# STATE ESTIMATION IN INDUCTION MACHINES

by

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Submitted to the Department of
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## ABSTRACT

This thesis focuses on the estimation of the electromagnetic state variables of the induction machine model from the point of view of observer theory. The latter viewpoint has not been sufficiently exploited in the previous work on estimation for electric machines. Furthermore, nearly all the previously developed estimation schemes for electric machines that have taken the approach of observer theory have neglected the use of a corrective prediction-error term. It is shown here that existing estimation schemes, for instance those used in field oriented control, can be better understood in the context of observer theory, and that observer theory naturally leads to estimators with improved performance over the existing estimation schemes through the use of prediction-error.

In particular, a class of observers for the estimation of rotor flux that uses a prediction-error term, and that can therefore be made to exhibit an arbitrarily fast rate of estimation error decay under ideal conditions is developed. These observers are contrasted with existing estimation schemes. A second class of observers for the combined estimation of rotor flux and stator current is also derived. This second class of observers similarly can attain arbitrary rates of estimation error decay (under ideal conditions), in contrast to the dynamical constraints imposed by previously developed estimation schemes for induction machines. The performance of the proposed observers is verified with numerical simulations.

Sampled-data realizations for the above observers are developed to facilitate microprocessor implementation. The particular sampled-data implementations are shown to have satisfactory performance (via numerical simulation), and have the feature that very little new computation is required to update the observer implementations at each time step.

The effects of measurement disturbances on the performance of the proposed flux (and current) estimators is analyzed. In

particular, two cases are addressed. The cases where measurement disturbances are constant biases and where measurement disturbances can be modelled as zero-mean, white noise processes are considered.

Two nonlinear estimation problems for induction machines are also considered. Firstly, the rotor flux (or rotor flux and stator current) estimation problem is re-examined in the case where model uncertainties are present. Secondly, the estimation of the rotor speed (and machine fluxes) from the electrical terminal measurements is considered. Previous work on these problems is reviewed, and certain hopefully new approaches are presented.

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To my parents

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## LIST OF SYMBOLS

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$^*$ = transposition ($A^*$ = transpose of A, for any matrix A)

$\hat{}$ = estimate of associated variable ($\hat{x}$ = estimate of x)

$\lambda_r$ = two-component rotor flux vector

$\lambda_s$ = two-component stator flux vector

$\lambda = [\lambda_s{}^* \ \lambda_r{}^*]^*$

$i_r$ = two-component rotor current vector

$i_s$ = two-component stator current vector

$\theta$ = angular position of rotor

$w$ = angular velocity of rotor

$\phi$ = angular position of the defined reference frame

$\gamma$ = angular velocity of the defined reference frame

$M$ = mutual inductance

$L_s$ = stator inductance

$L_r$ = rotor inductance

$R_s$ = stator resistance

$R_r$ = rotor resistance

$T_r = L_r/R_r$ (rotor time constant)

$\sigma^2 = L_rL_s - M^2$

$p_1 = (L_r{}^2R_s + M^2R_r)/(\sigma^2L_r)$

$p_3 = L_r/\sigma^2$

$p_4 = L_rR_s/\sigma^2$

$H$ = moment of inertia associated with rotor

## Chapter 1: INTRODUCTION

### Background

Modern induction machine control systems demand complete and accurate state information. Field oriented control has emerged as an important approach to the control of AC machines, and continues to be discussed and developed in the literature (see [1-3] and rererences therein). The field oriented method relies on knowledge of the rotor flux vector (magnitude and angle) to regulate this flux, and to control the electromagnetic torque. Future control schemes may apply sliding mode theory to regulate the machine flux and mechanical state variables [4]. Again, accurate and complete state information is a necessity for this application.

The flux linkage of the rotor of a squirrel cage induction machine is not directly measurable, and hence, rotor flux estimation from the terminal variables (stator voltage and current, and rotor speed) is a key step in the control schemes mentioned above. In the face of noisy measurement data, and even if measurements of all state variables are available, a state estimator can produce smoothed measures of the actual state variables. Thus, one may be led to construct an estimator for stator current as well as rotor flux if measurements are corrupted by noise.

### Scope of Thesis

This thesis will focus on the estimation of the electrical state variables (rotor flux and stator current) from the point of view of observer theory. The latter viewpoint has not been sufficiently exploited in this application area. We shall show that better insight into presently used methods, and ideas for improved methods, emerge naturally from the perspective of observer theory. In particular, a class of observers for the estimation of rotor flux using the terminal measurements (stator current and voltage, and speed) is derived in Chapter 2. These observers can be made to exhibit an arbitrarily fast rate of convergence of the estimate to the underlying state (i.e. of error decay) under ideal conditions; this is in contrast to the schemes presently used in field oriented control. The principles applied in the rotor flux estimation problem are extended to an observer for rotor flux <u>and</u> stator current, also in Chapter 2. Here again, arbitrary convergence rates can be achieved, in contrast to the dynamical constraints imposed by nearly all the previously developed estimation schemes for induction machines. Sampled-data implementations for the observers derived in Chapter 2 are presented in Chapter 3, to facilitate microprocessor implementation. In addition, the effects of errors in the measurement of current, voltage, and speed on the derived observers are analyzed in Chapter 4.

Considerable attention in the literature on field oriented control [1,2,5] has been devoted to the rotor flux estimation problem in the face of unknown or time-varying machine parameters. In particular, [1] and [5] have devised effective schemes to estimate the rotor time constant, which is known to vary by 40 percent as the temperature varies by 100C. Some methods to attack this problem are presented in Chapter 5, which is devoted to nonlinear estimation for induction machines. In particular, the extended Kalman filter algorithm, a method termed here "bounded nonlinearity", and a model reference adaptive identifier/observer approach are to be considered.

In speed control applications, the replacement of a mechanical speed sensor with additional signal processing may improve system reliability and reduce cost. There are significant initiatives in the literature [6,7] that deal with this nonlinear estimation problem. Hillenbrand [6] has cast the system equations into a form where the extended Kalman filter algorithm [8] can be effectively applied. We shall consider various approaches to the construction of a speed estimator in Chapter 5. This task, although more demanding than the static parameter estimation problem, will be treated with similar methods.

The remainder of this chapter introduces the induction machine model we shall be dealing with, and then reviews the fundamentals of observer theory from a standard, systems viewpoint.

### Induction Machine Model

An idealized two-axis model for the squirrel cage induction machine will be considered in this thesis. For more details, see [9]-[11]. The v-i relationship for the electrical machine terminals is given by:

$$\begin{bmatrix} v_s \\ 0 \end{bmatrix} = \begin{bmatrix} R_s I & 0 \\ 0 & R_r I \end{bmatrix} \begin{bmatrix} i_s \\ i_r \end{bmatrix} + \begin{bmatrix} \lambda_s' \\ \lambda_r' \end{bmatrix} \tag{1.1}$$

where $v_s$, $i_s$, $\lambda_s$, $i_r$, and $\lambda_r$ are two-component vector representations of stator voltage, stator current, stator flux, rotor current, and rotor flux, respectively. Here stator quantities are measured on axes fixed to the stator, and rotor quantities on rotor-fixed coordinates. $R_s$ and $R_r$ are the stator and rotor winding resistances in each axis. For the symmetric, smooth air gap machine, in the absence of saturation, the flux and current vectors are related by the position-dependent inductance matrix,

$$L(\theta) = \begin{bmatrix} L_s & 0 & M\cos\theta & -M\sin\theta \\ 0 & L_s & M\sin\theta & M\cos\theta \\ M\cos\theta & M\sin\theta & L_r & 0 \\ -M\sin\theta & M\cos\theta & 0 & L_r \end{bmatrix}, \tag{1.2}$$

through

$$\begin{bmatrix} \lambda_s \\ \lambda_r \end{bmatrix} = L(\theta) \begin{bmatrix} i_s \\ i_r \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} i_s \\ i_r \end{bmatrix} = L(\theta)^{-1} \begin{bmatrix} \lambda_s \\ \lambda_r \end{bmatrix}, \tag{1.3a,b}$$

where $\theta$ is the angle between axis-1 on the rotor and axis-1 on the stator, and $L_s$, $L_r$, and M are the stator, rotor, and mutual inductances, respectively. Fig. 1.1 shows the orientation of the induction machine windings.

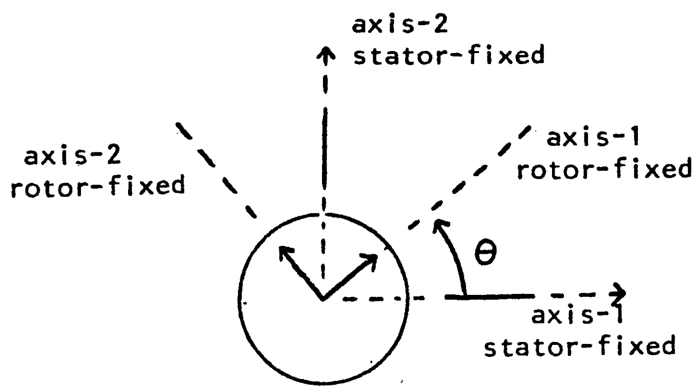



Fig. 1.1

Note that if the position $\theta$ is viewed as a parameter, we can immediately obtain a state-space representation for the flux as follows:

$$\begin{bmatrix} \lambda_s' \\ \lambda_r' \end{bmatrix} = -RL(\theta)^{-1} \begin{bmatrix} \lambda_s \\ \lambda_r \end{bmatrix} + \begin{bmatrix} v_s \\ 0 \end{bmatrix}, \quad \text{where} \quad R = \begin{bmatrix} R_s I & 0 \\ 0 & R_r I \end{bmatrix} \quad . \qquad (1.4)$$

We shall introduce some more notation and Park's transformation before discussing the behavior of the mechanical state variables. The inductance matrix $L(\theta)$ can be written as

$$L(\theta) = \begin{bmatrix} L_s I & M\exp\{J\theta\} \\ M\exp\{-J\theta\} & L_r I \end{bmatrix} \qquad (1.5)$$

where

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad , \qquad J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \qquad \text{and}$$

$$\exp\{J\theta\} = I + J\theta + 1/2\, J^2\theta^2 + \ldots = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} .$$

An elegant coordinate transformation, due to Park, can be applied to represent both stator and rotor variables in coordinates located at some arbitrary instantaneous angular position $\phi$. For this, consider the coordinate transformation:

$$\begin{bmatrix} x_s \\ x_r \end{bmatrix} = P \begin{bmatrix} \lambda_s \\ \lambda_r \end{bmatrix} \quad \text{where} \quad P = \begin{bmatrix} \exp\{-J\phi\} & 0 \\ 0 & \exp\{J(\theta-\phi)\} \end{bmatrix} . \tag{1.6}$$

See the diagram of Fig. 1.2 for an explanation of the new coordinate representation.

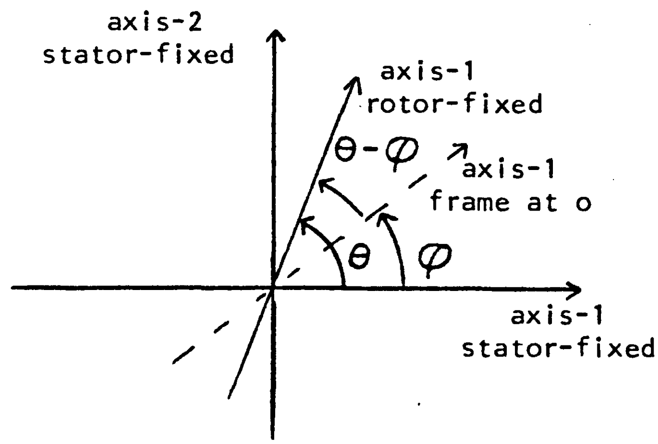



Fig. 1.2

The transformed representation is obtained as follows:

$$\begin{bmatrix} x_s' \\ x_r' \end{bmatrix} = P' \begin{bmatrix} \lambda_s \\ \lambda_r \end{bmatrix} + P \begin{bmatrix} \lambda_s' \\ \lambda_r' \end{bmatrix} = P'P^{-1} \begin{bmatrix} x_s \\ x_r \end{bmatrix} + P \begin{bmatrix} \lambda_s' \\ \lambda_r' \end{bmatrix} \tag{1.7}$$

or

$$\begin{bmatrix} x_s' \\ x_r' \end{bmatrix} = \left\{ -RL^{-1} + w \begin{bmatrix} 0 & 0 \\ 0 & J \end{bmatrix} - \gamma \begin{bmatrix} J & 0 \\ 0 & J \end{bmatrix} \right\} \begin{bmatrix} x_s \\ x_r \end{bmatrix} + \begin{bmatrix} \exp\{-J\phi\} v_s \\ 0 \end{bmatrix}, \qquad (1.8)$$

where $w = \theta'$ is the angular velocity of the rotor, and $\gamma = \phi'$ is the angular velocity of the reference frame. Note that this model does not have any position dependence apart from the input term, and its dynamics is linear and time-invariant if $w$, the speed of the rotor, and $\gamma$, the speed of the reference frame, are constant.

This state space model can be augmented to include the mechanical variable $w = \theta'$ by considering the net torque on the rotor shaft. Newton's law relates the angular acceleration to the net torque by

$$Hw' = T_{em} - T_L, \qquad (1.9)$$

where H is the total inertia of the shaft, and $T_{em}$ and $T_L$ are the electromagnetic and load torques, respectively. An expression for the electromagnetic torque can be derived by considering the partial derivative with respect to position of the stored magnetic energy, i.e.,

$$T_{em} = -\frac{\partial}{\partial\theta} \lambda^* L(\theta)^{-1} \lambda = \lambda^* \begin{bmatrix} 0 & -(M/\sigma^2) J \exp\{J\theta\} \\ (M/\sigma^2) J \exp\{-J\theta\} & 0 \end{bmatrix} \lambda \qquad (1.10)$$

where $\lambda = [\lambda_s^* \; \lambda_r^*]^*$ and $\sigma^2 = L_r L_s - M^2$. Note that the symbol ($^*$) indicates transposition. Introducing the change of coordinates due to Park's transformation to express the torque in terms of $x_s$ and $x_r$, we obtain:

$$T_{em} = \mathbf{x}^* \begin{bmatrix} 0 & -(M/\sigma^2)J \\ (M/\sigma^2)J & 0 \end{bmatrix} \mathbf{x} \quad \text{where } \mathbf{x} = [x_s{}^* \; x_r{}^*]^* . \tag{1.11}$$

Note that this torque expression is independent of both the reference frame position $\phi$ and the rotor position $\theta$. The state equation for speed w is finally obtained as

$$w' = (1/H)(M/\sigma^2)\, \mathbf{x}^* \begin{bmatrix} 0 & -J \\ J & 0 \end{bmatrix} \mathbf{x} - (1/H)T_L \quad . \tag{1.12}$$

Note that throughout the remainder of this thesis, we shall use the symbols $\lambda_s$ and $\lambda_r$ to represent the stator and rotor flux vectors in Park-transformed coordinates.

## Complex Notation for the Induction Machine Model

The structure of the state space system can be exploited to yield a simplified representation, if we consider the well known isomorphism between matrices of the form $[aI + bJ]$ and complex numbers $(a + jb)$ [12]. The complex number $(a + jb)$ may be viewed as an operator on a two component vector represented as a complex number $(x + jy)$, in the same sense that the matrix $[aI + bJ]$ is an operator on a two component vector $[x,y]^*$. The algebraic properties of these two representations are identical, as indicated below:

**Operation on a Vector**

$$[aI + bJ] \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} ax-by \\ bx+ay \end{bmatrix} \tag{1.13a, b}$$

$$(a + jb)(x + jy) = (ax-by) + j(bx+ay)$$

**Inverse**

$$[aI + bJ]^{-1} = [aI - bJ]/(a^2 + b^2)$$
$$(a + jb)^{-1} = (a - jb)/(a^2 + b^2) \qquad (1.14a,b)$$

The complex operator/vector notation may be applied to the induction machine model as follows. If each of the 2x2 matrix blocks of the system equation (1.8), which is composed of a linear combination of the elementary matrices I and J, is represented by the corresponding scalar complex operator, and if the two-component variables $\lambda_s$, $\lambda_r$, $i_s$, $i_r$, and $v_s$ are taken as complex numbers instead, significant simplification of algebraic computations can be achieved. In particular, for constant speed operation, the eigenvalues of the 4x4 system matrix in (1.8) can be computed by considering this matrix as a 2x2 matrix with complex elements. This 2x2 complex matrix is given by

$$\begin{bmatrix} -R_sL_r/\sigma^2 & R_sM/\sigma^2 \\ R_rM/\sigma^2 & -R_rL_s/\sigma^2 + jw \end{bmatrix} . \qquad (1.15)$$

Two of the eigenvalues of the system (1.8) may be computed by finding the roots of the second-order characteristic polynomial,

$$s^2 + s((R_sL_r + R_rL_s)/\sigma^2 - jw) + R_sR_r/\sigma^2 - jwR_sL_r/\sigma^2 \qquad , (1.16)$$

of the above 2x2 matrix. The other two eigenvalues of the system (1.8) are the complex conjugates of those obtained by solving (1.16).

Another important result of the complex notation is that we may obtain a closed form complex transfer function $T(s) = I(s)/V(s)$, where $I(s)$ and $V(s)$ are complex-variable Laplace transforms of the stator current and voltage (if speed $w$ is viewed as a constant parameter). This will be of interest when parameter and speed estimation are discussed in Chapter 5.

We shall find it convenient to use one point of view sometimes, and the other at other times.

**State Estimation with Observers**

A well developed approach to estimation of the state of a dynamical system is based on observer theory [13]. The discussion in [13] is addressed to linear, time-invariant systems, but many other results are known for linear, time-varying and nonlinear systems [14-16]. An observer for a known linear system takes the form of a real-time simulation of the system, except that, in addition to being driven by the known inputs, it is also driven by the error between actual outputs of the system and the predicted outputs. Thus, consider a linear system modeled by the state-space description

$$x'(t) = A(t)x(t) + B(t)u(t) \;;\; x(0) = x_0 \tag{1.17a}$$

(where $x$ is an n-dimensional state vector and $u$ an m-dimensional vector of known inputs), with outputs modeled by

$$y(t) = C(t)x(t) \;. \tag{1.17b}$$

(where $y$ denotes a p-dimensional vector of measured outputs). An observer for this system is given by

$$\hat{x}'(t) = A(t)\hat{x}(t) + B(t)u(t) + K(t)[\hat{y}(t) - y(t)] \tag{1.18}$$
$$\hat{y}(t) = C(t)\hat{x}(t) \quad .$$

The term in brackets is called the <u>prediction error</u>, and the matrix $K(t)$ is the <u>observer gain</u>. Given $u(\cdot)$ and $y(\cdot)$, the system (1.18) can be solved by integrating forwards in time from some specified initial condition $\hat{x}(0)$. Typically, we shall use the initial condition $\hat{x}(0) = 0$ if no information is available on $x(0)$.

