Problem Set 4 Solutions Fall, 2003

1. a) Metal - Oxide - Semiconductor - n-type semiconductor.
   
   Accumulation
   
   Positive bias to metal ($V_g > 0$)
   
   M O S

   Depletion
   
   Negative bias to metal ($V_g < 0$)
   
   M O S

   Inversion
   
   Large negative bias to metal ($V_g < V_T$)
   
   M O S
Apply $V_G < V_T$ to obtain n-channel inversion layer. (note, $V_T$ is negative).

Then, as we apply an increasingly negative $V_D$, the current between the source & drain increases linearly, up to pinch-off (at $V_D,_{sat} = V_G - V_T$)

Then, as we apply an increasingly negative $V_G < V_T$.

C)

Symmetrically doped, there will be a single $V_T$ to induce inversion on either side.

$-V_T < V_G < 0$

$V_T > V_G > 0$

$V_G < -V_T$

$V_G > V_T$
When the MOSFET is turned on, we can model its n-channel as a resistor (for \( 0 \leq V_D \leq V_{D, on} \))

\[
\begin{array}{c}
\text{R} \\
\text{C}
\end{array}
\rightarrow
\begin{array}{c}
\text{R} \\
\text{C}
\end{array}
\]

In order to estimate how long it takes to read/write a bit,

- Assume \( N_e \) in n-channel ~ \( N_e \) in n-type material
- \( n = 10^{16} \text{ carriers/cm}^2 \) in channel
- Seeert p. 86 gives resistivity \( \rho = \frac{1}{q N_e m_e} = .5 \text{ [ohm-cm]} \)

\[ R = \frac{\rho L}{A} = \frac{.5 \cdot 10^{-\text{11}}}{(5 \cdot 10^{-16})} = 2 \text{ [k\Omega]} \]

\[ C = \frac{\varepsilon_0 \varepsilon A}{9} = \frac{8.85 \cdot 10^{-12} \cdot 4 \cdot (10^{-3})^2}{5 \cdot 10^{-12}} = 7.08 \cdot 10^{-9} \text{ [Farads]} \]

Consider the RC time constant of a closed circuit:

\[ \text{Note, } V_R = IR \text{ & } I = C \frac{dV_c}{dt} \]

\[ V_c \]

Also, in a closed ckt, \( V_R + V_c = 0 \)

\[ I \rightarrow IR + \frac{1}{C} \int I \, dt = 0 \]

The solution to this differential equation will be exponential (the exact form of which depends on the equation's boundary conditions, defined by whether we are reading or writing a bit), characterized by a time constant:

\[ \tau = RC \]

\[ = 1.42 \text{ ns} \]
b) The presence of the shaded region in the band diagram, without the application of any voltage, indicates that there will be a conducting path from source to drain via electrons.

To turn the conducting path "off," apply a negative bias to the gate, assuming the bulk is grounded.

This device will operate very quickly, as the conduction layer is being formed in lightly doped GaAs, which has a high electron mobility. Additionally, the generation process of the conducting layer will be much faster now that electrons can simply be supplied directly to the heterostructure interface from the donor-rich AlGaAs.
b.) The suggested way of looking at this problem was to realize that photocurrent is proportional to the light intensity. The intensity ratio will then just be the ratio of the short circuit currents.

\[
\frac{I_{SC,\text{ilium}}}{I_{SC,\text{exp}}} = \frac{3.1A}{0.72A} = 4.3
\]

So the sun must be around 4.3 times brighter in order to reach full illumination. This is possible during the summer months. Many of you also made the point that because of the damaged condition of the solar cell that the cell will never reach full illumination which is probably true.

c.) Maximum power is determined by the largest possible IV product.

\[
P_{\text{max}} = (IV)_{\text{max}} = 0.67A \times 18.9V = 12.7W
\]

Operation time is then simply the desired energy divided by the maximum power.

