# Modeling a Vehicle Suspension System as an Educational Laboratory Exercise

by

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## ABSTRACT

In order to teach specific mathematical and applicable concepts of modeling a vehicle suspension system to students in the 2.672 class at MIT, a design of a lab apparatus and plan of the procedure for a laboratory exercise featuring the necessary materials was completed. Specifically, the exercise had the goal of teaching providing students with an insight to vehicle suspension systems and the potential for finding an ideal system by varying system parameters. Using a scaled model of a suspension system that that can be modeled as a $2^{nd}$ order differential system with a variable damper, the lab allows students to study different characteristics of the system and learn to apply mathematics and control to the system.

The laboratory exercise will feature a scaled suspension system of a standard vehicle with a variable damping and variable oscillatory input force. These conditions will allow students working the exercise create different scenarios and to vary the damping, allowing them to predict and model the system as a $2^{nd}$-order differential equation. Analysis indicates that the model will work for the setup and allow for students to predict the behavior of the model vehicle suspension system.

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# INTRODUCTION

This thesis project will teach and familiarize students with vehicle suspension systems which are in use daily by millions of people, and their function to provide stability and comfort require a significant design process. The design of this laboratory exercise provides a relatively restricted environment where the variations of a vehicle suspension system are available and the effects of the variations are observed. At the same time, the design of the exercise will maintain enough control of the system as to guarantee a focus on the learning goals. Suspension systems are so widely applied, and an understanding of the support and damping is helpful for those in the fields of design, manufacturing, engineering, and mathematics. Modeling a "real-world" system with the use of mathematic studies allows for students to gain a better understanding of the world around them and help enforce important mathematical and engineering concepts.

## MOTIVATIONS

The laboratory setups in the 2.672 Project Laboratory class have begun to be updated and replaced for the first time in decades. Many of the procedures have been around longer than the professors teaching the course have been at MIT. As a result, the emergence of new technologies and updated curriculums in other mechanical engineering studies demands that the old laboratory procedures be scrutinized and reconsidered. The decision has been made for new laboratory setups to be designed and used in the 2.672 Project Laboratory class. These labs, including the one outlined here, seek to offer a wider range of subject material, chances for students to investigate subject areas deeper, and a new level of relevance to real-world applications. This lab hopes to expand students' learning beyond the damped harmonic systems by adding focus on product design and the variety of damping available to suspension systems.

## THE LAB EXPERIENCE

Students will be able to vary inputs to the system to simulate different driving speeds and/or different driving surfaces. The students will also be able to vary the damping aspect of the system, which provides two advantages. First, it will allow simulation of different suspension systems, and second, to allow students to selectively decide what type of damping design is the "best" given a set of conditions. This lab is a hands-on experience for the students, part of the transitional studies for students getting ready to make the jump from the classroom chalkboards to the real world challenges. The lab also provides a learning environment where the student is engaged in a hands-on exercise and can obtain a view of the real-world application. A successful experience will be one where the student has related their understanding of 2nd-order differential equations to the model, and has gained an understanding of the design options to choose the damping necessary for requested conditions.

## BUILDING A USABLE LABORATORY EXPERIMENT

To develop this laboratory exercise, a thorough design process is necessary for an easy-to-use laboratory procedure where gaining familiarity with the hardware is a process that does not distract from the goals of the laboratory exercise. Detailed plans will provide a guideline for an intelligently designed setup that can accurately model a vehicle suspension system.

The materials for the exercise setup will need to be purchased and assembled, and instructions for students will need to be prepared. The materials will be purchased specifically for the function of the laboratory exercise and assembled within the constraints of the class setup (table size, time allotted for each class, etc.). The durability of the experimental setup is paramount, as it protects against accidental student error, provides for consistent results in the laboratory procedures, and accomplished the goal of implementing a "quality lab." After full assembly, the hardware setup will need to undergo sufficient testing to ensure predictable and consistent results. While partial analysis is feasible prior to assembly, it will be necessary to test the setup after its installation and implementation to complete full analysis of the laboratory setup and attain a comprehensive collection of the system information. This testing can be recorded for reference by the educators of the laboratory procedure in the years to come.

