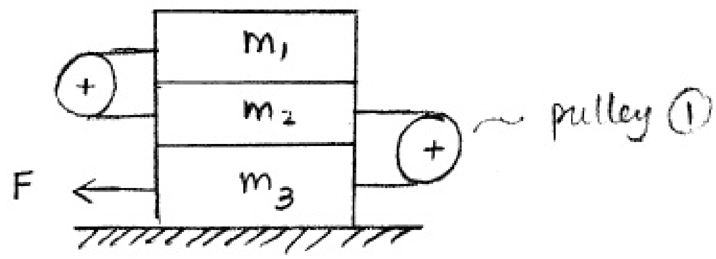
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Mechanical Engineering

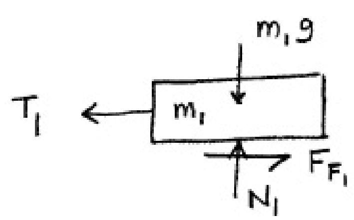
2.001 - Mechanics of Materials I
Spring, 2003

Solutions for Quiz # 1

Problem 1



a) Draw FBD of each mass and write down eq. of equilibrium

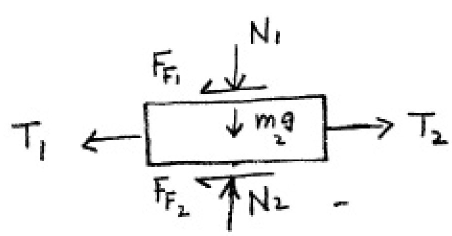


ΣF_y : $N = m_1 g$

$\Rightarrow F_{F_1} = \mu m_1 g$

ΣF_x : $-T_1 + F_{F_1} = 0$

$\Rightarrow T_1 = \mu m_1 g$ — ①



ΣF_y: $N_2 = N_1 + m_2 g$

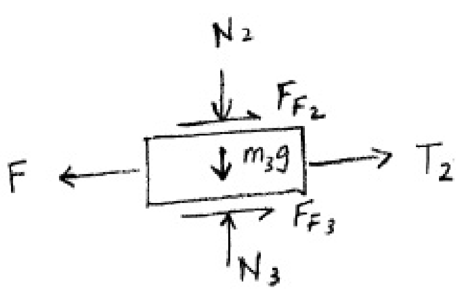
$= (m_1 + m_2) g$

$\Rightarrow F_{F_2} = \mu (m_1 + m_2) g$

ΣF_x : $T_2 = T_1 + F_{F_1} + F_{F_2}$ use ①

$= \mu m_1 g + \mu m_1 g + (m_1 + m_2) \mu g$

$T_2 = 3 \mu m_1 g + \mu m_2 g$ — ②



ΣF_y: $N_3 = N_2 + m_3 g = (m_1 + m_2 + m_3) g$

$F_{F_3} = \mu (m_1 + m_2 + m_3) g$

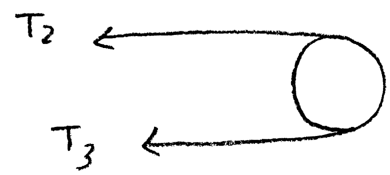
ΣF_x : $F = T_2 + F_{F_2} + F_{F_3}$

$\Rightarrow$

$$F = 3\mu m_1 g + \mu m_2 g + \mu(m_1 + m_2)g + \mu(m_1 + m_2 + m_3)g$$

$$\boxed{F = 5\mu m_1 g + 3\mu m_2 g + \mu m_3 g}.$$

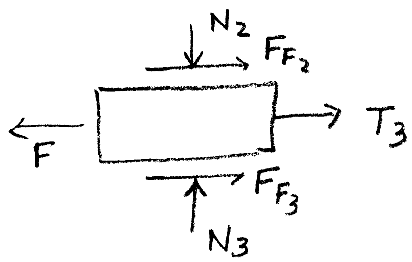
(b) Pulley ① freezes.



$$T_2 = T_3 e^{\mu_1 \pi}$$

$$\Rightarrow T_3 = T_2 e^{-\mu_1 \pi}.$$

Redrawing the FBD of m_3



ΣF_x :