The effectiveness of the observer is assessed by examining the dynamics of the error between the states of the observer and the actual system. We denote the estimation error,

$$e(t) = \hat{x}(t) - x(t), \tag{1.19}$$

and subtract (1.17) from (1.18) to obtain the error dynamics:

$$e'(t) = [A(t) + K(t)C(t)]e(t) \tag{1.20}$$

The initial condition for (1.20) is the initial estimation error $e(0)$, which (even if small) is invariably nonzero because of uncertainties regarding the initial state of (1.17). Lyapunov theory is called for to establish the stability of the homogeneous time-varying linear system (1.20). Quadratic Lyapunov functions of the error $e(t)$ will be devised in Chapter 2 for the flux observers discussed therein. In the case where the matrices A, C, and K are time-invariant, however, the behavior of (1.20) is governed by the eigenvalues of $[A + KC]$. It is evident from (1.20) that if $K=0$, i.e. if the real-time simulation is not corrected by a prediction error term, then the error dynamics is the same as that of the underlying system (1.17), and is therefore governed by the eigenvalues of A. We shall sometimes refer to the K=0 case of (1.18) as being an open-loop observer, and the K=0 case as being a closed-loop observer. Under a so-called observability condition on the pair of time-invariant matrices [A,C], the dynamics of (1.20) can in principle be made arbitrarily fast by appropriate choice of K. In practice, however, the presence of system disturbances, noise in the sensors, and model uncertainties set limits on how fast one may reasonably make the observer. The celebrated Kalman filter, see references in [13], in fact results from picking the gain K that gives minimum mean square estimation error for a specific noise model.

There are many variants of the above development. For example, the measured outputs may be given by a more complicated form than (1.17b), perhaps involving the inputs and the derivatives of the states. In this case, one may construct the observer with modified variables to avoid the differentiation of signals. This will be done for a class of observers derived in Chapter 2. The details will be provided in that chapter. The brief review of observer theory given here is only intended to introduce the concept of "real-time simulation corrected by prediction error".

## Chapter 2: FLUX ESTIMATION

### Introduction

This chapter deals with the estimation of the electromagnetic states (i.e. the rotor flux) of the induction machine model which are not directly measurable at the machine terminals. The observers for rotor flux discussed herein will sometimes be referred to as reduced-order observers because they only attempt to estimate some of the state variables of the induction machine model. We shall review the "indirect" rotor flux estimation scheme used in field oriented control, and then, show how to modify its error dynamics with a prediction-error term. A second flux estimation scheme used in field oriented control, termed the "direct" method, will also be discussed, and it will be shown how a prediction-error term can be used to improve the dynamics of this estimation scheme.

We shall also consider the possibility of filtering the measurements of machine states which are directly accessible (i.e. the stator current), in the case where these measurements are corrupted by noise. This will lead us to the construction of an observer for rotor flux and stator current using measurements of stator current and voltage, and rotor speed. The resulting observer will be characterized by arbitrary (i.e. specified by design) rates of convergence of the estimates to the underlying states of the induction machine model. This is in contrast to the dynamical constraints imposed by nearly all the previously developed estimation schemes for induction machines.

Pioneering work along the directions pursued in this chapter can be found in the papers of Bellini et. al. [17] and [18]. However, the development to be presented here was carried out independently of that work, and although our results are in some respects similar to those of [17] and [18], our presentation is perhaps more transparent. (Also, see the 1985 PESC paper of G. C. Verghese and S. R. Sanders [21].) We shall begin this chapter by giving a more detailed critique of the previous work in the area.

**Assessment of Literature on Flux Estimation**

The literature on flux estimation for electrical machines rarely makes a clear distinction between the state of a model of the system being studied and the state of the estimator itself, thus obscuring the issue of the behavior of the estimation error. The most explicit analyses of the flux estimators (including analysis of the estimation error dynamics) are in [17-20], while [5] comes close in the course of examining the effects of unknown parameters.

A further striking fact is that, apart from [17] and [18], all the existing flux estimation schemes that we know of correspond essentially to real-time simulations that have no corrective feedback derived from prediction error. These include [19] and [20], even though the word "observer" is used in their titles. Papers such as [6] and [22], which proceed via an extended Kalman filter or least squares approach, have estimators that inherently have a corrective prediction error term, and may therefore be considered exceptions.

The discussion of observer theory in Chapter 1 suggests that the dynamics of the estimation error in all these open-loop observer realizations is governed by that of the underlying physical system. For example, the so-called "indirect" estimation flux estimation scheme used in field oriented control [1], which is simply an uncorrected real-time simulation of the rotor flux dynamics, has error behavior that is governed by the rotor time constant. This scheme will be explored in greater detail in this chapter, and it will be shown how to obtain faster error decay with the application of a corrective prediction error signal.

There are perhaps two main reasons for the neglect of error dynamics in the literature on flux estimation. Firstly, existing implementations (in particular, the "indirect" method) of field oriented control schemes have been found to be satisfactorily robust and effective in practice. Secondly, existing theoretical treatments of the error dynamics, such as [17-20], are not easily penetrated. Undoubtedly, there will be applications where error decay at a rate limited by the underlying physical system will not be sufficient. The lack of estimation error analysis in the literature and the likelihood of a need in some applications for improved error decay compel the examination of error dynamics in this chapter.

**Rotor Flux Observer based on the "Indirect" Scheme**

An induction machine can be characterized by a fifth order state-space model. When measurements of rotor velocity and (the two components of) stator current are available, control applications that require state feedback demand the reconstruction of only two state variables, for example (the two components of) rotor current or rotor flux. The indirect method in field oriented control is based explicitly or implicitly on an observer for the two-component rotor flux vector. The works of Garces [5], Gabriel and Leonhard [1], Dote [19], and others include an observer based on the rotor flux dynamics:

$$\lambda_r' = [(-1/T_r)I + wJ]\lambda_r + (M/T_r)i_s \quad , \tag{2.1}$$

where $\lambda_r$ and $i_s$ are two axis representations of rotor flux and stator current, respectively, w is the rotor speed, $T_r$ $(=L_r/R_r)$ is the rotor time constant, M is the mutual inductance, and the matrices J and I are as follows

$$J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \quad , \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad . \tag{2.2a, b}$$

Note that (2.1) can be obtained from the second equation of the state-space model (1.8) of the induction machine, using stator-fixed coordinates. These reduced-order observers for rotor flux have been implemented as open-loop simulators:

$$\hat{\lambda}_r' = [(-1/T_r)I + wJ]\hat{\lambda}_r + (M/T_r)i_s \; , \tag{2.3}$$

where $\hat{\lambda}_r$ is the estimate of the rotor flux vector. The error system that results from subtracting (2.1) from (2.3) is as follows:

$$e' = [(-1/T_r)I + wJ]e \; , \tag{2.4}$$

where $e = \hat{\lambda}_r - \lambda_r$ .

As previously mentioned, the literature on flux estimation typically neglects to examine this error system; exceptions are [17-20] (though [5] does display the error system in the course of analyzing the effects of uncertainties in $T_r$).

With the speed w viewed as a known parameter, (2.4) is a linear system. However, since w is in general a time-varying quantity, the convergence properties of (2.4) cannot generally be studied by simply taking the eigenvalues of the matrix in brackets. When the speed w is <u>constant</u>, (2.4) becomes time-invariant. Since its eigenvalues are $-1/T_r \pm jw$ (which is most easily seen if one considers the isomorphism between matrices of $aI + bJ$ and complex numbers $a + jb$, see Chapter 1), the two scalar components of e display an oscillation at the frequency w (the constant rotor speed) that is damped with a time constant of $T_r$ (the rotor time constant). Numerical simulations performed with the simulation language Simnon on the MIT Joint Computation Facility of the induction machine model (1.8) and this rotor flux observer are shown in Fig. 2.1. The machine (whose parameters are given in the figure) was considered to be excited by a 60 Hz sinusoidal voltage. Fig. 2.1 shows traces of the axis-1 and axis-2 components of induction machine rotor flux in (i) and (ii), while traces of the observer estimates of these two components of rotor flux are shown in (iii) and (iv). Waveforms representing the two components of the rotor flux estimation error are shown in (v) and (vi).

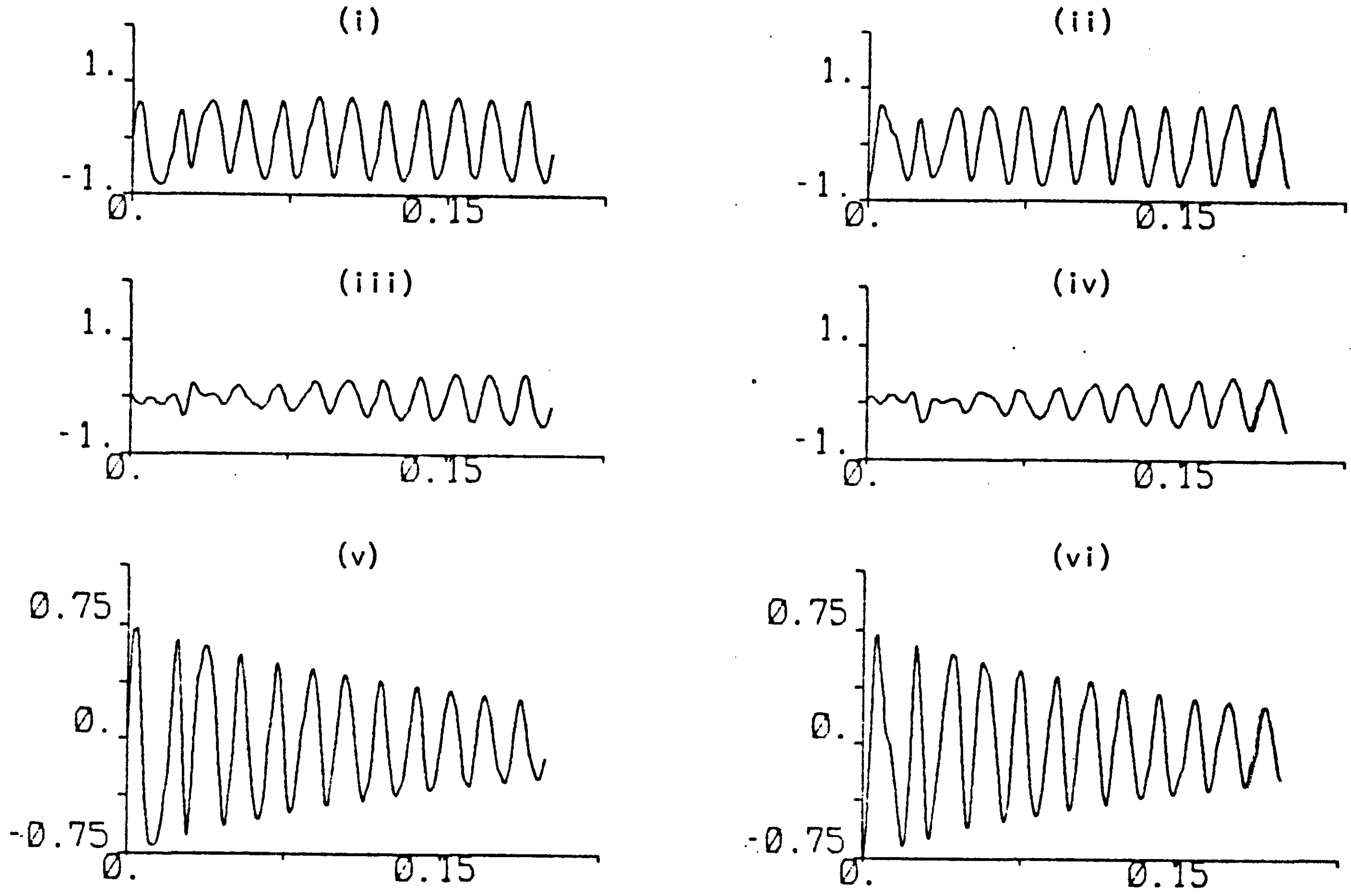



Fig. 2.1

Numerical Simulation of Open-Loop Rotor Flux Observer

(i) Axis-1 rotor flux

(ii) Axis-2 rotor flux

(iii) Estimate of axis-1 rotor flux

(iv) Estimate of axis-2 rotor flux

(v) Estimation error in axis-1 rotor flux estimate

(vi) Estimation error in axis-2 rotor flux estimate

<u>Parameters of machine used in numerical simulations:</u>

$R_s$ = 0.3 ohms $\quad$ $L_s$ = 0.0553 H $\quad$ M = 0.0533 H

$R_r$ = 0.3 ohms $\quad$ $L_r$ = 0.0546 H

Significant insight into the general case where speed w is time-varying can be obtained through a Lyapunov analysis. The stability of the error system can be assessed by considering the Lyapunov candidate $V = e^*e$. Differentiation of V with respect to time and using (2.4) yields the following:

$$V' = 2e^*e' = -2(1/T_r)\, e^*e \;=\; -2(1/T_r)V \quad . \tag{2.5}$$

This relationship is easily obtained from (2.4) by noting that J is skew-symmetric. The function V is positive definite, decrescent, and radially unbounded. (See [23] for details on Lyapunov stablility theory.) Clearly, V' is negative definite since it is equal to $-2(1/T_r)V$. Hence, the error system is exponentially stable for any initial condition, and the magnitude of the error decays with the time constant of the rotor. This analysis is essentially equivalent to that given in [19], with $e^*e$ as the Lyapunov function.

## Using Prediction Error

A method to improve the rate of convergence of an observer for rotor flux is to introduce a prediction error term based upon stator voltage measurements. Except in [17], this possibility has apparently not been considered in the literature on flux estimation. To modify the convergence rate, we would construct the following observer system:

$$\hat{\lambda}_r' = [-1/T_r)I + wJ]\hat{\lambda}_r + (M/T_r)i_s + K(\hat{v}_s - v_s) \ , \tag{2.6}$$

where K is a 2x2 observer gain matrix and $\hat{v}_s$ is the estimated prediction of the stator voltage, given by

$$\hat{v}_s = (M/L_r)\hat{\lambda}_r' + ((L_rL_s - M^2)/L_r)i_s' + R_si_s \,. \tag{2.7}$$

Equation (2.7) is obtained by rearranging the first equation of the state-space representation of (1.8), using stator-fixed coordinates. For the present, we assume the known current waveform is differentiable, and that we can obtain the derivative of stator current exactly in implementing (2.6-2.7). Later, it will be shown that this restriction can be removed. The dynamics of the obtained error system are described by

$$e' = [(-1/T_r)I + wJ]e - K(M/L_r)e' \,, \quad \text{where} \tag{2.8}$$
$$e = \hat{\lambda}_r - \lambda_r.$$

For purpose of illustration, let K be chosen to have the form K = kI. Then, the error system simplifies to

$$e' = (1 + kM/L_r)^{-1}[(-1/T_r)I + wJ]e. \tag{2.9}$$

If the rotor speed w is constant, then (2.9) is a time-invariant linear system, and the eigenvalues that govern it are seen (again, using the isomorphism with complex numbers) to be

$$(1\text{-}kM/L_r)^{-1}(-1/T_r \pm jw) \quad . \tag{2.10}$$

Thus, the eigenvalues of the error dynamics are scaled up by the factor $(1\text{-}kM/L_r)^{-1}$, i.e. the time constant that governs the error decay is scaled <u>down</u> by this factor, while the frequency of oscillation of oscillation in the error decay waveform is scaled <u>up</u> by the same factor.

The behavior of this observer with the gain k selected to obtain error decay twice as fast as that of the open-loop observer has been verified by digital simulation with the simulation program Simnon on the MIT Joint Computation Facility. The simulation results shown in Fig. 2.2 were obtained using the same operating conditions as were used for the simulation of Fig. 2.1. For purpose of comparison, the axis-1 rotor flux waveform of Fig. 2.1 is repeated in Fig. 2.2 (i), the axis-1 open-loop rotor flux estimate of Fig. 2.1 is repeated in (ii), and the corresponding axis-1 open-loop estimation error is repeated in (iii). The trace in (iv) shows the axis-1 estimate produced by our improved rotor flux observer, while the trace in (v) shows the axis-1 error produced by the improved observer. We see that the observer using a prediction error term does indeed have a faster decay of the estimation error.

