A Simulation Method for Calculating the Path Travel Time in Dynamic Transportation Network
G.C. Lin, J. Peraire, B.C. Khoo, and G. Perakis

Abstract—The calculation of path travel times is an essential component for the dynamic traffic assignment and equilibrium problems. This paper presents a simulation method for calculating actual path travel times for the traffic network with dynamic demands. The method is based on a path-based macroscopic simulation model of network traffic dynamics. There is no need to explicitly model intersection delays in this method. Discontinuity in the travel time caused by traffic light control can be captured by this method. It is flexible in terms that the model is not limited to a specific velocity-density relationship. Some numerical results for signalized and unsignalized networks are reported.

Keywords—Traffic flow, transportation network, path travel time, dynamic user equilibrium

I. INTRODUCTION

Two main methods have been proposed for the modeling of dynamic transportation networks. These two approaches differ both in underlying mathematical description and physical implications. The first approach can be called simulation method, which can be microscopic or macroscopic. Along with proper mechanism for route choices at junctions, simulation models can predict the evolution of traffic flows over time and space in networks given initial and boundary conditions. Simulation models are extensively used for evaluating the performance of traffic controls, traffic management and traffic facilities. Microscopic simulation models (e.g., Nagel et al. [1992]) usually are characterized by that every vehicle's information is kept track of and each vehicle's behavior is modeled according to neighboring vehicles' information, like velocities. Microscopic models have some deficiencies which is pointed out in Daganzo [1994]. In this paper, we are using macroscopic models. Macroscopic models use an aggregate way to describe the dynamics of the vehicles. The hydrodynamic theory (kinematic wave model) of traffic flow (Lighthill and Whitham [1955]; Richards [1956]) is the foundation of many macroscopic models. Years' research in traffic network simulations lead to a variety of simulation models, some of which are developed into software tools (e.g., TRANSYT, NETSIM, INTERS, TEXAS, PASSER and DYNASMART).

The second approach can be called optimization methods. This method is assuming the traffic flow evolves following some kinds of optimum principles (system optimum or user optimum/equilibrium). Researchers seek to model the equilibrium conditions and obtain the equilibrium traffic flow pattern by solving the problem. This kind of models is generally formulated into optimal control or variational inequality problems. The research on the equilibrium of static transportation network has been conducted for a long time. Recently many research interests are devoted to the user equilibrium for dynamic networks. A comprehensive summary of dynamic network models can be found in Ran et al. [1994]. Other recent research efforts include Eisez et al. [1993], Ran [1996]. It was pointed out by Friess [1988] that although the user equilibrium has been employed extensively as the main assumption for route choices, evidences are still lacking that the real world network flow patterns evolved into such user equilibrium patterns. But with the advent of information era and the construction of Intelligent Vehicle Highway Systems (IVHS) kind of equilibrium models may find applications in Advanced Traveler Information Systems (ATIS) by evaluating the optimal strategies for departure times, modes and routes in real time and providing real-time route guidances for drivers.

In the models of dynamic user equilibrium, researchers have frequently assumed that the link travel time can be described as a function of the current state, which is mainly identified by the inflow of the link, the outflow of the link and the current traffic volume on the link. This method of calculating travel time has some deficiencies which is pointed out in Daganzo [1995]. And functional forms of link travel times are insufficient to reflect the dynamic features of realistic transportation network. For example, in dynamic user equilibrium, a sudden accident in a road will cause important impact to the evolution of the traffic flow and the travel time. This is hardly captured by static form of the travel time functions. Conversely, simulation methods can predict the complexities of the real world if accidents information is incorporated into models in real time. In this research, we try to combine a simulation method of path travel times into the dynamic user equilibrium. Instead of using the simulation models in other literatures for the dynamic network, we have developed a path-based simulation model. It can be integrated into the dynamic user equilibrium framework naturally and easily. In this paper, we focus on the description of the path-based simulation model. We will present the integration of this simulation model into the dynamic user equilibrium framework in another paper.

The path-based simulation seeks to predict the evolution of the traffic flow in a congested dynamic network and calculate the time-dependent path travel times corresponding to given path-flow departure rate. This problem is termed as Dynamic Networking Loading Problem in some litera-
ture (see Astarita [1988], Xu et al. [1999]). In Xu [1999], an analytical method for the problem is proposed. But the link travel time function is assumed, which is relaxed in our work.

Comparing with other simulation models like Jayakrishnan [1994] and Daganzo [1995], the path-based model has been developed to work as a component of the dynamic user equilibrium. Further efforts can be made to extend the model for general applications by relaxing the assumption that path-flow departure rate is given and choosing proper route choice mechanism according to specific requirements.

