

# Particle Image Velocimetry Study on the Stripe Formation in Vertically Vibrated Granules

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**Abstract**—Recently, granules under vertical vibrations receive many attentions due to their importance in theoretical research and engineering application. In this paper, a two-dimensional Particle Image Velocimetry (PIV) system was used to examine the  $f/2$  stripe pattern forming in a vertically vibrated granular layer. Since the PIV sampling frequency does not match with the vibrating frequency, a special identification-coupling method was adopted to combine the images taken in different cycles to offer the information in one complete cycle. The measured velocity vectors showed exactly the particle motions at various stages of a motion cycle, illustrating the alternating peaks and valleys on the layer top. Furthermore, quantitative results on the temporal evolution of velocity profiles were obtained and some other interesting phenomena were observed, such as the appearance of local structures (e.g. dual-phase layer structure) and the moving feature of the ‘standing point’. The mechanism accounting for the occurrence of stripes on the surface is also discussed. This work will be of interest to a better understanding on pattern formation in the vibrating granular bed.

**Index Terms**—Granule, Particle Image Velocimetry, Stripe, Vibrating bed

## I. INTRODUCTION

IN recent years, granular materials under vertical vibrations receive many attentions due to their importance in theoretical research and engineering application. For instance, the occurrence of fluidlike wave patterns (stripe, square, hexagon, oscillations, etc.) on the free surface has been studied both experimentally and theoretically [1-9]. In order to find out the detailed story of such instabilities in the granular systems, it is important to investigate the motion of particles at different stages in each vibrating cycle.

Particle Image Velocimetry (PIV) is a useful tool to directly measure the instant velocity in single- or multi-phase flow systems. Successful applications of the PIV technique to granular materials were reported recently for some systems, such as rolls in a vertically shaken granular layer [10], avalanches on a rough inclined surface [11], and

flowing granular layer in a rotating tumbler [12]. Lueptow *et al.* discussed the principles governing the PIV applications in granular materials and tested them in the heap formation [13]. However, few results from PIV measurements were reported on the wave patterns in the vertically vibrated bed.

In the present work, a 2D-PIV system is applied to the granular layer in a vibrating bed in order to collect information about the formation of stripe pattern in this system. For serving such a purpose, the understanding of the particle motion during each cycle and velocity distribution in the bed is necessary and important.

## II. METHODOLOGY

### A. Experimental Set-up

Fig. 1(a) shows the apparatus used in our experiments. 148.5 grams of glass beads with the density of  $2500\text{kg/m}^3$  and the size ( $d_p$ ) of 0.5mm were placed in a square-base ( $13.7$  by  $13.7\text{cm}^2$ ) Perspex vessel which was vertically oscillated by a LDS Ling V408 vibrator (Ling Dynamic Systems Ltd., Royston, Herts, UK). The input signal was created by a Lodster function generator and amplified by a LDS 100E Power Amplifier (Ling Dynamic Systems Ltd., Royston, Herts, UK). A PowerView 2-D PIV system from TSI Company (Shoreview, MN) was applied to investigate the motions of particles in the vessel. The laser lightsheet generated by the Laser Pulse<sup>TM</sup> Solo Mini Dual Nd:YAG laser was introduced from the sidewall to lighten the particles near the wall. Images of the granular layer were captured by a PowerView<sup>TM</sup> 4M 2K x 2K camera at the direction perpendicular to the lightsheet and sent to the computer for processing. A LaserPulse<sup>TM</sup> Computer Controlled Synchronizer was adopted to coordinate the operation of the whole PIV system. The image data were analyzed in the Insight<sup>TM</sup> Parallel Processing Ultra PIV software to obtain the velocity profiles.

