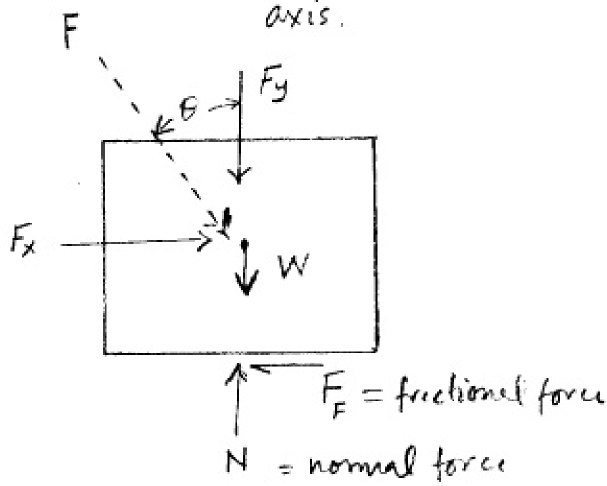
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Mechanical Engineering

2.001 - Mechanics of Materials I
Spring, 2003

Solutions for Problem Set 3

Problem 1

Draw FBD of the block, with applied force resolved into its two components along x and y axis.



$m = 10$ kg

$W = mg$

$F_x = F\sin\theta$

$F_y = F\cos\theta$

$\mu_s = 0.9$

$\boxed{\sum F_y = 0}$

$N - W - F_y = 0$

$N = W + F_y$ — I

$\boxed{\sum F_x = 0}$

$-F_F + F_x = 0$ — II

$F_F = \mu_s N$

$F_F = \mu_s (W + F_y)$ (from

— III

putting III in II

$-\mu_s(W + F_y) + F_x = 0$

$-\mu_s(mg + F\cos\theta) + F\sin\theta = 0$

$-\mu_s mg - F[\mu_s\cos\theta - \sin\theta] = 0$

$$F = \frac{\mu_s mg}{[\sin\theta - \mu_s\cos\theta]}$$

a) $\theta = 20^\circ$

$$F = \frac{0.9 \times 10 \times 9.81}{\sin 20 - 0.9\cos 20}$$

$= -ve$

b) $\theta = 40^\circ$

$F = -ve$ } angle not la[rge] enou[gh] for sliding [to] take pla[ce]

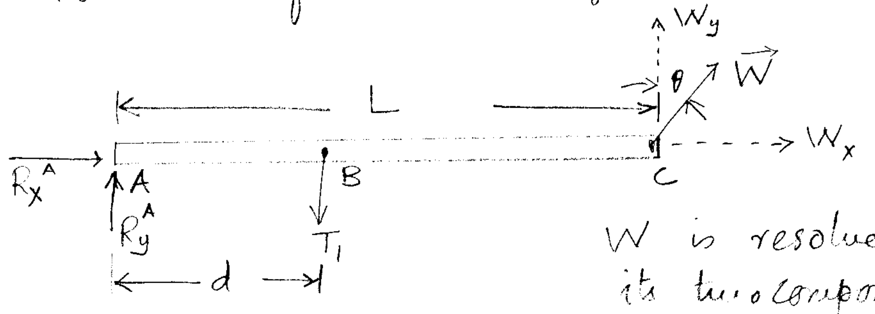
(c) $\theta = 80^\circ$

$F = 106.56$ N

PROBLEM 2.

(a) Draw the FBD of the beam for calculating T_1

FBD



W is resolved into its two components W_x and W_y

$$\vec{W} = W_x \hat{i} + W_y \hat{j}$$

$$W_x = W \sin\theta$$
$$W_y = W \cos\theta$$

$\Longrightarrow$ $\boxed{\sum M^A = 0}$

$$(T_1)(d) - (W_y)(L) = 0$$

$$T_1 = \frac{W_y L}{d}$$

$$= \frac{W \cos\theta \cdot L}{d} \Leftarrow \text{Ans}$$

(b) $T_1 = T_2 e^{-f\theta}$

$\theta = 720^\circ = 4\pi$ rad $\qquad f = \mu$

$$T_1 = T_2 e^{-4\mu\pi} \Rightarrow T_2 = \frac{1}{e^{-4\mu\pi}} \frac{W \cos\theta L}{d}$$