For the more general case of time-varying rotor speed w, the stability of this modified observer can be analyzed, again, with the Lyapunov candidate V = e*e. Differentiation of V yields the following:

$$\begin{aligned} V' = 2e^*e' &= -2(1+kM/L_r)^{-1}(1/T_r)e^*e \\ &= -2(1+kM/L_r)^{-1}(1/T_r)V \end{aligned} \tag{2.11}$$

It is clear that the magnitude of the error decays as an exponential function of time, and the exponential rate is specified by

$$-(1+kM/L_r)^{-1}(1/T_r) \, , \tag{2.12}$$

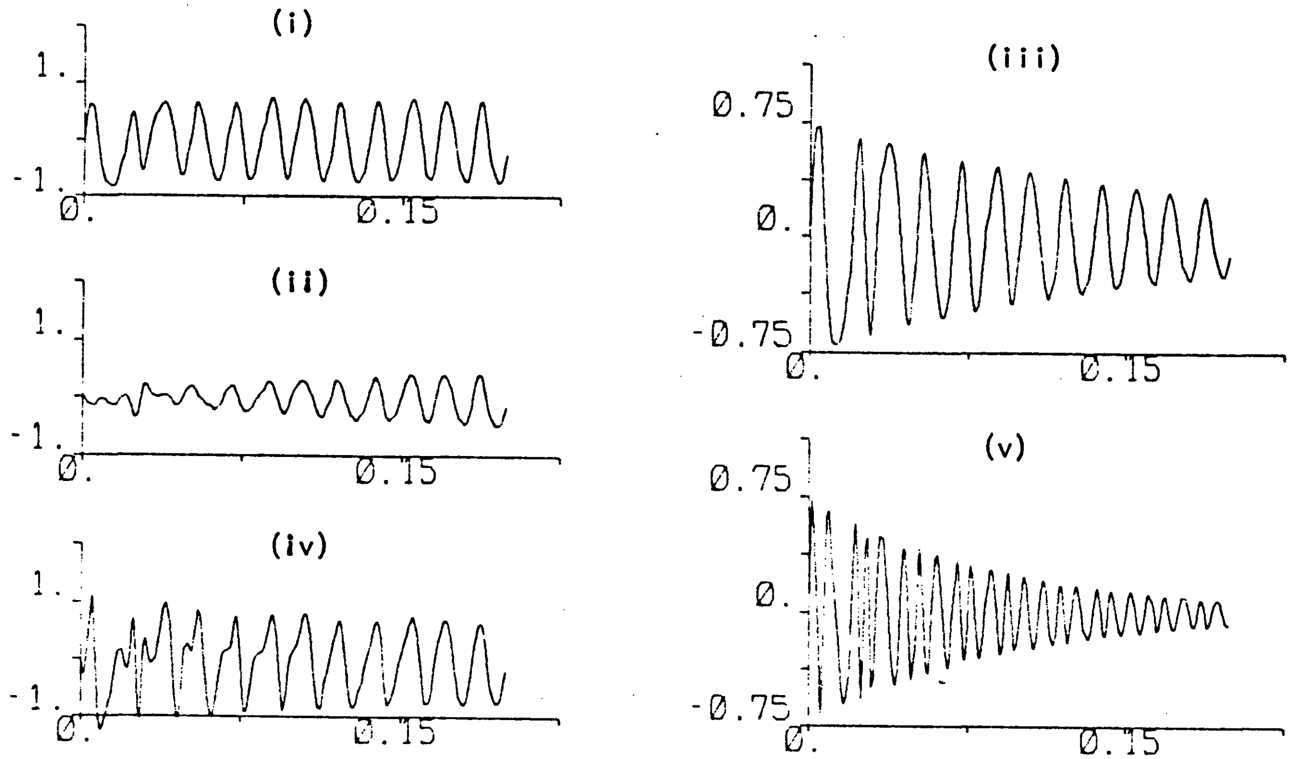



Fig. 2.2
Numerical simulation of rotor flux observer using a prediction-error term

(i) Axis-1 rotor flux

(ii) Open-loop estimate of axis-1 rotor flux

(iv) Estimate of axis-1 rotor flux using prediction-error

(iii) Estimation error in open-loop estimate of axis-1 rotor flux

(v) Estimation error in estimate of axis-1 rotor flux using prediction-error where k is a free design parameter. In principle, we can <u>arbitrarily</u> specify the rate of error decay.

To eliminate the differentiation of current waveforms in implementing (2.6-2.7), we would group together (additively) all the terms which involve a derivative, and rename this sum as a new variable. For this purpose, we define the auxiliary variable z by

$$z = \hat{\lambda}_r + k[(M/L_r)\hat{\lambda}_r + ((L_rL_s - M^2)/L_r)i_s], \tag{2.13}$$

and implement the following system:

$$z' = [(-1/T_r)I + wJ]\hat{\lambda}_r + (M/T_r)i_s + k(v_s - R_si_s) \text{ , where} \tag{2.14}$$

$$\hat{\lambda}_r = (z - k[(L_rL_s - M^2)/L_r]i_s)/(1 + kM/L_r) . \tag{2.15}$$

Equation (2.14) can be integrated forward in time, and the rotor flux estimate can be recovered from the relationship between $\hat{\lambda}_r$ and z in (2.15). Note that no differentiation of signals is required.

The particular gain in (2.9) was chosen for ease of illustration. A more general gain, suggested by the results in [17], is given by

$$K = k_1I + k_2J . \tag{2.16}$$

With this choice of observer gain, the estimation error is governed by the system equation:

$$e' = [(g_1/T_r + g_2w)I + (g_1w - g_2/T_r)J]e, \text{ where} \tag{2.17}$$

$$g_1 = (1-Mk_1/L_r)/[(1-Mk_1/L_r)^2 + (Mk_2/L_r)^2] \text{ , and} \tag{2.18}$$

$$g_2 = (Mk_2/L_r)/[(1-Mk_1/L_r)^2 + (Mk_2/L_r)^2] . \tag{2.19}$$

If the rotor speed w is constant, this error system is linear and time-invariant, and the governing eigenvalues are given by

$$-[g_1/T_r + g_2 w] \pm j[g_1 w - g_2/T_r]. \tag{2.20}$$

It can be seen from (2.18-2.20) that by proper choice of $k_1$ and $k_2$, these eigenvalues can be placed at any specified pair of conjugate locations. It is in principle also possible to vary $k_1$ and $k_2$ as a function of (a slowly varying) operating speed, to control the variation of error dynamics with operating point in some desired fashion.

With a general time-varying speed function w, a Lyapunov analysis similar to that performed for (2.9) can be used to demonstrate the stability of (2.17). Once again, the Lyapunov function $V = e^*e$ can be used. Differentiation of V yields the result:

$$V' = -2(g_1/T_r + g_2 w)V . \tag{2.21}$$

The rate of convergence of the error magnitude is seen to depend on the speed w for general constants $g_1$ and $g_2$. However, as previously indicated, the gains $k_1$ and $k_2$ could be selected as functions of the speed w to obtain an invariant convergence rate. This possibility remains for future investigation.

**Rotor Flux Observer Based on the "Direct" Scheme**

There is an alternative flux estimation scheme, sometimes called the "direct" method, that is based on rewriting the first equation of the state-space representation (1.8) (using stator-fixed coordinates) as

$$\lambda_r' = (L_r/M)(v_s - R_s i_s) - [(L_r L_s - M^2)/M] i_s' , \tag{2.22}$$

which leads to the real time simulation

$$\hat{\lambda}_r' = (L_r/M)(v_s - R_s i_s) - [(L_r L_s - M^2)/M] i_s' . \tag{2.23}$$

The limitation of this scheme that is typically quoted in the literature is the poor behavior at low speeds. Note the additional fact, however, that the estimation error <u>remains constant</u>, because the derivatives of actual and estimated flux are equal! Any initial error in the estimate therefore persists. One can now attempt to improve this estimator by feeding in a corrective prediction error term. In this case, it is (2.1) (or the second equation of the state-space representation (1.8)) that we turn to for the prediction error term. The resulting observer then has the form:

$$\hat{\lambda}_r' = (L_r/M)(v_s - R_s i_s) - [(L_r L_s - M^2)/M] i_s' + K(\hat{i}_s - i_s), \tag{2.24}$$

where $v_s$ and $i_s$ are measured, and $\hat{i}_s$ is obtained from (2.1) as

$$\hat{i}_s = T_r/M \, \{\hat{\lambda}_r' - [(-1/T_r)I + wJ]\hat{\lambda}_r\}. \tag{2.25}$$

For this observer, we obtain the error system:

$$e' = K(\hat{i}_s - i_s) = KT_r/M \, \{e' - [(-1/T_r)I + wJ]e\} \text{ , or} \tag{2.26}$$

$$e' = -(I - KT_r/M)^{-1}(T_r/M)K \, [(-1/T_r)I + wJ]e \quad . \tag{2.27}$$

The parallel between this error model and the one in the previous section is evident. Once again, simple choices of K such as those in (2.9) or (2.16) will lead to error dynamics that is substantially different from that of the uncorrected system. Also, by use of appropriate auxiliary variables, it is again straight forward to implement the observer without the necessity of differentiating measured signals.

The question that now arises is how to choose between the observers based on the "direct" and "indirect" schemes. The two are closely related, since they actually result from using the same sets of equations, and the error dynamics obtained by any particular choice of gain matrix in one scheme can typically be obtained by some appropriate choice of gain matrix in the other. However, it is possible that one scheme is more easily implemented than the other in a given situation. For example, if an error decay rate of $1/T_r$ was desired with a gain of the form $K = kI$, then k would be zero in the first scheme, but infinite in the second.

**Fourth-Order Observer**

The observers discussed in the preceding sections are actually examples of what are termed <u>reduced-order</u> observers, because those observers do not create estimates for all the state variables of the induction machine model. A full order observer would produce estimates for $i_s$ and $w$, in addition to $\lambda_r$, since these state variables (or independent combinations of them) characterize the dynamic behavior of the model. Usually $i_s$ and $w$ are more easily measured than $\lambda_r$, and therefore are often considered known. The assumption that these variables are known motivates the construction of a reduced-order observer. However, there are costs associated with this assumption. Firstly, the unfiltered measurements of $i_s$ and $w$ contain all the measurement disturbances. Secondly, the estimate $\hat{\lambda}_r$ produced by the reduced order observer may be more sensitive to the measurement noises in $i_s$ and $w$; as an example, (2.15) shows that any noise in $i_s$ appears (scaled but) unfiltered in $\hat{\lambda}_r$. For control applications, e.g. in current- or speed-control loops, the filtered estimates of $i_s$, $w$, and $\lambda_r$ may be preferable to the raw or noisy measurements of these variables.

For the reasons described above, the potential merits of a full order observer are apparent, even when direct measurements of $i_s$ and $w$ are available. A full order observer that estimated the speed $w$ would necessarily enter the realm of nonlinear estimation because of the way $w$ enters the state-space model (1.8). Consideration of the speed estimation problem will be reserved for the discussion of nonlinear estimation in Chapter 5.

Here, we shall consider a fourth-order observer that produces filtered estimates of $i_s$ and $\lambda_r$ using measurements of w, $i_s$, and $v_s$. We could equivalently construct an observer for $\lambda_s$ and $\lambda_r$, as was done in [20]. A review of the observer in [20] will be offered for comparative purposes before we develop the fourth order observer of this section.

<u>The Observer in [20]</u>

The observer proposed in [20] is an uncorrected fourth-order real-time simulation that generates estimates of both stator and rotor fluxes, using measured values of stator voltage and rotor velocity. The fourth-order system model for flux that is used in this real-time simulation is given by:

$$\begin{matrix} \lambda_s' \\ \lambda_r' \end{matrix} = \{-RL^{-1} + w\begin{bmatrix} 0 & 0 \\ 0 & J \end{bmatrix}\} \begin{matrix} \lambda_s \\ \lambda_r \end{matrix} \quad \begin{matrix} v_s \\ 0 \end{matrix} \tag{2.28}$$

where $\lambda_s$, $\lambda_r$, $v_s$ are 2-component vectors representing stator flux, rotor flux, and stator voltage, respectively. The matrices R and L which represent the winding resistances and machine inductances, respectively, have the form:

$$R = \begin{bmatrix} R_s I & 0 \\ 0 & R_r I \end{bmatrix} \quad , \quad L = \begin{bmatrix} L_s I & MI \\ MI & L_r I \end{bmatrix} \quad . \tag{2.29a, b}$$

For the uncorrected fourth-order observer based on (2.28), we obtain the following error dynamics:

$$e' = [-RL^{-1} + w\begin{bmatrix} 0 & 0 \\ 0 & J \end{bmatrix}]e, \tag{2.30}$$

where e is a four-component vector representing the error in the flux estimate.

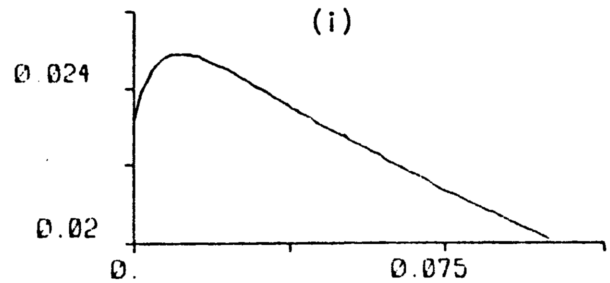
If w is constant, the dynamics of the error system is governed by its eigenvalues. As discussed in Chapter 1 of this thesis, the particular structure of (2.30) allows the simplified calculation of these eigenvalues. These are given by the roots of the characteristic polynomial

$$s^2 + s((R_sL_r + R_rL_s)/\sigma^2 - jw) + R_sR_r/\sigma^2 - jwR_sL_r/p_2 , \qquad (2.31)$$

along with the their complex conjugates.

Fig. 2.3 shows results of numerical simulations of this fourth-order observer using the same machine model as was used in the simulations of Figs. 2.1 and 2.2. In the vicinity of zero speed (w = 0), the above eigenvalues are approximately -2.77 (twice) and -182.0 (twice). These values are reflected in the error transient associated with the rotor flux estimate on axis-1, shown in (i); the response is ultimately dominated by the larger time constant, which is 0.36 sec. At a speed of 377 rad/sec, the eigenvalues are computed to be $-93.0 \pm j354.0$ and $-91.7 \pm j22.7$. Again, these values are reflected in the rotor flux error waveform for axis-1, shown in (ii); the time constant of the envelope is now 0.01 sec.

In the general case where the speed w is a time-varying function of time, Lyapunov analysis is called for to determine the stability properties of the time-varying system (2.30). We consider the Lyapunov candidate



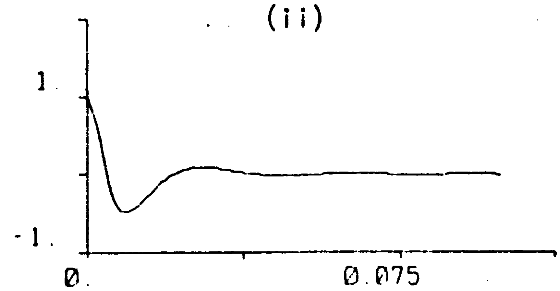




Fig. 2.3

Numerical simulation of open-loop fourth-order observer

(i) Estimation error in the estimate of axis-1 rotor flux for a speed of approximately zero (w = 0)

(ii) Estimation error in the estimate of axis-1 rotor flux for a speed of approximately 377 rad/s

$$V = 1/2\, e^T R^{-1} e \quad , \tag{2.32}$$

which is an energy type function since $R^{-1}$ is positive definite. Differentiation yields the following:

$$V' = -e^T L^{-1} e \tag{2.33}$$

(The above is derived by noting that $R^{-1}\begin{bmatrix} 0 & 0 \\ 0 & J \end{bmatrix}$ is skew-symmetric, and that $x^T Ax=0$ for any vector x if A is skew-symmetric.) The matrix $L^{-1}$ is positive definite, and hence $V'<0$ for e not equal to zero. The dynamic behavior of this observer is dictated by the entries of the R and L matrices, or by the physical machine parameters. The rate of error decay can be bounded by noting that (2.32) and (2.33) lead to the following inequalities:

$$V' = -e^T L^{-1} e \leq -(\text{min. eigenvalue of } L^{-1}) e^* e \tag{2.34}$$

$$V = 1/2\, e^T R^{-1} e \leq 1/2\, (\text{max. eigenvalue of } R^{-1}) e^* e \tag{2.35}$$

These inequalities can be combined to obtain the following:

$$V' \leq -2 \frac{(\text{min. eigenvalue of } L^{-1})}{(\text{max. eigenvalue of } R^{-1})} V \tag{2.36}$$

or, equivalently,

$$V' \leq -2 \frac{(\text{min. eigenvalue of } R)}{(\text{max. eigenvalue of } L)} V \quad . \tag{2.37}$$

This bound guarantees that all components of the estimation error will decay to zero at a rate at least as fast as an exponential function of time with a time constant dictated by the induction machine parameters (0.36s for our example) under any operating conditions.

Though the above bound is independent of the rotor speed w, the dynamic behavior of this observer is strongly dependent on w. Our simulations of this observer indicate the error convergence is much faster than the bound given above at speeds corresponding to 60 Hz in electrical frequency; however, at zero speed, the convergence rate is nearly identical to this bound. See the simulations in Fig. 2.3.

## Using Prediction Error

It is now possible to consider the use of a corrective prediction error term based on measurements of stator current, to modify the estimation error dynamics. For this purpose, it will be convenient to consider a state-space model with state variables of stator current and rotor flux. This is also a practical choice, since many control applications require accurate estimates of precisely these variables. This system model is given by:

$$\begin{bmatrix} i_s' \\ \lambda_r' \end{bmatrix} = \left\{ \begin{bmatrix} -p_1 I & \{M/(\sigma^2 T_r)\} I \\ (M/T_r) I & (-1/T_r) I \end{bmatrix} + w \begin{bmatrix} 0 & -(M/\sigma^2) J \\ 0 & J \end{bmatrix} \right\} \begin{bmatrix} i_s \\ \lambda_r \end{bmatrix} + \begin{bmatrix} (L_r/\sigma^2) I \\ 0 \end{bmatrix} v_s , \tag{2.38}$$

where

$\sigma^2 = L_r L_s - M^2$, $\quad p_1 = (L_r^2 R_s + M^2 R_r)/(\sigma^2 L_r)$, and

$i_s$, $\lambda_r$, $v_s$ are 2-component vector representations of stator current, rotor flux, and stator voltage, respectively.

