LAGRANGE’S EQUATIONS (CONTINUED)

Mechanism in “uncoupled” inertial coordinates: (innermost box in the figure)

\[ \tau = \frac{d}{dt} \mathbf{p} = \mathbf{F} \]

Mechanism in generalized coordinates: (middle box in the figure)

\[ \tau = \frac{d}{dt} \eta - \frac{\partial E^*_k}{\partial \theta}; \quad \eta = I(\theta) \omega; \quad E^*_k(\theta, \omega) = \frac{1}{2} \omega^t I(\theta) \omega \]

or

\[ \frac{d}{dt} \left[ \frac{\partial L}{\partial \omega} \right] - \frac{\partial L}{\partial \theta} = \tau \quad \text{with} \quad L(\theta, \omega) = E^*_k(\theta, \omega) \]

Add elastic elements in generalized coordinates: (outermost box in the figure)

\[ \tau = \tau_{\text{inertial}} + \tau_{\text{elastic}} = \frac{d}{dt} \eta - \frac{\partial E^*_k}{\partial \theta} + \frac{\partial E_p}{\partial \theta} \]

or

\[ \frac{d}{dt} \left[ \frac{\partial L}{\partial \omega} \right] - \frac{\partial L}{\partial \theta} = \tau \quad \text{with} \quad L(\theta, \omega) = E^*_k(\theta, \omega) - E_p(\theta) \]