Essays on Political Institutions and Macroeconomics

by

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A.B. Economics
Harvard University, 2001

Submitted to the Department of Economics
in partial fulfillment of the requirements for the degree of

Doctor of Philosophy in Economics

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

June 2007

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Abstract

This dissertation consists of three chapters on the interaction of political institutions and macroeconomic activity in dynamic environments. Chapter 1 studies the optimal management of taxes and debt in a framework which relaxes the standard assumption of a benevolent government. We assume instead the existence of a self-interested ruler who manages the government budget. Unlike in the standard economy, temporary economic shocks generate persistent changes in taxes and debt along the equilibrium path so as to optimally limit rent-seeking by the ruler. The presence of political economy distortions causes the debt market which is complete to behave as if it were incomplete. In contrast to an incomplete market economy, taxes are positive in the long run. A numerical exercise suggests that the welfare cost of political economy distortions is high if the government chooses suboptimal politically sustainable policies which do not respond persistently to shocks. This is because the government over-saves and resources are wasted on rents.

Chapter 2 studies the dynamics of war and peace in an environment with two groups seeking resources from each other. Peaceful compromise is subject to limited commitment and informational frictions since groups cannot commit to concession-making and the private cost of concession-making can be extremely high. We show that phases of war enforce phases of peace along the equilibrium path. Even though fluctuations between war and peace can occur in the short run, long run convergence to permanent war is inevitable since this maximizes the duration of peace in the short run. In an extension, we allow each group to waste resources during war to inflict additional damage on its enemy. Under some conditions, phases of peace occur even in the long run, since phases of peace enforce phases of war.

Chapter 3 is joint work with Daron Acemoglu, Simon Johnson, and James A. Robinson. We revisit the conventional wisdom which views high levels of income as a prerequisite for democracy. We show that existing evidence for this view is based on cross-country correlations which disappear once we look at within-country variation. Rather than reflecting causality, the cross-country correlation between income and democracy reflects longer-run changes, in particular, a positive correlation between changes in income and democracy over the past 500 years. We suggest a possible explanation for this pattern based on the idea that societies may have embarked on divergent political-economic development paths at certain critical junctures.

Thesis Supervisor: Daron Acemoglu
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Acknowledgments

I am deeply indebted to Daron Acemoglu for his mentoring. In his boundless enthusiasm for economics, he has left an indelible mark on me and is a true inspiration. I am also deeply indebted to Mike Golosov for being a great advisor. He has taught me to pay attention to detail, to organize my ideas, and to see the big picture. I am grateful to Ricardo Caballero, Simon Johnson, James A. Robinson, Aleh Tsyvinski, and Iván Werning for their instrumental roles in my education and research. I thank the MIT economics department for financial support. I also thank my friends in the department: Miriam Bruhn, Sylvain Chassang, Alex Debs, Fadi Kanaan, Jin Li, Dimitris Papanikolaou, Heikki Rantakari, Catarina Reis, Olga Shurchkov, and Ding Wu for making my time at graduate school an even more enjoyable one.

For Chapter 1, I specifically thank: Daron Acemoglu, Manuel Amador, Marios Angeletos, Roc Armenter, Abhijit Banerjee, Ricardo Caballero, Larry Christiano, Alex Debs, Mike Golosov, Dimitris Papanikolaou, James Poterba, Catarina Reis, Aleh Tsyvinski, Iván Werning, and participants at the seminars at Brown, Columbia, INSEAD, Kellogg MEDS, Minnesota, MIT, Northwestern, NY Fed, NYU, Princeton, and Yale for comments.

For Chapter 2, I specifically thank: Daron Acemoglu, Sylvain Chassang, Alex Debs, Mike Golosov, James A. Robinson, Iván Werning, and seminar participants at the MIT macro lunch and NYU Conflict Conference for comments.

For Chapter 3, Daron Acemoglu, Simon Johnson, James A. Robinson, and I thank: David Autor, Robert Barro, Sebastián Mazzuca, Robert Moffitt, Jason Seawright, four anonymous referees, and seminar participants at the Banco de la República de Colombia, Boston University, the Canadian Institute for Advanced Research, the CEPR annual conference on transition economics in Hanoi, MIT, and Harvard for comments.

Finally, I thank my parents and my little sister for their support and their love.

To my mother and father

Chapter 1

Politicians, Taxes, and Debt

1.1 Introduction

The traditional understanding of fiscal policy holds that tax smoothing is optimal and that budget balancing is suboptimal. In their seminal paper, Lucas and Stokey (1983) argue that in a complete market economy under a benevolent government, tax smoothing implies a labor income tax rate which co-moves with public spending shocks. An implication of this argument is that if a government anticipates a temporary war, it should save prior to the war, borrow during the war, and pay back its debt after the war. As a result, the tax rate should only potentially adjust during the war and return to its original level thereafter.

A natural question is whether observed tax rates are optimal as defined by the theory. Figure 1.1 depicts the response of the actual tax rate in the United States to the First World War and compares it to the response of the optimal tax rate.[1] Traditional theory predicts that receipts to GDP should only change during the war. However, receipts to GDP increased during the war and remained higher even after the war. This particular example is consistent with other research showing that observed taxes do not behave in a theoretically optimal manner.[2] Thus, a potential interpretation of these data is that fiscal policy is suboptimal and improvable.[3]

[1] The optimal tax revenue is merely drawn to co-move with public spending shocks and is not calibrated to a model or to any quantities.

[2] Data for the US is from Barro (1990). The deviation of the empirical time path of taxes from the optimal time path of taxes is discussed more generally in the following work: Marcet and Scott (2003), Sargent and Velde (1995), Huang and Lin (1993), and Kingston (1987), and Bohn (1990).

[3] As has been emphasized by Barro (1979) and Aiyagari, Marcet, Sargent, and Seppala (2002), the observed

In this paper, we ask whether observed fiscal policy could be optimal once political economy constraints–which are ignored by the traditional theory–are taken into account. Current theories assume that a benevolent government chooses policies, even though, in practice, policies are chosen by politicians with objectives other than social welfare, such as reelection and personal enrichment. Empirically, there is evidence that constraints on politicians matter for macroeconomic outcomes.[4] Theoretically, politicians can appropriate part of the government budget as rents, creating a tradeoff between the benefit of public goods and the cost of rent-seeking which is ignored by the traditional theory.[5]

Our intention is to describe fiscal policy prescriptions which are politically sustainable, and this is in the spirit of Buchanan (1987) who writes that

> "Economists should cease proffering policy advice as if they were employed by a benevolent despot, and they should look to the structure within which political decisions are made." (p.243)

To this end, we consider a closed economy with no capital, with shocks to the productivity of public spending, and with complete markets, which is equivalent to the economy of Lucas and Stokey (1983). We depart from their model by relaxing the assumption of a benevolent government. Instead, a self-interested ruler allocates the budget between rents and public spending, and households choose the tax rate so as to control the ruler. This simple framework captures an important friction present in many political economy settings: the incentives of politicians and of citizens are not always aligned.

In the one period version of the model, the ruler devotes the entire government budget to rents, and there is no public spending. Since the tax revenue does not benefit them, households choose zero taxes. We consider an infinitely repeated game with double sided lack of commit-

policies may be optimal in the absence of contingent debt. Nevertheless, even when contingent debt is not available, it can be replicated by the use of other policy instruments which are available in practice such as a capital tax (Chari, Christiano, and Kehoe, 1994), an inflation tax (Bohn, 1988), or long debt maturities (Angeletos, 2002 and Buera and Nicolini, 2004).

[4] See Acemoglu, Johnson, and Robinson (2004).

[5] For a discussion of the consequences of the self-interested behavior of governments, see Buchanan and Tullock (1962), North (1981), Dixit (2004), Persson and Tabellini (2000), and Acemoglu (2003) among others. For a discussion of corruption and its required structure for the implementation of public goods, see Acemoglu and Verdier (2000), Banerjee (1997), Becker (1968), Becker and Stigler (1974), Shleifer and Vishny (1993), and Rose-Ackerman (1978).

ment in which reputation sustains more efficient outcomes. We examine sustainable competitive equilibria, meaning competitive equilibria which satisfy the incentive compatibility constraints of the ruler and of the households in every period. In the efficient sustainable competitive equilibrium, the ruler sets aside a fraction of the national budget for rents in every period, and if the ruler deviates from this implicit agreement by increasing rents, households choose zero taxes in the future, thereby punishing the ruler. Moreover, if households deviate from the implicit agreement to choose positive taxes, the ruler stops implementing public projects in the future, thereby punishing the households. The efficient sustainable competitive equilibrium is a solution to the standard problem subject to the addition of two incentive compatibility constraints in every period. These incentive compatibility constraints operate like endogenous debt limits for the government. Specifically, debt cannot be too negative relative to the ruler's equilibrium continuation value, since this increases the continuation value to the ruler off the equilibrium path. Analogously, debt cannot be too positive relative to the household's equilibrium continuation value, since this increases the continuation value to the household off the equilibrium path.

Our main result is that, even though markets are complete, taxes and debt adjust persistently to shocks along the equilibrium path so as to optimally limit rent-seeking by the ruler, and this is in contrast to the policies of Lucas and Stokey (1983). Public spending shocks create variation in the opportunity for the ruler to appropriate rents, and therefore create variation in the need for society to provide incentives for the ruler. Optimal incentive provision is intertemporal, and relaxing the incentive compatibility constraint of the ruler can be achieved by changing rents and debt into the future. In general equilibrium, this manifests itself in more persistent tax rates. The time path of taxes in our economy is qualitatively similar to that generated under incomplete markets, so that political economy distortions introduce a form of endogenous market incompleteness.

To illustrate the equilibrium path behavior of the model, consider the example of an economy experiencing a temporary war. According to the standard analysis, the government should save during peace and borrow during war by trading state-contingent debt. As a consequence, taxes should only potentially adjust during the war and return to their original level thereafter. Now imagine an economy subject to political economy distortions with the associated incentive compatibility constraints on the ruler and on the households. The ruler's incentive compatibility constraint is the tightest when war is taking place and there are more resources to steal. It can be relaxed in one of two ways. First, the ruler's wealth can be reduced by increasing government debt during the shock. This increase in debt reduces the value of deviation by the ruler in the future, and it can be implemented with a persistent increase in the tax rate to service the debt. Second, current and future rents for the ruler can be increased. This increase in rents increases the value of cooperation by the ruler in the future, and it can be implemented with a persistent increase in the tax rate to finance these rents. However, higher taxes and rents may not be tolerated by households in all states of the world, and it is important to consider the incentive compatibility constraint on the households. This constraint is the tightest when the war has ended, since the ruler is less productive and more of the government's resources are used to pay for rents and to service debt. Incentives for the households must be provided by reducing taxes and by reducing rents, though the tax rate need not drop all the way down to its original level under peace.

In the long run, if the discount factor on the ruler and on the households is sufficiently high, the efficient sustainable policy is not persistent and qualitatively similar to that of Lucas and Stokey (1983). For instance, in the best sustainable competitive equilibrium for the households, the ruler accumulates debt and rents along the equilibrium path, until the incentive compatibility constraint on the ruler stops binding. Households are sufficiently patient so as to accept the gradual increase in the tax rate and the decrease in public spending which accompany the government's accumulation of debt and rents. In contrast, if the discount factor is low, the tax rate may respond persistently to shocks even in the long run. The long run behavior of our economy with endogenously incomplete markets is different than the long run behavior of an economy with exogenously incomplete markets. Specifically, Aiyagari, Marcet, Sargent, and Seppala (2002) show that in an economy managed by a benevolent ruler without state-contingent debt, the government accumulates assets along the equilibrium path, until it is able to finance the entire stream of public spending with zero taxes.[6]

To understand why traditional policies are inefficient along the equilibrium path, imagine if we impose suboptimal policies comparable to those of Lucas and Stokey (1983) in our economy.

[6] Werning (2006a) shows that this is true for a broad class of utility function.

Under a benevolent ruler, tax revenues are the same before and after the war. Tax revenue before the war is used by the government to accumulate assets, and tax revenue after the war is used to service debt. Such an arrangement is inefficient under a self-interested ruler, since he must receive rents prior to the war in order to be trusted to pledge the accumulated assets for public use. As a consequence, more resources than necessary are diverted away from public spending and towards rents prior to the war. This is inefficient since it is more optimal to delay the distortions associated with providing incentives to the ruler until the war actually takes place.

In addition to providing a theoretical characterization of the model, we illustrate the logic of the model using a numerical example which explores the welfare cost of political economy. Our example suggests that this welfare cost is relatively small because the value of rents paid to the ruler is small. We consider the welfare loss due to the imposition of a suboptimal policy appropriate to a benevolent government by finding the efficient sustainable smooth tax rate and smooth public spending profile. Our example suggests that the prescription of suboptimal policies appropriate to a benevolent government can generate sizeable welfare losses by forcing the government to save too much and by diverting too many resources away from public spending towards rents.

We explore an application of our framework in a setting in which the government experiences a stochastic windfall revenue. This extension is relevant for many countries in which natural resource revenue is a significant portion of the federal budget. In an economy managed by a benevolent planner, fiscal policies are acyclical, meaning they are independent of the shocks to the revenue. The government saves during high revenue shocks and it borrows during low revenue shocks. In constrast, in an economy subject to political economy distortions, taxes increase (decrease) and public spending decreases (increases) during low (high) revenue shocks.

This paper is most closely related to the political economy literature on debt. Battaglini and Coate (2006) also study the dynamics of taxes and debt in a political economy model, but they focus on the Markov Perfect Equilibrium in an environment with competing groups and incomplete markets. The distinguishing feature of the current paper is the focus on efficient sustainable allocations in complete markets, which enables us to obtain different predictions for the short and long run.[7] In this respect, our paper is also similar to the work of Acemoglu, Golosov and Tsyvinski (2005) who characterize the efficient sustainable allocations and taxes in a dynamic economy with a self-interested ruler and who show that political economy distortions may disappear in the long run. The current paper is different from their work in two important respects. First, it focuses on the dynamics of government debt, which is an essential element of macroeconomic fiscal policies and is ruled out in their model. Second, it introduces aggregate shocks, which are not present in their work. This paper is also related to the large literature on dynamic optimal taxation in the Ramsey setting with and without commitment, but it departs from previous papers by relaxing the assumption of a benevolent government.[8] As in the work of Chari and Kehoe (1993a,1993b) and Sleet and Yeltekin (2006), this paper relates limited commitment to financial constraints on the government.[9] Furthermore, the insights of our model along with its solution technique are close to the literature on consumption risk sharing with two-sided lack of commitment (see Alvarez and Jermann, 2001 and Kocherlakota, 1996), and the key distinction is that the agents' resources in our political environment are not exogenous endowments but are dynamically generated through government policies and competitive markets.[10] Finally, our discussion of optimal incentive provision is related to the work of Ray (2002) who shows that rents to an agent should be increased into the future in order to increase the value of cooperation. Although our economy captures this insight, changes in the timing of rents in our economy must be made in conjunction with changes in the timing of debt, since debt–which is used to finance rents–affects the value of deviation.

The paper is organized as follows. Section 1.2 describes the model. Section 1.3 defines a sustainable competitive equilibrium. Section 1.4 characterizes the best sustainable competitive equilibrium. Section 1.5 discusses a numerical example. Section 1.6 considers an application with shocks to the government budget. Section 1.7 generalizes our main result to a broad class

[7] In contrast to their work, we find that, under i.i.d. shocks, the government becomes less responsible after high shocks. For related work on the political economy of debt, see Aghion and Bolton (1990), Persson and Svensson (1989), Lizzeri (1999), and Alesina (1990).

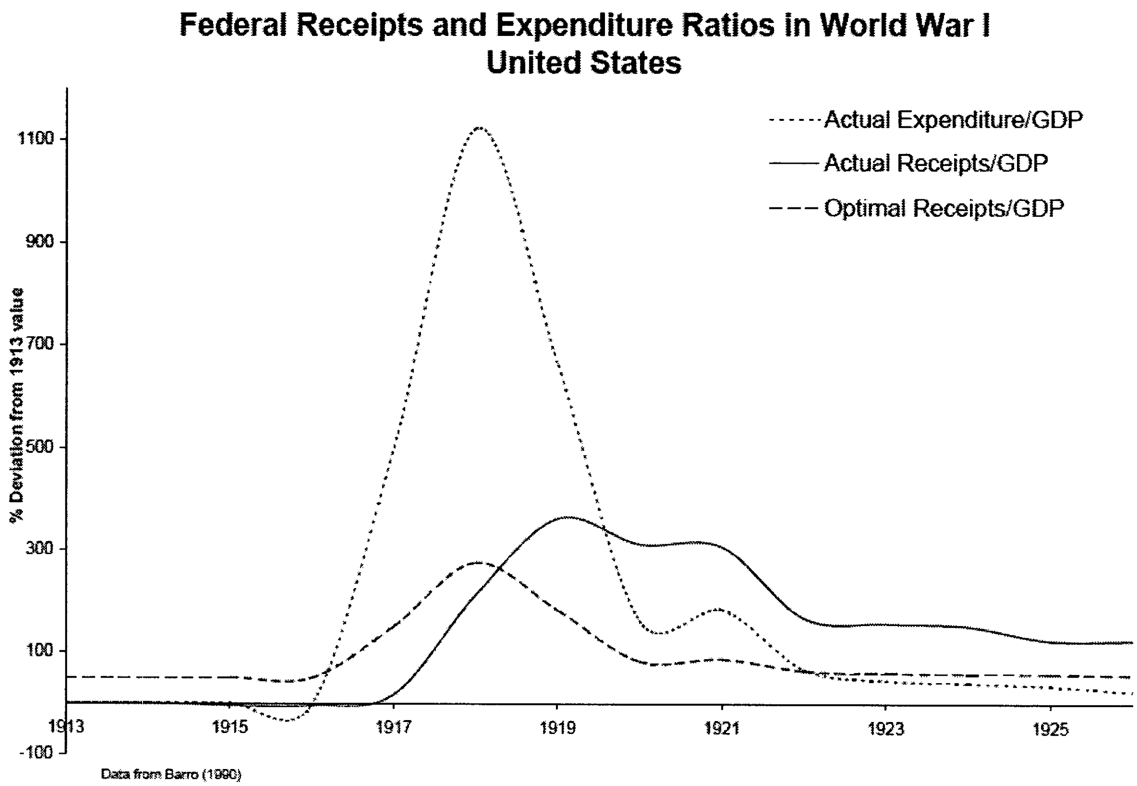
[8] Related papers which assume a benevolent government include Chari, Christiano, and Kehoe (1995), Kydland and Prescott (1977), Werning (2006a,2006b), Phelan and Stacchetti (2001), Sleet (2004), and Aguiar, Amador, and Gopinath (2006), Angeletos (2002), and Buera and Nicolini (2004), among others.

[9] In contrast to this work, endogenous market incompleteness emerges purely from the non-benevolence of the government and not from the possibility of default.

[10] See also Kletzer and Wright (2000) for an application to sovereign debt and Dixit, Grossman, and Gul (2000) for an application to political compromise.

of preferences. Section 1.8 concludes, and the Appendix contains all of the proofs and additional material.

Figure 1.1



1.2 Model

1.2.1 Economic Environment

Time and Uncertainty

There are discrete time periods $t = \{0, ..., \infty\}$ and a stochastic state $s_t \in S \equiv \{1, ..., N\}$ which follows a first order Markov process. There are no absorbing states. Let $\Pr\{s_{t+1} = k | s_t = s\} = \pi_{ks}$ with s_0 given. Let $s^t = \{s_0, ..., s_t\} \in S^t$ represent a history, let $\pi\left(s^k | s^t\right)$ represent the probability of s^k conditional on s^t for $k \geq t$, and let $\pi\left(s^k\right) = \pi\left(s^k | s^0\right)$.

Households

There is a continuum of mass 1 of identical households with the utility function:

$$\mathbf{E}_0\left(\sum_{t=0}^{\infty}\beta^t u\left(c_t, n_t, g_t, s_t\right)\right),\ \beta \in (0,1). \tag{1.1}$$

c_t is consumption, n_t is labor, and g_t is government spending. Our model considers a special case of this preference: $u\left(c_t, n_t, g_t, s_t\right) = c_t - \eta\frac{n_t^\gamma}{\gamma} + \theta\left(s_t\right)\frac{g_t^\alpha}{\alpha}$, for $0 < \alpha < 1 < \gamma$ and $\theta\left(s_t\right) > 0$. $\theta\left(s_t\right)$ is high (low) when public spending is more (less) productive. This utility function allows us to abstract from bond price manipulation by the government and to focus on labor market distortions.[11] See Section 1.7 for a generalization of our main results to a broad class of risk averse utility functions.

Ruler

We depart from the economy of Lucas and Stokey (1983) and most of the literature on optimal taxation by relaxing the assumption of a benevolent ruler and by introducing a self-interested ruler with the utility function:

$$\mathbf{E}_0\left(\sum_{t=0}^{\infty}\beta^t v\left(x_t\right)\right). \tag{1.2}$$

x_t represents inefficient government projects only beneficial to the ruler which we refer to as rents. $v\left(x_t\right)$ is increasing and weakly concave, and for simplicity, let $v\left(x_t\right) = x_t$. The self-interested ruler can be interpreted as an individual politician or a group of bureaucrats with the power to control the government budget. Like the benevolent ruler, the self-interested ruler has the unique ability to improve household welfare by financing and implementing public spending, but unlike the benevolent ruler, he derives no utility from this endeavor.

Markets

Household wages are normalized to 1 and are taxed at a linear rate τ_t. $b_t^h\left(s_{t+1}\right) \gtreqless 0$ represents debt owned by the household at t, which is a promise to repay 1 unit of consumption at

[11] An equivalent formulation of our problem is to allow for risk aversion in consumption and to consider an open economy with exogenous bond prices, and this does not change any of our results.

$t+1$ conditional on the realization of s_{t+1}, and $q_t(s_{t+1})$ is its price at t. We ignore bonds of longer maturity structure only for notational simplicity. At every t, the household's allocation $\omega_t = \left\{c_t, n_t, \{b_t^h(s_{t+1})\}_{s_{t+1}\in S}\right\}$ must satisfy the household's dynamic budget constraint

$$c_t + b_{t-1}^h(s_t) = (1-\tau_t)\, n_t + \sum_{s_{t+1}\in S} q_t(s_{t+1})\, b_t^h(s_{t+1}), \tag{1.3}$$

subject to $n_t \geq 0$.

$b_t^g(s_{t+1}) \gtreqless 0$ represents debt owned by the government at t, analogously defined to the household's debt. At every t, government policies $\left\{\tau_t, \{b_t^g(s_{t+1})\}_{s_{t+1}\in S}, g_t, x_t\right\}$ must satisfy the government's dynamic budget constraint

$$g_t + x_t + b_{t-1}^g(s_t) = \tau_t n_t + \sum_{s_{t+1}\in S} q_t(s_{t+1})\, b_t^g(s_{t+1}), \tag{1.4}$$

subject to $g_t \geq 0$. The only difference between these budget constraints and those of the standard economy is that the rent x_t is included on the left hand side of (1.4). We discuss the implications of allowing for default in Section 1.4.1.

The economy is closed, and bonds are in zero net supply:

$$b_t^g(s_{t+1}) + b_t^h(s_{t+1}) = 0, \tag{1.5}$$

which combined with (1.3) and (1.4) implies the aggregate resource constraint

$$c_t + g_t + x_t = n_t. \tag{1.6}$$

For notational simplicity, we let $b_t^g(s_{t+1}) = b_t(s_{t+1})$ for the remainder of the discussion. $b_{-1}(s^0)$ is exogenous. The following debt limits rule out Ponzi schemes

$$b_t^g(s_{t+1}) \in \left[\underline{b}, \overline{b}\right]. \tag{1.7}$$

Let $\underline{b}$ to be sufficiently low and $\overline{b}$ to be sufficiently high so that (1.7) does not bind.

1.2.2 Political Environment

At every t, s_t is realized. Households collectively choose a tax rate $\tau_t \in [0,1]$.[12] The ruler then chooses non-tax policies $\rho_t = \left\{ \{b_t^g(s_{t+1})\}_{s_{t+1}\in S}, g_t, x_t \right\}$. The structure of this game is identical to one in which the ruler proposes a tax rate which the households accept or reject through a formal process or through a tax riot.[13]

An economy managed by a benevolent ruler is equivalent to one in which households choose $\left\{ \tau_t, \{b_t^g(s_{t+1})\}_{s_{t+1}\in S}, g_t, x_t \right\}$ in every period subject to $x_t = 0$. Our political economy environment therefore captures the fact that households cannot control the spending decisions of politicians.[14]

1.2.3 Repeated Game Interaction

The interaction between households and the ruler is a game with the following form:

1. Nature chooses the state s_t.

2. Households choose the tax rate τ_t.

3. The ruler chooses non-tax policies ρ_t.

4. Markets open and clear.

When households pay taxes, they cannot control the ruler's budget allocation decision. When the ruler pledges part of the budget for public spending, he cannot control the household's future tax decisions. For example, if the discount factor is 0, the ruler does not implement public projects. As a consequence, households choose $\tau_t = 0$. This double-sided commitment problem can be resolved by considering history dependent strategies.

[12] Subsidies can be allowed as long as there is a lower bound on the tax rate. Our main results do not depend on households behaving strategically, since the ruler continues to behave strategically.

[13] Bassetto and Phelan (2006) formally study equilibria with tax evasion.

[14] One can also interpret our environment as consisting of a benevolent branch of government (the tax minister) and a non-benevolent branch of government (the spending minister). Our main results do not change if households do not choose the tax rate but can instead replace the ruler. This is discussed in greater detail in Section 1.3.3.

1.3 Sustainable Competitive Equilibria

1.3.1 Definition

Our definition of a sustainable competitive equilibrium builds on the formal work of Chari and Kehoe (1993a,1993b). Individual households are anonymous and non-strategic in their private market behavior, though they are strategic in their choice of the tax rate. The ruler is strategic in his choice of policies, and he must ensure that the government's dynamic budget constraint is satisfied given the anonymous and non-strategic market behavior of households.

Define $h_t^0 = \{s^t, \tau^{t-1}, \rho^{t-1}\}$ as the history of shocks, tax rates, and non-tax policies after the realization of s_t. Define $h_t^1 = \{s^t, \tau^t, \rho^{t-1}\}$ and $h_t^2 = \{s^t, \tau^t, \rho^t\}$. Let $\zeta_t(h_t^2) = \{q_t(s_{t+1})(h_t^2)\}_{s_{t+1}\in S}$ denote a vector of state prices at h_t^2 and let $\zeta = \{\zeta_t(h_t^2)\}_{t=0}^{\infty}$ denote a sequence of such vectors.

At every t, households choose taxes τ_t as a function of h_t^0 together with a contingency plan for choosing τ_k's for all possible h_k^0's for $k > t$. Let $\Upsilon_t(h_t^0)$ represent the choice of τ_t. The ruler chooses non-tax policies ρ_t as a function of h_t^1 together with a contingency plan for choosing ρ_k's for all possible h_k^1's for $k > t$. Let $\sigma_t(h_t^1)$ represent the choice of ρ_t. $\zeta_t(h_t^2)$ is then revealed, and households privately choose allocations ω_t as a function of h_t^2 together with a contingency plan for choosing future ω_k's for all possible h_k^2's for $k > t$. Let $f_t(h_t^2)$ represent the household's allocation. Because households are anonymous, public decisions are not conditioned on their allocation but only on τ_t and on ρ_t which are public. Define $\Upsilon = \{\Upsilon_t(h_t^0)\}_{t=0}^{\infty}$, and define σ and f analogously.

Strategies induce histories as follows. Given h_t^0, Υ induces $h_t^1 = \{h_t^0, \Upsilon_t(h_t^0)\}$, and given h_t^1, σ induces $h_t^2 = \{h_t^1, \sigma_t(h_t^1)\}$ and $h_{t+1}^0 = \{h_t^1, \sigma_t(h_t^1), s_{t+1}\}$, and so on. Continuation strategies are generated as follows. Given h_t^0 and σ, a continuation of Υ is

$$\{\Upsilon_t(h_t^0), \Upsilon_{t+1}(h_t^0, \Upsilon_t(h_t^0), \sigma_t(h_t^0, \Upsilon_t(h_t^0)), s_{t+1}), \ldots\}.$$

Given h_t^1 and Υ, a continuation of σ is

$$\{\sigma_t(h_t^1), \sigma_{t+1}(h_t^1, s_{t+1}, \Upsilon_{t+1}(h_t^1, \sigma_t(h_t^1), s_{t+1})), \ldots\}.$$

Given h_t^2, Υ, and σ, a continuation of f is

$$\left\{f_t\left(h_t^2\right), f_{t+1}\left(h_t^2, s_{t+1}, \Upsilon_{t+1}\left(h_t^2, s_{t+1}\right), \sigma_{t+1}\left(h_t^2, s_{t+1}, \Upsilon_{t+1}\left(h_t^2, s_{t+1}\right)\right)\right), \ldots\right\}.$$

Given h_t^2, Υ, and σ, a continuation of ζ is defined analogously.

In every period, continuations of Υ and σ must be best responses to each other given continuations of f and ζ, and the continuation of f must maximize the welfare of households given the continuation of Υ, σ, and ζ. Consider the private household solving its market problem in period t. Given h_t^2, Υ, σ, and ζ, a household chooses a continuation of f to maximize:

$$\mathbf{E}\left\{\sum_{k=t}^{\infty} \beta^{k-t}\left[u\left(c_k\left(h_k^2\right), n_k\left(h_k^2\right), g_k\left(h_k^1\right), s_k\right)\right] | h_t^2, \Upsilon, \sigma, f, \zeta\right\}$$

s.t.

$$c_t\left(h_t^2\right) + b_{t-1}^h\left(s_t\right)\left(h_{t-1}^2\right) = \left(1-\tau_t\right) n_t\left(h_t^2\right) + \sum_{s_{t+1}\in S} q_t\left(s_{t+1}\right)\left(h_t^2\right) b_t^h\left(s_{t+1}\right)\left(h_t^2\right),$$

$$c_k\left(h_k^2\right) + b_{k-1}^h\left(s_k\right)\left(h_{k-1}^2\right) = \left(1-\tau_k\left(h_k^0\right)\right) n_k\left(h_k^2\right) + \sum_{s_{k+1}\in S} q_k\left(s_{k+1}\right)\left(h_k^2\right) b_k^h\left(s_{k+1}\right)\left(h_k^2\right)$$

for $k > t$, and $n_t\left(h_t^2\right), n_k\left(h_k^2\right) \geq 0$. τ_t is given in h_t^0, and for $k > t$ all future histories are induced by Υ and σ from h_t^2. Since future histories do not depend on f, households are non-strategic in this decision. Let $W\left(h_t^2; \Upsilon, \sigma, \zeta\right)$ denote the solution to this problem.

Consider the ruler at t. Given h_t^1, Υ, f, and ζ, the ruler chooses a continuation of σ to maximize:

$$\mathbf{E}\left\{\sum_{k=t}^{\infty} \beta^{k-t} v\left(x\left(h_k^1\right)\right) | h_t^1, \Upsilon, \sigma, f, \zeta\right\},$$

s.t.

$$g_k\left(h_k^1\right) + x_k\left(h_k^1\right) + b_{k-1}^g\left(s_k\right)\left(h_{k-1}^1\right) = \tau_k\left(h_k^0\right) n_k\left(h_k^2\right) + \sum_{s_{k+1}\in S} q_k\left(s_{k+1}\right)\left(h_k^2\right) b_k^g\left(s_{k+1}\right)\left(h_k^1\right),$$

$g_k\left(h_k^1\right) \geq 0$, and $b_k^g\left(s_{k+1}\right) \in \left[\underline{b}, \overline{b}\right]$. τ_t is given in h_t^0, and for all $k \geq t$, future history are induced by Υ and σ from h_t^1.

Consider the public household solving its political problem in period t. Given h_t^0, σ, f, and

ζ, the household chooses $\tau_t\left(h_t^0\right) \in [0,1]$ to maximize

$$W\left(h_t^0, \tau_t\left(h_t^0\right), \sigma_t\left(h_t^0, \tau_t\left(h_t^0\right)\right); \Upsilon, \sigma, \zeta\right), \tag{1.8}$$

for future histories which are induced by Υ and σ from h_t^0.

Given h_t^2, Υ, σ, and f, ζ must clear the bond market:

$$b_t^g\left(s_{t+1}\right)\left(h_t^1\right) + b_t^h\left(s_{t+1}\right)\left(h_t^2\right) = 0 \ \forall s_{t+1} \in S.$$

Definition 1 *A sustainable competitive equilibrium is a 4-tuple* $\{\Upsilon, \sigma, f, \zeta\}$ *that satisfies the following conditions:*

1. *Given* $\{\sigma, f, \zeta\}$, Υ *solves the household's political problem for every history* h_t^0.

2. *Given* $\{\Upsilon, f, \zeta\}$, σ *solves ruler's problem for every history* h_t^1.

3. *Given* $\{\Upsilon, \sigma, \zeta\}$, f *solves the household's market problem for every history* h_t^2.

4. *Given* $\{\Upsilon, \sigma, f\}$, ζ *clears the bond market for every history* h_t^2.

The competitive equilibrium characterized by Lucas and Stokey (1983) considers a sequence f and ζ which satisfy the third and fourth condition of the above definition for a given policy sequence $\{\Upsilon, \sigma\}$ chosen by a benevolent government.

An important assumption which allows sustainable competitive continuation equilibria to exist at every h_t^0, h_t^1, and h_t^2 is that consumption c_t and rents x_t can be negative so that debts can be repaid.

1.3.2 Competitive Equilibria

We characterize competitive equilibria using the primal approach developed by Ramsey (1927) and Lucas and Stokey (1983). This is useful for determining necessary and sufficient conditions for a sustainable competitive equilibrium. Households choose equilibrium allocations $\omega = \left\{\omega\left(s^t\right)\right\}_{t=0}^{\infty}$ which maximize their utility as a function of the equilibrium sequence of tax rates $\tau = \left\{\tau\left(s^t\right)\right\}_{t=0}^{\infty}$ and state prices $q = \left\{q\left(s^{t+1}|s^t\right)\right\}_{t=0}^{\infty}$, for $q\left(s^{t+1}|s^t\right)$ which represents the price of a bond traded at s^t with payment conditional on the realization of s^{t+1}. Moreover, the bond market must clear. Let

$$\xi = \left\{c\left(s^t\right), n\left(s^t\right), g\left(s^t\right), x\left(s^t\right)\right\}_{t=0}^{\infty},$$

a sequence of consumption, labor, public spending, and rents. All proof are in the Appendix.

Proposition 1 ***(Competitive Equilibrium)*** *ξ is a competitive equilibrium if and only if it satisfies*

$$c\left(s^t\right) + g\left(s^t\right) + x\left(s^t\right) = n\left(s^t\right) \;\; \forall s^t, \text{ and} \tag{1.9}$$

$$\sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t \pi\left(s^t\right)\left(R\left(n\left(s^t\right)\right) - g\left(s^t\right) - x\left(s^t\right)\right) = b_{-1}\left(s^0\right), \tag{1.10}$$

$$\text{for } R(n) = n - \eta n^{\gamma}.$$

(1.9) is the resource constraint of the economy. $R(n)$ is the revenue generated by labor n derived from the household's intratemporal condition. It is independent of consumption because of risk neutrality in consumption. (1.10) is the present value budget constraint of the government. It states that total public spending, rents, and initial government debt are serviced by total revenues. Present values are calculated using probabilities because of risk neutrality in consumption. Together with (1.9), (1.10) implies the satisfaction of the household's present value budget constraint. Because of the completeness of financial markets, the satisfaction of (1.9) and (1.10) are sufficient to imply the satisfaction of the government's present value budget constraint in the future:

$$\sum_{k=t}^{\infty} \sum_{s^k \in S^k} \beta^{k-t} \pi\left(s^k | s^t\right)\left(R\left(n\left(s^k\right)\right) - g\left(s^k\right) - x\left(s^k\right)\right) = b\left(s^t | s^{t-1}\right) \;\; \forall s^t, \tag{1.11}$$

for $b\left(s^t|s^{t-1}\right)$ representing a bond traded at s^{t-1} with payment conditional on the realization of s^t. (1.11) means the government holds assets ($b\left(s^t|s^{t-1}\right) < 0$) when it is planning to run deficits in the future and it holds debt ($b\left(s^t|s^{t-1}\right) > 0$) when it is planning to run surpluses in the future.

1.3.3 Sustainable Competitive Equilibria

A competitive sequence ξ need not be sustainable since the ruler may wish to deviate from the prescribed sequence of non-tax policies ρ and households may wish to deviate from the prescribed sequence of taxes τ. Using the methods developed by Abreu (1988), we derive the most severe punishment strategies to sustain equilibrium behavior.

Proposition 2 ***(Sustainable Competitive Equilibrium)*** *ξ is a sustainable competitive equilibrium if and only if it satisfies* (1.9), (1.10),

$$\sum_{k=t}^{\infty}\sum_{s^k\in S^k}\beta^{k-t}\pi\left(s^k|s^t\right)x\left(s^k\right)\geq R\left(n\left(s^t\right)\right)-b\left(s^t|s^{t-1}\right)\ \forall s^t, \tag{1.12}$$

$$\sum_{k=t}^{\infty}\sum_{s^k\in S^k}\beta^{k-t}\pi\left(s^k|s^t\right)\left(c\left(s^k\right)-\eta\frac{n\left(s^k\right)^{\gamma}}{\gamma}+\theta\left(s_k\right)\frac{g\left(s^k\right)^{\alpha}}{\alpha}\right)\geq U^{AUT}+b\left(s^t|s^{t-1}\right)\ \forall s^t, \tag{1.13}$$

$$\textit{and } n\left(s^t\right)\in\left[0,n^{fb}\right] \tag{1.14}$$

$$\textit{for } n^{fb}=\left(\frac{1}{\eta}\right)^{1/(\gamma-1)},\ U^{AUT}=\frac{n^{fb}-\eta\frac{n^{fb\gamma}}{\gamma}}{1-\beta},\ \textit{and } b\left(s^t|s^{t-1}\right)\ \textit{determined by } (1.11).$$

To understand (1.12) and (1.13) imagine the following punishment strategy. Whenever the ruler or the households deviate from prescribed policies, the ruler and the households revert to the repeated static equilibrium in which taxes and public spending are set to zero forever. Given this punishment, the ruler's best deviation is to choose zero public spending today so as to achieve a continuation value equal to the right hand side of (1.12), since the ruler repays his debts off the equilibrium path. The ruler achieves a continuation value equal to the left hand side of (1.12) along the equilibrium path, which means this deviation is weakly dominated.

Moreover, given this punishment, the household's best deviation is to choose zero taxes today, since the ruler is going to use all of the tax revenue towards rents off the equilibrium path. Since the households supply the first best level of labor n^{fb} forever, and since public spending is zero forever, the household's best deviation generates a continuation payoff equal to the right hand side of (1.13), since households are repaid their debt off the equilibrium path. Households achieve a continuation value equal to the left hand side of (1.13) along the equilibrium path, which means that this deviation is weakly dominated. In the Appendix, we show that this punishment strategy represents the worst punishment for the ruler and the households. Equations (1.12) and (1.13) make it clear that the introduction of political economy distortions generates endogenous debt limits. Neither the ruler nor the households can be too wealthy since this tightens their incentive compatibility constraints. Constraint (1.14) ensures that tax rates are non-negative.[15]

Definition 2 Λ *is the set of sustainable competitive allocations.*

1.4 Efficient Sustainable Competitive Equilibria

1.4.1 Program

We now consider efficient equilibria. Define $U(\xi)$ and $V(\xi)$ as the values of (1.1) and (1.2), respectively, implied by an allocation ξ.

Definition 3 $\xi \in \Lambda$ *is an efficient equilibrium if* $\nexists \xi' \in \Lambda$ *s.t.* $U(\xi') > (\geq) U(\xi)$ *and* $V(\xi') \geq (>) V(\xi)$.

Such an equilibrium is a solution to

$$\max_{\xi} U(\xi) \text{ s.t. } V(\xi) \geq V_0,\ \xi \in \Lambda \tag{1.15}$$

for some V_0. In comparison, the original problem of Lucas and Stokey (1983) sets $x(s^t) = 0$ for all s^t and ignores incentive compatibility constraints (1.12) and (1.13).

In order to simplify our problem, we make use of the risk neutrality of agents with respect to consumption in order to reduce the dimensionality of the problem.

[15] If households could remove the ruler or if the ruler could resign, the ruler would additionally acquire the maximal amount of debt $\bar{b}$ at price β off the equilibrium path. If additionally, the ruler chooses the tax rate, the incentive compatibility constraint of the ruler becomes

$$\sum_{k=t}^{\infty} \sum_{s^k \in S^k} \beta^{k-t} \pi\left(s^k | s^t\right) x\left(s^k\right) \geq R^{\max} - b\left(s^t | s^{t-1}\right) + \beta \bar{b},$$

for $R^{\max} = \max_n R(n)$ and the incentive compatibility constraint of the household never binds. This modification increases equilibrium rents and affects the long run properties of the equilibrium but it does not affect our main results.

Lemma 1 *(i) If* $\xi \in \Lambda$, $\exists \xi' \in \Lambda$ *s.t.* $\{n'(s^t), g'(s^t)\} = \{n(s^t), g(s^t)\}$ $\forall s^t$, $x'(s^t) = \overline{x}$ $\forall s^t$, $U(\xi') = U(\xi)$, *and* $V(\xi') = V(\xi)$, *and (ii)* $\exists \xi'' \in \Lambda$ *s.t.* $\{n''(s^t), g''(s^t)\} = \{n(s^t), g(s^t)\}$ $\forall s^t$, $x''(s^t) = R(n''(s^t)) - g''(s^t)$ $\forall s^t$ *and* $t \geq 1$, $x''(s^0) = R(n''(s^0)) - g''(s^0) - b_{-1}(s^0)$, $U(\xi'') = U(\xi)$, *and* $V(\xi'') = V(\xi)$.

Given a sustainable competitive allocation with a particular n and g, there exists a sustainable competitive allocation with same n and g with either (i) a constant rent to the ruler or (ii) a balanced budget for $t \geq 1$, and both of these alternative allocations generate the same welfare as the original allocation. Lemma 1 is implied by the substitution of (1.9) and (1.11) into (1.12) and (1.13) to achieve:

$$\sum_{k=t}^{\infty} \sum_{s^k \in S^k} \beta^{k-t} \pi\left(s^k | s^t\right) \left(R\left(n\left(s^k\right)\right) - g\left(s^k\right)\right) \geq R\left(n\left(s^t\right)\right) \ \forall s^t, \text{ and} \tag{1.16}$$

$$\sum_{k=t}^{\infty} \sum_{s^k \in S^k} \beta^{k-t} \pi\left(s^k | s^t\right) \left(n\left(s^k\right) - \eta \frac{n\left(s^k\right)^{\gamma}}{\gamma} - R\left(n\left(s^k\right)\right) + \theta\left(s_k\right) \frac{g\left(s^k\right)^{\alpha}}{\alpha}\right) \geq U^{AUT} \ \forall s^t. \tag{1.17}$$

(1.16) and (1.17) do not depend on the sequences of c or x. This is because the ruler is invested in the assets used to pay for his rents, and households are invested in the assets used to pay for their consumption. More specifically, it is possible to relax (1.12) by increasing rents (i.e., the value of cooperation) while holding debt constant or by increasing public debt (i.e., the punishment from deviation) while holding rents constant. Because of risk neutrality, both of these methods are equivalent from a welfare perspective and have the same implications for the incentive compatible sequence of taxes and public spending.[16] Analogous arguments hold with respect to the relaxation of (1.13).

An important implication of Lemma 1 is there is no deep substance behind our ruling out of default in our benchmark economy. In particular, Lemma 1 means that any sustainable competitive equilibrium in an economy without default under $b_{-1}(s^0) \leq 0$ is a sustainable competitive equilibrium in an economy in which the ruler can default, since debt can be chosen to be equal to zero along the equilibrium path, and there is no deviation by the ruler which

[16] In Section 1.7, we generalize our result to an economy with risk aversion and we show with a simulation that both payments and debt are used to provide incentives.

involves default.[17]

Lemma 1 is in contrast with the work of Ray (2002) and Acemoglu, Golosov, and Tsyvinski (2005) who show that payments must be backloaded to provide incentives. There are two reasons why this is not the case here. First, there are financial markets, so that incentives must be provided by changing the timing of payments as well as the timing of debt. Second, the ruler and the households are risk neutral and can have negative consumption, so that there is no sense in which payments have to be smooth across time or across states, which leads to multiple sequences of consumption and rents which are optimal despite a *unique* optimal sequence of tax rates and public spending.[18]

Given the flexibility in choosing consumption and rents, we focus our attention to an allocation in which rents are constant, keeping in mind that many other sequences of consumption and rents exist which yield the same unique optimal time path for taxes and public spending. Let $Q(s,b)$ represent the solution to the period 0 problem subject to $s = s_0$, $b = b_{-1}(s^0)$, and $x(s^t) = \overline{x}\ \forall s^t$. As a reminder, $\pi_{ks} = \Pr\{s_{t+1} = k | s_t = s\}$. The recursive program which characterizes the solution is:

$$Q(s,b) = \max_{c,n\in[0,n^{fb}],g,\{b_k\}_{k\in S}} c - \eta\frac{n^\gamma}{\gamma} + \theta(s)\frac{g^\alpha}{\alpha} + \beta\sum_{k\in S}\pi_{ks}Q(k,b_k) \tag{1.18}$$

s.t.

$$c + g + \overline{x} = n \tag{1.19}$$

$$b = R(n) - g - \overline{x} + \beta\sum_{k\in S}\pi_{ks}b_k \tag{1.20}$$

$$\overline{x}/(1-\beta) \geq R(n) - b, \text{ and} \tag{1.21}$$

$$Q(k,b_k) \geq U^{AUT} + b_k\ \forall k \in S. \tag{1.22}$$

$Q(s,b)$ represents the highest possible welfare to households that can be achieved conditional on the state s and on the value of debt being equal to b. (1.18) represents this program

[17] There is nevertheless no guarantee that such an equilibrium is efficient in an economy under default. This is because the possibility of default by the ruler can relax the incentive compatibility constraint of the households (1.13) and can improve efficiency.

[18] A version of our model with default which imposes non-negative rents can be solved without changing the main results. Details available upon request.

written in a recursive fashion. c, n, and g, represent consumption, labor, and public spending today, respectively. b_k represents the value of debt conditional on the realization of the state k following state s. (1.19) is the resource constraint. (1.20) ensures that the value of debt is b. (1.21) represents (1.12) and (1.22) represents (1.13). In our recursive formulation, we take the ruler's incentive compatibility constraint into account in today's period and we take the household's incentive compatibility constraint into account in tomorrow's period. This is useful for the characterization of the solution since (1.16) and (1.17) suggest that changes in the tax rate today do not affect the ruler's incentive compatibility constraint today, but they affect the household's incentive compatibility constraint today.

We can use the techniques developed by Thomas and Worrall (1988) to characterize $Q(s, b)$.

Lemma 2 $Q(s, b)$ *(i) is defined over a compact interval* $[\underline{b}_s, \overline{b}_s]$, *and (ii) is strictly decreasing, strictly concave, and continuously differentiable in* $b \in (\underline{b}_s, \overline{b}_s)$.

Remark 1 *The Lucas and Stokey (1983) solution is a solution to* $(1.18) - (1.20)$ *for* $\overline{x} = 0$.

This problem adds two constraints to the original problem of Lucas and Stokey (1983). First, the generation of rents creates a strain on the resource constraint. Second, there are endogenous debt limits. Government assets cannot be so large that the ruler prefers to stop providing public goods and government debt cannot be so large that households would like to stop paying taxes.

1.4.2 Efficient Sustainable Policy

In order to take first order conditions, we make the following assumption.

Assumption 1 $\underline{b}_s < \overline{b}_s$ $\forall s$.

This assumption implies that $[\underline{b}_s, \overline{b}_s]$ is non-degenerate. It is implied by the existence of an equilibrium with positive taxes and positive public spending. We show in the Appendix that this assumption holds for a high enough discount factor β.

Use (1.19) to substitute in for c in (1.18), and rewrite (1.21) after substituting (1.20) in for b. Let λ, ϕ, and $\beta\pi_{ks}\psi_k$ represent the Lagrange multipliers for (1.20), (1.21), and (1.22), respectively. First order conditions and the envelope condition yield:

$$\begin{aligned} n &: \eta n^{\gamma-1} = \frac{1+\lambda}{1+\gamma\lambda} && (1.23) \\ g &: \theta(s) g^{\alpha-1} = 1+\lambda+\phi && (1.24) \\ b_k &: Q_b(k, b_k) = -\frac{\lambda+\phi-\psi_k}{1+\psi_k} && (1.25) \\ b &: Q_b(s, b) = -\lambda && (1.26) \end{aligned}$$

Equation (1.23) pins down output, and by consequence the tax rate and the revenue as a function λ, the slope of the welfare function $Q(\cdot)$. Specifically, the higher the debt b conditional on the state s, the lower the output and the higher the tax rate. Public spending is a function of λ as well as ϕ, since it is related to the provision of incentives for the ruler. Equations (1.25) and (1.26) show how the sequence of future debt is related to the provision of incentives. Specifically, the slope of $Q(\cdot)$ only changes if an incentive compatibility contraint binds. Under a benevolent ruler, the slope of $Q(\cdot)$ never changes, and this has implications for policy. We can use this observation to characterize the motion of the efficient tax rate and public spending. Define

$$\widetilde{g}\left(s^t\right) = \frac{g\left(s^t\right)}{\theta\left(s_t\right)^{1/(1-\alpha)}},$$

the value of public spending normalized by its productivity.