In our simulation model, we are assuming that O-D (origin-destination) pairs and the path-flow departure rates are given.

Our contributions are:

- Developing a methodology of incorporating simulation components into the dynamic user equilibrium problems.
- Developing a simple but reasonable simulation model for traffic flow and a simulation method for path travel times.

This paper is organized as follows: In section 2, the model for a single link is introduced. We extend the model to the network level in section 3. In section 4, we present a model for the path travel time based on the traffic network flow model. Numerical schemes for the travel time model are discussed, too. Some numerical experiments are shown in section 5, which verifies the validity of our model.

II. PATH BASED DYNAMICS FOR ONE SINGLE LINK

In this section, we discuss about our path-based hydrodynamic model for a one-way road which has no intermediate traffic supplies and sinks.

Here we define some notations. Subscript $i$ means link $i$, subscript $p$ means path $p$, $ip$ will means the path $p$ on link $i$.

- $\rho_i$: Link $i$'s density
- $q_i$: Link $i$'s flow rate
- $u_i$: Link $i$'s velocity
- $\rho_{pi}$: Path $p$'s density on link $i$
- $q_{pi}$: Path $p$'s flow rate on link $i$
- $u_{pi}$: Path $p$'s velocity on link $i$

All these variables are time and space dependent. So a notation $X(x, t)$ will denote the value at time $t$, position $x$. For example, $\rho_i(x, t)$ mean the link density at time $t$, at position $x$. Even for the path, the position $x$ is the position in link.

In this paper, we are using the conservation equation of the traffic flow (Lighthill and Whitham [1955]; Richards [1956]). This conservation equation in terms of link flows can be written as:

$$\frac{\partial \rho_i}{\partial t} + \frac{\partial q_i}{\partial x} = 0$$

And the traffic flow is related to density and velocity by the definition:

$$q_i = \rho_i u_i$$

The physical meaning for the conservation equation is clearly explained in Haberman [1977]. Equation 1 and 2 both are time and space dependent. For convenience, they are omitted in these two equations and some equations after this. Another main assumption in traffic flow's hydrodynamic theory is that the velocity is a function of the density, as

$$u_i = f(\rho_i)$$

where function $f$ can be linear (first order model), or non-linear (higher order model). Although linear velocity-density relationships have some well-known deficiencies (see TRB (Transportation Research Board) report, "Traffic Flow Theory" [1977]). For the sake of simplicity, Greenshields' linear velocity-density relationship is used here. It can be written as:

$$u_i = u_i^{max} \left(1 - \frac{\rho_i}{\rho_i^{max}}\right)$$

This equation shows the basic property that the velocity reaches the maximal value at zero density, and the velocity is zero for the case with maximal density.

Similar to the link traffic flows, each individual path's traffic flow also satisfies the conservation property. Here's our proposition.

Proposition 1: The path flow satisfies the conservation equation. It can be written as:

$$\frac{\partial \rho_{pi}}{\partial t} + \frac{\partial q_{pi} u_{pi}}{\partial x} = 0$$

Proof: Following the similar procedure presented in Haberman [1977] for the link flow's conservation equation, this proposition comes out naturally.

As noted, the velocity used in equation (3) is the link flow's velocity. This is justified by the knowledge that for the aggregate description of traffic, all the paths' flow on the same link share the same velocity at the same position and the same time. So it is always true that $u_i = u_p$. And the link flow's density is the sum of all the path flows' density, it's written as:

$$\rho_i = \sum_{p \epsilon P} \rho_{pi}$$

The link flow's velocity is calculated by using the link flow velocity-density relationship: $u_i = f(\rho_i)$.

It may be pointed out that we are using the simple continuum model for traffic flow. In this research, we have no intention to improve the basic hydrodynamic theory for traffic flow. We aim to give a new way for evaluating the traffic flow in the network, which can be incorporated into dynamic user equilibrium framework naturally and easily.