\[
t = \frac{\text{Energy}}{P_{\text{max}}} = \frac{1\text{KWhr}}{12.7W} = 79\text{hrs}
\]
We want to match the load resistance such that the voltage dropped across the load corresponds exactly to the voltage supplied by the cell. This is an impedance matching problem. Resistance is then found by taking the voltage at maximum power and dividing by the current at maximum power.

\[
R_{\text{load, opt}} = \frac{V}{I} = \frac{17.3V}{2.53A} = 6.8\Omega
\]

For comparison, let's take a 40W light bulb.

\[
P = \frac{V^2}{R} \Rightarrow R = \frac{V^2}{P} = \frac{(120V)^2}{40W} = 360\Omega
\]

The optimal load of our solar cell is much less than for a typical light bulb.

c.) Connecting the 36 cells in series or parallel will change the voltage and current applied to the load.

With the cells connected in parallel, \( P_{\text{max}} \) is the same but \( V \) is \( 1/36 \) of the original value and \( I \) is 36 times the original value.

\[
R_{\text{parallel}} = R_{\text{series}} \div (36)^2 = 5.3m\Omega
\]

In parallel the cell can be used to power low impedance devices.

f.) The difference in current is due mainly to the large difference in junction area between a diode and the solar cell. Current densities are roughly equivalent for both but the large area of the solar cell allows for a much larger current.

From relations \( V = IR \) \( \Rightarrow \) \( R = \frac{E}{I} \)

\[
\text{(a) Starting from } J_0 = q \left( \frac{D_n}{e \mu_n} \frac{P_v}{L_{p}} \right) = 2.5 \times 10^{-12} A/cm^2
\]

and assuming reasonable doping concentrations \( n_f = 10^5 \text{ cm}^{-3} \) \( \Rightarrow \) \( N_d = 10^{17} \text{ cm}^{-3} \)

and \( N_A \gg N_d \Rightarrow p \gg n_f \Rightarrow J_0 \) can be simplified to:

\[
J_0 = q \left( \frac{D_P}{e \mu_P} \frac{P_v}{L_{p}} \right) = 2.5 \times 10^{-12} A/cm^2
\]

using \( L_p = 1.0 \times 2 \) we get \( J_0 = q \left( \frac{1.0}{2 \times \mu_P} \frac{P_v}{1.0} \right) \)

using Einstein's relation we get \( J_0 = q \left( \frac{1.6 \times 10^{-15}}{1.0 \times 1.0} \frac{P_v}{1.0} \right) = \frac{2.5 \times 10^{-12}}{1.2 \times 10^{-15}} \Rightarrow 9.4 \times 10^{10} \text{ cm}^2/V\cdot s
\]

Solving for the ratio of mobility to lifetime (what were looking for), we get:

\[
\frac{\mu_P}{\tau_P} = \left( \frac{2.5 \times 10^{-12}}{1.0 \times 1.0} \right)^2 = \left( \frac{2.5 \times 10^{-12}}{1.0 \times 1.0} \right)^2 = 9.4 \times 10^{10} \text{ cm}^2/V\cdot s
\]

In microcrystalline silicon, typically \( \frac{\mu_P}{\tau_P} \approx \frac{10^{-10}}{10^{-15}} \Rightarrow 10^{-10} \text{ cm}^2/V\cdot s
\]

\( \Rightarrow \) Solar cell Si is of higher quality.
There are many possible solutions to this problem. A number of factors must be considered:

The principle of operation of a pin diode relies on a large amount of light being absorbed in the i-region, which means this region should be wider than any other layer through which light passes. However, a shorter \( W_i \) will enable charge carriers to cross the region in less time.

When used in conjunction with a resistor, an RC time constant also affects the frequency response \( \alpha W_i \). (considering the pin structure to be similar to a capacitor)

The device will operate at frequencies up to a certain cut-off, limited by these considerations.

Devices of smaller width may have trouble absorbing longer wavelengths. To compensate for this, we can employ material layers that help confine the light to the i-region by reflection.

Device doping must be arranged such that during operation, the entire i-region is depleted.

