Identification of Convection Constants for Electronic Packages Using Modified Genetic Algorithm and Reduced-Basis Method

Zhenglin Yang, Jung Hong Lee, Gui Rong Liu, Anthony T. Patera and KhinYong Lam

***Abstract*--A new inverse analysis method is presented to identify parameters of heat convection in microelectronic packages. This approach adopts a modified Micro Genetic Algorithm (μGA) in finding the global optimum of parameters. A reduced-basis approach is introduced in the forward heat transfer analysis so as to significantly improve the efficiency in the calculation. Different identification procedures are employed to identify heat convection coefficients of a typical microelectronic package. Comparisons between different algorithms are performed. Results show that the use of the reduced-basis method together with the modified μGA outperforms the conventional GAs significantly. The presented method of coefficient identification is ideal for practical applications. It is efficient enough even for online analysis of both forward and inverse problem.**

***Index Terms*--Inverse analysis, microelectronic package, modified μGA, parameter identification, reduced-basis method.**

I. INTRODUCTION

A. Problems in Microelectronic Packages

In modern times, electronic devices are getting smaller and the operating frequencies are getting higher, leading to a higher heat generation rate. As such, it becomes increasingly more important to ensure proper thermal dissipation from the silicon die to the ambient. However, there is often not enough space to install a sophisticated cooling system, to ensure a proper thermal dissipation path to lower the PN junction temperature. Especially in mobile devices, this is the case and special design in structure and thermal system may be needed for electronic packages to meet the PN junction temperature requirement and thus to ensure the system reliability.

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Many attempts have been made to the accurately predict the PN junction temperature when the ambient or the case temperature is given. The main difficulties lie in the trade-off between the accuracy of the solution and the computational complexity. Some interesting work has been reported by Bar-Cohen [1,2], using a junction-to-case thermal resistance, R_{jc}. A lumped model based on R_{jc} with a calibrated surface temperature gives an excellent prediction of the junction temperature. This work was extended by Ortega [3] to cope with chips cooled by conduction through a PCB, when the convection heat transfer from the top surface is blocked. But as soon as convection comes into the picture, this simplified model breaks down, especially, when there is an order of difference among the convection coefficients [2].

B. Reduced-Basis Technique

In order to overcome the above mentioned shortcomings, a newly developed reduced-basis method [4] can be adopted. This method requires many finite element analyses in the development stage. Once the model is developed, the computational effort involved in obtaining the output for a different set of parameters is much lower than the full finite element analysis. This method is not only computationally inexpensive but also provides a sharp error bound [5] with respect to the full finite element analysis.

The above method is applicable for heat transfer simulation of electronic packages if the boundary conditions are given. However, the boundary heat convection coefficients are usually unknown for electronic devices because of the complex situation of its surroundings. It is also very difficult to directly measure these constants experimentally. In such cases, an inverse procedure is ideal for the identification of these constants. And, the reduced-basis method is particularly useful in reducing the computation time for forward analysis in the inverse process.

C. Inverse Analysis

Inverse analysis is a technique that can identify system parameters from known system responses. Before using inverse techniques, an objective function must be defined. The desired system parameters are then determined through minimizing this objective function. Generally, the objective functions can be defined as sum of responds difference squared between the measured value and the values computed

using a set of trial parameters. As inverse techniques develop, more and more engineering identification problems can be solved using the method. The work in this article aims to develop an inverse procedure for the identification of the boundary heat convection coefficients, using temperatures obtained at special locations in system. The objective function is defined as sum of squared differences between the temperatures obtained from experiments and the ones from trial calculations. The system parameters (the heat convection coefficients) at specific boundary can be obtained by minimizing the objective function. A genetic algorithm is used for finding the global minimum that corresponds to the parameter to be identified.

D. Micro Genetic algorithm

Genetic algorithm was introduced by Holland [6] as a method of searching for global optimum in complex systems. There are several different versions of genetic algorithms. The micro genetic algorithm (μGA) is one of the most widely used GAs. This algorithm produces fewer individuals in each generation and the individuals of each generation are created through two operations: selection and crossover. μGA is a very robust algorithm in finding the global optimum rather than local optimum for a given domain. This advantage is especially important in finding the global optimum for multi-minimum or multi-maximum problems. However, as the selection procedure is totally random, the time required to find the desired solution is usually very long. The searching time will also increase very rapidly as the number of genes in the individuals increase. It is commonly believed that the μGA is impractical for finding the global optimum for real life problems with large number of parameters, unless measures are taken to speed up the searching process.

One of the methods used for speeding up the search process of GA is the hill climbing technique [7]. This method combines the general GA global search procedure with locally optimized search using hill climbing. This local optimized technique greatly improves the local searching performance of conventional GAs. However, due to the fact that a large number of function evaluations is necessary in the local search, the method encounters difficulties for problems where variables are large and/or a single function evaluation takes considerable computational time. As most of the time is spent on function evaluation, the desired searching method should be the one that requires only a small number of function evaluations.