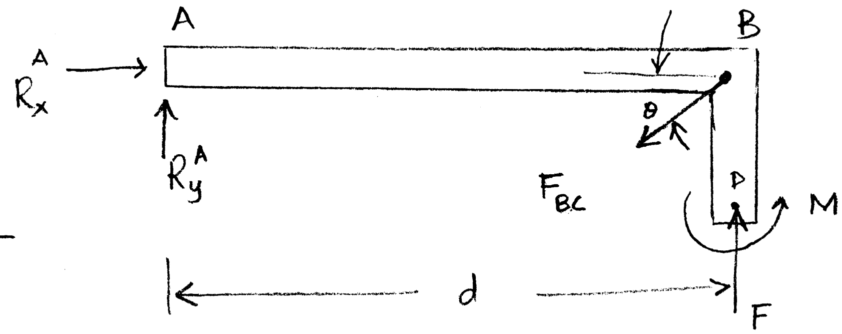
$$F = T_3 + F_{F_2} + F_{F_3}$$

$$F = T_2 e^{-\mu_1 \pi} + \mu(m_1 + m_2)g + \mu(m_1 + m_2 + m_3)g$$

$$F = (3\mu m_1 g + \mu m_2 g)e^{-\mu_1 \pi} + 2\mu(m_1 + m_2)g + \mu m_3 g$$

PROBLEM 2

(a) Draw the FBD of the member ACD



Assumming two force member BC is in tension

$$\theta = \tan^{-1}\frac{h}{d}$$

$\underline{\Sigma^B M}$:

$$M - R_y^A \cdot d = 0$$

$$R_y^A = M/d \qquad \text{——— (1)}$$

$\underline{\Sigma F_y}$:

$$R_y^A + F - F_{BC}\sin\theta = 0$$

$$F_{BC} = \frac{F + M/d}{\sin\theta}$$

$\underline{\Sigma F_x}$:

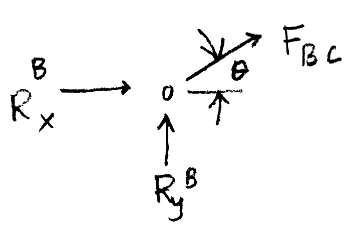
$$R_x^A - F_{BC}\cos\theta = 0$$

$$R_x^A = F_{BC}\cos\theta$$

$$= \frac{F + M/d}{\sin\theta} \cdot \cos\theta$$

$$R_x^A = \frac{F + M/d}{\tan\theta} \qquad \text{——— (2)}$$

Draw FBD of Support B



$\underline{\Sigma F_x}$:

$$R_x^B + F_{BC}\cos\theta$$

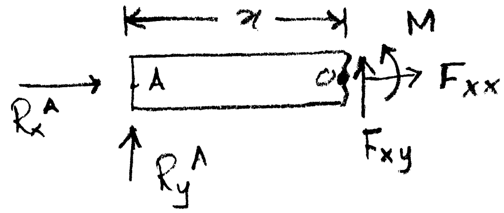
$$R_x^B = -\frac{F + M/d}{\tan\theta}$$

$\underline{\Sigma F_y}$:

$$R_y^B + F_{BC}\sin\theta = 0$$

$$\Rightarrow R_y^B = -(F + M/d)$$

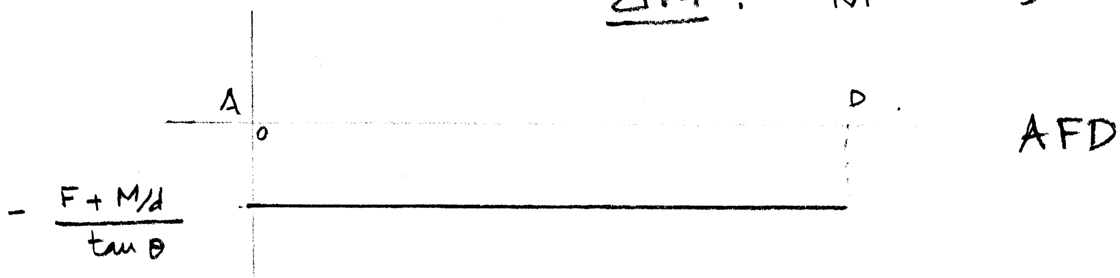
(b)