Although a mixture of light and dark particles can be helpful to provide a clear image of particles [13], the problem is that the granular layer here is so shallow (the layer thickness  $N=H/d_p$  is about 10, where  $H$  and  $d_p$  are the layer thickness and particle diameter, respectively) that the adding of dark particles may affect both the measurements on local velocities and the brightness of the pictures. Fortunately, the experimental results show that strong laser and high-resolution camera adopted can bring clear enough pictures. Furthermore, as shown in Fig. 1(b), stripes show a quasi 2D structure so that the cross-section can be

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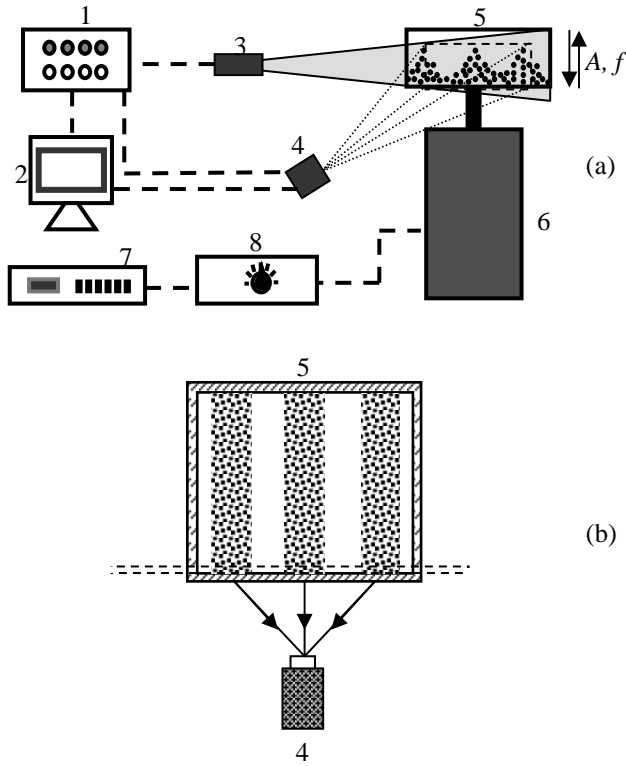


Fig. 1: Schematic diagram of the experimental set-up: (a) Major components used in the system (1) Synchronizer (2) Computer (3) Laser generator (4) CCD camera (5) Vessel (6) Vibrator (7) Function generator (8) Power amplifier; (b) Top view of the vibrating bed.

observed from the sidewall with one camera to have an insight into the velocity profile of the particles near the wall; however other patterns like squares and hexagons are more difficult to study due to the nature of their 3D structures. Thus, the present study focuses on the  $f/2$  stripes [1]. For convenience, in the following we refer to the oscillation cycle of the vessel and the layer as “vibrating cycle” and “motion cycle”, respectively.

The coordinate system adopted in the study is described as follows: The  $X$  and  $Y$  axes are set in the horizontal and vertical directions, respectively. The left-bottom corner of the picture has the pixel coordinate of  $(0, 0)$  and the upper-right corner  $(2047, 2047)$ . Because the position of the camera is fixed, the images captured at different moments will have the same coordination system (e.g. the origin is set at the same point in the space), and the spatial coordinates (i.e.  $X$ - $Y$ ) of all the objects in the images can be determined.

### B. Data acquisition and processing

In order to calculate the velocity, two frames are to be taken at a very short time interval separated by two laser beams. Since the particle velocity here is less than  $0.5\text{m/s}$ , the time difference between two laser pulses is set at  $500\mu\text{s}$ . The images are then processed using the cross-correlation method and a validation step is followed to filter out the spurious vectors. Fig. 2 shows an example of the velocity vectors obtained. It can be seen that some areas are without particles (above the dash-dot or

below the dash curve) and should be removed from velocity analysis; only the motion of particles enclosed by the above two curves is taken into consideration. The velocities (displacements) are then transferred from the unit of pixel/s (pixel) into  $\text{m/s}$  (millimeter) by adopting a calibration step, which is done by photographing a ruler placed at the site where the laser lightsheet passes by and comparing the pixel distance in the image and the corresponding real length of the object.

