3.15 Problem Set 2 Solutions
Fall, 2003

1)

a)

\[ \text{Length } \sqrt{D_p T_p} \]

assumed \( \sqrt{D_p T_p} < L \).

b) No: Excess carriers present, \( np \neq n_i^2 \).

c) \[ \frac{\partial p}{\partial x} = \frac{\partial p}{\partial t} \mid_{\text{drift}} + \frac{\partial p}{\partial t} \mid_{\text{diff}} + \frac{\partial p}{\partial t} \mid_{\text{recomb}} = 0 \]

\[ \uparrow \]

zero: \( \varepsilon = 0 \) inside.

Insider material: \( G = r n_i^2 \), \( R = r n_p = r p_e N_0 \). \( n_i^2 = p_e N_0 \).

\[ R - G = r (p_e N_0 - p_e N_0) \]

\[ = r N_0 (p_e - p_e) \]

\[ = r N_0 \Delta p_e \quad \Delta p = \text{excess # of holes.} \]

So \( 0 = \frac{1}{8} D_p \frac{d^2 p}{dx^2} - r N_0 \Delta p_e \)

assumed \( D_p = \text{const} \)

\[ \Rightarrow \text{eqn is } D_p \frac{d^2 p_e}{dx^2} = \frac{p_e}{r N_0} \frac{D_p}{D_e} \]

boundary conditions: \( p_e = 0.001 \ N_0 \) at \( x = 0, \ x = L \)

solution of the form: \( p_e = p_e(x=0) \ exp\left( -\frac{x}{\sqrt{D_e}} \right) + p_e(x=L) \ exp\left( -\frac{(L-x)}{\sqrt{D_e}} \right) \)

\[ = 0.001 \ N_0 \left( \exp\left( -\frac{2}{\sqrt{D_e}} \right) + \exp\left( -\frac{(L-2)}{\sqrt{D_e}} \right) \right) \]

valid for \( 0 \leq x \leq L \).
2) \( N_A = N_D \) so \( d_p = d_n \)

charge density

\[
\varepsilon = \frac{N_A e x}{d_p} \text{ from } x = -d_n \text{ to } x = +d_p
\]

\[
\epsilon = \frac{1}{\varepsilon_0 r} \int p(x) \, dx = \frac{1}{\varepsilon_0} \int d_n^{+} d_n^{-} \frac{e N_A x \, dx}{d_n} \uparrow \quad \text{up to } x = d_p
\]

\[
\epsilon = -\frac{e N_A}{2\varepsilon_0 r d_p} (x^2 - d_n^2) \quad \text{Parabolic}
\]

Potential \( V = -\int_{0}^{x} \epsilon(x) \, dx \)

\[
= -\int_{-d_n}^{x} \frac{e N_A}{2\varepsilon_0 r d_p} (x^2 - d_n^2) \, dx
\]

\[
= -\frac{e N_A}{2\varepsilon_0 r d_p} \left( \frac{x^3}{3} - d_n x^2 - \frac{2d_n^3}{3} \right)
\]

This is cubic.
Equilibrium: again, fluxes in deep region are equal to opposite biasing the one on left leads to a large electron current.

Bias other way: large hole diffusion current.

Characteristic shows exponential current increase in both directions.
Problem 4

(a)

I define \( p_{n,0} \) to be the expression for equilibrium hole concentration in the \( n^+ \) material. \( p_{n,0} = n_t^2/N_D \). Note that since the material is \( n^+ \), \( N_D \) is assumed to be very high (\( > 10^{19} \)), so \( p_{n,0} \) will be very small.

The peak-height is determined readily from the "law of the junction" for carrier concentration at the junction edges in non-equilibrium conditions. Law of the junction:
\[
np = n_t^2 \exp(qV_A/kT) \quad \text{[see Pierret p. 245]}
\]

So, our peak-height will be:
\[
p_n(x = 0) = p_{n,0} \exp(qV_A/kT)
\]
where \( V_A \) is the forward bias voltage being applied to the junction.

Minority carrier concentration will fall exponentially from \( p_n(x = 0) \) to \( p_{n,0} \), along the length from the depletion region edge into \( n^+ \) bulk. \( L_p \) is diffusion length of holes that characterizes that fall, defined as \( L_p = \sqrt{\tau_p D_p} \), where \( \tau_p \) is minority carrier lifetime bulk and \( D_p \) is the diffusion coefficient. From Pierret page 116, we know that a typical value for \( \tau_p \) is .5 micro-seconds. The \( n^+ \) region is heavily doped, so if we assume \( N_D \) to be equal to \( 10^{19} \), mobility (according to the plot on Pierret p. 80) would be about 70 [cm²/V·s]. \( D_p = kT/q \ast \mu \ast 0.26 \ast 70 = 1.82 \) [cm²/sec]. We can then estimate
\[
L_p = \sqrt{\tau_p \ast D_p} = 9.5 \, \mu m
\]

(b) In the ideal diode, we assume there is no voltage drop and resultantly no electric field in the bulk. Carrier movement in this region is entirely driven by diffusion, as carriers flow down the concentration gradient.

(c) \( L_p \) will decrease by a factor of \( \sqrt{10} \), causing this exponential decrease of carrier concentration in the material to occur over a shorter distance. See above plot. \[
L_p' = \frac{L_p}{\sqrt{10}}
\]

(d) The boundary condition at the end of the \( n^+ \)-side is that of an ohmic contact, which forces carrier concentrations to their equilibrium value. This will result in an approximately linear carrier concentration profile:

(e) The boundary condition at the ohmic contact forces a larger carrier-concentration gradient in the short diode, which will result in a larger current.
parts a and b (1pt)
part b is shown with dashed lines.
Assume $N_A = 10^{17} \text{ cm}^{-3}$ and $N_D = 10^{16} \text{ cm}^{-3}$.

Note that since the semiconductor has $N_A$ greater than $N_D$, $E_f$ is closer to $E_v$ in the p-region than the distance from $E_f$ to $E_c$ in the n-region.

Assuming a step junction, acceptors will leave behind immobile negative charge and donors will leave behind immobile positive charge. In the depletion region where there are no carriers to compensate for the ion charge the charge density is as shown above. Note the depletion region extends farther into the lightly doped region in order to maintain overall charge neutrality.

$$N_A dp = N_D dn$$

Electric field is obtained from the following equation:

$$E = \frac{1}{\varepsilon} \int \rho dx$$
Voltage is given by

\[ V = -\int E \, dx \]

Note that in forward bias, the depletion region width decreases. Also note that in order to apply a forward bias, the positive terminal must be connected to the p-side of the diode.

Parts c and d (1 pt total)
Very similar to the above situation. Forward bias is shown with a dashed line.

The charge density in the metal is represented as a delta function.

Note that the positive terminal must be applied to the metal side of the diode. Forward bias again decreases the depletion width.