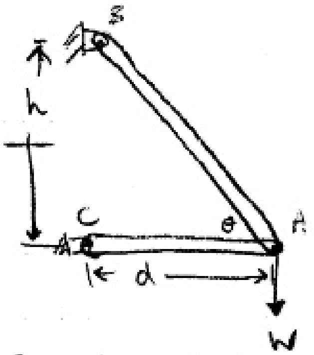
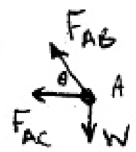
SOLUTIONS FOR PROBLEM SET 2

PROBLEM 1



Let us consider equilibrium about the pins to solve this problem:

Pin A:



$\Sigma F_x = 0: \; -F_{AB}\cos\theta - F_{AC} = 0$ (I)

$\Sigma F_y = 0: \; F_{AB}\sin\theta - W = 0$ (II)

From (II) → $\boxed{F_{AB} = \dfrac{W}{\sin\theta} = \dfrac{W(h^2+d^2)^{1/2}}{h}}$

From geometry:

$\tan\theta = \dfrac{h}{d}$

$\overline{AB}^2 = h^2 + d^2$

$\overline{AB} = (h^2+d^2)^{1/2}$

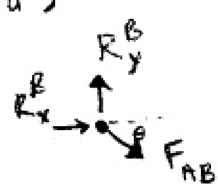
$\sin\theta = \dfrac{h}{(h^2+d^2)^{1/2}}$

From (I) → $F_{AC} = -F_{AB}\cos\theta$

Combining (I) (II) → $F_{AC} = -\dfrac{W}{\tan\theta}$

$\boxed{F_{AC} = \dfrac{-Wd}{h}}$

Pin B:



* Note: F_{AB} on Pin B acts equal and opposite than the direction shown on Pin A. This is the nature of internal forces in Two-Force Members

$\Sigma F_x = 0: \; R_x^B - F_{AB}\cos\theta = 0$ (III)

$\Sigma F_y = 0: \; R_y^B - F_{AB}\sin\theta = 0$ (IV)

From (III) → $R_x^B = -F_{AB}\cos\theta$

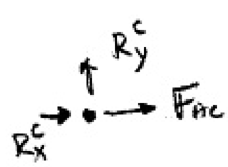
Combine (I) & (III) → $\boxed{R_x^B = -\dfrac{W}{\tan\theta} = -\dfrac{Wd}{h}}$

* NOTE: $R_y^B = W$, which would result if a Free Body Diagram was done of the entire truss structure

From (IV) → $R_y^B = F_{AB}\sin\theta$

Combine (II) & (IV) → $\boxed{R_y^B = W}$

Pin C:



$\Sigma F_x = 0: \; F_{AC} + R_x^C = 0$ (V)

$\Sigma F_y = 0: \; R_y^C = 0$ (VI)

From (V) → $\boxed{R_x^C = -F_{AC} = \dfrac{Wd}{h}}$

What dimensions (d ; h) would minimize these loads?

Considering that $\tan\theta = \frac{h}{d}$; $\sin\theta = \frac{h}{(h^2+d^2)^{1/2}}$

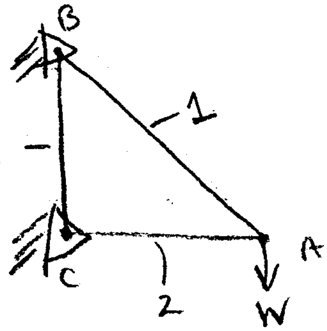
AND that $R_x^B = -\frac{W}{\tan\theta}$; $F_{AB} = \frac{W}{\sin\theta}$

we see that As $d >> h$, $\theta \to 0$ and $R_x^B \to -\infty$; $F_{AB} \to \infty$

As $h >> d$, $\theta \to 90°$ and $R_x^B \to 0$; $F_{AB} \to W$

Therefore $\theta = 90°$ is the most optimal position ; $h >> d$.

