MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Mechanical Engineering

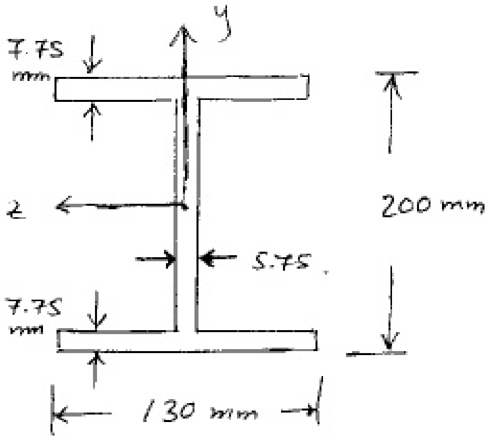
2.001 - Mechanics of Materials I
Spring, 2003

Solutions for Problem Set 9

Problem 1

7.8 CDL

Calculate the moment of inertia I_{zz} for the beam cross-section.



This section is symmetric about both y and z axis. The neutral axis coincides with the z-axis in the figure.

The section consists of three parts one vertical "web" and two horizontal flanges. We calculate moments of inertia of each part independently, then transfer these to the centroidal axis by "Parallel Axis Theorem" (See book pg. 430, section 7.5)

Web.

$$I_{web} = 5.75 \times (200 - 2\times 7.75)^3/12$$
$$= 3009370.85 \text{ mm}^4$$

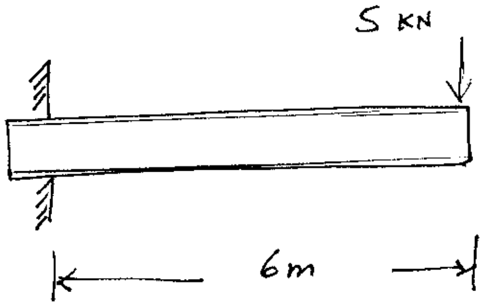
Flanges

$$I_{flange} = 130 \times 7.75^3/12 = 5042.75 \text{ mm}^4$$

$$I_z = 3009370.85 + 2\times 5042.75 + 2\times\left[7.75\times 130 \times \left(100 - \frac{7.75}{2}\right)^2\right]$$
$$= 21638087.83 \text{ mm}^4 = 2.164\times 10^7 \text{ mm}^4.$$

Problem 2.

7.14 CDL.



The section of the beam is the one in previous problem (7.18)

$$M_{max} = -5 \times 6 = -30 \text{ KN}\cdot\text{m}$$
$$= -30 \times 10^3 \times 10^3 \text{ N}\cdot\text{mm}$$
$$= -3 \times 10^7 \text{ N}\cdot\text{mm}.$$

$$\sigma_{bending} = -\frac{My}{I}$$

$y = c$ for maximum stress.

$c = 100$ mm.

$$\sigma\Big|_{c=100} = -\frac{(-3\times10^7)(100)}{2.164\times10^7}$$

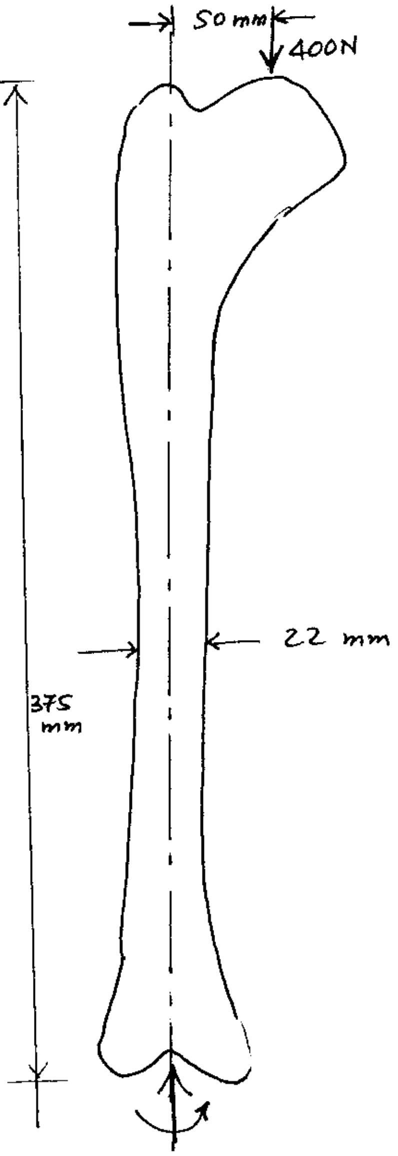
$$= 138.6 \text{ N/mm}^2 \quad (\text{tensile}).$$

