Bond graphs of the cable hoist models help to develop insight about how the electrical R-C filter affects the mechanical system dynamics.

Equivalent mechanical system:

- velocity source (equivalent to switch voltage)
- viscous damper (equivalent to electrical resistor)
- mass equivalent of capacitor
- spring (cable compliance)
- mass of elevator cage
- force source (elevator weight)
Resonant oscillation:

due to *out-of-phase* motion of the two masses opposed by the spring

Consider the two masses and the spring in isolation

No external forces

— net momentum = zero

— the masses move in opposite directions

\[ m_1 v_1 = -m_2 v_2 \]

(subscripts as indicated in the diagram)

\[ v_2 = \frac{m_1}{m_2} v_1 \]

**Kinetic (co-) energy:**

\[ E_k^* = m_1 \frac{v_1^2}{2} + m_2 \frac{v_2^2}{2} = m_1 \frac{v_1^2}{2} + \frac{m_1^2 v_1^2}{m_2^2} = m_1 \left[ 1 + \frac{m_1}{m_2} \right] \frac{v_1^2}{2} \]
Potential energy:

\[ \Delta x_2 = -\frac{m_1}{m_2} \Delta x_1 \]

\[ \Delta x_{spring} = \Delta x_1 - \Delta x_2 = \left(1 + \frac{m_1}{m_2}\right) \Delta x_1 \]

\[ E_p = \frac{k}{2} \Delta x_{spring}^2 = \frac{k}{2} \left(1 + \frac{m_1}{m_2}\right)^2 \Delta x_1^2 \]

Undamped natural frequency:

\[ w_n^2 = \frac{k \left(1 + \frac{m_1}{m_2}\right)^2}{m_1 \left(1 + \frac{m_1}{m_2}\right)} = \frac{k}{m} \left(1 + \frac{m_1}{m_2}\right) \]

Thus the undamped natural frequency will be *increased* by the factor

\[ \sqrt{1 + \frac{m_1}{m_2}} \]
Check the numbers:
The parameters used in the MATLAB simulations were as follows:

\[ R = 10 \text{ ohms} \]
\[ C = 0.1 \text{ farads} \]
\[ K_{\text{motor}} = 0.03 \text{ Newton-meters/amp} \]
\[ n_{\text{gear}} = 0.02 \]
\[ r_{\text{drum}} = 0.05 \text{ meters} \]
\[ k_{\text{cable}} = 200000 \text{ Newton/meter} \]
\[ m_{\text{cage}} = 200 \text{ kilograms} \]

Undamped natural frequency without the R-C filter:

\[
\sqrt{\frac{k_{\text{cable}}}{m_{\text{cage}}}} = 31.6 \text{ radian/second} = 5 \text{ Hertz}
\]

This agrees with the numerical simulation.
The mass equivalent of the capacitor is

\[
m_2 = \left( \frac{K_{\text{motor}}}{r_{\text{drum}} n_{\text{gear}}} \right)^2 \ C = 90 \text{ kilograms} \!
\]

Undamped natural frequency with the R-C filter:

\[
\sqrt{\frac{k}{m} \left( 1 + \frac{m_1}{m_2} \right) } = 56.8 \text{ radians/second} = 9 \text{ Hertz}
\]

This agrees quite well with the numerical simulation.
Decay time constant:
both masses move in unison, opposed by the damper

The viscous damping equivalent of the resistor is
\[
b = \left( \frac{K_{\text{motor}}}{r_{\text{drum}} n_{\text{gear}}} \right)^2 \frac{1}{R} = 90 \text{ Newton-seconds/meter}
\]

If the equivalent mass and equivalent damper were isolated, the decay time constant would be
\[
t_{\text{isolated}} = m_2/b = RC = 1 \text{ second}
\]
which is the time constant of the electrical filter—as it should be.

In the coupled electro-mechanical system, both masses interact with the damper and the decay time constant is
\[
t_{\text{coupled}} = (m_1 + m_2)/b = 3.2 \text{ seconds}
\]
This also agrees quite well with the numerical simulation.
Less can be better
The designer’s original objective may be achieved without increasing the frequency of oscillation by eliminating the capacitor. The system becomes second order.