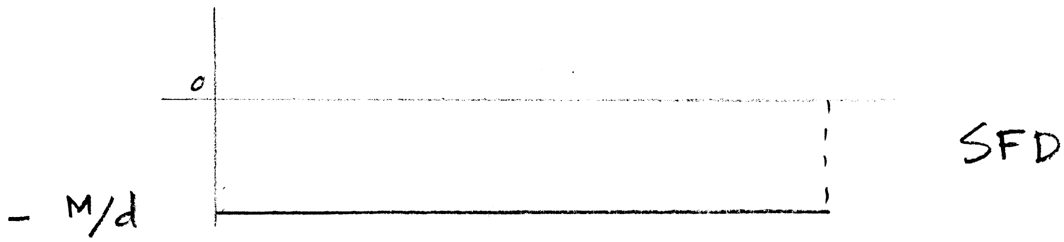


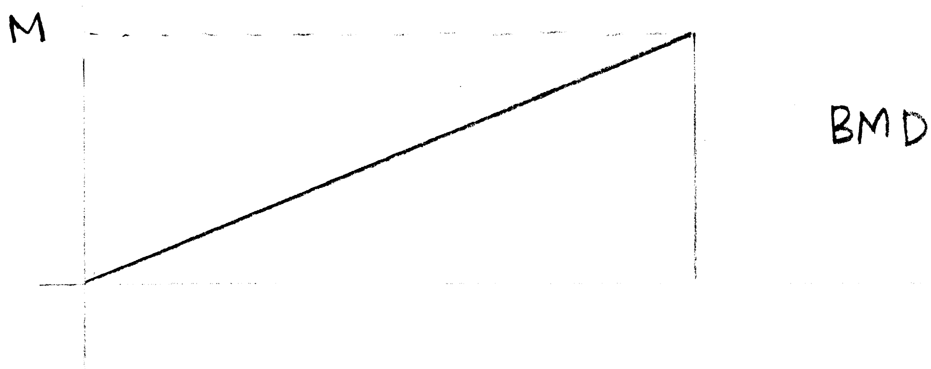
$\underline{\sum F_x}$: $F_{xx} = -R_x^A$

$\underline{\sum F_y}$: $F_{xy} = -R_y^A$

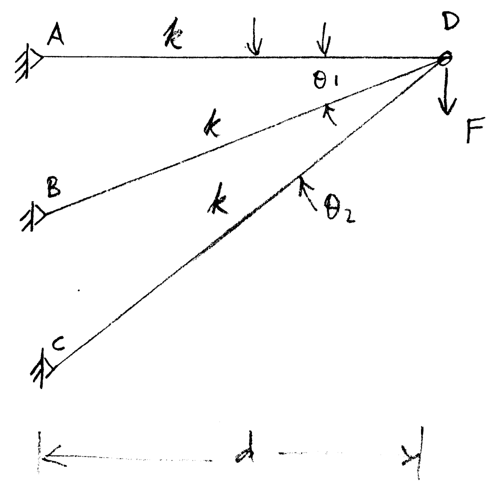
$\underline{\sum^{O} M}$: $M = R_y^A \cdot x$



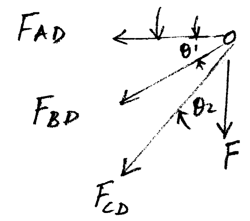




Problem 3



Equilibrium of Joint D.



$$\Sigma F_x: \quad -F_{AD} - F_{BD}\cos\theta_1 - F_{CD}\cos\theta_2 = 0 \quad (1)$$