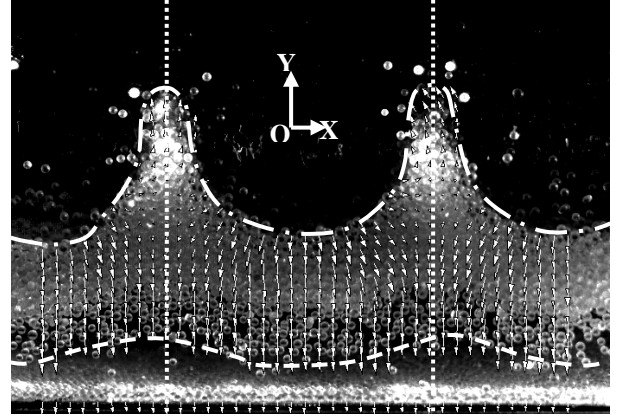


Fig. 2: Velocity profile obtained by PIV measurements. The dash-dot and dash curves denote the up and bottom boundaries of the granular layer, respectively. The dot lines denote the central positions of two peaks forming in the free surface.

A problem to be solved is that the PIV sampling frequency does not match the vibrating frequency of the bed. The maximum frame rate of the CCD camera used is 30fps (frames per second), while the vibrating frequency ranges from 10 to 40HZ in which apparent patterns can be observed. As a result it is impossible to trace the particle motions for such a periodic system because the camera can only take one or two pictures for one vibrating cycle. The solution is to make use of the periodicity of the patterns — we can take pictures in continuous vibrating cycles and then combine them together to investigate the evolution of the layer in one motion cycle. Note that although an individual particle may not return to its original position after two external cycles [8], the macroscopic wave pattern itself would (which has been proved by our repeated tests) — at least in several minutes otherwise the patterns can not be viewed as stable. And because PIV adopts the interrogation window (32 pixels by 32 pixels in our study) as a processing unit, it uses the information of all the particles in this window (the number of particles is about 5 to 8) and can provide the macroscopic velocity that meets our needs. In order to carry out the combination mentioned above, the layer state corresponding to each image is to be specified. This can be achieved by using the information in Fig. 2, which provides not only the velocity profile of granular materials but also the displacement and velocity of the plate as well as the position of the peaks (indicated by the vertical dotted lines). For example, if the vertical displacement of bottom plate is  $Y_p$ , we can find four corresponding points ( $P_1, P_2, P_3$  and  $P_4$ ) in Fig. 3, among which  $P_1$  and  $P_3$  have a negative plate velocity while  $P_2$  and  $P_4$  have a positive plate velocity. Because the peaks in the

first half of the motion cycle change to valleys in the second half for the  $f/2$  patterns,  $P_1$  and  $P_3$  (or  $P_2$  and  $P_4$ ) can be distinguished by specifying the positions of the peaks. After identifying the statuses of all these images, we can then combine them together and obtain a full history of the layer movement in one motion cycle.

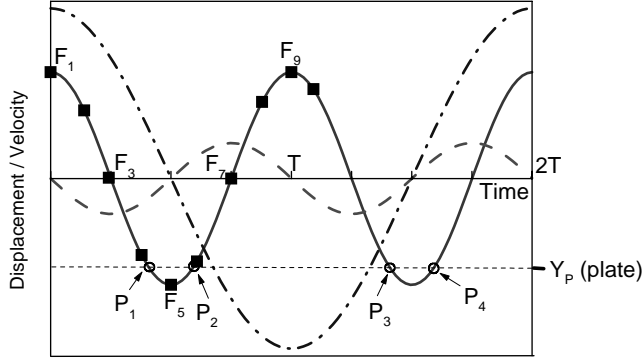


Fig. 3: Variation of plate displacement, plate velocity and layer displacement with time in two consecutive vibrating cycles. The solid and dash curves denote the temporal displacement and velocity of the bottom plate, respectively, and the dash-dot curve denotes the oscillation amplitude of the granular layer. From left to right, the points indicated by solid squares are numbered as  $F_1$ - $F_{10}$ . “ $T$ ” denotes the vibrating period of the bottom plate.  $P_1$ - $P_4$  correspond to the states when the bottom plate has a vertical position of  $Y_p$ .