Part B:



1) Each Joint allows 2 degrees of Freedom:

$$2 \times \underset{\#\text{ of Joints}}{3} = 6 \text{ DOF}$$

2) B ; C are constrained in 2 directions each:

$$2 \times 2 = 4$$

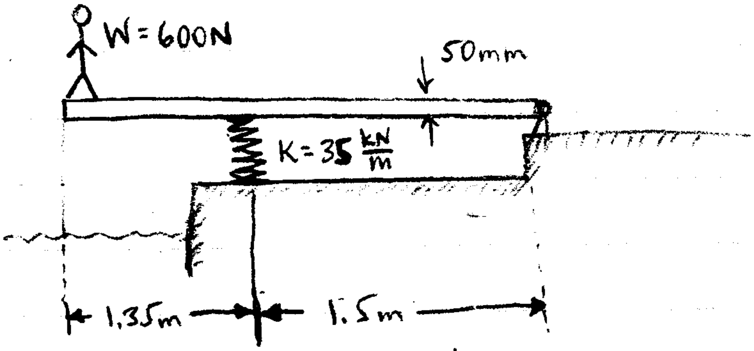
\# of directions   \# of constraints

3) Members provide additional constraints each

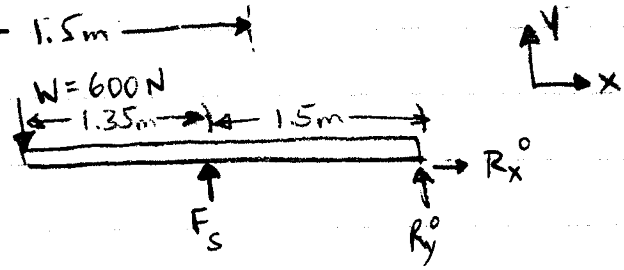
Total Members = 3

$$\underbrace{6 \text{ DOF}}_{\text{Motion allowed}} - \underbrace{7 \text{ constraints}}_{\text{Motion Prohibited}} = -1 \Rightarrow \text{STATICALLY INDETERMINATE}$$

Problem 2: CDL 2.1



FBD:



$\Sigma {}^{O}M = 0$: $-F_s(1.5m) + (600N)(2.85m) = 0$

$F_s = 1140N$

$F_s = k\delta$

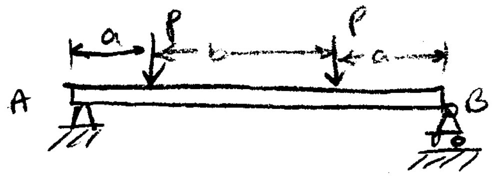
$1140N = 35\ kN/m \cdot \delta$

$1.14kN = 35\ kN/m \cdot \delta$

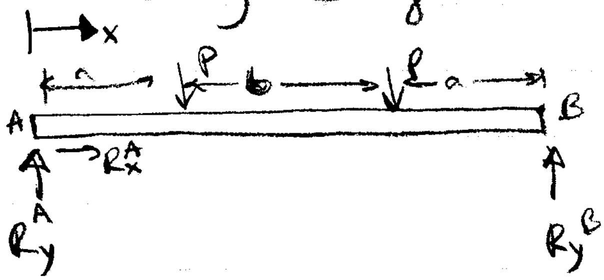
$\delta = 0.0326m = \underline{32.6\ mm}$

The deflection would be slightly less is the board was extremely rigid.

Problem 3: CDL 3.5



Draw the Free Body Diagram:



$\Sigma F_x = 0: \quad R_x^A = 0$ I

$\Sigma F_y = 0: \quad R_y^A + R_y^B - 2P = 0$ II

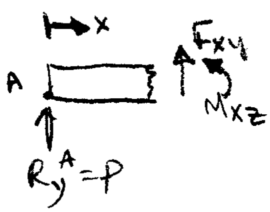
$\Sigma M^A = 0: \quad Pa + P(a+b) - (2a+b)R_y^B = 0$

$P\cancel{(2a+b)} - \cancel{(2a+b)}R_y^B = 0$

III $\underline{P = R_y^B}$ (Can be determined using the symmetry argument)