Remark 2 *The solution to the Lucas and Stokey (1983) problem is*

$$\tau\left(s^t\right) = \tau\left(s^{t-1}\right) \ \textit{and}\ \widetilde{g}\left(s^t\right) = \widetilde{g}\left(s^{t-1}\right) \forall s^t.$$

Proposition 3 ***(Optimal Tax Dynamics)*** *The unique τ which solves* (1.15) *has the following property:*

$$\tau\left(s^t\right) = \begin{cases} \overline{\tau}(s_t) & \text{if } \tau\left(s^{t-1}\right) > \overline{\tau}(s_t) \ \text{ or } \underline{\tau}(s_{t-1}) > \overline{\tau}(s_t) \\ \tau\left(s^{t-1}\right) & \text{if } \tau\left(s^{t-1}\right) \in \left[\underline{\tau}(s_{t-1}), \overline{\tau}(s_t)\right] \\ \underline{\tau}(s_{t-1}) & \text{if } \tau\left(s^{t-1}\right) < \underline{\tau}(s_{t-1}) \leq \overline{\tau}(s_t) \end{cases} \qquad (1.27)$$

for $\underline{\tau}(s)$ and $\overline{\tau}(s)$ independent of $b_{-1}\left(s^0\right)$ and V_0.

Corollary 1 ***(Optimal Public Spending Dynamics)*** *The unique g which solves* (1.15) *has* *the following property:*

$$\widetilde{g}\left(s^{t}\right)=\begin{cases}\overline{\widetilde{g}}\left(s_{t}\right) & \text{if } \widetilde{g}\left(s^{t-1}\right)>\overline{\widetilde{g}}\left(s_{t}\right)\\ \widetilde{g}\left(s^{t-1}\right) & \text{if } \widetilde{g}\left(s^{t-1}\right)\in\left[\underline{\widetilde{g}}\left(s_{t}\right),\overline{\widetilde{g}}\left(s_{t}\right)\right]\\ \underline{\widetilde{g}}\left(s_{t}\right) & \text{if } \widetilde{g}\left(s^{t-1}\right)<\underline{\widetilde{g}}\left(s_{t}\right)\end{cases} \tag{1.28}$$

for $\underline{\widetilde{g}}(s)$ and $\overline{\widetilde{g}}(s)$ independent of $b_{-1}\left(s^{0}\right)$ and V_0.

(1.27) and (1.28) are dynamic equations which characterize the time path of the tax rate and public spending for $t \geq 1$, and the optimal levels of $\tau\left(s^{0}\right)$ and $\widetilde{g}\left(s^{0}\right)$ are determined by initial conditions V_0 and $b_{-1}\left(s^{0}\right)$.

(1.27) means that the tax rate cannot be above a state dependent upper bound $\overline{\tau}\left(s_{t}\right)$. This ensures that the incentive compatibility constraint of the households is satisfied at s^t. Conditional on the tax rate being below this upper bound, it must be above a lower bound $\underline{\tau}\left(s_{t-1}\right)$ determined by yesterday's state. This lower bound ensures that the incentive compatibility constraint of the ruler is satisfied at s^{t-1}. The reason why taxes must change tomorrow to provide incentives for the ruler today is that increasing taxes today actually tightens the ruler's incentive compatibility constraint by increasing the amount of resources that he controls. (1.28) implies that public spending normalized by productivity cannot be above a state dependent upper bound $\overline{\widetilde{g}}\left(s_{t}\right)$ in order to satisfy the ruler's incentive compatibility constraint, and it cannot be below a state dependent lower bound $\underline{\widetilde{g}}\left(s_{t}\right)$ in order to satisfy the household's incentive compatibility constraint.

Changes in the tax rate and in public spending are always smoothed into the future to the extent that such smoothing is possible given future incentive compatibility constraints. In addition to reducing economic distortions, this smoothing relaxes future incentive compatibility constraints. This is because the implied change in debt serves to reduce the value of deviation to the households and to the ruler in the future. Operationally, satisfaction of incentive compatibility constraints requires

$$b\left(s^{t}|s^{t-1}\right)\in\left[\underline{b}\left(s_{t},s_{t-1}\right),\overline{b}\left(s_{t}\right)\right],$$

so that there is a history dependent lower bound on debt at s^t required to satisfy the incentive compatibility constraint of the ruler at s^{t-1}, and there is a state dependent upper bound on debt required to satisfy the incentive compatibility constraint of the households at s^t.

An additional understanding of Proposition 3 and Corollary 1 can emerge by taking Lemma 1 into account. Specifically, under a balanced budget for $t \geq 1$, rents can be chosen so that $x\left(s^t\right) = R\left(n\left(s^t\right)\right) - g\left(s^t\right)$. Rents must be sufficiently high so that the ruler does not wish to steal the revenue, but they cannot be so high that households wish to pay zero taxes. If public spending is to increase, current rents decrease, and incentive compatibility for the ruler puts upward pressure on future rents and therefore on future taxes. Furthermore, if public spending decreases, current rents increase, and incentive compatibility for the households put downward pressure on current and future rents and on future taxes. Moreover, when incentive compatibility constraints do not bind, rents covary negatively with public spending under a balanced budget.

As an example, consider a deterministic economy with an increase in the productivity of public spending which lasts several periods. The model Lucas and Stokey (1983) predicts a flat tax rate.[19] This is illustrated in Figure 1.2. In this economy, revenue is constant, and the government accumulates assets in preparation for the spending increase and gradually depletes its assets during the spending increase. With the exception of debt, nothing fundamentally changes about the economy before and after the spending increase.

In an economy managed by a self-interested ruler, this strategy is not incentive compatible. The ruler cannot credibly pledge any accumulated assets towards public spending since any such assets would be used for rents, which means that government assets net of rents cannot increase prior to the increase in public spending.[20] The efficient way of paying for the more productive spending is therefore to raise taxes in the period *after* the increase in productivity, since raising taxes during the productivity increase does not relax the ruler's incentive compatibility constraint. This increase in taxes is best made permanent, since it implies a smaller and

[19]Since the economy is deterministic, then Barro (1979) also predicts a flat tax rate.

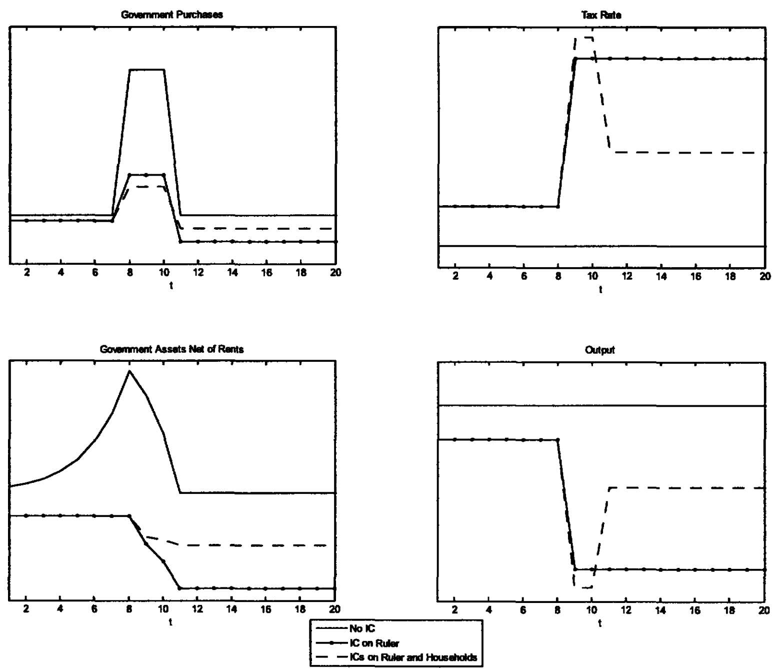
[20]We refer to assets and assets net of rents interchangeably. Mathematically, this corresponds to

$$-\sum_{k=t}^{\infty}\sum_{s^k \in S^k} \beta^{k-t}\pi\left(s^k|s^t\right)\left(R\left(n\left(s^k\right)\right) - g\left(s^k\right)\right),$$

which is uniquely determined by the program.

less distortionary change in the tax rate. This can be achieved by the government issuing debt during the spending increase, and this debt serves to relax even further the incentive compatibility constraint on the ruler in the future. Nevertheless, when the government becomes less productive, households may not accept this increase in taxes along with the increase in national debt. Therefore, taxes must to some extent decline after the spending increase so as to provide incentives for households.

Figure 1.2



In this example, when public spending is very productive, the government is wealthier and there is a greater need to provide incentives to the ruler, so that taxes and debt must increase and public spending must decrease. When public spending is not very productive, households are wealthier, and there is a greater need to provide incentives to households so that taxes and debt must decrease and public spending must increase. To the extent that it is possible, these changes should persist into the future. This is generally true in a setting with i.i.d. shocks.

Proposition 4 ***(i.i.d. Tax Intervals)*** *If* $\pi_{ks} = \pi_k \ \forall k, s \in S$, *then* $\underline{\tau}(s) < \underline{\tau}(k)$ *and* $\overline{\tau}(s) < \overline{\tau}(k)$ *if* $\theta(s) < \theta(k)$.

This means that under i.i.d. shocks, the tax rate weakly increases when θ increases and it weakly decreases when θ decreases, so that the current tax rate reflects the last binding incentive compatibility constraint. Therefore, the tax rate is more likely to change tomorrow if θ tomorrow significantly differs from θ today.

More generally, the tax rate reflects the history of incentive compatibility constraints, and it may not always be flat as in a setting without political economy constraints. This is because of the need to provide incentives and the fact that shocks to the productivity of public spending create variation in the tightness of incentive compatibility constraints. To simplify the discussion, consider the case for which the Markov process has full support.

Theorem 1 ***(Short Run Persistence)*** *If* $\pi_{ks} > 0 \ \forall s, k \in S$, $\exists V_0, s_0, s^t, s^k$ *s.t.* $s_t = s_k$ *and the solution to* (1.15) *admits* $\tau\left(s^t\right) \neq \tau\left(s^k\right)$.

This theorem establishes that there are initial conditions under which the tax rate experiences permanent changes after incentive compatibility constraints bind. Because incentive compatibility constraints are tied to the time path of shocks, tax rates and policies more generally depend on the history of shocks.

To understand why persistence is efficient, consider the solution to (1.15) subject $\tau\left(s^t\right) = \tau\left(s^{t-1}\right)$ and $\widetilde{g}\left(s^t\right) = \widetilde{g}\left(s^{t-1}\right)$ for all s^t and subject to V_0 chosen to be arbitrarily low. A tradeoff emerges between taxation and rent-seeking since a higher flat tax rate raises the amount of government resources yet it also increases the temptation by the ruler to steal. This puts upward pressure on rents so as to satisfy the incentive compatibility constraint of the ruler and it creates distortions which could instead be avoided by raising taxes only when it becomes necessary to do so.

1.4.3 Long Run

Figure 1.2 shows that if we ignore the incentive compatibility constraint of the household, the tax rate permanently adjusts following a one time shock. We use this observation to explore the long run properties of the tax rate. We examine whether it is possible for the tax rate to converge to a constant level in the long run so that this economy subject to political economy distortions is qualitatively similar to that of Lucas and Stokey (1983). From Proposition 3, we can see that the tax rate converges if the sustainable intervals for the tax rate have an overlapping region. Such a tax rate satisfies all incentive compatibility constraints.

Theorem 2 *(Long Run Convergence) If $\pi_{ks} > 0$ $\forall s, k \in S$, $\exists \beta^* \in (0,1)$ s.t. $\forall \beta > \beta^*$, the solution to* (1.15) *admits* $\lim_{t \to \infty} \tau_t = \tau^{LR}$.

Incentive provision for the ruler puts upward pressure on the tax rate and on debt under some shocks and incentive provision for the households puts downward pressure on the tax rate and on debt under some shocks. When both the ruler and the household are sufficiently patient, the ruler and the households can tolerate the same fixed tax rate under all shocks.

For instance, in the best sustainable competitive equilibrium for the households (with V_0 chosen arbitrarily low), the ruler accumulates debt along the equilibrium path, and households are sufficiently patient so as to accept the gradual increase in the tax rate and decrease in public spending which accompany the government's accumulation of debt. In the long run, this economy is qualitatively similar to an economy managed by a benevolent ruler but with more debt than that associated with a benevolent ruler. The long run behavior of this economy stands in contrast to that of Aiyagari, Marcet, Sargent, and Seppala (2002). They show that in an economy managed by a benevolent ruler without state-contingent debt, the government accumulates assets along the equilibrium path, until it is able to finance the entire stream of public spending with zero taxes.

In general, it is not possible to show that for intermediate values of β that the tax rate does not converge. The exception is if shocks are i.i.d. In this situation, it is the case that the tax intervals do not overlap for intermediate values of β, so that a flat tax rate cannot satisfy all incentive compatibility constraints. Given the updating rule in (1.27), this means that the tax rate is volatile and there is history dependence even in the long run.

Proposition 5 *(Long Run Volatility) If $\pi_{ks} = \pi_k$ $\forall k, s \in S$, $\exists \beta^{**} \in (0, \beta^*)$ s.t. $\forall \beta \in (\beta^{**}, \beta^*)$, the solution to* (1.15) *does not admit* $\lim_{t \to \infty} \tau_t = \tau^{LR}$ *and admits* $\tau_t > 0$ $\forall t \geq 1$.

Intuitively, households may tolerate a high tax rate when the government is very productive, but when the government ceases to be productive, they require a decrease in the tax rate. This decrease in the tax rate is nevertheless not sustainable if the government becomes productive again in the future, since the ruler's incentive compatibility constraint will bind. Therefore, even in the long run, this economy will differ from that of Lucas and Stokey (1983).

1.4.4 Predicting Tax Rate Movements

Our model predicts that tax rates should sometimes adjust persistently to shocks, and this is in line with what we observe empirically. As mentioned in the introduction, both Barro (1979) and Aiyagari, Marcet, Sargent, and Seppala (2002) also predict persistent tax rates, and they achieve this by ruling out state-contingent debt. A natural question is how the stochastic process of tax rates in our economy compares to theirs both along the equilibrium path and in the long run.

To simplify the discussion, let the shock θ map one to one with the state s, so that the observation of θ is equivalent to the observation of s. According to Lucas and Stokey (1983), the tax rate covaries one to one with θ (in the quasi-linear model, the covariance is zero), which means that tax rates tomorrow are best predicted by today's shock used to forecast tomorrow's shock:

$$\mathbf{E}\left(\tau_t|\tau_{t-1},...,\tau_0,\theta_{t-1},...,\theta_0\right) = \mathbf{E}\left(\tau_t|\theta_{t-1}\right).$$

In contrast, according to Barro (1979)'s intuitions, taxes are a random walk, which means that yesterday's tax rate alone can predict today's tax rate:

$$\mathbf{E}\left(\tau_t|\tau_{t-1},...,\tau_0,\theta_{t-1},...,\theta_0\right) = \mathbf{E}\left(\tau_t|\tau_{t-1}\right).$$

Our model combines features of both of these statistical processes. Given the updating rule in (1.27), one needs both past tax rates as well as past shocks in order to forecast tomorrow's tax rate:

$$\mathbf{E}\left(\tau_t|\tau_{t-1},...,\tau_0,\theta_{t-1},...,\theta_0\right) = \mathbf{E}\left(\tau_t|\tau_{t-1},\theta_{t-1}\right).$$

This statistical process for the tax rate in our model is qualitatively similar to that of Aiyagari, Marcet, Sargent, and Seppala (2002), even though there are no exogenous limits on the contingency of government debt in our model. The crucial distinction between our model and theirs is in the *long run* implications for the tax rate. In their model, the tax rate converges to zero and the government holds more assets in the long run than would be implied under a benevolent ruler with complete markets. In our model, the tax rate may not converge, and if it converges, it converges to a positive level, and the government holds more debt (net of rents) in the long run than would be implied under a benevolent ruler. Our model therefore links political economy to the endogenous incompleteness of markets, and it provides different implications for the long run behavior of the economy in comparison to a model which assumes exogenous incompleteness.

1.5 Numerical Example

We illustrate the mechanics of our model using a numerical example. Let

$$(\eta, \gamma, \alpha, b_0, V_0) = (.75, 2, .5, 0, 0), \ \beta = \{.95, .65\}, \text{ and } \theta_t = \{4, 5, 6\}.$$

$V_0 = 0$ is chosen so as to compare our results to an economy not subject to political economy distortions. Normalize the resource constraint of the economy so that $c + g + x = 10n$. The transition matrix for the three shocks is

$$\begin{bmatrix} .98 & .02 & 0 \\ .01 & .98 & .01 \\ 0 & .02 & .98 \end{bmatrix},$$

so that each shock is very persistent, and any path from the highest to the lowest shock (and vice versa) must pass through the middle shock. Let $\theta_0 = 4$.

Figure 1.3 considers an economy with $\beta = .95$ and describes the dynamics of policies for a realized sequence of θ shocks. We compare an economy under a benevolent ruler who extracts zero rents to an economy under a self-interested ruler. Under a benevolent ruler, public spending and government assets vary only with the state, and taxes and output are constant. Since revenue is constant, the government has low funds when θ is low and has high funds when θ is high. This economy is however not incentive compatible for a self-interested ruler. Increasing assets when θ increases is not incentive compatible since the ruler prefers to use these assets personally. Therefore, there is a need to increase in the tax rate when θ increases, which decreases output. Because the discount factor is high, the household's incentive compatibility constraints do not bind and these increases can be made permanent. This means that along the equilibrium path, policies reflect the last binding incentive compatibility constraint on the ruler, so that they depend on the entire history of shocks. In the long run, the tax rate reaches a maximum and the economy is qualitatively identical to one managed by a benevolent ruler.

Figure 1.3: $\beta = .95$

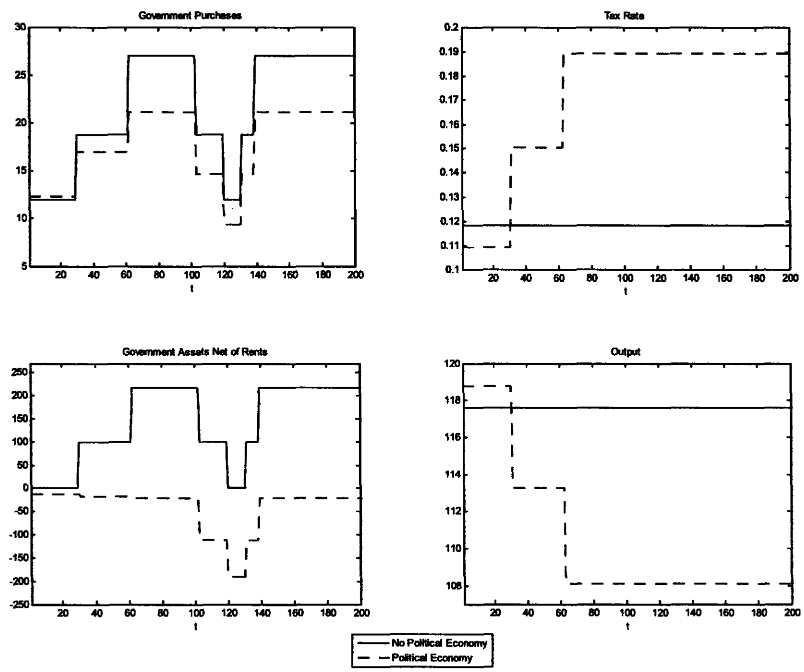
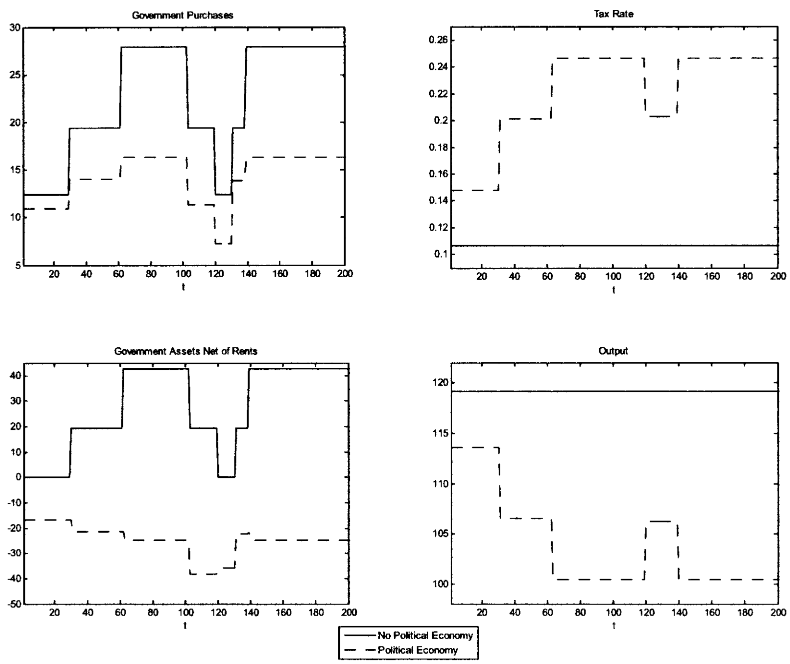


Figure 1.4 considers an economy with $\beta = .65$. The dynamics of an economy under a benevolent ruler are similar to those under $\beta = .95$. In contrast to the economy under a self-interested ruler in Figure 1.3, the household's incentive compatibility constraint binds in the lowest state in a transition path from the high state to the low state. As a consequence, the tax rate in the middle state depends on whether the highest or the lowest state occurred most recently. This means that even in the long run, the tax rate and output continue to be volatile and continue to reflect the history of shocks. Policies more generally also reflect the history of

shocks.

Figure 1.4: $\beta = .65$



In Figures 1.5 and 1.6, we compare the long run properties of our economy to that of Aiyagari, Marcet, Sargent, and Seppala (2002) for which state contingent debt is not available. While the tax rate exhibits persistence in their economy along the equilibrium path, in the long run, the government accumulates assets worth over several multiples of GDP and the tax rate converges to zero and output and public spending converge to first best. In our economy, the government does not accumulate such a large asset position since it is politically unsustainable and since it is economically unnecessary under complete markets. Moreover, the long run tax rate is positive in our economy.

Figure 1.5: Long Run Comparison to Incomplete Markets $\beta = .95$

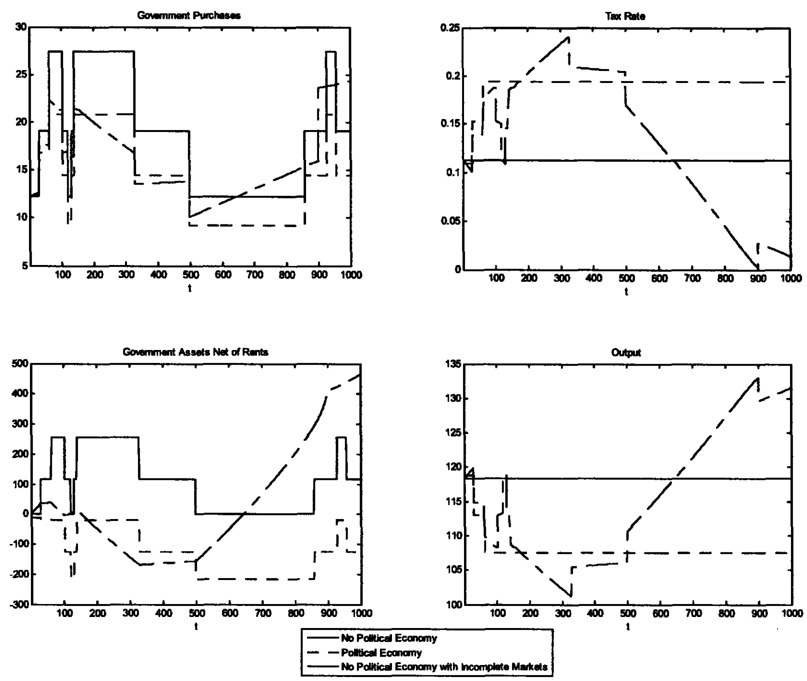
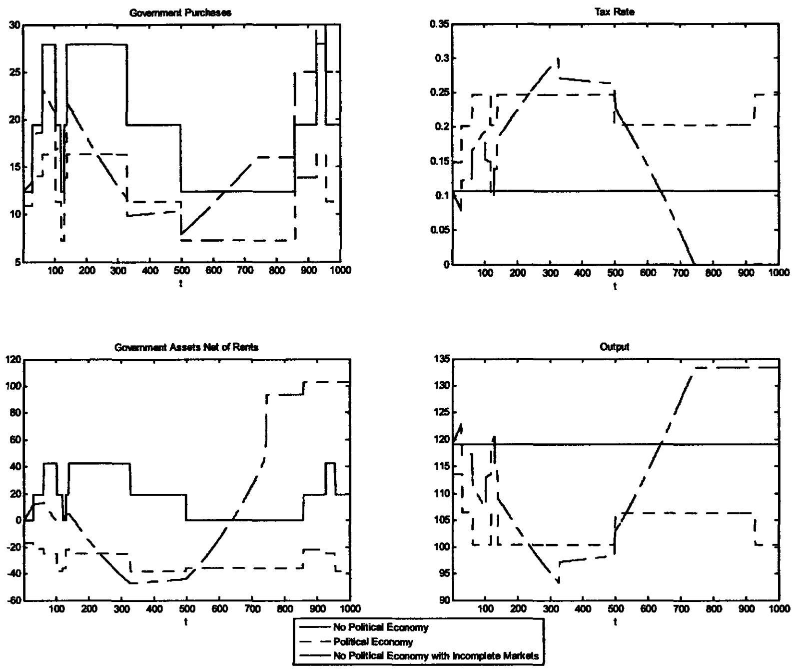


Figure 1.6: Long Run Comparison to Incomplete Markets $\beta = .65$



In Table 1, we calculate the additional units of total consumption required to make a household indifferent between an economy with political economy constraints and an economy without political economy constraints. These additional units are measured relative to the value of total consumption in an economy not subject to political economy distortions. For comparison, we calculate the welfare cost of constraining the benevolent government to balance the budget in every period and the welfare cost of constraining the benevolent government to trading non-contingent debt. This table suggests that the cost of political economy is low in absolute terms, though it is slightly higher than the cost of running a balanced budget or of not using contingent debt. Our result is in many ways not so surprising given that rents are so low in equilibrium (they have a present discounted value of 0). Therefore, any distortions emerging from political economy will be due to the endogenous constraints on the use of financial markets, and these distortions are not very large.

The spirit of this exercise is depicted in Figure 1.7. The y-axis represents the welfare of the households and the x-axis represent the welfare of the ruler. The competitive frontier represents the frontier of the set of possible values to the households and to the ruler when incentive compatibility constraints are ignored. The best competitive policy corresponds to the policy of Lucas and Stokey (1983) which sets $V_0 = 0$. The sustainable frontier represents the frontier of the set of possible values to the households and to the ruler when incentive compatibility constraints are taken into account. The best sustainable policy corresponds to the policy described in this model subject to $V_0 = 0$. Welfare costs are calculated as distances on the y-axis.

We also calculate the cost of choosing suboptimal policies appropriate to the benevolent ruler in an environment subject to political economy distortions. Specifically, we consider the solution to (1.15) subject to $\tau\left(s^t\right) = \overline{\tau}\ \forall s^t$ and $\widetilde{g}\left(s^t\right) = \overline{\widetilde{g}}\ \forall s^t$, taking into account that a fixed rent $\overline{x}$ may be generated from this exercise. This policy generates welfare inside the sustainable frontier in Figure 1.7. Table 1.1 suggests that the cost of inappropriate policies is very large, and Table 1.2 illustrates the source of inefficiency. Consider the case with $\beta = .95$. While $\left(\overline{\tau}, \overline{\widetilde{g}}, \overline{x}\right)$ are $(.12, .75, 0)$ under a benevolent ruler, they are $(.07, .24, 4)$ under a self-interested ruler, so that the government collects less revenue, implements fewer public projects, but wastes more resources on rents. This is because the self-interested ruler cannot be trusted to pledge accumulated revenues for public use, and this simultaneously limits the size of the government while making the government more wasteful.[21]

Table 1.1: Welfare Costs

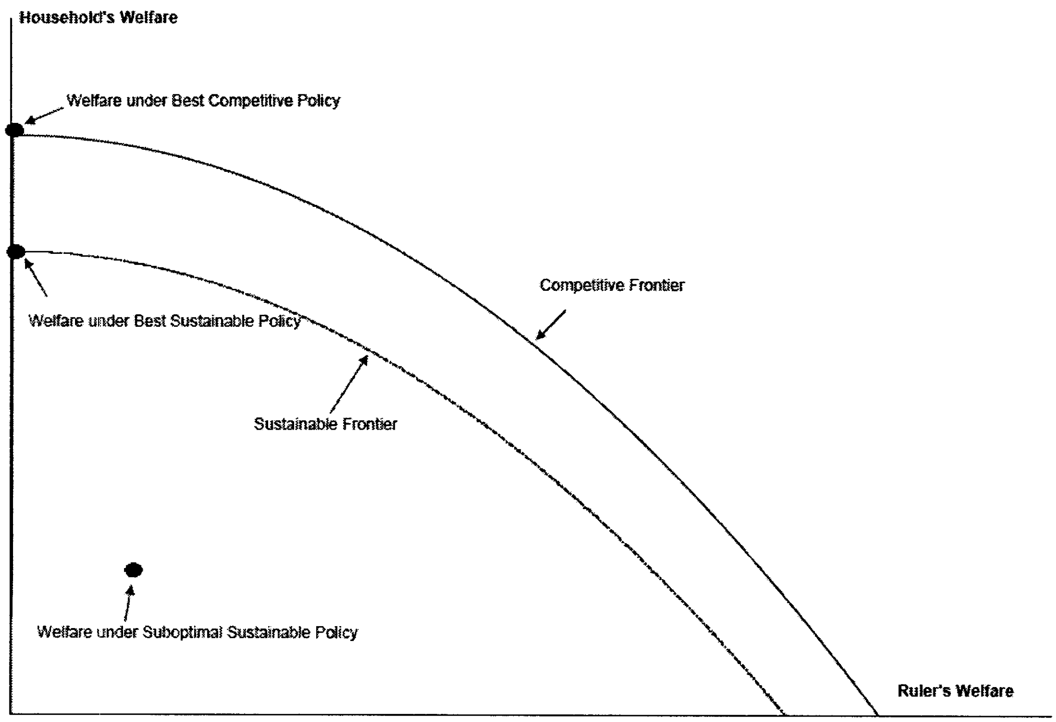
	$\beta = .95$	$\beta = .65$
Balanced Budget	0.07%	0.01%
Incomplete Markets (AMSS)	0.04%	0.01%
Best Sustainable Policy (This Model)	0.13%	0.48%
Suboptimal Sustainable Policy (Hypothetical)	7.48%	9.92%

[21] The pure transfer cost of the rents is 3.8% for $\beta = .95$ and 3.0% for $\beta = .65$.
AMSS refers to Aiyagari, Marcet, Sargent, and Seppala (2002).

Table 1.2: Policies $\left(\overline{\tau}, \overline{\overline{g}}, \overline{x}\right)$

	$\beta = .95$	$\beta = .65$
Best Competitive Policy	$(.12, .75, 0)$	$(.11, .78, 0)$
Suboptimal Sustainable Policy	$(.07, .24, 4)$	$(.04, .08, 3.12)$

Figure 1.7



1.6 Application: Government Budget Windfall

In practice, a government has multiple sources of revenue outside of wage income. In many economies, the government owns natural resources which it can use to service debts and to pay for public projects, and the size of these resources fluctuates with economic shocks. Optimal policy response in this setting is the subject of tremendous policy debates, and a natural question is what political economy implies for policy.

To simplify our discussion, consider again the economy of Section 1.2. There are 2 states and let $\theta(1) = \theta(2) = 1$, so that public spending is always as productive. Associated with each state is a government endowment of size $e(s^t) > 0$. After s_t is realized, households choose τ_t and they choose whether to appropriate the endowment from the ruler. Let the latter decision be denoted by $P_t = \{0, 1\}$ with $P_t = 0$ representing appropriation. If $P_t = 1$, $e(s_t)$ enters additively on the right hand side of the government budget constraint (1.4). If $P_t = 0$, $e(s_t)$ enters additively on the right hand side of (1.3). Let $e(1) < e(2)$, and let $\pi_{11} = \pi_{22} = \pi > 1/2$, so that shocks are persistent. Let $E(s_t) = e(s_t) + \beta \sum_{s_{t+1}=1,2} \pi(s_{t+1}|s_t) E(s_{t+1})$, so that $E(1) < E(2)$.

Due to space restrictions, and for simplicity, we describe efficient equilibria subject to $P_t = 1$ for all t, so that appropriation of the endowment by the households does not occur along the equilibrium path. This allows us to redefine the revenue function $R(n)$ and the value of autarky U^{AUT} as:

$$\begin{aligned} \widehat{R}(n,s) &= n - \eta\frac{n^{\gamma}}{\gamma} + e(s) \\ \widehat{U}^{AUT}(s) &= U^{AUT} + E(s) \end{aligned}$$

We consider equilibria under which $P_t = 1$ for all t. Off the equilibrium path, the ruler loses access to tax revenue and to the endowment forever, and his best deviation it to take the tax revenue and the endowment prior to the punishment. Off the equilibrium path, households never receive public projects, and their best deviation is to choose zero taxes and to appropriate the national resource from the ruler forever. An equivalent proposition to Proposition 2 holds under $\widehat{R}(n,s)$ and $\widehat{U}^{AUT}(s)$, and the problem can be written recursively as in $(1.18) - (1.22)$ and we do not do so here to preserve space.

The economy managed by a benevolent ruler maintains a flat tax rate with fixed public spending so that Remark 2 applies. When the endowment increases, the government runs a surplus and when the endowment decreases, it runs a deficit. This however is not the case in an economy managed by a self-interested ruler, since the tax rate fluctuates according to the same rules as in Proposition 3. We can determine how the tax intervals $[\underline{\tau}(s), \overline{\tau}(s)]$ and public spending intervals $\left[\underline{\tilde{g}}(s), \bar{\tilde{g}}(s)\right]$ depend on s.

Proposition 6 $\underline{\tau}(2) < \underline{\tau}(1)$, $\overline{\tau}(2) < \overline{\tau}(1)$, $\underline{\tilde{g}}(2) > \underline{\tilde{g}}(1)$, *and* $\bar{\tilde{g}}(2) > \bar{\tilde{g}}(1)$.

This means that the tax rate must increase and public spending decrease when the endowment decreases. The reason is that there are not enough funds in the national treasury to pay for public spending since it is not possible for the government to effectively save, so that the tax rate must adjust. Moreover, when the endowment increases, households demand that the ruler use this resource as opposed to taxes to service government debts and to pay for public spending, so that it is efficient for the tax rate to decrease and for public spending to increase. Therefore, fiscal policy is procyclical in the shock to the national resource. While a flat tax rate with flat public spending is efficient in an economy managed by a benevolent ruler, in this economy a flat tax rate would cause too many resources to be diverted towards rents, and it is important for the tax rate to respond to shocks so as to be able to provide the right incentives to the ruler and to households.[22]

1.7 Extension: Risk Averse Preferences

1.7.1 Model and Main Result

In this section we show that the main insights presented in our quasi-linear economy translate to an economy with risk averse preferences. Let

$$u(c, n, g, s) = h(c, n) + z(g, s)$$

[22]For other theoretical work on the cyclicality of fiscal policies, see Aizenman, Gavin, and Hausmann (1996), Alesina and Tabelini (1990), Lane and Tornell (1999).

for $u(c, n, g, s)$ which is increasing in c, decreasing in $n \geq 0$, and increasing in $g \geq 0$, and which is globally concave. The ruler has an increasing and concave utility function $v(x)$. As in the quasi-linear model, x may be arbitrarily negative so that government debts are always guaranteed. c may be constrained to be non-negative or may be arbitrarily negative, since household debts are guaranteed by the fact that n can be chosen to be large. Without any loss of generality, we constrain c to be weakly positive. Let $\lim_{c \to 0} u_c(\cdot) = -\lim_{n \to \infty} u_n(\cdot) = \lim_{g \to 0} u_g(\cdot) = \lim_{x \to -\infty} v'(\cdot) = \infty$, and let $\lim_{c \to \infty} u_c(\cdot) = \lim_{n \to 0} u_n(\cdot) = \lim_{g \to \infty} u_g(\cdot) = \lim_{x \to \infty} v'(\cdot) = 0$ so that the solution is interior. Furthermore, $u_{cc}(\cdot) + u_{cn}(\cdot) < 0$ and $u_{nn}(\cdot) + u_{cn}(\cdot) < 0$ so that leisure is a normal good.

As in the quasi-linear economy, in the efficient sustainable competitive equilibrium, any deviation by the ruler results in the worst punishment for the ruler, and this punishment is a function of the equilibrium tax rate and the debt portfolio of the government in the period of deviation. In principle, one can compute this punishment and the efficient sustainable competitive equilibrium using the recursive techniques of Abreu, Pearce, Stacchetti (1990). Nevertheless, the theoretical characterization of this punishment cannot be achieved given the additional difficulties related to the fact that the ruler can manipulate the price of contingent claims off the equilibrium path. In order to simplify our discussion, we exclude long term debt, and we leave the study of how debt maturity can be altered to provide the right incentives for future research.[23]

In our economy, off the equilibrium path, the ruler can always choose to set public spending to zero and to trade zero debt forever. Since tax revenue is weakly positive, his consumption in the periods following the period of deviation is weakly positive. Therefore, off the equilibrium path, his continuation value can never be below

$$\underline{V}(\tau_t, b_{t-1}(s_t)) = v(\tau_t \widetilde{n}(\tau_t, b_{t-1}(s_t)) - b_{t-1}(s_t)) + \beta \frac{v(0)}{1-\beta}$$

for $\widetilde{n}(\tau_t, b_{t-1}(s_t))$ which represents the labor market decision of households who are not trading state contingent claims for the future conditional on $(\tau_t, b_{t-1}(s_t))$. In the Appendix, we characterize $\widetilde{n}(\tau_t, b_{t-1}(s_t))$. Given that $\underline{V}(\tau_t, b_{t-1}(s_t))$ is always achievable, there cannot exist

[23]Krusell, Martin, and Rios-Rull (2006) discuss the commitment problems faced by a benevolent ruler who can manipulate interest rates.

a sustainable competitive equilibrium which delivers a continuation value to the ruler which is below $\underline{V}(\tau_t, b_{t-1}(s_t))$. Without additional structure, however, we cannot characterize the off equilibrium strategies and beliefs of households which would generate $\underline{V}(\tau_t, b_{t-1}(s_t))$ in a sustainable competitive equilibrium.[24]

In order that $\underline{V}(\tau_t, b_{t-1}(s_t))$ be sustainable, we assume that households have the ability to collectively decide to stop trading claims with the ruler. Formally, households choose $L_t = \{0, 1\}$ so that the relevant state-contingent debt traded in period t becomes $L_t b_t(s_{t+1})$. Furthermore, households have the ability to commit to their strategy, so that we can ignore their incentive compatibility constraint. This means that off the equilibrium path, it is possible to achieve $\underline{V}(\tau_t, b_{t-1}(s_t))$ if households choose zero taxes and choose to trade zero debt forever.

The order of events is as follows:

1. Nature chooses the state s_t.
2. Households choose the tax rate τ_t.
3. The ruler chooses non-tax policies ρ_t.
4. Households choose trading decision L_t.
5. Markets open and clear.

Define:

$$\Gamma(c, n) = h_c(c, n)\, c - h_n(c, n)\, n.$$

$\Gamma(c, n)$ represents the primary surplus of the government multiplied by the household's marginal utility of consumption, and it is useful in the characterization of debt. In the quasi-linear economy $\Gamma(c, n) = c - n + R(n) = R(n) - g - x$. For simplicity, we focus on equilibria for which $b_{-1}(s^0) = 0$, so that the government holds zero initial liabilities.

[24]In the quasi-linear economy, $\underline{V}(\tau_t, b_t(s_{t+1}))$ is sustainable since state prices are exogenous.

Proposition 7 *ξ is a sustainable competitive equilibrium if and only if it satisfies* (1.9),

$$\sum_{t=0}^{\infty}\sum_{s^t\in S^t}\beta^t\pi\left(s^t\right)\Gamma\left(c\left(s^t\right),n\left(s^t\right)\right)=0, \tag{1.29}$$

$$\sum_{k=t}^{\infty}\sum_{s^k\in S^k}\beta^{k-t}\pi\left(s^k|s^t\right)v\left(x\left(s^k\right)\right)\geq\underline{V}\left(\tau\left(s^t\right),b\left(s^t|s^{t-1}\right)\right)\ \forall s^t, \tag{1.30}$$

$$\text{and } \tau\left(s^t\right)=1+\frac{h_n\left(c\left(s^t\right),n\left(s^t\right)\right)}{h_c\left(c\left(s^t\right),n\left(s^t\right)\right)}\in[0,1] \tag{1.31}$$

$$\text{for } \sum_{t=k}^{\infty}\sum_{s^k\in S^k}\beta^{k-t}\pi\left(s^k|s^t\right)\Gamma\left(c\left(s^k\right),n\left(s^k\right)\right)=b\left(s^t|s^{t-1}\right)\forall s^t \tag{1.32}$$

Equations (1.29), (1.30), (1.31), and (1.32) are analogous to (1.10), (1.12), (1.14), and (1.11), respectively.

With some abuse of notation, let Λ represent the set of allocations which satisfy (1.9) and (1.29) – (1.32). We can write the general program as:

$$\max_{\xi}U\left(\xi\right)\ \text{s.t.}\ V\left(\xi\right)\geq V_0,\ \xi\in\Lambda. \tag{1.33}$$

The solution to (1.33) which ignores (1.30) and subject to $x\left(s^t\right)=0$ corresponds to the solution for the Lucas and Stokey (1983) economy. Such an allocation varies with respect to the state s_t and not the history of states.[25] In the Appendix, we use the methods of Marcet and Marimon (1998) to characterize the solution to (1.33), and we prove an analogous version to Theorem 1. To make the problem interesting, we consider an economy in which taxes are not 1 and are not 0 forever.

Assumption 2 The solution to (1.33) admits $\tau\left(s^t\right)<1$ for all s^t and $\tau\left(s^t\right)>0$ for some s^t.

Assumption 3 The solution to (1.33) admits $\underline{V}_\tau\left(\tau\left(s^0\right),0\right)\neq 0$.

[25]If the program is non-concave, lotteries can be used to improve the allocation, and the *expected* allocation only depends on the state. Our results are robust to the introduction of lotteries and we ignore this possibility due to space constraints.

Assumption 3 means that the initial tax rate is not chosen at a critical point on the Laffer curve. It is always guaranteed to be true in economies for which the Laffer curve is upward sloping. In environments in which this is not the case, small perturbations to the economy yield a tax rate not at this critical point. We can prove an analogous result to Theorem 1.

Theorem 3 *If $\pi_{ks} > 0$ $\forall s, k \in S$, $\exists V_0, s_0, s^t, s^k$ s.t. $s_t = s_k$ and the solution to* (1.33) *admits*

$$\left\{c\left(s^t\right), n\left(s^t\right), g\left(s^t\right), x\left(s^t\right)\right\} \neq \left\{c\left(s^k\right), n\left(s^k\right), g\left(s^k\right), x\left(s^k\right)\right\}.$$

1.7.2 Numerical Example

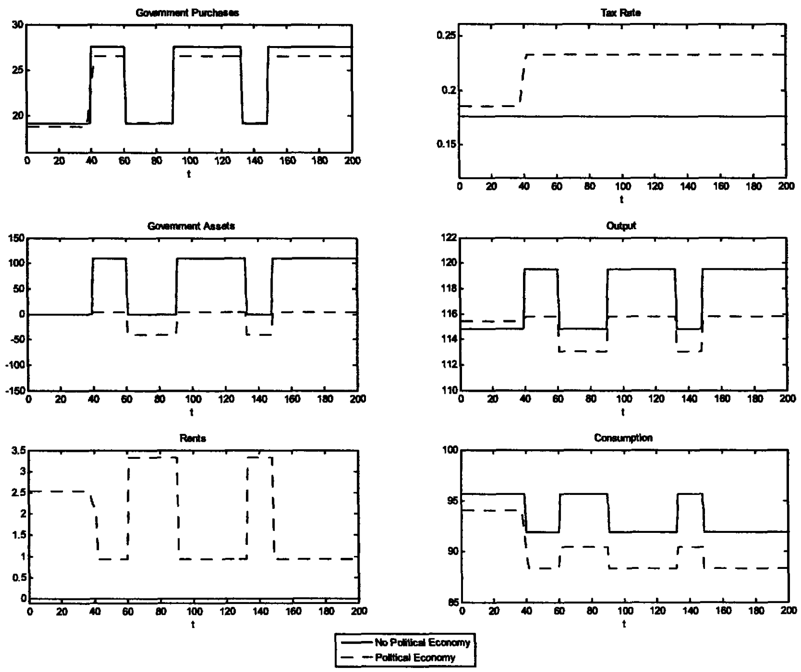
We provide a numerical example to illustrate the mechanics of the model under risk averse preferences. Let

$$\begin{aligned} u\left(c, n, g, s\right) &= \log c - \eta \frac{n^\gamma}{\gamma} + \theta\left(s\right) \log g \\ v\left(x\right) &= -\frac{1}{\sigma} \exp\left(-\sigma x\right). \end{aligned}$$

Let $(\eta, \gamma, \sigma, \beta) = (.75, 2, 1, .95)$ and set $V_0 = 0$. Normalize the resource constraint so that $c + g + x = 100n$. Let $\theta_t = \{.2, .3\}$ and $\Pr\{\theta_t = \theta_{t-1}\} = .99$. The dynamics of this economy are illustrated in Figure 1.8. As in the risk neutral case, the tax rate is flat under a benevolent government, though output now responds to public expenditure shocks because of the need to smooth consumption since households are risk averse.

In the economy managed by a self-interested ruler, debt as well as rents are increased in order to provide incentives to the ruler in response to shocks, and this causes the tax rate to increase along the equilibrium path. Moreover, responses to shocks occur somewhat more gradually given that agents care about intertemporal consumption smoothing. In the long run, the ruler receives greater rents when public spending is low and when the economy is less constrained, and the tax rate is flat as it would be under a benevolent government.

Figure 1.8



1.8 Conclusion

In this paper, we have developed a theoretical framework which studies the optimal management of taxes and debt in an environment subject to political economy distortions. In doing this, we argued that these distortions manifest themselves in the form of endogenous debt limits on the government, and that these distortions serve as a channel for macroeconomic persistence and volatility. Our model predicts that in an environment subject to political economy distortions, taxes will respond persistently to shocks even though financial markets are complete, and long run taxes will be positive, which is in contrast to an economy with exogenously incomplete financial markets. Our numerical simulations provide us with a lesson. The cost of political economy distortions is not very high subject to the government choosing optimal sustainable taxes which respond persistently to shocks. Nevertheless, suboptimal sustainable taxes which resemble those under a benevolent government and which do not respond persistently to shocks are very costly. This suggests that it is important to make policy prescriptions which take political realities into account.

Our analysis leaves some natural directions for future research. We have assumed the perfect observability of the ruler's actions, although in practice rent seeking is a private activity. Relaxing this assumption would generate even further distortions in our economy and provide more limits on financial markets, though it may generate different long run implications. Second, our model ignores the important interaction between fiscal policy and monetary policy by focusing on the real economy. We plan to explore these extensions in future research.

1.9 Appendix

1.9.1 Proofs of Section 1.3

Proof of Proposition 1

Let $b\left(s^t|s^{t-1}\right)$ represent a bond traded at s^{t-1} with a payment contingent on the realization of s^t. The intratemporal and intertemporal conditions for the household at s^t, respectively, are:

$$\eta n\left(s^t\right)^{\gamma-1} = 1-\tau\left(s^t\right) \tag{1.34}$$

$$q\left(s^{t+1}|s^t\right) = \beta\frac{\pi\left(s^{t+1}\right)}{\pi\left(s^t\right)}. \tag{1.35}$$

(1.34) implies the function $R(n)$. (1.9) follows from (1.6). For the necessity of (1.10), let $q\left(s^t\right) = q\left(s^t|s^{t-1}\right) \times ... \times q\left(s^1|s^0\right)$. By (1.35), $q\left(s^t\right) = \beta^t\pi\left(s^t\right)$. Substitute $\beta\pi\left(s^{t+1}|s^t\right)$ and $R\left(n\left(s^t\right)\right)$ into (1.4) at s^t, multiply both sides of (1.4) at s^t by $\beta^t\pi\left(s^t\right)$, and take the sum of all constraints (1.4) subject to the transversality condition implied by (1.7)

$$\lim_{t\to\infty}\beta^t\pi\left(s^t\right)b\left(s^t|s^{t-1}\right) = 0, \tag{1.36}$$

to achieve (1.10). Similar arguments imply (1.11). For sufficiency, choose $\tau\left(s^t\right)$ which satisfies (1.34). Let $b^g\left(s^t|s^{t-1}\right) = -\,b^h\left(s^t|s^{t-1}\right) = b\left(s^t|s^{t-1}\right)$ satisfy (1.11). (1.4) is satisfied. Given (1.9), (1.3) is satisfied by Walras's law. **Q.E.D.**

Proof of Proposition 2

A sustainable competitive equilibrium must be competitive so as to satisfy (1.9) and (1.10). It must furthermore satisfy (1.14) so that $\tau\left(s^t\right) \in [0,1]$. For sufficiency, construct the following equilibrium. Any deviation from the prescribed $\rho\left(s^t\right)$ or the prescribed $\tau\left(s^t\right)$ results in a permanent reversion to the static equilibrium in which $\tau_t = g_t = 0$ forever. By (1.11), the ruler's continuation value from choosing his best deviation of $g'\left(s^t\right) = 0$ at s^t is equal to $R\left(n\left(s^t\right)\right) - b\left(s^t|s^{t-1}\right)$, and this deviation is weakly dominated by (1.12). By (1.9) and (1.11), the household's continuation value from choosing its best deviation of $\tau'\left(s^t\right) = 0$ at s^t is equal to $U^{AUT} + b\left(s^t|s^{t-1}\right)$, and this deviation is weakly dominated by (1.13).