### III. RESULTS AND DISCUSSION

In order to examine the particle motion at different moments in one motion cycle, some points are selected for investigation as shown in Fig. 3. The experiments were carried out at  $f$  (vibrating frequency) = 16HZ and  $\Gamma$  (dimensionless acceleration, defined as  $4\pi^2 f^2 A/g$ , where  $A$  is the amplitude of vibration, and  $g$  is the gravity acceleration) = 4.50. The vibrating period  $T$  is then calculated to be 0.0625s and the resultant stripes have a wavelength  $\lambda$  of 24.8mm.

Fig. 4 shows five characteristic stages during the period plate moving downwards. According to Fig. 3, the plate corresponding to point  $F_1$  is at its top position, so its velocity measured in panel (a) is close to zero. The particles near the bottom have a low velocity; however those in the upper parts show a complex situation. The particles in peak  $T_2$  flow downwards, denoting a deforming stage happens here. The downflowing particles press those upflowing from lower parts and cause them flow to the left or the right; as a result the upflowing particles tend to concentrate in the places denoted by  $T_1$  and  $T_3$ . The closer these particles are to the center of  $T_1$  (or  $T_3$ ), the high velocities they have. In panel (b), the bottom plate moves downwards; the particles near the bottom, although downflowing similarly, can not catch up with the speed of the plate and consequently a detachment space forms between the granular layer and the bottom plate. Now  $T_1$  and  $T_3$  serve as the converging positions where the particles move upwards and form new peaks, while peak  $T_2$  diminishes very quickly. The plate velocity in panel (c) is very high and the detachment space becomes larger than

before. All the particles in the lower part of the granular layer downflow. For the upper part, peaks  $T_1$  and  $T_3$  keep on growing while peak  $T_2$  totally disappears. The plate velocity decreases in panel (d) and finally reaches zero in panel (e), but the particles still accelerate under the action of gravity. In these two panels, more and more particles change from a positive vertical velocity to a negative one and only those at the top of peaks  $T_1$  and  $T_3$  still move upwards, making these peaks more significant in height.

After point  $F_5$ , the bottom plate begins to move upwards while the lower part of granular layer is still under free falling; the countercurrent motion causes the detachment space to dwindle very fast. Finally the two meet and a strong collision happens. As a result, the particles near the bottom change their velocity direction after the collision and move along with the plate however the particles in the upper tier remain falling down and collide with the upflowing particles. Such a collision is referred to as a “tier-tier collision” in the following. With time increasing (point  $F_6$ ), more particles join the upward movements along with the plate, and the collision curve appears at a higher position. If the vertical distance between the crests and the troughs is defined as the peak height, we can see