(c) To draw Shear Force, Bending Moment and Axial Force diagram

$\boxed{\sum F_y = 0}$

$$R_y^A - T_1 + W_y = 0$$
$$R_y^A = T_1 - W_y$$
$$= W \cos\theta \left[\frac{L}{d} - 1\right]$$

$\boxed{\sum F_x = 0}$

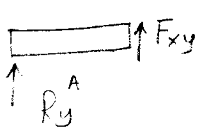
$$R_x^A + W_x = 0$$
$$R_x^A = -W_x$$
$$= -W \sin\theta \Leftarrow$$

Shear Force Diagram

$0 < x < d$.

$R_y^A + F_{xy} = 0$

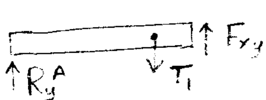
$F_{xy} = -R_y^A$



$\Rightarrow$ $F_{xy} = -W \cos\theta \left[\frac{L}{d} - 1\right]$

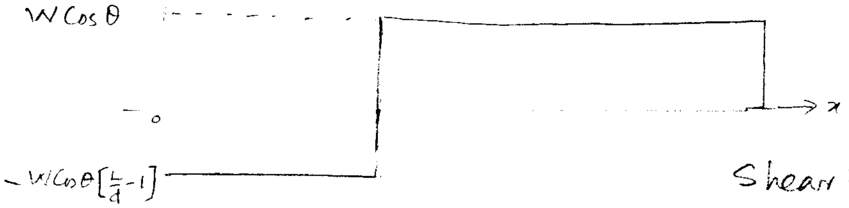
$d < x < L$

$F_{xy} = T_1 - R_y^A$



$\Longrightarrow$ $F_{xy} = W \cos\theta \left[\frac{L}{d} - \frac{L}{d} + 1\right]$

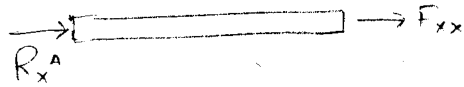
$= W \cos\theta$



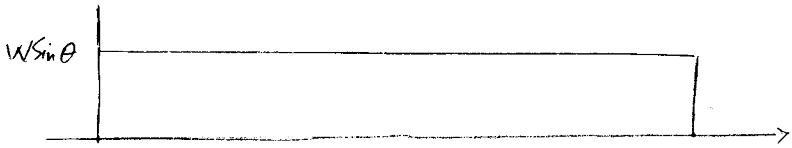
Shear Force Diagram

Axial Force Diagram.

$0 < x < L$



$$R_x^A + F_{xx} = 0 \quad \Rightarrow \quad F_{xx} = -R_x^A = W \sin\theta$$

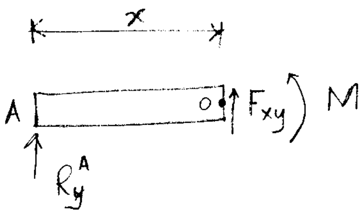


Shear Force Diagram

Bending Moment Diagram.