E. Work in this paper

In this paper, the theory of reduced-basis approach is introduced for heat transfer analysis of electronic packages. An inverse analysis with modified μGA search algorithm is then presented to reduce the required number of forward analyses. Numerical examples for identifying the heat convection constants are performed using forward solvers of both general finite element method and reduced-basis method. Advantages of the present algorithm are subsequently shown from these examples.

II. REDUCED-BASIS FORMULATIONS

The reduced-basis method is a member of the family of model reduction methods, and can perform very efficient calculations by introducing parameter space and incorporating the results of previous finite element calculations. It can be applied to a variety of linear problems. The following part describes its application in boundary value problem dealing with heat transfer.

A. Normalization and Superposition for Boundary Value Problem

When applying the reduced-basis approach to boundary value problems, it is important to normalize the given problem and use superposition. The aim of using normalization is to reduce the number of system parameters in the solution space and remove the dependency on the absolute dimensions of the problem. Usually, fewer parameters make the reduced-basis approach more efficient. Additionally, superposition can divide a complicated problem to many simplified sub-problems, further reducing the number of effective parameters. In order to show this idea, we begin with the one-dimensional formulation.

1) One-Dimensional Formulation

Consider a simple one-dimensional heat conduction bar of length L with a conductance k and a uniform heat source Q, within the bar. On the boundary, the two end points of the bar, heat convection is assumed with different coefficients h_1 and h_2, and corresponding ambient temperatures of T_1 and T_2, respectively. The boundary problem can be formulated as [8]:

$$\begin{aligned} -k\frac{d^2T}{dx^2} &= Q \qquad in \quad \Omega \\ -k\frac{dT}{dx} &= h_1(T-T_1) \qquad on \quad \Gamma_1 \\ -k\frac{dT}{dx} &= h_2(T-T_2) \qquad on \quad \Gamma_2 \end{aligned} \tag{1}$$

where the domain Ω is $[0,L]$ and the boundaries Γ_1 and Γ_2 correspond to 0 and L, respectively. For this one-dimensional problem, there are total of seven parameters, which are k, Q, L, h_1, h_2, T_1 and T_2.

a) Normalization

Let us first look at the differential equation and the boundary condition of Γ_1. The boundary is non-homogeneous due to the presence of T_1 on the right-hand side of Eq. (1). We can change this boundary condition into a homogeneous one by defining $\theta' = T - T_1$. In addition, a normalized variable $x^* = x/L$ is defined, leading to

$$-k\frac{d\theta'}{d(x^*)L} = h_1\theta' \qquad on \quad \Gamma_1 \tag{2}$$

Introducing the Biot number, $Bi_i = h_iL/k$ for $i=1,2$, we get

$$\frac{d\theta'}{dx^*} + Bi_1\theta' = 0 \qquad on \quad \Gamma_1 \tag{3}$$

The normalized governing equation thus becomes:

$$-k\frac{d^2\theta'}{d(x^*)^2L^2} = Q \tag{4}$$

Assuming $\theta''=\theta'/L$, yields

$$\begin{aligned} &-k\frac{d^2\theta''}{d(x^*)^2}=Q \qquad in \quad \Omega^* \\ &\frac{d\theta''}{dx^*}+Bi_1\theta''=0 \qquad on \quad \Gamma_1^* \end{aligned} \tag{5}$$

where Ω^*=[0,1] and $\Gamma_1{}^*$ are the new normalized domain and the corresponding new boundary, respectively. Finally, defining $\theta=k\frac{\theta^*}{Q}=k\frac{T-T_1}{L^2Q}$, the equations for the problem become

$$\begin{aligned} &-\frac{d^2\theta}{d(x^*)^2}=1 \qquad in \quad \Omega^* \\ &\frac{d\theta}{dx^*}+Bi_1\theta=0 \qquad on \quad \Gamma_1^* \\ &\frac{d\theta}{dx^*}+Bi_2\theta=Bi_2\delta_2 \qquad on \quad \Gamma_2^* \end{aligned} \tag{6}$$

where $\delta_j=k\frac{T_j-T_1}{L^2Q}$. In this final form, there remain only 3 independent parameters, Bi_1, Bi_2 and δ_2.

b. Application of Linear Superposition

The above formulation shows that certain parameter dependencies can be removed using the normalization technique. Additionally, linear superposition enables further reduction in the dependency on the right-hand side parameters, which divides original complex problem into simpler sub-problems.