The proposed observer will have the form:

$$\begin{bmatrix} \hat{i}_s' \\ \hat{\lambda}_r' \end{bmatrix} = \left\{ \begin{bmatrix} -p_1 I & \{M/(\sigma^2 T_r)\} I \\ (M/T_r) I & (-1/T_r) I \end{bmatrix} + w \begin{bmatrix} 0 & -(M/\sigma^2) J \\ 0 & J \end{bmatrix} \right\} \begin{bmatrix} \hat{i}_s \\ \hat{\lambda}_r \end{bmatrix} + \begin{bmatrix} (L_r/\sigma^2) I \\ 0 \end{bmatrix} v_s + \begin{bmatrix} k_i I + k_{ij} wJ \\ k_\lambda I + k_{\lambda j} wJ \end{bmatrix} (\hat{i}_s - i_s) \quad , \tag{2.39}$$

where $k_i$, $k_{ij}$, $k_\lambda$, and $k_{\lambda j}$ are scalars. The effectiveness of this choice of corrective feedback structure is apparent when one considers the resulting error system:

$$\begin{bmatrix} e_i' \\ e_\lambda' \end{bmatrix} = \left\{ \begin{bmatrix} (-p_1+k_i) I & \{M/(\sigma^2 T_r)\} I \\ \{(M/T_r)+k_\lambda\} I & (-1/T_r) I \end{bmatrix} + w \begin{bmatrix} k_{ij} J & (-M/\sigma^2) J \\ k_{\lambda j} J & J \end{bmatrix} \right\} \begin{bmatrix} e_i \\ e_\lambda \end{bmatrix} \tag{2.40}$$

where $e_i = \hat{i}_s - i_s$ and $e_\lambda = \hat{\lambda}_r - \lambda_r$. Note that we can freely determine the coefficients of the left-hand blocks of the two matrices in (2.40). If $k_i$ and $k_\lambda$ are selected such that $k_i = -k_{ij}/T_r + p_1$ and $k_\lambda = -k_{\lambda j}/T_r - M/T_r$, the error dynamics become:

$$\begin{bmatrix} e_i' \\ e_\lambda' \end{bmatrix} = A\, Q(w) \begin{bmatrix} e_i \\ e_i \end{bmatrix} \quad \text{, where} \tag{2.41}$$

$$A = \begin{bmatrix} k_{ij} I & (-M/\sigma^2) I \\ k_{\lambda j} I & I \end{bmatrix} , \quad Q(w) = \begin{bmatrix} (-1/T_r) I + wJ & 0 \\ 0 & (-1/T_r) I + wJ \end{bmatrix} . \tag{2.42a, b}$$

The freedom in the selection of $k_{ij}$ and $k_{\lambda j}$ allows the eigenvalues of A (in pairs) to be placed arbitrarily. This can be seen by considering the characteristic polynomial of A,

$$p_A(s) = \{ s^2 - (1+k_{ij}) s + k_{ij} + (M/\sigma^2) k_{\lambda j} \}^2 \quad . \tag{2.43}$$

Clearly, the proper choice of the gains can arbitrarily determine the two coefficients of $p_A(s)^{1/2}$.

The stability properties of the error system (2.41) are dictated by the eigenvalues, $u_1$ and $u_2$, of A. In the case of constant speed operation, the error dynamics (2.41) is governed by the eigenvalues of the matrix AQ(w) which can be shown to be

$$[(-1/T_r) \pm jw]u_1 \text{ (twice)} \quad \text{and} \tag{2.44a}$$

$$[(-1/T_r) \pm jw]u_2 \text{ (twice)} . \tag{2.44b}$$

It is therefore clear that in this case the observer error may be made to decay to zero exponentially fast, with a specified time constant. Fig. 2.4 shows the results of numerical simulations of this observer, with $u_1=2$ and $u_2=10$. Here, again, we have used the same machine model as in the previous simulations. The waveforms in (i) and (ii) are obtained for a rotor speed near zero, and correspond respectively to errors in the axis-1 estimates of stator current and rotor flux. Note that the envelope of the error decay has time constant 0.09s ($=T_r/u_1$). The traces in (iii) and (iv) correspond to these same two quantities, but are obtained for a speed near 377 rad/s. The results correlate well with the above analysis, in that the visible oscillation frequencies are 2x377 rad/s and 10x377 rad/s, while the envelope ultimately decays with time constant 0.09s, as before.

If the rotor speed w is time-varying, the eigenvalues do not directly give information on the dynamics, and it is natural to attempt a Lyapunov analysis. We shall show that the stability properties of the error dynamics (2.41) are still controlled by

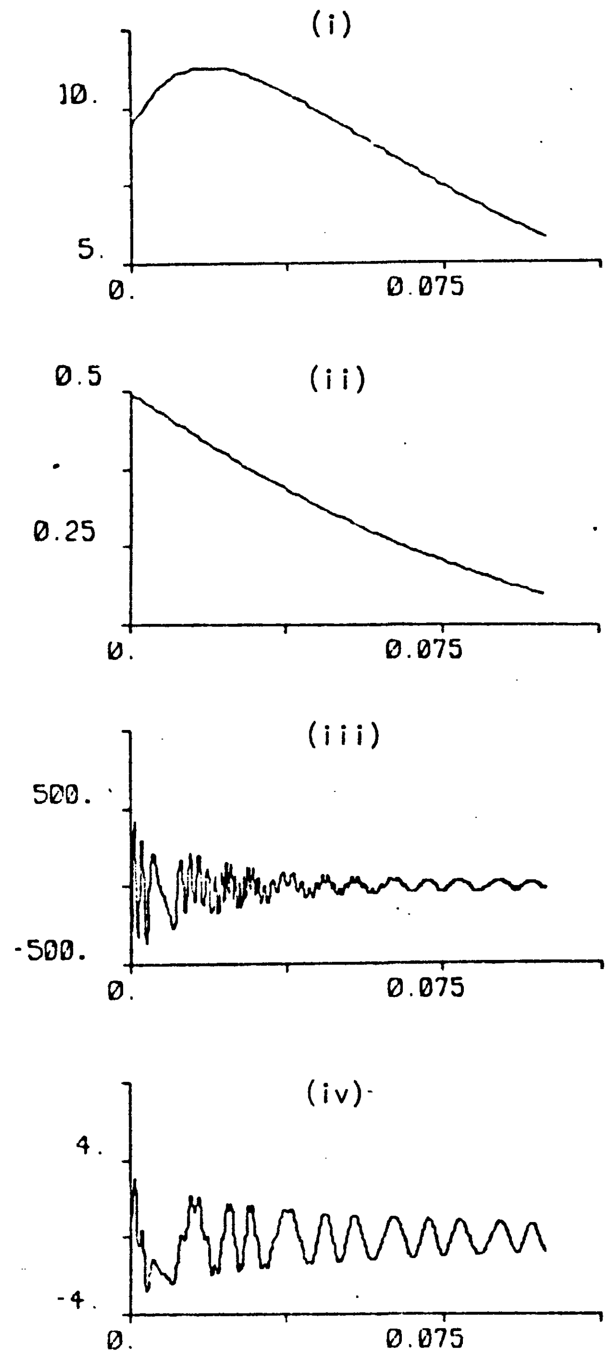



Fig. 2.4

Numerical simulation of fourth-order observer using prediction-error

(i) Estimation error in the estimate of axis-1 stator current for a speed of approximately zero (w = 0)
(ii) Estimation error in the estimate of axis-1 rotor flux for a speed of approximately zero (w = 0)
(iii) Estimation error in the estimate of axis-1 stator current for a speed of approximately 377 rad/s
(iv) Estimation error in the estimate of axis-1 rotor flux for a speed of approximately 377 rad/s

the eigenvalues, $u_1$ and $u_2$, of A. In particular, exponential stability with a rate at least as fast as $\min\{u_1, u_2\}/T_r$ is guaranteed if $u_1$ and $u_2$ are strictly positive. This can be seen by noting that A is similar to a Jordan form matrix through a block diagonal transformation P. We have the following relationship:

$$P\,A\,P^{-1} = \begin{bmatrix} u_1 I & 0 \\ 0 & u_2 I \end{bmatrix} \quad , \text{ where} \tag{2.45}$$

$$P = \begin{bmatrix} p_{11} I & p_{12} I \\ p_{21} I & p_{22} I \end{bmatrix} \tag{2.46}$$

must commute with Q(w) since P has blocks of 2x2 scaled identity matrices. Then, we consider the weighted error function

$$V = 1/2[e_i^* \; e_\lambda^*]P^*P \begin{matrix} e_i \\ e_\lambda \end{matrix} \quad , \tag{2.47}$$

as a candidate for a Lyapunov function. Differentiation of V yields the following:

$$V' = [e_i^* \; e_\lambda^*]P^* \begin{bmatrix} u_1 I & 0 \\ 0 & u_2 I \end{bmatrix} Q(w)P \begin{matrix} e_i \\ e_\lambda \end{matrix} \tag{2.48}$$

which is negative definite if $u_1$ and $u_2$ are positive. Furthermore, the stability is exponential since

$$V' \leq -2 \min\{u_1, u_2\}/T_r \; V \tag{2.49}$$

Since V is a quadratic function of the error, the <u>magnitude</u> of the error must decay with time constant $T_r/\min\{u_1, u_2\}$.

The fact that the system matrix $AQ(w)$ is similiar to a matrix of the form $\{\text{diagonal } (u_1 I,\ u_2 I)\}Q(w)$ for all $w$ will be of use when sampled-data models are discussed in Chapter 3.

# Chapter 3: SAMPLED-DATA IMPLEMENTATION OF FLUX OBSERVERS

## Introduction

A favorable approach to the hardware realization of the observers discussed in Chapter 3 is via a microprocessor implementation. In this case, state estimation would be one function of an overall digital control scheme. Critical to this, however, is some attention to the task of putting the observer equations in a form that is naturally and efficiently handled by a microprocessor, and this is our focus here. We shall begin this chapter by presenting a well known method [24] for obtaining an exact sampled-data model of a continuous-time, linear system under the restriction that the inputs and the underlying system itself are piecewise constant over intervals of length T. The application to the observers of Chapter 2 will then be discussed.

Consider a time-varying, linear system of the form

$$x'(t) = A(t)x(t) + B(t)u(t), \tag{3.1}$$

where

$$u(t) = u(nT),\ A(t) = A(nT),\ \text{and}\ B(t) = B(nT), \qquad (3.2a,b,c)$$
$$\text{for}\ nT \leq t < (n+1)T\ ,\ n = 0,1,...$$

It is then well known that the evolution of the sampled state x(nT) is described by the linear, time-varying, discrete-time model

$$x(nT + T) = F_n x(nT) + G_n u(nT), \tag{3.3}$$

where

$$F_n = \exp(A_n T) \quad \text{and} \quad G_n = \int_0^T \exp(A_n s) B_n ds, \tag{3.4a, b}$$

with

$$\exp(At) = I + At + A^2 t^2/2! + \ldots \tag{3.5}$$

The latter matrix is called the matrix exponential. Note that if the underlying continuous-time system is time-invariant, the resulting sampled-data model is also time-invariant, i.e. if the matrices $A_n$ and $B_n$ are constant, then so are the matrices $F_n$ and $G_n$.

The observers derived in the preceding sections are, in general, time-varying (for time-varying speed w). We shall assume that the time variations in $i_s$, $v_s$, and w are slow enough for the models and their inputs to be considered piecewise constant over each sampling interval of duration T, so that the development associated with (3.3) and (3.4) can be followed. One might, however, expect a large computational burden associated with recomputing the matrices in (3.4) at each time step to obtain the appropriate model for that step. Nevertheless, it turns out that the observers we are considering have the feature that very little new computation is required at each step. This will be demonstrated in the subsequent sections.

**Sampled-Data Model for the Rotor Flux Observer**

With the assumption that all measurements, $v_s$, $i_s$, and w, are constant over each sample interval, the elements of the sampled-data model given in (3.3) and (3.4) corresponding to the observer (2.6-2.7) are easily obtained. This is because a matrix exponential of the form $\exp\{aI + bJ\}$ is equal to the product of the two matrix exponentials, $\exp\{aI\}$ and $\exp\{bJ\}$, i.e.

$$\exp\{aI + bJ\} = \exp\{aI\}\exp\{bJ\} \quad , \tag{3.6}$$

where a and b are arbitrary real scalars. This relationship holds because the two matrices, I and J, commute (i.e. IJ = JI). The matrix exponential associated with the sampled-data model for the reduced-order observer (2.6-2.7) is thus seen to be given by

$$\begin{aligned} &\exp\{(1-kM/L_r)^{-1}\,[(-1/T_r) + wJ]t\} \\ &= \exp(ct)\begin{bmatrix} \cos(dt) & -\sin(dt) \\ \sin(dt) & \cos(dt) \end{bmatrix} \quad , \end{aligned} \tag{3.7}$$

where $c = -(1-kM/L_r)^{-1}/T_r$ and $d = (1-kM/L_r)^{-1}w$ . Because the matrix exponential is a simple function of the speed w, this sampled-data model is easily recomputed with each new measurement of w.

Figure 3.1 compares the performance of the continuous-time and sampled-data implementations of the rotor flux observer of Chapter 2. The waveform in (i) is just Figure 2.2(v) repeated, showing the error in the rotor flux estimate produced by the continuous-time observer, while (ii) shows the estimation error produced by the sampled-data observer with a sampling interval of 0.1ms. It is evident that the sampled-data implementation performs well.

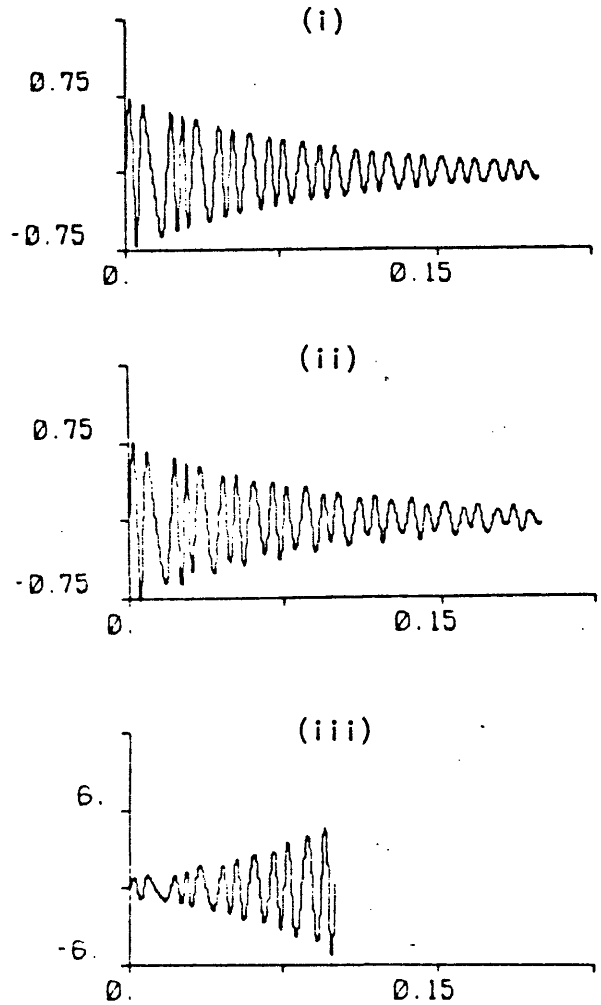



Fig. 3.1

Numerical simulation of sampled-data implementation of rotor flux observer

(i) Estimation error in continuous-time estimate of axis-1 rotor flux with observer gain selected to obtain an error convergence rate twice that of the open-loop observer

(ii) Estimation error in sampled-data estimate of axis-1 rotor flux with observer gain as in (i)

(iii) Estimation error in forward Euler discrete-time estimate of axis-1 rotor flux with observer gain as in (i)

For comparison, the waveform in (iii) shows the error obtained if one attempts to get away without computing matrix exponentials at all, but simply uses forward differences to approximate derivatives (i.e. uses the "forward Euler" method). The result in this instance is a disaster! Note that the resulting instability could have been predicted (under the assumption of time-invariance) by computing the eigenvalues of the matrix [I + AT] where A is the underlying system matrix of the error dynamics (2.9), since it is this matrix that governs propagation of the sampled state under the forward difference scheme. These eigenvalues are given by

$$u_i\{I + AT\} = 1 + u_i\{A\}T \quad . \tag{3.8}$$

For the present case with T=0.1ms, $T_r$=.182s, and w=377rad/s, we find

$$u_{1,2} = 0.999 \pm j0.0754 \tag{3.9}$$

and

$$|u_1| = |u_2| = 1.002 \; > 1 \; .$$

<u>Some Errors due to Assumption of Piecewise Constant Variables</u>

It is evident that the sampled-data observer (that uses the matrix exponential) has an error transient that is not identical to that of the continuous-time observer. Additional simulations of the sampled-data observer with the gain selected to obtain faster error decay display more clearly a steady state oscillation in the error waveforms. This behavior occurs when the machine current and voltage waveforms are in a sinusoidal steady state and the rotor speed is constant. Figure (3.2) shows the axis-1 error waveforms in the sampled estimates of rotor flux for convergence rates of five (i), ten (ii), and twenty (iii) times that of the open-loop observer. Also shown in Figure (3.2) is the axis-1 stator current (iv), while the last .03 seconds of the traces in (i) and (ii) are expanded and shown superimposed in (v), and the last .03 seconds of the stator current trace in (iv) are expanded in (vi).

One might speculate from the traces of Fig. 3.2 (v and vi) that the sinusoidal steady state error waveforms result from a sinusoid (i.e. the stator current) driving a stable LTI error plant. This can be seen to be true by considering that these observers use sampled values of the actual current, voltage, and speed (which is known to be constant in this case), stored in a zero-order hold over the sample interval. A typical waveform of the effective input to the sampled-data observer is compared to the actual continuous-time waveform of the induction machine in Fig. (3.3).

