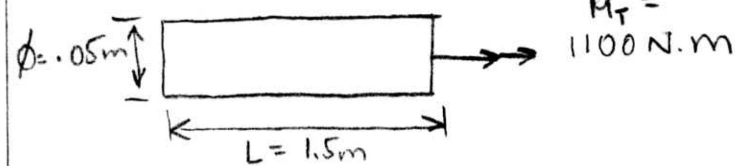
6.1



$J = \frac{\pi r^4}{4} = \frac{\pi d^4}{32} = 6.14 \times 10^{-7}\ m^4$

$G_{steel} \approx 80 GPa$

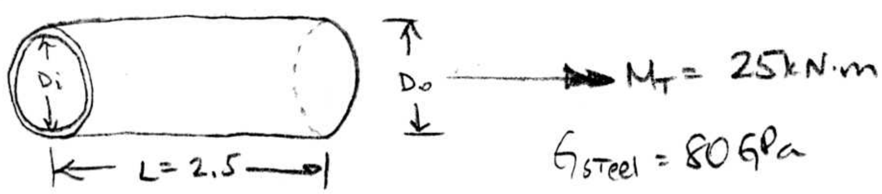
$\theta = \frac{M_T L}{GJ}$ $\qquad \tau = \frac{M_T r}{J}$

$\theta = \frac{(1100 N.m)(1.5m)}{(80 \times 10^9 \frac{N}{m^2})(6.14 \times 10^{-7} m^4)} = \underline{.034\ rad}$

$\tau = \frac{(1100 N.m)(.025m)}{6.14 \times 10^{-7} m^4} = 44.79 \times 10^6\ Pa$

$= \underline{44.79\ MPa}$

6.2



$R = \frac{D_o}{2}$

$\theta_{max} = 2.0° = .0349\ rad$

$\tau_{max} = 82 MPa$

$J = \frac{\pi d_o^4}{32} - \frac{\pi d_i^4}{32}$

II. $\tau = \frac{M_T R}{J} = \frac{M_T D_o}{2J} \Longrightarrow D_o = \frac{2\tau J}{M_T} = \frac{2(82 \times 10^6 Pa)(2.239 \times 10^{-5} m^4)}{25 \times 10^3 N.m}$

$\boxed{D_o = .1469m}$

I. $\rightarrow \theta = \frac{M_T L}{GJ}$

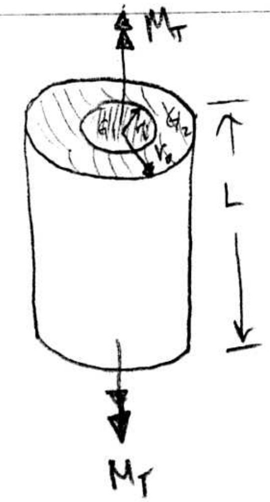
$.0349 = \frac{(25 \times 10^3 N.m)(2.5m)}{80 \times 10^9 N/m^2 \cdot J}$

$\boxed{J = 2.239 \times 10^{-5}\ m^4}$

$J = \frac{\pi}{32}(D_o^4 - D_i^4)$

III. $2.239 \times 10^{-5} = \frac{\pi}{32}((.1469)^4 - D_i^4) \longrightarrow \boxed{D_i = .1242m}$

6.3



From 6.3 in CDL, we know that

$$\sigma_r = \sigma_\theta = \sigma_z = \sigma_{r\theta} = \sigma_{rz} = 0$$

$$\sigma_{\theta z} = G_1 r \frac{d\phi}{dz} \quad \text{for } 0 < r < r_i$$

$$\sigma_{\theta z} = G_2 r \frac{d\phi}{dz} \quad \text{for } r_i < r < r_o$$

Similar to 6.4, $M_z = \int_A r \, \sigma_{\theta z} \, dA$

$$= \frac{d\phi}{dz} \left\{ 2\pi G_1 \int_0^{r_i} r^3 dr + 2\pi G_2 \int_{r_i}^{r_o} r^3 dr \right\}$$

$$= \frac{\pi}{2} \frac{d\phi}{dz} \left\{ (G_1 - G_2) r_i^4 + G_2 r_o^4 \right\}$$