For necessity, let $\underline{V}(s,b,\tau)$ represent the lowest continuation value achievable by the ruler conditional on $s_t = s$, $b_{t-1}(s_t) = b$, and $\tau_t = \tau$. Let R represent the revenue associated with τ. At (s,b,τ), the ruler can always choose $\rho' = \left\{\{0\}_{s_{t+1}\in S}, 0, R - b\right\}$. Sustainability of $\underline{V}(s,b,\tau)$ requires

$$\underline{V}(s,b,\tau) \geq R - b + \beta \sum_{k\in S} \pi_{ks} \underline{V}\left(k, 0, \tau'_k(s,b,\tau)\right), \tag{1.37}$$

for $\tau'_k(s,b,\tau)$ which represents the household's response to the ruler's deviation. Forward iteration on (1.37) implies that $\underline{V}(s,b,\tau) \geq R - b$, since $R \geq 0$, which implies (1.12). Let $\underline{U}(s,b)$ represent the lowest continuation value achievable by the households conditional on $s_t = s$ and $b_{t-1}(s_t) = b$. Households at (s,b) can always choose $\tau' = 0$. Using (1.3), (1.5), and (1.35), the sustainability of $\underline{U}(s,b)$ requires

$$\begin{aligned} \underline{U}(s,b) \geq\; & n^{fb} - \eta \frac{n^{fb^{\gamma}}}{\gamma} + \theta(s) \frac{g'(s,b)^{\alpha}}{\alpha} + b \\ & + \beta \sum_{k\in S} \pi_{ks} \left(\underline{U}\left(k, b'_k(s,b)\right) - b'_k(s,b) \right), \end{aligned} \tag{1.38}$$

for $g'(s,b)$ and $b'_k(s,b)$ which represent the ruler's response to the household's deviation. Forward iteration on (1.38) taking (1.36) into account implies that $\underline{U}(s,b) \geq U^{AUT} + b$, which implies (1.13). **Q.E.D.**

1.9.2 Proofs of Section 1.4

Proof of Lemma 1

(i) Consider ξ' s.t. $c'(s^t) = n'(s^t) - g'(s^t) - x'(s^t)$, $n'(s^t) = n(s^t)$, $g'(s^t) = g(s^t)$, and $x'(s^t) = \overline{x} = (1-\beta)\sum_{t=0}^{\infty}\sum_{s^t\in S^t}\beta^t\pi(s^t)x(s^t)$. Since $\sum_{t=0}^{\infty}\sum_{s^t\in S^t}\beta^t\pi(s^t)c'(s^t) = \sum_{t=0}^{\infty}\sum_{s^t\in S^t}\beta^t\pi(s^t)c(s^t)$ and $\sum_{t=0}^{\infty}\sum_{s^t\in S^t}\beta^t\pi(s^t)x'(s^t) = \sum_{t=0}^{\infty}\sum_{s^t\in S^t}\beta^t\pi(s^t)x(s^t)$, then $U(\xi') = U(\xi)$ and $V(\xi') = V(\xi)$. ξ' satisfies (1.9) and (1.10) since ξ satisfies them. If ξ' does not satisfy (1.12) and/or (1.13), then it does not satisfy (1.16) and/or (1.17), but this contradicts the fact that ξ satisfies (1.16) and (1.17).

(ii) Consider ξ'' s.t. $c''(s^t) = n''(s^t) - g''(s^t) - x''(s^t)$, $n''(s^t) = n(s^t)$, $g''(s^t) = g(s^t)$, $x''(s^t) = R(n''(s^t)) - g''(s^t)$ $\forall s^t$ and $t \geq 1$, and $x''(s^0) = R(n''(s^0)) - g''(s^0) - b_{-1}(s^0)$. Since $\sum_{t=0}^{\infty}\sum_{s^t\in S^t}\beta^t\pi(s^t)c''(s^t) = \sum_{t=0}^{\infty}\sum_{s^t\in S^t}\beta^t\pi(s^t)c(s^t)$ and $\sum_{t=0}^{\infty}\sum_{s^t\in S^t}\beta^t\pi(s^t)x''(s^t) =$

$\sum_{t=0}^{\infty}\sum_{s^t\in S^t}\beta^t\pi\left(s^t\right)x\left(s^t\right)$, then $U\left(\xi''\right) = U\left(\xi\right)$ and $V\left(\xi''\right) = V\left(\xi\right)$. ξ'' satisfies (1.9) and (1.10) since ξ satisfies them. If ξ'' does not satisfy (1.12) and/or (1.13), then it does not satisfy (1.16) and/or (1.17), but this contradicts the fact that ξ satisfies (1.16) and (1.17).

Q.E.D.

Additional Lemmas

Define

$$R^{\max} = \max_{n} R\left(n\right)$$

associated with maximizer $n^{\max}$.

Lemma 3 *The solution to* (1.15) *sets* $n\left(s^t\right) \geq n^{\max} = \left(\frac{1}{\gamma\eta}\right)^{1/(\gamma-1)}$ $\forall s^t$.

Proof. *Consider a solution ξ for which $n\left(s^t\right) < n^{\max}$ for some s^t. Then $R\left(n\left(s^t\right)\right) \geq 0$, since the range of $R\left(\cdot\right)$ over the feasible domain $\left[0, n^{\max}\right)$ is $\left[0, R^{\max}\right)$. Consider ξ' identical to ξ with the exception that $n'\left(s^t\right) \in \left[n^{\max}, n^{fb}\right]$ which solves $R\left(n'\left(s^t\right)\right) = R\left(n\left(s^t\right)\right)$ and let $c'\left(s^t\right) = c\left(s^t\right) + \left(n'\left(s^t\right) - n\left(s^t\right)\right)$, which is possible by the fact that the range of $R\left(\cdot\right)$ over the domain $\left[n^{\max}, n^{fb}\right]$ is $\left[R^{\max}, 0\right]$. By* (1.11), *$b\left(s^t|s^{t-1}\right)$ is unchanged $\forall s^t$. Therefore, $\xi' \in \Lambda$. However, $U\left(\xi'\right) > U\left(\xi\right)$ and $V\left(\xi'\right) = V\left(\xi\right)$, so that ξ is not the solution.* ■

By Lemma 3, $R\left(n\right)$ is decreasing over the relevant range $n \in \left[n^{\max}, n^{fb}\right]$. Define $\widehat{n}\left(R\right)$ as the value of labor associated with revenue R. Define

$$\widehat{u}\left(R\right) = \widehat{n}\left(R\right) - \eta\frac{\widehat{n}\left(R\right)^{\gamma}}{\gamma}. \tag{1.39}$$

Lemma 4 $\widehat{u}''\left(R\right) < 0$, $\widehat{u}'\left(R^{\max}\right) = -\infty$ *and* $\widehat{u}'\left(0\right) = 0$.

Proof. *By the implicit function theorem,*

$$\widehat{u}'\left(R\right) = \frac{1 - \eta\widehat{n}\left(R\right)^{\gamma-1}}{1 - \eta\gamma\widehat{n}\left(R\right)^{\gamma-1}} < 0, \tag{1.40}$$

implying

$$\widehat{u}''\left(R\right) = \frac{\left(\gamma-1\right)^2\eta\widehat{n}\left(R\right)^{\gamma-2}}{\left(1 - \eta\gamma\widehat{n}\left(R\right)^{\gamma-1}\right)^2}\widehat{n}'\left(R\right) = \frac{\left(\gamma-1\right)^2\eta\widehat{n}\left(R\right)^{\gamma-2}}{\left(1 - \eta\gamma\widehat{n}\left(R\right)^{\gamma-1}\right)^2}\frac{1}{1 - \eta\gamma\widehat{n}\left(R\right)^{\gamma-1}} < 0,$$

and the inequalities follow from $\widehat{n}(R) \in [n^{\max}, n^{fb}]$. *Plugging* $n^{\max}$ *and* n^{fb} *into* (1.40) *yields* $\widehat{u}'(R^{\max}) = -\infty$ *and* $\widehat{u}'(0) = 0$, *respectively.* ■

Lemma 5 *Any solution to* (1.15) *is a solution to a modification of* (1.15) *which replaces* (1.10) *with its slack form*

$$\sum_{t=0}^{\infty} \sum_{s^{kt} \in S^t} \beta^t \pi(s^t) \left(R\left(n\left(s^t\right)\right) - g\left(s^t\right) - x\left(s^t\right)\right) \geq b_{-1}\left(s^0\right). \tag{1.41}$$

Proof. *By Lemma 3,* $n(s^t) \in [n^{\max}, n^{fb}]$. *Consider the solution* ξ *to the relaxed program. Let* (1.41) *not bind. Using* (1.12), *substitute in for* $\sum_{t=0}^{\infty} \sum_{s^{kt} \in S^t} \beta^t \pi(s^t) x(s^t)$ *in* (1.41) *to achieve*

$$\sum_{t=0}^{\infty} \sum_{s^{kt} \in S^t} \beta^t \pi(s^t) \left(R\left(n\left(s^t\right)\right) - g\left(s^t\right)\right) > R\left(n\left(s^t\right)\right). \tag{1.42}$$

If $n(s^0) < n^{fb}$, *then* $\exists \xi'$ *identical to* ξ *with the exception that* $n'(s^0) = n(s^0) + \epsilon$ *and* $c'(s^0) = c(s^0) + \epsilon$ *for* $\epsilon > 0$ *which is sufficiently small which strictly improves welfare and satisfy all constraints, since* (1.42) *is an inequality. Therefore,* $n(s^0) = n^{fb}$. *Analogous arguments imply that* $g(s^0) = g^{fb}(\theta(s_0))$ *for* $g^{fb}(\theta) = \theta^{1/(1-\alpha)}$, *the first best level of public spending, and that* $n(s^1) = n^{fb}$. *Since* $\beta R(n^{fb}) - g^{fb}(\theta(s)) < 0$, *then for* (1.42) *to hold,* (1.16) *is slack for some* s^1. *We refer to all* s^1 *for which* (1.16) *is slack as* $\widehat{s}^1$. *Now consider some* $\widehat{s}^k$ *for* $k \geq 1$ *for which* (1.16) *does not bind for all subhistories* $\widehat{s}^l \subset \widehat{s}^k$ *for* $l < k$ *(histories along the path which lead to* $\widehat{s}^k$*). If* (1.16) *does not bind at* $\widehat{s}^k$, *then* $n(\widehat{s}^{k+1}) = n^{fb}$ *and* $g(\widehat{s}^k) = g^{fb}(\theta(\widehat{s}_k))$ *by analogous reasoning to the* $t = 0$ *case. Also,* (1.16) *does not bind for some* $\widehat{s}^{k+1}$ *with* $\widehat{s}^k \subset \widehat{s}^{k+1}$. *Iterating forward on this argument, for any history* s^k, (1.16) *binds and if it does not bind,* $-g(s^k) + \beta \sum \pi(s^{k+1}|s^k) R(n(s^{k+1})) < 0$, *but this violates* (1.42) *leading to a contradiction.*
■

For the rest of the analysis, it is useful to define

$$\widehat{g}(\theta, R) = \left(\frac{\theta}{1 - \widehat{u}'(R)}\right)^{1/(1-\alpha)} \tag{1.43}$$

for $\widehat{u}(R)$ defined in (1.39).

Proof of Lemma 2

(i) This proof uses many of the results in Thomas and Worrall (1988), Lemma 1. Define the program in terms of a sequence $\{R, g\}$ so that the instantaneous utility to the household is $\widehat{u}(R) - g - x + \theta \frac{g^\alpha}{\alpha}$. Fix $x(s^t) = \overline{x}$, and let b_s be the set of feasible values of b for our program. By Lemma 5, (1.20) can be relaxed. If $b' \in b_s$ then $b'' \in b_s$ for all $\underline{b}_s \leq b'' < b' \leq \overline{b}_s$, since the constraint set is no smaller. To show that b_s is closed, consider a sequence $b_s'^j \in b_s$ such that $\lim_{j\to\infty} b_s'^j = b_s'$. There is a corresponding stochastic sequence $\{R^j, g^j\}$ for each $b_s'^j$. Because each element of R is contained in $[0, R^{\max}]$ and each element of g is contained in $[0, g^{\max}]$ for some arbitrarily large chosen value $g^{\max}$, a stochastic sequence $\{R^j, g^j\}$ specifies a countable number of allocations, and the space of sequences which includes $\{R^j, g^j\}$ is sequentially compact in the product topology. Then there is a sub-sequence of allocations converging pointwise to a limiting sequence $\{R^\infty, g^\infty\}$. Given the continuity of the utility function and the continuity of debt in R and g and since $\beta \in (0, 1)$, then by the Dominated Convergence Theorem, the limit of the social welfare function equals the social welfare achieved under the limiting sequence, and the limit of the total value of debt equals the total value of debt achieved under the limiting sequence. Then $\{R^\infty, g^\infty\}$ is sustainable since $\{R^j, g^j\}$ is sustainable, and it implies $\{R^\infty, g^\infty\}$ achieves b_s'. This establishes the compactness of the feasible values b.

(ii) The fact that the frontier $Q(s, b)$ is strictly decreasing in b conditional on some $\overline{x}$ follows from the fact that our program is equivalent to a relaxed program by Lemma 5. To show that the frontier is strictly concave, consider $b', b'' \in [\underline{b}_s, \overline{b}_s]$ with associated sequences $\{R', g'\}$ and $\{R'', g''\}$, respectively. Let $R''' = \kappa R' + (1-\kappa) R''$ for $\kappa \in (0,1)$ and define g''' analogously. This new sequence is sustainable by the convexity of the constraint set implied by the fact that $\widehat{u}(\cdot)$ is concave established in Lemma 4. This sequence also provides a welfare greater than $\kappa Q(s, b') + (1-\kappa) Q(s, b'')$ by the concavity of the utility function, establishing the strict concavity of $Q(\cdot)$ in b. To prove differentiability, consider a sequence $\{R, g\}$ associated with the solution for some $b \in (\underline{b}_s, \overline{b}_s)$. Consider the sequence $\{R^\epsilon, g^\epsilon\}$ for which the only difference between $\{R, g\}$ and $\{R^\epsilon, g^\epsilon\}$ is that $R_0^\epsilon = R_0 + \epsilon$, meaning the allocation in the initial period is different but the continuation allocation is identical, and assume that $R_0 \in (0, R^{\max})$ so that one can choose $\epsilon \gtreqless 0$ of arbitrarily small magnitude. Define the function $F(s, b, \epsilon) = \widehat{u}(R_0^\epsilon) - g_0 - \overline{x} + \theta(s) \frac{g_0^\alpha}{\alpha} + \beta \sum \pi_{ks} Q(k, J_k)$, so that $F(s, b, 0) = Q(s, b)$. Optimality implies that $F(s,b,\epsilon) \leq Q(s,b+\epsilon)$, for $F(s,b,\epsilon)$ which is concave and differentiable and is associated with $b+\epsilon$. By Lemma 1 of Benveniste and Scheinkman (1979) $Q(\cdot)$ is differentiable. Lemma 4 mean that $R_0 = R^{\max}$ is suboptimal if $g_0 > 0$, since it is possible to reduce both R_0 and g_0 equally while continuing to satisfy all constraints and increasing welfare. If $g_0 = 0$, then for (1.22) to hold in period 0, (1.22) must be slack in some future period, which means that it is optimal to reduce R_0 while increasing R_t or decreasing g_t at some future date, while continuing to satisfy all constraints and increasing welfare. If R_t cannot be increased or g_t cannot be decreased at any future date, then (1.22) cannot be satisfied since this implies that $R_t = R^{\max}$ and $g_t = 0$ for all t. Imagine if $R_0 = 0$. Then if (1.21) does not bind a similar exercise can be performed for $g_0^\epsilon = g_0 + \epsilon$ analogously defined. If (1.21) binds and $R_0 = 0$, then $b = \underline{b}_s$, since no element of R can be decreased or of g can be increased without violating (1.21) today or in the future. **Q.E.D.**

Sufficient Conditions for Assumption 1

Imagine if $\underline{b}_s = \overline{b}_s$ for some s. This means that from state $s_t = s$ at history s^t, (1.21) and (1.22) both bind. In order that no debt below $\underline{b}_s$ be sustainable, it must be that $R\left(n\left(s^t\right)\right) = 0$. In order that (1.22) bind, it must therefore be that $g\left(s^t\right) = 0$. This implies that

$$\beta \sum_{s_{t+1} \in S} \pi\left(s_{t+1}\right) b\left(s^{t+1}|s^t\right) + \beta\overline{x}/\left(1-\beta\right) = 0.$$

Since (1.21) is satisfied at $t+1$, then $R\left(n\left(s^{t+1}\right)\right) = 0$ for (1.21) to bind at t, and given this fact, since (1.22) is satisfied at $t+1$, then $g\left(s^{t+1}\right) = 0$ in order that (1.22) bind at t. Forward iteration on this argument implies that $R\left(n\left(s^k\right)\right) = g\left(s^k\right) = 0\ \forall s^k$. We provide sufficient conditions under which there exists a value of b_s which yields a higher welfare.

Define $R^*(s)$ as the solution to $R^*(s) - \widehat{g}\left(\theta(s), R^*(s)\right) = (1-\beta) R^{\max}$, and consider the value of

$$\widehat{u}\left(R^*(s)\right) - \widehat{g}\left(\theta(s), R^*(s)\right) - (1-\beta) R^{\max} + \theta(s) \frac{\widehat{g}\left(\theta(s), R^*(s)\right)^{\alpha}}{\alpha} - U^{AUT}(1-\beta). \quad (1.44)$$

(1.44) strictly increases in β and (1.44) it becomes strictly positive as $\beta \to 1$. This means there exists a value of $\widehat{\beta}$ above which (1.44) is strictly positive. Consider $\beta > \widehat{\beta}$, and construct an equilibrium in which $n\left(s^t\right) = \widehat{n}\left(R^*\left(s_t\right)\right)$ for $\widehat{n}\left(\cdot\right)$ defined in (1.39) and in which $g\left(s^t\right) = \widehat{g}\left(\theta\left(s_t\right), R^*\left(s_t\right)\right)$ $\forall s^t$. This equilibrium is associated with constant debt $R^{\max} - \frac{\overline{x}}{1-\beta}$ for any chosen $\overline{x}$. Since (1.44) is strictly positive, this satisfies all sequential incentive compatibility constraints of the household and yields a strictly higher welfare than U^{AUT}. The definition of $R^*\left(s\right)$ means that the sequential incentive compatibility of the ruler are also satisfied. **Q.E.D.**

Proof of Proposition 3

Define $\overline{b}_s$ as the solution to $Q\left(s, \overline{b}_s\right) - \overline{b}_s = U^{AUT}$ and let $\overline{\lambda}_s = -Q_b\left(s, \overline{b}_s\right)$. Define $\widehat{b}_k\left(\lambda_k\right)$ as the solution to $-Q_b\left(k, \widehat{b}_k\left(\lambda_k\right)\right) = \lambda_k$. (1.25) implies that $b_k = \widehat{b}_k\left(\min\left\{\lambda + \phi, \overline{\lambda}_k\right\}\right)$. Define $\widehat{\widehat{g}}_s\left(\lambda + \phi\right)$ as the solution to $\theta\left(s\right)\widehat{\widehat{g}}_s\left(\lambda + \phi\right)^{\alpha - 1} = 1 + \lambda + \phi$. Finally, define $\underline{\lambda}_s$ as the solution to

$$\beta \sum_{k \in S} \pi_{ks} \widehat{b}_k\left(\min\left\{\underline{\lambda}_s, \overline{\lambda}_k\right\}\right) + \beta\overline{x}/\left(1 - \beta\right) = \widehat{\widehat{g}}_s\left(\underline{\lambda}_s\right),$$

which is unique since $\widehat{b}_k\left(\cdot\right)$ is increasing and $\widehat{\widehat{g}}_s\left(\cdot\right)$ is decreasing. Given these definitions, if (1.21) binds then $\lambda + \phi = \underline{\lambda}_s$ and if (1.22) binds then $\frac{\lambda + \phi - \psi_k}{1 + \psi_k} = \overline{\lambda}_k$. Moreover, $\lambda + \phi$ can only either equal λ or $\underline{\lambda}_s$ in equilibrium.

Let $\lambda\left(s^t\right)$, $\phi\left(s^t\right)$, and $\psi\left(s^t\right)$ represent sequential values of λ,ϕ, and ψ, so that from (1.25),

$$\lambda\left(s^t\right) = \frac{\lambda\left(s^{t-1}\right) + \phi\left(s^{t-1}\right) - \psi\left(s^t\right)}{1 + \psi\left(s^t\right)}. \tag{1.45}$$

We argue that $\lambda\left(s^t\right)$ follows this updating rule:

$$\lambda\left(s^t\right) = \begin{cases} \overline{\lambda}\left(s_t\right) & \text{if } \lambda\left(s^{t-1}\right) > \overline{\lambda}\left(s_t\right) \text{ or } \underline{\lambda}\left(s_{t-1}\right) > \overline{\lambda}\left(s_t\right) \\ \lambda\left(s^{t-1}\right) & \text{if } \lambda\left(s^{t-1}\right) \in \left[\underline{\lambda}\left(s_{t-1}\right), \overline{\lambda}\left(s_t\right)\right] \\ \underline{\lambda}\left(s_{t-1}\right) & \text{if } \lambda\left(s^{t-1}\right) < \underline{\lambda}\left(s_{t-1}\right) \leq \overline{\lambda}\left(s_t\right) \end{cases}, \tag{1.46}$$

for $\underline{\lambda}\left(s_t\right)$ and $\overline{\lambda}\left(s_t\right)$ defined above, and by (1.23) this implies (1.27). If $\lambda\left(s^t\right) > \overline{\lambda}\left(s_t\right)$, equation (1.26) and the concavity of $Q\left(\cdot\right)$ imply that $Q\left(s_t, b\left(s^t|s^{t-1}\right)\right) - b\left(s^t|s^{t-1}\right) < U^{AUT}$, which violates (1.13). Therefore, if $\lambda\left(s^{t-1}\right) + \phi\left(s^{t-1}\right) > \overline{\lambda}\left(s_t\right)$, from (1.45) it must be that $\psi\left(s^t\right) > 0$ so that (1.13) binds at s^t and $\lambda\left(s^t\right) = \overline{\lambda}\left(s_t\right)$. If $\lambda\left(s^t\right) < \underline{\lambda}\left(s_{t-1}\right) \leq \overline{\lambda}\left(s_t\right)$, then $\psi\left(s^t\right) = 0$

since (1.13) cannot bind by the definition of $\overline{\lambda}(s_t)$, and by (1.45) $\lambda(s^t) = \lambda(s^{t-1}) + \phi(s^{t-1}) < \underline{\lambda}(s_{t-1})$, but this means that (1.12) is violated at s^{t-1} by the definition of $\underline{\lambda}(s_{t-1})$. Therefore, if $\lambda(s^{t-1}) < \underline{\lambda}(s_{t-1}) \leq \overline{\lambda}(s_t)$, from (1.45) it must be that $\phi(s^{t-1}) > 0$ so that (1.12) binds at s^{t-1} and $\lambda(s^t) = \underline{\lambda}(s_{t-1})$. Now consider if $\lambda(s^{t-1}) \in [\underline{\lambda}(s_t), \overline{\lambda}(s_t)]$ but $\lambda(s^t) \neq \lambda(s^{t-1})$. If $\lambda(s^t) > \lambda(s^{t-1})$, equation (1.45) implies that $\phi(s^{t-1}) > 0$, so that (1.12) binds at s^{t-1}, which means that $\lambda(s^t) = \underline{\lambda}(s_{t-1}) \leq \lambda(s^{t-1})$, leading to a contradiction. If $\lambda(s^t) < \lambda(s^{t-1})$, equation (1.45) implies that $\psi(s^t) > 0$, so that (1.13) binds at s^t, which means that $\lambda(s^t) = \overline{\lambda}(s_t) \geq \lambda(s^{t-1})$, leading to contradiction. To see why $\underline{\lambda}(s_t)$ and $\overline{\lambda}(s_t)$ are independent of $\overline{x}$ note that (1.18) – (1.22) can be rewritten with the following change of variable: $Q^*(s, b^*) = Q(s, b) + \overline{x}/(1-\beta)$ and $b^* = b + \overline{x}/(1-\beta)$, so that the problem admits the same solution independently of $\overline{x}$. **Q.E.D.**

Proof of Corollary 1

Using the notation and the results of the proof of Proposition 3, and given (1.24), this is a consequence of the law of motion for $\lambda(s^t) + \phi(s^t)$:

$$\lambda(s^t) + \phi(s^t) = \begin{cases} \overline{\lambda}(s_t) & \text{if } \lambda(s^{t-1}) + \phi(s^{t-1}) > \overline{\lambda}(s_t) \\ \lambda(s^{t-1}) + \phi(s^{t-1}) & \text{if } \lambda(s^{t-1}) + \phi(s^{t-1}) \in [\underline{\lambda}(s_t), \overline{\lambda}(s_t)] \\ \underline{\lambda}(s_t) & \text{if } \lambda(s^{t-1}) + \phi(s^{t-1}) < \underline{\lambda}(s_t) \end{cases},$$

where we use the fact that Assumption 1 implies that (1.12) and (1.13) cannot bind simultaneously. **Q.E.D.**

Proof of Proposition 4

(1.27) and (1.28) in an i.i.d. setting imply that we can define

$$\hat{\hat{b}}(\lambda) = \sum_{t=1}^{\infty} \sum_{s^t \in S^t} \beta^{t-1} \pi(s^t | s^0) \left(R(n(s^t)) - g(s^t) \right)$$

$$\widehat{Q}(\lambda) = \sum_{t=1}^{\infty} \sum_{s^t \in S^t} \beta^{t-1} \pi(s^t | s^0) \left(\widehat{u}(R(n(s^t))) - R(n(s^t)) + \theta(s^t) \frac{\widehat{g}(\theta(s_t), R(n(s^t)))^{\alpha}}{\alpha} \right)$$

for $\widehat{g}(\cdot)$ defined in (1.43) and for λ representing the slope of $Q(\cdot)$ at s, and for $\widehat{\widehat{b}}(\lambda)$ and $\widehat{Q}(\lambda)$ which do not depend on s_0 by the i.i.d. assumption. The updating rules imply that $\widehat{\widehat{b}}(\lambda)$ is weakly increasing in λ and $\widehat{Q}(\lambda)$ is weakly decreasing in λ. Use (1.23) and (1.24) together with the definitions of $\widehat{\widehat{b}}(\lambda)$ and $\widehat{Q}(\lambda)$ to write (1.16) at $\underline{\lambda}(s)$ and (1.17) at $\overline{\lambda}(s)$ respectively:

$$-\left(\frac{\theta(s)}{1+\underline{\lambda}(s)}\right)^{1/(1-\alpha)} + \beta\widehat{\widehat{b}}(\underline{\lambda}(s)) = 0 \tag{1.47}$$

$$\eta\left(1-\frac{1}{\gamma}\right)\left(\frac{1}{\eta}\left(\frac{1+\overline{\lambda}(s)}{1+\gamma\overline{\lambda}(s)}\right)\right)^{\gamma/(\gamma-1)} + \theta(s)\frac{\left(\frac{\theta(s)}{(1+\overline{\lambda}(s))^{\alpha}}\right)^{\alpha/(1-\alpha)}}{\alpha} + \beta\widehat{Q}\left(\overline{\lambda}(s)\right) = U^{AUT}. \tag{1.48}$$

The satisfaction of equation (1.47) requires $\underline{\lambda}(s)$ to be increasing in $\theta(s)$, and the satisfaction of (1.48) requires $\overline{\lambda}(s)$ to be increasing in $\theta(s)$. The implications for $\underline{\tau}(s)$ and $\overline{\tau}(s)$ follow from the proof of Proposition 3. **Q.E.D.**

Proof of Theorem 1

Optimality requires that $\overline{x}$ be chosen such that (1.12) binds in period 0, otherwise it is possible to reduce $\overline{x}$ which increases welfare by Lemma 5. Together, equations (1.10) and (1.12) in period zero imply that

$$\sum_{t=0}^{\infty}\sum_{s^t\in S^t}\beta^t\pi\left(s^t\right)\left(R\left(n\left(s^t\right)\right)-g\left(s^t\right)\right) = \max\left\{V_0, -b_{-1}\left(s^0\right)\right\} + b_{-1}\left(s^0\right). \tag{1.49}$$

From (1.23) and (1.24), it follows that $g\left(s^t\right) = \widehat{g}\left(\theta\left(s_t\right), R\left(s^t\right)\right)$ for $\widehat{g}(\cdot)$ defined in (1.43) whenever $\tau\left(s^{t-1}\right) \in \left[\underline{\tau}\left(s_{t-1}\right), \overline{\tau}\left(s_t\right)\right]$ and this fact together with (1.27) and (1.28) allow us to determine $R\left(n\left(s^0\right)\right)$ and $g\left(s^0\right)$ which satisfy (1.49) as a function of V_0. One can choose V_0 low enough s.t. that $\tau\left(s^0\right) \leq \underline{\tau}\left(s_0\right)$ and $\tau\left(s^1\right) = \underline{\tau}\left(s_0\right)$. Given (1.27), it is sufficient to prove that $\underline{\tau}(s) < \underline{\tau}(k)$ for some $s \neq k$. This means that given two paths $\{s,s,s,k\}$ and $\{s,s,k,k\}$, $\tau_4 = \underline{\tau}(s)$ under the first path, and $\tau_4 = \underline{\tau}(k)$ under the second path so that they are not equal. Imagine if $\nexists s,k \in S$ such that $\underline{\tau}(s) < \underline{\tau}(k)$. This would imply that the same minimal tax rate $\underline{\tau}$ associated with $\underline{R}$ would cause (1.16) to bind in all states, so that $\beta\underline{R} - \widehat{g}(\theta,\underline{R}) = 0$ $\forall\theta$. But this is not possible since $\widehat{g}(\theta,\underline{R})$ is increasing in θ. **Q.E.D.**

Proof of Theorem 2

Define

$$\widehat{\widehat{\widehat{b}}}\left(s, R^{LR}\right) = R^{LR} - \widehat{g}\left(\theta\left(s\right), R^{LR}\right) + \beta \sum_{k \in S} \pi_{ks} \widehat{\widehat{\widehat{b}}}\left(k, R^{LR}\right)$$
$$\widehat{\widehat{Q}}\left(s, R^{LR}\right) = \widehat{u}\left(R^{LR}\right) - R^{LR} + \theta\left(s\right) \frac{\widehat{g}\left(\theta\left(s\right), R^{LR}\right)^{\alpha}}{\alpha} + \beta \sum_{k \in S} \pi_{ks} \widehat{\widehat{Q}}\left(k, R^{LR}\right).$$

for $\widehat{g}\left(\cdot\right)$ defined in (1.43) which satisfies (1.24). In order that R^{LR} satisfy (1.16) and (1.17) $\forall s$, this requires

$$\widehat{\widehat{\widehat{b}}}\left(s, R^{LR}\right) \geq R^{LR} \ \forall s \text{ and} \tag{1.50}$$
$$\widehat{\widehat{Q}}\left(s, R^{LR}\right) \geq U^{AUT} \ \forall s. \tag{1.51}$$

We first show that if $\exists R^{LR}$ which satisfies (1.50) and (1.51), then $\lim_{t \to \infty} \tau_t = \tau^{LR}$. Assume and later prove that $\exists \tau^{LR} \in \{\cap_{s \in S} \left[\underline{\tau}\left(s\right), \overline{\tau}\left(s\right)\right]\} = [\max_{s \in S} \underline{\tau}\left(s\right), \min_{s \in S} \overline{\tau}\left(s\right)]$. Since $\max_{s \in S} \underline{\tau}\left(s\right) \leq \min_{s \in S} \overline{\tau}\left(s\right)$, (1.27) implies $\tau\left(s^{t-1}\right) \leq \tau\left(s^{t}\right) \leq \max_{s \in S} \underline{\tau}\left(s\right)$ or $\tau\left(s^{t-1}\right) \geq \tau\left(s^{t}\right) \geq \min_{s \in S} \overline{\tau}\left(s\right)$. Since it is bounded, τ_t converges. Imagine if $\{\cap_{s \in S} \left[\underline{\tau}\left(s\right), \overline{\tau}\left(s\right)\right]\} = \emptyset$. Choose the minimal possible R^{LR} which satisfies (1.50) and (1.51). Denote the state under which (1.50) binds under R^{L} by s. Consider the solution to (1.15) s.t. $s = s_0$, $\widehat{\widehat{\widehat{b}}}\left(s, R^{LR}\right) - \frac{\overline{x}}{1-\beta} = b_{-1}\left(s^0\right)$, and $x\left(s^t\right) = \overline{x} \ \forall s^t$. This solution must coincide with $Q\left(s, \widehat{\widehat{\widehat{b}}}\left(s, R^{LR}\right) - \frac{\overline{x}}{1-\beta}\right)$. Compare this solution to that of the relaxed problem which ignores (1.12) and (1.13) for $t \geq 1$. The relaxed solution sets $R\left(n\left(s^t\right)\right) = R^{LR} \ \forall s^t$, and this solution to the relaxed problem satisfies (1.12) and (1.13) for $t \geq 1$ so that it is the solution to the constrained problem. Therefore $\{\cap_{s \in S} \left[\underline{\tau}\left(s\right), \overline{\tau}\left(s\right)\right]\} \neq \emptyset$.

Second, we show that if $\exists R^{LR}$ which strictly satisfies (1.50) and (1.51) for β', then $\exists R^{LR}$ which satisfies (1.50) and (1.51) for $\beta'' > \beta'$. Let $\widehat{\widehat{\widehat{b}}}\left(s, R^{LR}\right) | \beta'$ and $\widehat{\widehat{\widehat{b}}}\left(s, R^{LR}\right) | \beta''$ represent the value of $\widehat{\widehat{\widehat{b}}}\left(s, R^{LR}\right)$ associated with β' and β'', respectively. Note that

$$\left| \widehat{\widehat{\widehat{b}}}\left(s, R^{LR}\right) | \beta' - \widehat{\widehat{\widehat{b}}}\left(s, R^{LR}\right) | \beta'' \right| < \infty$$

for $\beta', \beta'' \in (0,1)$. It follows that

$$\widehat{\widehat{\widehat{b}}}\left(s, R^{LR}\right)|\beta'' - \widehat{\widehat{\widehat{b}}}\left(s, R^{LR}\right)|\beta' = \begin{array}{c} \left(\beta''-\beta'\right)\left(\sum \pi_{ks}\widehat{\widehat{\widehat{b}}}\left(k, R^{LR}\right)|\beta''\right) + \\ \beta' \sum \pi_{ks}\left(\widehat{\widehat{\widehat{b}}}\left(k, R^{LR}\right)|\beta'' - \widehat{\widehat{\widehat{b}}}\left(k, R^{LR}\right)|\beta'\right) \end{array}. \tag{1.52}$$

The continuity of $\widehat{\widehat{\widehat{b}}}\left(k, R^{LR}\right)|\beta''$ in β'' implies that for β'' sufficiently close to β', $\widehat{\widehat{\widehat{b}}}\left(k, R^{LR}\right)|\beta'' > 0$, since $\widehat{\widehat{\widehat{b}}}\left(k, R^{LR}\right)|\beta' > R^{LR} \geq 0$. Forward iteration on (1.52) implies that $\widehat{\widehat{\widehat{b}}}\left(s, R^{LR}\right)|\beta'' - \widehat{\widehat{\widehat{b}}}\left(s, R^{LR}\right)|\beta' > 0$ since $\beta'' > \beta'$. Analogous arguments imply that

$$\left(\widehat{\widehat{Q}}\left(s, R^{LR}\right)|\beta'' - U^{AUT}|\beta''\right) - \left(\widehat{\widehat{Q}}\left(s, R^{LR}\right)|\beta' - U^{AUT}|\beta'\right) > 0,$$

for $\widehat{\widehat{Q}}\left(s, R^{LR}\right)|\beta''$, $\widehat{\widehat{Q}}\left(s, R^{LR}\right)|\beta'$, $U^{AUT}|\beta''$, and $U^{AUT}|\beta'$ analogously defined. Then R^{LR} satisfies (1.50) and (1.51) for $\beta = \beta''$.

Finally, $\exists R^{LR}$ which strictly satisfies (1.50) and (1.51) for $\beta^* \in (0,1)$. Let $\widetilde{\pi}_s = \lim_{t\to\infty} \Pr\{s\}$ which is unique since $\pi_{ks} > 0\ \forall k, s$ and let $\widetilde{\theta} = \left(\widetilde{\mathbf{E}}\left(\theta(s)^{1/(1-\alpha)}\right)\right)^{1-\alpha}$ for $\widetilde{\mathbf{E}}(\cdot)$ which represent the expectations operator under the long run probability. Subtract R^{LR} from both sides of (1.50) and U^{AUT} from both sides of (1.51). Multiply both sides of (1.50) and (1.51) by $1-\beta$. It follows that

$$\lim_{\beta\to 1}\left(\widehat{\widehat{\widehat{b}}}\left(s, R^{LR}\right)|\beta - R^{LR}\right)(1-\beta) = R^{LR} - \widehat{g}\left(\widetilde{\theta}, R^{LR}\right), \tag{1.53}$$

$$\lim_{\beta\to 1}\left(\widehat{\widehat{Q}}\left(s, R^{LR}\right)|\beta - U^{AUT}|\beta\right)(1-\beta) = \widehat{u}\left(R^{LR}\right) - R^{LR} + \frac{\widetilde{\theta}\widehat{g}\left(\widetilde{\theta}, R^{LR}\right)^{\alpha}}{\alpha} - \widehat{u}(0). \tag{1.54}$$

Choose R^{LR} s.t. (1.53) equals 0. By the definition of $\widehat{g}\left(\widetilde{\theta}, R^{LR}\right)$, (1.54) strictly exceed 0. By the continuity of $\widehat{\widehat{\widehat{b}}}\left(s, R^{LR}\right)$ and $\widehat{\widehat{Q}}\left(s, R^{LR}\right)$ in β, $\exists\beta^* \in (0,1)$ s.t. (1.50) and (1.51) are strict inequalities under R^{LR} since they are satisfied for $\beta \to 1$. **Q.E.D.**

Proof of Proposition 5

Under i.i.d. shocks, $\widehat{\widehat{\widehat{b}}}\left(s,R^{LR}\right)$ and $\widehat{\widehat{Q}}\left(s,R^{LR}\right)$ from the proof of Theorem 2 can be written as

$$\begin{aligned}
\widehat{\widehat{\widehat{b}}}\left(s,R^{LR}\right) &= R^{LR}-\widehat{g}\left(\theta\left(s\right),R^{LR}\right)+\beta\frac{\sum_{k\in S}\pi_k\left(R^{LR}-\widehat{g}\left(\theta\left(k\right),R^{LR}\right)\right)}{1-\beta} \\
\widehat{\widehat{Q}}\left(s,R^{LR}\right) &= \widehat{u}\left(R^{LR}\right)-R^{LR}+\theta\left(s\right)\frac{\widehat{g}\left(\theta\left(s\right),R^{LR}\right)^{\alpha}}{\alpha} \\
&\quad +\beta\frac{\sum_{k\in S}\pi_k\left(\widehat{u}\left(R^{LR}\right)-R^{LR}+\theta\left(k\right)\frac{\widehat{g}\left(\theta(k),R^{LR}\right)^{\alpha}}{\alpha}\right)}{1-\beta}.
\end{aligned}$$

By the i.i.d assumption and Proposition 4, the minimum $\widehat{\widehat{\widehat{b}}}\left(s,R^{LR}\right)$ for a given R^{LR} is associated with the highest $\theta\left(s\right)$ which we label as θ_H associated with s_H, and the minimum $\widehat{\widehat{Q}}\left(s,R^{LR}\right)$ for a given R^{LR} is associated with the lowest $\theta\left(s\right)$ which we label as θ_L associated with s_L. Define $\underline{R}\left(\beta\right)$ as the solution to $\widehat{\widehat{\widehat{b}}}\left(s_H,\underline{R}\left(\beta\right)\right)=\underline{R}\left(\beta\right)$. Define $\overline{R}\left(\beta\right)$ as the solution to $\widehat{\widehat{Q}}\left(s_L,\overline{R}\left(\beta\right)\right)=U^{AUT}$. For R^{LR} to exist, it must be that $R^{LR}\in\left[\underline{R}\left(\beta\right),\overline{R}\left(\beta\right)\right]$. $\underline{R}\left(\beta\right)$ monotonically decreases in β and $\overline{R}\left(\beta\right)$ monotonically decreases in β and by the arguments of Theorem 2, there exist some β under which $\overline{R}\left(\beta\right)>\underline{R}\left(\beta\right)$. Given the monotonicity, there exists a value $\beta^*>0$ such that $\underline{R}\left(\beta^*\right)=\overline{R}\left(\beta^*\right)$ above which R^{LR} exists and below which R^{LR} does not exist. We are left to show that equilibria with positive taxes and positive public spending which generate welfare above U^{AUT} exist for all $\beta\in\left(\beta^{**},\beta^*\right)$ for some $\beta^{**}<\beta^*$. Consider the following perturbation to the above equilibrium under β^*. Consider an allocation which is identical for all $s\neq s_L,s_H$. However, let $R\left(s_L\right)=R^{LR}-\epsilon_L$ and $R\left(s_H\right)=R^{LR}+\epsilon_H$ with the associated $\widehat{g}\left(\theta\left(s_L\right),R^{LR}-\epsilon_L\right)$ and $\widehat{g}\left(\theta\left(s_H\right),R^{LR}+\epsilon_H\right)$ for $\epsilon_H,\epsilon_L>0$ both chosen to be arbitrarily small and for all states s_L and s_H along the equilibrium path. Since incentive compatibility constraint do not bind for the original allocation under $s\neq s_L,s_H$, they continue to not bind. It can be shown that the perturbed allocation relaxes the incentive constraint of the ruler under $s=s_H$ and it relaxes the incentive constraint of the households under $s=s_L$ which means that β can be reduced without violating any incentive compatibility constraint. Specifically, it can be verified by the definition of $\widehat{g}\left(\cdot\right)$ that

$$\frac{1-\widehat{g}_R\left(\theta_H,R^{LR}\right)}{1-\widehat{g}_R\left(\theta_L,R^{LR}\right)}=\frac{\widehat{u}'\left(R^{LR}\right)-1+\theta_H\frac{\widehat{g}_R\left(\theta_H,R^{LR}\right)^{\alpha}}{\alpha}}{\widehat{u}'\left(R^{LR}\right)-1+\theta_L\frac{\widehat{g}_R\left(\theta_L,R^{LR}\right)^{\alpha}}{\alpha}}=\varrho.$$

Letting π_H and π_L represent the probabilities of the high state and the low state, respectively, one can choose $\epsilon_L = \varrho \frac{\pi_H}{\pi_L} \epsilon_H$ and small enough perturbations in ϵ_H will relax (1.16) under s_H and (1.17) under s_L. **Q.E.D.**

1.9.3 Proofs of Section 1.6

Proof of Proposition 6

The equivalent versions of (1.12) and (1.13) in this economy are, respectively:

$$\sum_{k=t}^{\infty} \sum_{s^k \in S^k} \beta^{k-t} \pi \left(s^k | s^t\right) x \left(s^k\right) \geq \widehat{R} \left(n \left(s^t\right), s_t\right) - b \left(s^t | s^{t-1}\right) \; \forall s^t, \text{ and} \tag{1.55}$$

$$\sum_{k=t}^{\infty} \sum_{s^k \in S^k} \beta^{k-t} \pi \left(s^k | s^t\right) \left(c \left(s^k\right) - \eta \frac{n \left(s^k\right)^{\gamma}}{\gamma} + \theta \left(s_k\right) \frac{g \left(s^k\right)^{\alpha}}{\alpha} \right) \geq \widehat{U}^{AUT} \left(s_t\right) + b \left(s^t | s^{t-1}\right) \; \forall s^t, \tag{1.56}$$

Let $[\underline{\tau}(1), \overline{\tau}(1)] \cap [\underline{\tau}(2), \overline{\tau}(2)] \neq \emptyset$. If $\underline{\tau}(2) > \underline{\tau}(1)$, then $\tau\left(s^t\right) = \underline{\tau}(2) \; \forall s^t$ satisfies (1.55) and (1.56) $\forall s^t$. By definition, $\underline{\tau}(2)$ is associated with $\underline{R}(2)$ under which (1.55) binds if $s = 2$:

$$\begin{aligned} \frac{\beta \underline{R}(2) - \widehat{g}(2, \underline{R}(2))}{1-\beta} &= -\beta \left((1 - \pi_{22}) E(1) + \pi_{22} E(2) \right) \\ &< -\beta \left(\pi_{11} E(1) + 1 - \pi_{11} E(2) \right), \end{aligned}$$

and the last inequality implies (1.55) is not satisfied if $s = 1$, yielding a contradiction. If $\overline{\tau}(2) > \overline{\tau}(1)$, then $\tau\left(s^t\right) = \overline{\tau}(1) \; \forall s^t$ satisfies (1.55) and (1.56) $\forall s^t$. By definition, $\overline{\tau}(1)$ is associated with $\overline{R}(1)$ under which (1.56) binds if $s = 1$:

$$\begin{aligned} \frac{\widehat{u}\left(\overline{R}(1)\right) - \overline{R}(1) + \frac{\widehat{g}\left(1, \overline{R}(1)\right)^{\alpha}}{\alpha}}{1-\beta} &= \widehat{U}^{AUT}(1) = U^{AUT} + E(1) \\ &< U^{AUT} + E(2), \end{aligned}$$

and the last inequality implies (1.56) is not satisfied if $s = 2$, yielding a contradiction. Let $[\underline{\tau}(1), \overline{\tau}(1)] \cap [\underline{\tau}(2), \overline{\tau}(2)] = \emptyset$. $\underline{\tau}(2) > \underline{\tau}(1)$ if and only if $\overline{\tau}(2) > \overline{\tau}(1)$. (1.27) implies that in the long run, $\tau\left(s^t\right) = \overline{\tau}(1)$ if $s_t = 1$ and $\tau\left(s^t\right) = \underline{\tau}(2)$ if $s_t = 2$ since $\overline{\tau}(1) < \underline{\tau}(2)$. Since shocks are persistent, then the value of debt net of rents and net of the endowment in state 2 exceeds the value of debt net of rents and net of the endowment in state 1. Since (1.55) binds in state 2, the value of debt net of rents and net of the endowment is $-\beta\left(\left(1-\pi_{22}\right)E\left(1\right)+\pi_{22}E\left(2\right)\right)$. However, this value does not exceed $-\beta\left(\pi_{11}E\left(1\right)+1-\pi_{11}E\left(2\right)\right)$ which is the minimum value of debt net of rents and net of the endowment in state 1 for (1.55) to be satisfied. Therefore, $\underline{\tau}\left(2\right)<\underline{\tau}\left(1\right)$ and $\overline{\tau}\left(2\right)<\overline{\tau}\left(1\right)$. **Q.E.D.**

1.9.4 Proofs of Section 1.7

We first formally characterize $\underline{V}\left(\tau,b\right)$.

$$\underline{V}\left(\tau,b\right)=v\left(\tau\widetilde{n}\left(\tau,b\right)-b\right)+\beta\frac{v\left(0\right)}{1-\beta}$$

$$\text{for } \widetilde{n}\left(\tau,b\right) \text{ which solves}$$

$$-\frac{h_n\left(\left(1-\tau\right)\widetilde{n}\left(\tau,b\right)+b,\widetilde{n}\left(\tau,b\right)\right)}{h_c\left(\left(1-\tau\right)\widetilde{n}\left(\tau,b\right)+b,\widetilde{n}\left(\tau,b\right)\right)}=1-\tau \tag{1.57}$$

Equation (1.57) represents the intratemporal condition for a household not trading claims for the future. Since the left hand side of (1.57) is continuously differentiable and defines a unique $\widetilde{n}\left(\tau,b\right)$, $\underline{V}\left(\tau,b\right)$ is continuously differentiable.