From II & III, we find that $\underline{R_y^A = P}$ IV

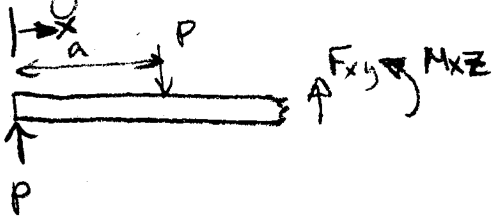
In the region where $0 < x < a$:



$\Sigma F_y = 0: \quad \underline{F_{xy} = -P}$ V

$\Sigma M^A = 0: \quad F_{xy} \cdot x + M_{xz} = 0 \quad \underline{M_{xz} = Px}$ VI

In the region where $a < x < a+b$:



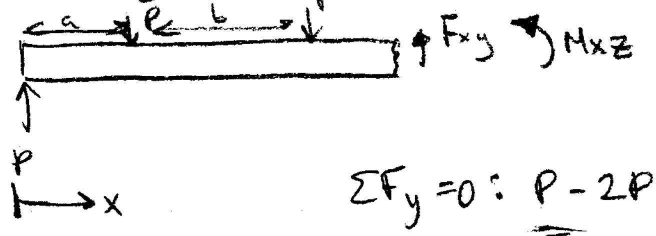
$\Sigma F_y = 0: \quad F_{xy} - P + P = 0 \quad \underline{F_{xy} = 0}$ VII

$\Sigma M_{xz} = 0: \quad -Pa + F_{xy} \cdot x + M_{xz} = 0$

$M_{xz} = Pa$ VIII

$a+b < x < 2a+b$

Approach 1: looking at beam from left to right



$$\Sigma F_y = 0: \; P - 2P + F_{xy} = 0$$

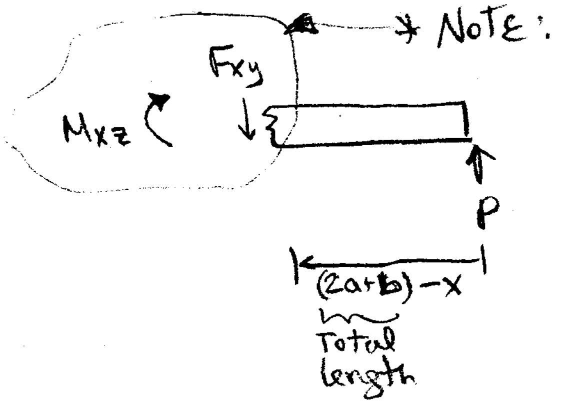
$$F_{xy} = P \qquad \text{IX}$$

$$\Sigma \overset{A}{M} = 0: \; -Pa - P(a+b) + F_{xy} \cdot x + M_{xz} = 0$$

$$M_{xz} = 2Pa + Pb - Px$$

$$M_{xz} = P(2a+b-x)$$

Approach 2: looking at beam from right to left



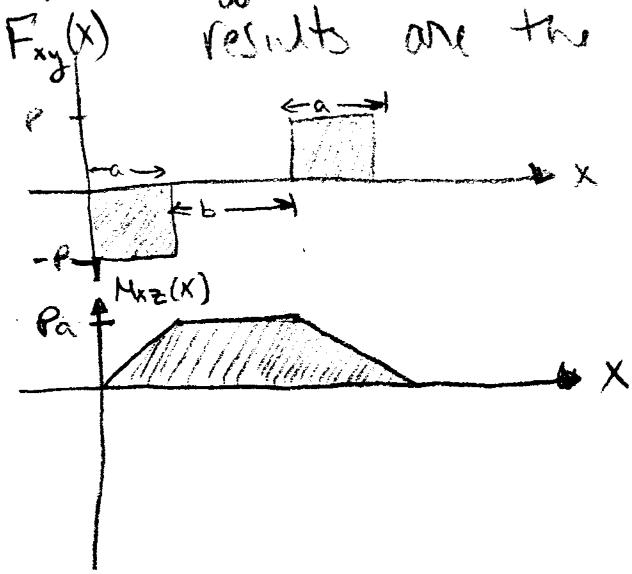
* NOTE: EQUAL & OPPOSITE TO THE FORCES SHOWN ABOVE