As is well known that the hydrodynamic model for the traffic flow can capture the non-linear features like shock wave phenomena provided proper numerical scheme is used to solve the hyperbolic partial differential equation. Here we use Godunov's scheme to solve the PDE by discretizing the problem both in time and space domains.
For Godunov's scheme, the discretized equation is written as:
\[
\rho_i(k, n + 1) = \rho_i(k, n) - \frac{\Delta t}{\Delta x} [F_{\rho i}(k + \frac{1}{2}, n) - F_{\rho i}(k - \frac{1}{2}, n)]
\]

Here \( F \) is the corresponding flow, which is calculated using Godunov's scheme:
\[
F_{\rho i}(k + \frac{1}{2}, n) = f(\rho_{\rho i}(k + \frac{1}{2}, n))
\]

If \( \rho_{\rho i}(k, n) < \rho_{\rho i}(k + 1, n) \),
\[
f(\rho_{\rho i}(k + \frac{1}{2}, n)) = \min_{\rho \in [\rho_{\rho i}(k, n), \rho_{\rho i}(k + 1, n)]} f(\rho)
\]

If \( \rho_{\rho i}(k, n) > \rho_{\rho i}(k + 1, n) \),
\[
f(\rho_{\rho i}(k + \frac{1}{2}, n)) = \max_{\rho \in [\rho_{\rho i}(k, n), \rho_{\rho i}(k + 1, n)]} f(\rho)
\]

where the flow rate is calculated:
\[
f(\rho_{\rho i}) = \rho_{\rho i} u_i
\]

For Greenshield's velocity-density relationship, \( f \) can be written as:
\[
f(\rho_{\rho i}) = \rho_{\rho i} U_{\max}^i \left(1 - \frac{\rho_{\rho i}}{\rho_{\max}^i} \right)
\]

The numerical flow between two discretized points depends on both the flow rate of the upstream point and the flow rate of the downstream point. This reflects the reality that the flow depends on both the downstream's capacity and upstream's maximal possible flow.

In numerical treatments, we have to take care of the link's boundary points. The discretized equation for updating density will be applicable, too. But we need to change the calculation of the flow. For the head of the link,
\[
F_{\rho i}(k + \frac{1}{2}, n) = f_{\rho i}^{IN}
\]

The inflow \( f_{\rho i}^{IN} \) is provided according to the path departure rate or the models for the intersections. For the tail of the link,
\[
F_{\rho i}(k - \frac{1}{2}, n) = f_{\rho i}^{OUT}
\]

The outflow \( f_{\rho i}^{OUT} \) is calculated according to the intersection models (discussed in next section) or basing on the assumption that the position just after the tail of the link has zero density.

For a traffic network, at each timestep, we update the path flow's variables one by one instead of link flow's. So additional bookkeeping efforts are needed for coding. We developed a simulator using C++. The implementation details are not given in this paper.

III. PATH BASED DYNAMICS FOR A WHOLE NETWORK

For the network model, we are assuming that there is no interaction between links except at the position where links intersect. For network modelling, three main components are needed. They are the evolution of the traffic on the link, the route choice models at intersections and the intersection models. These three components are coupled in the sense that the route choice models and the intersection models will affect the inflow and outflow of a single link. Additionally, we may incorporate the incident models considering the complexities of the real world. These components are discussed in the following paragraphs.

A. Network initial traffic and traffic supplies

Here we are considering the input of the network model. We are given the initial traffic distribution in the network. And as also mentioned, we know about all the possible routes for any O-D pair. For each path, the path departure rate is provided. To make the network model more comprehensive, we may consider the information like, the category of the intersection (signalized or unsignalized), and the setting of traffic light for signalized intersections. Also if possible, real-time accidents information may be reported.

B. Path traffic flow evolution

The path traffic flow evolution is calculated one link by link. On each link, the link variables' evolution is updated according to the method discussed in Section II. Special procedures are used to evaluate the inflow and outflow of the links.

C. The intersection modelling

Intersections can be signalized or unsignalized. Intersections are also classified into merging junctions or diverging junctions.

For unsignalized intersections, consider the merging junctions, we denote \( \rho_k(T) \) as the link \( k \) tail's density, and \( \rho_k(H) \) as link \( k \) head's density. Suppose link \( i \) is an upstream approach of a merging junction and link \( j \) is the downstream approach, \( f_{\rho i}^{\text{max}} \) is the maximal inflow to the downstream approach \( j \), it's calculated by \( f_{\rho j}^{\text{max}} = \frac{1}{2} \rho_{\text{max}} U_{\text{max}} \) for linear velocity-density relationship given in equation (4). The outflow \( f_j \) of link \( j \) is evaluated as:
\[
f_j^{\text{OUT}} = \min\{f_j^{\text{OUT}}, f_j^{\text{max}} \frac{\rho_i}{\sum_{i \in L} \rho_i} \}
\]

Here \( f_j^{\text{OUT}} \) is evaluated by the Godunov's scheme discussed in section II as:
\[
f_j^{\text{OUT}} = F(\rho_j(T), \rho_k(H))
\]