Proof of Proposition 7

The necessity and sufficiency of (1.9) and (1.29) for a competitive equilibrium is analogously derived as in the proof of Proposition 1 given the observation that (1.34) and (1.35) respectively become:

$$-\frac{h_n\left(c\left(s^t\right),n\left(s^t\right)\right)}{h_c\left(c\left(s^t\right),n\left(s^t\right)\right)}=1-\tau\left(s^t\right)$$

$$q\left(s^{t+1}|s^t\right)=\beta\frac{\pi\left(s^{t+1}\right)h_c\left(c\left(s^{t+1}\right),n\left(s^{t+1}\right)\right)}{\pi\left(s^t\right)h_c\left(c\left(s^t\right),n\left(s^t\right)\right)}.$$

Furthermore, the same methods establish (1.32).

For the sufficiency of (1.30), consider an equilibrium in which, off the equilibrium path, households choose $L_k=0\ \forall k\geq t$ and $\tau_k=0\ \forall k>t$. The ruler's best response to the household's policy decision is to choose zero public spending forever which yields $\underline{V}\left(\tau_t,b_{t-1}\left(s_t\right)\right)$ off the equilibrium path. Equation (1.30) implies that the deviation is weakly dominated. For the necessity of (1.30), the ruler can always choose to trade zero debt and to choose zero public spending forever. By definition, $\underline{V}(\tau_t, b_{t-1}(s_t))$ represents the lowest possible continuation value which is competitive that could be achieved from this strategy starting from a given $(\tau_t, b_{t-1}(s_t))$. Allocations must also satisfy (1.31) for taxes to be weakly positive. **Q.E.D.**

Proof of Theorem 3

Let $\beta^t \pi(s^t)\kappa(s^t)$, $\beta^t \pi(s^t)\phi(s^t)$, $\beta^t \pi(s^t)\Psi(s^t)$, and $\beta^t \pi(s^t)\upsilon(s^t)$ represent the Lagrange multipliers on (1.9), (1.30), (1.31), and (1.32), respectively, where we only consider the lower bound on the tax rate given Assumption 2. Let

$$\begin{aligned}\lambda(s^t) &= \lambda(s^{t-1}) + \phi(s^t) \\ \mu(s^t) &= \mu(s^{t-1}) + \upsilon(s^t)\end{aligned}$$

for $\lambda(s^{-1}) \geq 0$ representing the exogenous Pareto weight assigned to the ruler for the problem and let $\mu(s^{-1}) = 0$. In order to prove our result, we examine first order conditions at $t = 0$ and at $t = 1$ for $s^1 = s_0$. Defining $\tau(c, n) = 1 + h_n(c, n)/h_c(c, n)$ derived from the intratemporal condition, the first order conditions with respect to consumption, labor, public spending, and rents at $t = 0$ imply the following equalities:

$$\begin{aligned}u_c(s^0) + u_n(s^0) + \mu(s^0)\left(\Gamma_c(s^0) + \Gamma_n(s^0)\right) \\ + \left(\Psi(s^0) - \phi(s^0)\underline{V}_\tau(s^0)\right)\left(\tau_c(s^0) + \tau_n(s^0)\right)\end{aligned} = 0 \tag{1.58}$$

$$z'(s^0) = \lambda(s^0)v'(s^0). \tag{1.59}$$

Furthermore, at $t = 1$, first order conditions with respect to consumption, labor, public spending, rents, and debt imply the following equalities:

$$
\begin{aligned}
u_c\left(s^1\right)+u_n\left(s^1\right)+\mu\left(s^1\right)\left(\Gamma_c\left(s^1\right)+\Gamma_n\left(s^1\right)\right) \\
+\left(\Psi\left(s^1\right)-\phi\left(s^1\right)\underline{V}_\tau\left(s^1\right)\right)\left(\tau_c\left(s^1\right)+\tau_n\left(s^1\right)\right)
\end{aligned}
=0 \tag{1.60}
$$

$$
z'\left(s^1\right)=\lambda\left(s^1\right)v'\left(s^1\right) \tag{1.61}
$$

$$
-\phi\left(s^1\right)\underline{V}_b\left(s^1\right)=v\left(s^1\right)u_c\left(s^k\right). \tag{1.62}
$$

In order that $\left\{c\left(s^0\right),n\left(s^0\right),g\left(s^0\right),x\left(s^0\right)\right\}=\left\{c\left(s^1\right),n\left(s^1\right),g\left(s^1\right),x\left(s^1\right)\right\}$, equations (1.59) and (1.61) require that $\lambda\left(s^0\right)=\lambda\left(s^1\right)$, which means that $\phi\left(s^1\right)=0$, so that equation (1.62) implies that $v\left(s^1\right)=0$, so that $\mu\left(s^0\right)=\mu\left(s^1\right)$. Imagine if (1.31) does not bind. In order that (1.58) and (1.60) hold, it is therefore necessary that

$$
\phi\left(s^0\right)\underline{V}_\tau\left(s^0\right)\left(\tau_c\left(s^0\right)+\tau_n\left(s^0\right)\right)=0.
$$

Imagine if $\phi\left(s^0\right)=0$, so that $\lambda\left(s^0\right)=0$. Equation (1.59) would require that $v'\left(s^0\right)=\infty$, so that $x\left(s^0\right)=-\infty$. However, if this the case, then $v\left(x\left(s^0\right)\right)=-\infty$, since $v\left(\cdot\right)$ is a concave function, yet since $\underline{V}\left(\tau\left(s^0\right),0\right)>-\infty$, this allocation does not satisfy (1.30). Moreover, by Assumption 3, $\underline{V}_\tau\left(s^0\right)\neq 0$. Imagine if $\tau_c\left(s^0\right)+\tau_n\left(s^0\right)=0$. By the Implicit Function Theorem,

$$
\tau_c\left(s^0\right)+\tau_n\left(s^0\right)=\frac{h_c\left(s^0\right)\left(h_{nn}\left(s^0\right)+h_{cn}\left(s^0\right)\right)-h_n\left(s^0\right)\left(h_{cc}\left(s^0\right)+h_{cn}\left(s^0\right)\right)}{h_c\left(s^0\right)^2}<0, \tag{1.63}
$$

which yields a contradiction. Now imagine if (1.31) binds in periods 0 and 1. Substitution of (1.63) and the fact that $h_c\left(s^t\right)+h_n\left(s^t\right)=0$ into equations (1.58) and (1.60) implies that $\mu\left(s^0\right)=\mu\left(s^1\right)<0$, since $\Psi\left(s^1\right)>0$. Moreover, in order that (1.61) hold for all t, it is necessary that $\lambda\left(s^t\right)=\lambda\left(s^0\right)$ for all t. Combined with (1.62), this implies that $v\left(s^t\right)=0$ for all t so that $\mu\left(s^t\right)=\mu\left(s^1\right)<0$ for all t. However, for this to be true, it is necessary that $\Psi\left(s^t\right)>0$ for all s^t, so that taxes are zero forever, contradicting Assumption 2. **Q.E.D.**

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Chapter 2

The Efficient Use of War

2.1 Introduction

Many violent confrontations between groups are episodic and do not lead to a permanent resolution. For example, the disputes between England and the IRA and between Israel and the Palestinians date back to over half a century, and neither is fully resolved. In contrast to more deliberate and conclusive conflicts such as the world wars, diplomacy is often interrupted by fighting, and fighting is often interrupted by temporary concessions.

A common feature of these conflicts is imperfect information in the diplomatic process. Consider, for example, the case of the Second Palestinian Intifada during which the Israeli government urged the Palestinian Authority leader Yasser Arafat to crack down on militants. During this episode, it was unclear to the Israelis whether Arafat could not control the militants, or whether he privately endorsed the militants while publicly apologizing for their behavior. Raanan Gissin, a spokesman for Israeli Prime Minister Ariel Sharon, argued that:

> "Arafat is responsible since he encourages terrorists to commit suicide acts."[1]

Arafat defended himself by claiming to have no control over the militants:

> "There are those who claim that I am not a partner in peace...I condemn terrorism. I condemn the killing of innocent civilians...But condemnations do not stop

[1] BBC News, January 31, 2002.

terrorism. To stop terrorism, we must understand that terrorism is simply the symptom, not the disease."[2]

This example provides us with a useful lens for considering the informational problems inherent in diplomacy. Diplomatic concessions can fail because leaders put no effort in making them. Alternatively, diplomatic concessions can fail because they are too expensive, and sometimes impossible to make. Rival groups cannot distinguish between these two possibilities, and this breeds mistrust which can lead to war.

This paper uses these observations to study the dynamics of war in a game-theoretic environment. We consider an infinitely repeated game in which two groups seek resources from each other. In every period, either group can choose to fight, and this leads to a war in which some resources are destroyed. Alternatively, if neither group chooses to fight, peace ensues. Under peace, each group privately concedes some resources to the other group, and this concession fails with an exogenous probability. Rival groups only observe successful concessions from each other. Though war is destructive, it is prefered by one group to not receiving a concession.

This simple framework captures commitment and informational frictions which lead to war. In the one period version of the model, neither group can commit to making a concession under peace, and war is the only equilibrium. In contrast, in an infinitely repeated environment, reputation sustains more efficient outcomes. We study public perfect equilibria. We show that if war is sufficiently destructive, phases of peace can be supported by phases of war along the equilibrium path.

We then ask whether fluctuations between war and peace characterize an efficient public perfect equilibrium. We show that in the efficient public perfect equilibrium, each group must sometimes refrain from war and must sometimes make a concession. If either group publicly deviates from this implicit agreement, then the two groups revert to the repeated static equilibrium with war forever. If either group privately deviates from this implicit agreement by making no concession under peace, then its concession fails with certainty, and this group is punished with a reduction in future utility.

Our main result is that fluctuations between phases of peace and phases of war are only

[2]New York Times, February 3, 2002

efficient in the short run. In the long run, war occurs in every period. While this seems like an undesirable outcome, the threat of permanent war enables to two groups to reach a higher welfare by prolonging peace in the short run.

This result emerges because of the existence of a breaking point. More specifically, while phases of war are used to enforce phases of peace along the equilibrium path, the transition from phases of war back to phases of peace occurs at a breaking point. If concessions are successful at the breaking point, further peace ensues, and if concessions are unsuccessful at the breaking point, permanent war ensues. Because the two groups always reach the breaking point with positive probability, and because concessions have a positive probability of failing at the breaking point, long run convergence to permanent war is inevitable. The model therefore predicts that groups that fluctuate between phases of war and phases of peace eventually reach a stage at which war must be used to permanently resolve their dispute.

This result appears puzzling given that fluctuations between war and peace have lasted for several decades in the examples which motivate this study. In order to explore this issue further, we consider an extension of our model which allows for the use of excessive force during war by either group. This extension is particularly relevant given that, in practice, groups can often decide the amount of harm they wish to inflict on their rival. Excessive force is defined as a group's ability to inflict additional damage on its rival at a cost to itself. A naive intuition suggests that excessive force is both irrational and inefficient. Though this insight is correct in the one period version of the model in which the static equilibrium is unchanged, in a dynamic environment, the use of excessive force during phases of war can be supported by phases of peace along the equilibrium path. We show that if it is sufficiently cheap, excessive force is always used during phases of war in the efficient public perfect equilibrium. Therefore, fluctuations between phases of peace and phases of war occur both in the short run and in the long run, so that convergence to permanent war no longer takes place. This is because war in the future sustains peace today, and peace in the future sustains war today.

This paper is related to the large neorealist literature in political science on the causes of war dating back to the work of Waltz (1959) and Schelling (1966). In line with this approach, we assume that wars are the outcome of rational actions chosen by representative rulers of groups, which means that we abstract from any behavioral theories or theories which focus on war's domestic dimension.[3] We build on the work of Powell (1999, 2004a, 2004b) and Fearon (1995) who show that there are two fundamental frictions which lead to war: imperfect information and limited commitment.[4] We provide an alternative explanation for war which combines these two frictions in a dynamic setting, and our explanation can account for the episodic realization of war. We also differ from their work by focusing on the characterization of efficient equilibria which means that we consider general history dependent strategies.

Our theory of war is also related to Green and Porter (1984)'s theory of oligopolistic competition. As in their model, non-cooperation is used to enforce cooperation along the equilibrium path. In contrast to their work, we consider efficient equilibria with non-symmetric strategies, and we explicitly characterize the dynamics of actions and continuation values. To do this, we use the recursive techniques developed in Abreu, Pearce, and Stacchetti (1986,1990). These authors argue that extreme points in the set of continuation values can be used to sustain efficient equilibria. Our paper asks whether permanent war–which represents such an extreme point–can be *avoided* in an efficient equilibrium. In related work, Sannikov (2006a,2006b) shows that in a class of continuous time games, convergence to the static Nash equilibrium occurs in the long run if the repeated static Nash equilibrium payoffs are on the contour of set of continuation values. This is because continuation values can travel continuously to all points of the contour if they have not yet reached an absorbing state. In our discrete time model, continuation values need not reach all points on the contour, though they must reach the breaking point after a temporary phase of war, and this drives our long run result.[5]

The paper is organized as follows. Section 2.2 describes the model. Section 2.3 defines and provides necessary and sufficient conditions for a public perfect equilibrium. Section 2.4 analyzes peaceful public perfect equilibria. Section 2.5 analyzes efficient public perfect equilibria. Section 2.6 analyzes an extension in which we allow for excessive force. Section 2.7 concludes, and the Appendix contains additional proofs not included in the text.

---

[3] We abstract from issues involving group formation which are explored in Alesina and Spolaore (2004), and Caselli and Coleman (2006), and Esteban and Ray (1994,1999).

[4] See also Hirschleifer (1995), Skarpedas (1992), and Dixit (1987) for work along this approach.

[5] For related work on dynamic games with imperfect information, see Atkeson and Lucas (1992,1995), Hauser and Hopenhayn (2004), Phelan and Townsend (1991), Prescott and Townsend (1984), Spear and Srivastava (1987), and Thomas and Worrall (1990).

2.2 Model

Consider two groups $i = \{1, 2\}$ and time periods $t = \{0, ..., \infty\}$. In every date t, each group i publicly chooses $F_{it} = \{0, 1\}$, and $F_{it} = 1$ represents a decision to go to war. If $W_t = \max\{F_{1t}, F_{2t}\} = 1$, war takes place, group 1 receives w_1, group 2 receives w_2, and the period ends. Alternatively, if $W_t = 0$, peace occurs, and each group i *privately* makes a concession to group $-i$ of size $x_{it} \in [0, \overline{x}_i]$. This concession succeeds with exogenous probability $\pi_i \in (0, 1)$ and fails with probability $1 - \pi_i$. Let $s_{it} = \{0, 1\}$ be an indicator which equals 1 if a concession by i succeeds and which equals 0 otherwise.[6] Each group i receives a flow utility of $s_{-it}x_{-it} - s_{it}x_{it}$.[7]

Group i observes x_{it}, s_{it}, and $s_{-it}x_{-it}$, but it does not observe x_{-it}. Therefore, if $W_t = 0$ and if a concession by $-i$ does not occur $(s_{-it}x_{-it} = 0)$, group i cannot tell if group $-i$ attempted to make a concession $(x_{-it} > 0)$ or not $(x_{-it} = 0)$. This captures the fact that group i is uncertain about group $-i$'s commitment to diplomacy. The shock s_{it} can be interpreted at capturing a private shock to the cost of a concession, so that under some circumstances, concessions by one group become impossible, but this cannot be verified by the rival group.

Let $z_t \in [0, 1]$ represent an i.i.d. random variable independent of the s_{it}'s and all actions drawn from a continuous c.d.f. $G(\cdot)$ at the beginning of every period t. z_t is observed by both groups and can be used as a randomization device which can improve efficiency by allowing the two groups to probabilistically go to war.

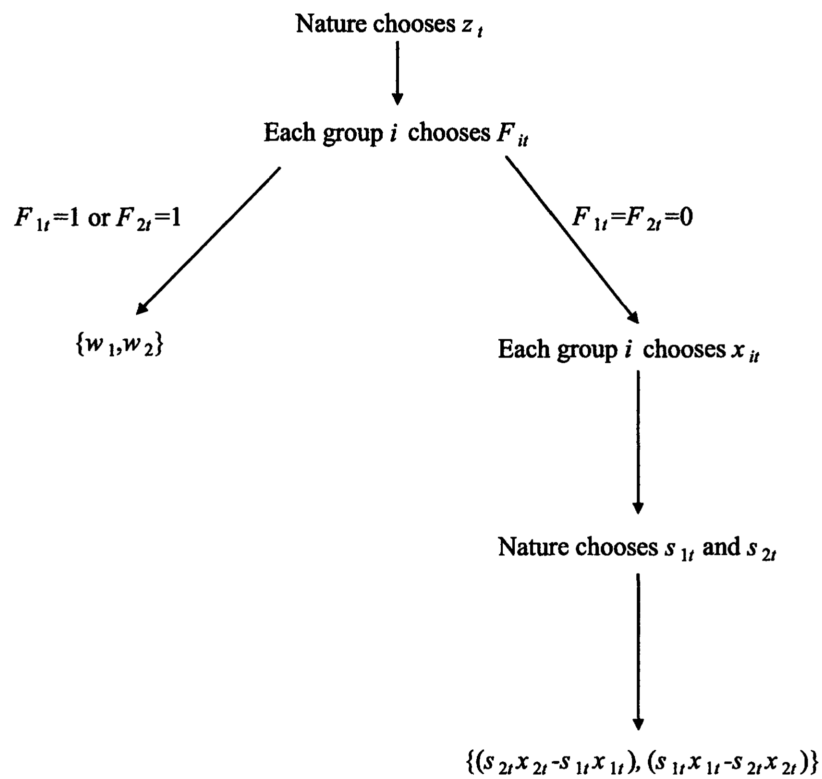
---

[6] All of our results only depend on $\pi_2 \in (0, 1)$. We can relax the independence of s_{1t} and s_{2t} without any loss of generality.

[7] None of our result change if a group's utility is $f_i(s_{-it}x_{-it} - s_{it}x_{it})$ for $f_i(\cdot)$ which is increasing and concave.

The game form is displayed in Figure 2.1.

Figure 2.1: Game Form



Let $u_i(F_{it}, F_{-it}, x_{it}, x_{-it})$ represent the expected payoff to i after the realization of z_t and prior to the choice of F_{it}'s. Each group i has a period zero welfare

$$\mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t u_i(F_{it}, F_{-it}, x_{it}, x_{-it}), \ \beta \in (0,1).$$ [8]

Assumption 1 (inefficiency of war) $\exists x_1 \in [0, \overline{x}_1)$ and $\exists x_2 \in [0, \overline{x}_2)$ s.t.

$$\pi_2 x_2 - \pi_1 x_1 > w_1 \text{ and } \pi_1 x_1 - \pi_2 x_2 > w_2.$$

Assumption 2 (necessity of war) $w_1 > 0$.

[8] Without loss of generality, $x_{it} = s_{it} = 0$ when $W_t = 1$ (both are payoff irrelevant).

Assumption 3 (maximal size of concessions) $\beta w_2 > -\overline{x}_2$.

Assumption 1 means that war is inefficient, since groups can make concessions which, despite a positive failure rate, make both groups better off. Assumption 2 means that, although war is inefficient, group 1 prefers war to not receiving any concessions. Naturally, if $w_1 > 0$ and $w_2 > 0$, then this violates Assumption 1, which means that $w_2 < 0$. Assumption 3 means that the concessions requested from group 2 by group 1 can be large.[9] In a dynamic setting, it ensures that it is more efficient for group 1 to punish the failed concessions of group 2 by requesting higher concessions versus choosing war. This assumption is required for the long run characterization of our equilibrium. We provide a direct interpretation of Assumption 3 in Remark 5.

Remark 3 *$W = 1$ is the only equilibrium in the one shot version of this game. Conditional on $W = 0$, group 2 chooses $x_2 = 0$. Therefore, by Assumption 2, group 1 chooses $F_1 = 1$.*

Group 2 would like to commit to making a concession to group 1 in order to deter group 1 from using violent force. However, if group 1 is committed to not fighting, group 2 actually prefers to not make a concession. By examining history dependent strategies it may be possible to improve upon this equilibrium. Though imperfect information plays no role in the one shot version of this game, in a dynamic setting, a group always has the option of *privately* making zero concessions, and in this regard, the success and failure of a concession affects a group's reputation, and the continuation sequence of play.

2.3 Public Perfect Equilibria

We consider equilibria in which each group conditions its strategy on past public information. Specifically, let $h_t = \left\{z^{t-1}, F_1^{t-1}, F_2^{t-1}, (s_1x_1)^{t-1}, (s_2x_2)^{t-1}\right\}$, the history of public information at t prior to the realization of z_t.[10] Define a strategy $\sigma_i = \{F_{it}(h_t, z_t), x_{it}(h_t, z_t)\}_{t=0}^{\infty}$ with $\sigma = \{\sigma_i, \sigma_{-i}\}$.

[9] There is no need for $\overline{x}_1$ to be large, and it is even possible for $\overline{x}_1 = 0$.

[10] Recent theoretical work departs from this equilibrium concept by considering private strategies, for example in Kandori and Obara (2006). We choose to focus on public strategies here since equilibria with private strategies are difficult to characterize in our framework.

Definition 4 *σ is feasible if* $\forall t \geq 0$ *and* $\forall z_t \in [0,1]$,

$$\{F_{it}(h_t, z_t), x_{it}(h_t, z_t)\}_{i=1,2} \in \{\{0,1\}, [0, \overline{x}_i]\}_{i=1,2}.$$

For a particular σ, define the equilibrium continuation value for i at (h_t, z_t) as:

$$U_i(\sigma|_{h_t,z_t}) = u_i(F_{it}(h_t, z_t), F_{-it}(h_t, z_t), x_{it}(h_t, z_t), x_{-it}(h_t, z_t)) + \beta \mathbf{E}\left\{U_i\left(\sigma|_{h_{t+1},z_{t+1}}\right)|h_t, z_t\right\}$$

for $\sigma|_{h_t,z_t}$ which is the continuation of a strategy after (h_t, z_t) has been realized. Let $\Sigma_i|_{h_t,z_t}$ denote the entire set of feasible public continuation strategies for i after (h_t, z_t) has been realized.

Definition 5 *σ is public perfect if it is feasible and if* $\forall (h_t, z_t)$

$$U_i(\sigma|_{h_t,z_t}) \geq U_i\left(\sigma'_i|_{h_t,z_t}, \sigma_{-i}|_{h_t,z_t}\right) \ \forall \sigma'_i|_{h_t,z_t} \in \Sigma_i|_{h_t,z_t} \text{ for } i = 1, 2. \tag{2.1}$$

In a public perfect equilibrium, each group dynamically chooses its best response given the strategy of its rival. Because group $-i$'s strategy is public, a deviation by group i to a non-public strategy is irrelevant (see Fudenberg, Levine, and Maskin, 1994).

We now consider how we can build a public perfect equilibrium. Note that by the structure of the game depicted in Figure 2.1, war can always be enforced, since neither group can affect its realization on the margin if both groups choose to fight.

Definition 6 *A symmetric fighting public perfect equilibrium is a public perfect equilibrium for which* $F_{1t}(h_t, z_t) = F_{2t}(h_t, z_t) \ \forall (h_t, z_t)$.

Remark 4 *For every public perfect σ, there exists a symmetric fighting public perfect equilibrium σ' which delivers the same sequence of war, peace, concessions, and utilities. This is because* $u_i(1, 0, x_i, x_{-i}) = u_i(0, 1, x_i, x_{-i}) = u_i(1, 1, x_i, x_{-i})$, *so that conditional on* $F_i = 1$, $-i$ *is indifferent between* $F_{-i} = 0$ *and* $F_{-i} = 1$.

Our interest is in characterizing the dynamics of war, and our model is such that the realization and payoff of war is independent of the instigator of war. Remark 4 implies that we can focus without loss of generality on equilibria with symmetric fighting decisions, and these will characterize the entire set of public perfect equilibria. For the rest of our discussion we will refer to public perfect equilibria and to symmetric fighting public perfect equilibria interchangeably.

In order to build a symmetric fighting public perfect equilibrium allocation, let

$$q_t = \left\{z^{t-1}, (s_1x_1)^{t-1}, (s_2x_2)^{t-1}\right\},$$

the *equilibrium* history of public signals prior to the realization of z_t in period t. Define an *equilibrium* allocation

$$\alpha = \{W_t(q_t, z_t), x_{1t}(q_t, z_t), x_{2t}(q_t, z_t)\}_{t=0}^{\infty}.$$

Let $U_i(\alpha|_{q_t,z_t})$ represents the equilibrium continuation value at a public history (q_t, z_t) under this sequence of actions. Define

$$\underline{U}_i = \frac{w_i}{1-\beta},$$

the payoff to i from permanent war. By our discussion in Remark 3, there exists an equilibrium with permanent war which generates $\{\underline{U}_1, \underline{U}_2\}$.

The next proposition explains that public perfect equilibria can be enforced by reverting to permanent war after any observable deviation. Naturally, either group will always be able to unobservably deviate in periods of peace by making zero concessions. Let

$$\mathbf{E}\left\{U_i\left(\alpha|_{q_{t+1},z_{t+1}}\right)|q_t, z_t, s_{it}x_{it} = x_{it}(q_t, z_t)\right\}, \text{ and}$$
$$\mathbf{E}\left\{U_i\left(\alpha|_{q_{t+1},z_{t+1}}\right)|q_t, z_t, s_{it}x_{it} = 0\right\}$$

represent the expected continuation value to i at $t+1$ conditional on q_t, z_t, and a successful or unsuccessful concession by agent i, respectively. Unless specified otherwise, all proofs are in the Appendix.

Proposition 8 *(public perfect equilibria) α is public perfect if and only if* $\forall (q_t, z_t)$

$$U_i(\alpha|_{q_t,z_t}) \geq \underline{U}_i \text{ for } i = 1,2 \text{ and} \tag{2.2}$$

$$\beta \begin{pmatrix} \mathbf{E}\left\{U_i\left(\alpha|_{q_{t+1},z_{t+1}}\right)|q_t, z_t, s_{it}x_{it} = x_{it}(q_t, z_t)\right\} \\ -\mathbf{E}\left\{U_i\left(\alpha|_{q_{t+1},z_{t+1}}\right)|q_t, z_t, s_{it}x_{it} = 0\right\} \end{pmatrix} \geq x_{it}(q_t, z_t) \tag{2.3}$$

$$\text{for } i = 1,2 \text{ if } W_t(q_t, z_t) = 0.$$

Proposition 8 establishes conditions which are necessary and sufficient for an allocation to be public perfect. Either group can always fight forever, and this creates a lower bound for its continuation values, which establishes (2.2). Furthermore, whenever group i is prescribed a concession, group i can choose zero concessions and proceed with the same strategy after the unobservable deviation. In order for this deviation to be suboptimal, the marginal benefit into the future of making a successful concession today (the left hand side of (2.3) multiplied by π_i) must weakly exceed the instantaneous marginal cost of making a concession today (the right hand side of (2.3) multiplied by π_i). Note that under perfect information, (2.3) is an unnecessary constraint, and all deviations are observable and punished by reversion to permanent war.[11]

Definition 7 Λ *is the set of public perfect allocations.*

Definition 8 $V \in R^2$ *is the set of public perfect period zero continuation values.*

By the stationarity of the game, $\{U_1(\alpha|_{q_t,z_t}), U_2(\alpha|_{q_t,z_t})\} \in V \; \forall (q_t, z_t)$, and this observation is useful for the recursive representation of the equilibrium which is discussed in Section 2.5. The presence of the public signal z to be used as a randomization device implies that V is convex.

Lemma 6 *V is convex and compact.*

[11]Under perfect information, there are two types of efficient public perfect equilibria: permanent war and permanent peace. Permanent peace is achievable under a high enough discount factor.

2.4 Peaceful Public Perfect Equilibria

Remark 3 establishes that in the static version of our game, peace is not achievable. We now show that peace is always achievable in the repeated game if war is sufficiently costly to group 2. This is because group 1 can provide dynamic incentives for group 2 to make concessions today by threatening to fight in the future if concessions fail today. In proving this result, we take into account that the cost of war cannot be so large as to violate Assumption 3.

Definition 9 *α is peaceful if $\exists\,(q_t, z_t)$ s.t. $\Pr\{q_t, z_t\} > 0$, and $W_t(q_t, z_t) = 0$.*

Proposition 9 ***(possibility of peace)*** *$\exists w_2^* \in (-\overline{x}_2/\beta, 0)$ s.t. $\forall w_2 \leq w_2^*$, $\exists \alpha \in \Lambda$ which is peaceful.*

Proof. *Construct the following equilibrium. If $s_{2t-1}x_{2t-1} = x$, then $W_t = 0$, $x_{1t} = 0$ and $x_{2t} = x$, for $x \in (0, \overline{x}_2)$ which satisfies the inequalities of Assumption 1. If $s_{2t-1}x_{2t-1} = 0$, both groups revert to the repeated static public perfect equilibrium with symmetric fighting forever. Let $W_0 = 0$, $x_{10} = 0$, and $x_{20} = x$. By Assumption 1, both groups i prefer equilibrium continuation values to $\underline{U}_i$, so that* (2.2) *is satisfied. To check* (2.3)*, let $U_2|_{s_2=1}$ represent the continuation value to group 2 conditional on successful concessions yesterday. The stationarity of the equilibrium implies*

$$U_2|_{s_2=1} = -\pi_2 x + \beta\left(\pi_2 U_2|_{s_2=1} + (1-\pi_2)\,\underline{U}_2\right),$$

so that (2.3) *which requires $\beta\left(U_2|_{s_2=1} - \underline{U}_2\right) \geq x$ becomes $-x \geq \beta w_2$. Since $x \in (0, \overline{x}_2)$, it follows that there exists a range between $-\overline{x}_2/\beta$ and $-x/\beta$ for values of w_2 which satisfy this condition.* ■

Remark 5 *Assumption 3 implies that the equilibrium described in the proof of Proposition 9 cannot be public perfect if $x = \overline{x}_2$. More specifically, if group 1 requests $\overline{x}_2$ from group 2, then group 2 must be rewarded with an increase in continuation value.*

Proposition 10 ***(necessity of war)*** *$\nexists \alpha \in \Lambda$ s.t. $W_t(q_t, z_t) = 0\ \forall\,(q_t, z_t)$.*

War must be expected in the future in all periods, since it is is required to provide incentives for group 2 to make concessions. Without war, group 2 makes zero concessions, and by Assumption 2, group 1 cannot be satisfied by zero concessions. Together with Proposition 9, Proposition 10 means that any periods of peace are necessarily followed by periods of war. Moreover, in going to war, groups realize that cooperation occurred in the past, so that *war is by no means ex-post necessary, though it is ex-ante required for the enforcement of peace.* In this sense, the equilibria we describe are not renegotiation proof.[12]

2.5 Efficient Public Perfect Equilibria

2.5.1 Definition

We now characterize efficient public perfect equilibria in order to describe the optimal dynamics of war and peace. Let $U_i(\alpha)$ represents the period 0 continuation value to i implied by α prior to the realization of z_0.

Definition 10 *$\alpha \in \Lambda$ is an efficient public perfect equilibrium if $\nexists \alpha' \neq \alpha$ s.t. $\alpha' \in \Lambda$, $U_i(\alpha') > U_i(\alpha)$, and $U_{-i}(\alpha') \geq U_{-i}(\alpha)$ for $i = 1, 2$.*

We can write our program as maximizing the welfare of group 1 subject to providing group 2 with a minimum welfare of v_0:

$$\max_{\alpha} U_1(\alpha) \text{ s.t. } U_2(\alpha) \geq v_0 \text{ and } \alpha \in \Lambda. \tag{2.4}$$

In order to solve this program, we first characterize the set of continuation values V. Second, we argue that continuation values travel along the contours of V in the solution to (2.4). As a consequence, it is possible to write (2.4) as a recursive program. We use this recursive program to ask two questions. First, are fluctuations between war and peace along the equilibrium path features of an efficient equilibrium? Second, does the efficient equilibrium admit fluctuations between war and peace in the long run?

[12] There is no renegotiation proof equilibrium with peace in our model, since war is always required to sustain it.

2.5.2 Shape of V

Our first step is to show that the convex hull of V is inverse U-shaped. Let $K(v)$ represent the solution to (2.4) subject to $U_2(\alpha) = v \geq v_0$. By Lemma 6, it follows that $K(v)$ is defined for $v \in [\underline{U}_2, \overline{U}_2]$, for some $\overline{U}_2 \geq \underline{U}_2$ which represents the highest possible period zero continuation value which can be assigned to group 2.

Lemma 7 $K(\underline{U}_2) = K(\overline{U}_2) = \underline{U}_1$.

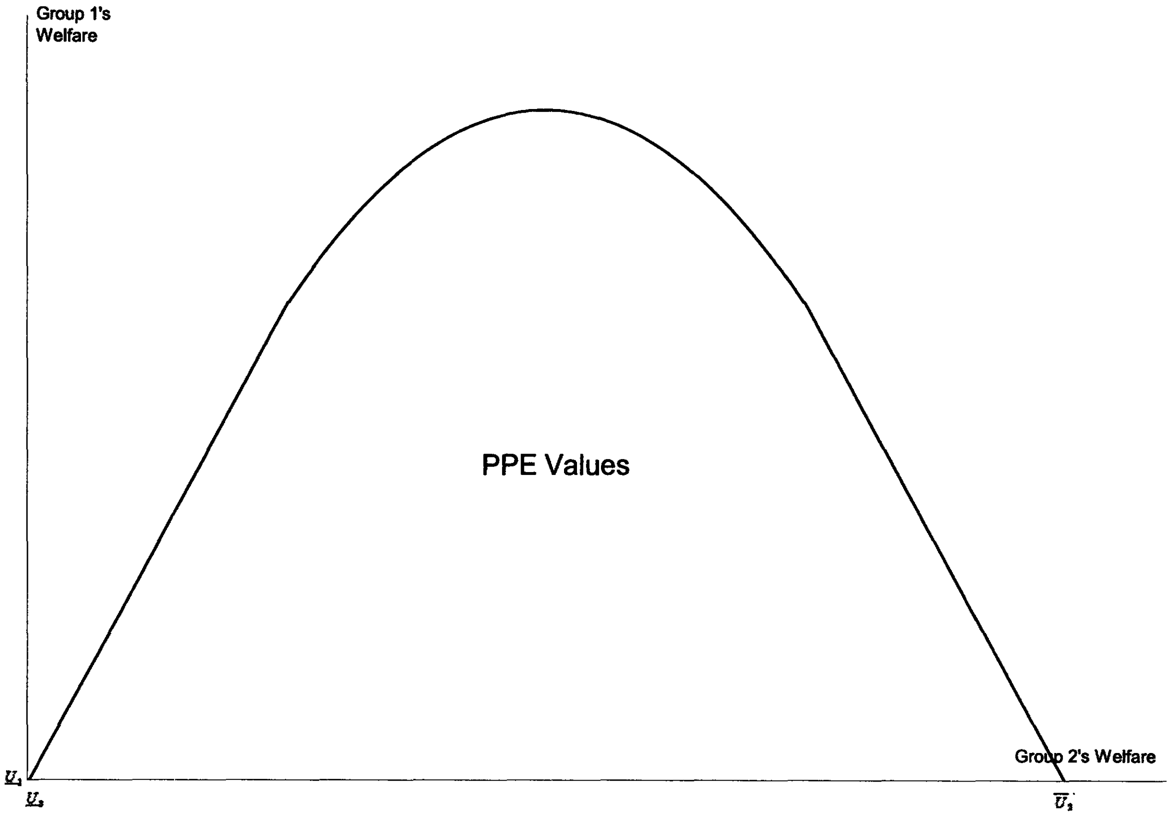
The reasoning of Lemma 7 is as follows. The first part of the lemma is a consequence of the fact that the unique method of providing group 2 with a continuation value of $\underline{U}_2$ is through permanent war which provides group 1 with a continuation value of $\underline{U}_1$. $\underline{U}_2$ cannot be delivered to group 2 via peace, because it is not possible to provide incentives for group 2 to make concessions. This is because the continuation value to group 2 cannot decline in the event of a failed concession since the continuation value is already at a minimum.

The second part of the lemma states that the highest continuation value to group 2 is associated with the lowest continuation value to group 1. If this were not the case, then it would be possible to increase the continuation value of group 2 while reducing the continuation value of group 1. This is because providing incentives to group 1 to make a bigger concession or to accept a lower concession today (in order to increase group 2's current utility) does not generate an efficiency loss since this does not increase the probability of future war. More specifically, Assumption 2 implies that, while war is a useful device for disciplining group 2 into making concessions since concessions are preferable to war (i.e., $w_2 < 0$), war is not a useful device for disciplining group 1 into making concessions since concessions are not preferable to war (i.e., $w_1 > 0$).

The implications of Lemmas 6 and 7 are displayed in Figure 2.2. The y-axis represents $K(v)$ and the x-axis represents v. $K(v)$ is increasing for low values of v and decreasing for high values of v. All of the points underneath $K(v)$ and above the x-axis represent the space of public perfect continuation values V. Any efficient equilibrium must begin on the downward sloping portion of $K(v)$ where it is not possible to make one group strictly better off without making the other group strictly worse off. The increasing portion of $K(v)$ is the consequence of the "value burning" associated with the use of war to discipline concession-making by group

2.[13]

Figure 2.2: Set of Values



Assumption 4 $\underline{U}_2 < \overline{U}_2$.

A necessary and sufficient condition for Assumption 4 is that there exist a public perfect equilibrium with peace, so that V is not a singleton associated with permanent war.

2.5.3 Recursive Program

We now use the shape of V to argue that (2.4) can be written as a recursive program, so that continuation values travel along the contours of the set of continuation values along the equilibrium path. More specifically, let $\pi^{HH} = \pi_1\pi_2$, $\pi^{LL} = (1-\pi_1)(1-\pi_2)$, $\pi^{HL} = \pi_1(1-\pi_2)$, and

[13]Because "value burning" is used to provide incentives only to group 2, this set of values is not oval shaped as in Sannikov (2006b).

$\pi^{LH} = (1-\pi_1)\pi_2$, and let $S = \{HH, LL, HL, LH\}$. Consider the following recursive program in which the state variable v–the continuation value assigned to group 2–subsumes the entire history of the game:

$$K(v) = \max_{\left\{W_z, x_{1z}, x_{2z}, v_z^W, \{v_z^j\}_{j\in S}\right\}_{z\in[0,1]}} \int_0^1 \left(\begin{array}{c} W_z \left[w_1 + \beta K\left(v_z^W\right)\right] + \\ (1-W_z)\left[\pi_2 x_{2z} - \pi_1 x_{1z} + \beta \sum_{j\in S} \pi^j K\left(v_z^j\right)\right] \end{array} \right) dG_z \tag{2.5}$$

s.t.

$$v = \int_0^1 \left(W_z \left[w_2 + \beta v_z^W\right] + (1-W_z)\left[\pi_1 x_{1z} - \pi_2 x_{2z} + \beta \sum_{j\in S} \pi^j v_z^j\right]\right) dG_z, \tag{2.6}$$

$$K\left(v_z^m\right) \geq \underline{U}_1 \ \forall z \in [0,1] \text{ and } \forall m \in \{W, S\}, \tag{2.7}$$

$$v_z^m \geq \underline{U}_2 \ \forall z \in [0,1] \text{ and } \forall m \in \{W, S\}, \tag{2.8}$$

$$\beta\left(\pi_2\left(K\left(v_z^{HH}\right) - K\left(v_z^{LH}\right)\right) + (1-\pi_2)\left(K\left(v_z^{HL}\right) - K\left(v_z^{LL}\right)\right)\right) \geq x_{1z} \ \forall z \in [0,1], \tag{2.9}$$

$$\beta\left(\pi_1\left(v_z^{HH} - v_z^{HL}\right) + (1-\pi_1)\left(v_z^{LH} - v_z^{LL}\right)\right) \geq x_{2z} \ \forall z \in [0,1], \tag{2.10}$$

$$v_z^{Hm} = v_z^{Lm} \text{ for } m = H, L \text{ if } x_{1z} = 0 \ \forall z \in [0,1], \tag{2.11}$$

$$v_z^{mL} = v_z^{mH} \text{ for } m = H, L \text{ if } x_{2z} = 0 \ \forall z \in [0,1], \tag{2.12}$$

$$W_z \in \{0,1\} \ \forall z \in [0,1], \text{ and } x_{iz} \in [0, \overline{x}_i] \ \forall z \in [0,1] \text{ and } i = 1, 2. \tag{2.13}$$

Proposition 11 (2.4) *subject to* $U_2(\alpha) = v$ *is equivalent to* (2.5) – (2.13).

In order to understand Proposition 11, we first describe the recursive formulation in (2.5) – (2.13). $K(v)$ is defined in Section 2.5.2 and it represents the highest possible welfare that can be achieved by group 1 conditional on providing group 2 with a continuation value equal to v. Equation (2.5) represents this program written in a recursive fashion, taking into account that the signal z can be used to randomize across allocations. W_z represents the decision to go to war, and x_{iz} represents the size of a concession by i, each for a given realization of z. v_z^W

represents the value promised to group 2 for tomorrow conditional on war taking place today for a given realization of z. v_z^{HH}, v_z^{LL}, v_z^{HL}, and v_z^{LH} represent the values promised to group 2 for tomorrow conditional on peace taking place today. v_z^{HH} follows successful concessions by both groups, v_z^{LL} follows unsuccessful concessions by both groups, v_z^{HL} follows a successful concession by group 1 and an unsuccessful concession by group 2, and v_z^{LH} follows a successful concession by group 2 and an unsuccessful concession by group 1.

Equation (2.6) represents the promise keeping constraint which ensures that group 2 is achieving a continuation value of v. Equations (2.7) and (2.8) represent the recursive version of (2.2) for $i = 1$ and $i = 2$, respectively. Equations (2.9) and (2.10) represent the recursive versions of (2.3) for $i = 1$ and $i = 2$, respectively. Equations (2.11) and (2.12) are informational constraints which ensure that continuation equilibria do not depend on the success or failure of concessions if those concessions are zero and cannot be observed to fail by definition. Constraints (2.13) ensure that the allocation is feasible.[14]

Proposition 11 implies that, in the efficient equilibrium which solves (2.4), continuation value pairs always travel along the convex hull of V and therefore along the contours of the set of values in Figure 2.2. This is a consequence of the structure of our game which implies that concessions in any period need only be made by one group at a time, so that incentive provision is only necessary for one group at a time. For example, imagine an equilibrium in which each group i makes a concession $x_{iz} > 0$. One can construct a continuation equilibrium in which each group i makes a concession of size $\max\{x_{iz} - \pi_{-i}x_{-iz}/\pi_i, 0\}$ so that only one group makes concessions, and this equilibrium yields the same sequence of utilities. Therefore, incentives need only be provided to the group which is making concessions, which means that continuation values should not move to the interior of V after any history since this unnecessarily punishes both groups at the same time.

2.5.4 Dynamics of War and Peace

Using the recursive formulation, we can characterize the dynamics of war and peace. We are specifically interested in determining whether fluctuations between war and peace are efficient.

[14]Note that if $W_z = 1$, then the value of v_z^j's is irrelevant, and analogously, if $W_z = 0$, then the value of v_z^W is irrelevant.

Let us define fluctuations formally. Let $(q_t, z_t) \in (q_k, z_k)$ for $k > t$ imply that (q_t, z_t) is a subhistory of (q_k, z_k), meaning a history which occurs along the path associated with (q_k, z_k).

Definition 11 *α generates fluctuations between war and peace if* $\exists (q_t, z_t) \in (q_k, z_k) \in (q_l, z_l)$ *s.t.* $l > k > t$, $\Pr\{q_l, z_l\} > 0$, *and*

1. $W_t(q_t, z_t) = 0$, $W_k(q_k, z_k) = 1$, $W_l(q_l, z_l) = 0$, *or*

2. $W_t(q_t, z_t) = 1$, $W_k(q_k, z_k) = 0$, $W_l(q_l, z_l) = 1$.

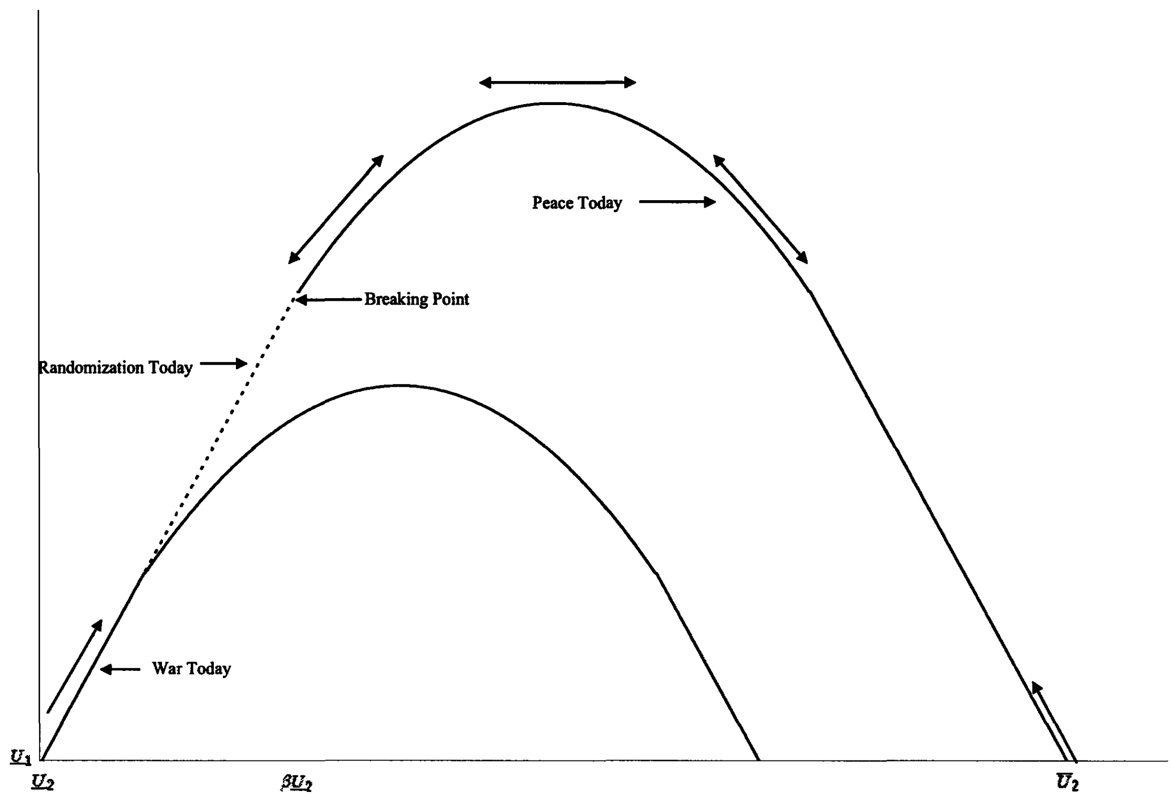
By Proposition 10, war must follow peace, so fluctuations will occur if peace follows war along the equilibrium path. Let

$$\left\{W_z^*(v), v_z^{W*}(v), x_{1z}^*(v), x_{2z}^*(v), \left\{v_z^{j*}(v)\right\}_{j\in S}\right\}_{z\in[0,1]}$$

represent a solution to (2.5) – (2.13) for a given v. Such a solution need not be unique since the program is not strictly concave.

In order to characterize the solution to (2.5) – (2.13), it is useful to characterize the solution to (2.5) – (2.13) subject to the additional constraint that war is assured today (i.e., $\int_0^1 W_z dG_z = 1$) as well as the solution to (2.5) – (2.13) subject to the additional constraint that peace is assured today (i.e., $\int_0^1 W_z dG_z = 0$). The solution to (2.5) – (2.13) will represent a convex combination of these two constrained programs (by choosing any feasible set of W_z's). This exercise is performed in the Appendix and is displayed in Figure 2.3 which depicts the solution to the two constrained programs along with their convex combination denoted by the dotted line.

Figure 2.3: Values Generated by War and Peace Today



Equations (2.8) and (2.10) imply that $\beta \underline{U}_2$ represents the lowest continuation value that can ever be promised to group 2 starting from assured peace today, since $\beta \underline{U}_2$ is the continuation value received by group 2 if group 2 makes zero concessions today and receives its lowest possible continuation value tomorrow as a punishment. Therefore, all continuation values below $\beta \underline{U}_2$ are generated either by assured war today or by randomization between war and peace today. In the Appendix, we argue that the portion of $K(\cdot)$ to the left of $\beta \underline{U}_2$ is a straight line, and this leads to the following result.

Proposition 12 ***(efficient permanent war)*** $v_z^{W*}(v) = \underline{U}_2$ $\forall z$ *is a solution* $\forall v \in [\underline{U}_2, \overline{U}_2]$.

Together with Proposition 10, Proposition 12 means that there exist efficient equilibria *without* fluctuations between war and peace. Such an equilibrium begins with peace, and it stochastically transitions to a state of permanent war. More specifically, the continuation value to group 2 can stochastically increases or decreases along the equilibrium path, and the sequential failure of group 2's concessions lead group 2's continuation value to decrease into the range between $\underline{U}_2$ and $\beta\underline{U}_2$. In this range, the two groups randomize between temporary peace and permanent war. In the long run, permanent war is inevitable since permanent peace is not sustainable by Proposition 10.