$0 < x < d$

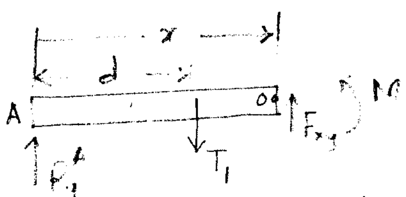
$\boxed{\sum {}^{0}M = 0}$



$$-R_y^A x + M = 0$$

$$M = R_y^A x = W \cos\theta \left(\frac{L}{d} - 1\right) \cdot x$$

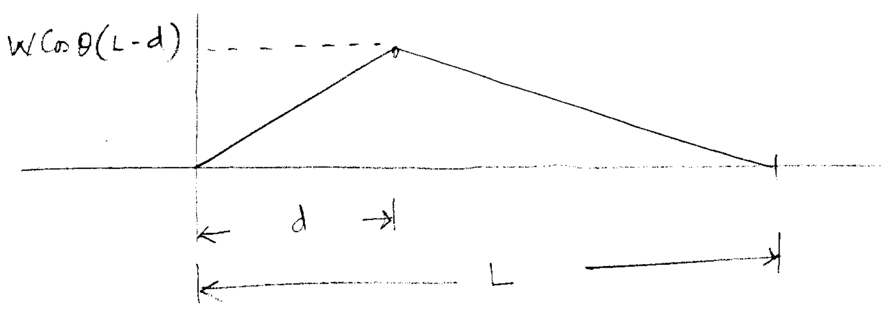
$d < x < L$



$\boxed{\sum {}^{0}M = 0}$

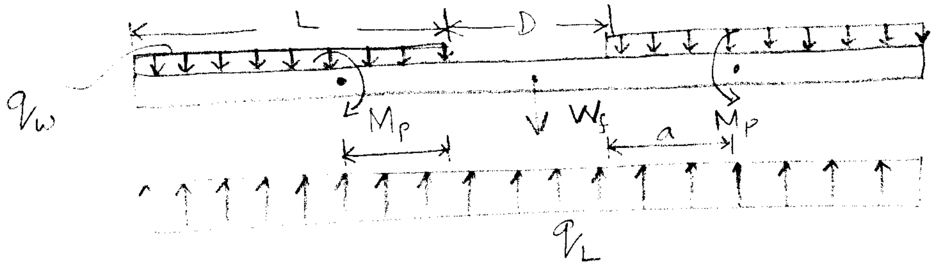
$$M + T_1(x - d) - R_y^A x = 0$$

$$M = R_y^A x - T_1(x - d)$$



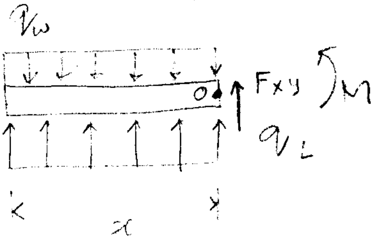
Bending Moment Diagram

PROBLEM 3. Idealizing the aircraft as a beam



$q_L > q_w$

$0 < x < (L-a)$



$\boxed{\sum F_y = 0}$

$$\int_0^x q_L\,dx - \int_0^x q_w\,dx + F_{xy} = 0$$

$$F_{xy} = -\int_0^x q_L\,dx + \int_0^x q_w\,dx$$

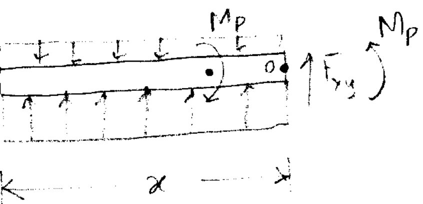
$$= -(q_L - q_w)\,x$$

$\boxed{\sum^{o} M = 0}$

$$\int_0^x q_w(x-\bar{x})\,d\bar{x} - \int_0^x q_L(x-\bar{x})\,d\bar{x} + M = 0$$

$$M = (q_L - q_w)\frac{x^2}{2}$$

$(L-a) < x < L$



$\boxed{\sum F_y = 0}$

$$\int_0^x q_L\,dx - \int_0^x q_w\,dx + F_{xy} = 0$$

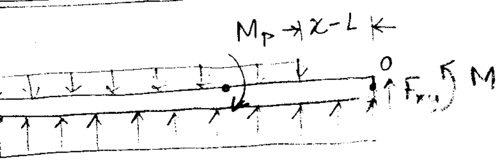
$$F_{xy} = -(q_L - q_w)\,x$$

$\boxed{\sum^{o} M = 0}$

$$\int_0^x q_w(x-\bar{x})\,d\bar{x} - \int_0^x q_L(x-\bar{x})\,d\bar{x} - M_p + M = 0$$

