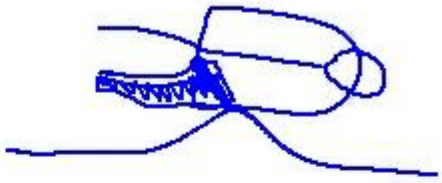
# Notes on Bones

## Dynamics - Fall on Hip



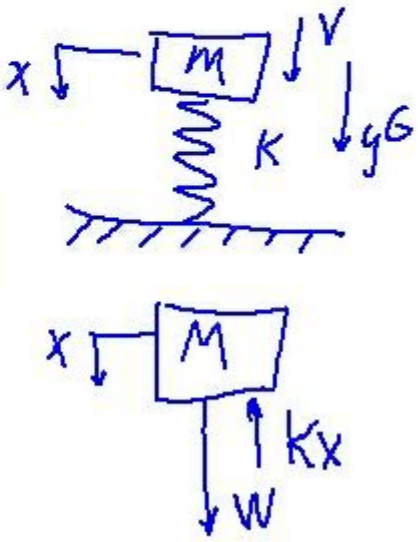
$$F_{MAX} = KX_{MAX}$$

$$KX = MgG - M\ddot{x}$$

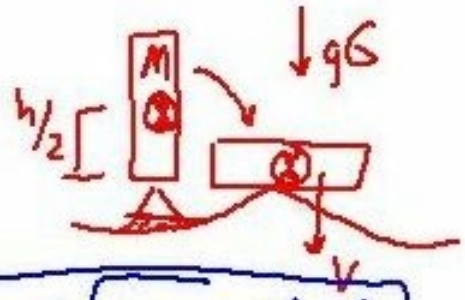
$$\Rightarrow M\ddot{x} + KX - MgG = 0$$

$$F_{MAX} = MgG\left[\frac{V\omega_0}{gG}\sin(\omega_0 t)_{MAX} - \cos(\omega_0 t) + 1\right]$$