The insight to this proposition is related to the work of Abreu, Pearce, and Stacchetti (1990), who argue that extreme points in the set continuation values can sustain all equilibria. Nevertheless, since there are multiple solutions to the problem, Proposition 12 does not mean that fluctuations between war and peace are inefficient per se, it just means that we can always construct efficient equilibria without fluctuations.

We are nevertheless interested in whether fluctuations between war and peace could also be efficient. In order to answer this question, we first determine when it is efficient for peace to occur. The following proposition shows that if assured peace today is possible (i.e., if $v \geq \beta\underline{U}_2$), then it strictly dominates all allocations with probabilistic war.

Proposition 13 ***(efficient peace)*** *If $v \geq \beta\underline{U}_2$, then $W_z^*(v) = 0 \; \forall z$ is the unique solution.*

To understand this proposition, note that there are potentially two means of punishing group 2 for failed concessions. The first is for group 1 to ask for larger concessions. The second is by fighting. However, war is costly to both groups, whereas peace with high concessions by group 2 is only costly to group 2, which explains Proposition 13. More specifically, group 1 can request a very high concession today and promise peace tomorrow if the concessions requested today succeed, and this is beneficial to both groups since it reduces the probability of war in the future. The ability for group 1 to request such a high concession is assured by Assumption 3, which implies that concessions can be made large enough so as to reward their success with an increase in group 2's (and by consequence group 1's) continuation value. Technically, this means that there is a drop in the slope of $K(\cdot)$ at $v = \beta\underline{U}_2$. We will refer to $\beta\underline{U}_2$ as a *breaking point.*

We can now more thoroughly describe the dynamics of continuation values.

Proposition 14 ***(efficient transitions)***

(i) If $v \in (\underline{U}_2, \beta\underline{U}_2)$*,* $\exists$ *a solution s.t.* $v_z^{W*}(v) > \underline{U}_2$ $\forall z$ *s.t.* $W_z^*(v) = 1$.

(ii) If $v \in (\underline{U}_2, \beta\underline{U}_2)$*,* $\nexists$ *a solution s.t.* $v_z^{j*}(v) \neq \underline{U}_2$ $\forall j \in S$ *and for any* z *s.t.* $W_z^*(v) = 0$.

(iii) If $v \in [\beta\underline{U}_2, \overline{U}_2]$*,* $\nexists$ *a solution s.t.* $v_z^{j*}(v) \geq v$ $\forall j \in S$ *and* $\forall z$ *s.t.* $W_z^*(v) = 0$.

The arrows on the contours of the set in Figure 2.3 represent the implied direction of continuation values for tomorrow. The first part of Proposition 14 along with Proposition 12 means that there are potentially two ways of punishing a series of failed concessions by group 2. The first is to use war today with a high probability, but to return to peace in the future. The alternative way is to use war today with a low probability, but to never return to peace in the future. Both are equivalent methods of *partial* punishment. The second part of Proposition 14 means that the failure of concessions in the region of partial punishment must be punished with permanent war. While costly, this allows the success of concessions to be rewarded with a longer peace. The third part of Proposition 14 means that continuation values to group 2 always have a positive probability of decreasing under peace. If group 2 is making a concession, its continuation value must decrease if its concession fails. If group 1 is making a concession, the continuation value to group 2 must decrease if group 1's concession succeeds. Therefore, there is a constant downward pressure on group 2's continuation value.

Our propositions implies the following path for group 2's continuation value. After a certain number of sequential failures of concessions by group 2, war is necessary, so that continuation values are in the range $[\underline{U}_2, \beta\underline{U}_2)$. If values are in $(\underline{U}_2, \beta\underline{U}_2)$, then there are two possible outcomes. The first outcome is randomization between permanent war at $\underline{U}_2$ and peace at $\beta\underline{U}_2$ and is due to Proposition 12. The second outcome is randomization between some value in the range $(\underline{U}_2, w_2 + \beta^2\underline{U}_2)$ and $\beta\underline{U}_2$, given that $w_2 + \beta^2\underline{U}_2$ represents the highest continuation value that can efficiently be promised with assured war today and some transition to peace in the future at $\beta\underline{U}_2$. Therefore, all paths which do not hit $\underline{U}_2$ eventually hit the breaking point $\beta\underline{U}_2$. Group 2 is never forgiven beyond this point. If a concession by group 2 fails here, permanent war ensues. If a concession succeeds, future peace ensues. However, even if peace ensues, there is always a probability of reaching the breaking point again in the future, since group 2 will be punished again, and this leads to our main theorem regarding convergence to permanent war.

Definition 12 α *converges to permanent war if* $\lim_{t\to\infty} \Pr\{W_t(q_t, z_t) = 1\} = 1$.

Theorem 4 *(short run fluctuations and long run permanent war)*

(i) If $\overline{U}_2 > \beta \underline{U}_2$*,* $\exists v_0 \in [\underline{U}_2, \overline{U}_2]$ *with a solution with fluctuations.*

(ii) $\forall v_0 \in [\underline{U}_2, \overline{U}_2]$*, all solutions converge to permanent war.*

The first part of Theorem 4 establishes that fluctuations can emerge under some initial conditions, and the requirement that $\overline{U}_2 > \beta \underline{U}_2$ means that continuation values associated with peace are not a singleton, and this condition is easily satisfied.[15] Given that many continuation values begin from peace, one can choose a continuation value v_0 from which a path to $(\underline{U}_2, \beta \underline{U}_2)$ must occur with positive probability. Intuitively, partial punishment occurs in this region, and it is possible to inflict partial punishment by choosing war and returning to peace in the future.

While convergence to permanent war seems like an undesirable outcome, the threat that it poses enables to two groups to reach a higher welfare in the short run by prolonging peace. As time passes, the probability that diplomacy fails increases, which increases the probability of a phase of war, and which makes it more likely to reach the breaking point.[16] Though transitions to permanent war in our model are required in the long run, the probability with which this occurs goes to 0 as the discount factor β goes to 1. This is because in the limit, continuation values do not decline after a failed concessions and permanent war is never reached.

Our long run result is related to that of Sannikov (2006a,2006b). He shows that in a class of continuous time games in which information follows a diffusion process, convergence to the static Nash equilibrium occurs in the long run if the repeated static Nash equilibrium payoffs are on the contours of the set of continuation values. This result in our discrete time model in which information follows a Poisson process also depends on Assumption 3. If instead $\beta w_2 < -\overline{x}_2$, then Proposition 13 fails, and probabilistic war is efficient over a larger set of continuation values, since peace only strictly dominates war for some $v > \beta \underline{U}_2$. More specifically, it is not always possible for group 1 to request high enough concessions so as to be able to reward their success with peace tomorrow. Once Assumption 3 is relaxed, transitions to permanent war continue to be efficient, though they are not *required* for efficiency.

[15] As long as it is possible to improve the welfare of group 1 over $\underline{U}_1$ then this is possible. This is implied if $-w_1 > \beta w_2$.

[16] We remind the reader that Theorem 4 is a consequence of efficiency, and not a consequence of subgame perfection, since inefficient equilibria with permanent fluctuations between peace and war can generally be constructed.

2.6 Excessive Force

Theorem 4 provides us with an empirically untestable hypothesis, and it is to some extent puzzling given that fluctuations between war and peace have lasted for several decades in the examples which motivate our study. In order to explore this issue further, we consider a realistic extension of our model which allows for the use of excessive force. This is particularly relevant given that, in practice, groups can often decide the amount of harm they wish to inflict on their rival. We model excessive force as the ability to inflict additional damage on the rival group at a cost.

Formally, imagine if while choosing F_{1t}, group 1 chooses $D_t = \{0, 1\}$, and we continue to define W_t as in the benchmark model. If $W_t = 0$, then the game ensues as in the benchmark model. However if $W_t = 1$, then payoffs depend on D_t. Specifically, payoffs are:

$$\{w_1 - D_t e, w_2 - D_t \chi\},$$

for $e, \chi > 0$. e represents the cost to group 1 of using excessive force and χ represents the additional damage inflicted to group 2 from group 1's use of excessive force. For simplicity, we only allow group 1 to use excessive force, though a full extension of the model in which both groups can use excessive force is possible. Such an extension is analytically more complicated but leads to the same results as those presented in this section.[17]

Since excessive force is costly to group 1, the one shot equilibrium is the same as in the benchmark model.

Remark 6 $W = 1$ *and* $D = 0$ *is the only equilibrium in the one shot version of this game. Conditional on* $W = 0$*, each group* i *chooses* $x_i = 0$*. By Assumption 2 and since* $e > 0$*, group 1 prefers* $W = 1$ *and* $D = 0$*.*

A naive intuition would suggest that excessive force is both irrational and inefficient. However, it can be incentive compatible in a dynamic setting. Define an allocation α analogously to before, and incorporate the choice of D_t into group 1's decision. We can generate necessary and sufficient conditions under which an allocation constitutes a public perfect equilibrium.

[17]Details available upon request.

Proposition 15 ***(public perfect equilibria)*** *α is public perfect if and only if $\forall (q_t, z_t)$*

$$U_i\left(\alpha|_{q_t,z_t}\right) \geq \underline{\underline{U}}_i \text{ for } i = 1,2 \text{ and} \tag{2.14}$$

$$\beta \left(\begin{array}{c} \mathbf{E}\left\{U_i\left(\alpha|_{q_{t+1},z_{t+1}}\right)|q_t, z_t, s_{it}x_{it} = x_{it}\left(q_t, z_t\right)\right\} \\ -\mathbf{E}\left\{U_i\left(\alpha|_{q_{t+1},z_{t+1}}\right)|q_t, z_t, s_{it}x_{it} = 0\right\} \end{array} \right) \geq x_{it}\left(q_t, z_t\right) \tag{2.15}$$

$$\text{for } i = 1,2 \text{ if } W_t\left(q_t, z_t\right) = 0$$

for $\underline{\underline{U}}_1 = \underline{U}_1$ and $\underline{\underline{U}}_2 \leq \underline{U}_2$.

The only difference between Proposition 8 and Proposition 15 is that the lowest possible promised value to group 2 may actually be below $\underline{U}_2$, the payoff generated from the repeated static equilibrium. This follows from basic arguments due to Abreu (1988) that punishments which are worse than those associated with the repeated static equilibrium can be sustained along the equilibrium path. Such a punishment can emerge if group 1 exerts excessive force.[18]

As an example, imagine if group 1 requests a constant concession from group 2 during peace. Group 1 punishes failed concessions by group 2 by temporarily expending excessive force and by promising peace tomorrow. Group 2 makes concessions during peace since it prefers to not be punished. Moreover, group 1 is willing to temporarily expend excessive force because it looks forward to future peace, since failure to expend excessive force during war by group 1 would lead to a breakdown of the equilibrium and a reversion to the repeated static equilibrium with permanent war. Therefore, it is precisely the positive probability of future peace which provides an incentive for group 1 to expend excessive force. Such an equilibrium is possible if the cost of expending excessive force is not too high and if the damage caused by the use of excessive force is very high. This is expressed formally in Proposition 16. With some abuse of notation we continue to refer to the set of public perfect allocations as Λ.

Definition 13 *α exhibits excessive force if $\exists (q_t, z_t)$ s.t. $\Pr\{q_t, z_t\} > 0$ and $D_t(q_t, z_t) = 1$.*

Proposition 16 ***(possibility of excessive force)*** *$\exists \chi^* \in (0, \infty)$ and $\exists e^* \in (0, \infty)$ s.t. $\forall \chi \geq \chi^*$ and $\forall e \leq e^*$, $\exists \alpha \in \Lambda$ which exhibits excessive force.*

[18] Our previous model is a special case of this extended model with $\chi = 0$.
Note that since group 1 is never tempted to use excessive force when it is not prescribed, the proofs of Propositions 9 and 10 continue to hold here.

Proposition 17 ***(necessity of peace)*** *If* $D_t(q_t, z_t) = 1$, $\exists (q_k, z_k)$ *s.t.* $k \geq t$, $\Pr\{q_k, z_k | q_t, z_t\} > 0$ *and* $W_k(q_k, z_k) = 0 \; \forall \alpha \in \Lambda$.

Proposition 17 means that any equilibrium which experiences the exertion of excessive force must experience peace with positive probability in the future. The reason is that group 1 cannot exert excessive force without additional incentives, otherwise it would never choose to do so, since it is statically inefficient.

Next we consider efficient public perfect equilibria to see if the exertion of excessive force is actually efficient. We can write our program as maximizing the welfare of group 1 subject to providing group 2 with a minimum welfare of v_0:

$$\max_{\alpha} U_1(\alpha) \text{ s.t. } U_2(\alpha) \geq v_0 \text{ and } \alpha \in \Lambda. \tag{2.16}$$

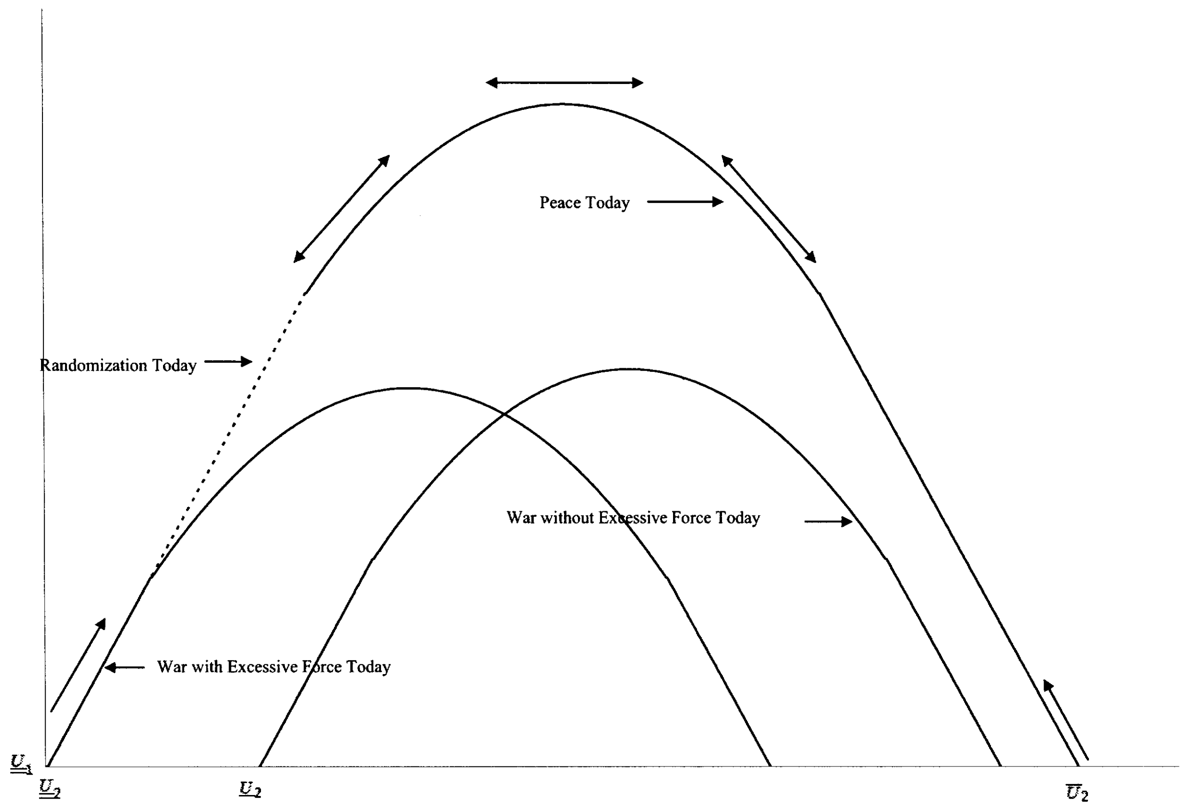
In the Appendix, we show that this extended model can be solved using similar methods to those of the benchmark model. Let $D_z^*(v)$ correspond to an indicator parameterizing the excessive use of force conditional on the realization of war which solves the recursive version of (2.16) in which the continuation value to group 2, v, serves as the state variable.

Proposition 18 ***(efficiency of excessive force)*** $\exists \chi^* \in (0, \infty)$ *and* $\exists e^* \in (0, \infty)$, *s.t.* $\forall \chi \geq \chi^*$ *and* $\forall e \leq e^*$, *then* $D_z^*(v) = 1 \; \forall z$ *is the unique solution.*

Corollary 2 ***(efficiency of long run fluctuations)*** *Under these conditions,* $\forall v_0 \in \left[\underline{\underline{U}}_2, \overline{U}_2\right]$, *there does not exist a solution to* (2.16) *which converges to permanent war.*

To understand Proposition 18, perform a similar exercize as in Figure 2.3 on the extended model by considering the set of continuation values generated by the use of excessive force by group 1 today. We consider the case for χ chosen to be high and e to be chosen to be low so that $\underline{\underline{U}}_2 < \underline{U}_2$.

Figure 2.4: Values Generated by War and Peace Today



The set of values generated under war without excessive force today is inefficient and never intersects $K(v)$ because $\underline{\underline{U}}_2 < \underline{U}_2$. Intuitively, excessive force is a more efficient means of inducing group 2 to cooperate. Therefore, wars always occur with excessive force. This means that it is not possible for the equilibrium to converge to permanent war, since if this were the case, group 1 would never choose to use excessive force. As a consequence, fluctuations between phases of war and phases of peace will occur both in the short run and in the long run. Our result is therefore related to that of Sannikov (2006b) who shows that if the values associated with the repeated static Nash equilibrium are not on the contour of the set of continuation values, then they never occur along the equilibrium path.

In the efficient equilibrium, groups engage in peaceful cooperation which is enforced by the threat of future war, and they engage in excessively destructive war which is enforced by the possibility for future peace. While the distribution of continuation values converges to the lower bound in the benchmark model of Section 2.5, the distribution of continuation values in this extension is the same in the long run as in the short run. There is no longer a sense in which a reduction in welfare in the future sustains an increase in welfare in the present. Moreover, both groups weakly prefer to live in a world in which the use of excessive force is possible from an ex-ante point of view. The reason is that the set of values which can be generated by an equilibrium without excessive force can always be generated by an equilibrium with excessive force.

2.7 Conclusion

We have analyzed a dynamic model in which two groups seeking resources from each other suffer from limited commitment and from informational frictions which lead to war. We have shown that phases of war sustain phases of peace along the equilibrium path. Though war is by no means ex-post necessary, it is ex-ante required for the enforcement of peace. Moreover, fluctuations between phases of peace and phases of war may be efficient in the short run. This model suggests that some of the observed behavior in the motivating examples discussed in the introduction is in fact efficient.

Nonetheless, the benchmark model also predicts that a transition to permanent war in the long run is necessary for efficiency. This is because a reduction in welfare in the long run is required to provide the right incentives to increase welfare in the short run. We show that this transition to permanent war emerges because of the existence of a breaking point which separates phases of war from phases of peace. Our extension explains that our long run result is in part due to the limitation on the use of force. If groups can use excessive force which is sufficiently cheap and sufficiently damaging, then fluctuations between war and peace are efficient in the long run, and excessive force is always used in periods of war. In equilibrium, groups are convinced to use excessive force by the possibility of peace in the future, and groups are even more deterred from not making concessions in light of the excessive destruction which might ensue if these concessions fail.

We highlight some important caveats in interpreting our results. First, we have ignored the role of armament by assuming that the payoff from war is exogenous in the benchmark model. This is apparent in Assumption 2 which effectively states that one group has a military advantage which is not commensurable with the resources it controls. In actuality, the military advantage of groups is partly endogenous and can be affected by the amount of resources at a group's disposal. One can imagine an extended model in which more resources today can increase a group's bargaining power in the future, and taking this into account would certainly affect the long run properties of the equilibrium.[19] Second, we have assumed that there is a single resource over which the two groups bargain, and this has allowed for analytic tractability due to the recursive structure of the program we solve. One can imagine a natural extension of this framework in which each group specializes in the production of a good, so that *bilateral* transfers become necessary under peace.[20] Third, we have ignored the potential role of shocks to bargaining power or to the size of available resources, and an interesting extension would investigate the relationship between these shocks and the realization of war.

[19]Fearon (1996) provides such a model with perfect information and without fluctuations between war and peace.

[20]Our framework allows us to focus on unilateral transfers so that only a single agent requires incentives in a given period, and this allows us to achieve a recursive structure to our program.

2.8 Appendix

2.8.1 Proofs of Section 2.3

Proof of Proposition 8

The necessity of (2.2) follows from the fact that either group i can choose $F'_{ik}(q_k, z_k) = 1$ $\forall k \geq t$ and $\forall (q_k, z_k)$ and this delivers continuation value $\underline{U}_i$. The necessity of (2.3) follows from the fact that conditional on $W_t(q_t, z_t) = 0$, group i can deviate to $x'_{it}(q_t, z_t) = 0$. For sufficiency, consider an allocation which satisfies (2.2) and (2.3), and construct the following off-equilibrium strategy. Any observable deviation results in a reversion to the static equilibrium with permanent war forever. We only need to consider single period deviations for a given (q_t, z_t) since $\beta < 1$ and since payoffs are bounded. Consider periods for which $W_t(q_t, z_t) = 1$. Neither group can affect the realization of war, so that there are no deviations which change payoffs. Consider periods for which $W_t(q_t, z_t) = 0$. Either group i can choose $F'_{it}(q_t, z_t) = 1$ and this leads to permanent war. This is weakly dominated by (2.2). Alternatively, either group can choose another level of $x'_{it}(q_t, z_t) \neq x_{it}(q_t, z_t)$. Any deviation to $x'_{it}(q_t, z_t) > 0$ is strictly dominated by a deviation to $x'_{it}(q_t, z_t) = 0$ since $\mathbf{E}\left\{U_i\left(\alpha|_{q_{t+1},z_{t+1}}\right)|q_t, z_t, s_{it}x_{it} = 0\right\} \geq \underline{U}_2$. Moreover, if $x'_{it}(q_t, z_t) = 0$, then this deviation is weakly dominated by (2.3). **Q.E.D.**

Proof of Lemma 6

Consider two continuation value pair $\{v'_1, v'_2\} \in V$ and $\{v''_1, v''_2\} \in V$ with corresponding allocations α' and α''. It must be that $\{\kappa v'_1 + (1-\kappa) v''_1, \kappa v'_2 + (1-\kappa), v''_2\} \in V$ for $\kappa \in (0,1)$ since $\{\kappa v'_1 + (1-\kappa) v''_1, \kappa v'_2 + (1-\kappa), v''_2\}$ can be achieved by using the signal z_0 to choose the allocation α' with probability κ and the allocation α'' with probability $(1-\kappa)$.

To show that V is closed, let W denote a sequence $\{W_t(q_t, z_t)\}_{t=0}^{\infty}$ and define x_1 and x_2 analogously. Now consider a sequence $V'_j \in V$ such that $\lim_{j\to\infty} V'_j = V'$. There is a corresponding stochastic sequence $\left\{W^j, x_1^j, x_2^j\right\}$. Because each element of this sequence is contained in $\{0,1\} \times [0, \overline{x}_1] \times [0, \overline{x}_2]$, the space of sequences which includes $\left\{W^j, x_1^j, x_2^j\right\}$ is sequentially compact in the product topology. This means that there is a sub-sequence of allocations converging pointwise to a limiting sequence $\{W^\infty, x_1^\infty, x_2^\infty\}$. Given the continuity of the expected utility function of group 1 over the range $[\min\{w_1, -\pi_1\overline{x}_1\}, \pi_2\overline{x}_2]$, the continuity of the expected utility of group 2 over the range $[\min\{w_2, -\pi_2\overline{x}_2\}, \pi_1\overline{x}_1]$, and the fact that $\beta \in (0,1)$, then by the Dominated Convergence Theorem, the limit of the welfare of group 1 equals the welfare of group 1 achieved under the limiting sequence, and the limit of the welfare of group 2 equals the welfare of group 2 achieved under the limiting sequence. This implies that $\{W^{\infty}, x_1^{\infty}, x_2^{\infty}\}$ is public perfect since $\left\{W^j, x_1^j, x_2^j\right\}$ is public perfect, and it means $\{W^{\infty}, x_1^{\infty}, x_2^{\infty}\}$ achieves V'.

2.8.2 Proofs of Section 2.4

Proof of Proposition 10

Imagine if $W_t(q_t, z_t) = 0 \; \forall (q_t, z_t)$. Group 2 can always deviate to $x'_{2t}(q_t, z_t) = 0 \; \forall (q_t, z_t)$, which delivers a continuation value to group 2 equal to 0. Therefore, $U_2(\alpha|_{q_t,z_t}) \geq 0 \; \forall (q_t, z_t)$. Since the size of the surplus cannot exceed zero, $U_1(\alpha|_{q_t,z_t}) + U_2(\alpha|_{q_t,z_t}) \leq 0$, which means that $U_1(\alpha|_{q_t,z_t}) \leq 0 \;\; \forall (q_t, z_t)$, but this violates (2.2) by Assumption 2. **Q.E.D.**

2.8.3 Proofs of Section 2.5

We establish some preliminary results useful for the proofs of this section. In order to establish these results, we note that (2.4) subject to $U_2(\alpha) = v$ can be written in a fashion which takes into account the fact that $\{U_1(\alpha|_{q_1,z_1}), U_2(\alpha|_{q_1,z_1})\} \in V$ so that continuation values are always selected from V. Define the success and failures of concessions as states $S = \{HH, LL, HL, LH\}$ with associated probabilities π^j for $j \in S$ as in Section 2.5.3. Given the set V, the program becomes:

$$\max_{\left\{W_z,x_{1z},x_{2z},\left\{U_{iz}^W,\{U_{iz}^j\}_{j\in S}\right\}_{i=1,2}\right\}_{z\in[0,1]}} \int_0^1 \left(\begin{array}{c} W_z\left[w_1+\beta U_{1z}^W\right]+ \\ (1-W_z)\left[\pi_2 x_{2z}-\pi_1 x_{1z}+\beta\sum_{j\in S}\pi^j U_{1z}^j\right] \end{array} \right) dG_z \tag{2.17}$$

s.t.

$$v=\int_0^1 \left(W_z\left[w_2+\beta U_{2z}^W\right]+(1-W_z)\left[\pi_1 x_{1z}-\pi_2 x_{2z}+\beta\sum_{j\in S}\pi^j U_{2z}^j\right]\right) dG_z, \tag{2.18}$$

$$\{U_{1z}^m, U_{2z}^m\}\in V\ \forall z\in[0,1]\ \text{ and } \forall m\in\{W,S\}, \tag{2.19}$$

$$\beta\left(\pi_2\left(U_{1z}^{HH}-U_{1z}^{LH}\right)+(1-\pi_2)\left(U_{1z}^{HL}-U_{1z}^{LL}\right)\right)\geq x_{1z}\ \forall z\in[0,1], \tag{2.20}$$

$$\beta\left(\pi_1\left(U_{2z}^{HH}-U_{2z}^{HL}\right)+(1-\pi_1)\left(U_{2z}^{LH}-U_{2z}^{LL}\right)\right)\geq x_{2z}\ \forall z\in[0,1], \tag{2.21}$$

$$U_{iz}^{Hm}=U_{iz}^{Lm} \text{ for } m=H,L \text{ if } x_{1z}=0\ \forall z\in[0,1]\ \text{ and } i=1,2, \tag{2.22}$$

$$U_{iz}^{mL}=U_{iz}^{mH} \text{ for } m=H,L \text{ if } x_{2z}=0\ \forall z\in[0,1]\ \text{ and } i=1,2\ , \tag{2.23}$$

$$W_z\in\{0,1\}\ \ \forall z\in[0,1]\,,\text{ and } x_{iz}\in[0,\overline{x}_i]\ \ \forall z\in[0,1]\ \text{ and } i=1,2. \tag{2.24}$$

(2.19) – (2.24) are equivalent to the constraint that $\alpha\in\Lambda$ in the sequence problem. (2.22) and (2.23) are informational constraints which state that no information about the success or failure of a concession can emerge if a concession is not made. Denote the solution to (2.17) – (2.24) by

$$\left\{W_z^*(v)\,,x_{1z}^*(v)\,,x_{2z}^*(v)\,,\left\{U_{iz}^{W*}(v)\,,\left\{U_{iz}^{j*}(v)\right\}_{j\in S}\right\}_{i=1,2}\right\}_{z\in[0,1]}.$$

Definition 14 *Concessions in period t are unilateral if* $x_{it}\geq 0$ *and* $x_{-it}=0$ *for* $i=1,2$.

We argue that initial concessions need only be unilateral, and this implies that the continuation value to either group which depends on the success or failure of this concession will be located on the convex hull of V.

Lemma 8 *There is a solution to (2.4) subject to $U_2(\alpha) = v$ with the following properties:*

(i) It exhibits unilateral concessions by group i at $t = 0$, and

(ii) If $W_0(q_0, z_0) = 0$ for some z_0, then $x_{i0}(q_0, z_0) = x_{i0}(q_0)\ \forall z_0$ for $i = 1, 2$.

Proof. *(i) Consider the solution to (2.4) for which concessions are not unilateral at $t = 0$ for some z_0. Rewrite the problem as in (2.17) − (2.24). Imagine if $\pi_1 x^*_{1z}(v) \geq \pi_2 x^*_{2z}(v)$. Perturb the equilibrium in the following fashion. Let group 1 make a transfer of size $\widehat{x}^*_{1z}(v) = x^*_{1z}(v) - \pi_2 x^*_{2z}(v)/\pi_1$ and let group 2 make a transfer of size 0, so that concessions are unilateral. Moreover, let*

$$\widehat{U}^{HH*}_{iz}(v) = \widehat{U}^{HL*}_{iz}(v) = \pi_{-i} U^{HH*}_{iz}(v) + (1 - \pi_{-i}) U^{HL*}_{iz}(v)$$
$$\widehat{U}^{LH*}_{iz}(v) = \widehat{U}^{LL*}_{iz}(v) = \pi_{-i} U^{LH*}_{iz}(v) + (1 - \pi_{-i}) U^{LL*}_{iz}(v)$$

and these values satisfy (2.19) by Lemma 6. The perturbed allocation satisfies (2.18) − (2.24) and yields the same welfare. Analogous arguments can be made if $\pi_1 x^*_{1z}(v) \leq \pi_2 x^*_{2z}(v)$.

(ii) Rewrite the problem as in (2.17) − (2.24). Perturb the allocation in the following fashion. Let $\widehat{x}^*_{iz}(v) = \frac{\int_0^1 (1 - W^*_z(v)) x^*_{iz}(v) dG_z}{\int_0^1 (1 - W^*_z(v)) dG_z}$ for $i = 1, 2$. Let $\widehat{U}^{j*}_{iz}(v) = \frac{\int_0^1 (1 - W^*_z(v)) U^{j*}_{iz}(v) dG_z}{\int_0^1 (1 - W^*_z(v)) dG_z}$ for $i = 1, 2$ and $j \in S$. The perturbed allocation satisfies (2.18) − (2.24) and yields the same welfare. ■

Lemma 9 *All solutions to (2.4) subject to $U_2(\alpha) = v$ have the following properties:*

$$\{\mathbf{E}\{U_1(\alpha|_{q_1, z_1}) | q_0, z_0, m\}, \mathbf{E}\{U_2(\alpha|_{q_1, z_1}) | q_0, z_0, m\}\} \in coV$$

$$\text{for } m = \left\{ \begin{array}{c} \{W_0(q_0, z_0) = 1\}, \\ \{W_0(q_0, z_0) = 0, s_{10} x_{10} = x_{10}(q_0, z_0), s_{20} x_{20} = x_{20}(q_0, z_0)\}, \\ \{W_0(q_0, z_0) = 0, s_{10} x_{10} = x_{10}(q_0, z_0), s_{20} x_{20} = 0\} \\ \{W_0(q_0, z_0) = 0, s_{10} x_{10} = 0, s_{20} x_{20} = x_{20}(q_0, z_0)\}, \\ \{W_0(q_0, z_0) = 0, s_{10} x_{10} = 0, s_{20} x_{20} = 0\}, \end{array} \right\}$$

Proof. *Rewrite the problem as in (2.17) − (2.24). It is sufficient to show that $\{U^{m*}_{1z}(v), U^{m*}_{2z}(v)\} \in coV\ \forall z \in [0, 1]$ and $\forall m \in \{W, S\}$. If $\{U^{W*}_{1z}(v), U^{W*}_{2z}(v)\} \notin coV$, then a perturbation to $\left\{\widehat{U}^{W*}_{1z}(v) + \epsilon, U^{W*}_{2z}(v)\right\}$ for $\epsilon > 0$ which is sufficiently low will satisfy (2.18) − (2.24) and strictly* *increase the welfare of group* 1*. Now consider the case for which* $m \in S$ *and* $\{U_{1z}^{m*}(v), U_{2z}^{m*}(v)\} \notin coV$*. Perturb the allocation under peace as in Lemma 8. Consider the case for which for which* $x_{1z}^*(v) = 0$ *and* $x_{2z}^*(v) > 0$ *so that* $U_{iz}^{HH*}(v) = U_{iz}^{LH*}(v) = U_{iz}^{H*}(v)$ *and* $U_{iz}^{HL*}(v) = U_{iz}^{LL*}(v) = U_{iz}^{L*}(v)$ *for* $i = 1, 2$*. If* $U_{1z}^{H*}(v) < K(U_{2z}^{H*}(v))$ *or if* $U_{1z}^{L*}(v) < K(U_{2z}^{L*}(v))$ *then the welfare of group* 1 *can be strictly improved by perturbing the equilibrium to* $\widehat{U}_{1z}^{H*}(v) = U_{1z}^{H*}(v) + \epsilon$ *or* $\widehat{U}_{1z}^{L*}(v) = U_{1z}^{L*}(v) + \epsilon$*, respectively, for* $\epsilon > 0$ *which is sufficiently low. Analogous arguments apply for the case in which* $x_{1z}^*(v) > 0$ *and* $x_{2z}^*(v) = 0$*.* ■

Lemma 10 *There is a solution to* (2.4) *subject to* $U_2(\alpha) = v$ *with the property that if* $W_0(q_0, z_0) = 0$*,* $x_{i0}(q_0, z_0) \geq 0$*, and* $x_{-i0}(q_0, z_0) = 0$*, then*

$$U_i(\alpha|_{q_0,z_0}) = \beta \mathbf{E}\{U_i(\alpha|_{q_1,z_1})|q_0, z_0, s_{i0}x_{i0} = 0\}.$$

Proof. *Perturb the allocation as in Lemma 8. Consider the case for which* $x_{1z}^*(v) = 0$ *and* $x_{2z}^*(v) > 0$ *so that* $U_{iz}^{HH*}(v) = U_{iz}^{LH*}(v) = U_{iz}^{H*}(v)$ *and* $U_{iz}^{HL*}(v) = U_{iz}^{LL*}(v) = U_{iz}^{L*}(v)$ *for* $i = 1, 2$*. By the arguments of Lemma 9, it must be that* $U_{1z}^{H*}(v) = K(U_{2z}^{H*}(v))$ *and* $U_{1z}^{L*}(v) = K(U_{2z}^{L*}(v))$*. Imagine if* (2.21) *does not bind. Consider the following perturbation. Let* $\widehat{U}_{2z}^{HL*}(v) = \widehat{U}_{2z}^{LL*}(v) = \widehat{U}_{2z}^{L*}(v) = U_{2z}^{L*}(v) + \epsilon$ *and* $\widehat{U}_{2z}^{HH*}(v) = \widehat{U}_{2z}^{LH*}(v) = \widehat{U}_{2z}^{H*}(v) = U_{2z}^{H*}(v) - \epsilon\left(\frac{1-\pi_2}{\pi_2}\right)$ *for* $\epsilon > 0$ *small enough so as to continue to satisfy* (2.19) − (2.24)*. Let* $\widehat{U}_{1z}^{HL*}(v) = \widehat{U}_{1z}^{LL*}(v) = \widehat{U}_{1z}^{L*}(v) = K\left(\widehat{U}_{2z}^{L*}(v)\right)$ *and* $\widehat{U}_{1z}^{HH*}(v) = \widehat{U}_{1z}^{LH*}(v) = \widehat{U}_{1z}^{H*}(v) = K\left(\widehat{U}_{2z}^{H*}(v)\right)$*. For this perturbation to be weakly suboptimal, it is necessary that*

$$\frac{K(U_{2z}^{H*}(v)) - K\left(\widehat{U}_{2z}^{H*}(v)\right)}{\epsilon\left(\frac{1-\pi_2}{\pi_2}\right)} \geq \frac{K(U_{2z}^{L*}(v)) - K\left(\widehat{U}_{1z}^{L*}(v)\right)}{\epsilon}. \tag{2.25}$$

It is the case that $U_{2z}^{H*}(v) > \widehat{U}_{2z}^{H*}(v) > \widehat{U}_{2z}^{L*}(v) > U_{2z}^{L*}(v)$*, so that* (2.25) *must be an equality by the concavity of* V*. This means that the perturbation yields the same welfare. Analogous arguments apply for the case in which* $x_{1z}^*(v) > 0$ *and* $x_{2z}^*(v) = 0$*.* ■

Proof of Lemma 7

It is not possible that $K(\cdot) < \underline{U}_1$ since this violates (2.2) for $i = 1$.

Imagine if $K(\underline{U}_2) > \underline{U}_1$. To show this is not possible consider the allocation associated with $\{K(\underline{U}_2), \underline{U}_2\}$. This allocation requires $U_2(\alpha|_{q_0,z_0}) = \underline{U}_2$ for every z_0 in order to satisfy $U_2(\alpha) = \underline{U}_2$ and (2.2) for $i = 2$. This means that if $W_0(q_0, z_0) = 1$ for some z_0, then this requires $\mathbf{E}\{U_2(\alpha|_{q_1,z_1})|q_0, z_0\} = \underline{U}_2$ for such z_0. If alternatively $W_0(q_0, z_0) = 0$ for some z_0, then consider the an allocation satisfying Lemma 8. It is not possible that $x_1(q_0, z_0) > 0$, since this violates the fact that $v = \underline{U}_2$ since $\mathbf{E}\{U_2(\alpha|_{q_1,z_1})|q_0, z_0\} \geq \underline{U}_2$. Therefore, $x_1(q_0, z_0) = 0$ and $x_2(q_0, z_0) \geq 0$ and by (2.2) and (2.3) for $i = 2$,

$$\underline{U}_2 = U_2(\alpha|_{q_0,z_0}) \geq \beta\mathbf{E}\{U_2(\alpha|_{q_1,z_1})|q_0, z_0, s_{20}x_{20} = 0\} \geq \beta\underline{U}_2, \tag{2.26}$$

since group 2 can always choose to make zero concessions and receive at least $\underline{U}_2$ in the future. (2.26) violates Assumptions 1 and 2 which imply that $w_2 < 0$. Therefore, $W_0(q_0, z_0) = 1$ and $\mathbf{E}\{U_2(\alpha|_{q_1,z_1})|q_0, z_0\} = \underline{U}_2$ for all z_0 and forward induction on this argument implies that $W_t(q_t, z_t) = 1$ $\forall t$ and $\forall(q_t, z_t)$ so that $K(\underline{U}_2) = \underline{U}_1$.

Imagine if $K(\overline{U}_2) > \underline{U}_1$. To show this is not possible consider the allocation associated with $\{K(\overline{U}_2), \underline{U}_2\}$. This allocation requires $U_2(\alpha|_{q_0,z_0}) = \overline{U}_2$ for every z_0, since it is possible to increase $U_2(\alpha)$ otherwise. Moreover, this allocation requires that $W_0(q_0, z_0) = 0$, since if $W_0(q_0, z_0) = 1$, this would imply that $\overline{U}_2 = w_2 + \beta\mathbf{E}\{U_2(\alpha|_{q_1,z_1})|q_0, z_0\} \leq w_2 + \beta\overline{U}_2$, which contradicts Assumption 4. Now consider a solution to the problem which satisfies the properties of Lemma 8 with $K(\overline{U}_2) > \underline{U}_1$. This requires that $x_{20}(q_0, z_0) = 0$, otherwise it is possible to reduce $x_{20}(q_0, z_0)$ while maintaining (2.2) and (2.3) and strictly increasing $U_2(\alpha)$. Consider the case with $x_{10}(q_0, z_0) = 0$. This requires that $\mathbf{E}\{U_2(\alpha|_{q_1,z_1})|q_0, z_0\} = \overline{U}_2$, otherwise it is possible to increase $\mathbf{E}\{U_2(\alpha|_{q_1,z_1})|q_0, z_0\}$ while maintaining (2.2) and (2.3) and strictly increasing $U_2(\alpha)$. Therefore, if $x_{10}(q_0, z_0) = 0$, then by Lemma 10, $\overline{U}_2 = \beta\overline{U}_2$, but this violates the fact that $\overline{U}_2 < 0$, since $\overline{U}_2 = 0$ is associated with permanent peace by the arguments of Proposition 10. Now consider the case with $x_{10}(q_0, z_0) > 0$. We can show that this requires that $\mathbf{E}\{U_1(\alpha|_{q_1,z_1})|q_0, z_0, s_{10}x_{10} = 0\} = K(\overline{U}_2)$. Assuming this is the case, then by Lemma 10, $K(\overline{U}_2) = \beta K(\overline{U}_2)$, but this violates the fact that $K(\overline{U}_2) \geq \underline{U}_1 > 0$ by (2.2) and Assumption 2.

To complete the argument, we show that if $K(\overline{U}_2) > \underline{U}_1$ and $x_{10}(q_0, z_0) > 0$, then

$\mathbf{E}\left\{U_1\left(\alpha|_{q_1,z_1}\right)|q_0, z_0, s_{10}x_{10}=0\right\} = K\left(\overline{U}_2\right)$. Assume and later prove that

$$\begin{aligned} \mathbf{E}\left\{U_2\left(\alpha|_{q_1,z_1}\right)|q_0, z_0, s_{10}x_{10}=x_{10}\left(q_0,z_0\right)\right\} &\leq \mathbf{E}\left\{U_2\left(\alpha|_{q_1,z_1}\right)|q_0, z_0, s_{10}x_{10}=0\right\} \quad (2.27) \\ \mathbf{E}\left\{U_1\left(\alpha|_{q_1,z_1}\right)|q_0, z_0, s_{10}x_{10}=0\right\} &\geq K\left(\overline{U}_2\right). \quad (2.28) \end{aligned}$$

Under these conditions, given Lemma 9, it is the case that

$$\mathbf{E}\left\{U_1\left(\alpha|_{q_1,z_1}\right)|q_0, z_0, s_{10}x_{10}\right\} = K\left(\mathbf{E}\left\{U_2\left(\alpha|_{q_1,z_1}\right)|q_0, z_0, s_{10}x_{10}\right\}\right).$$

(2.27) and (2.28) along with (2.3) imply that $K(\cdot)$ is downward sloping in the range between $\mathbf{E}\left\{U_2\left(\alpha|_{q_1,z_1}\right)|q_0, z_0, s_{10}x_{10}=0\right\}$ and $\overline{U}_2$. Therefore, it follows that it is possible to increase $\mathbf{E}\left\{U_2\left(\alpha|_{q_1,z_1}\right)|q_0, z_0, s_{10}x_{10}=0\right\}$ while reducing $\mathbf{E}\left\{U_1\left(\alpha|_{q_1,z_1}\right)|q_0, z_0, s_{10}x_{10}=0\right\}$ by the relevant amount implied by $K(\cdot)$ so as to continue to satisfy (2.2) and (2.3) and to strictly increase the welfare of group 2. Now imagine if (2.27) and (2.28) do not hold. If (2.27) does not hold, then given (2.3), $K(\cdot)$ is upward sloping in the range between $\mathbf{E}\left\{U_2\left(\alpha|_{q_1,z_1}\right)|q_0, z_0, s_{10}x_{10}=0\right\}$ and $\mathbf{E}\left\{U_2\left(\alpha|_{q_1,z_1}\right)|q_0, z_0, s_{10}x_{10}=x_{10}\left(q_0,z_0\right)\right\}$. Given the convexity of V, it is possible to increase $\mathbf{E}\left\{U_2\left(\alpha|_{q_1,z_1}\right)|q_0, z_0, s_{10}x_{10}=0\right\}$ by $\epsilon > 0$ for ϵ sufficiently low while preserving the value of $\mathbf{E}\left\{U_1\left(\alpha|_{q_1,z_1}\right)|q_0, z_0, s_{10}x_{10}=0\right\}$ so as to increase the welfare of group 2. Therefore, (2.27) must hold. (2.28) is implied by Lemma 10 and Assumption 2. **Q.E.D.**

Proof of Proposition 11

This follows directly from the representation of the problem in (2.17) – (2.24), Lemma 7, and Lemma 9. **Q.E.D.**

Proof of Proposition 12

We establish some preliminary results useful for the proofs of this section. Let $K^W(v)$ represent the solution to (2.5) – (2.13) subject to $W_z = 1\ \forall z$ and let $K^P(v)$ be the solution to (2.5) – (2.13) subject to $W_z = 0\ \forall z$. Constraints (2.6), (2.8), and (2.10) imply that $K^W(v)$ is defined over $\left[\underline{U}_2, w_2 + \beta\overline{U}_2\right]$ and that $K^P(v)$ is defined over $\left[\beta\underline{U}_2, \overline{U}_2\right]$. It follows that (2.5) – (2.13) can be rewritten as:

$$K(v) = \max_{W, v_-^W, v_-^P} WK^W\left(v_-^W\right) + (1-W) K^P\left(v_-^P\right) \tag{2.29}$$

s.t.

$$v = Wv_-^W + (1-W) v_-^P, \text{ and } W \in [0,1]. \tag{2.30}$$

We can let $W^*(v), v_-^{W*}(v)$, and $v_-^{P*}(v)$ represent a solution to (2.29) – (2.30). Note that since $K^W(v)$ is defined over $\left[\underline{U}_2, w_2 + \beta\overline{U}_2\right]$, it must be that $K^P\left(\overline{U}_2\right) = K\left(\overline{U}_2\right)$ if $\underline{U}_2 < \overline{U}_2$, which is the case by Assumption 4. Define $v^{\max} = \arg\max_v K(v)$. If this value is not unique, we let it be the minimum v of all maximizers. The following lemma establishes many useful technical results, some of which simply rewrite Lemma 8 recursively.

Lemma 11 *If* $\beta\underline{U}_2 < \overline{U}_2$,

(i) There is a solution to (2.5) – (2.13) *s.t. concessions are unilateral,* $x_{iz}^*(v) = x_i^*(v)$, *and* $v_z^{j*}(v) = v^{j*}(v)$ $\forall z$ *and* $\forall j \in \{W, S\}$,

(ii) There is a solution to (2.5) – (2.13) *s.t.* (2.9) *and* (2.10) *bind* $\forall z$,

(iii) $K(v^{\max}) > \underline{U}_1$,

(iv) $\exists\widetilde{U} \in \left(\underline{U}_2, \overline{U}_2\right)$ *s.t.* $K(v) > K^W(v)$ $\forall v > w_2 + \beta\widetilde{U}$,

(v) $K(v) = \underline{U}_1 + \frac{K(\widetilde{U}) - \underline{U}_1}{\widetilde{U} - \underline{U}_2}\left(v - \underline{U}_1\right)$ *if* $v \in \left[\underline{U}_2, \widetilde{U}\right]$,

(vi) $K(v) < \underline{U}_1 + \frac{K(\widetilde{U}) - \underline{U}_1}{\widetilde{U} - \underline{U}_2}\left(v - \underline{U}_1\right)$ *if* $v \in \left(\widetilde{U}, \overline{U}_2\right]$, *and*

(vii) The unique solution to (2.5) – (2.13) *sets* $W_z^*(v) = 0$ $\forall v \geq \widetilde{U}$.

Proof. *(i) This is a direct application of Lemma 8.*

(ii) This is a directly application of Lemma 10.