$$\Sigma F_y = 0: \quad F_{xy} = P \qquad \text{XI}$$

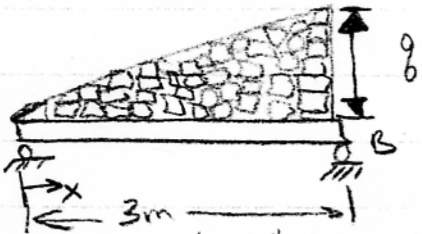
$$\Sigma \overset{B}{M}_z = 0: \quad -M_{xz} + F_{xy}(2a+b-x) = 0$$

$$M_{xz} = P(2a+b-x) \qquad \text{XII}$$

Note: Approach 2 is simpler than Approach 1 and results are the same.



Problem 4: CDL 3.10



TOTAL LOAD = 400 · 60 = 24 kN, however, this is a distributed load.

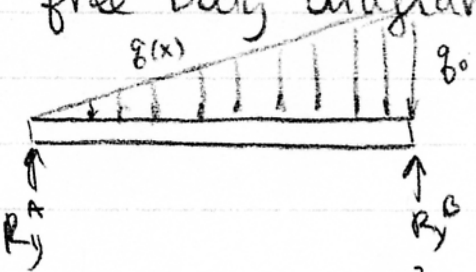
Let us determine the magnitude q_0:

$$\text{Total Load} = \int_0^l \frac{q_0 x}{l}\,dx = \int_0^3 \frac{q_0 x}{3}\,dx$$

$$24\,kN = \left.\frac{q_0 x^2}{6}\right|_0^3 = \frac{3}{2} q_0 \Rightarrow q_0 = 16\,kN$$

$$\boxed{q(x) = \frac{16x}{3}}$$

Now, do a free body diagram:



$\Sigma F_y = 0$: $R_y^A + R_y^B = \int_0^3 q(x)\,dx$

$$R_y^A + R_y^B = \int_0^3 \frac{16x}{3}\,dx = \left.\frac{8}{3}x^2\right|_0^3 = 24\,kN$$

$\overset{A}{\Sigma} M = 0$: $3 \cdot R^B_y - \int_0^3 q(x) \cdot x\,dx = 0$

$$3 \cdot R_y^B - \int_0^3 \frac{16x^2}{3}\,dx = 0$$

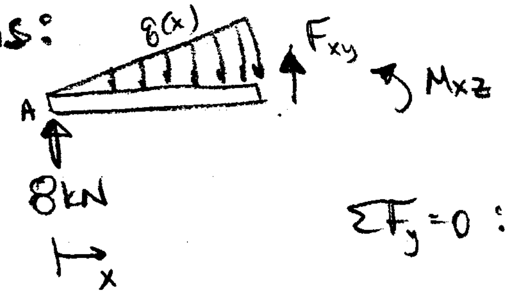
$$3 \cdot R_y^B = \left.\frac{16}{9}x^3\right|_0^3 = 48 \Rightarrow \boxed{R_y^B = 16\,kN}$$

From $24 = R_y^A + R_y^B \rightarrow \boxed{R_y^A = 8\,kN}$

⑦

Problem 4 (cont'd)

Determine Shear force & Bending Moment Expressions:



$$\Sigma F_y = 0: \quad F_{xy} - \int_0^x q(x')\,dx' + 8 = 0$$

$$F_{xy} = 8 - \int_0^x \frac{16x'}{3}\,dx' = \boxed{8 - \frac{8}{3}x^2}$$

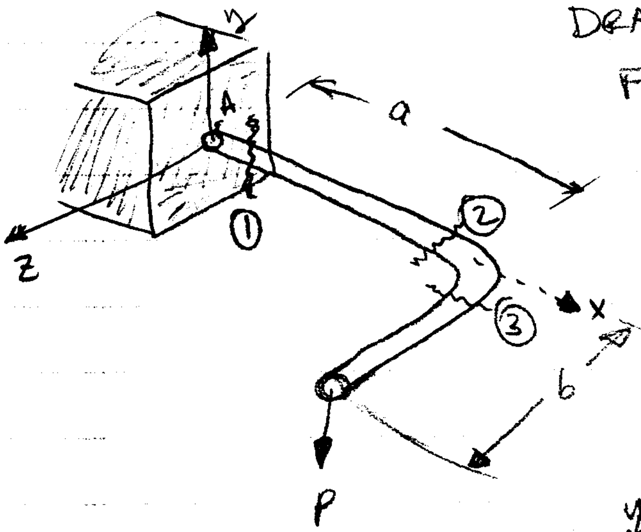
$$\overset{A}{\Sigma} M = 0: \quad F_{xy} \cdot x + M_{xz} - \int_0^x q(x') \cdot x'\,dx' = 0$$

$$\left(8 - \frac{8}{3}x^2\right)x + M_{xz} - \int_0^x \frac{16(x')^2}{3}\,dx'$$

$$M_{xz} = \frac{16}{9}x^3 + \frac{8}{3}x^3 - 8x$$

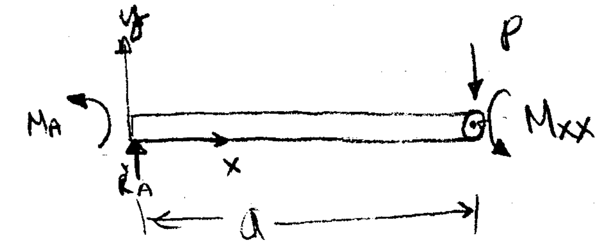
$$\boxed{M_{xz}(x) = \frac{40}{9}x^3 - 8x}$$

Problem 5: 3.13



DRAW SKETCHES SHOWING INTERNAL FORCES & MOMENTS ACTING AT SECTIONS 1, 2, & 3 :

FBD of System:

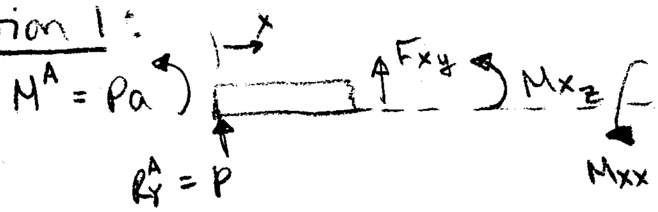


$\Sigma F_y = 0 \quad R^A_y = P$

$\Sigma M^A = 0 \ : \ M^A = Pa$

Reactions

Section 1:



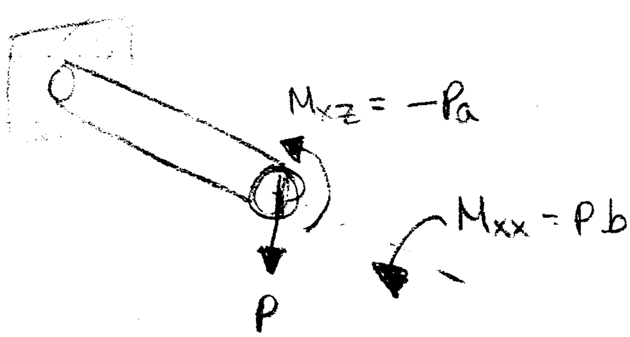
$\underline{F_{xy} = -P} \ : \ \Sigma F_y$

$M_{xz} + F_{xy} \cdot x + Pa = 0$

$M_{xz} = -Pa + Px$

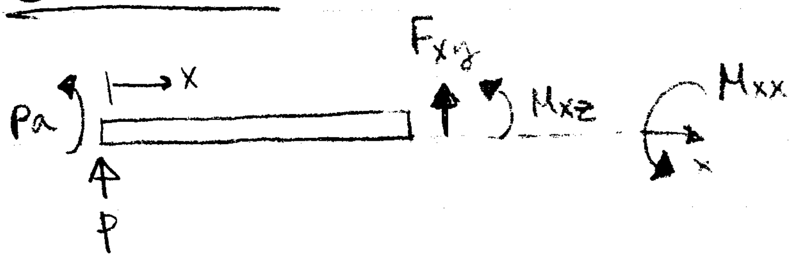
$= P(x-a)$

$x_1 \sim 0 \rightarrow \ = -Pa$



CDL 3.13 (cont'd)