$$\sigma\Big|_{c=-100} = -\frac{(-3\times10^7)(-100)}{2.164\times10^7}$$

$$= -138.6 \text{ N/mm}^2 \quad (\text{compressive})$$

Problem 3

7.20 CDL



a) Distribution of stress at section BB assuming that the circular section is solid.

Axial force at BB

$$F = 400 \ N.$$

Moment at BB

$$M = 400 \times 50 = 20{,}000 \ N \cdot mm$$

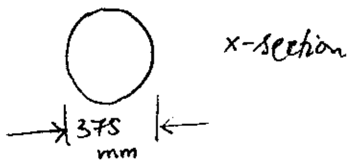
This force and moment is together producing stresses in the bone at BB.

Axial stress $\sigma_{axial} = \dfrac{F}{A}$

Bending stress $\sigma_{bending} = \dfrac{M \cdot y}{I}$.

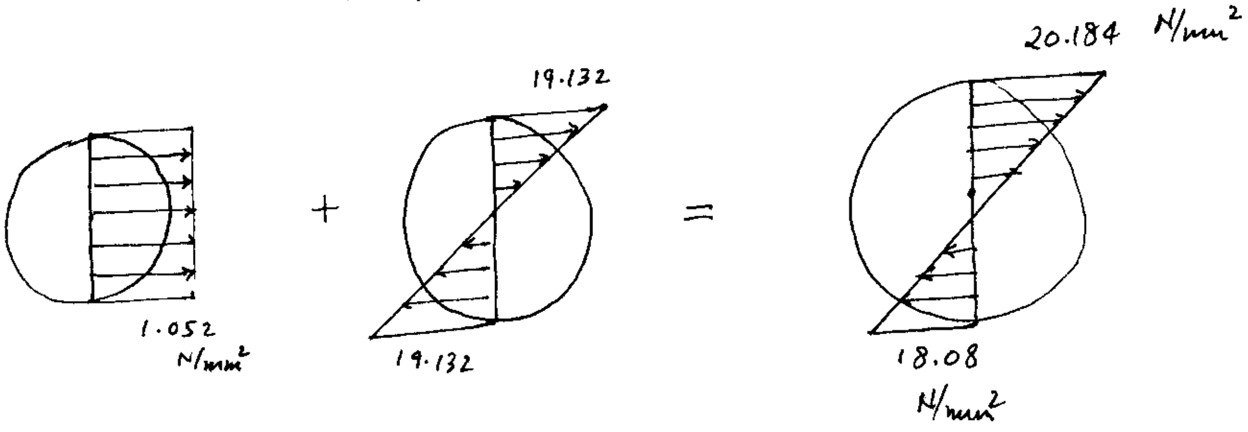
$$A = \frac{\pi}{4}(22)^2 = 380.13 \ mm^2$$

$$I = \frac{\pi}{64}(22)^4 = 11{,}499 \ mm^4$$



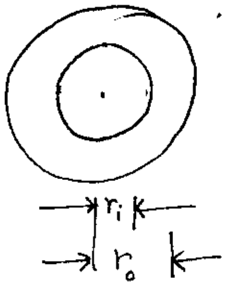
$$\sigma_{axial} = \frac{400}{380.13} = 1.052 \ N/mm^2$$

$$\sigma_{bending} = \frac{20{,}000 \times 11}{11\,499} = 19.132 \ N/mm^2$$



Notice that "neutral" axis has shifted down from centroidal axis.