If θ_Γ is assumed to be the solution for the problem when the $Bi_2\delta_2=1$ and $Q=0$, it holds that

$$\begin{aligned} &-\frac{d^2\theta_\Gamma}{d(x^*)^2}=0 \qquad in \quad \Omega^* \\ &\frac{d\theta_\Gamma}{dx^*}+Bi_1\theta_\Gamma=0 \qquad on \quad \Gamma_1^* \\ &\frac{d\theta_\Gamma}{dx^*}+Bi_2\theta_\Gamma=1 \qquad on \quad \Gamma_2^* \end{aligned} \tag{7}$$

Thus, the general solution for arbitrary $Bi_2\delta_2$ can be obtained as $(Bi_2\delta_2)\theta_\Gamma$. Similarly, assuming θ_Ω to be the solution for the following problem,

$$\begin{aligned} &-\frac{d^2\theta_\Omega}{d(x^*)^2}=1 \qquad in \quad \Omega^* \\ &\frac{d\theta_\Omega}{dx^*}+Bi_1\theta_\Omega=0 \qquad on \quad \Gamma_1^* \\ &\frac{d\theta_\Omega}{dx^*}+Bi_2\theta_\Omega=0 \qquad on \quad \Gamma_2^* \end{aligned} \tag{8}$$

Then, the solution for original problem (6) can be obtained from sub-problems (7) and (8) by direct substitution

$$\theta=\theta_\Omega+(Bi_2\delta_2)\theta_\Gamma \tag{9}$$

Once the values of θ_Ω and θ_Γ, which depend only on Bi_1 and Bi_2, are found, the remaining process is computationally cheap. Therefore, the number of parameters in the new system is further reduced to two: Bi_1 and Bi_2.

2) Three-Dimensional Formulation

Now, considering a three-dimensional domain Ω. Assume it consists of α sub-domains, Ω_i, $i=1,\ldots,\alpha$. In each sub-domain, there is a constant constitutive matrix $\mathbf{D}_i=\left[k^i_{p,q}\right]$ and a uniform heat generation constant Q_i. On the boundary of Ω, which is denoted as Γ, there are β different boundary conditions Γ_j, $j=1,\ldots,\beta$. Each sub-boundary Γ_j is specified by three constant parameters h_j, $\mathbf{D}_{c(j)}$ and T_j. Here, *c(j)* stands for a mapping from set $\{1,\ldots,\beta\}$ to set $\{1,\ldots,\alpha\}$. Therefore, the governing equation for above problem can be stated as [8]

$$\begin{aligned} &\nabla\cdot(\mathbf{D}_i\nabla T)+Q_i=0 \qquad in \quad \Omega_i \quad i=1\ldots\alpha, \\ &\left(-\mathbf{D}_{c(j)}\nabla T\right)\cdot\hat{\mathbf{n}}=h_j(T-T_j) \qquad on \quad \Gamma_j \quad i=1\ldots\beta \end{aligned} \tag{10}$$

a. Normalization

Assuming the normalization of the three-dimensional problem is performed with respect to the direction x. The dimensions in all directions are normalized with L_x, where L_x is usually taken to be the maximum dimensions of the problem in x direction. The transformation between the original coordinate system x and the normalized system x^* is

$$x^*=\mathbf{L}_1^{-1}x \qquad where \quad \mathbf{L}_1=L_x\mathbf{I} \tag{11}$$

For the gradient operator, we have

$$\nabla^*=\left(\frac{\partial}{\partial x^*},\frac{\partial}{\partial y^*},\frac{\partial}{\partial z^*}\right)=\mathbf{L}_1\nabla \tag{12}$$

Then the governing equation becomes

$$\nabla^*\cdot[(\mathbf{L}_1^{-1})^T\frac{\mathbf{D}_i}{k_1}\mathbf{L}_1^{-1}\nabla^*\frac{T-T_1}{Q_1}k_1]+\frac{Q_i}{Q_1}=0 \tag{13}$$

Define $\mathbf{\Lambda}_i=\mathbf{D}_i/k_1$ and $\sigma_i=Q_i/Q_1$, and it leads to

$$\nabla^*\cdot[(\mathbf{L}_1^{-1})^T\mathbf{\Lambda}_i\mathbf{L}_1^{-1}\nabla^*\frac{T-T_1}{Q_1}k_1]+\sigma_i=0 \tag{14}$$

In order to make the equation independent from the absolute dimensions of the problem, each $\mathbf{L}_1^{-1}$ is multiplied by L_x (which leads to identity matrix $\mathbf{I}$), and Eq.(14) becomes

$$\nabla^*\cdot[\mathbf{\Lambda}_i\nabla^*\frac{T-T_1}{L_x^2Q_1}k_1]+\sigma_i=0 \tag{15}$$

or

$$\nabla^*\cdot\left(\mathbf{\Lambda}_i\nabla^*\theta\right)+\sigma_i=0 \tag{16}$$

where $\theta=\frac{T-T_1}{L_x^2Q_1}k_1$ is the normalized temperature.