Section 2

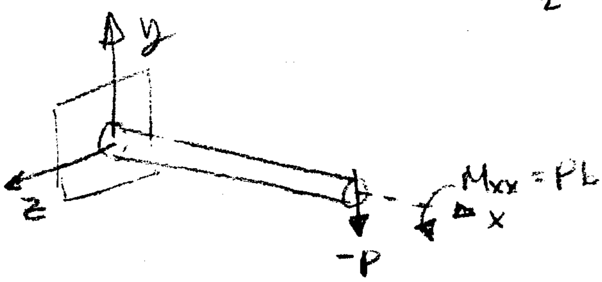


$$\Sigma F_y = 0: \quad P + F_{xy} = 0 \qquad F_{xy} = -P$$

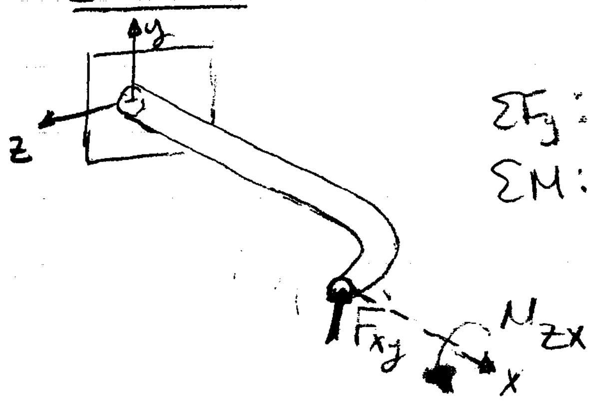
$$\Sigma M_z = 0: \quad Pa + F_{xy}X + M_{xz} = 0$$

$$M_{xy} = P(x-a)$$

$$x_2 \sim a \Rightarrow M_{xy} = 0$$



Section 3

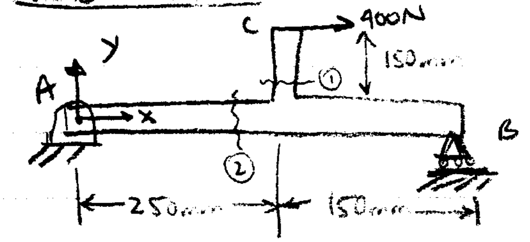


$$\Sigma F_y: \quad F_{xy} = -P$$

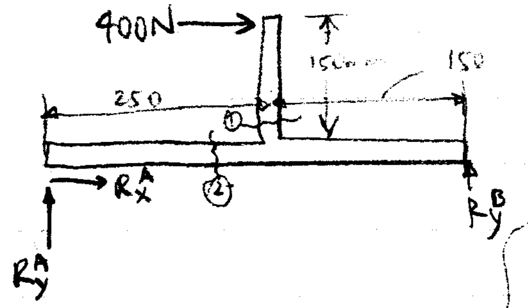
$$\Sigma M: \quad -M_{zx} + Pb = 0$$

$$\underline{M_{zx} = Pb}$$

Problem 6: CDL 3-14



Calculate Internal Forces & Moments @ Sections 1 & 2:



$\Sigma F_x = 0: R_x^A + 400N = 0$

$\boxed{R_x^A = -400N}$

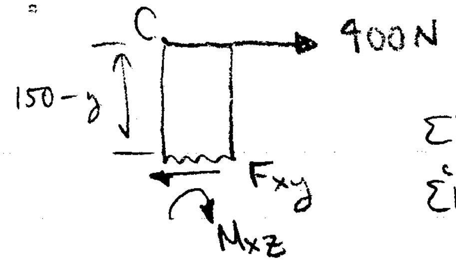
$\Sigma F_y = 0: R_y^A + R_y^B = 0$

$\Sigma M^A = 0: (400)(150) - R_y^B(400) = 0$

$\boxed{R_y^B = 150N}$

$\therefore \boxed{R_y^A = -150N}$

Section ①:



$\Sigma F_x = 0: \quad F_{xy} = 400N$

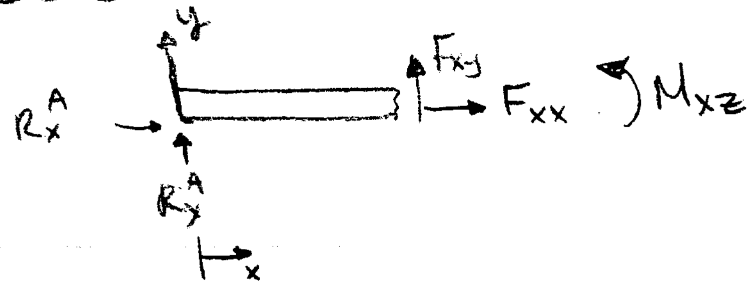
$\Sigma M = 0: \quad F_{xy}(150-y) + M_{xz} = 0$

$M_{xz} = -F_{xy}(150-y)$

$= -400(150-y)$

$y \sim 0 \implies \underline{M_{xz} = -60N \cdot m}$

Section ②:



$\Sigma F_x = 0: \quad R_x^A + F_{xx} = 0 \implies F_{xx} = 400N$

$\Sigma F_y = 0: \quad R_y^A + F_{xy} = 0 \implies F_{xy} = 150N$

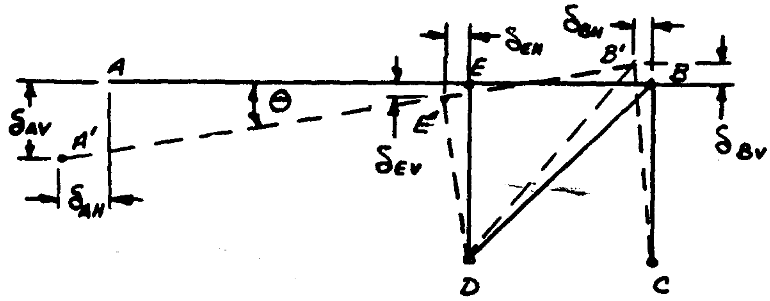
$\Sigma M^A = 0: \quad F_{xy} \cdot x + M_{xz} = 0 \implies M_{xz} = -150x$

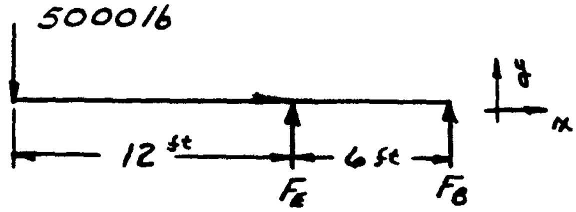
$x \sim 250mm \implies \boxed{M_{xz} = 37.5Nm}$

EXTRA CREDIT

2-6

GEOMETRY OF DEFORMATION





$\Sigma F_y = 0$

$F_E + F_B - 5000 = 0$

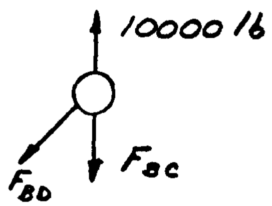
$+\circlearrowleft \Sigma M_E = 0$

$(5\times10^3)(12\,ft) + F_B\,(6\,ft) = 0$

$F_B = -10000\ lb$

$F_E = 15000\ lb$

FOR THE PIN B:



$\Sigma F_x = 0$

$F_{BD} = 0$

$\Sigma F_y = 0$

$F_{BC} = 10000\,lb$ TENSION

BY A SIMILIAR ANALYSIS $F_{DE} = 15000$ COMP.