(b) The cross-section is $\quad r_i = \frac{r_o}{2} \qquad r_o = \frac{22}{2} = 11$ mm.



$\Rightarrow r_i = \frac{11}{2} = 5.5$ mm.

$$A = \frac{\pi}{4}\left(d_o^2 - d_i^2\right) = \pi\left(r_o^2 - r_i^2\right)$$

$$= \pi\left(11^2 - 5.5^2\right) = 285.1 \text{ mm}^2$$

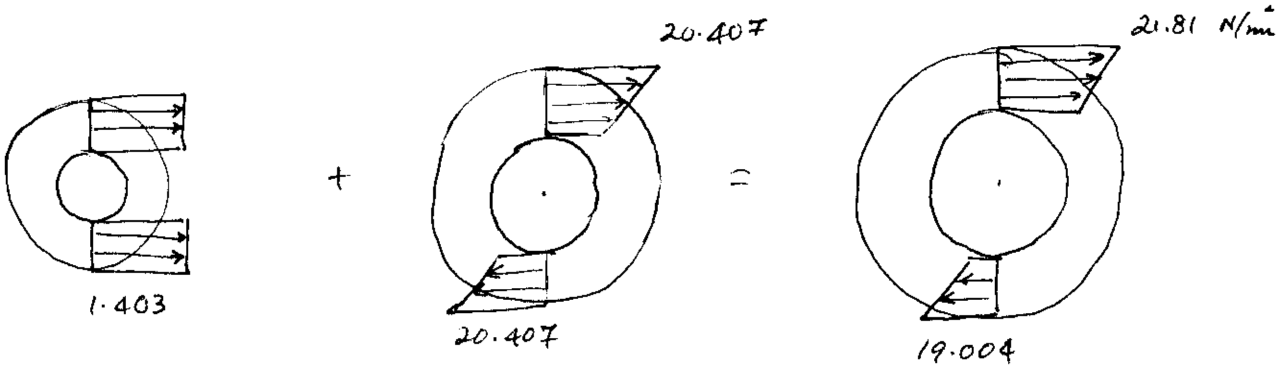
$$I_{zz} = \frac{\pi}{64}\left(d_o^4 - d_i^4\right)$$

$$= \frac{\pi}{64}\left(22^4 - 11^4\right) = 10780.3 \text{ mm}^4$$

$$\sigma_{axial} = \frac{400}{285.1}$$

$$= 1.403 \text{ N/mm}^2.$$

$$\sigma_{bending} = \frac{20{,}000 \times 11}{10780.3} = 20.407 \text{ N/mm}^2$$



(c) % increase in maximum stress.

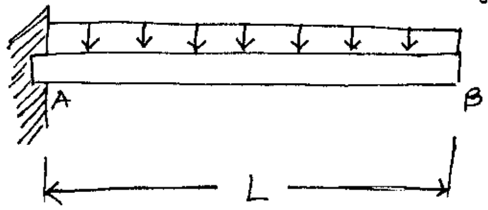
$$\% \text{ increase} = \frac{21.81 - 20.184}{20.184} \times 100$$

$$= 8.06\ \%.$$

Problem 4

8.1 (f): Find deflection of neutral axis of beam in prob. 3.6. EI = constant.

w_o = load / unit length



The reaction at A

$$R_y^A = w_o L \qquad \underline{R_x^A = 0}.$$

$$F_{xy} = -w_o L + w_o x$$
$$= -w_o(L - x).$$

$$^A M_z = -\frac{w_o L^2}{2}.$$

$$M = -{}^A M_z + R_y^A \cdot x - w_o \frac{x^2}{2}$$

$$M = -\frac{w_o L^2}{2} + w_o L \cdot x - w_o \frac{x^2}{2}.$$

$$EI \frac{d^2 u_y}{dx^2} = M.$$

$$EI \frac{d^2 u_y}{dx^2} = -\frac{w_o L^2}{2} + w_o L x - \frac{w_o x^2}{2}$$

Integrate the above equation

$$EI \frac{du_y}{dx} = -\frac{w_o L^2}{2} x + \frac{w_o L x^2}{2} - \frac{w_o x^3}{6} + C_1$$