$$M = (q_L - q_w)\frac{x^2}{2} + M_p$$

$L < x < (L + D/2)$



$\boxed{\sum F_y = 0}$

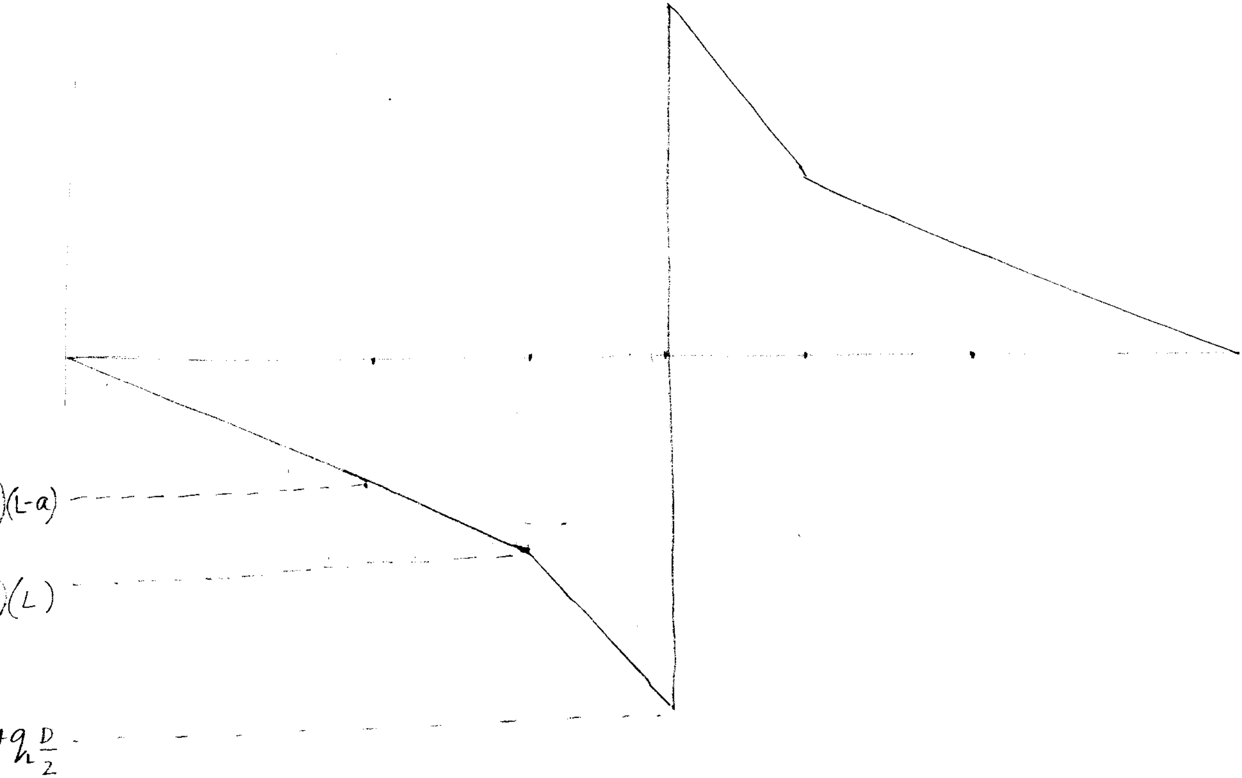
$$\int_0^x q_L\,dx + F_{xy} - \int_0^L q_w\,dx = 0$$