Function \( F \) is the Godunov's method for evaluating the flow between two neighboring densities. This way of evaluating guarantees that the flow would not exceed the capacity meanwhile one link will send as many flows as possible.
if other upstream links can send very little flow. The down-
stream approach j's inflow is calculated as:

\[ f_{tn}^{IN} = \sum_{i \in L} f_{tn}^{OUT} \]

For diverging junctions, L denotes the set of the down-
stream approaches. \( a \) denotes the ratio of the upstream's density which will go to downstream. It's calculated as:

\[ a = \frac{\rho(T)}{\sum_{i \in L} \rho_i(T)} \]

where \( \rho(T) \) is the sum of path's density on upstream link i which will go to downstream approach j. The outflow for the upstream approach j is calculated as:

\[ f_{tn}^{OUT} = \min_{i \in L} \frac{F(\rho_i(T), \rho_j(B))}{a_i} \]

After calculating \( f_{tn}^{OUT} \), each downstream approach i's inflow is calculated as \( a_i f_{tn}^{OUT} \). We can see that this model can capture the spillback phenomena if there's an accident at one downstream approach which causes the blockage of the road. A similar procedure is found in Dagarze (1995).

At signalized intersections, for the stopped links, the out-
flow is set to zero, and the inflow for the downstream link is also set to zero. For the links in green color phase, the link's inflow and outflow are equal, and are calculated simply as:

\[ f = F(\rho_i(T), \rho_j(B)) \]

where \( \rho_i(T) \) denotes the density at upstream link i's tail, and \( \rho_j(B) \) denotes the density at downstream link j's head.

And all above discussions are using link variables. To calculate the inflow and outflow for each path on one link, we multiply the path's density's ratio among the link density.

D. Route choice at intersections

For a general-purpose simulation model of traffic networks, a proper mechanism for modeling drivers' route choice at intersections is necessary, since there exist multiple routes for a single destination. But in our model, this component is simplified. We are assuming that path departure rate is known. This means we are knowing every vehicle's destination and the route followed. So there's no route choice modelling here. This is also the reason why this model is supposed to work as a component of the dynamic user equilibrium problem.

E. Incidents modelling

Now we only consider the situations that accidents caus-
ing the blockage a road. The modelling of accidents is simple by setting the flow at this point to zero.

IV. SIMULATION METHOD FOR CALCULATING PATH TRAVEL TIME

The path travel time is very important both in simu-
lation models and optimization models of the dynamic traffic network. In simulation models, the path travel times are used to evaluate the efficiencies of traffic controls. In optimization models, path travel times are costs which the models want to minimize. User equilibrium problems use it to describe the equilibrium conditions. In most literatures, a functional form of link travel times is assumed, which may depend on the inflow rate, outflow rate and the current traffic volume on the considered links. This method of path travel time evaluation faces some critiques from Dagarze (1994). And Rao, et al (1994) points out that traditional SPER (Share of Public Roads) volume-delay function is not sufficient to reflect the dynamic features of dyna-
mic traffic networks. It's also not trivial to evaluate actual path travel times even if the functional forms of link travel times are assumed, see Xu (1999).

In this paper, we are calculating the actual path travel time. Comparing with instantaneous path travel time, actual path travel time is defined as the time really experienced by the driver. In calculation, it's evaluated according to the prediction of the flow evolutions. It's more reasonable way for path travel time evaluation comparing with instantaneous path travel time which is calculated based on the assumption that prevailing traffic conditions will last.

Here we present a way for calculating the path travel time by simulation method. Considering the residence time \( \tau \), which is dependent on time \( t \) and position \( z \). The material derivative of the resident time is:

\[ \frac{D \tau}{Dt} = 1 \]  

The derivative can be expanded as:

\[ \frac{\partial \tau}{\partial t} + u \frac{\partial \tau}{\partial x} = 1 \]  

If we set the boundary condition at entering point as zero for \( \tau \), the \( \tau(x, t) \) will represent the time the vehicle has been in the link. The entering time for the vehicle, can be determined by

\[ t_e = t - \tau(x, t) \]  

For numerical treatments of the hyperbolic PDE for the travel time \( \tau \), a simple upwind scheme is used. The discretized equation can be written as:

\[ \tau^{n+1}_i = \tau^n_i - \frac{\Delta x}{\Delta t} (\tau^n_i - \tau^n_{i-1}) + \Delta t \]  

At each timestep, we evaluate the traffic density according to the conservation equation for the traffic flow, then calculate the velocity using velocity-density relationship. Finally, we get the \( \tau \) using the calculated velocity and \( \tau \) from previous iteration. It is very important to show this model will satisfy the first-in-first-out (FIFO) condition. Here is the proposition for the conservation law of path flows. We define Courant number \( C_t^* \) as \( C_t^* = \frac{\Delta x}{\Delta t} \).