For Robin conditions, using a similar transformation, the normal vector can be expressed as

$$\hat{\mathbf{n}}=L_x\mathbf{L}_1^{-1}\mathbf{n}^*=\mathbf{n}^* \tag{17}$$

The boundary condition can be transformed as

$$(-\mathbf{\Lambda}_{c(j)}\nabla^*\theta)\cdot\mathbf{n}^*=Bi_j(\theta-\delta_j) \tag{18}$$

Thus, the normalized three-dimensional problem formulation becomes

$$\begin{aligned} &\nabla^*\cdot\left(\mathbf{\Lambda}_i\nabla^*\theta\right)+\sigma_i=0 \qquad in \quad \Omega_i^*, \quad i=1,\ldots,\alpha \\ &(-\mathbf{\Lambda}_{c(j)}\nabla^*\theta)\cdot\mathbf{n}^*=Bi_j(\theta-\delta_j) \qquad on \quad \Gamma_j^*, \quad j=1,\ldots,\beta \end{aligned} \tag{19}$$

In this case, $\Omega^*=\sum_{i=1}^{\alpha}\Omega_i^*$ is the normalized domain with the normalized boundary $\Gamma^*=\sum_{j=1}^{\beta}\Gamma_j^*$.

b. Application of linear superposition

As in the one-dimensional case, linear superposition is applied to the normalized problem described in Eq.(19). Define $\theta_{\Omega_p^*}$ for $p=1,\ldots\alpha$ and $\theta_{\Gamma_q^*}$ for q=1,…,β as the solutions for the following sub-problems:

$$\nabla^*\cdot\left(-\boldsymbol{\Lambda}\nabla^*\theta_{\Omega_p^*}\right)=\begin{cases}1 & in\quad \Omega_p^*\\ 0 & otherwise\end{cases}$$

$$(\boldsymbol{\Lambda}\nabla^*\theta_{\Omega_p^*})\cdot\mathbf{n}^*+Bi\theta_{\Omega_p^*}=0 \qquad on\quad \Gamma^* \tag{20}$$

and

$$\nabla^*\cdot\left(-\boldsymbol{\Lambda}\nabla^*\theta_{\Gamma_q^*}\right)=0 \qquad in\quad \Omega^*$$

$$(\boldsymbol{\Lambda}\nabla^*\theta_{\Gamma_q^*})\cdot\mathbf{n}^*+Bi\theta_{\Gamma_q^*}=\begin{cases}1 & \text{on } \Gamma_q^*\\ 0 & otherwise\end{cases} \tag{21}$$

Then the solution for the original problem (19) is

$$\hat{\boldsymbol{J}}(k)\in R^p+\sum_{q=1}^{\beta}(Bi_q\delta_q)\theta_{\Gamma_q^*} \tag{22}$$

When $\frac{\partial\theta_i}{\partial n^*}=0$ on Γ^* and $\theta_{\Gamma_q^*}$ are obtained, the solution for the original problem can be easily obtained from Eq.(22). Hence, the effective parameters are only the Biot numbers.

B. Finite Element Formulations

The normalized boundary value problem in Eq.(19) can be expressed in the following form

$$\begin{aligned}&\nabla^*\cdot\left(-\boldsymbol{\Lambda}\nabla^*\theta\right)=\sigma \qquad in\quad \Omega^*\\ &(-\boldsymbol{\Lambda}_c\nabla^*\theta)\cdot\mathbf{n}^*=Bi(\theta-\delta) \qquad on\quad \Gamma^*\end{aligned} \tag{23}$$

For the finite element analysis, the corresponding weak form function for the above problem is

$$J(\theta)=\frac{1}{2}\int_{\Omega^*}\left(\nabla^*\theta\right)^T\cdot\left(\boldsymbol{\Lambda}\nabla^*\theta\right)dV+\int_{\Gamma^*}Bi\cdot\theta^2dS-\int_{\Omega^*}\sigma\theta dV-\int_{\Gamma^*}Bi\delta\cdot\theta dS \tag{24}$$

To get the solution of this problem (23), θ should lead to the minimization of function *J(θ)*. That is

$$\theta=\min J(\theta) \tag{25}$$

In the finite element method (FEM), a set of nodal based piecewise linear functions are used as basis functions:

$$\boldsymbol{\Phi}\left(x^*\right)=\left[\phi_1(x^*),...,\phi_N(x^*)\right] \tag{26}$$

which leads to

$$\theta=\boldsymbol{\Phi}(x^*)\{\theta\}^{node} \tag{27}$$

Here, *N* is the number of dimensions for the solution space and $\{\theta\}^{node}$ is the vector of nodal values.