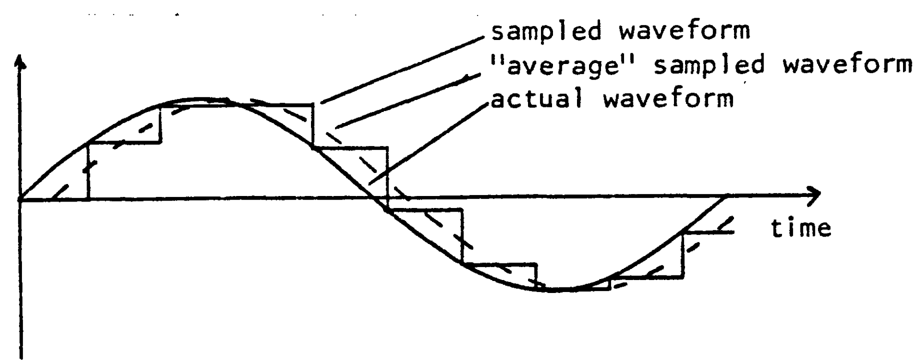



Fig. 3.3

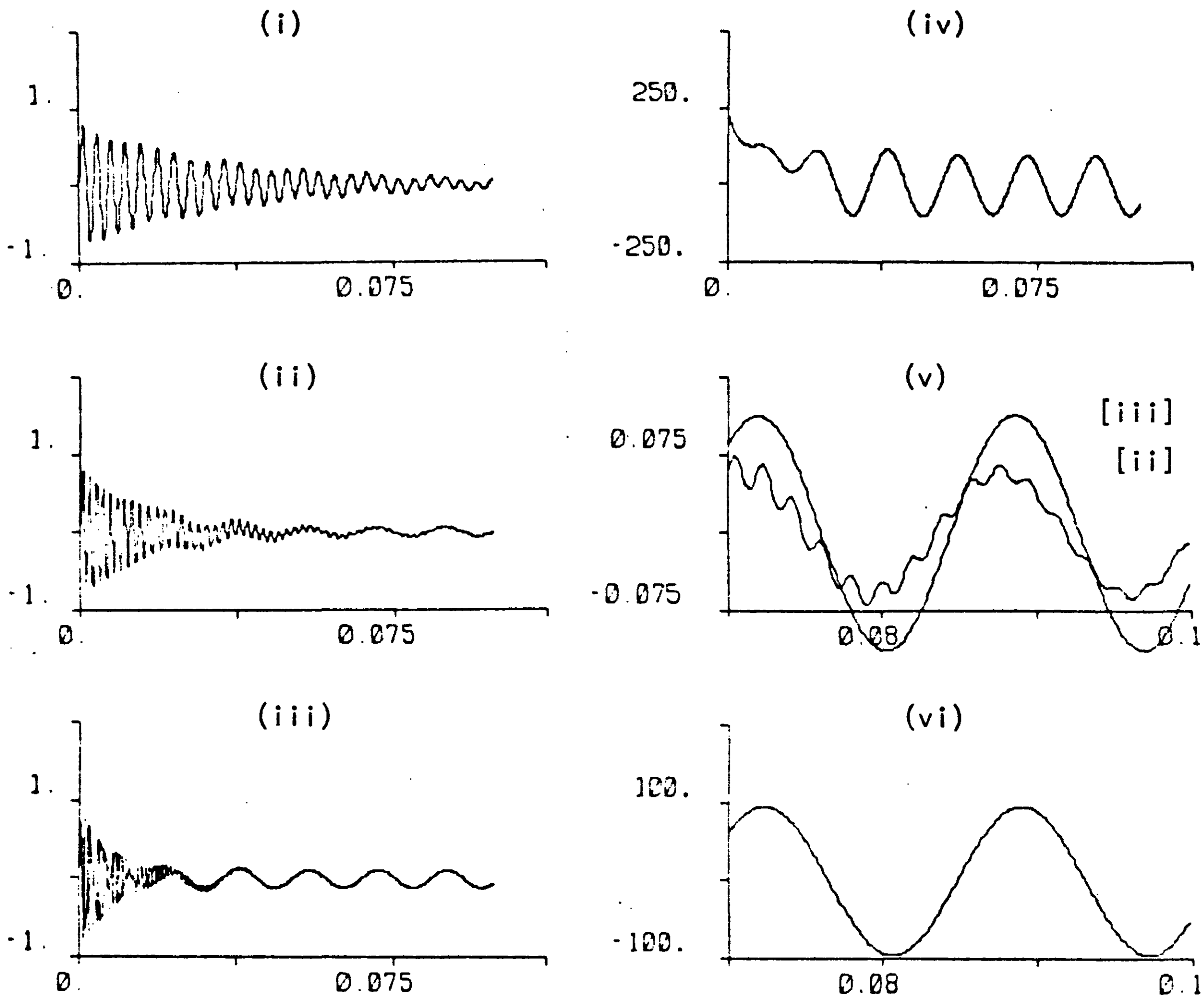



Fig. 3.2

Simulation study of asymptotic error behavior in sampled-data implementation of rotor flux observer

(i) Estimation error for observer covergence rate of five times the open-loop observer

(ii) Estimation error for observer convergence rate of ten times the open-loop observer

(iii) Estimation error for observer convergence rate of twenty times the open-loop observer

(iv) Actual axis-1 stator current

(v) Last 0.03 seconds of (ii) and (iii) superimposed

(vi) Last 0.03 seconds of (iv)

If we consider the rough average of the sampled waveform (shown with a dashed line) to be the effective input drive to the observer, we see that this waveform is essentially the continuous-time waveform delayed by one-half sample interval (T/2). The error between the two (sinusoidal) waveforms is given by

$$
\begin{aligned}
e_i &= I_s \sin(w_o t) - I_s \sin(w_o(t - T/2)) \\
&= (w_o T/2)\, I_s \cos(w_o t) \,.
\end{aligned} \tag{3.10}
$$

It is thus quite plausible that there is a steady sinusoidal input drive to the LTI error system. Now, we can study the effect of this input drive on the steady state behavior of the sampled-data error system, for varying observer convergence rates, as follows. The implementation of the sampled-data observer with auxiliary variable z can be approximated by

$$
\begin{aligned}
z' = {} & (1-kM/L_r)^{-1}[(-1/T_r)I + wJ](z + (k\sigma^2/L_r)\bar{i}_s) + \\
& (M/T_r)\bar{i}_s + k(R_s\bar{i}_s - \bar{v}_s) \,,
\end{aligned} \tag{3.11}
$$

where $\bar{i}_s$ and $\bar{v}_s$ are the "averages" of the sampled waveforms as shown in Fig. 3.3. Now, consider an error system in terms of the variable

$$
e_z = z - \{(1-kM/L_r)\lambda_r - (k\sigma^2/L_r)i_s\} \,. \tag{3.12}
$$

This is given by

$$
\begin{aligned}
e_z' = {} & (1-kM/L_r)^{-1}[(-1/T_r)I + wJ](e_z + (k\sigma^2/L_r)e_i) + \\
& (M/T_r)e_i + k(R_s e_i - e_v) \,,
\end{aligned} \tag{3.13}
$$

where $e_i$ and $e_v$ are the (approximately sinusoidal) differences between the inputs to the sampled-data observer $(\bar{i}_s, \bar{v}_s)$ and the actual continuous-time induction machine variables $(i_s, v_s)$.

We can easily obtain the steady state form for $e_z$ in (3.13) by realizing that we have a stable LTI system driven by steady sinusoidal inputs. In general, the steady state solution to the equation

$$x' = Ax + b[\exp(jw_o t)] \tag{3.14}$$

for stable A is given by

$$x = X_o[\exp(jw_o t)] \quad \text{, where} \tag{3.15a, b}$$
$$X_o = [jw_o I - A]^{-1} b \quad .$$

We shall only consider an approximate steady state solution for our problem, in the case where the driving frequency is small compared to the eigenvalues of the matrix A. This will give valuable insight into the behavior at accelerated convergence rates. The approximate solution for the generic problem just considered is

$$X_o = -A^{-1} b \; . \tag{3.16}$$

For the observer error system under consideration, with a fast convergence rate, i.e. with $(1-kM/L_r)^{-1}/T_r \gg w_o$ where $w_o$ is the drive frequency (377 rad/sec), the term in (3.13) multiplying $(1-kM/L_r)^{-1}$ must be approximately zero. This leads to the relation

$$e_z = -(k\sigma^2/L_r)e_i \; . \tag{3.17}$$

We see that as the convergence rate is increased, $e_z$, the error in the auxiliary variable becomes approximately constant, since k must tend toward $L_r/M$ in order that $(1-kM/L_r)^{-1}$ tend toward infinity. However, the error in the rotor flux estimate is related to the error $e_z$ by the gain factor $(1-kM/L_r)^{-1}$, and hence the rotor flux estimation error will grow linearly with the convergence rate for accelerated convergence rates. This is consistent with what is seen in Fig. 3.2. We conclude that extreme care must be taken in selecting the observer gain for the sampled-data realization of this observer.

**Sampled-Data Model for Fourth-Order Observer**

We would like to obtain a similar sampled-data model for the fourth-order observer for rotor flux and stator current proposed in Chapter 2. Fortunately, for constant speed w this observer has a matrix exponential that is, again, a simple function of the speed. The fundamental matrix **F** for this observer can be written as the product of two simple matrices (as shown in Chapter 2, eq. (2.41)), i.e.

$$\mathbf{F} = AQ(w) \text{ , } \quad \text{where} \tag{3.18}$$

$$A = \begin{bmatrix} k_{ij}I & (-M/\sigma^2)I \\ k_{\lambda j}I & I \end{bmatrix} \text{ , } Q(w) = \begin{bmatrix} (-1/T_r)I + wJ & 0 \\ 0 & (-1/T_r)I + wJ \end{bmatrix} . \tag{3.19a, b}$$

To compute the matrix exponential corresponding to **F**, we recall the similarity transformation discussed in Chapter 2 (see eq.(2.45)),

$$AQ(w) = P^{-1}DQ(w)P \text{ , where} \tag{3.20}$$

$$D = \begin{bmatrix} u_1 I & 0 \\ 0 & u_2 I \end{bmatrix} \quad \text{and} \quad P = \begin{bmatrix} p_{11} I & p_{12} I \\ p_{21} I & p_{22} I \end{bmatrix}. \tag{3.21a,b}$$

Then, the single step transition matrix for time interval T is given by the matrix exponential

$$\exp\{AQ(w)T\} = P^{-1}\exp\{DQ(w)T\}P \text{ ,} \tag{3.22}$$

and the matrix $\exp\{DQ(w)\}$ is a simple function of the speed w, given by

$$\begin{bmatrix} \exp(-u_1 T/T_r) \begin{bmatrix} \cos(u_1 wT) & -\sin(u_1 wT) \\ \sin(u_1 wT) & \cos(u_1 wT) \end{bmatrix} & 0 \\ 0 & \exp(-u_2 T/T_r) \begin{bmatrix} \cos(u_2 wT) & -\sin(u_2 wT) \\ \sin(u_2 wT) & \cos(u_2 wT) \end{bmatrix} \end{bmatrix}. \tag{3.23}$$

Fig. 3.4 compares the axis-1 sampled-data estimation error in rotor flux (i) and in stator current (ii) with the respective continuous-time estimation errors in these variables (iii and iv). The simulations are generated with a sampling interval of 0.1ms, and with the parameters $u_1 = 2$ and $u_2 = 10$. Note the close correspondence between the estimates produced by the sampled-data and continuous-time observers; however, it is again possible to obtain a steady state error residual in the sampled-data estimate by altering appropriate parameters. The analysis for this problem will not be carried further, but see the discussion for the sampled-data model of the rotor flux observer.

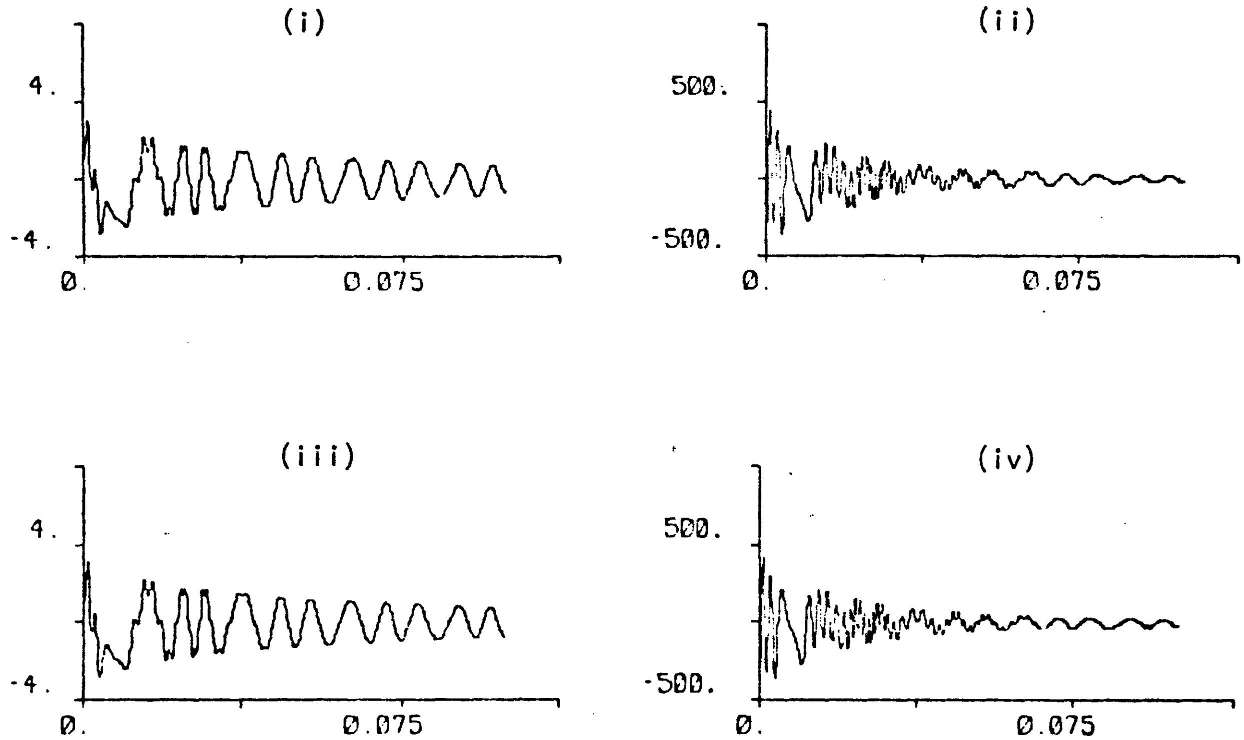



Fig. 3.4

Numerical simulation of sampled-data implementation of fourth-order observer

(i) Estimation error in sampled-data estimate of rotor flux

(ii) Estimation error in sampled-data estimate of stator current

(iii) Estimation error in continuous-time estimate of rotor flux

(iv) Estimation error in continuous-time estimate of stator current

## Chapter 4: EFFECTS OF DISTURBANCES

### Introduction

The analysis of the observers in Chapter 2 was based on a deterministic problem formulation. We have demonstrated the stability of those observers by computing the eigenvalues of the system equations that govern their estimation error for the time-invariant case (i.e. constant speed), or by showing that an appropriate Lyapunov function could be associated with the error dynamics for the general time-varying case. In the case where measurements of voltage, current, and speed are corrupted by additive noise, it is of interest to analyze the behavior of the proposed state estimators. For the case where the speed w is measured exactly, but the measurements of stator current and voltage are corrupted by noise (with known statistics), our analysis could follow the approach of the Kalman filter [8], since in this case, the underlying state-space model for flux dynamics (1.8) is linear. This approach would naturally lead to a fourth-order estimator that minimizes mean square estimation error.

We shall not follow this route, but shall restrict attention to the observers already proposed. We shall show that exact perturbation models can be obtained for these observers in the case where rotor speed (and stator current and voltage) measurements are corrupted by additive noise. In this case, the observer error dynamics will not be governed by homogeneous state-space equations, as drive terms associated with the disturbances will enter the state-space models for error dynamics. For the case where the measurement disturbances are constant biases, we shall show that simple steady state solutions to the observer perturbation models exist. We shall also consider the case of white, zero-mean disturbances with stationary and known statistics. To perform a proper analysis, we would enter the realm of Ito stochastic differential equations [8]. (Also, see [25] for a treatment of bilinear stochastic differential equations of this type.) We shall not introduce an Ito analysis, but instead, we shall present a state-space equation that approximately governs the propagation of the covariance of the estimation error. This equation will be shown to have a simple steady state solution when the underlying induction machine is also in steady state.

In this chapter, we shall consider measurements of the form:

$$\bar{v}_s = v_s + d_v, \quad \bar{i}_s = i_s + d_i, \quad \bar{w} = w + d_w. \qquad (4.1a,b,c)$$

For the rotor flux observer, we shall restrict attention to the case where measurements of voltage and current are exact, and focus on the nonlinear problem posed by uncertain speed measurements. A complete analysis including all measurement uncertainties will be performed for the fourth-order observer since it is believed to be the preferred implementation when current measurements are noisy.

**Rotor Flux Observer**

The rotor flux observer based on the "indirect" method (see

Chapter 2) with uncertain speed measurement takes the form:

$$\hat{\lambda}_r' = [(-1/T_r)I + \bar{w}J]\hat{\lambda}_r + (1/T_r)Mi_s + kI(\hat{v}_s - \bar{v}_s) \tag{4.2}$$

where $\hat{v}_s$ is computed from (2.7). The corresponding error dynamics becomes:

$$e' = (-1/T_r)e + \bar{w}J\hat{\lambda}_r - wJ\lambda_r + k(M/L_r)e', \text{ or}$$

$$e' = (1 - kM/L_r)^{-1} \{(-1/T_r)e + (\bar{w}J\hat{\lambda}_r - wJ\lambda_r)\} . \tag{4.3}$$

A direct calculation shows that the nonhomogeneous term of (4.3) can be written as follows:

$$\begin{aligned} \bar{w}J\hat{\lambda}_r - wJ\lambda_r &= \bar{w}J\hat{\lambda}_r - wJ\hat{\lambda}_r + wJ\hat{\lambda}_r - wJ\lambda_r \\ &= d_wJ\hat{\lambda}_r + wJe , \quad \text{or equivalently,} && (4.4a) \\ &= d_wJ\lambda_r + \bar{w}Je . && (4.4b) \end{aligned}$$

The dynamics for the rotor flux estimation error is then given by

$$e' = (1 - kM/L_r)^{-1} \{[(-1/T_r)I + wJ]e + d_wJ\hat{\lambda}_r\} . \tag{4.5}$$

Now, the disturbance on the speed measurement $d_w$ is seen to drive a stable error plant as a linear input. Of course, the input gain is dependent upon the flux estimate.

It is possible to compute the steady state error e(ss) for the case when $d_w$ is a constant bias, and the induction machine is in steady state. The result is as follows:

$$e(ss) = [(-1/T_r)I + (w-w_s)J]^{-1} J\hat{\lambda}_r d_w , \quad \text{or} \tag{4.6a}$$

$$e(ss) = [(-1/T_r)I + (\bar{w}-w_s)J]^{-1} J\lambda_r d_w , \tag{4.6b}$$

where $w_s$ is the frequency of the electrical drive. Note that the sensitivity to a constant bias in the speed measurement is independent of the gain factor $(1 - kM/L_r)^{-1}$.

In the case where $d_w$ is a white zero-mean random disturbance with stationary and known statistics, we would enter the realm of Ito stochastic differential equations [8] in carrying out a proper analysis, since the disturbance on the speed measurement enters the state-space error model (4.5) multiplicatively with the rotor flux estimate. See [25] for a treatment of bilinear stochastic differential equations of this type. Here, we shall limit ourselves to an approximate analysis of the temporal evolution of the covariance of the estimation error, based on the idea that the random component of the flux estimate is small compared to the magnitude of this estimate. In this case, the error covariance matrix $X = E\{ee^*\}$ can be propagated as if $\hat{\lambda}_r$ in (4.5) was <u>not</u> random, using the equation

$$X' = (1 - kM/L_r)^{-1} \{[(-1/T_r)I + wJ]X + X[(-1/T_r)I + wJ]^*\} + (1 - kM/L_r)^{-2} J\hat{\lambda}_r\hat{\lambda}_r^* J^* q_w . \quad (4.7)$$

where $E\{d_w(t)d_w(s)^*\} = q_w\delta(t-s)$. See a discussion of the behavior of linear systems driven by white noise such as that given in [8].