(iii) (2.7) *implies* $K(v^{\max}) \geq \underline{U}_1$. *Imagine if* $K(v^{\max}) = \underline{U}_1$. *This means that* $K(v) = \underline{U}_1$ $\forall v \in \left[\underline{U}_2, \overline{U}_2\right]$. *Performing the perturbations of part i and ii, this means that* $x_2^*(v) = w_1/\pi_2$ $\forall v \in \left[\underline{U}_2, \overline{U}_2\right]$ *and that* $v^{mH*}\left(\overline{U}_2\right) = \frac{\overline{U}_2 + w_1/\pi_2}{\beta} = \overline{U}_2$, *since if* $v^{mH*}\left(\overline{U}_2\right) < \overline{U}_2$, *then it would be possible to increase* $v^{mH*}\left(\overline{U}_2\right)$ *while continuing to satisfy* (2.7) – (2.13) *so as to increase* v. *Moreover, it must be that* $v^{mL*}\left(\overline{U}_2\right) = \underline{U}_2$. *This because if* $v^{mL*}\left(\overline{U}_2\right) > \underline{U}_2$, *then it is possible to increase group 1's welfare by increasing* $x_2^*(v)$ *while decreasing* $v^{mL*}\left(\overline{U}_2\right)$. *Since* (2.10) *binds, then* $\beta\underline{U}_2 = \overline{U}_2$, *yielding a contradiction.*

(iv) By (2.6) *and the weak concavity of* $K(\cdot)$, $K^W(v) = w_1 + \beta K\left(\frac{v-w_2}{\beta}\right)$, *which means that for* $v'' > v'$,

$$\begin{aligned} \frac{K^W(v'') - K^W(v')}{v'' - v'} &= \frac{K\left(\frac{v''-w_2}{\beta}\right) - K\left(\frac{v'-w_2}{\beta}\right)}{\frac{v''-v'}{\beta}} \\ &\leq \frac{K(v'') - K(v')}{v'' - v'}. \end{aligned} \tag{2.31}$$

The weak inequality follows from the concavity of $K(v)$ *established in Lemma 6 and the fact that* $v', v'' \geq \underline{U}_2$ *by* (2.8). *Equation* (2.31) *means that if* $K(v') > K^W(v')$, *then* $K(v'') > K^W(v'')$ $\forall\ v'' > v'$. *Define* $\widetilde{U}$ *s.t.* $K(v) > K^W(v)\ \forall v > w_2 + \beta\widetilde{U}$. *Imagine if* $\widetilde{U} = \underline{U}_2$. *Then the formulation of* (2.29) – (2.30), *implies that the frontier is linear between* $\underline{U}_2$ *and some* $\widetilde{\widetilde{U}} \in \left[\beta\underline{U}_2, \overline{U}_2\right]$, *yielding a contradiction, since* $\underline{U}_2 + \epsilon$ *for* $\epsilon > 0$ *sufficiently small can be efficiently generated by* $W = 1$. *Imagine if* $\widetilde{U} = \overline{U}_2$. *Given the linearity implied by* (2.31) *this means that* $K(v^{\max}) = K\left(\overline{U}_2\right)$, *contradicting part iii.*

(v) This is implied by equation (2.31).

(vi) Imagine if $K(v) = \underline{U}_1 + \frac{K(\widetilde{U}) - \underline{U}_1}{\widetilde{U} - \underline{U}_2}(v - \underline{U}_1)$ *for some* $v \in \left(\widetilde{U}, \overline{U}_2\right]$. *Since* $\widetilde{U} < \overline{U}_2$ *by part iv, this would imply that* $\exists\epsilon > 0$ *which is sufficiently small, such that* $K^W\left(w_2 + \beta\widetilde{U} + \epsilon\right) = w_2 + \beta K\left(\widetilde{U} + \frac{\epsilon}{\beta}\right) = K\left(w_2 + \beta\widetilde{U} + \epsilon\right)$, *contradicting the definition of* $\widetilde{U}$ *in part iv.*

(vii) Write the problem as in (2.29) – (2.30). *Imagine if* $W^*(v) > 0$. *This means that*

$$K(v) = W^*(v)\, K^W\left(v_-^{W*}(v)\right) + \left(1 - W^*(v)\right) K^P\left(v_-^{P*}(v)\right),$$

for $v_-^{W*}(v) \leq w_2 + \beta\widetilde{U} < v < v_-^{P*}(v)$, *which by the concavity of* $K(\cdot)$ *and by part v means that* $K\left(v_-^{P*}(v)\right) = \underline{U}_1 + \frac{K(\widetilde{U}) - \underline{U}_1}{\widetilde{U} - \underline{U}_2}\left(v_-^{P*}(v) - \underline{U}_1\right)$, *contradicting part vi.* ■

If $v = \underline{U}_2$, Lemma 7 establishes that $v_z^{W*}(v) = \underline{U}_2$. Now consider $v > \underline{U}_2$. Imagine if $\beta\underline{U}_2 < \overline{U}_2$, then we only require examining the case in which $W_z^*(v) = 1$ for some z, which by Lemma 11 part vii means the case in which $v \in \left(\underline{U}_2, \widetilde{U}\right)$. Perform the perturbations of Lemma 11 parts i and ii. By Lemma 11 part v, it is the case that $K(v) = \left(\frac{\widetilde{U} - v}{\widetilde{U} - \underline{U}_1}\right)\underline{U}_1 + \left(\frac{v - \underline{U}_1}{\widetilde{U} - \underline{U}_1}\right) K^P\left(\widetilde{U}\right)$, so that a perturbation to $\widehat{W}^*(v) = \frac{\widetilde{U} - v}{\widetilde{U} - \underline{U}_1}$, $\widehat{v}_-^{W*}(v) = \underline{U}_2$, and $\widehat{v}_-^{P*}(v) = \widetilde{U}$ is incentive compatible and provides the same welfare. If $\beta\underline{U}_2 = \overline{U}_2$, then this means $\widetilde{U} = \overline{U}_2$ and the linearity implied by Lemma 11 part v applies. Therefore, the same perturbation as in the previous case can be used. **Q.E.D.**

Proof of Proposition 13

The following Lemma is useful in proving this result and we use the definition of $\widetilde{U}$ and $v^{\max}$ provided in the proof of Lemma 11.

Lemma 12 *If* $\beta \underline{U}_2 < \overline{U}_2$, *then* $\int_0^1 v_z^{mH*}\left(\widetilde{U}\right) dG_z > \widetilde{U}$.

Proof. *Apply the perturbations of Lemma 11 parts i and ii. By Lemma 11 part vii,* $W_z^*\left(\widetilde{U}\right) = 0$ $\forall z$. *Moreover, it is the case that* $x_1^*\left(\widetilde{U}\right) = 0$. *This is because since* (2.9) *binds, then it it is not possible for* $x_1(v) > 0$ *for* $v < \widehat{v}$ *for* $\widehat{v} > v^{\max}$ *chosen such that* $K(v^{\max}) = K(\widehat{v})/\beta$. *Therefore,* $x_1^*(v) = 0$ $\forall v \leq \widehat{v}$. *If* $v^{mH*}\left(\widetilde{U}\right) < \widetilde{U}$, *this would imply that* $x_2^*\left(\widetilde{U}\right) < \overline{x}_2$ *by Assumption 3. Consider a perturbation to* $\widehat{x}_2^*\left(\widetilde{U}\right) = x_2^*\left(\widetilde{U}\right) + \epsilon$ *and* $\widehat{v}^{mH*}\left(\widetilde{U}\right) = v^{mH*}\left(\widetilde{U}\right) + \epsilon/\beta$ *for* $\epsilon > 0$ *which satisfies* (2.6) $-$ (2.13). *For such a perturbation to be weakly suboptimal, it is necessary that*

$$\frac{K\left(v^{mH*}\left(\widetilde{U}\right) + \epsilon/\beta\right) - K\left(v^{mH*}\left(\widetilde{U}\right)\right)}{\epsilon/\beta} \leq -1, \tag{2.32}$$

but this is not possible since $v^{mH*}\left(\widetilde{U}\right) < \widetilde{U} \leq v^{\max}$ *and* $K(v^{\max}) > K\left(v^{mH*}\left(\widetilde{U}\right)\right)$ *by the definition of* $v^{\max}$ *and by Lemma 11 part iii. If* $\widetilde{U} < v^{\max}$, *this argument implies that* $v^{mH*}\left(\widetilde{U}\right) > \widetilde{U}$. *If* $\widetilde{U} = v^{\max}$, *we must consider the case for which* $v^{mH*}\left(\widetilde{U}\right) = v^{\max}$. *For the same perturbation as before to be weakly suboptimal, equation* (2.32) *implies that*

$$\frac{K(v'') - K(v')}{v'' - v'} < -1 \text{ for } v'' > v' > v^{\max}. \tag{2.33}$$

Moreover, our analysis implies that for v' *and* v'' *sufficiently close to* $v^{\max}$ *so that* $v''/\beta, v'/\beta < v^{\max}$, *that* $v^{mH*}(v') = v^{mH*}(v'') = v^{\max}$. *This implies that* $x_2^*(v'') - x_2^*(v') = v' - v''$, *so that*

$$\frac{K(v'') - K(v')}{v'' - v'} = -\pi_2 + (1 - \pi_2)\frac{K(v''/\beta) - K(v'/\beta)}{(v'' - v')/\beta} \geq -\pi_2,$$

which contradicts (2.33). ■

Given the proof of Proposition 12 and Lemma 11 part vii, Proposition 13 is equivalent to the claim that $\widetilde{U} = \beta \underline{U}_2$. Clearly, it is not possible that $\widetilde{U} < \beta \underline{U}_2$, since $K^W(v)$ is defined over $\left[\beta \underline{U}_2, \overline{U}_2\right]$. Now consider the case for which $\beta \underline{U}_2 < \overline{U}_2$, and consider the possibility that $\widetilde{U} > \beta \underline{U}_2$. Apply the perturbations Lemma 11 parts i and ii. By Lemma 12 this means that $x_2^*\left(\widetilde{U}\right) > 0$ and that $\frac{\widetilde{U}+x_2^*(\widetilde{U})}{\beta} > \widetilde{U}$. Define $K^P\left(v|x_2^*\left(\widetilde{U}\right)\right)$ as the solution to (2.5) – (2.13) subject to $W_z = 0$ and $x_{2z} = x_2^*\left(\widetilde{U}\right)$ $\forall z$, and this frontier is weakly concave by the weak concavity of $K(\cdot)$. Given the optimality of $x_2^*(v)$ and given Lemma 11 parts iv and v, it is necessary that for $v' \in \left(\beta \underline{U}_2, \widetilde{U}\right)$,

$$\begin{aligned}\frac{K^P\left(\widetilde{U}|x_2^*\left(\widetilde{U}\right)\right) - K^P\left(v'|x_2^*\left(\widetilde{U}\right)\right)}{\widetilde{U} - v'} &\geq \frac{K\left(\widetilde{U}\right) - K(v')}{\widetilde{U} - v'} \\ &= \frac{K\left(\widetilde{U}\right) - \underline{U}_1}{\widetilde{U} - \underline{U}_2}.\end{aligned} \tag{2.34}$$

Choose v' arbitrarily close to $\widetilde{U}$ so that $\frac{v'+x_2^*(\widetilde{U})}{\beta} > \widetilde{U}$. Using Lemma 11 parts i and ii so as to substitute in for $v^{mH}(\cdot)$, it follows that

$$\begin{aligned}&\frac{K^P\left(\widetilde{U}|x_2^*\left(\widetilde{U}\right)\right) - K^P\left(v'|x_2^*\left(\widetilde{U}\right)\right)}{\widetilde{U} - v'} = \\ &= \pi_2\left(\frac{\left(K\left(\left(\widetilde{U} + x_2^*\left(\widetilde{U}\right)\right)/\beta\right) - K\left(\left(v' + x_2^*\left(\widetilde{U}\right)\right)/\beta\right)\right)}{\left(\widetilde{U} - v'\right)/\beta}\right) + (1-\pi_2)\left(\frac{K\left(\widetilde{U}\right) - \underline{U}_1}{\widetilde{U} - \underline{U}_2}\right) \\ &< \frac{K\left(\widetilde{U}\right) - \underline{U}_1}{\widetilde{U} - \underline{U}_2}.\end{aligned}$$

The last inequality follows from Lemma 11 part vi. However, this contradicts (2.34). Now imagine if $\beta \underline{U}_2 = \overline{U}_2$. Arguments made in the proof of Proposition 12 establish that $W^*\left(\overline{U}_2\right) = 0$ in the formulation of the problem in (2.29) – (2.30). **Q.E.D.**

Proof of Proposition 14

(i) Let $v \in (\underline{U}_2, \beta \underline{U}_2)$. If $v_z^{W*}(v) = \underline{U}_2$ $\forall z$, then this means that $v_{\underline{}}^{W*}(v) = \underline{U}_2$ in the problem formulated in (2.29) – (2.30), and this means that $W^*(v) < 1$. Given that $v_{\underline{}}^{P*}(v) = \beta \underline{U}_2$ by

Proposition 13, then a perturbation to $\widehat{v}_{-}^{W*}(v) = v_{-}^{W*}(v) + \epsilon$ and $\widehat{W}(v) = W(v) \frac{v_{-}^{W*}(v) - \beta \underline{U}_2}{v_{-}^{W*}(v) + \epsilon - \beta \underline{U}_2}$ for $\epsilon > 0$ sufficiently small so as to continue to satisfy (2.30) yields the same welfare to group 1 by Lemma 11 part v conditional on $K^W(\underline{U}_2 + \epsilon) = K(\underline{U}_2 + \epsilon)$. The perturbation can yield $v_z^{W*}(v) > \underline{U}_2$. Using Lemma 11 part v,

$$\begin{aligned} K^W(\underline{U}_2 + \epsilon) &= w_1 + \beta K(\underline{U}_2 + \epsilon/\beta) \\ &= \underline{U}_1 + \frac{K(\beta \underline{U}_2) - \underline{U}_1}{\beta \underline{U}_2 - \underline{U}_2} \epsilon \\ &= K(\underline{U}_2 + \epsilon). \end{aligned}$$

(ii) Let $v \in (\underline{U}_2, \beta \underline{U}_2)$. Given the formulation of the problem in (2.29)–(2.30), it is necessary that $v_{-}^{P*}(v) = \beta \underline{U}_2$. This is because $v_{-}^{P*}(v) \geq \beta \underline{U}_2$ given the range of $K^P(\cdot)$, and because choosing $v_{-}^{P*}(v) > \beta \underline{U}_2$ is suboptimal by Lemma 11 parts v and vi. Perform the perturbations of Lemma 11 parts i and ii, taking into account that $x_2^*(\beta \underline{U}_2) > 0$ since $v^{mH*}(\beta \underline{U}_2) > \beta \underline{U}_2$ by Lemma 12. Using (2.6),(2.8), and (2.10), this means that

$$\beta \underline{U}_2 = \int_0^1 \left(-\pi_2 x_{2z}^*(v) + \beta\left(\pi_2 v_z^{mH*}(v) + (1-\pi_2) v_z^{mL*}(v)\right)\right) dG_z \geq \beta \int_0^1 v_z^{mL*}(v)\, dG_z \geq \beta \underline{U}_2,$$

which means that $\int_0^1 v_z^{mL*}(v)\, dG_z = \underline{U}_2$, which by the fact that $v_z^{mL*}(v)\, dG_z \geq \underline{U}_2$ means that $v_z^{mL*}(v)\, dG_z = \underline{U}_2\ \forall z$.

(iii) Let $v \in \left[\beta \underline{U}_2, \overline{U}_2\right]$. By Proposition 13, $W_z^*(v) = 0\ \forall z$. Imagine if $v_z^{j*}(v) \geq v$ for a given z and for some j. If this were the case that $v_z^{j*}(v) \geq 0$ for all j, then (2.6) would imply that

$$v(1-\beta) \geq \int_0^1 \left(\pi_1 x_{1z}^*(v) - \pi_2 x_{2z}^*(v)\right) dG_z,$$

which means that $\int_0^1 \left(\pi_1 x_{1z}^*(v) - \pi_2 x_{2z}^*(v)\right) dG_z < 0$, since $v < 0$ by the arguments made in the proof of Proposition 10. Therefore, $\int_0^1 x_{2z}^*(v)\, dG_z > 0$, yet if this is the case, then (2.10) implies that

$$\int_0^1 \left(\pi_1 v_z^{HL} + (1-\pi_1) v_z^{LL}\right) dG_z \leq v/\beta < v,$$

which yields a contradiction.

Q.E.D.

Proof of Theorem 4

(i) Imagine if fluctuations do not take place. Given Proposition 14, this means that

$$\Pr\left\{v_t\left(q_t, z_t\right) \in \left(\underline{U}_2, \beta\underline{U}_2\right)\right\} = 0.$$

Use the perturbations of Lemma 11 parts i and ii. Note since (2.9) binds, then it is not possible that if $x_1^*(v) > 0$ for $v < \widehat{v}$ for $\widehat{v} > v^{\max}$ chosen such that $K(v^{\max}) = K(\widehat{v})/\beta$. Therefore, $x_1^*(v) = 0\ \forall v \leq \widehat{v}$. Since (2.10) binds, it follows that $v^{mL*}(v) = v/\beta < v$. If there are no fluctuations, then that

$$\Pr\left\{v_k\left(q_k, z_k\right) \in \left(\beta\underline{U}_2, \min\left\{\widehat{v}, \beta^2\underline{U}_2\right\}\right)\right\} = 0,$$

and if $\beta^2\underline{U}_2 > \widehat{v}$, this implies that

$$\Pr\left\{v_k\left(q_k, z_k\right) \in \left(\beta^2\underline{U}_2, \min\left\{\widehat{v}, \beta^3\underline{U}_2\right\}\right)\right\} = 0, \text{and so on.}$$

Therefore, for flucuations to not occur, paths which go between $\underline{U}_2$ and $\widehat{v}$ (which must occur with positive probability) must pass through a finite set of points included in $\left\{\underline{U}_2, \beta\underline{U}_2, \beta^2\underline{U}_2, ...\right\}$. If $\beta\underline{U}_2 < v^{\max}$, one can choose $v_0 \in (v^{\max}, \widehat{v})$ so that $U_2(\alpha) = v_0$ and so that $v_0 \notin \{\underline{U}_2, \beta\underline{U}_2, ...\}$, and this will yield fluctuations.

(ii) Imagine if there exists a solution which does not converge to permanent war. This means that $\Pr\left\{v_k\left(q_k, z_k\right) = \underline{U}_2\right\} = 0$. By Propositions 10 and 13, it must be that

$$\Pr\left\{v_k\left(q_k, z_k\right) \in \left(\underline{U}_2, \beta\underline{U}_2\right)\right\} > 0$$

for some $k \geq t$. Given Proposition 14, then

$$\begin{aligned} \Pr\left\{v_{k+1}\left(q_{k+1}, z_{k+1}\right) \leq \beta\underline{U}_2 | W_k\left(q_k, z_k\right) = 1, v_k\left(q_k, z_k\right) \in \left(\underline{U}_2, \beta\underline{U}_2\right)\right\} &= 1, \\ \Pr\left\{v_{k+1}\left(q_{k+1}, z_{k+1}\right) \neq \underline{U}_2 | W_k\left(q_k, z_k\right) = 0, v_k\left(q_k, z_k\right) \in \left(\underline{U}_2, \beta\underline{U}_2\right)\right\} &= 0. \end{aligned}$$

This means that in order that $\Pr\{v_{k+1}(q_{k+1}, z_{k+1}) = \underline{U}_2\} = 0$, then it must be that

$$\Pr\{W_k(q_k, z_k) = 0 | v_k(q_k, z_k) \in (\underline{U}_2, \beta \underline{U}_2)\} = 0,$$

but this implies convergence to permanent war, yielding a contradiction. **Q.E.D.**

2.8.4 Proofs of Section 2.6

Proof of Proposition 15

Necessity follows by similar arguments as the proof of Proposition 8, but we take into account that it is possible for $\underline{\underline{U}}_2 < \underline{U}_2$. For sufficiency, let $\underline{\underline{U}}_2$ represent the lowest possible value that can ever be promised to group 2. Construct the following off-equilibrium strategy. Any deviation from the prescribed $W_t(q_t, z_t)$ and/or $D_t(q_t, z_t)$ results in a transition to the repeated static equilibrium with permanent warx. Any observable deviation from the prescribed transfers by group 1 at t results with the promise of $\underline{U}_1$ at $t+1$, and any observable deviation from the prescribed transfers by group 2 at t results with the promise of $\underline{\underline{U}}_2$ at $t+1$. We only need to consider single period deviations for a given (q_t, z_t) since $\beta \in (0, 1)$ and since payoffs are bounded. Consider periods for which $W_t(q_t, z_t) = 1$. If $D_t(q_t, z_t) = 0$, then the same arguments as those of Proposition 8 apply. If $D_t(q_t, z_t) = 1$, then Group 1 can only make itself better off by choosing $D'_t(q_t, z_t) = 0$, but this is weakly suboptimal by (2.14). Group 2 cannot affect the current flow utility. Now consider periods for which $W_t(q_t, z_t) = 0$. The same arguments as those of Proposition 8, imply that neither group deviates from prescribed transfers. **Q.E.D.**

Proof of Proposition 16

Construct the following equilibrium. If $s_{2t-1}x_{2t-1} = x$ or if $W_{t-1} = 1$, then $W_t = 0$, $x_{1t} = 0$, and $x_{2t} = x$, for $x \in (0, \overline{x}_2)$ which satisfies the inequalities of Assumption 1. If $s_{2t-1}x_{2t-1} = 0$ and $W_{t-1} = 0$, then $W_t = D_t = 1$. We let $W_0 = 0$ and $x_{20} = x$. The continuation values implied by the strategies are:

$$
\begin{aligned}
U_1|_{s=1} &= \pi_2 x + \beta\left(\pi_2 U_1|_{s=1} + (1-\pi_2) U_1|_{s=0}\right), \\
U_2|_{s=1} &= -\pi_2 x + \beta\left(\pi_2 U_2|_{s=1} + (1-\pi_2) U_2|_{s=0}\right), \\
U_1|_{s=0} &= w_1 - e + \beta U_1|_{s=1}, \text{ and} \\
U_2|_{s=0} &= w_2 - \chi + \beta U_2|_{s=1}.
\end{aligned}
$$

We first check (2.15). This requires that $\beta\left(U_2|_{s=1} - U_2|_{s=0}\right) \geq x$ which simplifies to $-\beta\left(w_2 - \chi\right)/\left(1+\beta\right) \geq x$, which always holds for a sufficiently high χ below ∞. We next check (2.14) for group 1. This requires $U_1|_{s=1} \geq \underline{U}_1$ and $U_1|_{s=0} \geq \underline{U}_1$, and both simplify to $\pi_2 x - e\left(1-\beta\pi\right)/\beta \geq w_1$, which always hold for e sufficiently low and above 0. By definition of $\underline{\underline{U}}_2$, $U_2|_{s=0} \geq \underline{\underline{U}}_2$ so that (2.14) holds for group 2. **Q.E.D.**

Proof of Proposition 17

Imagine if $W_k\left(q_k, z_k\right) = 1$ forever. The highest possible continuation value awarded to group 1 starting from $t+1$ is $\underline{U}_1$ since peace occurs with zero probability. However, this violates (2.14) for $i = 1$. **Q.E.D.**

Proof of Proposition 18

Let us write the problem recursively for D_z which is an indicator for the use of excessive force conditional on war taking place and define v_z^D as the continuation value for tomorrow conditional on excessive force today.

$$K(v) = \max_{\left\{D_z, W_z, x_{1z}, x_{2z}, v_z^D, v_z^W, \{v_z^j\}_{j\in S}\right\}_{z\in[0,1]}}$$

$$\int_0^1 \begin{pmatrix} D_z W_z \left[w_1 - e + \beta K\left(v_z^D\right)\right] + \\ \left(1 - D_z\right) W_z \left[w_1 + \beta K\left(v_z^W\right)\right] + \\ \left(1 - W_z\right)\left[\pi_2 x_{2z} - \pi_1 x_{1z} + \beta \sum_{j\in S} \pi^j K\left(v_z^j\right)\right] \end{pmatrix} dG_z$$

s.t.

$$v = \int_0^1 \begin{pmatrix} D_z W_z \left[w_2 - \chi + \beta v_z^D\right] + \\ \left(1 - D_z\right) W_z \left[w_2 + \beta v_z^W\right] + \\ \left(1 - W_z\right)\left[\pi_1 x_{1z} - \pi_2 x_{2z} + \beta \sum_{j\in S} \pi^j v_z^j\right] \end{pmatrix} dG_z,$$

$$K\left(v_z^m\right) \geq \underline{U}_1 \ \forall z \in [0,1] \text{ and } \forall m \in \{D, W, S\},$$

$$v_z^m \geq \underline{U}_2 \ \forall z \in [0,1] \text{ and } \forall m \in \{D, W, S\},$$

$$\beta\left(\pi_2\left(K\left(v_z^{HH}\right) - K\left(v_z^{LH}\right)\right) + \left(1 - \pi_2\right)\left(K\left(v_z^{HL}\right) - K\left(v_z^{LL}\right)\right)\right) \geq x_{1z} \ \forall z \in [0,1],$$

$$\beta\left(\pi_1\left(v_z^{HH} - v_z^{HL}\right) + \left(1 - \pi_1\right)\left(v_z^{LH} - v_z^{LL}\right)\right) \geq x_{2z} \ \forall z \in [0,1],$$

$$v_z^{Hm} = v_z^{Lm} \text{ for } m = H, L \text{ if } x_{1z} = 0 \ \forall z \in [0,1],$$

$$v_z^{mL} = v_z^{mH} \text{ for } m = H, L \text{ if } x_{2z} = 0 \ \forall z \in [0,1],$$

$$D_z \in \{0,1\} \ \forall z \in [0,1], \ W_z \in \{0,1\} \ \forall z \in [0,1], \text{ and } x_{iz} \in [0, \overline{x}_i] \ \forall z \in [0,1] \text{ and } i = 1, 2.$$

Using the same arguments as in the proof of Proposition 12, we can write the problem as:

$$K(v) = \max_{D, W, v_-^D, v_-^W, v_-^P} \left(DWK^D\left(v_-^D\right) + (1 - D) WK^W\left(v_-^W\right) + (1 - W) K^P\left(v_-^P\right)\right) \quad (2.35)$$

s.t.

$$v = DWv_-^D + (1 - D) Wv_-^W + (1 - W) v_-^P, \ D \in [0,1], \text{ and } W \in [0,1]. \quad (2.36)$$

D represents the probability of excessive force conditional on war and W represent the probability of war. Imagine if $\underline{\underline{U}}_2 < \underline{U}_2$ which is always be true in an equilibrium constructed as in Proposition 16 for a sufficiently high χ. Note that $K^D\left(\underline{\underline{U}}_2\right) = K\left(\underline{\underline{U}}_2\right)$ since $K^W(v)$ is only defined for $v \geq w_2 + \beta\underline{\underline{U}}_2 > \underline{\underline{U}}_2$ and $K^P(v)$ is only defined for $v \geq \beta\underline{\underline{U}}_2 > \underline{\underline{U}}_2$. Moreover, note that $K\left(\underline{\underline{U}}_2\right) = \underline{U}_1$, since if this were not the case, then it would be possible to generate values below $\underline{\underline{U}}_2$ which are public perfect by reducing $v^{D*}\left(\underline{\underline{U}}_2\right)$. Moreover, it must be that $K(v^{\max}) > \underline{U}_1$, since if this is not the case, the use of excessive force would not be incentive compatible for group 1. This fact and the concavity of $K(\cdot)$ and $K^W(\cdot)$ means that $K(\underline{U}_2) > \underline{U}_1 = K^W(\underline{U}_2)$. The same arguments those used in the proof of Lemma 11 establish equation (2.31), which means that $K(v) > K^W(v)\ \forall v \geq \underline{U}_2$, so that the solution to (2.35) – (2.36) requires $D^*(v) = 1$ for all v. **Q.E.D.**

Proof of Corollary 2

If there exists a solution which converges to permanent war, then by Proposition 18, this solution requires $\lim_{t\to\infty} \Pr\{D_t(q_t, z_t) = 1\} = 1$. However, this violates Proposition 17. **Q.E.D.**

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Chapter 3

Income and Democracy

3.1 Introduction

One of the most notable empirical regularities in political economy is the relationship between income per capita and democracy. Today all OECD countries are democratic, while many of the nondemocracies are in the poor parts of the world, for example sub-Saharan Africa and Southeast Asia. The positive cross-country relationship between income and democracy in the 1990s is depicted in Figure 3.1. This relationship is not only confined to a cross-country comparison. Most countries were nondemocratic before the modern growth process took off at the beginning of the 19th century. Democratization came together with growth. Barro (1999, S. 160), for example, summarizes this as: "increases in various measures of the standard of living forecast a gradual rise in democracy. In contrast, democracies that arise without prior economic development ... tend not to last."[1]

This statistical association between income and democracy is the cornerstone of the influential modernization theory. Lipset (1959) suggested that democracy was both created and consolidated by a broad process of 'modernization' which involved changes in "the factors of industrialization, urbanization, wealth, and education [which] are so closely interrelated as to form one common factor. And the factors subsumed under economic development carry with it the political correlate of democracy" (Lipset, 1959, p. 80). The central tenet of the modernization

[1]Also see, among others, Lipset (1959), Londregan and Poole (1996), Przeworski and Limongi (1997), Barro (1997), Przeworski, Alvarez, Cheibub, and Limongi (2000), and Papaioannou and Siourounis (2006).

theory, that higher income per-capita causes a country to be democratic, is also reproduced in most major works on democracy (e.g., Dahl, 1971, Huntington, 1991, Rusechemeyer, Stephens and Stephens, 1992).

In this paper, we revisit the relationship between income per capita and democracy. Our starting point is that existing work, which is based on cross-country relationships, does not establish causation. First, there is the issue of reverse causality; perhaps democracy causes income rather than the other way round. Second, and more important, there is the potential for omitted variable bias. Some other factor may determine both the nature of the political regime and the potential for economic growth.

We utilize two strategies to investigate the causal effect of income on democracy. Our first strategy is to control for country-specific factors affecting both income and democracy by including country fixed effects. While fixed effect regressions are not a panacea against omitted variable biases,[2] they are well-suited to the investigation of the relationship between income and democracy, especially in the postwar era. The major source of potential bias in a regression of democracy on income per capita is country-specific, historical factors influencing both political and economic development. If these omitted characteristics are, to a first approximation, time-invariant, the inclusion of fixed effects will remove them and this source of bias. Consider, for example, the comparison of the United States and Colombia. The United States is both richer and more democratic, so a simple cross-country comparison, as well as the existing empirical strategies in the literature, which do not control for fixed country effects, would suggest that per capita income causes democracy. The idea of fixed effects is to move beyond this comparison and investigate the "within-country variation", that is to ask whether Colombia is more likely to *become* (relatively) democratic as it *becomes* (relatively) richer. In addition to improving inference on the causal effect of income on democracy, this approach is also more closely related to modernization theory as articulated by Lipset (1959), which emphasizes that individual countries should become more democratic if they are richer, not simply that rich countries should be democratic.

[2]Fixed effects would not help inference if there are time-varying omitted factors affecting the dependent variable and correlated with the right-hand side variables (see the discussion below). They may also make problems of measurement error worse because they remove a significant portion of the variation in the right-hand side variables. Consequently, fixed effects are certainly no substitute to instrumental-variables or structural estimation with valid exclusion restrictions.

Our first result is that once fixed effects are introduced, the positive relationship between income per capita and various measures of democracy disappears. Figures 3.2 and 3.3 show this diagrammatically by plotting changes in our two measures of democracy, the Freedom House and Polity scores for each country between 1970 and 1995 against the change in GDP per capita over the same period (see Section 3.2 for data details). These figures confirm that there is no relationship between *changes* in income per capita and *changes* in democracy.

This basic finding is robust to using various different indicators for democracy, to different econometric specifications and estimation techniques, in different subsamples, and to the inclusion of additional covariates. The absence of a significant relationship between income and democracy is *not* driven by large standard errors. On the contrary, the relationship between income and democracy is estimated relatively precisely. In many cases, two-standard error bands include only very small effects of income on democracy and often exclude the OLS estimates. These results therefore shed considerable doubt on the claim that there is a strong causal effect of income on democracy.[3]

While the fixed effects estimation is useful in removing the influence of long-run determinants of both democracy and income, it does not necessarily estimate the causal effect of income on democracy. Our second strategy is to use instrumental-variables (IV) regressions to estimate the impact of income on democracy.[4] We experiment with two potential instruments. The first is to use past savings rates, while the second is to use changes in the incomes of trading partners. The argument for the first instrument is that variations in past savings rates affect income per capita but should have no direct effect on democracy. The second instrument, which we believe is of independent interest, creates a matrix of trade shares and constructs predicted income for each country using a trade-share-weighted average income of other countries. We show that this predicted income has considerable explanatory power for income per capita. We also argue that it should have no direct effect on democracy. Our second major result is that

[3]It remains true that over time there is a general tendency towards greater incomes and greater democracy across the world. In our regressions, time effects capture these general (world-level) tendencies. Our estimates suggest that these world-level movements in democracy are unlikely to be driven by the causal effect of income on democracy.

[4]A recent creative attempt is by Miguel, Satyanath and Sergenti (2004), who use the weather conditions as an instrument for income in Africa to investigate the impact of income on civil wars. Unfortunately, weather conditions are only a good instrument for relatively short-run changes in income, thus not ideal to study the relationship between income and democracy.

both IV strategies show no evidence of a causal effect of income on democracy. We recognize that neither instrument is perfect, since there are reasonable scenarios in which our exclusion restrictions could be violated (e.g., saving rates might be correlated with future anticipated regime changes; or democracy scores of a country's trading partners, which are correlated with their income levels, might have a direct effect on its democracy). To alleviate these concerns, we show that the most likely sources of correlation between our instruments and the error term in the second stage are not present.

We also look at the relationship between income and democracy over the past 100 years using fixed effects regressions and again find no evidence of a positive impact of income on democracy. These results are depicted in Figure 3.4, which plots the change in Polity score for each country between 1900 and 2000 against the change in GDP per capita over the same period (see Section 3.6 for data details). This figure confirms that there is no relationship between income and democracy conditional on fixed effects.

These results naturally raise the following important question: why is there a cross-sectional correlation between income and democracy? Or in other words, why are rich countries democratic today? At a statistical level, the answer is clear; even though there is no relationship between changes in income and democracy in the postwar era or over the past hundred years or so, there is a positive association over the past 500 years. Most societies were nondemocratic 500 years ago and had broadly similar income levels. The positive cross-sectional relationship reflects the fact that those that have become more democratic over this time span are also those that have grown faster. One possible explanation for the positive cross-sectional correlation is therefore that there is a causal effect of income on democracy, but it works at *much* longer horizons than the existing literature has posited. Although the lack of a relationship over 50 or 100 years sheds some doubt on this explanation, this is a logical possibility.

We favor another explanation for this pattern. Even in the absence of a simple causal link from income to democracy, political and economic development paths are interlinked and are jointly affected by various factors. Societies may embark on *divergent political-economic development paths*, some leading to relative prosperity and democracy, others to relative poverty and dictatorship. Our hypothesis is that the positive cross-sectional relationship and the 500-year correlation between changes in income and democracy are caused by the fact that countries have embarked on divergent development paths at some *critical junctures* during the past 500 years.[5]

We provide support for this hypothesis by documenting that the positive association between changes in income and democracy over the past 500 years are largely accounted for by a range of historical variables. In particular, for the whole world sample, the positive association is considerably weakened when we control for date of independence, early constraints on the executive and religion.[6] We then turn to the sample of former European colonies, where we have better proxies for factors that have influenced the development paths of nations. Acemoglu, Johnson and Robinson (2001, 2002) and Engerman and Sokoloff (1997) argue that differences in European colonization strategies have been a major determinant of the divergent development paths of colonial societies. This reasoning suggests that in this sample, the critical juncture for most societies corresponds to their experience under European colonization. Furthermore, Acemoglu, Johnson and Robinson (2002) show that the density of indigenous populations at the time of colonization has been a particularly important variable in shaping colonization strategies and provide estimates of population densities in 1500 (before the advent of colonization). When we use information on population density as well as on independence year and early constraints on the executive, the 500-year relationship between changes in income and democracy in the former colonies sample disappears. This pattern is consistent with the hypothesis that the positive cross-sectional relationship between income and democracy today is the result of societies embarking on divergent development paths at certain critical junctures during the past 500 years (though other hypotheses might also account for these patterns).

A related question is whether income has a separate causal effect on transitions to and away from democracy. Space restrictions preclude us from investigating this question here, and the results of such an investigation are presented in our followup paper, Acemoglu, Johnson, Robinson and Yared (2006). Using both linear regression models and double-hazard models that simultaneously estimate the process of entry into and exit from democracy, we find no

[5]See, among others, North and Thomas (1973), North (1981), Jones (1981), Engerman and Sokoloff (1997), Acemoglu, Johnson and Robinson (2001, 2002) for theories that emphasize the impact of certain historical factors on development processes during critical junctures, such as the collapse of feudalism, the age of industrialization or the process of colonization.

[6]See Weber (1930), Huntington (1991), Fish (2002) for the hypothesis that religion might have an important effect on economic and political development.

evidence that income has a causal effect on either the transitions to or from democracy. The IV strategies and the focus on the long run relationship are unique to the current paper.

The paper proceeds as follows. In Section 3.2 we describe the data. Section 3.3 presents our econometric model. Section 3.4 presents the fixed effects results for the post-war sample. Section 3.5 contains our IV results for the post-war sample, while the fixed effects results for the 100-year sample are presented in Section 3.6. Section 3.7 discusses the sources of the cross-country relationship between income and democracy we observe today. Section 3.8 concludes. The Appendix contains further information on the construction of the instruments used in Section 3.5. Section 3.11 contains all of the tables and figures.

3.2 Data and Descriptive Statistics

Our first and main measure of democracy is the Freedom House Political Rights Index. A country receives the highest score if political rights come closest to the ideals suggested by a checklist of questions, beginning with whether there are free and fair elections, whether those who are elected rule, whether there are competitive parties or other political groupings, whether the opposition plays an important role and has actual power, and whether minority groups have reasonable self-government or can participate in the government through informal consensus.[7] Following Barro (1999), we supplement this index with the related variable from Bollen (1990, 2001) for 1950, 1955, 1960, and 1965. As in Barro (1999), we transform both indices so that they lie between 0 and 1, with 1 corresponding to the most democratic set of institutions.

The Freedom House index, even when augmented with Bollen's data, only enables us to look at the postwar era. The Polity IV dataset, on the other hand, provides information for all independent countries starting in 1800. Both for pre-1950 events and as a check on our main measure, we also look at the other widely-used measure of democracy, the composite Polity index, which is the difference between Polity's Democracy and Autocracy indices (see Marshall and Jaggers, 2004). The Polity Democracy Index ranges from 0 to 10 and is derived

[7] The main checklist includes 3 questions on the electoral process, 4 questions on the extent of political pluralism and participation, and 3 questions on the functioning of government. For each checklist question, 0 to 4 points are added, depending on the comparative rights and liberties present (0 represents the least, 4 represents the most) and these scores are combined to form the index. See Freedom House (2004), http://www.freedomhouse.org/research/freeworld/2003/methodology.htm

from coding the competitiveness of political participation, the openness and competitiveness of executive recruitment and constraints on the chief executive. The Polity Autocracy Index also ranges from 0 to 10 and is constructed in a similar way to the democracy score based on competitiveness of political participation, the regulation of participation, the openness and competitiveness of executive recruitment and constraints on the chief executive. To facilitate comparison with the Freedom House score, we normalize the composite Polity index to lie between 0 and 1.

Using the Freedom House and the Polity data, we construct five-year, ten-year, twenty-year, and annual panels. For the five-year panels, we take the observation every fifth year. We prefer this procedure to averaging the five-year data, since averaging introduces additional serial correlation, making inference and estimation more difficult (see footnote 11). Similarly, for the ten-year and twenty-year panels, we use the observations from every tenth and twentieth year. For the Freedom House data, which begin in 1972, we follow Barro (1999) and assign the 1972 score to 1970 for the purpose of the five-year and ten-year regressions.

The GDP per capita (in PPP) and savings rate data for the postwar period are from Heston, Summers, and Atten (2002), and GDP per capita (in constant 1990 dollars) for the longer sample are from Maddison (2003). The trade-weighted world income instrument is built using data from International Monetary Fund Direction of Trade Statistics (2005). Other variables we use in the analysis are discussed later (see also Appendix Table 3.A1 for detailed data definitions and sources).

Table 3.1 contains descriptive statistics for the main variables. The sample period is 1960-2000 and each observation corresponds to five-year intervals. The table shows these statistics for all countries and also for high- and low-income countries, split according to median income. The first panel refers to the baseline sample we use in Table 3.2, while the other panels are for samples used in other tables. In each case, we report means, standard deviations, and also the total number of countries for which we have data and the total number of observations. The comparison of high- and low-income countries in columns 2 and 3 confirms the pattern in Figure 3.1 that richer countries tend to be more democratic.

3.3 Econometric Model

Consider the following simple econometric model, which will be the basis of our work both for the post-war and in the 100-year samples:

$$d_{it} = \alpha d_{it-1} + \gamma y_{it-1} + \mathbf{x}'_{it-1}\beta + \mu_t + \delta_i + u_{it}, \tag{3.1}$$

where d_{it} is the democracy score of country i in period t. The lagged value of this variable on the right-hand side is included to capture persistence in democracy and also potentially mean-reverting dynamics (i.e., the tendency of the democracy score to return to some equilibrium value for the country). The main variable of interest is y_{it-1}, the lagged value of log income per capita. The parameter γ therefore measures the causal effect of income per capita on democracy. All other potential covariates are included in the vector $\mathbf{x}_{it-1}$. In addition, the δ_i's denote a full set of country dummies and the μ_t's denote a full set of time effects that capture common shocks to (common trends in) the democracy score of all countries. u_{it} is an error term, capturing all other omitted factors, with $E(u_{it}) = 0$ for all i and t.[8]

The standard regression in the literature, for example, Barro (1999), is pooled OLS, which is identical to (3.1) except for the omission of the fixed effects, δ_i's. In our framework, these country dummies capture any time-invariant country characteristic that affect the level of democracy. As is well known, when the true model is given by (3.1) and the δ_i's are correlated with y_{it-1} or $\mathbf{x}_{it-1}$, then pooled OLS estimates are biased and inconsistent. More specifically, let x^j_{it-1} denote the jth component of the vector $\mathbf{x}_{it-1}$ and let Cov denote population covariances. Then, if either $\text{Cov}(y_{it-1}, \delta_i + u_{it}) \neq 0$ or $\text{Cov}\left(x^j_{it-1}, \delta_i + u_{it}\right) \neq 0$ for some j, the OLS estimator will be inconsistent. In contrast, even when these covariances are nonzero, the fixed effects estimator will be consistent if $\text{Cov}(y_{it-1}, u_{it}) = \text{Cov}\left(x^j_{it-1}, u_{it}\right) = 0$ for all j (as $T \to \infty$). This structure of correlation is particularly relevant in the context of the relationship between income and democracy because of the possibility of underlying political and social forces shaping both equilibrium political institutions and the potential for economic growth.

Nevertheless, there should be no presumption that fixed effects regressions necessarily es-

[8]More generally, equation (3.1) can be combined with another equation that captures the effect of democracy on income. The simultaneous equation bias resulting from the endogeneity of democracy is addressed in Section 3.5. The estimation of the effect of democracy on income is beyond the scope of the current paper.

timate the causal effect of income on democracy. First, the regressor d_{it-1} is mechanically correlated with u_{is} for $s < t$ so the standard fixed effect estimator is biased (e.g., Wooldridge, 2002, chapter 11). However, it can be shown that the fixed effects OLS estimator becomes consistent as the number of time periods in the sample increases (i.e., as $T \to \infty$). We discuss and implement a number of strategies to deal with this problem in Section 3.4.

Second, even if we ignore this technical issue, it is possible that $\text{Cov}(y_{it-1}, u_{it}) \neq 0$ because of the reverse effect of democracy on income, because both changes in income and changes in democracy are caused by a third, time-varying factor, or because the correct model is one with fixed growth effects rather than fixed level effects (see the extended model in Section 3.7.1). In Section 3.5, we implement an instrumental variable strategy to account for these problems. It is worth noting, however, that almost all theories in political science, sociology and economics suggest that we should have $\text{Cov}(y_{it-1}, u_{it}) \geq 0$. Therefore, when it fails to be consistent, the fixed effects estimator of the relationship between income and democracy will be biased upwards. Our fixed effects results can thus be viewed as upper bounds on the causal effect of income on democracy. Consistent with this, instrumental-variables regressions in Section 3.5 lead to more negative estimates than the fixed effects results.

3.4 Fixed Effects Estimates

3.4.1 Main Results

We begin by estimating (3.1) in the post-war sample. Table 3.2 uses the Freedom House data and Table 3.3 uses the Polity data, in both cases for the period 1960-2000. All standard errors in the paper are fully robust against arbitrary heteroscedasticity and serial correlation at the county level (i.e., they are clustered at the country level, see Wooldridge, 2002).

The first columns of both Tables 3.2 and 3.3 replicate the standard pooled OLS regressions previously used in the literature using the five-year sample. These regressions include the (five-year) lag of democracy and log income per capita as the country variables, as well as a full set of time dummies. Lagged democracy is highly significant and indicates that there is a considerable degree of persistence in democracy. Log income per capita is also significant and illustrates the well-documented positive relationship between income and democracy. Though statistically significant, the effect of income is quantitatively small. For example, the coefficient of 0.072 (standard error = 0.010) in column 1 of Table 3.2 implies that a 10 percent increase in GDP per capita is associated with an increase in the Freedom House score of less than 0.007, which is very small (for comparison, the gap between the United States and Colombia today is 0.5). If this pooled cross-section regression identified the causal effect of income on democracy, then the long-run effect would be larger than this, because the lag of democracy on the right-hand side would be increasing over time, causing a further increase in the democracy score. The implied cumulative effect of log GDP per capita on democracy is shown in the fifth row. Since lagged democracy has a coefficient of 0.706, the cumulative effect of a 10% increase in GDP per capita is $0.007/(1\text{-}0.706) \approx 0.024$, which is still quantitatively small.

The remaining columns of Tables 3.2 and 3.3 present our basic results with fixed effects. Column 2 shows that the relationship between income and democracy disappears once fixed effects are included. For example, in Table 3.2 with Freedom House data, the estimate of γ is 0.010 with a standard error of 0.035, which makes it highly insignificant. With the Polity data in Table 3.3, the estimate of γ has the "wrong" (negative) sign, -0.006 (standard error=0.039). The bottom rows in both tables again show the implied cumulative effect of income on democracy, which are small or negative.

A natural concern is that the lack of relationship in the fixed effects regressions may result from large standard errors. This does not seem to be the case. On the contrary, the relationship between income and democracy is estimated relatively precisely. Although the pooled OLS estimate of γ is quantitatively small, the two standard error bands of the fixed effects estimates almost exclude it. More specifically, with the Freedom House estimate, two standard error bands exclude short-run effects greater than 0.008.

That these results are not driven by some unusual feature of the data is further shown by Figures 3.2 and 3.3, which plot the change in the Freedom House and Polity score for each country between 1970 and 1995 against the change in GDP per capita over the same period. These scatterplots correspond to the estimation of equation (3.9) with a start date at 1970 and end date at 1995 (and without lagged democracy on the right-hand side).[9] They show clearly

[9]These two dates are chosen to maximize sample size. The regression of the change in Freedom House score between 1970 and 1995 on change in log income per capita between 1970 and 1995 yields a coefficient of 0.032, with a standard error of 0.058, while the same regression with Polity data gives a coefficient estimate of -0.024,

that there is no strong relationship between income growth and changes in democracy over this period.

These initial results show that once we allow for fixed effects, per capita income is *not* a major determinant of democracy. The remaining columns of the tables consider alternative estimation strategies to deal with the potential biases introduced by the presence of the lagged dependent variable discussed in Section 3.3.

Our first strategy, adopted in column 3, is to use the methodology proposed by Anderson and Hsiao (1982), which is to time difference equation (3.1), to obtain

$$\Delta d_{it} = \alpha \Delta d_{it-1} + \gamma \Delta y_{it-1} + \Delta \mathbf{x}'_{it-1} \boldsymbol{\beta} + \Delta \mu_t + \Delta u_{it}, \tag{3.2}$$

where the fixed country effects are removed by time differencing. Although equation (3.2) cannot be estimated consistently by OLS, in the absence of serial correlation in the original residual, u_{it} (i.e., no second order serial correlation in Δu_{it}), d_{it-2} is uncorrelated with Δu_{it}, so can be used as an instrument for Δd_{it-1} to obtain consistent estimates and similarly, y_{it-2} is used as an instrument for Δy_{it-1}. We find that this procedure leads to negative estimates (e.g., -0.104, standard error = 0.107 with the Freedom House data), and shows no evidence of a positive effect of income on democracy.

Although the instrumental variable estimator of Anderson and Hsiao (1982) leads to consistent estimates, it is not efficient, since, under the assumption of no further serial correlation in u_{it}, not only d_{it-2}, but all further lags of d_{it} are uncorrelated with Δu_{it}, and can also be used as additional instruments. Arellano and Bond (1991) develop a Generalized Method-of-Moments (GMM) estimator using all of these moment conditions. When all these moment conditions are valid, this GMM estimator is more efficient than the Anderson and Hsiao's (1982) estimator. We use this GMM estimator in column 4. The coefficients are now even more negative and more precisely estimated, for example -0.129 (standard error = 0.076) in Table 3.2.[10] In this case, the two standard error bands comfortably exclude the corresponding OLS estimate of

with a standard error of 0.063.

[10]In addition, Arellano and Bover (1995) also use time-differenced instruments for the level equation, (3.1). Nevertheless, these instruments would only be valid if the time-differenced instruments are orthogonal to the fixed effect. Since this is not appealing in this context (e.g., five-year income growth is unlikely to be orthogonal to the democracy country fixed effect), we do not include these additional instruments.

γ (which, recall, was 0.072). In addition, the presence of multiple instruments in the GMM procedure allows us to investigate whether the assumption of no serial correlation in u_{it} can be rejected and also to test for overidentifying restrictions. With the Freedom House data, the AR(2) test and the Hansen J test indicate that there is no further serial correlation and the overidentifying restrictions are not rejected.[11]

With the Polity data, both the Anderson and Hsiao and GMM procedures lead to more negative (and statistically significant) estimates. However, in this case, though there continues to be no serial correlation in u_{it}, the overidentification test is rejected, so we need to be more cautious in interpreting the results with the Polity data.