Through standard FEM procedures, the linear equations can finally be obtained:

$$[K]\{\theta\}^{node}=\{F\} \tag{28}$$

If the domain can be separated to α sub-domains and the boundary consists of β sub-boundaries, then

$$\begin{aligned}[K]&=\sum_{i=1}^{\alpha}\left[K_{\Omega_i^*}\right]+\sum_{j=1}^{\beta}Bi_j\left[K_{\Gamma_j^*}\right]\\ \{F\}&=\sum_{i=1}^{\alpha}\sigma_i\left\{F_{\Omega_i^*}\right\}+\sum_{j=1}^{\beta}Bi_j\delta_j\left\{F_{\Gamma_j^*}\right\}\end{aligned} \tag{29}$$

where, $\left[K_{\Omega_i^*}\right]$ and $\left[K_{\Gamma_j^*}\right]$ are matrices of the volume integration for the *i*th sub-domain and the boundary integration for the *j*th sub-boundary, respectively. $\left\{F_{\Omega_i^*}\right\}$ and $\left\{F_{\Gamma_j^*}\right\}$ are vectors of the volume integration for the *i*th sub-domain and the boundary integration for the *j*th sub-boundary, respectively.

Solving linear equation (28), the normalized value at nodes $\{\theta\}^{node}$ can be obtained.

C. Reduced-basis simulation

Through previous normalization and superposition procedures, the reduced-basis method can then be easily performed as follows.

1) Equations

The first step of the method is to choose a set of parameters

$$\mathbf{M}=\left\{\mu^1,...,\mu^n\right\} \tag{30}$$

where $n<<N$ and μ^k $(k=1,...,n)$ is consisted of independent constants in the problem. For each $\mu^k\in\mathbf{M}$, has all the matrices and constants of $\boldsymbol{\Lambda}_i$ and σ_i for $i=1,...,\alpha$, and, Bi_j and δ_j, $j=1,...,\beta$. Solving Eq.(28), we can obtain the solution vector $\{\theta\}^{node}$.

Based on the sample solutions vectors, the reduced space is defined as

$$\mathbf{X_r}=span\left\{\theta(x^*;\mu^1),...,\theta(x^*;\mu^n)\right\} \tag{31}$$

and the corresponding matrix is

$$\boldsymbol{\Phi}_\mathbf{r}(x^*)=\left[\theta(x^*;\mu^1),...,\theta(x^*;\mu^n)\right] \tag{32}$$

This is related to the FEM matrix $\boldsymbol{\Phi}(x^*)$ through

$$\boldsymbol{\Phi}_\mathbf{r}(x^*)=\boldsymbol{\Phi}(x^*)[Z] \tag{33}$$

where, $[Z]=\left[\left\{\theta(\mu^1)\right\}^{node},...,\left\{\theta(\mu^n)\right\}^{node}\right]$ is an *N*-by-*n* FEM solution matrix. As a result, any $\theta_r\in\mathbf{X_r}$ can be written as

$$\theta_r=\boldsymbol{\Phi}_\mathbf{r}\{\alpha\}_r \tag{34}$$

where, $\{\alpha\}_r$ is an *n*-dimensional reduced solution vector.

Using the weak form in (24) over the reduced space $\mathbf{X_r}$, we can obtain equations similar to FEM as:

$$[K_r]\{\alpha\}_r=\{F_r\} \tag{35}$$

where

$$\begin{aligned}[K_r]&=[Z]^T[K][Z]\\ \{F_r\}&=[Z]^T\{F\}\end{aligned} \tag{36}$$

Here, $[K_r]$ is *n*-by-*n* matrix and $\{F_r\}$ is an *n* dimensional vector.

2) Decomposition

Assuming the original material is isotropic and one reference is used for normalization, Eq.(36) can be decomposed into

$$[K_r]=\sum_{i=1}^{\alpha}\kappa_i\left[K_{r,\Omega_i^*}\right]+\sum_{j=1}^{\beta}Bi_j\left[K_{r,\Gamma_j^*}\right] \tag{37}$$

$$\{F_r\}=\sum_{i=1}^{\alpha}\sigma_i\left\{F_{r,\Omega_i^*}\right\}+\sum_{j=1}^{\beta}Bi_j\delta_j\left\{F_{r,\Gamma_j^*}\right\} \tag{38}$$