Because $I_1 = \frac{\pi}{2} r_i^4$ (inner cylinder)

$I_2 = \frac{\pi}{2}(r_o^4 - r_i^4)$ (outer tube)

$$M_z \; \frac{d\phi}{dz} \cdot \frac{\pi}{2} \left\{ G_1 r_i^4 + G_2 (r_o^4 - r_i^4) \right\}$$

$$\underline{M_t = \frac{d\phi}{dz} \left\{ G_1 I_1 + G_2 I_2 \right\}}$$

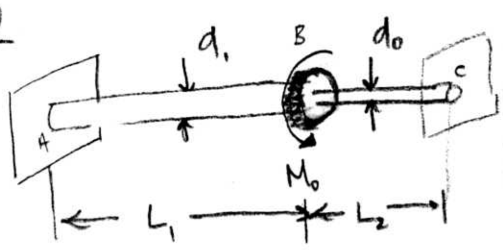
The Angle of twist ϕ is given by:

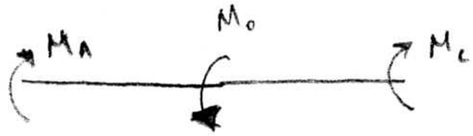
$$\frac{d\phi}{dz} = \frac{M_t}{G_1 I_1 + G_2 I_2} \longrightarrow \phi = \int_0^L \frac{d\phi}{dz} dz \rightarrow \phi = \frac{M_z L}{G_1 I_1 + G_2 I_2}$$

$$\sigma_{\theta z} = G_1 r \frac{d\phi}{dz} = \frac{G_1 M_t r}{G_1 I_1 + G_2 I_2}, \quad \text{for } 0 < r < r_i$$

$$\sigma_{\theta z} = G_2 r \frac{d\phi}{dz} = \frac{G_2 M_t r}{G_1 I_1 + G_2 I_2}, \quad \text{for } r_i < r < r_o$$

6.10





$$\Sigma M = 0: \; M_0 - M_A - M_C = 0 \qquad (1)$$

Compatibility: $\phi_{AB} + \phi_{BC} = 0 \qquad (2)$

$$\phi_{AB} = -\phi_{BC} \quad \text{OR} \quad \phi_{AB} = \phi_{CB}$$

$$\phi_{AB} = \frac{M_A L_1}{G \frac{\pi d_1^4}{32}} = \frac{32 M_A L_1}{G \pi d_1^4}$$

$$\phi_{CB} = \frac{M_C L_2}{G \frac{\pi d_2^4}{4}} = \frac{32 M_C L_2}{G \pi d_2^4}$$

$$\phi_{AB} = \phi_{CB}$$

$$\frac{\cancel{32} M_A L_1}{\cancel{G\pi} d_1^4} = \frac{\cancel{32} M_C L_2}{\cancel{G\pi} d_2^4}$$

$$\frac{M_A L_1}{d_1^4} = \frac{M_C L_2}{d_2^4} \rightarrow M_A = \left(\frac{d_1}{d_2}\right)^4 \frac{L_2}{L_1} M_C$$

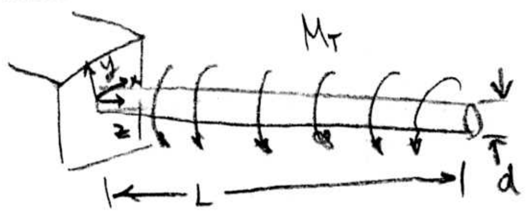
From (1), $\quad M_0 = M_A + M_C$

$$M_0 = \left(\frac{d_1}{d_2}\right)^4 \cdot \frac{L_2}{L_1} M_C + M_C = M_C \left[\left(\frac{d_1}{d_2}\right)^4 \frac{L_2}{L_1} + 1\right]$$

$$\boxed{M_C = \frac{M_0}{\left[\left(\frac{d_1}{d_2}\right)^4 \frac{L_2}{L_1} + 1\right]}}$$

$$M_A = M_0 - M_C = M_0 - \frac{M_0}{\frac{d_1^4 L_2}{d_2^4 L_1} + 1} = \boxed{\frac{M_0}{\left[\left(\frac{d_2}{d_1}\right)^4 \frac{L_1}{L_2} + 1\right]}} = M_A$$