Column 5 shows a simpler specification in which lagged democracy is dropped. With either the Freedom House or Polity measure of democracy there is again no evidence of a significant effect of income on democracy, and in this case, the two standard error bands comfortably exclude the corresponding OLS coefficient (the OLS estimate without lagged democracy, which is shown in the first column of Tables 3.5 and 3.6, is 0.233 with a standard error of 0.013).

Column 6 estimates (3.1) with OLS using annual observations. This is useful since the fixed effect OLS estimator becomes consistent as the number of observations becomes large. With annual observations, we have a reasonably large time dimension. However, estimating the same model on annual data with a single lag would induce significant serial correlation (since our results so far indicate that five-year lags of democracy predict changes in democracy). For this reason, we now include five lags of both democracy and log GDP per capita in these annual regressions. Column 6 in both tables reports the p-value of an F-test for the joint significance of these variables. There is no evidence of a significant positive effect of income on democracy either with the Freedom House or the Polity data (while democracy continues to be strongly predicted by its lags).

Columns 7 and 8 investigate the relationship between income and democracy at lower frequencies by estimating similar regressions using a dataset of ten-year observations. The results are similar to those with five-year observations and to the patterns in Figures 3.2 and 3.3, which

[11]We also checked the results with five-year averaged data rather than our dataset which uses only the democracy information every fifth year. The estimates in all columns are very similar, but in this case, the AR(2) test shows evidence for additional serial correlation, which is not surprising given the serial correlation that averaging introduces. This motivates our reliance on the five-yearly or annual data sets.

Our analysis with annual data in column 6 of Tables 3.2 and 3.3 makes use of all of the available data.

show no evidence of a positive association between changes in income and democracy between 1970 and 1995. Finally, column 9 in both tables presents a fixed effect regression using a smaller dataset consisting of twenty-year observations. Once again, there is no evidence of a positive effect of income on democracy.

Overall, the inclusion of fixed effects proxying for time-invariant country specific characteristics removes the cross-country correlation between income and democracy. These results shed considerable doubt on the conventional wisdom that income has a strong causal effect on democracy.

3.4.2 Robustness

Table 3.4 investigates the robustness of these results. To save space, we only report the robustness checks for the Freedom House data (the results with Polity are similar and are available upon request). Columns 1-4 examine alternative samples. Columns 1 and 2 show the regressions corresponding to columns 2 and 4 of Table 3.2 for a balanced sample of countries from 1970 to 2000. This is useful to check whether entry and exit of countries from the base sample of Tables 3.2 and 3.3 might be affecting the results. Both columns provide very similar results. For example, using the balanced sample of Freedom House data and the fixed effects OLS specification, the estimate of γ is -0.031 (standard error= 0.049). Columns 3 and 4 exclude former socialist countries, again with very similar results.

Columns 5-10 investigate the influence of various covariates on the relationship between income and democracy. Columns 5 and 6 include log population and age structure, and columns 7 and 8 add education. Columns 9 and 10 include the full set of covariates from Barro's (1999) baseline specification.[12] In each case, we present both fixed effects and GMM estimates. The results show that these covariates do not affect the (lack of) relationship between income and democracy when fixed effects are included. Age structure variables are significant in the

[12]Age structure variables are from United Nations Population Division (2003) and include median age and variables corresponding to the fraction of the population in the following four age groups: 0-15, 15-30, 30-45, and 45-60. Total population is from World Bank (2002). In our regressions we measure education as total years of schooling in the population aged 25 and above. Columns 9 and 10 add covariates from Barro (1999), the urbanization rate and the male-female education gap. For consistency, these columns also follow Barro's strategy by measuring education as primary years of schooling in the population aged 25 and above. Both education variables are from Barro and Lee (2000). For detailed definitions and sources see Appendix Table 3.A1.

specification that excludes education, but not when education is included. Education is itself insignificant with a negative coefficient. The causal effect of education on democracy, which is another basic tenet of the modernization hypothesis, is therefore also not robust to controlling for country fixed effects.

In addition, in regressions not reported here, we checked for non-linear and non-monotonic effects of income on democracy and for potential non-linear interactions between income and other variables and found no evidence of such relationships. We also checked and found no evidence of an effect of the volatility in the growth rate of income per capita on democracy.[13]

3.5 Instrumental Variable Estimates

As discussed in Section 3.3, fixed effects estimators do not necessarily identify the causal effect of income on democracy. The estimation of causal effects requires exogenous sources of variation. While we do not have an ideal source of exogenous variation, there are two promising potential instruments and we now present IV results using these.

3.5.1 Savings Rate Instrument

The first instrument is the savings rate in the previous five-year period, denoted by s_{it}. The corresponding first stage for log income per capita, y_{it-1}, in regression (3.1) is

$$y_{it-1} = \pi^F s_{it-2} + \alpha^F d_{it-1} + \mathbf{x}'_{it-1}\boldsymbol{\beta}^F + \mu^F_{t-1} + \delta^F_i + u^F_{it-1}, \tag{3.3}$$

where all the variables are defined in Section 3.3 and the only excluded instrument is s_{it-2}. The identification restriction is that $\text{Cov}(s_{it-2}, u_{it} \mid \mathbf{x}_{it-1}, \mu_t, \delta_i) = 0$, where u_{it} is the residual error term in the second-stage regression, (3.1).

We naturally expect the savings rate to influence income in the future. What about exclud-

[13]We also investigated the effect of growth accelerations using a definition similar to that in the recent paper by Hausmann, Pritchett and Rodrik (2005) and found no effect of growth accelerations on democracy. Interestingly, however, the incidence of crises are correlated with democracy once fixed effects are taken into account.

The only subsample where we find a positive association between income per capita and democracy conditional on fixed effects is the postwar sample with 18 West European countries. However, this relationship holds only with the Freedom House data and not with the Polity data, and also disappears when we look at a longer sample than the postwar period alone. Details are available upon request.

ability? While we do not have a precise theory for why the savings rate should have no direct effect on democracy, it seems plausible to expect that changes in the savings rate over periods of 5-10 years should have no direct effect on the culture of democracy, the structure of political institutions or the nature of political conflict within society.

Nevertheless, there are a number of channels through which savings rates could be correlated with the error term in the second-stage equation, u_{it}. First, the savings rate itself might be influenced by the current political regime, for example, d_{it-2}, and could be correlated with u_{it} if all the necessary lags of democracy are not included in the system. Second, the savings rate could be correlated with changes in the distribution of income or composition of assets, which might have direct effects on political equilibria. Below, we provide evidence that these concerns are unlikely to be important in practice.

With these caveats in mind, Table 3.5 looks at the effect of GDP per capita on democracy in IV regressions using past savings rates as instruments and using the Freedom House data (results using Polity data are similar and available upon request). The savings rate is defined as nominal income minus consumption minus government expenditure divided by nominal income.

We report a number of different specifications, with or without lagged of democracy on the right-hand side, and with or without GMM. The first three columns show the OLS estimates in the pooled cross section, the fixed effects estimates without lagged democracy on the right-hand side, and the fixed effects estimates with lagged democracy on the right-hand side. Without fixed effects, there is a strong association between income per capita and democracy (the relationship in column 1 is stronger than before because it does not include lagged democracy on the right-hand side). With fixed effects, this relationship is no longer present. The remaining columns look at IV specifications and the bottom panel shows the corresponding first stages.

Column 4 shows a strong first-stage relationship between income and the savings rate, with a t-statistic of almost 5. The 2SLS estimate of the effect of income per capita on democracy is -0.035 (standard error = 0.094). The two standard error bands comfortably exclude the OLS estimate from column 1. Column 5 adds lagged democracy to the right-hand side. The first stage is very similar and the estimate of γ is now -0.020 (standard error = 0.081). Column 6 uses the GMM procedure, again with the savings rate as the excluded instrument for income. Now the estimate of γ is again negative, relatively large and significant at 5%. These IV results, therefore, show no evidence of a positive causal effect of income on democracy.

The remaining columns investigate the robustness of this finding and the plausibility of our exclusion restriction. Column 7 adds labor share as an additional regressor, to check whether a potential correlation between the savings rate and inequality might be responsible for our results.[14] The first stage shows no significant effect of labor share on income per capita, and the 2SLS estimate of γ is similar to the estimate without the labor share. Column 8 includes further lags of democracy to check whether systematic differences in savings rates between democracies and dictatorships might have an effect on the results. The estimate of γ is similar to before and, if anything, a little more negative in this case. Finally, column 9 adds a further lag of the savings rate as an instrument. This is useful since it enables a test of the overidentifying restriction (namely, a test of whether the savings rate at t-3 is a valid instrument conditional on the savings rate at t-2 being a valid instrument). The 2SLS estimate of γ is again similar and the overidentification restriction that the instruments are valid is accepted comfortably (at the p-value of 1.00).

3.5.2 Trade-Weighted World Income Instrument

Our second instrument exploits trade linkages across countries. To develop this instrument, let $\Omega = [\omega_{ij}]_{i,j}$ denote the $N \times N$ matrix of (time-invariant) trade shares between countries in our sample, where N is the total number of countries. More precisely, ω_{ij} is the share of trade between country i and country j in the GDP of country i which measure using trade shares between 1980-1989 (which is chosen to maximize coverage).[15]

The transmission of business cycles from one country to another through trade (e.g., Baxter, 1995, Kraay and Ventura, 2001) implies that we can think of a statistical model for income of a country as follows:

$$Y_{it-1} = \zeta \sum_{j=1, j \neq i}^{N} \omega_{ij} Y_{jt-1} + \varepsilon_{it-1}, \tag{3.4}$$

for all $i = 1, ..., N$, where Y_{it-1} denotes log total income, so $y_{it-1} = Y_{it-1} - P_{it-1}$ where P_{it-1}

[14] This is the labor share of gross value added from Rodrik (1999). We use these data rather than the standard Gini indices, because they are available for a larger sample of countries. The results with Gini coefficients are very similar and are available upon request.

[15] We obtain similar results if we use predicted average trade shares from a standard gravity equation as in Frankel and Romer (1999). See the previous version of the paper for details.

is the log population of i at $t-1$. The parameter ζ measures the effect of the trade-weighted world income on the income of each country.

Given equation (3.4), the identification problem in the estimation of (3.1) can be restated as follows: the error term ε_{it-1} in (3.4) is potentially correlated with u_{it} in equation (3.1), and if so, the estimates of the effect of income on democracy, γ, will be inconsistent. The idea of the approach in this section is to purge Y_{it-1}, and hence y_{it-1}, from ε_{it-1} to achieve consistent estimation of γ. For this purpose, we construct

$$\widehat{Y}_{it-1} = \sum_{j=1, j\neq i}^{N} \omega_{ij} Y_{jt-1}, \tag{3.5}$$

to use as an instrument for y_{it-1}. Here $\widehat{Y}_{it-1}$ is a weighted sum of world income for each country, with weights varying across countries depending on their trade pattern. Given $\widehat{Y}_{it-1}$, we can consider a model for income per capita of the form:
$y_{it-1} = \tilde{\pi}^F \widehat{Y}_{it-1} + \alpha^F d_{it-1} + \mathbf{x}'_{it-1}\beta^F + \mu^F_{t-1} + \delta^F_i + u^F_{it-1}$. Substituting for (3.5), we obtain our first-stage relationship:

$$y_{it-1} = \pi^F \sum_{j=1, j\neq i}^{N} \omega_{ij} Y_{jt-1} + \alpha^F d_{it-1} + \mathbf{x}'_{it-1}\beta^F + \mu^F_{t-1} + \delta^F_i + u^F_{it-1}, \tag{3.6}$$

where the parameter π^F corresponds to $\zeta\tilde{\pi}^F$ (we do not need separate estimates of ζ and $\tilde{\pi}^F$). The identification assumption for this strategy is that $\widehat{Y}_{it-1}$ is orthogonal to u_{it}. A sufficient condition for this is for Y_{jt-1} to be orthogonal to u_{it} for all $j \neq i$.

There may be reasons for this identification assumption to be violated. For example, Y_{jt-1} may be correlated with democracy in country j at time t, d_{jt}, which may influence d_{it} through other, political, social or cultural channels.[16] Although we have no way of ruling out these channels of influence a priori, below we control for the direct effect of the democracy of trading partners and find no evidence to support such a channel.[17]

[16]Because ω_{ij} is time-invariant, it does not capture *changes* in trade patterns and in trade agreements, which could possibly have a direct effect on democracy.

[17]There is an econometric problem arising from the general equilibrium nature of equation (3.4). Since this equation also applies for country j, the disturbance term ε_{it-1}, which determines Y_{it-1}, will be correlated with Y_{jt-1}, inducing a correlation between Y_{jt-1} and ε_{it-1}, and thus between $\widehat{Y}_{it-1}$ and ε_{it-1}. However, under some regularity conditions, the problem disappears as $N \to \infty$. In exercises included in the previous version of our

The main results using the Freedom House data are presented in Table 3.6 (results using Polity data are similar and available upon request). In the bottom panel we report the first-stage relationships. The first three columns again report OLS regressions with and without fixed effects; the basic patterns are similar to those presented before. Column 4 shows our basic 2SLS estimate with the trade-weighted instrument. The instrument is constructed as in (3.5) using the average trade shares between 1980 and 1989. The bottom panel shows a strong first-stage relationship with a t-statistic of almost 5. The 2SLS estimate of γ is -0.213 (standard error= 0.150). When we add lag democracy in column 5, the estimate is slightly less negative and more precise, -0.120 (standard error = 0.105), and becomes a little more precise with GMM in column 6, -0.133 (standard error = 0.077).

Column 7 investigates whether the democracy of trading partners of country j might have a direct effect on d_{jt}. We construct a world democracy index, $\tilde{d}_{it}$ using the same trade shares as in equation (3.5) and include this both in the first and second stages. This democracy index, $\tilde{d}_{it}$, also varies across countries because of the differences in weights. We find that $\tilde{d}_{it}$ has no effect either in the first or the second stages, consistent with our identification assumption that $\widehat{Y}_{it-1}$ should have no effect on democracy in country i except through its influence on y_{it-1}. Column 8 uses Y_{jt-2} instead of Y_{jt-1} on the right-hand side of (3.5) as an alternative strategy. Finally, column 9 performs an overidentification test similar to that in column 9 in Table 3.5 by including both the instrument constructed using Y_{jt-2} and the instrument constructed using Y_{jt-1}. The estimate of γ is similar to the baseline estimate in column 4 and the overidentifying restriction that the twice-lagged instrument is valid conditional on the first instrument being valid is again accepted comfortably (at the p-value of 1.00).

Overall, our two IV strategies give results consistent with the fixed effects estimates and indicate that there is no evidence for a strong causal effect of income on democracy.[18]

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paper, we have estimated ζ adjusting for potential bias and found no change in our results. Details available upon request.

[18] We also performed overidentification tests using the savings rate as the base instrument and trade-weighted income as the additional instrument and the χ^2-statistic for a Hausman test takes the value of 1.63 which is accepted at the p-value of 1.00. The reverse procedure with trade-weighted income as the base instrument yields a χ^2-statistic for a Hausman test takes the value of 1.97 which is accepted at the p-value of 0.99.

3.6 Fixed Effects Estimates Over 100 Years

We have so far followed much of the existing literature in focusing on the post-war period, where the democracy and income data are of higher quality. Nevertheless, it is important to investigate whether there may be an effect of income on democracy at longer horizons.

Although historical data are typically less reliable, the Polity IV dataset extends back to the beginning of the 19th century for all independent countries and Maddison (2003) gives estimates of income per-capita for many countries during this period. To investigate longer-term relationship between income and democracy, we construct a 25-year dataset starting in 1875.[19] This dataset contains a balanced panel of 25 countries for which democracy, lagged democracy (calculated 25 years earlier), and lagged income (calculated 25 years earlier) are available for every 25th year between 1875 and 2000.[20] We also construct a larger dataset with 50-year observations that starts in 1900. This dataset contains a larger sample of 37 independent countries.[21]

Table 3.7 presents the basic fixed effects results with these two samples. The specifications of columns 1-4 in Table 3.7 are identical to the specifications of columns 1, 2, 4, and 5 of Table 3.2, but it uses the 25-year valid sample over 1875-2000 with the Polity index as the dependent variable. These results are very similar to those from the post-war panel presented in Tables 3.2-3.4. For example, without fixed effects, the coefficient on income per capita is positive and significant at 0.116 (standard error=0.034), and with fixed effects the coefficient has the wrong sign and is insignificant at -0.020 (0.093). Column 5 reports the baseline regression on a smaller sample excluding all countries with imputed income estimates (see footnote 19). The results are very similar to column 1. Columns 6-10 repeat the same regressions using the data in

[19]Since Maddison reports income estimates for 1820, 1870 and 1929, we assign income per capita from 1820 to 1850, income per capita in 1870 to 1875, and income per capita in 1929 to 1925. All of our results are robust to dropping the 1875 observation so as to not use the 1850 estimate of income per capita as the value of lag income. If income per capita is not available for a particular country-year pair, it is estimated at the lowest aggregation level at which Maddison's data are available (e.g., Costa Rica, Guatemala, and Honduras are assigned the same income per capita in 1850) and the standard errors are computed by clustering at the highest aggregation level assigned to a particular country.

[20]The countries included in this dataset are Argentina, Austria, Belgium, Brazil, Chile, China, Colombia, Costa Rica, Denmark, El Salvador, Greece, Guatemala, Honduras, Mexico, Netherlands, Nicaragua, Norway, Sweden, Switzerland, Thailand, Turkey, United Kingdom, United States, Uruguay, and Venezuela.

[21]In addition to the countries in the 25 year sample, this sample includes Bolivia, Dominican Republic, Ecuador, France, Haiti, Iran, Liberia, Nepal, Oman, Paraguay, Portugal, and Spain.

50-year intervals from 1900 to 2000, again with similar results. Once fixed effect are included, the coefficient on income is small and insignificant. Figure 3.4 depicts these results graphically and shows that there is little relationship between changes in democracy and income in this 100-year sample.

As emphasized in Section 3.3, these results do not necessarily correspond to the causal effect of income on democracy, since there may be omitted time-varying covariates.[22] Nevertheless, most plausible omitted variables (as well as potential reverse causality) would bias these estimates upwards, so it is safe to conclude that there is no evidence of causal effect of income on democracy over the past 100 years.

3.7 Sources of Income-Democracy Correlations

The results presented so far show no evidence of a causal effect of income on democracy. Nevertheless, there is a strong positive association between income and democracy today as shown in Figure 3.1. Since 500 years ago most (or all) societies were nondemocratic and exhibited relatively small differences in income, this current-day correlation suggests that over the past 500 years societies that have grown faster have also become democratic. We now investigate why this may have been and how to reconcile this 500-year pattern with our econometric results. We start with a variation on the econometric model presented in Section 3.3 to motivate our theoretical approach and empirical work.

3.7.1 Divergent Development Paths

We first extend the econometric model introduced in Section 3.3 and use it to clarify the notions of *divergent development paths* and *critical junctures.* Consider a simplified version of (3.1), without the lagged dependent variable and the other covariates and with contemporaneous income per capita on the right-hand side:

$$d_{it} = \gamma y_{it} + \delta_i^d + u_{it}^d \tag{3.7}$$

[22] We also looked at IV regressions on this sample using a version of trade-weighted income constructed as in Section 3.5. However, in this smaller sample of countries, the first-stage relationship was not strong enough to allow the estimation of meaningful second stage regressions.

Moreover suppose that the statistical process for income per capita is

$$y_{it} = \delta_i^y + u_{it}^y. \tag{3.8}$$

The parameter γ again represents the causal effect of income on democracy, while δ_i^d and δ_i^y correspond to fixed differences in levels of democracy and income across countries. These fixed differences have so far been taken out by country fixed effects.[23]

Imagine we have data for two time periods, $t = T - S$ and $t = T$. Time-differencing equations (3.7) and (3.8), we obtain:

$$d_{iT} - d_{iT-S} = \gamma\left(y_{iT} - y_{iT-S}\right) + u_{iT}^d - u_{iT-S}^d, \tag{3.9}$$

and

$$y_{iT} - y_{iT-S} = u_{iT}^y - u_{iT-S}^y.$$

Consider the fixed effects estimator $\hat{\gamma}_S^{FE}$ using only these two data points, where the time span is given by S. Standard arguments imply that the probability limit of this estimator using these two data points is:

$$\text{plim}\hat{\gamma}_S^{FE} = \gamma + \frac{\text{Cov}\left(u_{iT}^d - u_{iT-S}^d, u_{iT}^y - u_{iT-S}^y\right)}{\text{Var}\left(u_{iT}^y - u_{iT-S}^y\right)}. \tag{3.10}$$

Therefore, estimation of (3.9) would yield a consistent estimate of the effect of income on democracy only if $\text{Cov}\left(u_{iT}^d - u_{iT-S}^d, u_{iT}^y - u_{iT-S}^y\right) = 0$, that is, only if changes in income over the relevant time horizon are not correlated with changes in democracy through a third common factor.

The condition $\text{Cov}\left(u_{iT}^d - u_{iT-S}^d, u_{iT}^y - u_{iT-S}^y\right) = 0$ is restrictive, especially over long horizons. The presence of divergent political-economic development paths across countries implies that this covariance is likely to be positive. Intuitively, divergent development paths refer to processes of development whereby political and economic outcomes evolve jointly. This joint evolution implies that u_{it}^d and u_{it}^y are not orthogonal and that $\text{Cov}\left(u_{iT}^d - u_{iT-S}^d, u_{iT}^y - u_{iT-S}^y\right) \neq$

[23]Allowing democracy to influence income in equation (3.8) does not change the conclusions as long as the effect is nonnegative.

0.

As an example, let us contrast the development experience of the United States with that of Peru and Bolivia. The United States grew rapidly during the late 18th and 19th centuries and became gradually more democratic, while these Andean societies stagnated and did not show a tendency to become democratic. Nondemocracy and stagnation in the Andes cannot be separated; the hacienda system, based on labor repression and the control of the indigenous Indian communities, was not conducive to industrialization and rapid growth during the 19th century. This system and its continuation, even after the abolition of formal systems of Indian tribute and forced labor, were not consistent with democratic institutions and a relatively equal distribution of political power within the society. This contrasts with the small-holder society in the United States, which resulted from the process of European colonization based on settlements in relatively empty and healthy lands. This social structure dominated by small-holders was much more consistent with democratic representation.[24] Democratic representation was in turn conducive to an environment where new industries and new entrepreneurs could flourish with relatively little resistance from established interests.[25] This description suggests that beyond the impact of income on democracy or the impact of democracy on income, we may want to think of political and economic development taking place jointly.[26] These ideas in general and the contrast between Northeast United States and the Andes in particular are captured by our notion of divergent development paths.

This description naturally leads to the question of what determines whether a country embarks upon a specific development path and brings us to the notion of *critical juncture*. The colonization strategies brought about by the Europeans, ranging from the settler societies of Northeast United States to the repressive economies of the Andes, were clearly important for

[24]See Galenson (1996) and Keyssar (2000) on the development of Northeastern United States as a settler colony, with a relatively democratic and open institutions. See Lockhart (1968) and Jacobsen (1993) for the creation and persistence of colonial practices in Peru, and see Klein (1992) on Bolivia. For a contrast of these development paths, see, among others, Engerman and Sokoloff (1997) and Acemoglu, Johnson and Robinson (2001,2002).

[25]Sokoloff and Kahn (1990) and Kahn and Sokoloff (1993) show that many of the major U.S. inventors in the 19th century were not members of the already-established economic elite, but newcomers with diverse backgrounds.

[26]Examples of models in which democracy and economic outcomes are jointly determined include Acemoglu and Robinson (2000), Acemoglu (2003), Cervellati, Fortunato, and Sunde (2005), and Llavador and Oxoby (2005).

the kind of development paths these societies embarked upon. In this sense, we can think of the early stages of the colonization process as the critical juncture for these development paths.

In summary, the simple conceptual framework proposed here is one in which income and democracy evolvejointly. The development path a society embarks upon is partly influenced by its experience during certain critical junctures, which might include the early stages of colonization for former colonies, the aftermath of independence or the founding of a nation, the epoch of the collapse of feudalism for Western European nations, the age of industrialization, i.e., the 19th century, and the periods of significant ideological shocks such as the Reformation, the Enlightenment or the rise of Islam.

These ideas can be incorporated into the econometric model above in a simple way. Suppose that the critical juncture (for example, the early dates of European colonization) is denoted by T^*, and for notational simplicity, suppose that this is a single common date for all countries. Suppose moreover that each of the stochastic terms in (3.7) and (3.8), u_{it}^d and u_{it}^y, admits a unit-root representation:

$$\begin{aligned} u_{it}^d &= \eta_{it}^d + \xi_{it}^d \text{ and } u_{it}^y = \eta_{it}^y + \xi_{it}^y \\ \text{where } \eta_{it}^d &= \eta_{it-1}^d + v_{it}^d \text{ and } \eta_{it}^y = \eta_{it-1}^y + v_{it}^y, \end{aligned}$$

with $E\left(\xi_{it}^d\right) = E\left(\xi_{it}^y\right) = E\left(v_{it}^d\right) = E\left(v_{it}^y\right) = 0$. Let the variances of v_i^y and ξ_i^y be denoted by $\sigma_{v^y}^2$ and $\sigma_{\xi^y}^2$, and assume that the ξ's are independent of the v's. Moreover, let $\text{Cov}\left(\xi_{it}^d, \xi_{it+k}^y\right) = 0$, $\text{Cov}\left(v_{it}^y, v_{it+k}^y\right) = 0$, and $\text{Cov}\left(v_{it}^d, v_{it+k}^d\right) = 0$ for all i and $k \neq 0$. Given this formulation, our emphasis on political and economic development paths diverging at some critical juncture corresponds to large and correlated shocks v_{it}^d and v_{it}^y at some $t = T^*$, which will then have a persistent effect on democracy and prosperity because of the unit root in η_{it}^d and η_{it}^y. To capture this, let $\text{Cov}\left(v_{iT^*}^d, v_{iT^*}^y\right) = \sigma_{T^*}^2$ be positive and large (i.e., $\sigma_{T^*}^2 >> 0$), corresponding to the importance of a major event affecting both economic and political outcomes at this critical juncture. In contrast with the pattern during critical junctures, we have that $\text{Cov}\left(v_{it}^d, v_{it}^y\right) = \sigma_{\sim T^*}^2$ for $t \neq T^*$, which we presume to be positive but small (i.e., $\sigma_{\sim T^*}^2 \geq 0$ but $\sigma_{\sim T^*}^2 \simeq 0$). Suppose also that $\text{Cov}\left(v_{it}^d, v_{it+k}^y\right) = 0$ for all i and $k \neq 0$. With this additional structure, equation (3.10) implies the following probability limit for the fixed effect estimator $\hat{\gamma}_S^{FE}$,

$$\text{plim}\hat{\gamma}_S^{FE} = \begin{cases} \gamma + \frac{\sigma^2_{\sim T*}}{\sigma^2_{vy}+2\sigma^2_{uy}/S} & \text{if } T^* \notin [T-S,T] \\ \gamma + \frac{(\sigma^2_{T*}-\sigma^2_{\sim T*})/S+\sigma^2_{\sim T*}}{\sigma^2_{vy}+2\sigma^2_{uy}/S} & \text{if } T^* \in [T-S,T] \end{cases}, \tag{3.11}$$

where the second equality exploits the fact that v_i's and u_i's are serially uncorrelated.

Equation (3.11) emphasizes that the bias of $\hat{\gamma}_S^{FE}$ will crucially depend on whether or not the critical juncture T^* takes place between the dates $T-S$ and T. If it does not do so, the first-term applies and to the extent that $\sigma^2_{\sim T*} \simeq 0$, the estimator will be "approximately" consistent. Note, however, that as S increases, the denominator falls, so the potential bias in this estimator may increase when $\sigma^2_{\sim T*} > 0$. Nevertheless, by and large, the fixed effects estimator will be approximately consistent when the critical juncture does not take place during the sample period. This is the reason why we have some confidence in the results obtained using the fixed effects regressions in the postwar and 20th century samples.

However, as the second line in (3.11) illustrates, when the critical juncture T^* is in our sample, the estimate of γ will be more severely inconsistent, since $\sigma^2_{T*} >> 0$. This observation may be relevant in interpreting why we may see a positive relationship between these two variables during the past 500 years, where many major events affecting the ultimate development path of various societies have taken place, but not during the postwar era or the entire 20th century.

Equation (3.11) also suggests an empirical methodology for checking whether events during the critical junctures might indeed be responsible for the cross-country correlation between income and democracy that we observe today. If we can control for variables correlated with the common component in v^d_{iT*} and v^y_{iT*} (in practice, historical determinants of divergent development paths) while estimating (3.9), the positive association between changes in income and democracy should weaken significantly or disappear. We investigate this issue in the next subsection.

3.7.2 Income and Democracy over the Past 500 Years

As discussed above, the current cross-country correlation between income and democracy likely reflects the changes in income and democracy over the past 500 years. We now investigate this relationship and interpret it in the light of the econometric framework introduced in the previous subsection. The major hurdle against an analysis of the relationship between income and democracy over long horizons is the availability of data. Nevertheless, there exist rough estimates of income per capita for almost all areas of the world in 1500. Moreover, we also have information about the variation in political institutions around the turn of the 16th century. While no country was fully "democratic" according to current definitions, there were certain notable differences in the political institutions of countries around the world even at this date. In particular, most countries outside Europe were ruled by absolutist regimes while some European countries had developed certain constraints on the behavior of their monarchs.

Acemoglu, Johnson and Robinson (2005) provide a coding of constraint on the executive for European countries going back to 1500 (based on the Polity definition). Constraint on the executive is a key input to the Polity democracy score for European countries. In addition, it seems reasonable that constraint on the executive for non-European countries and the other components of the Polity index (competitiveness of executive recruitment, openness of executive recruitment, and competitiveness of political participation) both for European and non-European countries should take the lowest score in 1500. Based on this information, we construct estimates for the Polity Composite index for 1500 (details available upon request). We combine these data with Maddison's (2003) estimates of income per capita in 1500 and 2000 and Polity's democracy score for 2000.[27]

We first check whether the current income-democracy correlation is indeed caused by changes over the past 500 years by estimating (3.9) over this sample period. Table 3.8 column 1 provides estimates for our entire world sample and column 5 focuses on the sample of former European colonies, which will allow us to better control for potential determinants of divergent development paths. As conjectured above, in both samples, the coefficient on income is large and significant. For example, in this 500-year sample, the coefficient on change in income in the entire world sample is 0.134 (standard error=0.021) and for the former colonies sample, it is 0.136 (standard error=0.019). Figure 3.5 depicts the association between the changes in democracy and changes in income over the past 500 years for the entire world sample. These

[27]Countries that have become independent in the 1990s are excluded from the sample. If the Polity score for 2000 is missing we assign the 1995 score to the observation.

results suggest that the current cross-country correlation between income and democracy is indeed accounted for by the developments over the past 500 years.

We next investigate how the inclusion of proxies for the divergent development paths affects this relationship. For the entire world sample, we use two sets of proxies. The first set of variables include a measure of early political institutions, constraint on the executive at independence from Polity IV, and the independence year. Since the date of independence is a possible critical juncture for most countries, a direct measure of institutions immediately after the end of the colonial period (for former colonies) or at the date of national independence (for non-colonies) is a useful proxy for the nature of the development paths that these societies have embarked upon. This variable is constructed as the average score of constraint on the executive from Polity IV during the first ten years after independence. We again normalize this score to a 0 to 1 scale like democracy, with 1 representing the highest constraint on the executive. It is useful to control for date of independence as well, since this is also related to the development paths that societies may have embarked upon (with early independence more indicative of a pro-growth and pro-democracy development path). Moreover, constraint on the executive at the date of independence would not be comparable across countries if we did not control for date of independence, since the meaning of this constraint likely varies over time.[28]

Column 2 of Table 3.8 includes constraint on the executive at independence and independence year in the regression for our entire world sample. The coefficient on income is reduced from 0.134 (standard error=0.021) to 0.061 (standard error=0.023), and higher values of constraint on the executive at independence and earlier dates of independence significantly predict greater changes in democracy over the past 500 years (conditional on the change in income). The coefficient on the change in income is still significant in this regression, perhaps in part because constraint on the executive at independence and independence year are very crude proxies for the divergent development paths of nations.

For this reason, we look for additional potential determinants of development paths. An important candidate suggested in the literature is religion. Citing the experience of England as the primary example, Weber (1930) argued that the Protestant ethic was responsible for the

[28]Data on date of independence are from the CIA World Factbook (2004). For detailed data definitions and sources see Appendix Table 3.A1. The data on constraint on the executive from Polity begins in 1800 or at the date of independence. Countries independent prior to 1800 are coded as being independent in 1800.

development of an institutional structure conducive to the rise of democracy and capitalism. Other arguments pointing to religion as an important determinant of political and economic development have been articulated by Huntington (1991) and Fish (2002), who emphasize the importance of Islam as an institutional barrier to the economic and political development.

In column 3 we present estimates including the fractions of different religions (in particular, fractions of Protestants, Catholics, and Muslims in the population).[29] The coefficient on income is again reduced, now to 0.088 (standard error=0.020), and religious fractions are individually significant at the 10% level with the fraction Muslim being most significant and negative at -0.233 (standard error=0.083). Column 4 combines the religion variables with the proxies of early institutions and date of independence. Now there is a more substantial drop in the estimate of the effect of change in income on the change in democracy, to 0.047 (standard error=0.023), which is just significant at 5%. Figure 3.6 illustrates the significant weakening in the relationship between changes in income and democracy once we control for historical factors affecting divergent development paths. It depicts the residual plot of the regression in Table 3.8, column 4. It shows that the inclusion of historical factors significantly reduces the upward sloping relationship apparent in Figure 3.5. Recall also that this estimate is likely to be an upper bound on the effect of changes in income on changes in democracy over the past 500 years, since our historical measures are only crude proxies for the determinants of divergent development paths.

Although the change in income continues to be significant in this regression, the magnitude of the effect is very small. If the coefficient of 0.047 represented the causal effect of income on democracy, it would imply that an "average" dictatorship in 1500 with income per capita of $566 (average of world income in 1500 in 1990 Geary-Khamis dollars) would need to reach a per capita income of $984 billion to become democratic![30]

The rest of Table 3.8 turns to the former colonies sample. The advantage of this sample is that we have a better understanding of the factors that have shaped the divergent development

[29] Data from La Porta, Lopez-de-Silanes, Shleifer, and Vishny (1999)

[30] This follows since, given the estimates in column 4, a change from a score of democracy of 0 to 1 would require an increase in log GDP per capita of 1/0.047. This translates into a exp(1/0.047)-fold (i.e., $\simeq$ 1.7.billion-fold!) increase in GDP per capita starting from $566, which leads to an income per capita of $984 billion. In contrast, the coefficient of 0.134 in column 1 implies that a substantially smaller (though still large) 1742-fold increase in income per capita is necessary for a society to move from a democracy score of 0 to 1.

path during critical junctures. In particular, Acemoglu, Johnson and Robinson (2002) document that former colonies with high rates of indigenous population density in 1500 have experienced greater extraction of resources and repression by Europeans, and consequently have been more likely to embark on a development path leading to relative stagnation and nondemocracy. They also provide estimates for population density of the indigenous population in 1500.[31] Motivated by this reasoning, we use the estimates of the size of the indigenous population in 1500 (population density in 1500, for short) as an additional proxy for factors determining the divergent development paths of nations.

Columns 5-8 are similar to columns 2-4, but refer to the former colonies. They show that the inclusion of constraints on the executive, independence year and religion variables weakens the 500-year correlation between changes in democracy and income, but a significant relationship still remains (and is in fact stronger than was the case for the entire world sample). Column 9 turns to the effect of population density in 1500 by including the log of the population density of the indigenous population. This variable is significant and has the expected sign. Moreover, its inclusion reduces the coefficient on the change in income per capita substantially. Column 10 shows that the inclusion of this variable together with constraint on the executive and date of independence is sufficient to remove the significant association between changes in income and democracy over the past 500 years entirely. Now the coefficient on the change in income per capita, which was originally equal to 0.136 (standard error=0.019), is reduced to 0.025 (standard error=0.024), which is highly insignificant. Moreover, constraint on the executive, independence year, and population density in 1500 are each individually significant. For example, the coefficient on population density in 1500 is -0.059 (0.021). Therefore, in this sample, there is no evidence that changes in income causes changes in democracy once we condition on certain proxies for divergent development paths of former colonies. Finally, column 11 includes the religion variables as well and again, the coefficient on income is low and insignificant at 0.029 (standard error=0.026)

Overall, these results are encouraging for our hypothesis, since they indicate that once reasonable proxies for the divergent development paths are included, the 500-year correlation

[31] Population density in 1500 is calculated by dividing the historical measures of population from McEvedy and Jones (1975) by the area of arable land (see Acemoglu, Johnson and Robinson, 2002).

between changes in income and democracy disappears and the cross-country correlation between income and democracy can be largely accounted by these divergent development paths.

3.8 Conclusion

The conventional wisdom in the political economy literature is that income per capita has a causal effect on democracy. In this paper, we argue that, though income and democracy are positively correlated, there is no evidence of a causal effect. Instead, omitted, most probably historical, factors appear to have shaped the divergent political and economic development paths of various societies, leading to the positive association between democracy and economic performance. Consequently, regressions that include country fixed effects and/or instrumental variable regressions show no evidence of a causal effect of income on democracy over the postwar era or the past 100 years. These results shed considerable doubt on the conventional wisdom both in the academic literature and in the popular press that income per capita is a key determinant of democracy and that a general increase in income per capita will bring improvements in institutions.

These results raise the question of why there is a positive cross-country correlation between income and democracy today We provided evidence that this is likely to be because the political and economic development paths are interwoven. Some countries appear to have embarked upon a development path associated with democracy and economic growth, while others pursued a path based on dictatorship, repression and more limited growth. Consistent with this, we have showed that historical sources of variation in development paths are responsible for much of the statistical association between long-run economic and political changes.

In emphasizing the importance of historical development paths, we do not want to suggest that there is a historical determinism in political for economic institutions. The fixed effects in the regressions and the presence of divergent development paths create a tendency, but many other factors influence equilibrium political institutions.[32] The most important area for future research is a further investigation of the effect of these time-varying and human factors on the

[32] Many current factors could and in fact appear to influence democracy. In the previous versions of our paper, we showed how severe economic crises lead to the collapse of dictatorships, making democracy more likely. Jones and Olken (2006) show how deaths of autocratic leaders make subsequent democracy more likely.

evolution of equilibrium political institutions.

3.9 Appendix

This Appendix addresses the construction of trade-weighted instrument used in Section 3.5. We first measure the matrix $\Omega = [\omega_{ij}]_{i,j}$ using actual trade shares between 1980 and 1989. These dates are chosen to maximize coverage. Bilateral trade data are from from the International Monetary Fund Direction of Trade Statistics (DoT) (2005) CD-ROM. Let X_{ijs} denote the total trade flow between i and j in year s, meaning the sum of exports from i to j and exports from j to i in year s. We calculate X_{ijs} for all country pairs in year s for which both flows from i to j and from j to i are available. These flows can be measured using either FOB exports from i to j or CIF imports by j from i. When both are available, we take the average, and otherwise we use whichever measure is available. All trade data are deflated into 1983 US dollars using the US CPI from International Financial Statistics (2004).

Let Y_{is}^* denote the total GDP of country i in year s in 1983 US dollars obtained from Heston, Summers, and Aten (2002), and $\mathcal{I}_{ij}$ be the number of years between 1980 and 1989 for which bilateral data between i and j are available. Our main measure of $\Omega = [\omega_{ij}]_{i,j}$ is:

$$\omega_{ij} = \frac{1}{\mathcal{I}_{ij}} \sum_{s=1980}^{1989} \left(\frac{X_{ijs}}{Y_{is}^*} \right),$$

where $X_{iis} = 0$ by definition.

Since we have an unbalanced panel, we construct our instrument defined in (3.5) as follows. Define $I_{jt-1} = \{0, 1\}$ as an indicator for Y_{jt-1} being available in the dataset. Then

$$\widehat{Y}_{it-1} = \zeta \left(\sum_{j=1, j \neq i}^{N} \omega_{ij} I_{jt-1} Y_{jt-1} \right) \left(\frac{\sum_{j=1, j \neq i}^{N} \omega_{ij}}{\sum_{j=1, j \neq i}^{N} I_{jt-1} \omega_{ij}} \right), \tag{3.12}$$

where Y_{jt-1} is log income as before. The third term in (3.12) ensures that the sum of the weights ω_{ij} are the same across time for a given country i, and this adjustment term is equal to 1 in a balanced panel. We measure trade-weighted democracy $\tilde{d}_{it}$ in an analogous fashion using (3.12), where we substitute d_{jt} for Y_{jt-1} and let I_{jt-1} now represent an indicator referring to the availability of the variable d_{jt}.

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3.11 Tables and Figures

Table 3.1
Descriptive Statistics

	All countries	High Income Countries	Low Income Countries
	(1)	(2)	(3)
Panel A			
Freedom House Measure of Democracy$_t$	0.57	0.78	0.36
	(0.36)	(0.30)	(0.30)
Log GDP per Capita $_{t-1}$ (Chain Weighted 1996 Prices)	8.16	9.02	7.30
	(1.02)	(0.56)	(0.53)
Observations	945	473	472
Countries	150	93	98
Panel B			
Polity Measure of Democracy$_t$	0.57	0.79	0.36
	(0.38)	(0.31)	(0.31)
Observations	854	427	427
Countries	136	81	88
Panel C			
Log Population $_{t-1}$	9.10	9.13	9.07
	(1.54)	(1.56)	(1.52)
Education $_{t-1}$	4.57	6.62	2.52
	(2.86)	(2.36)	(1.53)
Observations	676	338	338
Countries	95	57	65
Panel D			
Savings Rate $_{t-2}$	0.17	0.22	0.11
	(0.13)	(0.10)	(0.14)
Observations	891	446	445
Countries	134	82	84
Panel E			
Trade-Weighted Log GDP $_{t-1}$	11.61	12.98	10.24
	(8.43)	(9.74)	(6.62)
Observations	895	448	447
Countries	124	75	85

Values are averages during sample period, with standard deviations in parentheses. Panel A refers to the sample in Table 3.1, column 1; Panel B refers to the sample in Table 3.2, column 1; Panel C refers to the sample in Table 3.4 column 7; Panel D refers to the sample in Table 3.5, column 3; Panel E refers to the sample in Table 3.6, column 3. Column 1 in each panel refers to the full sample and columns 2 and 3 split the sample in column 1 by the median income (from Penn World Tables 6.1) in the sample of column 1. The number of observations refers to the total number of observations in the unbalanced panel. The number of countries refers to the number of countries for which we use observations. Freedom House Measure of Democracy is the Political Rights Index, augmented following Barro (1999). Polity Measure of Democracy is Democracy Index minus Autocracy Index from Polity IV. GDP per capita in 1996 prices with PPP adjustment is from the Penn World Tables 6.1. Population is from the World Bank (2002). Education is average total years of schooling in the population aged 25 and over and is from Barro and Lee (2000). Nominal Savings Rate is from Penn World Tables 6.1 and is defined as nominal income minus consumption minus government expenditure divided by nominal income (not PPP). Trade-Weighted log GDP is constructed as in equation (5) using data from IMF Direction of Trade Statistics (2005) and Penn World Tables 6.1. For detailed definitions and sources, see Appendix Table 3.A1.

Table 3.2

Fixed Effects Results using Freedom House Measure of Democracy

	Base Sample, 1960-2000								
	5-year data					Annual data	10-year data		20-year data
	Pooled OLS (1)	Fixed Effects OLS (2)	Anderson-Hsiao IV (3)	Arellano-Bond GMM (4)	Fixed Effects OLS (5)	Fixed Effects OLS (6)	Fixed Effects OLS (7)	Arellano-Bond GMM (8)	Fixed Effects OLS (9)
	Dependent Variable is Democracy								
Democracy $_{t-1}$	0.706	0.379	0.469	0.489		[0.00]	-0.025	0.226	-0.581
	(0.035)	(0.051)	(0.100)	(0.085)			(0.088)	(0.123)	(0.198)
Log GDP per Capita $_{t-1}$	0.072	0.010	-0.104	-0.129	0.054	[0.33]	0.053	-0.318	-0.030
	(0.010)	(0.035)	(0.107)	(0.076)	(0.046)		(0.066)	(0.180)	(0.156)
Hansen J Test				[0.26]				[0.07]	
AR(2) Test				[0.45]				[0.96]	
Implied Cumulative Effect of Income	0.245	0.016	-0.196	-0.252				-0.411	-0.019
	[0.00]	[0.76]	[0.33]	[0.09]				[0.09]	[0.85]
Observations	945	945	838	838	958	2895	457	338	192
Countries	150	150	127	127	150	148	127	118	118
R-squared	0.73	0.80			0.76	0.93	0.77		0.89

Pooled cross-sectional OLS regression in column 1, with robust standard errors clustered by country in parentheses. Fixed effects OLS regressions in columns 2, 5, 6, 7, and 9, with country dummies and robust standard errors clustered by country in parentheses. Implied cumulative effect of income represents the coefficient estimate of log GDP per Capita $_{t-1}$/(1-Democracy$_{t-1}$) and the p-value from a non-linear test of the significance of this coefficient is in brackets. Column 3 uses instrumental variables method of Anderson and Hsiao (1982), with clustered standard errors, and columns 4 and 8 use GMM of Arellano and Bond (1991), with robust standard errors; in both methods we instrument for income using a double lag. Year dummies are included in all regressions. Dependent variable is Freedom House Measure of Democracy. Base sample is an unbalanced panel, 1960-2000, with data at 5-year intervals, where the start date of the panel refers to the dependent variable (i.e., t=1960, so t-1=1955); column 6 uses annual data from the same sample; a country must be independent for 5 years before it enters the panel. Columns 7 and 8 use 10-year data from the same sample, where as before the start date of the panel refers to the dependent variable (i.e., t=1960, so t-1=1950); a country must be independent for 10 years before it enters the panel. Column 9 uses 20-year data from the same sample, where as before the start date of the panel refers to the dependent variable (i.e., t=1980, so t-1=1960); a country must be independent for 20 years before it enters the panel. In column 6, each right hand side variable has five annual lags; we report the p-value from an F-test for the joint significance of all 5 lags. For detailed data definitions and sources see Table 3.1 and Appendix Table 3.A1.

Table 3.3

Fixed Effects Results using Polity Measure of Democracy

	Base Sample, 1960-2000								
	5-year data					Annual data	10-year data		20-year data
	Pooled OLS	Fixed Effects OLS	Anderson-Hsiao IV	Arellano-Bond GMM	Fixed Effects OLS	Fixed Effects OLS	Fixed Effects OLS	Arellano-Bond GMM	Fixed Effects OLS
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Dependent Variable is Democracy								
Democracy $_{t-1}$	0.749	0.449	0.582	0.590		[0.00]	0.060	0.309	-0.516
	(0.034)	(0.063)	(0.127)	(0.106)			(0.091)	(0.134)	(0.165)
Log GDP per Capita $_{t-1}$	0.053	-0.006	-0.413	-0.351	-0.011	[0.53]	0.007	-0.368	-0.126
	(0.010)	(0.039)	(0.163)	(0.127)	(0.055)		(0.070)	(0.190)	(0.164)
Hansen J Test				[0.03]				[0.01]	
AR(2) Test				[0.39]				[0.38]	
Implied Cumulative Effect of Income	0.211	-0.011	-0.988	-0.856			0.007	-0.533	-0.083
	[0.00]	[0.89]	[0.01]	[0.00]			[0.92]	[0.04]	[0.45]
Observations	854	854	747	747	880	3701	419	302	168
Countries	136	136	114	114	136	134	114	107	100
R-squared	0.77	0.82			0.77	0.96	0.77		0.87

Pooled cross-sectional OLS regression in column 1, with robust standard errors clustered by country in parentheses. Fixed effects OLS regressions in columns 2, 5, 6, 7, and 9, with country dummies and robust standard errors clustered by country in parentheses. Implied cumulative effect of income represents the coefficient estimate of log GDP per Capita $_{t-1}$/(1-Democracy$_{t-1}$) and the p-value from a non-linear test of the significance of this coefficient is in brackets. Column 3 uses instrumental variables method of Anderson and Hsiao (1982), with clustered standard errors, and columns 4 and 8 use GMM of Arellano and Bond (1991), with robust standard errors; in both methods we instrument for income using a double lag. Year dummies are included in all regressions. Dependent variable is Polity Measure of Democracy. Base sample is an unbalanced panel, 1960-2000, with data at 5-year intervals, where the start date of the panel refers to the dependent variable (i.e., t=1960, so t-1=1955); column 6 uses annual data from the same sample; a country must be independent for 5 years before it enters the panel. Columns 7 and 8 use 10-year data from the same sample, where as before the start date of the panel refers to the dependent variable (i.e., t=1960, so t-1=1950); a country must be independent for 10 years before it enters the panel. Column 9 uses 20-year data from the same sample, where as before the start date of the panel refers to the dependent variable (i.e., t=1980, so t-1=1960); a country must be independent for 20 years before it enters the panel. In column 6, each right hand side variable has five annual lags; we report the p-value from an F-test for the joint significance of all 5 lags. For detailed data definitions and sources see Table 3.1 and Appendix Table 3.A1.