where

$$\left[K_{r,\Omega_i^*}\right]=[Z]^T\left[K_{\Omega_i^*}\right][Z] \tag{39}$$

$$\left\{F_{r,\Omega_i^*}\right\}=[Z]^T\left\{F_{\Omega_i^*}\right\} \tag{40}$$

$$\left[K_{r,\Gamma_j^*}\right]=[Z]^T\left[K_{\Gamma_j^*}\right][Z] \tag{41}$$

$$\left\{F_{r,\Gamma_j^*}\right\}=[Z]^T\left\{F_{\Gamma_j^*}\right\} \tag{42}$$

Therefore, if the decomposition is possible, we can pre-

calculate $\left[K_{r,\Omega_i^*}\right]$, $\left\{F_{r,\Omega_i^*}\right\}$, $\left[K_{r,\Gamma_j^*}\right]$, and $\left\{F_{r,\Gamma_j^*}\right\}$, and then assemble $[K_r]$ and $\{F_r\}$ easily for different set of parameters. Additionally, these matrices and vectors are small in size, making them easier to store and solve. Thus, this approach is ideal for on-line calculation.

In reduced-basis calculation, the whole solving procedure consists of two stages:

- OFF-LINE STAGE:
  a. Choose sample parameter set **M**,
  b. Construct $[Z]$
  c. Construct $p=2$, $\left\{F_{r,\Omega_i^*}\right\}$, $\left[K_{r,\Gamma_j^*}\right]$ and $\left\{F_{r,\Gamma_j^*}\right\}$
- ON-LINE STAGE
  a. Form $[K_r]$, $\{F_r\}$
  b. Solve $[K_r]\{\alpha\}_r = \{F_r\}$
  c. Evaluate $\theta_r = \mathbf{\Phi_r}\{\alpha\}_r$
  d. Use superposition to all the sub-problems and get the final results

The idea is that the off-line stage is done once, generating a small data file with $[Z]$,$\left[K_{r,\Omega_i^*}\right]$, $\left\{F_{r,\Omega_i^*}\right\}$, $\left[K_{r,\Gamma_j^*}\right]$ and $\left\{F_{r,\Gamma_j^*}\right\}$; the online stage then accesses this data file to provide a quick response to a specific parameter μ.

III. INVERSE ANALYSIS

If the boundary condition is given, the above mentioned reduced-basis method can solve temperature boundary value problems very rapidly. However, in some cases, the boundary conditions are very complicated and cannot be detected directly through experiments. Inverse analysis is usually used in these cases for identifying some of the boundary conditions.

A. Fitness Function

The fitness function of the present inverse problem is defined as sum of squared difference between sample node temperatures obtained from measurements and those calculated from trial system parameters, which can be expressed as:

$$\varepsilon = \sum_{i=1}^{m}\left\{T_i - T_i^0\right\}^2 \tag{43}$$

where T_i^0 and T_i are measured and calculated temperature in the i^{th} node, respectively; m is the number of sample nodes considered. The estimated boundary heat convection constants are updated at each trial calculation. And when the desired boundary heat convection coefficients are found, the fitness function will reach its minimum.

For the forward calculations, any numerical method that evaluates the temperature field can be adopted. In order to show the efficiency of the present method, the finite element method and reduced-basis method will be used in the forward calculations.

B. Modified Micro Genetic Algorithm

The procedure of minimizing the fitness function in an inverse analysis is equivalent to the procedure finding the desirable system parameters. Generally, the ranges of parameters are given prior to analysis and the searching procedures are randomly chosen. Genetic algorithms (GAs) are very efficient algorithms for this purpose. But, as the individuals (a set of system parameters) in each generation are created at random, the time spent by general GAs to find the desired solutions is usually large.

Micro genetic algorithm (μGA) is one of the GAs that are widely used in engineering problems, but it still has the same advantages and disadvantages as other GAs. To overcome its shortcomings, a novel modified μGA with local and global optimized searching techniques will be presented in the following part.

1) Local Search

During the local search procedure, a Pattern-move direct search strategy is adapted. Suppose p_j and p_{j-1} are the two best individuals in the j^{th} and $(j-1)^{\text{th}}$ generation, respectively. In order to find out better individuals, two individuals, c_1 and c_2 , will be generated through the forward and internal interpolations, respectively. This can be expressed in the following equations:

$$\begin{aligned} c_1 &= p_j + \alpha(p_j - p_{j-1}) \\ c_2 &= p_{j-1} + \beta(p_j - p_{j-1}) \end{aligned} \tag{44}$$

where α and β are two nonnegative decimals whose values can be changed to adjust the distances between these new individuals and original individuals p_j and p_{j-1}. To get stable convergence, generally, the ranges of these parameters are: [0, 1.0]. For simplicity, in the rest of this paper, we fix the values of α and β to be 0.2 and 0.5, respectively.

2) Global Search

In global search, a pre-treatment procedure is made to ensure random selection of individuals that did not occur previously. Therefore, the domain of candidate individuals for random searching in this method becomes progressively smaller as the searching goes on. Hence, the modified μGA *greatly cuts down* the number of evaluations.