## TURNING THE LABORATORY SETUP INTO A MODEL

The laboratory procedure will walk the students through the methods necessary to understand how the hardware of the laboratory setup represents the vehicle suspension system common to everyday life. The laboratory procedure will isolate one wheel of the vehicle and model its oscillations under the assumption that the other three wheels of the vehicle will experience identical circumstances. The setup will be modeled as a mass-spring-damper system. The most considerable mass of the system is the mass of the vehicle. The connections from the body of the vehicle have a certain stiffness that will be modeled as the stiff spring of the system. As well, the suspension system contains a damping element that will be the main feature of the experiment. The damping element and the input force will be able to be varied by the students to change the parameters of the system, giving the students a change to effectively produce different suspension systems.

## ANALYSIS OF THE VEHICLE SUSPENSION SYSTEM MODEL

By modeling the laboratory setup system as a damped harmonic oscillator governed by a 2nd-order differential equation, students studying the system will be able to understand how changes in the parameters of the system will affect the response of the vehicle's modeled travel. With a variety of input driving forces representing different travel surfaces and speeds driving the system, students will seek an ideal damping parameter that provides riders of the modeled vehicle with an ideal feel of the ride for the theoretical passenger of the vehicle.

This understanding of the system will incorporate past the students' past learning about dynamics, damped oscillations. As well, it will provide an in-depth investigation into the nature of a variable damper and the design of a vehicle suspension system.

# EDUCATIONAL LABORATORY BACKGROUND

The design and installation of a laboratory exercise is an effort that will hopefully affect decades of student learning. In the case of this experiment, students taking 2.672 Project Laboratory have certainly been exposed to the learning necessary to understand the theory of the laboratory procedures. The class gives students the chance to begin to apply their theoretical learning to hands-on learning tools. This lab focuses on the application of background theory to an important area of study for mechanical engineers – control over a dynamic system. For students with a high level of interest in modeling, dynamic systems and variable damping of those systems, and seeking relevance of their learning to "real-life" applications, this lab provides a few options to study these areas at varying levels of intensity.

## THE IMPORTANCE OF "HANDS-ON" LEARNING

With limited resources, many teaching environments lack the ability to provide students with a great number of physical applications of their studies. This opportunity is a method not typically available to all universities and is therefore a unique privilege to the MIT student body. The presence of learning opportunities that connect the classrooms to reality is an invaluable educational resource. When a student completes their undergraduate studies, the lack of hands-on experience can be a serious drawback to further educational undertakings or occupational success. Physical systems where students can see first-hand the different characteristics of a laboratory setup respond to the changes they make are more similar to the variety of challenges facing them outside the classroom then the theoretical examples used to introduce students to the learning material.

## EDUCATIONAL GOALS OF THE LABORATORY SETUP

The overall goal of this lab is to provide students inside into the considerations necessary in the design of a vehicle suspension system. In this laboratory procedure, the students are given the opportunity to modify the damping parameter of the modeled system, as well as the input forces to the system. This range of options demands that students make educated predictions about the system prior to applying the system modifications. Students will need to be able to relate the driving surfaces with which they are familiar to the proper input forces from the voice coil. Students will also need to understand and relate the motions of the model vehicle system to the real-life experience of riding in an actual vehicle.

# BUILDING THE LABORATORY EXPERIMENT

The construction of the laboratory experiment requires a planned layout and the correct hardware to anticipate the steps of the laboratory procedure. SolidWorks assemblies of the predicted hardware can easily show how a laboratory setup will look and be employed. These designs show laboratory procedure progresses through a calibration of the variable damper used in the experimentation to the testing of the overall system.

## DAMPER CALIBRATION SETUP

In order to calibrate the key component of the laboratory setup, the variable damper, the damper must experience a controlled input of force with a measurable output of response. The best way to accomplish this is with a stabilized structure to support the damper over a force inputting voice coil. The displacement of the damper can be measured with a linear variable displacement transducer or an inductive position transducer. It makes sense for the LVDT to be stabilized against the voice coil or the laboratory setup table as a reference frame to measure the displacements of the damper. While the damper-supporting structure is fully stabilized to the laboratory workspace, the damper can be easily installed into and removed from the structure for use after calibration. As well, the voice coil and LVDT can be removed from their positions at the calibration setup for use in the rest of the laboratory procedure.