Table 3.4

Fixed Effects Results using Freedom House Measure of Democracy: Robustness Checks

	Balanced Panel, 1970-2000		Base Sample, 1960-2000, without Former Socialist Countries		Base Sample, 1960-2000					
	5-year data									
	Fixed Effects OLS (1)	Arellano-Bond GMM (2)	Fixed Effects OLS (3)	Arellano-Bond GMM (4)	Fixed Effects OLS (5)	Arellano-Bond GMM (6)	Fixed Effects OLS (7)	Arellano-Bond GMM (8)	Fixed Effects OLS (9)	Arellano-Bond GMM (10)
	Dependent Variable is Democracy									
$Democracy_{t-1}$	0.283	0.472	0.362	0.436	0.353	0.480	0.351	0.499	0.352	0.475
	(0.058)	(0.092)	(0.052)	(0.085)	(0.053)	(0.087)	(0.055)	(0.097)	(0.050)	(0.088)
Log GDP per $Capita_{t-1}$	-0.031	-0.262	0.005	-0.151	0.015	-0.008	-0.001	-0.121	-0.042	-0.126
	(0.049)	(0.128)	(0.035)	(0.078)	(0.041)	(0.139)	(0.049)	(0.182)	(0.045)	(0.130)
Log $Population_{t-1}$					-0.109	-0.001	-0.042	0.049	-0.070	-0.016
					(0.100)	(0.113)	(0.108)	(0.143)	(0.112)	(0.163)
$Education_{t-1}$							-0.007	-0.020	-0.006	-0.011
							(0.020)	(0.026)	(0.038)	(0.052)
Age $Structure_{t-1}$					[0.05]	[0.63]	[0.19]	[0.27]		
Barro (1999) Covariates	NO	NO	NO	NO	NO	NO	NO	NO	YES	YES
Hansen J Test		[0.40]		[0.34]		[0.08]		[0.15]		[0.25]
AR(2) Test		[0.73]		[0.49]		[0.43]		[0.88]		[0.75]
Implied Cumulative Effect of Income	-0.043	-0.496	0.008	-0.268	0.023	-0.015	-0.002	-0.242	-0.065	-0.240
	[0.53]	[0.03]	[0.89]	[0.05]	[0.72]	[0.96]	[0.98]	[0.50]	[0.35]	[0.33]
Observations	630	567	908	823	863	731	676	589	676	588
Countries	90	81	128	124	142	120	95	92	96	92
R-squared	0.80		0.79		0.80		0.77		0.76	

Fixed effects OLS regressions in columns 1, 3, 5, 7, and 9 with country dummies and robust standard errors clustered by country in parentheses. Columns 2, 4, 6, 8, and 10 use GMM of Arellano and Bond (1991), with robust standard errors; in this method we instrument for income using a double lag. Year dummies are included in all regressions. Implied cumulative effect of income represents the coefficient estimate of log GDP per $Capita_{t-1}/(1-Democracy_{t-1})$ and the p-value from a non-linear test of the significance of this coefficient is in brackets. Dependent variable is Freedom House Measure of Democracy. Base sample is an unbalanced panel, 1960-2000, with data at 5-year intervals in levels where the start date of the panel refers to the dependent variable (i.e., t=1960, so t-1=1955); a country must be independent for 5 years before it enters panel. Columns 1 and 2 use a balanced panel from 1970 to 2000. Columns 3 and 4 exclude Soviet bloc countries. Education is average years of total schooling in the population in columns 7 and 8. Education is average years of primary schooling in the population in columns 9 and 10. Columns 7-8 include but do not display the median age of the population at t-1 and 4 covariates corresponding to the percent of the population at t-1 in the following age groups: 0-15, 15-30, 30-45, and 45-60. The age structure F-test is gives the p-value for the joint significance of these variables. Columns 9 and 10 include but do not display additional covariates used by Barro (1999): male-female education gap and urbanization rate. For detailed data definitions and sources see Table 3.1 and Appendix Table 3.A1.

Table 3.5

Fixed Effects Results using Freedom House Measure of Democracy: Two Stage Least Squares with Savings Rate Instrument

	Base Sample, 1960-2000								
	All Countries								
	Pooled OLS	Fixed Effects OLS	Fixed Effects OLS	Fixed Effects 2SLS	Fixed Effects 2SLS	Arellano-Bond GMM	Fixed Effects 2SLS	Fixed Effects 2SLS	Fixed Effects 2SLS
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Panel A	*Dependent Variable is Democracy*								
Democracy $_{t-1}$			0.359		0.363	0.427		[0.00]	
			(0.054)		(0.056)	(0.100)			
Log GDP per Capita $_{t-1}$	0.233	0.044	0.009	-0.035	-0.020	-0.228	-0.036	-0.074	0.016
	(0.013)	(0.051)	(0.038)	(0.094)	(0.081)	(0.102)	(0.191)	(0.113)	(0.095)
Labor Share $_{t-1}$							0.250		
							(0.199)		
Panel B	*First Stage for Log GDP per Capita $_{t-1}$*								
Democracy $_{t-1}$					0.144			[0.24]	
					(0.066)				
Labor Share $_{t-1}$							0.329		
							(0.187)		
Savings Rate $_{t-2}$				1.356	1.343		1.202	1.173	1.022
				(0.277)	(0.270)		(0.315)	(0.254)	(0.218)
Savings Rate $_{t-3}$									0.720
									(0.182)
Hansen J Test						[0.34]			
AR(2) Test						[0.72]			
Implied Cumulative Effect of Income			0.014		-0.031	-0.398			
			[0.82]		[0.80]	[0.01]			
Observations	900	900	891	900	891	764	471	733	796
Countries	134	134	134	134	134	124	98	124	125
R-squared in First Stage				0.96	0.96		0.98	0.97	0.97

Pooled cross-sectional OLS regression in column 1, with robust standard errors clustered by country in parentheses. Fixed effects OLS regressions in columns 2 and 3 with country dummies and robust standard errors clustered by country in parentheses. Fixed effects 2SLS regressions in columns 4, 5, 7, 8, and 9 with country dummies and robust standard errors clustered by country in parentheses; first stage regressions are displayed in Panel B and include all second stage covariates (apart from income) on the right hand side with robust standard errors clustered by country in parentheses. GMM of Arellano-Bond in column 6 with robust standard errors; in this method we instrument for income in the first differenced equation with the first difference of the instrument. Year dummies are included in all regressions. Implied cumulative effect of income represents the coefficient estimate of log GDP per Capita $_{t-1}$/(1-Democracy$_{t-1}$) and the p-value from a non-linear test of the significance of this coefficient is in brackets. Dependent variable is Freedom House Measure of Democracy. Base sample is an unbalanced panel, 1960-2000, with data at 5-year intervals, where the start date of the panel refers to the dependent variable (i.e., t=1960, so t-1=1955); a country must be independent for 5 years before it enters the panel. In columns 4-9 instrument for Log GDP per Capita $_{t-1}$ with Savings Rate $_{t-2}$. Column 9 includes Savings Rate $_{t-3}$ as an additional instrument. Column 8 includes but does not display Democracy$_{t-1}$, Democracy$_{t-2}$, and Democracy$_{t-3}$; we report the p-value from an F-test for the joint significance of all 3 lags. For detailed data definitions and sources see Table 3.1 and Appendix Table 3.A1.

Table 3.6

Fixed Effects Results using Freedom House Measure of Democracy: Two Stage Least Squares with Trade-Weighted World Income Instrument

	Base Sample, 1960-2000								
	All Countries								
	Pooled OLS	Fixed Effects OLS	Fixed Effects OLS	Fixed Effects 2SLS	Fixed Effects 2SLS	Arellano-Bond GMM	Fixed Effects 2SLS	Fixed Effects 2SLS	Fixed Effects 2SLS
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Panel A	*Dependent Variable is Democracy*								
Democracy $_{t-1}$			0.376		0.393	0.478			
			(0.051)		(0.057)	(0.094)			
Log GDP per Capita $_{t-1}$	0.233	0.038	0.001	-0.213	-0.120	-0.133	-0.202	-0.198	-0.217
	(0.013)	(0.045)	(0.034)	(0.150)	(0.105)	(0.077)	(0.130)	(0.160)	(0.149)
Trade-Weighted Democracy$_t$							-0.137		
							(0.635)		
Panel B	*First Stage for Log GDP per Capita $_{t-1}$*								
Democracy $_{t-1}$					0.169				
					(0.063)				
Trade-Weighted Democracy$_t$							-1.195		
							(0.959)		
Trade-Weighted Log GDP $_{t-1}$				0.402	0.421		0.441		0.529
				(0.083)	(0.082)		(0.070)		(0.180)
Trade-Weighted Log GDP $_{t-2}$								0.341	-0.127
								(0.090)	(0.206)
Hansen J Test						[0.19]			
AR(2) Test						[0.50]			
Implied Cumulative Effect of Income			0.002		-0.198	-0.255			
			[0.98]		[0.28]	[0.07]			
Observations	906	906	895	906	895	812	906	906	906
Countries	124	124	124	124	124	122	124	124	124
R-squared in First Stage				0.95	0.96		0.95	0.95	0.95

Pooled cross-sectional OLS regression in column 1, with robust standard errors clustered by country in parentheses. Fixed effects OLS regressions in columns 2 and 3 with country dummies and robust standard errors clustered by country in parentheses. Fixed effects 2SLS regressions in columns 4, 5, 7, 8, and 9 with country dummies and robust standard errors clustered by country in parentheses; first stage regressions are displayed in Panel B and include all second stage covariates excluding income on the right hand side with robust standard errors clustered by country in parentheses. GMM of Arellano-Bond in column 6 with robust standard errors; in this method we instrument for income in the first differenced equation with the first difference of the instrument. Year dummies are included in all regressions. Implied cumulative effect of income represents the coefficient estimate of log GDP per Capita $_{t-1}$/(1-Democracy$_{t-1}$) and the p-value from a non-linear test of the significance of this coefficient is in brackets. Dependent variable is Freedom House Measure of Democracy. Base sample is an unbalanced panel, 1960-2000, with data at 5-year intervals, where the start date of the panel refers to the dependent variable (i.e., t=1960, so t-1=1955); a country must be independent for 5 years before it enters the panel. Columns 4-8 instrument for Log GDP per Capita $_{t-1}$ with Trade-Weighted World Log GDP $_{t-1}$. Column 9 uses Trade-Weighted World Log GDP $_{t-2}$ as an additional instrument. For detailed data definitions and sources see Table 3.1 and Appendix Table 3.A1. See Appendix for details on the construction of the instruments.

Table 3.7

Fixed Effects Results using Polity Measure of Democracy in the Long Run

	Balanced Panel, 1875-2000									
	25-year data					50-year data				
	Pooled OLS	Fixed Effects OLS	Arellano-Bond GMM	Fixed Effects OLS	Fixed Effects OLS	Pooled OLS	Fixed Effects OLS	Arellano-Bond GMM	Fixed Effects OLS	Fixed Effects OLS
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	Dependent Variable is Democracy									
Democracy $_{t-1}$	0.487	0.192	0.439		0.212	0.233	-0.248	0.319		-0.266
	(0.085)	(0.119)	(0.143)		(0.140)	(0.070)	(0.124)	(0.148)		(0.188)
Log GDP per Capita $_{t-1}$	0.116	-0.020	-0.495	0.003	0.074	0.191	0.039	-0.411	-0.004	0.028
	(0.034)	(0.093)	(0.266)	(0.092)	(0.118)	(0.043)	(0.110)	(0.194)	(0.092)	(0.222)
Hansen J Test			[0.27]					[0.87]		
AR(2) Test			[0.42]					N.A.		
Implied Cumulative Effect of Income	0.226	-0.025	-0.882		0.094	0.249	0.031	-0.604		0.031
	[0.00]	[0.84]	[0.02]		[0.53]	[0.00]	[0.72]	[0.04]		[0.90]
Observations	150	150	125	150	78	111	111	74	111	48
Countries	25	25	25	25	13	37	37	37	37	16
R-squared	0.55	0.65		0.63	0.72	0.49	0.72		0.70	0.69

Pooled cross-sectional OLS regression in columns 1 and 6, with robust standard errors clustered by highest level of aggregation for income data in parentheses. Fixed effects OLS regressions in columns 2, 4, 5, 7, 9, and 10, with country dummies and robust standard errors clustered by highest level of aggregation for income data in parentheses. Column 3 and 8 use GMM of Arellano and Bond (1991), with robus standard errors; we instrument for income using a double lag. Year dummies are included in all regressions. Implied cumulative effect of income represents the coefficient estimate of log GDP per Capita$_{t-1}$/(1-Democracy$_{t-1}$) and the p-value from a non-linear test of the significance of this coefficient is in brackets. Dependent variable is Polity Measure of Democracy. Base sample is a balanced panel 1875-2000. Columns 1-5 use 25-year data where the start date of the panel refers to the dependent variable (i.e., t=1875, so t-1=1850), and columns 6-10 use 50-year data where the start date of the panel refers to the dependent variable (i.e., t=1900, so t-1=1850), where the sample begins in 1900. In columns 5 and 10, we drop countries for which the level of aggregation for income data changes across the sample period. The AR (2) test is not possible in column 7 since there are two observation years. GDP per capita is from Maddison (2003). For detailed data definitions and sources see Table 3.1 and Appendix Table 3.A1.

Table 3.8

Democracy in the Very Long Run

	Base Sample, 1500-2000				Former Colonies Sample, 1500-2000						
	OLS	OLS	OLS	OLS	OLS	OLS	OLS	OLS	OLS	OLS	OLS
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
	Dependent Variable is Change in Democracy Over Sample Period										
Change in Log GDP per Capita Over Sample Period	0.134	0.061	0.088	0.047	0.136	0.067	0.099	0.057	0.081	0.025	0.029
	(0.021)	(0.023)	(0.020)	(0.023)	(0.019)	(0.012)	(0.012)	(0.013)	(0.027)	(0.024)	(0.026)
Constraint on the Executive at Independence		0.260		0.164		0.189		0.189		0.166	0.167
		(0.120)		(0.064)		(0.072)		(0.075)		(0.089)	(0.087)
Independence Year/100		-0.206		-0.133		-0.190		-0.105		-0.179	-0.128
		(0.063)		(0.036)		(0.032)		(0.075)		(0.023)	(0.074)
Fraction Muslim			-0.299	-0.233			0.023	0.059			0.038
			(0.097)	(0.083)			(0.101)	(0.105)			(0.088)
Fraction Protestant			0.191	0.180			0.508	0.491			0.411
			(0.112)	(0.091)			(0.258)	(0.154)			(0.196)
Fraction Catholic			0.155	0.117			0.306	0.277			0.200
			(0.073)	(0.069)			(0.130)	(0.229)			(0.221)
Log Population Density, 1500									-0.059	-0.049	-0.031
									(0.021)	(0.022)	(0.027)
Historical Factors F-test		[0.02]	[0.00]	[0.01]		[0.05]	[0.00]	[0.00]	[0.02]	[0.00]	[0.00]
Observations	135	135	131	131	87	87	87	87	83	83	83
R-squared	0.20	0.34	0.40	0.45	0.20	0.30	0.33	0.37	0.25	0.34	0.38

Cross-section OLS regression in all columns, with robust standard errors clustered by level of aggregation for 1500 income data in parentheses. Countries are included if independent prior to 1990, as determined by CIA (2006). Sample limited to former European colonies in columns 5-11. Changes are total differences between 1500 and 2000. GDP per capita is from Maddison (2003), and democracy is calculated using the Polity Measure of Democracy, which comprises in part constraint on the executive. All columns assume some values of democracy in 1500 in a few European countries, following Acemoglu et al (2004b) and assigns the lowest value of democracy for all other countries. The historical factors F-test reports the p-value for all variables other than change in income. For detailed data definitions and sources see text, Table 3.1, and Appendix Table 3.A1.

Appendix Table 3.A1

VARIABLE	DESCRIPTION	SOURCE
Freedom House Political Rights Index, also referred to here as Freedom House Measure of Democracy	Data for 1972-2000 in Freedom House Political Rights Index, original range 1,2,3,…,7 normalized 0-1. Data for 1972 used for 1970. Data for 1950, 1955, 1960 and 1965, in Bollen, original range 0.00,0.01,…0.99,1.	http://www.freedomhouse.org/ratings/, and Bollen (2001) "Cross National Indicators of Liberal Democracy 1950-1990" available on ICPSR
Polity Composite Democracy Index, also referred to here as the Polity Measure of Democracy	Data for 1850-2000 in Polity IV. The composite index is the democracy score minus the autocracy score. Original range -10,-9,...10, normalized 0-1. For the purposes of the historical regressions, countries for which data is not available in 2000 are assigned the data for 1995.	http://www.cidcm.umd.edu/inscr/polity/
Polity Composite Democracy Index in 1500	Constructed using constraint on the executive score from Acemoglu, Johnson, and Robinson (2004b) for the sample of European countries. Components of the index other than constraint on the executive are assigned a value of zero for all countries.	Acemoglu, Johnson, and Robinson (2004)
GDP per Capita (Chain Weighted 1996 Prices)	Data for 1950-2000 measured as Log Real GDP per Capita (Chain Method in 1996 prices) from Penn World Tables 6.1.	http://pwt.econ.upenn.edu/
GDP per Capita (1990 dollars)	Data for 1500-2000 measured as Log Real GDP per Capita (1990 Geary-Khamis dollars) from Maddison (2003). Countries are assigned values at the lowest possible aggregation. Data in 1820 is used for 1850. Data in 1870 is used for 1875. Data in 1929 is used for 1925.	http://www.eco.rug.nl/~Maddison/
Population	Total population in thousands.	World Bank (2002)
Education	Average total years of schooling in the population aged 25 and over. Data for 1960, 1965,…, 1995 from Barro and Lee. We include average years of primary schooling in the population aged 25 and over in specifications which include the same covariates as Barro (1999).	Barro and Lee (2000) available at http://www.cid.harvard.edu/ciddata/ciddata.html
Age Structure	Data for 1950, 1955,…, 2000 from United Nations Population Division (2002). These variables are median age of the population and fraction of the population 5 different age ranges: 0 to 15, 15 to 30, 30 to 45, 45 to 60, and 60 and above.	United Nations Population Division (2003)
Male-Female Education Gap	Gap between male and female primary schooling in the population aged 25 and over. Data for 1960, 1965,…,1995 from Barro and Lee.	Barro and Lee (2000) available at http://www.cid.harvard.edu/ciddata/ciddata.html
Urbanization Rate	Percent of population living in urban areas, 0-1 scale.	World Bank (2002)
Savings Rate	Data for 1950-2000 measured as (Y-G-C)/Y from Penn World Tables 6.1 where Y is nominal income, C is nominal consumption, and G is nominal government spending.	http://pwt.econ.upenn.edu/
Labor Share	Labor share of value added from Rodrik (1999). 0-1 scale.	Rodrik (1999)

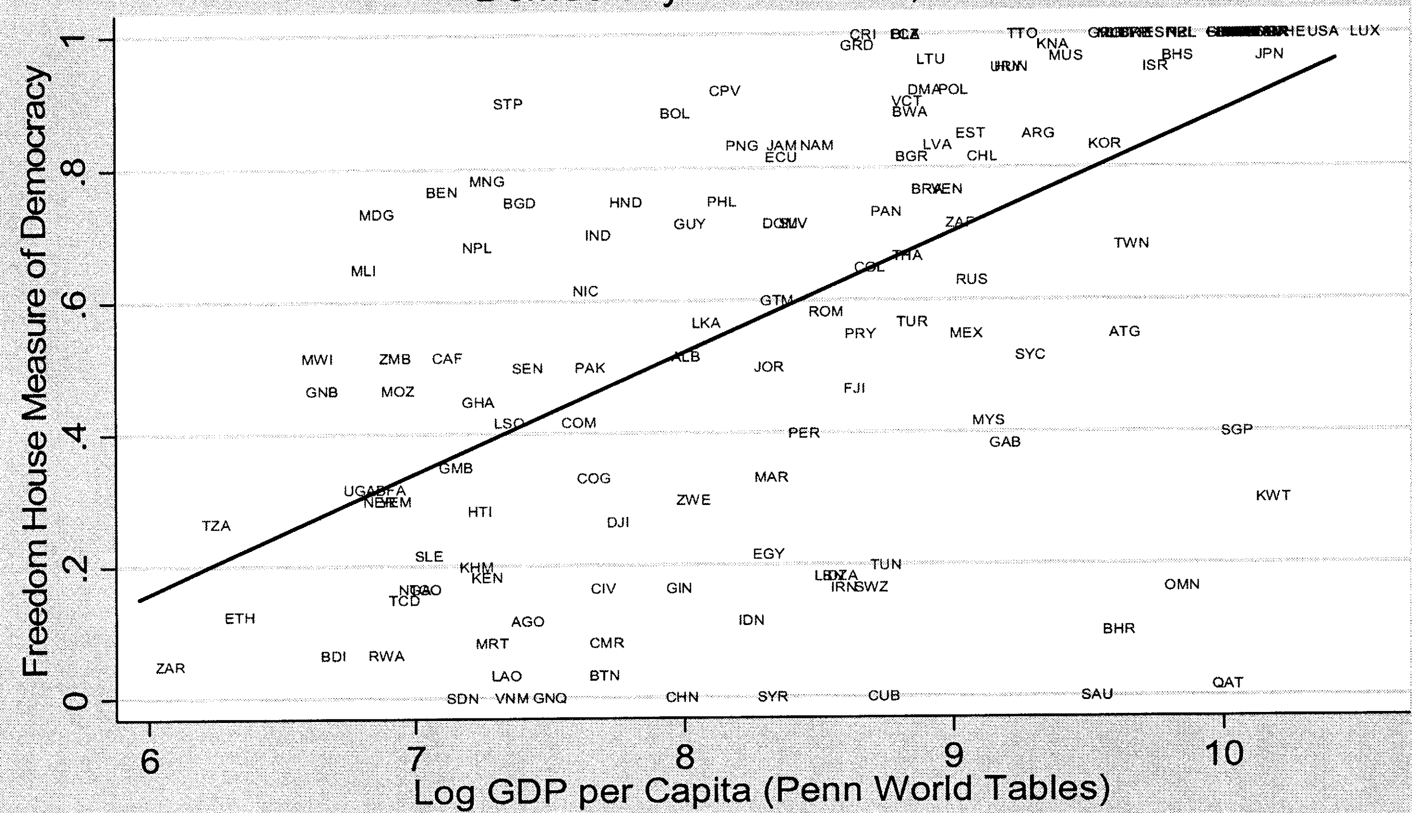
VARIABLE	DESCRIPTION	SOURCE
Trade-Weighted Log GDP	Constructed using GDP per Capita from Penn World Tables 6.1 and average trade shares between 1980 and 1989 from International Monetary Fund Direction of Trade Statistics (2005) according to procedures described in Appendix.	http://pwt.econ.upenn.edu/ and IMF DoTS CD-ROM (2005)
Trade-Weighted Democracy	Constructed using Freedom House Political Rights Index, GDP per Capita from Penn World Tables 6.1, and average trade shares between 1980 and 1989 from International Monetary Fund Direction of Trade Statistics (2005) according to procedures described in Appendix.	http://pwt.econ.upenn.edu/, IMF DoTS CD-ROM (2005), and http://www.freedomhouse.org/ratings/, and Bollen (2001) "Cross National Indicators of Liberal Democracy 1950-1990" available on ICPSR
Constraint on the Executive at Independence	Data in Polity IV, original range 1,2,3...7, normalized 0-1. Calculated as the average of constraint on the executive in a country during the first 10 years after its independence (ignoring missing data). If data for the first 10 years after independence is missing, we find the first year these data are available in Polity, then average over the following ten years (ignoring missing data).	http://www.cidcm.umd.edu/inscr/polity/
Independence year	Year when country became independent, with any year before 1800 coded as 1800. We coded Taiwan's independence year to 1948 and changed Zimbabwe's independence year to 1964. Classification of countries follows Polity.	CIA World Factbook (2004) available at http://www.cia.gov/cia/publications/factbook/
Population Density in 1500	Indigenous population divided by arable land in 1500.	Acemoglu et al (2002)
Religion	Percent of population in 1980 which is (1) Catholic, (2) Protestant, or (3) Muslim.	La Porta et al (1999)

Appendix Table 3.A2
Codes Used to Represent Countries in Figures

Country	Code	Country	Code	Country	Code
Andorra	ADO	Ghana	GHA	Netherlands	NLD
Afghanistan	AFG	Guinea	GIN	Norway	NOR
Angola	AGO	Gambia, The	GMB	Nepal	NPL
Albania	ALB	Guinea-Bissau	GNB	New Zealand	NZL
United Arab Emirates	ARE	Equatorial Guinea	GNQ	Oman	OMN
Argentina	ARG	Greece	GRC	Pakistan	PAK
Armenia	ARM	Grenada	GRD	Panama	PAN
Antigua	ATG	Guatemala	GTM	Peru	PER
Australia	AUS	Guyana	GUY	Philippines	PHL
Austria	AUT	Honduras	HND	Papua New Guinea	PNG
Azerbaijan	AZE	Croatia	HRV	Poland	POL
Burundi	BDI	Haiti	HTI	Korea, Dem. Rep.	PRK
Belgium	BEL	Hungary	HUN	Portugal	PRT
Benin	BEN	Indonesia	IDN	Paraguay	PRY
Burkina Faso	BFA	India	IND	Qatar	QAT
Bangladesh	BGD	Ireland	IRL	Romania	ROM
Bulgaria	BGR	Iran	IRN	Russia	RUS
Bahrain	BHR	Iraq	IRQ	Rwanda	RWA
Bahamas	BHS	Iceland	ISL	Saudi Arabia	SAU
Bosnia and Herzegovina	BIH	Israel	ISR	Sudan	SDN
Belarus	BLR	Italy	ITA	Senegal	SEN
Belize	BLZ	Jamaica	JAM	Singapore	SGP
Bolivia	BOL	Jordan	JOR	Solomon Islands	SLB
Brazil	BRA	Japan	JPN	Sierra Leone	SLE
Barbados	BRB	Kazakhstan	KAZ	El Salvador	SLV
Brunei	BRN	Kenya	KEN	Somalia	SOM
Bhutan	BTN	Kyrgyz Republic	KGZ	Sao Tome and Principe	STP
Botswana	BWA	Cambodia	KHM	Suriname	SUR
Central African Republic	CAF	Kiribati	KIR	Slovakia	SVK
Canada	CAN	St. Kitts and Nevis	KNA	Slovenia	SVN
Switzerland	CHE	Korea, Rep.	KOR	Sweden	SWE
Chile	CHL	Kuwait	KWT	Swaziland	SWZ
China	CHN	Lao PDR	LAO	Seychelles	SYC
Cote d'Ivoire	CIV	Lebanon	LBN	Syrian Arab Republic	SYR
Cameroon	CMR	Liberia	LBR	Chad	TCD
Congo, Rep.	COG	Libya	LBY	Togo	TGO
Colombia	COL	St. Lucia	LCA	Thailand	THA
Comoros	COM	Liechtenstein	LIE	Tajikistan	TJK
Cape Verde	CPV	Sri Lanka	LKA	Turkmenistan	TKM
Costa Rica	CRI	Lesotho	LSO	Tonga	TON
Cuba	CUB	Lithuania	LTU	Trinidad and Tobago	TTO
Cyprus	CYP	Luxembourg	LUX	Tunisia	TUN
Czech Republic	CZE	Latvia	LVA	Turkey	TUR
Germany	DEU	Morocco	MAR	Taiwan	TWN
Djibouti	DJI	Moldova	MDA	Tanzania	TZA
Dominica	DMA	Madagascar	MDG	Uganda	UGA
Denmark	DNK	Maldives	MDV	Ukraine	UKR
Dominican Republic	DOM	Mexico	MEX	Uruguay	URY
Algeria	DZA	Macedonia, FYR	MKD	United States	USA
Ecuador	ECU	Mali	MLI	Uzbekistan	UZB
Egypt, Arab Rep.	EGY	Malta	MLT	St. Vincent and the Grenadines	VCT
Eritrea	ERI	Myanmar	MMR	Venezuela, RB	VEN
Spain	ESP	Mongolia	MNG	Vietnam	VNM
Estonia	EST	Mozambique	MOZ	Vanuatu	VUT
Ethiopia	ETH	Mauritania	MRT	Western Samoa	WSM
East Timor	ETM	Mauritius	MUS	Yemen	YEM
Finland	FIN	Malawi	MWI	Yugoslavia - post 1991	YUG
Fiji	FJI	Malaysia	MYS	South Africa	ZAF
France	FRA	Namibia	NAM	Congo, Dem. Rep.	ZAR
Gabon	GAB	Niger	NER	Zambia	ZMB
United Kingdom	GBR	Nigeria	NGA	Zimbabwe	ZWE
Georgia	GEO	Nicaragua	NIC		

Figure 3.1

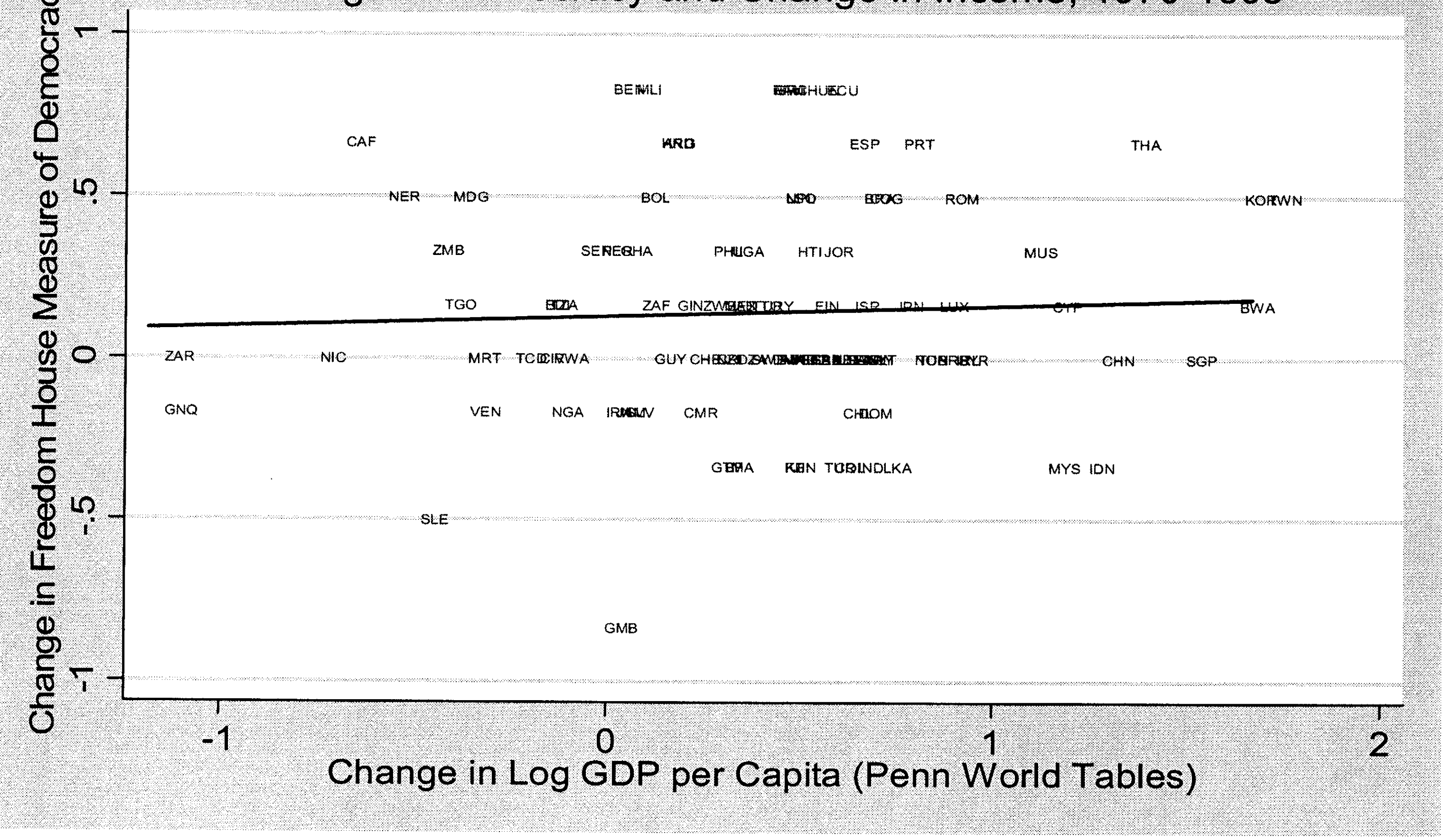
Democracy and Income, 1990s



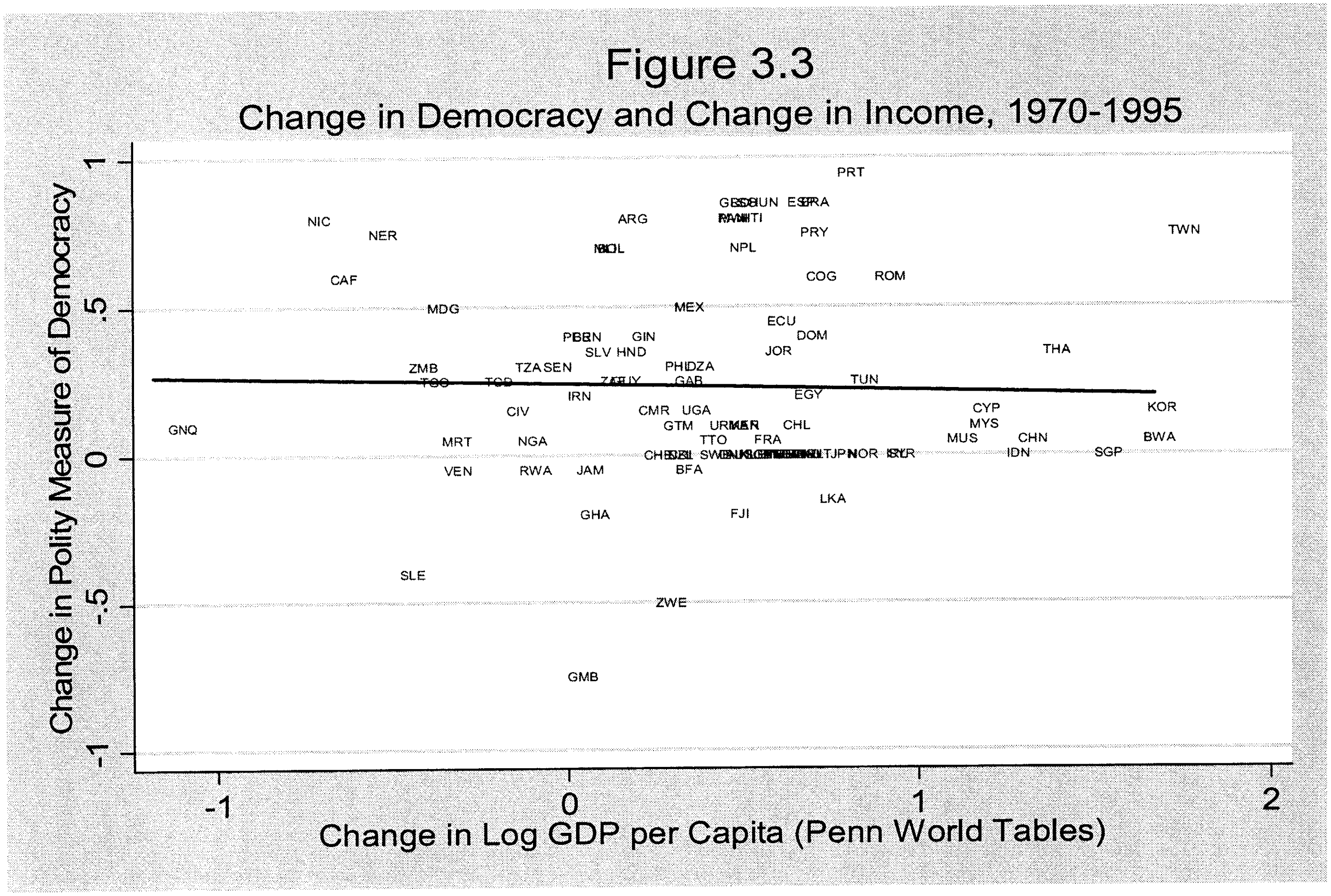
See Appendix Table 3.A1 for data definitions and sources. Values are averaged by country from 1990 to 1999. GDP per Capita is in PPP terms. The regression represented by the fitted line yields a coefficient of 0.181 (standard error=0.019), N=147, R^2=0.35.

Figure 3.2

Change in Democracy and Change in Income, 1970-1995



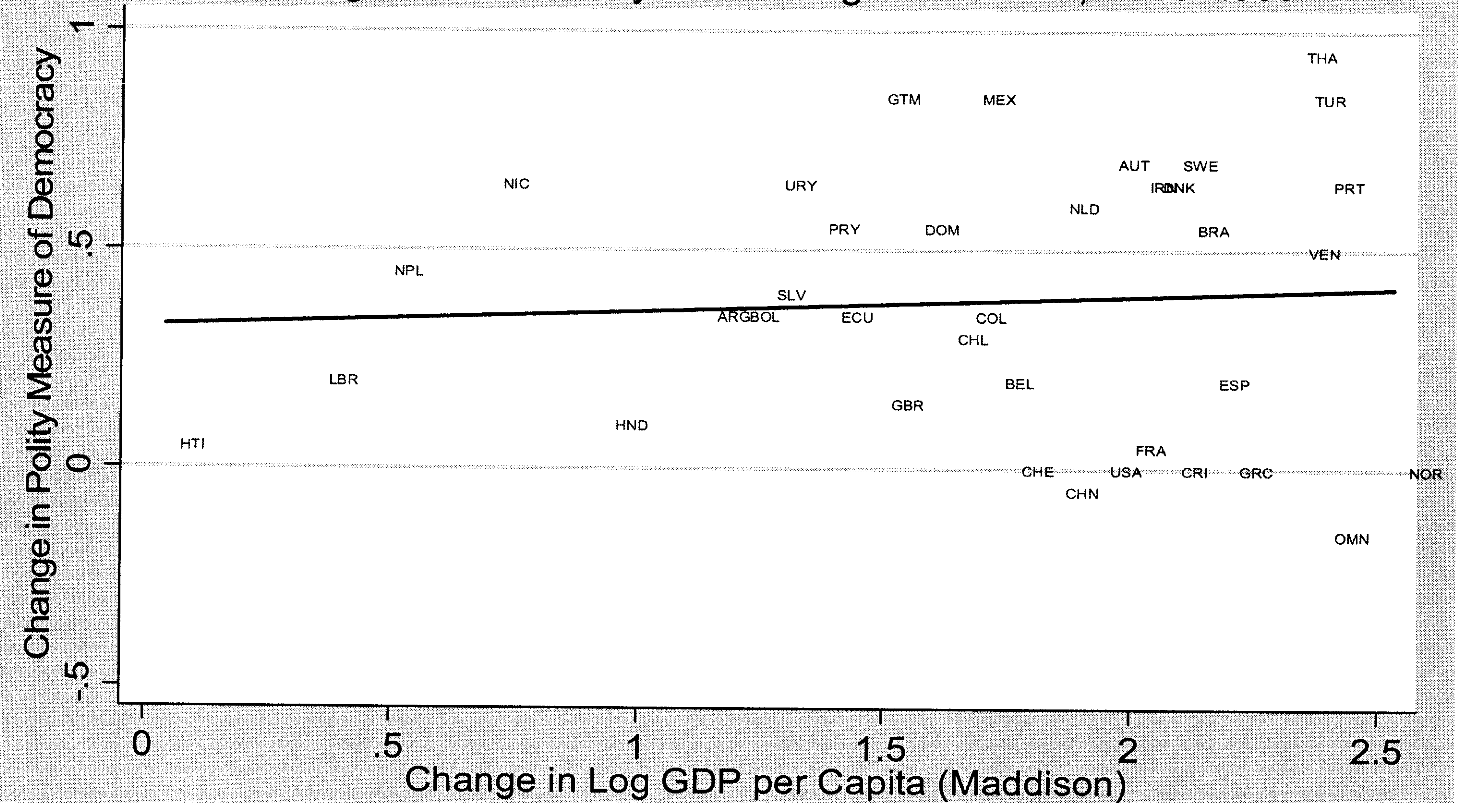
See Appendix Table 3.A1 for data definitions and sources. Changes are total difference between 1970 and 1995. Countries are included if they were independent by 1970. Start and end dates are chosen to maximize the number of countries in the cross-section. The regression represented by the fitted line yields a coefficient of 0.032 (standard error=0.058), N=102, R^2=0.00.



See notes to Figure 3.2. The regression represented by the fitted line yields a coefficient of -0.024 (standard error=0.063), N=98, R^2=0.00.

Figure 3.4

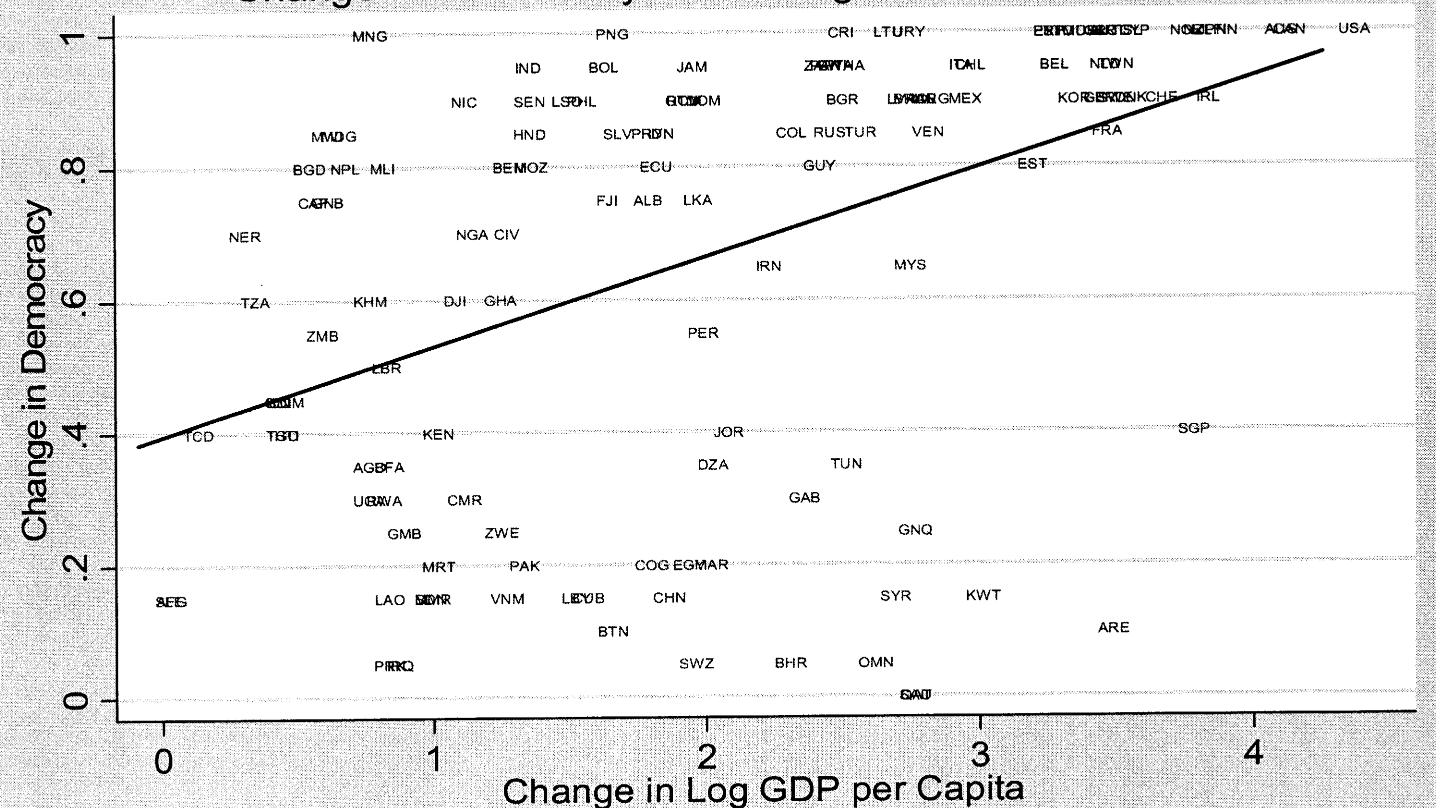
Change in Democracy and Change in Income, 1900-2000



Log GDP per Capita is from Maddison (2003). See Appendix Table 3.A1 for data definitions and sources. Changes are total difference between 1900 and 2000. Countries are included if they are in the 1900-2000 balanced 50 year panel discussed in Section 6 of the text. The regression represented by the fitted line yields a coefficient of 0.035 (standard error=0.049), N=37, R^2=0.00.

Figure 3.5

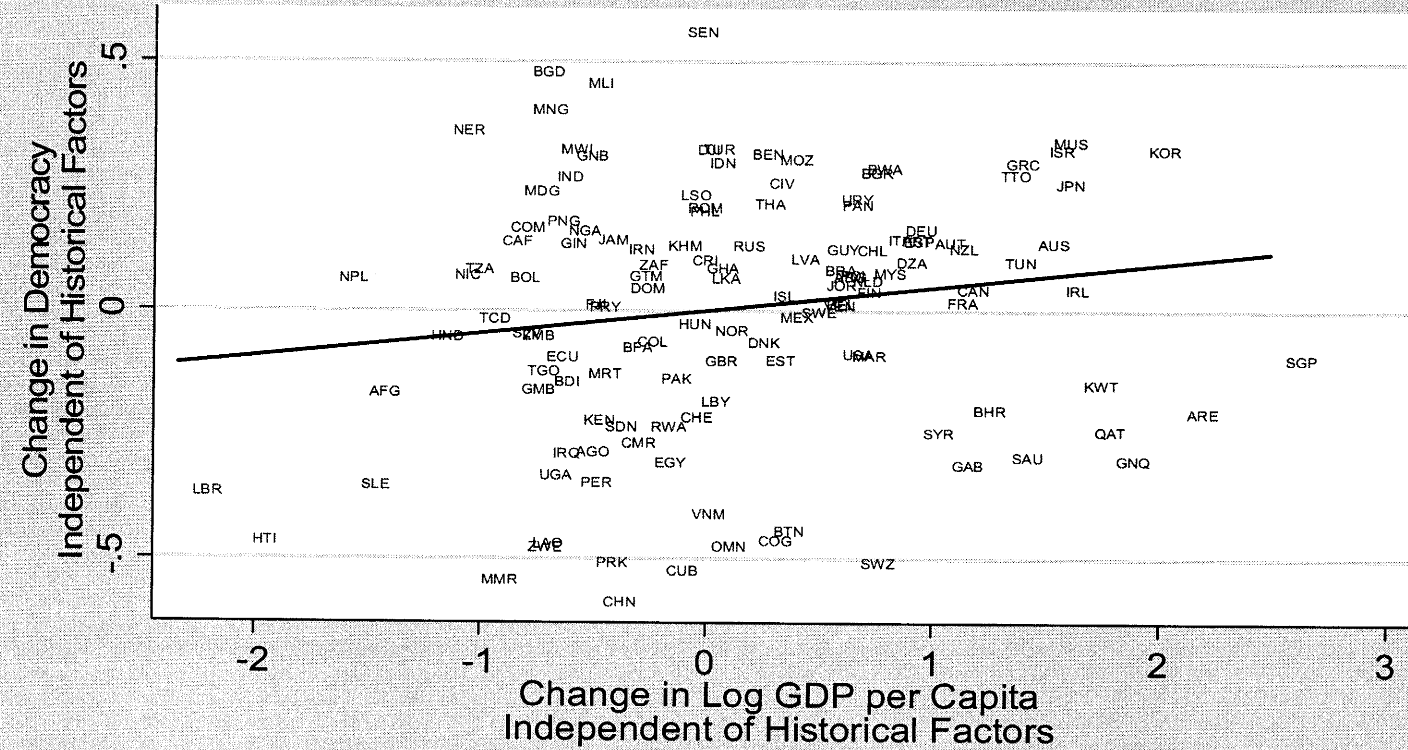
Change in Democracy and Change in Income, 1500-2000



See Appendix Table 3.A1 for data definitions and sources. Changes are total differences between 1500 and 2000. GDP per capita is from Maddison. Democracy is calculated using the Polity measure of democracy, which comprises in part constraint on the executive; data for 1500 from Acemoglu et al (2004b). The regression represented by the fitted line yields a coefficient of 0.134 (standard error=0.021), N=135, R^2=0.20, and corresponds to the specification of Table 3.8, column 1.

Figure 3.6

Change in Democracy and Change in Income, 1500-2000
Conditional on Historical Factors



See Appendix Table 3.A1 for data definitions and sources. Changes are total differences between 1500 and 2000 (see Figure 3.5 for the construction of these differences) which are not predicted in a linear regression by historical factors: Fraction Muslim, Fraction Protestant, Fraction Catholic, Constraint on the Executive at Independence, and Independence Year. This corresponds to the residual plot of the regression in Table 3.8, column 4 and it yields a coefficient of 0.047 (standard error=0.023), N=131, R^2=0.45.