IV. EXAMPLE RESULTS AND DISCUSSION

This section describes an example of identifying the boundary heat convection coefficients in microelectronic packages. An inverse analysis with two kinds of forward temperature analysis methods (FEM and reduced-basis method) and two kinds of searching algorithms (conventional μGA and modified μGA) are used in the following calculation. And, results are compared among the following four cases:

- Conventional μGA combined with conventional finite element method using ABACUS
- Modified μGA plus conventional finite element method using ABACUS
- Conventional μGA together with the introduced reduced-basis method

Fig. 1. A simplified internal structure of electronic packages

- Modified μGA combined with the same reduced-basis method.

For the example, a 20-pin plastic lead chip carrier (PLCC) is selected [9]. The internal structure is given in Fig. 1, which is a simplified version to make the meshing process easier, but it still captures the essential structure of a microelectronic package. In this structure, the outermost part with pins models the lead frame, and the innermost part stands for the silicon die. The part between the two is called the die attach, and provides mechanical support to the die and also acts as a heat spreader for high power devices. The gap between the lead frame and the die attach is filled with molding material and some bonding wires. For our model, 0.5W heat generation is assumed in the silicon die, with SUMITOMO 1033D die attach, 1Zr/99Cu lead frame and SUMITOMO EME-6600CS

TABLE I
HEAT CONDUCTION CONSTANTS IN OUR ELECTRONIC PACKAGE

Parts	Silicon die	Die attach	Molding material	Lead Frame
Materials	Silicon	SUMITOMO 1033D	SUMITOMO EME-6600CS	1Z/99Cu
Values (W/mK)	153	1.5	0.8	380

for the molding material [9]. The corresponding heat conduction constants are listed in table I.

In order to simplify our method, different constant boundary temperatures and heat convection constants are assumed on the

TABLE II
BOUNDARY CONDITION AND HEAT GENERATION IN OUR STUDY

Items	Boundary heat convection coefficients (W/m^2K)			Boundary environment temperature (^{0}C)			Silicon die heat generation (W)
Marks	h_{top}	h_{side}	h_{btm}	T_{top}	T_{side}	T_{btm}	$Q_{silicon}$
Values	10	7	2	25	130	50	0.5

top, bottom and four sides of the chip. They are listed in table II.

Also, assuming that all the materials are isotropic, our normalized model with respect to k_{die}, T_{top} and L_x has $\boldsymbol{\mu}=\{\kappa_{att}, \kappa_{mold}, \kappa_{lead}, Bi_{top}, Bi_{side}, Bi_{btm}\}$ as parameters and $\delta=\{\delta_{side}, \delta_{btm}\}$ as coefficients for superposition. In this example $\mu=\{9.8E\text{-}3, 5.23E\text{-}3, 2.48, 3.25E\text{-}4, 2.28E\text{-}4, 6.51E\text{-}5\}$ and $\delta=\{0.38,1.91\}$.

In the forward analysis, the original heat transfer problem can be divided into three smaller sub-problems:

$$-\kappa\nabla^{*2}\theta_{die}=\begin{cases}1 & in\quad \Omega^*_{die}\\ 0 & otherwise\end{cases}$$
$$(\kappa\nabla^*\theta_{die})\cdot\mathbf{n}^*+Bi\theta_{die}=0 \quad on\quad \Gamma^* \tag{45}$$

$$-\kappa\nabla^{*2}\theta_{side}=0 \quad in\quad \Omega^*$$
$$(\kappa\nabla^*\theta_{side})\cdot\mathbf{n}^*+Bi\theta_{side}=\begin{cases}1 & on\quad \Gamma^*_{side}\\ 0 & otherwise\end{cases} \tag{46}$$

and

$$-\kappa\nabla^{*2}\theta_{btm}=0 \quad in\quad \Omega^*$$
$$(\kappa\nabla^*\theta_{btm})\cdot\mathbf{n}^*+Bi\theta_{btm}=\begin{cases}1 & on\quad \Gamma^*_{btm}\\ 0 & otherwise\end{cases} \tag{47}$$

Thus, the normalized solution for the original problem is

$$\theta=\theta_{die}+(Bi_{side}\delta_{side})\theta_{side}+(Bi_{btm}\delta_{btm})\theta_{btm} \tag{48}$$

and the actual temperature can be obtained from

$$T=\frac{L_x^{\,2}Q_{die}}{k_{die}}\theta+T_{top} \tag{49}$$

During each forward simulation, one-fourth of the 8.97mm-by-8.97mm device is meshed by introducing the symmetry of the problem. The mesh is done in MSC/PATRAN software, resulting in 5406 nodes and 4322 elements. To see the validity of the reduced-basis method, both the original FEM model and the reduced-basis model are simulated. The result of temperature field for the finite element model is shown in Fig. 2. Comparing the temperature distribution from the two cases,