The damper is slides onto the structure at the top and secured by a small bolt and nut. This suspends the damper over the voice coil and LVDT. The voice coil and LVDT unit must be relatively stable, but moveable for use in the rest of the laboratory procedure. The voice coil and LVDT structure are bolted to the laboratory setup table by the students for the calibration procedure.

## MAIN LABORATORY PROCEDURE SETUP

After the calibration of the system's variable damper, the student will continue to the main laboratory procedure. This part of the lab features the student testing the entire system model of one-fourth of a vehicle. The body of the vehicle is modeled by a 1 kilogram mass supported on a platform on linear bearings. These bearings constrain the platform in the horizontal plane, allowing only for vertical motion. The LVDT is secured in reference to the table to which the bearings are attached to measure the displacement of the oscillating mass. The mass is attached to a stiff connection to the damper and point of input force. These structures model the suspension system's connections from the chassis to the wheel of the vehicle.

In this setup, the linear bearings supporting the platform and mass are semi-permanently secured to the laboratory setup table. As well, the connecting structures do not need to be removed from the mass nor do the connecting structures from the damper to the point of force input. The damper is attached to the structures in this setup in a similar fashion to the calibration setup. The voice coil and LVDT from the calibration setup are secured again to the table in the appropriate location. The voice coil provides the input force to the appropriate contact on the structures. The LVDT, bolted securely, is able to measure the vertical displacement of the moving platform with the mass.

## THE LABORATORY PROCEDURE

Given the described laboratory setup, the students will proceed through the laboratory procedure. After initial familiarity with the hardware, the students will calibrate the damper. The variable damper will be the main parameter students can adjust to change the characteristics of the system. This is relatively consistent with the model of a vehicle suspension system. With a given mass of the vehicle, the stiffness of the connecting structure and the damper can be changed to modify the smoothness of the ride. In this lab, it would be nearly impossible for students to study and calculate the stiffness of even a simple series of connecting structures and construct and install multiple apparatuses in the time allocated to the laboratory procedure. However, variable dampers are easy to adjust and can produce significant system modifications.

With the variable damper in place, the students can analyze the system's response to multiple inputs. The voice coil can be employed to offer a known input to the system, causing an applied force to the model wheel of the system. The LVDT secured to measure the physical response of the mass is used to track the vertical motion of the model vehicle. The characteristics of the system can be obtained and further information concerning the system through a swept sine response if necessary. With this data, the students will need to make some insight to what type of response the mass should experience with given inputs. After making their predictions, students will test their hypothesis by varying the damper to appropriate values and testing the new system response.

### CALIBRATION OF THE VARIABLE DAMPER

The student will attach the damper to the calibration setup via a bolted connection at the top of the damper to the underside of the calibration setup structure. The voice coil's supporting structure is secured to the optical table placed below the calibration setup structure. The voice coil is used to apply known forces to the damper while the LVDT tracks the vertical displacement. After a sufficient amount of data is acquired, damping constant can be measured from the data. It is intended that the time necessary to complete this part of the laboratory procedure is short compared to the rest of the process with the damper installed to the full model vehicle system.

## IDENTIFYING THE SYSTEM PARAMETERS

After the damper has been calibrated such that the damping constant is known for a given setting on the damper, it should be installed on the full vehicle model setup. Additionally, the voice coil and LVDT should be reapplied to the part of the main part of the setup. The voice coil can be used to apply known forces to the model suspension system. With a step function driving force, the system parameters can be identified. By applying the step function to the model wheel of the system, the damped system will oscillate and settle. The mass will rise and oscillate on the platform constrained by the linear bearings. As the system is in response, the LVDT will be employed to track the motion of the mass. Analysis of the data can lead to measured values of the gain, damping, natural frequency and the related stiffness and damping constant.