Proposition 2: If Courant number \( C_t^* \leq 1 \) for all \( i \), and the initial solution, \( \tau_i^n \) is monotonically increasing, the calculated path travel time using equation (12) will be monotonically increasing, i.e., satisfies FIFO condition.
Proof: According to equation (12), we consider the difference of the actual travel time between two different positions with the same departing time as:

$$
\tau_{i+1}^{n+1} - \tau_{i}^{n+1} = \tau_{i}^{n} - \tau_{i-1}^{n} - C_n(\tau_{i}^{n} - \tau_{i-1}^{n}) \\
+ C_{n-1}(\tau_{i-1}^{n} - \tau_{i-2}^{n}) \\
= (1 - C_n)(\tau_{i}^{n} - \tau_{i-1}^{n}) + C_{n-1}(\tau_{i-1}^{n} - \tau_{i-2}^{n})
$$

Since initial actual travel time is monotone increasing, and constant number $C_n \leq 1$, so $\tau_{i+1}^{n+1} - \tau_{i}^{n+1} \geq 0$.

Although equation (10) is for the residence time on one link, we are interested in the path travel time. So when using numerical scheme, we are updating the paths rather than the links. And we need to take care by setting the initial residence time for the traffic in the path properly.

We also use another way to calculate the travel time, which is used to verify the results using the hyperbolic equation from the travel time. We keep track of the movement of an individual vehicle by updating the position at each iteration. Similar procedure can be found in Jayakrishnan et al [1994], where it serves for a different purpose. The position and velocity of the vehicle are updated as:

$$
x_{i+1} = x_i + u_i \times \Delta t \\
u_{i+1} = \alpha(x_{i+1}, t_{i+1})
$$

Suppose a vehicle starts at time $t_0$, the position of the vehicle is updated iteratively. Suppose the arrival time is $t_1$, the actual path travel time for the vehicle departing at time $t_0$ is $t_1 - t_0$. We give some results using these two ways. The results are reasonable by reflecting the discontinuity in travel time for the network with traffic light controls.

V. NUMERICAL RESULTS

In this section, we will show the computational results for two example networks. The first network is traffic-light controlled. The second one is not.

Figure 1 shows the example network 1. There's one intersection controlled by traffic light at node D. The traffic light is controlled simply by setting equal green time for the 3 connecting in-links at every period. Figure 2,3 show the numerical results for the path travel time by two different methods.

Here is the description of the network parameters. Link 1,2, and 3's length is 1 unit. Link 4's length is 3 units. The traffic light at intersection D is controlled in a simple way that every link has a 2 units green time. And the inflow to path 1, 2 and 3 is 0.2 units. The maximum velocity is set to 1 unit. The maximum density is set to 1 unit.

We can see the calculated path travel time is consistent with the intuitive sense in that there's discontinuity in path travel time, and the gap between the maximal path travel time and the minimal path travel time is equal to red light time, which is 4 units in this example.

Example network 2 is a bit more complex, and is shown in Figure 4. It has to be noted that this example network is taken from Xu et al [1999].

Table 1 shows all the O-D pairs with nonzero demands and all the routes in the network.

| Table 1 |
that the departure time for path 12 goes down to free cruise time a bit later than path 3 and 4. The reason is that the path 12 is the last segment of path 3 and 4. This is physically reasonable.

**VI. Conclusions**

In this paper, we propose a path-based simulation model for dynamic transportation networks with given path departure rate. Based on the traffic flow simulation model, we introduce a new method for calculating the path travel time.

Computational experiments show that the simulation model and travel time calculation can reflect the dynamic features of a dynamic traffic network. We expect this method to provide a more accurate way for calculating path travel time as compared to traditional link cost functions.

The work of incorporating simulation model to the dynamic user equilibrium model is ongoing. The extension from single-lane links to multiple-lane links will be considered. The model can be also extended to more general purposes by considering the demand generation, destination identification mechanism, and proper route choice mechanism.

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**References**


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Table 2

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The maximum density for each path is set to 50 units, and the maximum velocity is set to 0.3 unit.

Figure 5 shows the result by using the iterative update methods.

As we can see from Figure 5, the path travel times are dynamically changing. The path travel time reaches the maximal value after a period, then goes down to free cruise time when the input of traffic goes to zero. We also note

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