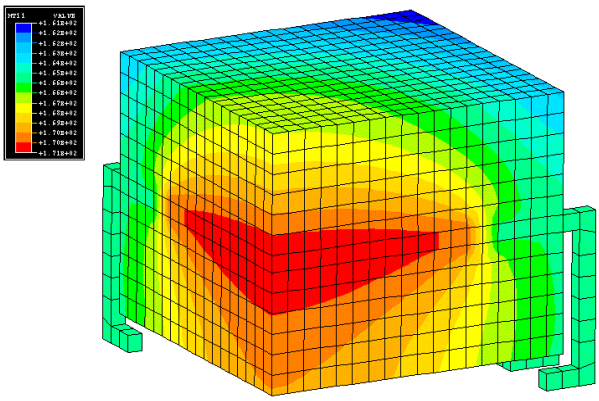

Fig. 2. Temperature field in electronic chip calculated by ABAQUS software

less than 3% difference can be observed. This shows the effectiveness of the reduced-basis method.

For the physical measurements, the temperature of ten sample nodes on the top surface of the device is taken into considerations. In the present calculation, we assume that all the constants except the bottom heat convection constant are known. The bottom convection constant is identified with a domain range of [0,10], which means the trial heat convection boundary condition can be changed from isolated to the same boundary condition that applies for the top surface. The four aforementioned calculation schemes are used, and the same HP-workstation is used for all the calculations. The time for forward evaluations and the time to achieve convergence for the cases are listed in table III, and the searching procedures

TABLE III
RESULTS OF DIFFERENT METHODS IN IDENTIFYING THE HEAT CONVECTION COEFFICIENTS

Cases for study	Case one	Case two	Case three	Case four
Time for one step forward calculation (seconds)	24.8	24.8	0.96	0.96
Generation for convergence (Generations)	64	5	40	5
Time for convergence (minutes)	107	11	3	0.5

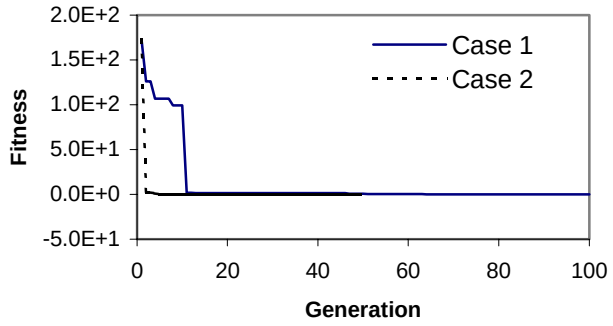

Fig. 3. Boundary heat convection coefficient identifying procedure for case one and two

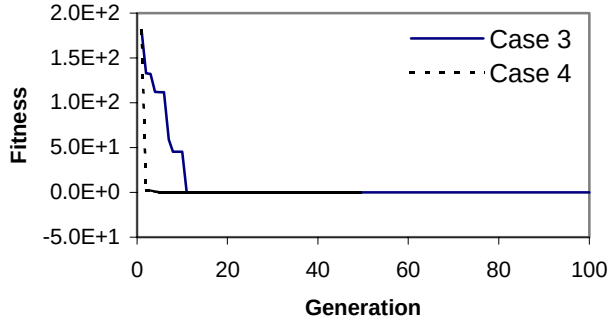

Fig. 4. Boundary heat convection coefficient identifying procedure for case three and four

are shown in Fig. 3 and Fig.4.

The time for a single forward calculation is greatly reduced from 24.8 seconds to 0.96 seconds by using the reduced-basis method instead of general finite element method. This indicates that reduced-basis is an appropriate forward calculation scheme. By using the optimized μGA, the number of generations necessary to converge is greatly reduced, compared to conventional μGA, the generation is shortened from 64 generations to 5 generations. Finally, if inverse analysis of μGA and reduced-basis is used, the time spent for finding the true convection constant can be largely reduced from 107 minutes to 0.5 minutes compared to the method of conventional μGA and finite element method. In this case, real-time and/or online identification can be properly performed.

V. CONCLUSIONS

From the above analyses, the following conclusions can be drawn:

- Heat transfer boundary value problem can be normalized and reduced-basis technique can be used for temperature calculation.
- The reduced-basis method is an approximation method which can greatly save calculation time compared to conventional finite element method,
- The modified *μGA* can greatly shorten the time required by the searching procedure due to its local and global optimization.
- Boundary heat convection coefficients can be identified with ease through inverse analysis in combination with modified *μGA*.

Therefore, the presented method that combines optimized μGA and reduced-basis method is a very efficient in terms of solving the inverse problem. It can be used for real-time and/or online identification.

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