## VARYING THE DAMPER TO CREATE AN IDEAL SYSTEM

After this is complete, students will have to apply their understanding of $2^{nd}$-order harmonic systems to choose a new damping constant. Their choice will be motivated by the objective of creating a responsive model of a vehicle that simulates an ideal ride for a theoretical passenger of the vehicle. A new damping constant should be chosen and employed with the variable damper used throughout the laboratory procedure. The smoothness of the ride can be achieved with an ideal range of damping resulting in a semi-damped system. With this new damping constant, the system should be tested again and tracked with the LVDT to ensure the mass responds in the desired fashion.

# ANALYZING THE MODEL

## THEORY

Modeling this setup as a lumped parameter 2nd-order system employs the use of the equation of motion for a spring-mass-damper system:

$$M(\ddot{x}) + B(\dot{x}) + Kx = F(t) \tag{1}$$

where x is the position of the mass (M), B is the damping constant, K is the stiffness of spring constant, and F(t) is the applied force. Mass is typically in kilograms, [kg]; the damping constant is measured in Newton-seconds per meter, [N-s/m]; the stiffness is in Newtons per meter, [N/m]; finally, the force is in Newtons, [N]. For the given equation of motion, the natural frequency of the system, $\omega_n$, can be found using K and M:

$$\omega_n = \sqrt{\frac{K}{M}} \tag{2}$$

where $\omega_n$ is in radians per second, [rad/sec]. Furthermore, the damping ratio $\zeta$ can be determined using the all three of the coefficients of the equation of motion:

$$\zeta = \frac{B}{2\sqrt{MK}} \tag{3}$$

The next step is to incorporate the measured displacement of the mass Y(t). With F(t) as the input force as a function of time, the transfer function H(t) of the system is:

$$H(t) = \frac{Y(t)}{F(t)} = \frac{Stiffness\ Factor}{Mt^2 + Bt + K} \tag{4}$$

where the Stiffness Factor is a dynamic of the stiffness of the supporting structure between the mass and the wheel. This factor is dependent on the specific structure included in the lab and theoretical derivation is dependent on the geometry of that structure. The transfer equation and be put in terms of the parameters Gain, natural frequency $\omega_n$, and the damping ratio $\zeta$:

$$H(t) = \frac{Gain * \omega_n^2}{t^2 + 2\zeta\omega_n t + \omega_n} \tag{5}$$

These equations can be manipulated to solve for B:

$$B = \frac{2 * \zeta * Stiffness\ Factor}{Gain * \omega_n} \tag{6}$$

Using this, one can determine a desired damping ratio $\zeta$ to select the type of damped response the system should experience for the final procedure of the lab.

## APPARATUS AND PROCEDURE

The given variable damper will be given and calibrated. The variable damper will be installed in a vehicle suspension system and the system will be tested to identify the system parameters. With the system parameters identified, the natural frequency of the system will be determined. As well, the behavior of the system operating under different damping constants will be predicted and an ideal damping constant will selected. The system will be tested once again with this ideal damping to model the ideal modeled ride of the modeled system.

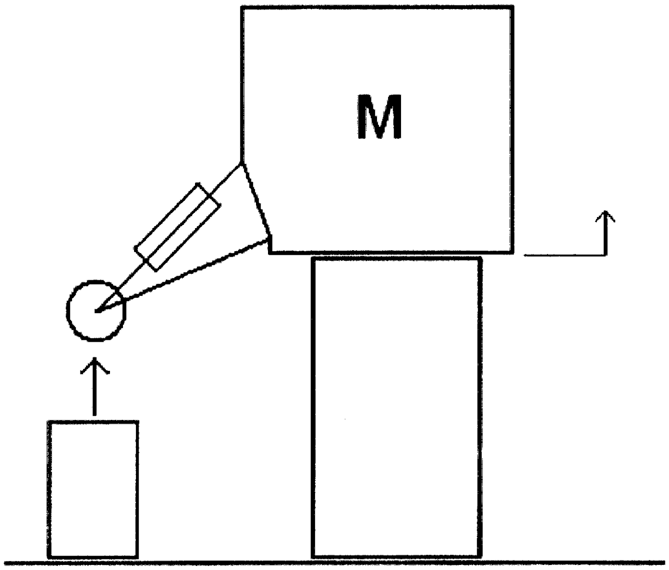

Figure 1: Laboratory Setup schematic

*DAMPING CALIBRATION*

In order to obtain reasonable information about the variable damper, it is calibrated with a voice coil and LVDT. Using the Calibration Setup area of the laboratory setup, a calibration process can be employed to calibrate the variable damper.

