EXAMPLE: ELECTRICAL TRANSFORMER

A two-port electrical transformer is sketched below. Two coils of wire are wound on a common core (typically laminated soft iron) so that (ideally) a magnetic flux is common to both coils.

![Idealized two-port electrical transformer](image)

1 In practice, this ideal is most closely approximated by winding the two coils on top of one another.

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The simplest useful model of the transformer is **power-continuous**.

- energy storage in the magnetic field is neglected
- power dissipation in the conductors is neglected
A bond graph of this model contains a gyrator for each coil.

\[
\begin{array}{c|c|c}
\text{electrical domain} & \text{magnetic domain} & \text{electrical domain} \\
\hline
\frac{e_1}{i_1} & \frac{F}{\phi} & \frac{e_2}{i_2}
\end{array}
\]

Power-continuous electrical transformer model depicting the magnetic domain.
POWER TRANSDUCTION WITHIN A SINGLE DOMAIN

The magnetomotive force generated by the current in one coil is related to the current in the other coil.

\[ i_2 = \frac{1}{N_2} F F = N_1 i_1 \]

Combining yields one constitutive equation of an ideal electrical transformer.

\[ i_2 = \frac{N_1}{N_2} i_1 \]

The other constitutive equation is:

\[ e_1 = N_1 \dot{\phi} \dot{\phi} = \frac{1}{N_2} e_2 \]

\[ e_1 = \frac{N_1}{N_2} e_2 \]

Equivalent all-electrical bond graph of the power-continuous transformer model
This device serves to “scale” voltage or current levels in an electrical circuit.

**Power-continuous transducing junction elements**
- gyrorator and transformer —
  may be used within a single energy domain.

Usually this is due to a combination of inter-domain transducers
- as in this example.

In general, two gyrators back-to-back are equivalent to a transformer.
"PARASITIC" DYNAMICS

Why bother with the two- gyrator representation at all?

All models are false

— we need to know when (and how) this model fails.

A real transformer requires a magnetic field to function.

A magnetic field stores energy.

— include magnetic energy storage in our model.

Add a (magnetic) capacitor between the gyrators.

One magnetic flux is common to both gyrators and the capacitor

— connector is a one-junction.

\[
\begin{align*}
\dot{e}_1 & \rightarrow \text{GY} \rightarrow \dot{F}_1 \rightarrow \dot{\phi} \rightarrow F_2 \rightarrow \text{GY} \rightarrow e_2 \\
\dot{i}_1 \rightarrow \text{GY} \rightarrow \dot{\phi} \rightarrow F_1 \rightarrow F_2 \rightarrow \dot{i}_2 \\
F_3 \downarrow \\
C:1/R
\end{align*}
\]

Bond graph of a transformer model including magnetic energy storage.
Constitutive equations:

Flux is common to both coils (as before)

\[ \phi = \frac{\lambda_1}{N_1} = \frac{\lambda_2}{N_2} \]

\[ \frac{e_1}{e_2} = \frac{d\lambda_1}{dt} = \frac{d\lambda_1}{d\lambda_2} \left( \frac{N_1}{N_2} \right) \]

Looks familiar ...

— but be careful!

In steady state, all voltages are identically zero.

This equation is not valid in steady state.
Relation between currents:
(assume magnetic linearity)

\[ i_2 = \frac{F_2}{N_2} = \frac{F_1 - F_3}{N_2} \]

\[ i_2 = \frac{N_1}{N_2} i_1 - \frac{R}{N_2^2} \lambda_2 \]

**Output current is reduced in proportion to output flux linkage.**

The physical meaning of flux linkage may be obscure.

—Time-differentiate to obtain a relation involving output voltage.

\[ \frac{di_2}{dt} = \frac{N_1}{N_2} \frac{di_1}{dt} - \frac{R}{N_2^2} e_2 \]
AN EQUIVALENT ALL-ELECTRICAL REPRESENTATION:

Bring the magnetic capacitor and one-junction through the output gyrator.

\[
\text{Sf} \xrightarrow{e_1} \text{TF} \xrightarrow{0} \text{Se}
\]

\[
\frac{N_1}{N_2} \quad \frac{i_1}{i_2} \quad \frac{i_{L2}}{\frac{N_2^2}{R}}
\]

Equivalent electrical network for the transformer model

Source elements represent assumed boundary conditions

—current input of the left and voltage input on the right.

Magnetic capacitor is equivalent to an electrical inductor.
An equation for the current output on the right may be read directly from the casual graph.

\[ i_2 = \frac{N_1}{N_2} i_1 - i_{L2} \]

where \( i_{L2} \) is the (equivalent) inductor current.

Inductor current determined from the inductor constitutive equation expressed in integral causal form.

\[ i_{L2} = \frac{R}{N_2^2} \lambda_2 \]

Time-differentiate:

\[ \frac{di_{L2}}{dt} = \frac{R}{N_2^2} e_2 \]

Time-differentiate the current output equation and substitute:

\[ \frac{di_2}{dt} = \frac{N_1}{N_2} \frac{di_1}{dt} - \frac{di_{L2}}{dt} \]

\[ \frac{di_2}{dt} = \frac{N_1}{N_2} \frac{di_1}{dt} - \frac{R}{N_2^2} e_2 \]

—Same equation as before.
ALTERNATIVELY:

Bring the magnetic capacitor and one-junction through the input gyrator.

\[ S_f | \begin{array}{c} e_1 \rightarrow 0 \rightarrow T_F \begin{array}{c} e_2 \rightarrow S_e \\ \frac{N_1}{N_2} \end{array} \\ i_{L1} \rightarrow \frac{N_1^2}{R} \end{array} \]

Alternative equivalent electrical network for the transformer model

The same equivalent network is obtained by bring the inductor and zero-junction through the transformer.

The two equivalent inductances are related by the square of the transformer coefficient.

\[ L_1 = \frac{N_1^2}{R} = \left[ \frac{N_1}{N_2} \right]^2 \frac{N_2^2}{R} = \left[ \frac{N_1}{N_2} \right]^2 L_2 \]
DISCUSSION:

Any of the above representations reduce to the power-continuous transformer if voltages are negligible compared to the rates of change of current.

Sufficiently rapidly changing currents
(e.g. sinusoids at high frequencies)
— energy storage effects may be neglected.

Sufficiently slowly changing currents
— energy storage effects become significant.

In steady state, no power transmission is possible
— voltages are identically zero.
PERSPECTIVE:
A two-port electrical transformer is the archetype of all transformers.

Power-continuous transformer model is not wrong;

It is a useful idealization
— a convenient fiction
— similar to an idealized “point mass” with inertia but no extent.

THE MOST IMPORTANT THING:

Know the limitations of these ideal model.

Know how real device behavior may deviate from this ideal.

FINAL NOTE:

Modeling errors are commonly assumed to occur under high frequency operation.

Real physical systems may deviate from their models at low frequencies
— or at all frequencies.