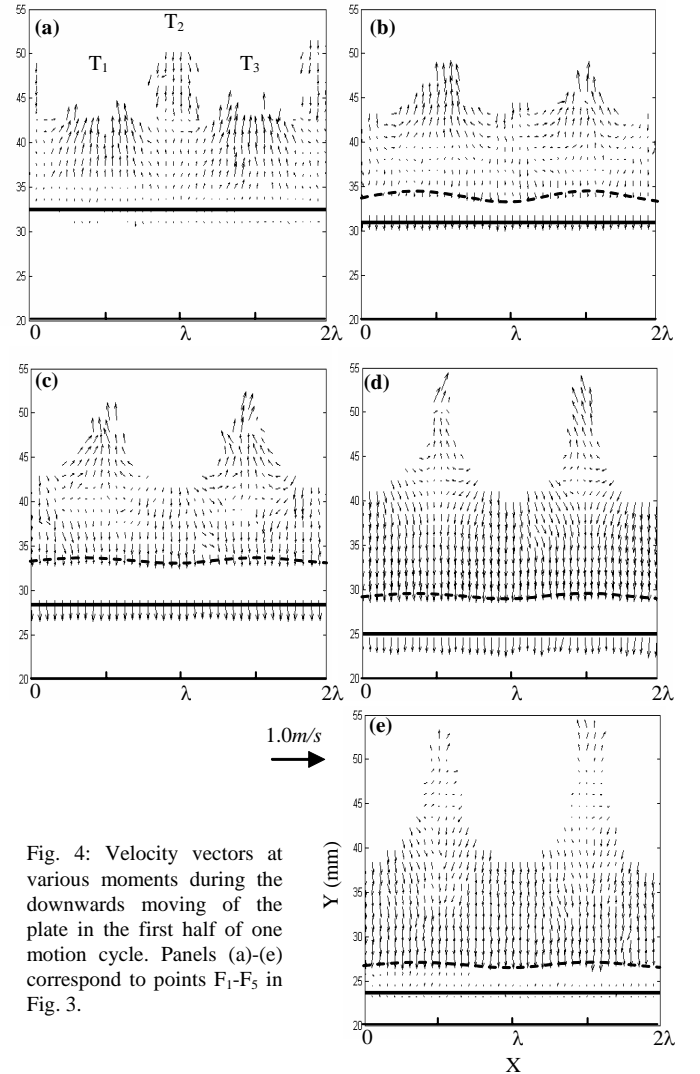


Fig. 4: Velocity vectors at various moments during the downwards moving of the plate in the first half of one motion cycle. Panels (a)-(e) correspond to points  $F_1$ - $F_5$  in Fig. 3.

that the peak height reaches the maximum here. When the collision curve passes through the troughs (point  $F_7$ ), most of the particles move upwards while those in peaks  $T_1$  and  $T_3$  continue to fall downwards. Here particles below the collision curve begin to have a lateral component of velocity pointing to the position of peak  $T_2$  indicated in panel (a) of Fig. 4. Such a tendency continues until the plate returns its top position (point  $F_8$ ), which is also accompanied by the reduction in the height of peaks  $T_1$  and  $T_3$ . Then, in the early stage of the second half of this motion cycle (point  $F_9$ ), the destruction of  $T_1$  and  $T_3$  and formation of  $T_2$  become more significant. Because the particle motion is similar to the first half except the difference in the positions of peaks and valleys, we need not to repeat the descriptions again.

Umbanhowar and Swinney identified three stages of layer motion during each cycle for  $\Gamma > 1$ : free-flight (layer not in contact with container), impact (layer and container collide) and contact (layer and container in contact and moving with the same vertical velocity), and stated that no contact stage exists above the wave onset [5]. It can be seen from the PIV measurements here that the free-flight stage in one cycle starts from the time when the bottom plate is at its top to a moment (e.g. between points  $F_5$  and  $F_6$ ) during its upward motion. Then, a strong collision happens between the downflowing layer and the upmoving plate. After that, the particles near the bottom start to move upwards together with the plate, which is similar to the contact stage (but not for the whole layer).

Actually, it should be pointed out that we can not find even one moment in a motion cycle that all particles move upwards or downwards together, thus the motion of particle at various parts in the granular layer must be identified separately. For example, the free flight stage (Fig. 4) is characterized by the upwards moving particles in the upper part of the layer and downwards moving particles in the lower part. Henceforth, the impact and contact stages are linked closely by the tier-tier collision. This face-to-face collision is not like the particle-wall collision while also different from the usual random particle-particle collision. The bottom tier of the granular layer completes its impact with the plate in a short time and moves along with the plate, indicating the beginning of the contact stage. This tier then collides with the upper downflowing tier and reverses the moving direction of the latter, so now two tiers are in the contact stage. Such a process happens for one tier after another, resulting in more and more tiers in the contact stage.

