Capstan—a mechanical amplifier

A schematic diagram of a basic capstan and a force diagram for a small segment of the rope are shown in the figures.

\[ F_{\text{normal}} = F \sin \Delta \theta/2 + (F + \Delta F) \sin \Delta \theta/2 \]

In the limit of small angles

\[ F_{\text{normal}} = F \, d\theta/2 + (F + dF) \, d\theta/2 \]
Assuming continuous slip and Coulomb friction between rope and drum,

\[ dF = \mu F_{\text{normal}} = \mu \frac{2F + dF}{2} \ d\theta \]

\[ dF = F \ d\theta \text{ or } d \ln F = \mu \ d\theta \]

Integrating from 0 to \( \theta \)

\[ F_{\text{out}} = e^{\mu \theta} F_{\text{control}} \]

Note that this relation is only valid if \( \omega r \geq v_{\text{control}} \)

From continuity: \( v_{\text{control}} = v_{\text{out}} \)

Torque required of capstan drive:

\[ \tau = (F_{\text{out}} - F_{\text{control}}) r = (e^{\mu \theta} - 1) r F_{\text{control}} \]

Power dissipated:

\[ P_{\text{dissipated}} = \tau \omega + F_{\text{control}} v_{\text{control}} - F_{\text{out}} v_{\text{out}} \]

\[ P_{\text{dissipated}} = (e^{\mu \theta} - 1) r F_{\text{control}} \omega - (e^{\mu \theta} - 1) F_{\text{control}} v_{\text{control}} \]

Note that \( P_{\text{dissipated}} \geq 0 \) with \( P_{\text{dissipated}} = 0 \) if \( \omega r = v_{\text{control}} \)

A bond graph follows:

\[ \begin{align*}
F_{\text{control}} & \quad 0 \quad \text{Fout} \\
\text{Coulomb friction} & \quad \text{R} \\
\text{Drum rim speed} & \quad \text{TF} : r
\end{align*} \]
This is a three-port resistor.

Typical boundary conditions result in the following causality.

Constitutive equations are

\[
\begin{bmatrix}
F_{\text{out}} \\
\tau \\
\nu_{\text{control}}
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & e^{\mu \theta} \\
0 & 0 & (e^{\mu \theta} - 1) r \\
1 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\nu_{\text{out}} \\
\omega \\
F_{\text{control}}
\end{bmatrix}
\]
This is all very well but what’s the point? Not all amplifiers are like transistors? duh ...

Two (?) broad classes of amplifiers; all are resistors; one uses modulated resistors – like the capstan, like the electro-magnetic brake. The other uses internal (?) coupling like a bipolar transistor. Transistor-like amplifiers may include ejector pumps, fluidic amplifiers and hydraulic tee/wye junctions.