$$\omega_0^2 = \frac{K}{M}$$




## Solve for V




$$MgG\frac{h}{2} = \frac{1}{2}I\omega^2$$

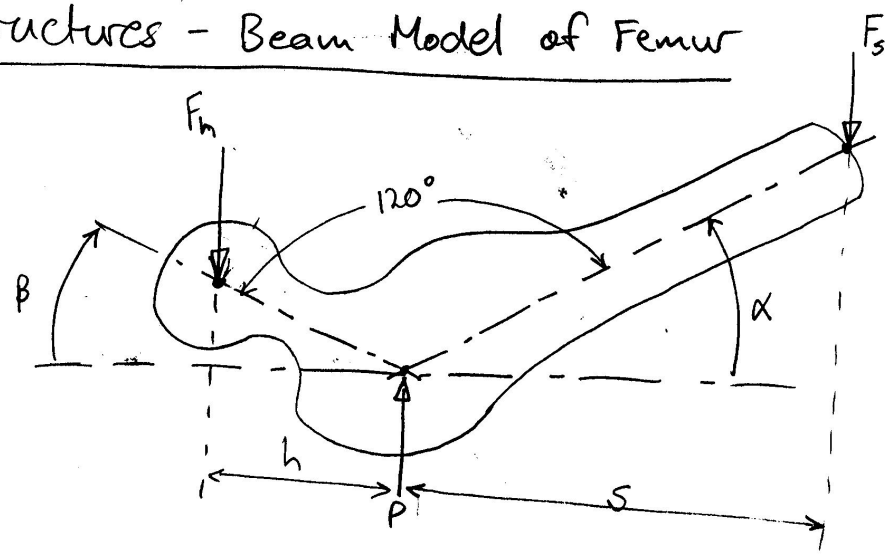
$$I = \frac{Mh^2}{3}$$

$$V = \frac{h}{2}\omega \Rightarrow \omega^2 = \frac{4V^2}{h^2}$$

$$F_{MAX} \approx V\sqrt{KM + MgG}$$

2) $$V = \frac{\sqrt{3gGh}}{2}$$

# II Structures - Beam Model of Femur




$$+\uparrow \Sigma F_V = 0: \quad P - F_h - F_s = 0 \qquad (1)$$

$$+\circlearrowleft \Sigma (M)_P = 0: \quad (F_h)h - (F_s)s = 0 \qquad (2)$$

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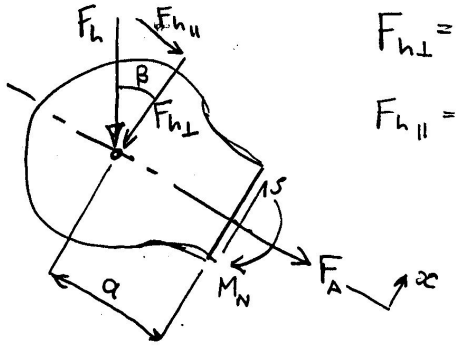
$(1) \rightarrow F_h = P - F_s$

$\rightarrow (2) \rightarrow \boxed{F_s = P\left(\frac{h}{s+h}\right)}$

$\Rightarrow \boxed{F_h = P\left(\frac{s}{s+h}\right)}$




$F_{h\perp} = F_h \cos\beta$

$F_{h\parallel} = F_h \sin\beta$

$$+\circlearrowleft \Sigma M_N = 0: \quad M_N - (F_{h\perp})a = 0 \quad \Rightarrow \boxed{M_N = P\left(\frac{as}{s+h}\right)\cos\beta}$$

$$+\rightarrow \Sigma F = 0: \quad F_A + F_{h\parallel} = 0 \quad \Rightarrow \boxed{F_A = -P\left(\frac{s}{s+h}\right)\sin\beta}$$

## Normal Stress

Due to Moment: $\sigma_M = \frac{M_N \cdot x}{I}$ Due to axial force: $\sigma_F = \frac{F_A}{A}$

Total Stress in Neck: $\sigma_N = \sigma_M + \sigma_F = \frac{M_N \cdot x}{I} + \frac{F_A}{A}$

$$\Rightarrow \boxed{\sigma_N = P\left(\frac{s}{sth}\right)\left(\frac{ax\cos\beta}{I} - \frac{\sin\beta}{A}\right)}$$

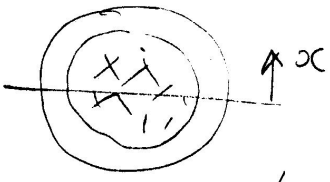
Values:

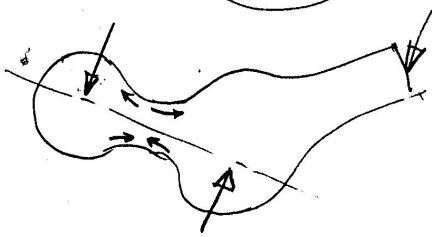
$I = 1\times10^{-8}\ m^4$ $A = 1.25\times10^{-4}\ m^2$

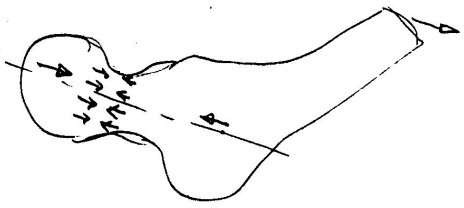
$h = .03\ m$ $s = .18\ m$

$a = .025\ m$ $\beta = 50°$

$P = F_{max} = 7.96\ kN$





Tension Side $x = +.015\ m$

$$\boxed{(\sigma_N)_t = 123\ MPa}$$

Compression Side $x = -.015\ m$

$$\boxed{(\sigma_N)_c = -206\ MPa}$$

From literature:

$\sigma_{ut} = 135\ MPa$

$\sigma_{uc} = 199\ MPa$

$\therefore$ FAILS ON COMPRESSION SIDE