If the induction machine is in steady state, the steady state covariance of the estimate X(ss) may be computed. The steady state covariance is in general a function of the operating speed w. We shall not give a solution for this quantity explicitly, but we shall examine the trace of the steady state covariance matrix, since the trace of this matrix corresponds to the variance in the magnitude of the rotor flux estimate. An interesting observation is that the trace of X(ss) (or the variance in the magnitude of $\hat{\lambda}_r(ss)$) assumes a speed independent form. This quantity is given by:

$$\mathrm{trace}\{X(ss)\} = 1/2\ (1 - kM/L_r)^{-1} T_r\ (\hat{\lambda}_r^{*}\hat{\lambda}_r) q_W\ . \qquad (4.8)$$

Thus, we expect the variance in the magnitude of the estimate to increase approximately linearly with increasing observer error convergence rate.

Simulations of the reduced-order observer with a piece-wise constant disturbance added to the speed measurement are shown in Fig. 4.1. The disturbance which has standard deviation of ten percent of the machine base speed (377 rad/s, here) is constructed by passing a sequence of white zero-mean gaussian random variables through a zero-order hold. The disturbance is held constant over intervals of 0.2ms which is short relative to the observer error dynamics. The traces in (i), (ii), and (iii) show the actual temporal evolution of the rotor flux estimation error for observer convergence rates of two, five, and ten times the convergence rate of the open-loop observer (see (2.4)). The traces in (iv), (v), and (vi) show the temporal evolution of the squared magnitude of the estimation error for observer convergence rates of two, five, and ten times that of the open-loop observer. It is indeed quite plausible from Fig. 4.1 that the steady state variance in the magnitude of the flux estimate increases linearly with the gain factor $(1 - kM/L_r)^{-1}$.

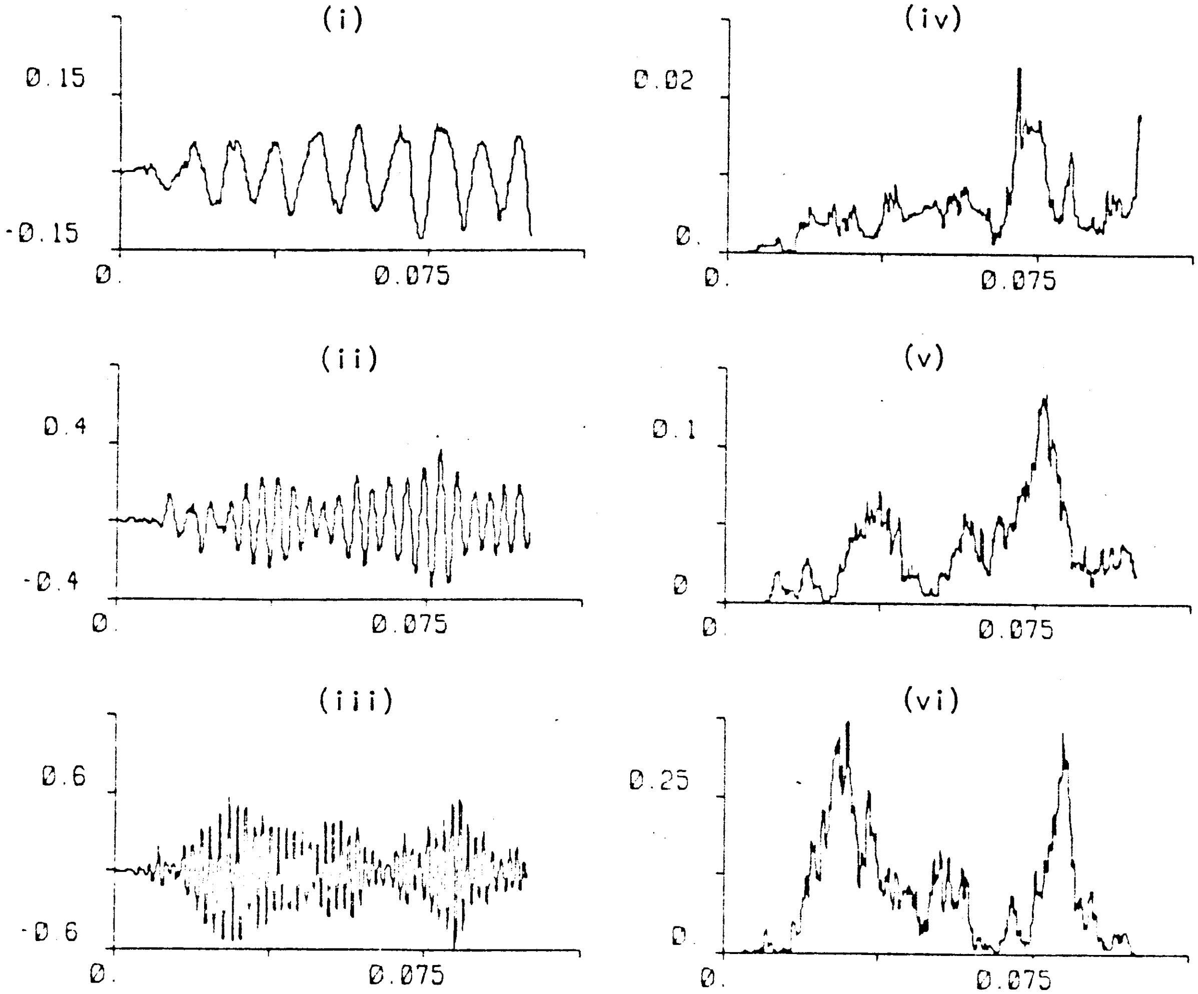



Fig. 4.1

Numerical simulation of rotor flux observer with "white" , zero-mean disturbance on the speed measurement

(i) Axis-1 estimation error for observer convergence rate of two times the open-loop observer

(ii) Axis-1 estimation error for observer convergence rate of five times the open-loop observer

(iii) Axis-1 estimation error for observer convergence rate of ten times the open-loop observer

(iv) Squared-magnitude of estimation error for observer of (i)

(v) Squared-magnitude of estimation error for observer of (ii)

(vi) Squared-magnitude of estimation error for observer of (iii)

**Fourth-Order Observer**

Under conditions where only uncertain measurements $\bar{v}_s, \bar{i}_s$, and $\bar{w}$ are available, the fourth order observer can be constructed as in (2.39), but with the available data as:

$$\begin{bmatrix} \hat{i}_s' \\ \hat{\lambda}_r' \end{bmatrix} = \begin{bmatrix} -p_1 I & \{M/(\sigma^2 T_r)\}I \\ (M/T_r)I & (-1/T_r)I \end{bmatrix} + \bar{w}\begin{bmatrix} 0 & -(M/\sigma^2)J \\ 0 & J \end{bmatrix} \begin{bmatrix} \hat{i}_s \\ \hat{\lambda}_r \end{bmatrix} + \begin{bmatrix} (L_r/\sigma^2)I \\ 0 \end{bmatrix} \bar{v}_s + \begin{bmatrix} k_i I + k_{ij}\bar{w}J \\ k_\lambda I + k_{\lambda j}\bar{w}J \end{bmatrix} (\hat{i}_s - \bar{i}_s) \ . \qquad (4.9)$$

The resulting error system for this estimator is obtained by subtracting (2.38) from (4.9). Using a calculation similiar to that performed on the bilinear term of the error system of the rotor flux observer, we find:

$$\begin{bmatrix} e_i' \\ e_\lambda' \end{bmatrix} = \left\{ \begin{bmatrix} (-p_1+k_i)I & \{M/(\sigma^2 T_r)\}I \\ \{(M/T_r)+k_\lambda\}I & (-1/T_r)I \end{bmatrix} + \bar{w}\begin{bmatrix} k_{ij}J & (-M/\sigma^2)J \\ k_{\lambda j}J & J \end{bmatrix} \right\} \begin{bmatrix} e_i \\ e_\lambda \end{bmatrix} + \qquad (4.10)$$

$$d_w \begin{bmatrix} 0 & (-M/\sigma^2)J \\ 0 & J \end{bmatrix} \begin{bmatrix} i_s \\ \lambda_r \end{bmatrix} + \begin{bmatrix} (L_r/\sigma^2)I \\ 0 \end{bmatrix} d_v - \begin{bmatrix} k_i I + \bar{w}k_{ij}J \\ k_\lambda I + \bar{w}k_{\lambda j}J \end{bmatrix} d_i \ .$$

And, if $k_i$ and $k_\lambda$ are selected such that $k_i = p_1 - k_{ij}/T_r$ and $k_\lambda = -M/T_r - k_{\lambda j}/T_r$, respectively, we obtain the following error dynamics:

$$\begin{bmatrix} e_i' \\ e_\lambda' \end{bmatrix} = AQ(\bar{w})\begin{bmatrix} e_i \\ e_\lambda \end{bmatrix} + b_w d_w + B_v d_v + B_i(\bar{w})d_i \ , \text{ where} \qquad (4.11)$$

$$A = \begin{bmatrix} k_{ij}I & (-M/\sigma^2)I \\ k_{\lambda j}I & I \end{bmatrix}, \quad Q(\bar{w}) = \begin{bmatrix} (-1/T_r)I + \bar{w}J & 0 \\ 0 & (-1/T_r)I + \bar{w}J \end{bmatrix},$$

$$b_w = \begin{bmatrix} 0 & (-M/\sigma^2)J \\ 0 & J \end{bmatrix} \begin{bmatrix} i_s \\ \lambda_r \end{bmatrix} , \quad B_v = \begin{bmatrix} (L_r/\sigma^2)I \\ 0 \end{bmatrix}, \quad \text{and}$$

$$B_i(\bar{w}) = \begin{bmatrix} k_i I + \bar{w} k_{ij} J \\ k_\lambda I + \bar{w} k_{\lambda j} J \end{bmatrix} . \tag{4.12a, b, c, d, e}$$

If $d_v, d_i$, and $d_w$ are constant offsets, it is straight forward to compute $e_i$ and $e_\lambda$ under steady state conditions. The explicit result will not be shown here. If $d_v$, $d_i$, and $d_w$ are uncorrelated, white, zero-mean random variables with (co)variances $q_v I$, $q_i I$, and $q_w$, respectively, and if we assume the random components of the estimate are relatively small, an approximate covariance of the estimate can be propagated according to:

$$Y' = AQ(\bar{w})Y + YQ(\bar{w})^* A^* + q_v B_v B_v^* + q_i B_i(\bar{w}) B_i(\bar{w})^* + q_w b_w b_w^* . \tag{4.13}$$

For steady state conditions and relatively small $q_w$, it is possible to compute an approximate steady state covariance Y(ss). This will not be given here. Further studies are necessary to address the problem of minimizing Y.

# Chapter 5: NONLINEAR ESTIMATION

## Introduction

This is intended to be an exploratory chapter that discusses some previous work in the area of nonlinear estimation for induction machines and suggests certain hopefully new approaches to the problems at hand. In particular, two problems will be addressed. Firstly, the flux estimation problem will be re-examined in the presence of static model parameter uncertainties. The effects of an unknown (or time-varying) rotor time constant on the "indirect" rotor flux estimation scheme have been widely studied in the literature, see for example [1,5,22,26]. We shall consider other uncertainties as well. Secondly, possible methods for the estimation of the rotor speed from the electrical machine terminals will be explored. Encouraging work in this area appears in [6], while other results appear in [7]. In general, the speed estimation problem is more demanding than the above parameter estimation problems, because speed can be viewed as a parameter that can vary more rapidly than, for example, the rotor time constant.

The observers for rotor flux (or rotor flux and stator current) discussed in Chapter 2 can all be classified as linear estimators. These observers reconstruct the state of the linear, time-varying dynamical system model for machine flux when speed is considered as a known, but time-varying parameter. For the two estimation problems introduced above, the system models must be augmented to include as state variables the unknown parameters and/or the unknown speed. The nonlinearity arises because these unknown parameters and the speed enter the system model multiplicatively with the usual state variables, namely rotor flux and stator flux (or current) vectors, see the state-space model (1.8) in Chapter 1.

Three approaches, which are in principle applicable to both estimation problems, will be considered. The first method involves the application of the extended Kalman filter algorithm [8], which is based on the method of linearization (small perturbation analysis). Dote and Anbo [22] have demonstrated the validity of a variation of this approach to the flux estimation problem with unknown winding resistances. The speed estimation problem has been attacked by Hillenbrand [6] and De Foenel et. al. [7] with this algorithm.

The second approach, to be termed here the "bounded nonlinearity" method, places a bound on the magnitude of the nonlinear components of the system , permitting the design of an observer with a linear prediction error term, based on the linear components of the system. Our discussion of this approach will be based on the paper [14] of Derese, Stevens, and Noldus. The development in [14] was strictly for bilinear systems, but here it will be extended to other nonlinearities (e.g. the quadratic terms arising in the induction machine model).

The third approach to be considered applies the Model Reference Adaptive Observer algorithms for linear, time-invariant systems with unknown parameters [27-33]. The recent doctoral thesis of Shih [34] provides a framework for this method, and suggests the extension to systems with slowly varying parameters. This will be of use in the speed estimation problem, as well as in the flux estimation problem with uncertain parameters.

### Extended Kalman Filter

A reasonable starting point for this investigation of nonlinear estimation techniques is with the extended Kalman filter. A brief description of the algorithm will be given here, while more details can be found in [8] and [35]. There are numerous variations of this algorithm, many of which are discussed in [8] and [35]. The particular application to the combined parameter and state estimation for linear, time-invariant systems is addressed in [36]. The algorithm is based on the application of the usual Kalman filter to a linearization of the system model and measurement equation about the estimated value of the state.

If we consider the nonlinear state-space equation

$$x' = F(x) + Bw \quad ; \quad x(0) = x_o \tag{5.1a}$$
$$y = H(x) + v \ , \tag{5.1b}$$

where w and v are zero-mean, independent, white noise processes with known statistics ($E\{ww^*\} = Q$, $E\{vv^*\} = R$), the linearization used in the filter algorithm is as follows:

$$x' = F(\hat{x}) + [F_x(\hat{x})](x-\hat{x}) + Bw \tag{5.2a}$$
$$y = H(\hat{x}) + [H_x(\hat{x})](x-\hat{x}) + v \ , \tag{5.2b}$$

where $[F_x(*)]$ and $[H_x(*)]$ denote the gradients of $F(*)$ and $H(*)$, respectively.

For the case where we have a continuous-time system model, but obtain sampled measurements, we shall implement the continuous-discrete extended Kalman filter [8,35]. During the prediction stage, the mean of the state estimate is propagated in accord with the nonlinear state-space model (5.1a) by

$$\hat{x}' = F(\hat{x}) \ , \tag{5.3}$$

and the error covariance of this estimate is propogated in accord with the approximate, linearized model

$$P' = [F_x(\hat{x})]P + P[F_x(\hat{x})]^* + BQB^* \ . \tag{5.4}$$

At the measurement incorporation times, the state estimate is updated using a Kalman gain derived from the gradient of the measurement equation (i.e. (5.2b)) and the approximate error covariance resulting from (5.4). The state estimate is updated in accord with the following law:

$$\hat{x}_k^+ = \hat{x}_k^- + K_k(y_k - H(\hat{x}_k^-)) \ , \tag{5.5}$$

where $K_k$ is given by

$$K_k = P_k^-[H_x(\hat{x}^-)]^*\{[H_x(\hat{x}^-)]P_k^-[H_x(\hat{x}^-)]^* + R\}^{-1} \ , \tag{5.6}$$

where all the quantities denoted with the symbol ($^-$) on the right-hand sides of (5.5) and (5.6) are evaluated at the instant before the measurement is aquired. The symbol ($^+$) indicates the associated quantity is evaluated after the incorporation of the measurement. The estimation error covariance is updated at the measurement incorporation time in accord with the following law:

$$P_k^+ = \{I - K_k[H_x(\hat{x})]\}P_k^- \quad . \tag{5.7}$$

A variation of this algorithm has been applied by Dote and Anbo [22] to the induction machine flux estimation problem with unknown stator and rotor winding resistances. The motivation for this application is that these resistances are temperature sensitive, and hence the operating values cannot, in general, be determined before the operation of the machine system. To cast the problem into the above framework, the state equation is augmented to include the winding resistances as state variables, and an appropriate noise model is chosen. The modified state equation is given by

$$\begin{aligned} \begin{bmatrix} \lambda_s' \\ \lambda_r' \end{bmatrix} &= \left\{ -RL^{-1} + w \begin{bmatrix} 0 & 0 \\ 0 & J \end{bmatrix} \right\} \begin{bmatrix} \lambda_s \\ \lambda_r \end{bmatrix} + \begin{bmatrix} v_s \\ 0 \end{bmatrix} + \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \\ \begin{bmatrix} R_s' \\ R_r' \end{bmatrix} &= \begin{bmatrix} u_3 \\ u_4 \end{bmatrix} \quad , \end{aligned} \tag{5.8}$$

where $u_1$, $u_2$, $u_3$, and $u_4$ represent the system drive noise.

The particular algorithm used by Dote and Anbo in [22] is termed the <u>second-order Gaussian</u> filter [8,35], which is applied by forming a Taylor expansion for the state equation about the <u>known</u> state estimate, and truncating after the second term. The particular algorithm also uses the assumption that measurement and drive noises have Gaussian distributions, and that the state itself always has a Gaussian distribution. The computations involved in the prediction and update stages analagous to (5.3-

5.7) are performed using both the first- and second-order terms of the Taylor series representation of the nonlinearities, and using the assumption that the state has a Gaussian distribution. Dote and Anbo [22] indicate that this may be a rather effective algorithm for the induction machine model, since this model has only linear and quadratic terms.

Encouraging simulation results are presented in [22]. However, there are obviously some critical difficulties. Firstly, if the estimate differs widely from the actual state, the Kalman gain computed by the estimator which assumes Gaussian statistics for the state may lead to divergence. (In the case of the proper extended Kalman filter, the Kalman gain computed by linearizing the system model about an inaccurate state estimate can also lead to divergence.) One may argue that reliable estimates of initial conditions for the flux state variables and winding resistances are always available so that the convergence of the estimator is generally guaranteed. However, this estimator cannot be guaranteed to be robust in the presence of large disturbances because, then, the estimate may be driven far from the true trajectory. A second difficulty lies in the construction of the noise model. It is not always clear how one should obtain reasonable statistics for the driving and/or measurement noise processes.

