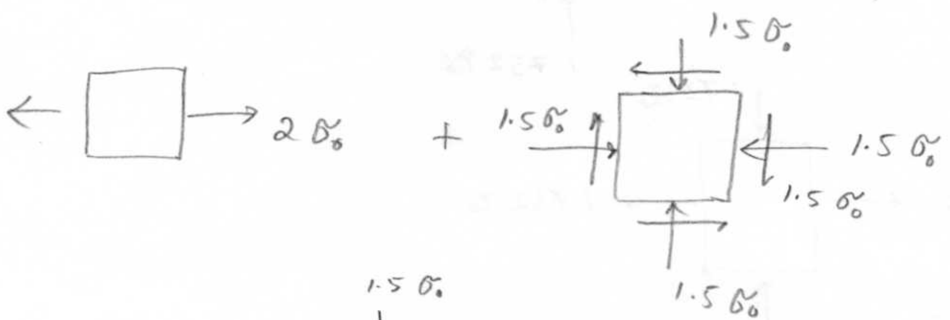
$$= -1.5\,\sigma_0$$

$$\sigma_{yy} = -\frac{3\sigma_0}{2} + \frac{3\sigma_0}{2}\cos(2\times 45)$$

$$= -1.5\,\sigma_0$$

$$\tau_{xy} = -\frac{3\sigma_0}{2}\sin(2\times 45) = -1.5\,\sigma_0$$

Therefore the two states can be represented by



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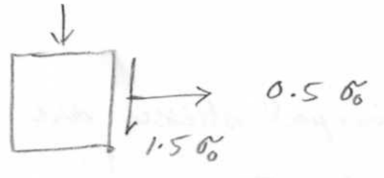


$\Rightarrow$ $\sigma_{xx} = 0.5\,\sigma_0$
$\sigma_{yy} = -1.5\,\sigma_0$
$\tau_{xy} = -1.5\,\sigma_0$

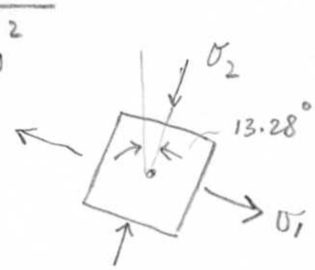
$$\sigma_1 = \sqrt{\left(\sigma_{xx} - \frac{\sigma_{xx}+\sigma_{yy}}{2}\right)^2 + \tau_{xy}^2} + \frac{\sigma_{xx}+\sigma_{yy}}{2}$$

$$= \sqrt{\sigma_0^2 + 2.25\,\sigma_0^2} = 1.8\,\sigma_0 + -0.5\,\sigma_0$$
$$= 1.3\,\sigma_0.$$

$$\sigma_2 = \frac{\sigma_{xx}+\sigma_{yy}}{2} - \sqrt{\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2 + \tau_{xy}^2}$$

$$= -2.3\,\sigma_0$$

$$\theta = 13.28$$

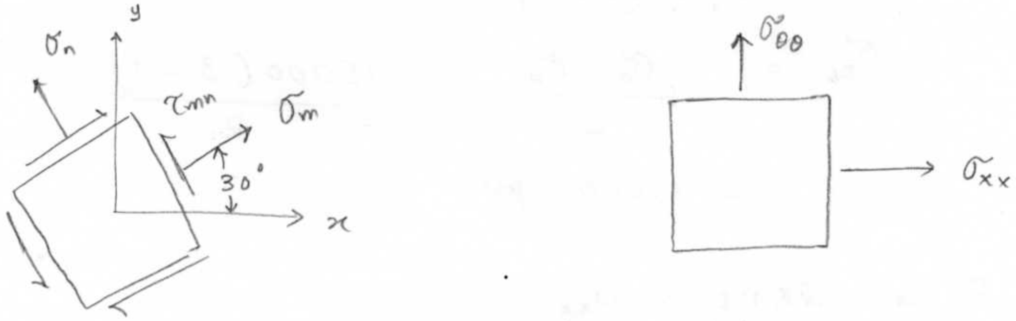


PROBLEM 5

a) $\sigma_m = 15000$ psi , $\sigma_n = 5000$ psi $\tau_{mn} = ?$

σ_{xx} and $\sigma_{\theta\theta}$ are principal stresses

since $\sigma_{x\theta} = \sigma_{\theta x} = 0$, from shown loading



$$\sigma_m = \frac{\sigma_{xx} + \sigma_{\theta\theta}}{2} + \frac{\sigma_{xx} - \sigma_{\theta\theta}}{2} \cos(2\times 30)$$

$$\sigma_m = 0.75\ \sigma_{xx} + 0.25\ \sigma_{\theta\theta} \quad — (1)$$

$$\sigma_n = \frac{\sigma_{xx} + \sigma_{\theta\theta}}{2} - \frac{\sigma_{xx} - \sigma_{\theta\theta}}{2} \cos(2\times 30)$$

$$\sigma_n = 0.25\ \sigma_{xx} + 0.75\ \sigma_{\theta\theta} \quad — (2)$$

From (1) and (2)

$$\sigma_{xx} = \frac{3}{2}\sigma_m - \frac{\sigma_n}{2}$$

$$\sigma_{\theta\theta} = \frac{3\sigma_n - \sigma_m}{2}$$

$$\Rightarrow \sigma_{xx} = 1.5 \times 15000 - 0.5 \times 5000$$
$$= 20{,}000 \text{ psi}$$

$$\sigma_{\theta\theta} = 0$$

$$\sigma_{xx} = \frac{F}{2\pi r t} \Rightarrow F = 2\pi r t \times 20{,}000$$
$$= 2\pi \times 10 \times 0.1 \times 20{,}000$$
$$= 125{,}663.7 \text{ lb}$$

$$\sigma_{\theta\theta} = 0 \Rightarrow p = 0.$$

b) $\sigma_m = 15{,}000$ psi $\qquad \sigma_n = 15000$ psi

$$\sigma_{xx} = \frac{3}{2}\sigma_m - \frac{\sigma_n}{2}$$

$$= 15000\left(\frac{3}{2} - \frac{1}{2}\right)$$

$$= 15000 \text{ psi}$$

$$\sigma_{\theta\theta} = \frac{3\sigma_n - \sigma_m}{2} = \frac{15000(3-1)}{2}$$

$$= 15000 \text{ psi}$$

$$F = 2\pi r t \times \sigma_{xx}$$

$$= 2\pi \times 10 \times 0.1 \times 15000$$

$$= 94{,}247.8 \text{ lb.}$$

$$\sigma_{\theta\theta} = \frac{pr}{2t} \Rightarrow p = \frac{2t}{r}\sigma_{\theta\theta}$$

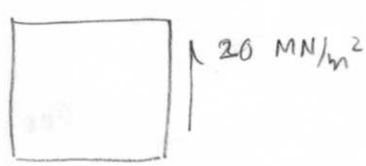
$$= \frac{2 \times \cancel{0.1}}{1\cancel{0}} \times 150\cancel{0}\cancel{0}$$

$$= 300 \text{ psi}$$

PROBLEM 6 $\quad$ 4.28.

$\sigma_1 = 20$ MN/m^2 $\qquad \sigma_2 = -45$ MN/m^2

$$\tau_{xy} = -\frac{\sigma_1 - \sigma_2}{2}\sin 2\theta$$



$$20 = -\frac{20 + 45}{2}\sin 2\theta$$

$$\sin 2\theta = -0.615$$

$$\Rightarrow \theta = -18.99^\circ$$