ROD BD DOES NOT DEFORM. IT ONLY ROTATES ABOUT D. THUS $\delta_{BH} = \delta_{BV}$ (FROM GEOMETRY)

$$\delta_{BV} = \left(\frac{F\ell}{AE}\right)_{BC} = \frac{(10^4\ lb)(6\,ft)(12\,\frac{in}{ft})}{(1\,in^2)\ (30\times10^6\,\frac{lb}{in^2})} = .024\ in$$

$$\delta_{EV} = \left(\frac{F\ell}{AE}\right)_{ED} = \frac{(1.5\times10^4\,lb)(6\,ft)(12\,\frac{in}{ft})}{(2\,in^2)\ (30\times10^6\,\frac{lb}{in^2})} = .018\,in$$

FOR E'B' TO EQUAL EB, $\delta_{EH} \approx \delta_{BH}$ BUT $\delta_{BH} = \delta_{BV} = .024\,in$

$\delta_{EH} \approx .024\ in$

SIMILARLY $\delta_{AH} = .024\,in$

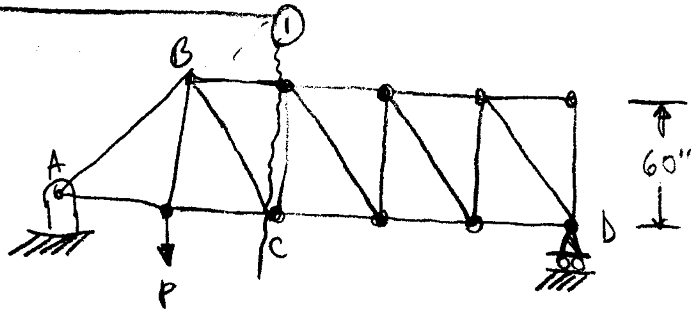
$$\theta \approx \frac{\delta_{BV} + \delta_{EV}}{72\ in} = \frac{.042}{72}\ RAD$$

$$\delta_{AV} = -\delta_{BV} + \theta\times(18\,ft)(12\,\frac{in}{ft}) = .126\,in - .024\,in$$
$$= .102\ in$$

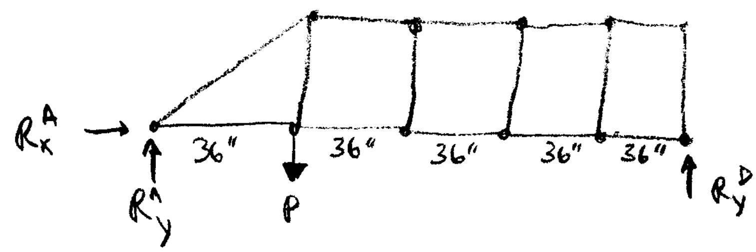
$\delta_{AH} = .024\,in$

EXTRA CREDIT

2.9



Determine Reactions from FBD:



$\Sigma F_y = 0: \quad R_y^A - P + R_y^D = 0$

$R_y^A + R_y^D = P = 100{,}000 \text{lb}$

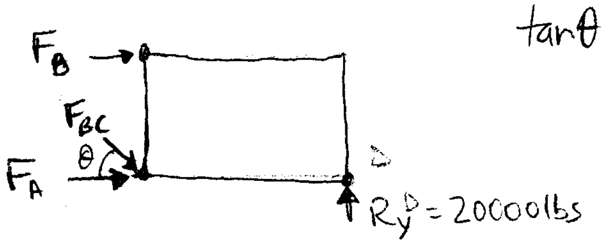
$\Sigma M^A = 0: \quad -36P + R_y^D \cdot 180 = 0$

$R_y^D = \frac{P}{5} = 20000 \text{lb}$

$R_y^A = 80000 \text{lb}$

Consider the section to the Right of Cut 1:
(TREAT IT AS A BEAM)

$\tan\theta = \frac{5}{3} \qquad \theta \approx 59^\circ$



$\Sigma F_y = 0$:

$-F_{BC} \sin\theta + 20000 \text{ lbs} = 0$

$$\boxed{F_{BC} = \frac{20000}{\sin(59)} = 23324 \text{lbs}}$$