The extended Kalman filter has also been applied to the speed estimation problem in [6] and [7]. Hillenbrand [6] uses a reduced order system model with state variables of stator flux and rotor speed, considering the measured stator current as exactly known. This form of the model is particularly effective for the use of the extended Kalman filter because the system dynamics are linear and very simple, and nonlinearities appear only in the output equation. In addition, Hillenbrand augments the system model to allow estimation of the (unknown) rotor time constant along with stator flux and rotor speed. With this model, the output is given as a function of the stator current and its derivative. The model in [6] is given by

$$\begin{aligned} \lambda_s' &= v_s - R_s i_s + u_1 \\ w' &= (1/H)\, i_s^{*} J \lambda_s - (1/H)\, T_L + u_2 \\ T_r' &= u_3 \;, \end{aligned} \qquad (5.9)$$

with a nonlinear measurement equation given in terms of $i_s$, $i_s'$, and $v_s$ by

$$y(i_s, i_s', v_s) = (R_r/\sigma^2)\lambda_s + wJi_s - (L_r/\sigma^2)wJ\lambda_s + u_4 \quad . \qquad (5.10)$$

If appropriate models are made for the load torque $T_L$, and for driving and measurement noises ($u_1$, $u_2$, $u_3$, and $u_4$), it is straightforward to apply the extended Kalman filter. With some algebraic manipulations, Hillenbrand is able to avoid differentiation of the current waveform, and derives a discrete-time representation corresponding to (5.9). The simulation results given in [6] are again encouraging, but we note that the exact initial condition for the rotor speed is assumed to be known. Neither global stability nor any particular region of convergence is established.

Another example of the application of the extended Kalman filter algorithm to the speed estimation problem can be found in [7]. The speed estimator of [7], which is similar in some respects to that of [6], is integrated into a speed-control loop. Here again, successful numerical simulation results are presented to verify this method of speed estimation.

The extended Kalman filter algorithm (and its variations) is clearly a viable approach to the parameter and speed estimation problems. Future work to establish the robustness of state estimators derived with this algorithm will be of value. The region of convergence (or the set of initial estimation errors that lead to convergent estimates) might be determined analytically, or more likely, by simulation and experimentation.

**Bounded Nonlinearity**

In this section, we shall study a class of observers developed by Derese, Stevens, and Noldus [14] for bilinear systems with bounded inputs. The principle for demonstrating stability of these observers, namely placing an appropriate bound on the input, can be extended to observers for other nonlinear systems by placing a bound on the nonlinear components of their state equations. Though the approach is promising, we have not yet obtained a successful application to the induction machine model.

The class of systems considered by Derese et. al. in [14] is characterized by a bilinear state-space equation of the form:

$$x' = A_o x + \sum_{i=1}^{m} u_i A_i x + Bu \; ; \qquad x(0) = x_o$$
$$y = Cx \; . \tag{5.11}$$

The observers proposed for these systems are constructed with a corrective linear-prediction error term, in the fashion described in Chapter 1:

$$\hat{x}' = A_o\hat{x} + \sum_{i=1}^{m} u_i A_i \hat{x} + Bu + H(\hat{y} - y) \; ; \; \hat{x}(0) = 0 \tag{5.12}$$
$$\hat{y} = C\hat{x} \; .$$

With this structure, the problem is to choose H so that the observer state $\hat{x}$ always converges to the underlying system state x.

To study this issue, we must consider the error dynamics obtained by subtracting (5.11) from (5.12). This is given by

$$e' = (A_o - HC + \sum_{i=1}^{m} u_i A_i)e \quad , \text{ where} \tag{5.13}$$
$$e = \hat{x} - x \; .$$

This error dynamics may be viewed as a linear, time-invariant system driven by a state-dependent disturbance, that is, we can rewrite (5.13) in the form:

$$e' = (A_o - HC)e + v \quad , \quad \text{where} \tag{5.14}$$
$$v = (\sum_{i=1}^{m} u_i A_i)e \quad .$$

Asymptotically stable behavior of (5.14) may be obtained if H is selected to make the homogeneous system

$$e' = (A_o - HC)e \tag{5.15}$$

converge sufficiently fast, and the driving disturbance v is sufficiently small.

We recall that the system dynamics (5.15) can be made arbitrarily fast if the pair $[A_o, C]$ satisfies an observability condition [13]. The requirement that the driving disturbance v be small is embodied in [14] via the constraint

$$v^* v \leq e^* S e \quad , \tag{5.16}$$

where S is symmetric and positive definite.

We shall consider a Lyapunov candidate that is quadratic in the observer error to establish the stability of the observer. Let

$$V = e^* K e \quad , \tag{5.17}$$

where K is symmetric and positive definite. Direct calculation yields the following:

$$\begin{aligned} V' &= e'^* K e + e^* K e' \\ &= e^*(A_o - HC)^* K e + e^* K(A_o - HC)e \; + \\ &\quad e^*\Big(\sum_{i=1}^{m} u_i A_i\Big)^* K e + e^* K\Big(\sum_{i=1}^{m} u_i A_i\Big)e \quad . \end{aligned} \tag{5.18}$$

Because of the constraint imposed on v (or the $u_i$), it follows that

$$V' \leq V' + e^* S e - e^*\Big(\sum_{i=1}^{m} u_i A_i\Big)^*\Big(\sum_{i=1}^{m} u_i A_i\Big)e \quad , \quad \text{or} \tag{5.19}$$

$$\begin{aligned} V' \leq\; & e^*(A_o - HC)^* K e + e^* K(A_o - HC)e + e^* S e + e^* K^2 e \; + \\ & - e^*\Big(K - \sum_{i=1}^{m} u_i A_i\Big)^*\Big(K - \sum_{i=1}^{m} u_i A_i\Big)e \; . \end{aligned} \tag{5.20}$$

The last term on the right-hand side of (5.20) is negative semi-definite, and hence, it follows that

$$V' \leq e^*(A_o - HC)^*Ke + e^*K(A_o - HC)e + e^*Se + e^*K^2e \ . \tag{5.21}$$

It follows that the stability of the observer is guaranteed if an observer gain matrix H can be found such that the right-hand side of (5.21) is always less than or equal to zero.

For practical applications where a specified rate of observer error convergence (say r) is required, we can look for a matrix H such that

$$V' \leq -2rV \tag{5.22}$$

is satisfied. This requirement can be summarized for this problem with the matrix inequality

$$(A_o - HC)^*K + K(A_o - HC) + S + K^2 \leq -2rK \text{ , or equivalently} \tag{5.23}$$

$$(A_o + rI - HC)^*K + K(A_o + rI - HC) + S + K^2 \leq 0 \ . \tag{5.24}$$

The problem remains to find an H that satisfies (5.24). The method proposed in [14] to solve this inequality is to select an H of the form

$$H = (1/2p)K^{-1}C^* \ . \tag{5.25}$$

It has been shown that if a solution H to (5.24) exists, then a solution of the form in (5.25) also exists. It suffices, then, to look for a solution of this form, so that the inequality (5.24) becomes

$$(A_o + rI)^*K + K(A_o + rI) + S + K^2 - (1/2p)C^*C \leq 0 \ . \tag{5.26}$$

The problem has now been transformed so that, if a positive definite, symmetric solution K to (5.26) can be found, then a matrix H that satisfies (5.24) is determined. We note that (5.26) has a positive definite solution K for equality if $S = 0$ and the pair $[(A_o + rI),C]$ is observable. This corresponds to the statement that the algebraic Riccati equation arising in the steady state Kalman filter (with an appropriate noise model) has a positive definite solution if the underlying system is observable. In general, (5.26) can be solved for equality with an iterative technique, just as is possible with the usual Ricatti equation. See the discussion in [14] for a characterization of the solution for (5.26).

To extend this method of observer design for bilinear systems to other nonlinear systems, we are led to impose a bound on the nonlinear terms of the resulting observer error models in the same way that a bound was placed on the bilinear components of the bilinear obsever error model, above. We shall consider the design of a fifth-order speed and flux observer for the induction machine model given in Chapter 1, (1.8) and (1.12). It will be assumed that the terminal variables, stator voltage and current, are measured, and that the load torque is known. Although, the last assumption can be quite unrealistic, it is convenient to illustrate the technique of observer design; it is possible to consider a more detailed (higher order) model for the mechanical load, and to then include additional states in the derived observer (see [13]). If we define the five-component state vector $\mathbf{x}$ by

$$\mathbf{x} = [\lambda_s{}^* \; \lambda_r{}^* \; w]^*, \tag{5.27}$$

then the system model can be written compactly as

$$\begin{aligned} \mathbf{x}' &= A\mathbf{x} + f(\mathbf{x}) + Bu \;;\; \mathbf{x}(0) = \mathbf{x}_o \\ y &= C\mathbf{x} \;, \end{aligned} \tag{5.28}$$

where

$$A = \begin{bmatrix} (-R_rL_s/\sigma^2)I - \gamma J & (M/\sigma^2)I & 0I \\ (M/\sigma^2)I & (-R_sL_r/\sigma^2)I - \gamma J & 0I \\ 0 & 0 & 0 \end{bmatrix},$$

(5.29a, b, c, d, e, f)

$$f(\mathbf{x}) = \begin{bmatrix} 0I \\ wJ\lambda_r \\ (2M/H\sigma^2)\lambda_r{}^*J\lambda_s \end{bmatrix}, \quad B = \begin{bmatrix} I & 0I \\ 0I & 0I \\ 0 & (1/H) \end{bmatrix},$$

$$C = [(L_r/\sigma^2)I \quad (-M/\sigma^2)I \quad 0] \;, \; u = [v_s{}^* \; T_L]^* \;,$$

$$y = i_s \;,$$

and the stator voltage is taken in the rotating reference frame at instantaneous position $\phi$.

We shall construct an observer for this system with a linear prediction-error term, and then examine the resulting error system to select the form of the observer gain. Consider the observer

$$\begin{aligned} \hat{\mathbf{x}}' &= A\hat{\mathbf{x}} + f(\hat{\mathbf{x}}) + Bu + H(\hat{y} - y) \;;\; \hat{\mathbf{x}}(0) = 0 \\ \hat{y} &= C\hat{\mathbf{x}}, \end{aligned} \tag{5.30}$$

which has its estimation error $e = \hat{\mathbf{x}} - \mathbf{x}$ governed by the following error dynamics:

$$e' = (A + HC)e + f(\hat{\mathbf{x}}) - f(\mathbf{x}) \; ; \; e(0) = -\hat{\mathbf{x}}_o \, . \tag{5.31}$$

We shall now attempt to place a bound on the nonlinear term $v = f(\hat{\mathbf{x}}) - f(\mathbf{x})$, which is viewed here as a disturbance. We proceed by performing a simple calculation that shows that v may be expressed similarly to the form given in the above development for bilinear observers:

$$v = f(\hat{\mathbf{x}}) - f(\mathbf{x})$$

$$= \begin{bmatrix} 0I - 0I + 0I - 0I \\ \{\hat{w}J\hat{\lambda}_r - \hat{w}J\lambda_r + \hat{w}J\lambda_r - wJ\lambda_r\} \\ (2M/H\sigma^2)\ \{\hat{\lambda}_r^{*}J\hat{\lambda}_s - \hat{\lambda}_r^{*}J\lambda_s + \hat{\lambda}_r^{*}J\lambda_s - \lambda_r^{*}J\lambda_s\} \end{bmatrix}$$

$$= \begin{bmatrix} 0I & 0I & 0I \\ 0I & \hat{w}J & J\lambda_r \\ (2M/H\sigma^2)\hat{\lambda}_r^{*}J & -(2M/H\sigma^2)\lambda_s^{*}J & 0 \end{bmatrix} e \quad . \tag{5.32}$$

It should be straightforward to place some bound of the form

$$v^{*}v \leq e^{*}Se \tag{5.33}$$

on v, because the state $\mathbf{x}$ trajectories are bounded under the mildest restrictions [20], and we assume the designed observer will also be stable, so that the observer state $\hat{\mathbf{x}}$ is also bounded. The remainder of the development for the specification of the observer gain H follows by the selection of a quadratic Lyapunov function in the observer error, and all the steps taken in the previous discussion for the bilinear observer design. Because the development is parallel to previous one, the details will be omitted. It is of great interest that this method may be applied to many other nonlinear systems, especially those with polynomial-type nonlinear terms.

There are a number of considerations that must be taken into account for the particular application to the induction machine observer. We note that the speed cannot be detected from the electrical machine terminals if the rotor flux is identically zero. This can be seen by considering the equation relating the derivative of stator current to stator current, stator voltage, rotor speed, and rotor flux, given by

$$i_s' = -p_1 i_s - (M/\sigma^2)[(-1/T_r)I+wJ]\lambda_r + (L_r/\sigma^2)v_s \text{ , where} \qquad (5.34)$$
$$\sigma^2 = L_r L_s - M^2 \text{ and } p_1 = (L_r^2 R_s + M^2 R_r)/(\sigma^2 L_r) \quad .$$

Clearly, when the rotor flux is zero, the rotor speed has no effect on the behavior detectable at the electrical terminals. Equivalently, we could obtain a perturbation model for the entire state space system about the operating point $\lambda_r = 0$, and find that the rotor speed is unobservable from the inputs and outputs (stator voltage and current).

This fact places a restriction on the region of state space inwhich the machine state $\mathbf{x}$ and the observer state $\hat{\mathbf{x}}$ are to be bounded. In particular, we must select a region of the state-space that does not contain $\lambda_r = 0$. The bounding regions defined by the inequality (5.33) are inherently convex, and therefore we must select a region that is biased away from the origin in the rotor flux components of the state vector. In the usual operation of the induction machine, the flux and current vectors all rotate (approximately synchronously) with respect to the fixed stator reference frame. If a rotating reference frame at instantaneous position $\phi$ is selected such that axis-1 aligns with the stator current vector, it is possible to place a reasonable bound on stator and rotor flux vectors defined in this frame. In many applications, the control algorithm requires essentially constant magnitude of the rotor flux. Depending on the torque that is commanded, the rotor flux vector may be situated at a positive or negative angle with respect to the axis defined by the stator current vector. This is depicted below in Fig. 5.1.

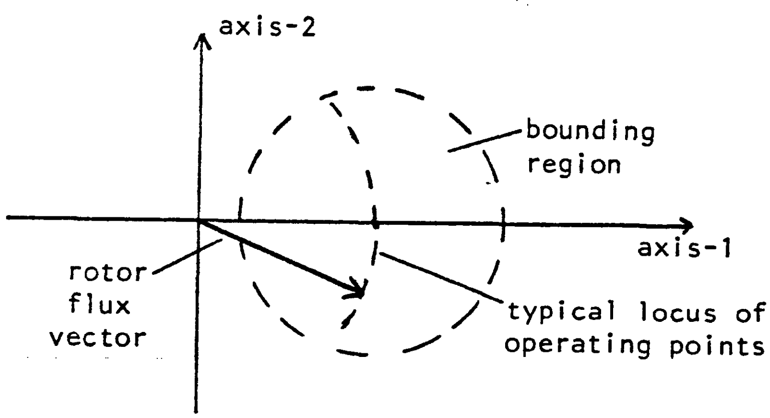



Fig. 5.1

Then, the rotor flux vector $\lambda_r$ might effectively be restricted to an circular region as shown in Fig. 5.1. Similarly, the stator flux vector could be bounded.

So far, we have been unable to solve the appropriate inequality to obtain an observer gain matrix that leads to convergent estimates when the state vector is bounded as described above. This technique might well be applied to the parameter identification problem when speed is considered to be known. Future work may lead to other valuable results. For instance, this method may lead to a simple adaptive observer scheme for LTI systems when bounds on the parameter uncertainties are known, and some knowledge of the typical state trajectories is available.

**Model Reference Adaptive Observer**

A well known approach to the design of observers for linear, time-invariant systems with unknown parameters is surveyed and developed further in the recent doctoral thesis of Shih [34]. We refer to [34] for a summary of the model reference adaptive observer algorithm and many extensions (e.g. to systems with unknown order or with time-varying parameters). As mentioned in Chapter 1, the state-space model for the stator and rotor flux vectors (or equivalently, stator current and rotor flux vectors) is a linear, time-invariant system when the rotor speed is constant. If some machine parameters or the rotor speed are unknown, it is then natural to apply the adaptive observer of [34] to this model. The resulting observer can identify the unknown parameters of an equivalent (canonical form) realization of the system, and simultaneously generate estimates of the states in this particular realization. It is possible to identify many of the parameters (including the rotor speed) of the usual induction machine model given in Chapter 1, and to obtain a scaled estimate of the rotor flux vector.