One possible solution is given as follows:
When optimally designed, a p-i-n diode will absorb as much light as possible in its completely depleted i-region so that generated carriers can be immediately accelerated to \( V_{sat} \):

\[
\frac{V}{V_{sat}} = \frac{1}{W_i/V_{sat}}
\]

Now, \( W_i \) = intrinsic layer thickness

Smaller \( W_i \) \( \Rightarrow \) faster frequency response, however if we continue to decrease \( W_i \), the frequency response will become dominated by the RC time constant of the electrical circuit:

\[
f = \frac{1}{RC} = \frac{1}{R \cdot E_r \cdot W_i / A} = \frac{W_i}{R \cdot E_r \cdot A}
\]

To find optimum intrinsic layer thickness with the given \( R \),

\[
\frac{V_{sat}}{W_i} = \frac{W_i}{R \cdot E_r \cdot A} \quad \Rightarrow \quad W_i = \frac{V_{sat} \cdot R \cdot E_r \cdot A}{10^7 \cdot 50 \cdot 8 \cdot 8 \cdot 10^{-10} \cdot 11.8 \cdot 0.1}
\]

\[
= \frac{23 \ \text{mm}}{}
\]

Note also that a wider \( W_i \) would allow more distance over which the light can be absorbed by the Si, however we can make up for this by employing an anti-reflective coating at the top of the device or a mirror at its bottom, to confine & absorb incident radiation.

Consider an optimal doping, we know that the i-region must be very lightly doped and the n+ & p+ regions must be heavily doped in order to completely deplete the i-region in equilibrium (more necessary, in reverse bias).

Let us assume a minimum achievable dopant concentration in the i-layer of \( 10^{10} \ \text{cm}^{-2} \) donors.
If we assume \( N_D = 10^{19} \, \text{cm}^{-3} \) in the \( n \)-region and \( N_A = 10^{19} \, \text{cm}^{-3} \) in the \( p \)-region, then we see that the \( i \)-region will deplete under the application of a small reverse bias. Considering the \( p^+ - i - n^+ \) junction,

\[
x_x = \sqrt{\frac{2 e_{i} E_0}{q N_D}} (V_b - V) \quad ; \quad x_d = 2.3 \, \mu \text{m} \implies V \approx 3.3 \, \text{volts}
\]

We will set the \( n^+ \) and \( p^+ \) regions to be short, to reduce light absorption, but long enough to support \( i \)-depletion. \( W_{n^+} = W_{p^+} = 1 \, \mu \text{m} \).

Neglecting absorption of incident light in the \( p^+ / n^+ \) regions, and assuming that incident photons are contained by the internally reflective layers, the wavelength range over which this device is effective is determined by the band structure. The optimum wavelength will be just lower than \( \lambda_{y} = \frac{1}{\epsilon_y} \approx 1 \, \mu \text{m} \).

![Graph of frequency response](image)

The expected frequency response will attenuate signals above the cutoff frequency \( f_{\text{max}} = \frac{V_{\text{in}}}{W_i} = 4.4 \, \text{GHz} \).

To estimate minimum detectable light intensity, note that the photogenerated current must exceed the minimum current that can be detected by whatever device we use to measure it. This will be limited by noise in the system. If we assume a typical minimum current of \( I_{\text{min}} = 10^{-7} \, \text{A} \), then the corresponding light intensity is given by

\[
10^{-7} \, \text{A} = 10^{-7} \, \text{Coulombs} \text{/ second} \quad ; \quad \eta ph = n = \frac{q}{h} \text{photons} \text{/ electron} \\
\eta ph \approx 6.25 \times 10^{-13} \, \text{photons} \text{/ electron} \quad ; \quad \text{Assume} \ E = 1.1 \, \text{eV} \text{ / photon} \\
\text{Intensity} = 6.25 \times 10^{-13} \times 1.1 \times 1.6 \times 10^{-19} = 1.1 \times 10^{-6} \, \text{Watts} \text{ / cm}^2 \text{ by conservation of energy}
\]