To find the value of the constant of integration C_1, we impose the boundary condition

$$\left.\frac{du_y}{dx}\right|_{x=0} = 0 \quad \text{i.e. no rotation.}$$

$$\Rightarrow \quad 0 = 0 + 0 - 0 + C_1$$
$$\Rightarrow C_1 = 0$$

$$\Rightarrow \quad EI \frac{du_y}{dx} = -\frac{w_o L^2 x}{2} + \frac{w_o L x^2}{2} - \frac{w_o x^3}{6}$$

Integrating the last equation again

$$EI \cdot u_y = -\frac{w_o L^2 x^2}{4} + \frac{w_o L x^3}{6} - \frac{w_o x^4}{24} + C_2$$

Again to find C_2, we impose the boundary condition $\left. u_y \right|_{x=0} = 0$

$$\Rightarrow 0 = 0 + 0 - 0 + C_2$$
$$\Rightarrow C_2 = 0$$

$$\Rightarrow$$

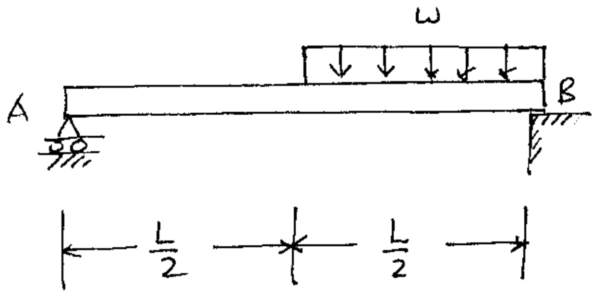
$$u_y = \frac{1}{EI}\left[+\frac{w_o L x^3}{6} - \frac{w_o x^4}{24} - \frac{w_o L^2 x^2}{4} \right]$$

Problem 5

8.2 Find the central deflection of the uniform, simply supported beam due to the uniformly distributed load over the right half of the beam.

$$R_y^A = \frac{wL}{2} \cdot \frac{\cancel{L}}{4} \frac{1}{\cancel{L}}$$

$$= \frac{wL}{8}$$



$0 < x < \frac{L}{2}$

$$M = \frac{wL}{8} x$$

$\frac{L}{2} < x < L$

$$M = \frac{wL}{8} x - \frac{w}{2}\left(x - \frac{L}{2}\right)^2$$

Lets write a generalized expression for the bending moment.

$$M = \frac{wL}{8} x - \frac{w}{2} \langle x - \frac{L}{2} \rangle^2$$

In this expression the brackets $\langle \; \rangle$ mean that any expression written in these, is taken to be "zero" if it is negative. If it is positive then the expression is treated in the ordinary way.

Therefore.

$$EI \frac{d^2 u_y}{dx^2} = M$$

$$= \frac{wLx}{8} - \frac{w}{2} \langle x - \frac{L}{2} \rangle^2$$

Integrating

$$EI \frac{du_y}{dx} = \frac{wLx^2}{16} - \frac{w}{6} \langle x - \frac{L}{2} \rangle^3 + C_1$$

$$EI \cdot u_y = \frac{wLx^3}{48} - \frac{w}{24} \langle x - \frac{L}{2} \rangle^4 + C_1 x + C_2$$

$$u_y\big|_{x=L} = 0$$

$$\Rightarrow 0 = wL^4/48 - \frac{wL^4}{384} + C_1 \cdot L + C_2$$

$$u_y\big|_{x=0} = 0$$

$$0 = 0 - 0 + 0 + C_2$$

$$\Rightarrow C_2 = 0$$

$$\Rightarrow C_1 = -\frac{7}{384} wL^3$$

$$\Rightarrow x = L/2$$

$$u_y = -\frac{5wL^4}{768\, EI}$$