There is a modification of the usual adaptive observer algorithm that is developed here: the algorithm in [34] for single-input, single-output systems is applied to the two-input, two-output induction machine flux model by collapsing the usual two-component vector representations to single-component, complex variables; see Chapter 1 for details on how this may be done. We shall consider the complex vector representation of the state-space model for stator current and rotor flux given in the section on fourth-order observers in Chapter 2. This is described by

$$\begin{bmatrix} i_s' \\ \lambda_r' \end{bmatrix} = \begin{bmatrix} -p_1 & (M/\sigma^2)(1/T_r - jw) \\ M/T_r & -1/T_r + jw \end{bmatrix} \begin{bmatrix} i_s \\ \lambda_r \end{bmatrix} + \begin{bmatrix} L_r/\sigma^2 \\ 0 \end{bmatrix} v_s \tag{5.35}$$

where $\sigma^2 = L_rL_s - M^2$, $p_1 = (L_r{}^2R_s + M^2R_r)/(\sigma^2L_r)$, and $i_s$, $\lambda_r$, and $v_s$ are complex representations of stator current, rotor flux, and stator voltage vectors, respectively. The complex transfer function $T(s) = I(s)/V(s)$ can be computed by taking the Laplace transform of (5.35). The result is given by

$$\frac{(s + 1/T_r - jw)\ L_r/\sigma^2}{s^2 + s(p_1 + 1/T_r - jw) + p_1(1/T_r - jw) - (M^2/\sigma^2T_r)(1/T_r - jwM/\sigma^2)}$$

or

$$\frac{(s + 1/T_r - jw)\ L_r/\sigma^2}{s^2 + s((L_rR_s + L_sR_r)/\sigma^2 - jw) - L_rR_s/\sigma^2\ (-1/T_r + jw)} \quad . \tag{5.36}$$

It is now straightforward to construct an adaptive observer for a non-minimal state-space realization of the transfer function $T(s)$, using the procedure described in [34]. Let the parameters of $T(s)$ be renamed so that $T(s)$ is given by

$$T(s) = \frac{Cs + D}{s^2 + Bs + W} \quad . \tag{5.37}$$

Then a non-minimal state-space realization for $T(s)$ can be constructed as follows:

$$
\begin{aligned}
i_s' &= (A-B)(i_s - Ay) - Wy + Dz + C(v_s - Az) \\
y' &= -Ay + i_s \\
z' &= -Az + v_s \quad ,
\end{aligned}
\tag{5.38}
$$

where the additonal state variables y and z, and the coefficients B, W, C, and D are, in general, complex valued. The design parameter A (which may also be complex) is selected to obtain a stable realization (i.e. $\mathrm{re}\{A\} < 0$), and possibly to filter high frequency noise. The adaptive observer is constructed with the state variables $\hat{i}_s$, $\hat{B}$, $\hat{W}$, $\hat{C}$, and $\hat{D}$, but also uses the known values of the states $i_s$, y, and z of the system (5.38). The adaptive observer is given by

$$
\begin{aligned}
\hat{i}_s' &= (A-\hat{B})(i_s - Ay) - \hat{W}y + \hat{D}z + \hat{C}(v_s - Az) - Re \\
\hat{B}' &= q_1 e^*(i_s - Ay) \\
\hat{W}' &= q_2 e^* y \\
\hat{C}' &= -q_3 e^*(v_s - Az) \\
\hat{D}' &= -q_4 e^* z \quad ,
\end{aligned}
\tag{5.39}
$$

where $e = \hat{i}_s - i_s$, $(^*)$ now denotes the complex conjugate, and R is a design parameter that should be selected to be real and positive.

To analyze the convergence properties of this observer, we consider the error system that results on subtracting (5.38) from (5.39):

$$
\begin{aligned}
e' &= -e_B(i_s - Ay) - e_W y + e_D z + e_C(v_s - Az) - Re \\
e_B' &= q_1 e^*(i_s - Ay) \\
e_W' &= q_2 e^* y \\
e_C' &= -q_3 e^*(v_s - Az) \\
e_D' &= -q_4 e^* z \qquad\qquad , \text{ where}
\end{aligned}
\tag{5.40}
$$

$$
e_B = \hat{B} - B , \quad e_W = \hat{W} - W , \quad e_C = \hat{C} - C , \quad \text{and} \quad e_D = \hat{D} - D .
$$

A Lyapunov function for this error dynamics is given by a weighted sum of the squared errors, as follows:

$$
V = 1/2 \, [e^* e + e_B^* e_B/q_1 + e_W^* e_W/q_2 + e_C^* e_C/q_3 + e_D^* e_D/q_4]. \tag{5.41}
$$

Differentiation of V with respect to time yields the result,

$$
V' = -Re^* e , \tag{5.42}
$$

which is negative semi-definite with respect to the variables $e$, $e_B$, $e_W$, $e_C$, and $e_D$. It has been shown that if the input ($v_s$ here) is sufficiently rich, the error will converge to zero at a rate bounded by an exponential function of time. The references contained in [34] specify conditions for sufficient richness in terms of the spectral content of the input (e.g. a certain number of independent sinusoids).

Now it is clear that with this method we can identify the following machine parameters:

$$
\begin{aligned}
B &= (L_r R_s + L_s R_r)/\sigma^2 - jw , \quad & W &= L_r R_s/\sigma^2 \, (1/T_r - jw) , \\
C &= L_r/\sigma^2 , \quad & D &= L_r/\sigma^2 \, (1/T_r - jw) .
\end{aligned}
\tag{5.43}
$$

With algebraic combinations of the above quantities, we can obtain many of the parameters of the state-space system (5.35), including the speed w, the rotor time constant $T_r$, $L_r/\sigma^2$, $p_1$, and $R_s$. We can also deduce the rotor flux vector $\lambda_r$ to within the multiplicative constant, $\sigma^2/M$. This can be done by comparing the two expressions for the derivative of the stator current given in the realizations (5.35) and (5.38), as follows:

$$i_s' = -p_1 i_s + (M/\sigma^2)(1/T_r - jw)\lambda_r + (L_r/\sigma^2)v_s \quad \text{, and} \tag{5.44}$$

$$i_s' = (a-B)(i_s-ay) - Wy + dz + c(v_s-az) \quad . \tag{5.45}$$

The parameters and states on the right-hand side of (5.45) are all known when the adaptive observer has run for a sufficient period of time, and hence the derivative of stator current is also known. The expression (5.44) may then be solved for $\lambda_r$ in terms of $i_s'$, $i_s$, $v_s$, and various parameters, as shown below:

$$\lambda_r = (\sigma^2/M)(1/T_r - jw)^{-1}[i_s' + p_1 i_s - (L_r/\sigma^2)v_s] \ . \tag{5.46}$$

Since $(\sigma^2/M)$ cannot be identified, we can obtain the direction of the rotor flux, but not the magnitude using only the electrical terminal measurements. In nearly all cases, a priori knowledge of some machine parameters (especially inductances that are not temperature sensitive) allows a reasonable estimate of both the magnitude and direction of $\lambda_r$ to be made.

Time-Varying Speed

The preceding development for speed and parameter identification may prove successful in the case where the speed w varies with time, but at a rate of variation characteristically slower than that of the convergence of the identification algorithm and the flux dynamics. Simulations in [34] show that the model reference adaptive observer can track parameter variations that are slow relative to the modelled dynamics. However, The situation will certainly arise where the speed varies at a rate comparable to the flux dynamics; existing implementations of field oriented control schemes (e.g. the General Electric AC-200 servo system) allow torque control bandwidths on the order of a few hundred hertz.

In this section, we shall consider a method to identify the unknown machine parameters, abandoning any attempt to observe the rotor speed from measurements at the electrical machine terminals. In particular, if we allow the direct measurement of speed and the direct measurement or computation of the derivatives of speed, stator current, and stator voltage, we can obtain a nonlinear canonical form realization for the induction machine dynamics for which we have complete state information. We shall develop and demonstrate (with digital simulations) the validity of a variation of the model reference adaptive identifier algorithm given in [34]. The algorithm introduced here can identify certain parameters of a nonlinear dynamical system using complete state information. The particular realization we shall consider for the induction machine dynamics will be presented first, and then the modified identification algorithm will be developed.

We shall consider a realization of the induction machine model that uses as state variables the stator current and the derivative of the stator current. This realization is derived simply by computing the second derivative of stator current in terms of the stator current, stator voltage, the rotor speed, and their derivatives. The model using complex variable notation is then given by

$$
\begin{aligned}
i_s' &= y \\
y' &= -p_1 y + (-1/T_r + jw)(y - p_3 + p_4 i_s) + \\
&\qquad jw'(-1/T_r + jw)^{-1}(y - p_3 v_s + p_1 i_s) + p_3 v_s' \ ,
\end{aligned}
\tag{5.47}
$$

where y is the derivative of stator current, w' and $v_s'$ are the derivatives of rotor speed and stator voltage, respectively, and the parameters in (5.47) are defined as follows:

$$
\begin{aligned}
p_1 &= (L_r^2 R_s + M^2 R_r)/(\sigma^2 L_r) \ , \quad & \sigma^2 &= L_r L_s - M^2 \\
p_3 &= L_r/\sigma^2 \ , & p_4 &= L_r R_s/\sigma^2 \ .
\end{aligned}
\tag{5.48}
$$

As already pointed out in the preceding section, we can express the rotor flux in terms of the stator current $i_s$, its derivative y, and the rotor speed w (see (5.46)).

The model reference adaptive identifier algorithm of [34] will now be modified for the nonlinear state-space system with parameter vector p

$$
x' = f(x, u, p) \ , \tag{5.49}
$$

where x and u are the state and input vectors, respectively. We are led by the previous development for linear, time-invariant systems in [34] to consider an adaptive identifier of the form:

$$\hat{x}' = f(x,u,p) + Ke$$
$$\hat{p}' = g(e,\hat{p},x,u) \quad , \tag{5.50}$$

where K is a stable matrix, $\hat{x}$ is an estimate of the state x, $\hat{p}$ is the estimate of the parameter vector p, and $e = \hat{x} - x$. If we also define $e_p = \hat{p} - p$, the following error system in e and $e_p$ results from subtracting the system dynamics (5.49) from the adaptive identifier (5.50):

$$e' = f(x,u,\hat{p}) - f(x,u,p) + Ke$$
$$e_p{}' = g(e,\hat{p},x,u) \quad . \tag{5.51}$$

To determine an appropriate parameter update law (i.e. the function $g(e,\hat{p},x,u)$) that leads to convergent estimates, we consider the Lyapunov candidate

$$V = 1/2\ \{e^{*}Re + e_p{}^{*}Pe_p\} \ , \tag{5.52}$$

where P and R are symmetric, positive-definite matrices. Differentiation of V yields the following:

$$V' = e^{*}R\{f(x,u,\hat{p}) - f(x,u,p)\} + e_p{}'^{*}Pe_p + 1/2\ e^{*}[RK+K^{*}R]e \ . \tag{5.53}$$

The matrix R should be chosen such that $[RK+K^{*}R]$ is negative definite, which can be done since K is stable. In this case, to guarantee that V' is negative semi-definite, we would like to select the update law so that the first two terms in the expression for V' cancel exactly. However, this is not generally possible if $f(\cdot,\cdot,\cdot)$ has a nonlinear dependence on p. We shall proceed to obtain an approximation for $f(x,u,\hat{p}) - f(x,u,p)$, namely

$$f(x,u,\hat{p}) - f(x,u,p) = [f_p(x,u,p)]e_p \,, \tag{5.54}$$

where $[f_p(*,*,*)]$ is the gradient of $f(*,*,*)$ with respect to the vector p. This will allow the specification of an update law that makes V' only approximately negative semi-definite. However, it is intuitive that this is a very good approximation if this gradient does not vary greatly in the neighborhood of (x,u,p), and we have a reasonable initial estimate of p. Note that this expression for the difference $f(x,u,\hat{p}) - f(x,u,p)$ is exact if $f(*,*,*)$ is linear in p. The resulting parameter update law is given by

$$p' = e_p' = -P^{-1}f_p^*(x,u,\hat{p})Re \,. \tag{5.55}$$

This algorithm has been applied to the induction machine model using the particular realization introduced in this section. Fig. 5.2 shows the results of a numerical simulation with the estimates of $p_1$ in (i), $1/T_r$ in (ii), $p_3$ in (iii), $p_4$ in (iv), the approximate Lyapunov function V in (v), and the axis-1 voltage drive in (vi). The matrices R, P, and K for this simulation were as follows:

$$P = 10^{-5}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \,, \quad R = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \,, \quad K = 1 \,. \tag{5.56}$$

The parameter estimates are seen to converge rapidly to their true values. Note that we have used a non-sinusoidal input drive to ensure that the excitation is sufficiently rich. Although this algorithm is effective without the assumption of constant speed, it has the costs of requiring the direct measurement of speed and either the measurement or computation of the derivatives of speed, stator current, and stator voltage.

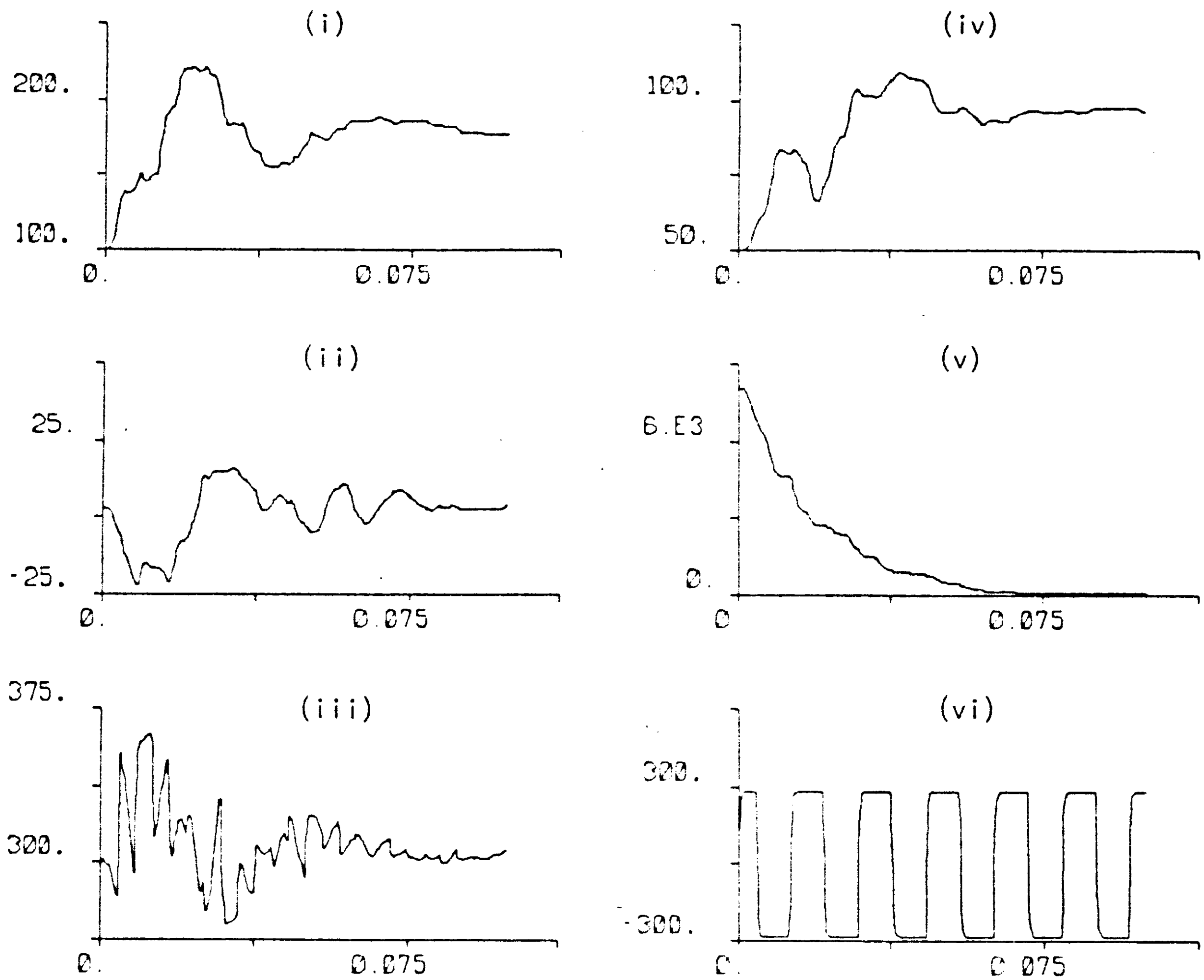



Fig. 5.2

Numerical simulation of modified model reference adaptive identifier applied to the induction machine model

(i) Estimate of $p_1$

(ii) Estimate of $(1/T_r)$

(iii) Estimate of $p_3$

(iv) Estimate of $p_4$

(v) Approximate Lyapunov function V

(vi) Axis-1 voltage drive

# Chapter 6: SUMMARY AND SUGGESTIONS FOR FURTHER WORK

## Summary

In this thesis we have focused on the estimation of the electromagnetic state variables of the induction machine model from the point of view of observer theory. The latter viewpoint has not been sufficiently exploited in the previous work in this area. We have noted the additional fact that nearly all the previously developed estimation schemes for electric machines have been implemented as open-loop simulators, neglecting the use of a corrective prediction-error term. We have shown that existing estimation schemes, for instance those used in field oriented control, can be better understood in the context of observer theory, and that observer theory naturally leads to estimators with improved performance over the existing estimation schemes through the use of prediction-error.

In particular, we have developed a class of observers for the estimation of rotor flux using a prediction-error term in Chapter 2. These observers can be made to exhibit an arbitrarily fast rate of estimation error decay under ideal conditions, which is in contrast to existing estimation schemes. The principles used in the rotor flux estimation problem have been applied to an observer for the combined estimation of rotor flux and stator current. The performance of the proposed observers has been verified with numerical simulations.

In Chapter 3, we have developed sampled-data realizations for the observers proposed in Chapter 2 to facilitate microprocessor implementation. The particular sampled-data realizations have been shown to have satisfactory performance (via numerical simulation), and have the feature that very little new computation is required to update the observer models at each time step.

We have analyzed the effects of measurement disturbances on the performance of the proposed flux (and current) estimators in Chapter 4. In particular, we have addressed the cases where measurement disturbances are constant biases and where the disturbances can be modelled as zero-mean, white noise processes.

In Chapter 5, we have considered two nonlinear estimation problems for induction machines. Firstly, we have re-examined the rotor flux (and stator current) estimation problem of Chapter 2 in the case where we have model uncertainties. Secondly, the estimation of rotor speed (and flux) from the electrical terminal measurements was considered. Previous work on these problems was reviewed, and certain hopefully new approaches were presented.

## Suggestions for Further Work

It will be of great interest to construct experimental models of the observers proposed in Chapter 2, possibly using a microprocessor implementation. For this application, we would propose the sampled-data implementations discussed in Chapter 3. The resulting state estimators can be integrated into an overall control algorithm that regulates the rotor flux (or other electromagnetic state variables) and the mechanical states (e.g. acceleration, speed, and position). A favorable approach may invoke sliding mode theory as suggested in [4].

In an experimental setting, the issues dealing with the effects of measurement uncertainties as discussed in Chapter 4 will become more concrete. One will be able to verify the error models given in that chapter, and with a more specific characterization of the measurement noise processes, further studies in the minimization of estimation error (co)variance may prove valuable.

The area of nonlinear estimation for electrical machines remains an interesting area where many problems have yet to be solved. Although, there are promising initiatives into the parameter identification and speed estimation problems (for induction machines) as discussed in Chapter 5, it will be of interest to experimentally verify the proposed estimation schemes. Many new schemes will certainly evolve, especially those that are less computationally intensive. Estimators that use constant observer gains may prove valuable (see [22]). In particular, the "bounded nonlinearity" method suggested in Chapter 5 bears further investigation for the induction machine speed estimation problem, and for other applications. One possible application may be for parameter identification in LTI systems.

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