6.14



Analogous to:

q = Intensity $\left[\frac{N}{m}\right]$

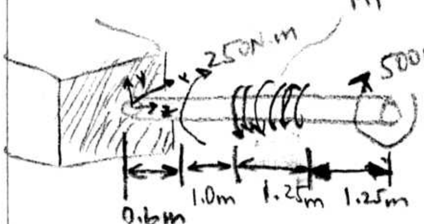
$M_T \rightarrow$ Moment Intensity … $\left[\frac{N\cdot m}{m} \rightarrow N\right]$

$$\frac{d\phi}{dz} = \frac{M_T}{GJ} = \frac{M(z)}{GJ} \qquad M(z) = M_T z \ , \quad J = \frac{\pi d^4}{32}$$

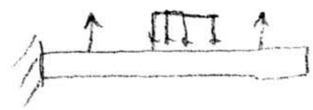
$$\phi = \int_0^L \frac{M(z)}{GJ}\,dz = \int_0^L \frac{M_T z}{GJ}\,dz = \frac{M_T z^2}{2GJ}\Big|_0^L = \frac{M_T L^2}{2GJ}$$

$$\phi = \frac{M_T L^2}{2GJ} = \frac{16 M_T L^2}{\pi d^4 G}$$

6-16

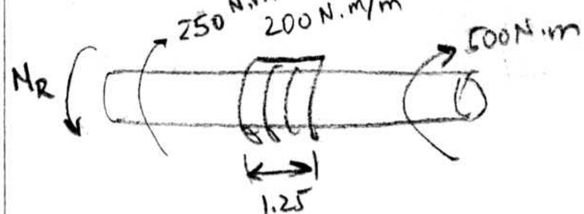


dia = .05m



Similar to Distributed loading on a beam:

1st Find the reaction Moment at the wall!

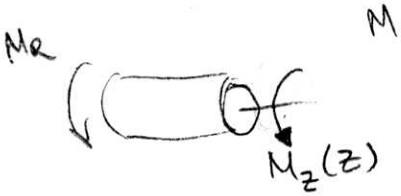


$$M_R - 250 + 200(1.25) - 500 = 0$$

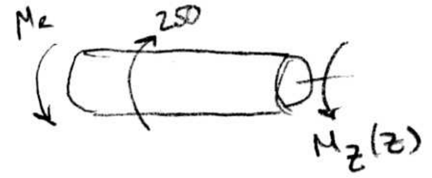
$$M_R = 500\,N.m$$

For $0 < z < 0.6$:

$$M_z(z) = -M_R = -500\,N.m$$

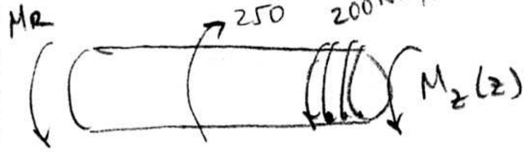


For $0.6 < z < 1.6$



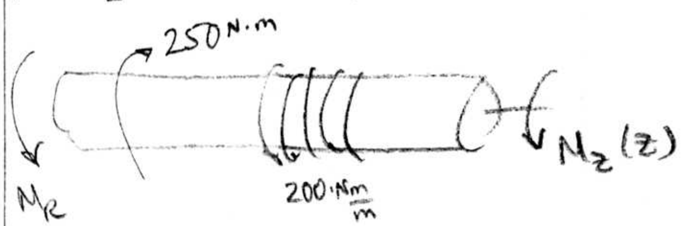
$$M_z(z) + M_R - 250 = 0$$
$$M_z(z) = -250\ \text{N·m}$$