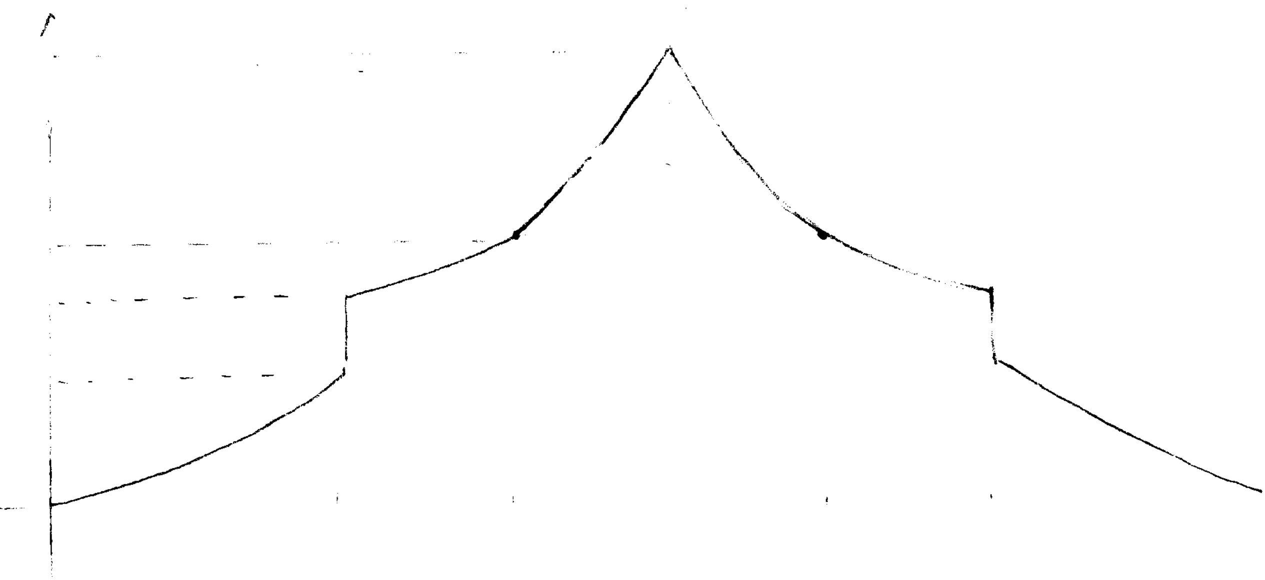
$$F_{xy} = -q_L x + q_w L$$

$\boxed{\sum^{o} M = 0}$

$$q_w L \cdot \frac{x}{2} - \int^x q_L(x-\bar{x})\,d\bar{x} - M_p + M = 0$$

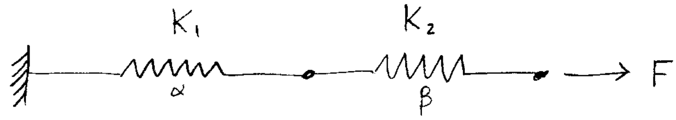
$$M = \frac{q_L x^2}{2} - q_w L\left(x - \frac{L}{2}\right) + M_p$$


(L-a)
(L)



PROBLEM 4

(a)

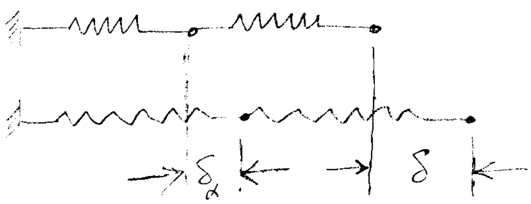


force in spring α = force in spring β = F —(1)

δ_α = elongation of α.

δ_β = elongation of β

δ = Total elongation



$\delta_\alpha + \delta_\beta = \delta$ — (2)

K_e = effective stiffness

$\Rightarrow$ $K_e \cdot \delta = F$ — (3)

from eq (1)

$K_1 \delta_\alpha = F \Rightarrow \delta_\alpha = F/K_1$

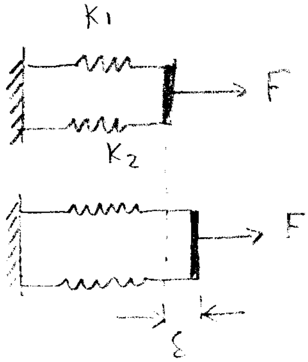
and $K_2 \delta_\beta = F \Rightarrow \delta_\beta = F/K_2$

from (3) $K_e = \dfrac{F}{\delta}$

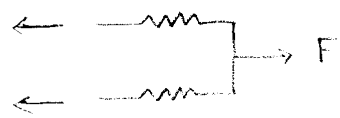
using (2) $K_e = \dfrac{F}{\delta_\alpha + \delta_\beta} = \dfrac{F}{F/K_1 + F/K_2}$

$K_e = \dfrac{K_1 K_2}{K_1 + K_2}$ $\Leftarrow$ Ans.