$$\Sigma F_y: \quad -F_{BD}\sin\theta_1 - F_{CD}\sin\theta_2 - F = 0 \quad (2)$$

Force Deformation Equations

$$\delta_{AD} = \frac{F_{AD}}{k} \Rightarrow F_{AD} = k\,\delta_{AD} \quad (3)$$

$$\delta_{BD} = \frac{F_{BD}}{k} \Rightarrow F_{BD} = k\,\delta_{BD} \quad (4)$$

$$\delta_{CD} = \frac{F_{CD}}{k} \Rightarrow F_{CD} = k\,\delta_{CD} \quad (5)$$

Compatibility conditions

$$\delta_{AD} = (u_x^D - \cancel{u_x^A})\cos 0 + (u_y^D - u_y^A)\sin 0$$

$$= u_x^D \quad (6)$$

$$\delta_{BD} = (u_x^D - u_x^B)\cos(\theta_1) + (u_y^D - u_y^B)\sin\theta_1$$

$$= u_x^D\cos\theta_1 + u_y^D\sin\theta_1 \quad (7)$$

$$\delta_{CD} = (u_x^D - u_x^C)\cos(\theta_2) + (u_y^D - u_y^C)\sin\theta_2$$

$$= u_x^D\cos\theta_2 + u_y^D\sin\theta_2 \quad (8)$$

(b) $d = 1$ $\theta_1 = 30°$ $\theta_2 = 60°$

eq. 6, 7 and 8 become.

$$\left.\begin{aligned} \delta_{AD} &= u_x^D \\ \delta_{BD} &= u_x^D \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot u_y^D \\ \delta_{CD} &= \frac{1}{2} u_x^D + \frac{\sqrt{3}}{2} u_y^D \end{aligned}\right\}$$ use these in eq. 3, 4 and F

$$\left.\begin{aligned} F_{AD} &= K u_x^D \\ F_{BD} &= K\left(u_x^D \frac{\sqrt{3}}{2} + \frac{1}{2} u_y^D\right) \\ F_{CD} &= K\left(\frac{1}{2} u_x^D + \frac{\sqrt{3}}{2} u_y^D\right) \end{aligned}\right\}$$ use These in 1 and 2

putting in 1

$$-\cancel{K} u_x^D - \cancel{K}\left(u_x^D \frac{\sqrt{3}}{2} + \frac{1}{2} u_y^D\right)\frac{\sqrt{3}}{2} - \cancel{K}\left(\frac{1}{2} u_x^D + \frac{\sqrt{3}}{2} u_y^D\right)\frac{1}{2} = 0$$

$$-u_x^D - \frac{3}{4} u_x^D - \frac{1}{4} u_x^D - \frac{\sqrt{3}}{4} u_y^D - \frac{\sqrt{3}}{4} u_y^D = 0$$

$$-2 u_x^D - \frac{\sqrt{3}}{2} u_y^D = 0 \quad \text{—— (A)}$$

From eq 2

$$-K\left(u_x^D \frac{\sqrt{3}}{2} + \frac{1}{2} u_y^D\right)\frac{1}{2} - K\left(\frac{1}{2} u_x^D + \frac{\sqrt{3}}{2} u_y^D\right)\frac{\sqrt{3}}{2} - F = 0$$

$$\Rightarrow \frac{F}{K} = -u_x^D \frac{\sqrt{3}}{4} - u_x^D \frac{\sqrt{3}}{4} - u_y^D \cdot \frac{1}{4} - u_y^D \frac{3}{4}$$

$$\Rightarrow -\frac{\sqrt{3}}{2} u_x^D - u_y^D = F/K \quad \text{—— (B)}$$

$$u_y^D = -\frac{\sqrt{3}}{2} u_x^D - \frac{F}{K} \quad \text{—— (C)}$$

put in (A)

$$-2u_x^D - \frac{\sqrt{3}}{2}\left(-\frac{\sqrt{3}}{2}u_x^D - \frac{F}{K}\right) = 0$$

$$-2u_x^D + \frac{3}{4}u_x^D + \frac{\sqrt{3}}{2}\frac{F}{K} = 0$$

$$-\frac{5}{4}u_x^D + \frac{\sqrt{3}}{2}\frac{F}{K} = 0$$

$$u_x^D = \sqrt{3}\frac{F}{K} \times \frac{2}{5}$$

$$= \frac{2\sqrt{3}}{5} \cdot \frac{F}{K} \quad \text{—— sol}$$

put in C

$$u_y^D = -\frac{\sqrt{3}}{2}\left(\frac{2}{5}\sqrt{3}\frac{F}{K}\right) - \frac{F}{K}$$

$$= -\frac{3}{5}\frac{F}{K} - \frac{F}{K}$$

$$= -\frac{8}{5}\frac{F}{K} \quad \text{—— sol}$$