For $1.6 < z < 2.85$

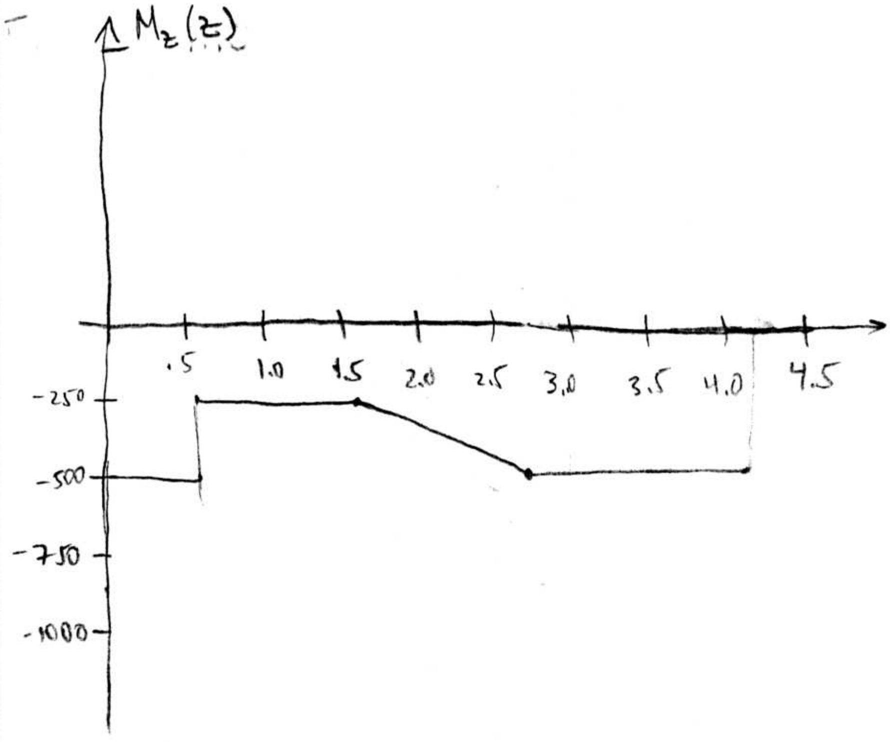


$$M_z(z) + M_R - 250 + 200(z - 1.6)$$
$$M_z(z) + 500 - 250 + 200z - 320$$
$$M_z(z) = 70 - 200z$$

For $2.85 < z < 4.10$



$$M_z(z) + M_R - 250 + 200(1.25) = 0$$
$$M_z(z) = -500$$



To find the twisting angle, we must integrate:

$$\phi = \int_0^L \frac{M(z)}{GJ}\, dz \qquad J = \frac{\pi d^4}{32} = 6.14\times 10^{-7}\, m^4 \qquad GJ = 4.909\times 10^4$$

$G = 80\times 10^9\, Pa$

$0 < z < 0.6$: $\phi_1 = \int_0^z \frac{-500}{GJ}\, dz' = -.0102z$

$0.6 < z < 1.6$: $\phi_2(z) = \int_{0.6}^{z} \frac{-250}{4.909\times 10^4}\, dz' = -.005(z-.6)$

$1.6 < z < 2.85$:

$$\phi_3(z) = \int_{1.6}^{z} \frac{M(z)}{GJ}\, dz = \int_{1.6}^{z} \frac{70-200z'}{4.909\times 10^4}\, dz'$$

$$= \frac{70z'}{4.909\times 10^4} - \frac{100z'^2}{4.909\times 10^4} \Bigg|_{1.6}^{z}$$

$$\phi(z) = .0014(z-1.6) - .002(z-1.6)^2$$

$2.85 < z < 4.10$

$$\phi_4(z) = \int_{2.85}^{z} \frac{M(z)}{GJ}\, dz = \int_{2.85}^{z} \frac{-500}{4.909\times 10^4}\, dz' = -.002(z-2.85)$$

$$\phi_{TOT} = \phi_1(z) + \phi_2(z) + \phi_2(z) + \phi_4(z)$$

The total angle of twist can be found by putting in the upper limits of integration and adding it all together. The equations above $\phi_1(z)$, $\phi_2(z)$, $\phi_3(z)$, $\phi_4(z)$ are the distribution of ϕ in the region specified by their limits.