(b)



$\delta = \delta_\alpha = \delta_\beta$ geometric compatibility



FBD

$\Sigma F_x = 0$

$\Rightarrow$ $F = F_1 + F_2$ — (1)

$$\left.\begin{array}{l} F_1 = K_1 \delta_\alpha = K_1 \delta \\ F_2 = K_2 \delta_\beta = K_2 \delta \end{array}\right] — (2)$$

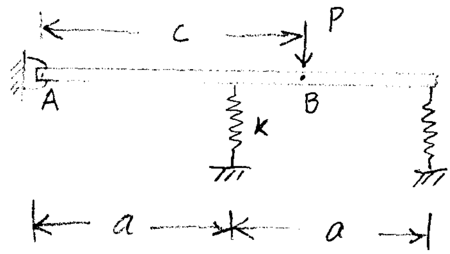
also $F = K_e \delta$ —(3)

using (2) and (3) in — (1).

$K_e \delta = K_1 \delta + K_2 \delta$

$\Rightarrow$ $K_e = K_1 + K_2$ $\Leftarrow$ Ans

PROBLEM 5. 2.19 from book

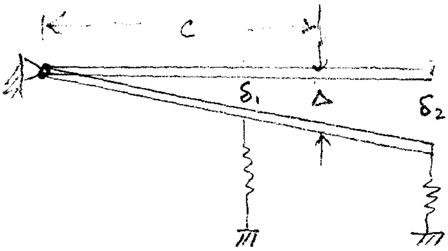


Let deflection at B = Δ

Then from problem statement

$$P = \frac{20}{9} K \cdot \Delta \qquad (1)$$

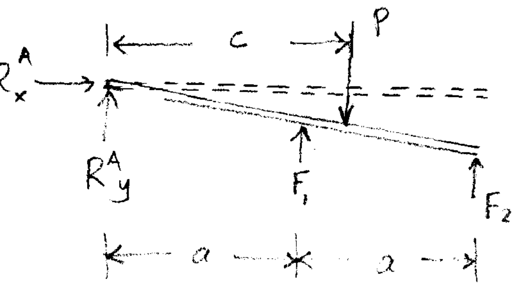
The beam is assumed to be rig



From similar triangles

$$\frac{\delta_1}{a} = \frac{\delta_2}{2a} \Rightarrow 2\delta_1 = \delta_2$$

(Geometric Compatibility)



From Equilibrium

$$\boxed{\Sigma^A M = 0}$$

$$F_1(a) + F_2(2a) - P(c) = 0$$

$$\Rightarrow F_1 a + 2F_2 a = Pc \qquad (3)$$

$$F_1 = K\delta_1 \quad \text{and} \quad F_2 = K\delta_2 \qquad (4)$$

using equations (4) and (1) in (3)

$$\cancel{K}\delta_1 a + 2\cancel{K}\delta_2 a = \frac{20\cancel{K}}{9} \cdot \Delta \cdot c \qquad (5)$$

also from geometric compatibility $\boxed{\frac{\Delta}{c} = \frac{\delta_1}{a} \Rightarrow \Delta = \frac{c}{a}\delta_1}$

Therefore eq. (5) becomes

$$\cancel{\delta_1} a + 2(2\cancel{\delta_1})a = \frac{20}{9} \cdot \frac{c}{a}\cancel{\delta_1} \cdot c$$

$$5a^2 = \frac{20}{9} c^2$$

$$\boxed{c = \sqrt{\frac{45}{20}}\, a} \Leftarrow \text{Solution}$$