#### IV. CONCLUSIONS

PIV is a powerful method in the study of granular materials by providing a full view on the collective behaviors. As shown above, the results from the PIV measurements on the vibrating system display exactly the particle motions in each cycle to form the stripe pattern, including both the formation and deformation of peaks.

Quantitative results on the velocity profile are also obtained for this granular system. The maximum vertical

velocity in the layer is less than 0.5m/s, comparable to the peak plate velocity; the maximum lateral velocity is 0.254m/s, resulting in the shifting of peak positions. It is also noticed that the velocity profile is quite non-uniform for both the bed moving upwards and downwards procedures. Actually, we can not find even one moment in a motion cycle that all particles have the vertical velocities in the same direction.

Besides the velocity profile, some special phenomena have also been observed, such as the dual-phase layer structure during the free-flight stage, the oscillation of the “standing” points and the periodically distributed balance positions along the horizontal dimension. These findings may be of interest to a better understanding on pattern formation in the vibrating granular bed.

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#### REFERENCES

- [1] F. Melo, P. B. Umbanhowar and H. L. Swinney, “Transition to parametric wave patterns in a vertically oscillated granular layers,” *Phys. Rev. Lett.*, vol. 72, no.1, pp172-176, 1994.
- [2] F. Melo, P. B. Umbanhowar and H. L. Swinney, “Hexagons, kinks and disorder in oscillated granular layers,” *Phys. Rev. Lett.*, vol. 75, no. 21, pp3838-3841, 1995.
- [3] P. B. Umbanhowar, “Patterns in the Sand,” *Nature*, vol. 389, pp541-542, 1997.
- [4] P. B. Umbanhowar, F. Melo and H. L. Swinney, “Periodic, aperiodic, and transient patterns in vibrated granular layers,” *Physica A*, vol. 249, pp1-9, 1998.
- [5] P. B. Umbanhowar and H. L. Swinney, “Wavelength Scaling and square/stripe and grain mobility transitions in vertically oscillated granular layers,” *Physica A*, vol. 288, pp344-362, 2000.
- [6] H. K. Pak, R. P. Behringer, “Surface waves in vertically vibrated granular materials,” *Phys. Rev. Lett.*, vol. 71, pp1832-1835, 1993.
- [7] E. Clement, L. Vanel, J. Rajchenbach and J. Duran, “Pattern formation in a vibrated granular layer,” *Phys. Rev. E*, vol. 53, no.3, pp2972-2976, 1996.
- [8] C. Bizon, M. D. Shattuck, J. R. deBruyn, J. B. Swift, W. D. McCormick and H. L. Swinney, “Convection and diffusion in vertically vibrated granular patterns,” *J. Stat. Phys.*, vol. 93, pp449-465, 1998.
- [9] C. Bizon, M. D. Shattuck, J. B. Swift, W. D. McCormick and H. L. Swinney, “Patterns in 3D vertically oscillated granular layers: simulation and experiment,” *Phys. Rev. Lett.*, vol. 80, no.1, pp57-60, 1998.
- [10] A. Garcimartin, D. Maza, J. L. Ilquimiche and I. Zuriguel, “Convective motion in a vibrated granular layer,” *Phys. Rev. E*, vol. 65, 031303, 2002.
- [11] B. Andreotti and S. Douady, “Selection of velocity profile and flow depth in granular flows,” *Phys. Rev. E*, vol. 63, 031305, 2001.
- [12] N. Jain, J. M. Ottino and R. M. Lueptow, “An experimental study of the flowing granular layer in a rotating tumbler,” *Phys. Fluids*, vol. 14, pp572-582, 2002.
- [13] R. M. Lueptow, A. Akonur and T. Shinbrot, “PIV for granular flows,” *Exp Fluids*, vol. 28, pp183-186, 2000.