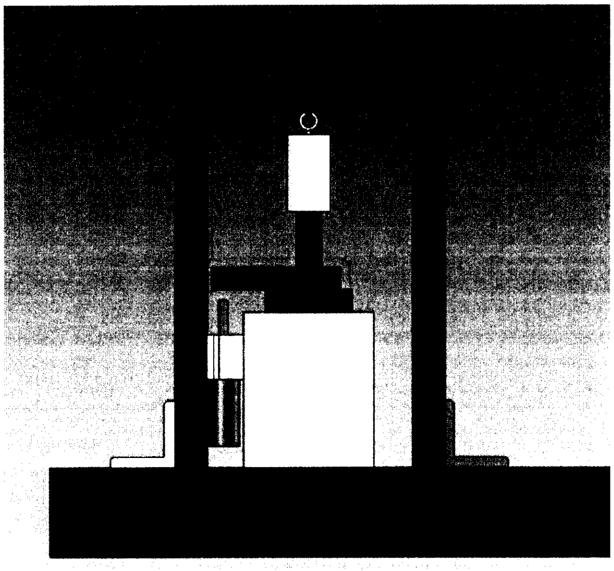

Figure 2: The Calibration Setup viewed from the front

The voice coil is secured to the optical table under the suspended damper. Additionally, the LVDT is placed to measure and record the displacement of the damper under various settings. The damper is attached to the calibration setup by securing a bolt at the top of the damper to the underside of the steel support arch.

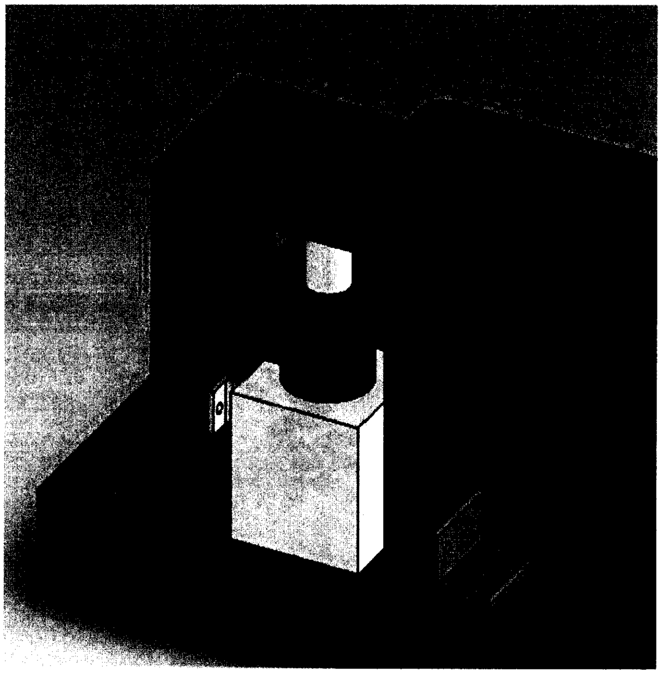

Figure 3: The Calibration Setup from an angled view

The voice coil is then employed to offer a known input force to the damper while the damper displacement is measured and recorded using the LVDT. When the calibration process is complete, the damper is removed from the Calibration Setup area for use in the Suspension Setup.

### *IDENTIFYING SYSTEM PARAMETERS*

The voice coil, LVDT, and variable damper are installed in the Vehicle Suspension setup area for testing of the vehicle suspension system model. The damper is secured with a bolted joint to the structural support from the mass platform to the model wheel of the system. The voice coil is secured to the optical table to provide an input force to the model wheel of the suspension system, modeling the varying surface over which a vehicle would travel. As well, the LVDT is secured to measure the vertical displacement of the mass platform supported by the linear bearing.

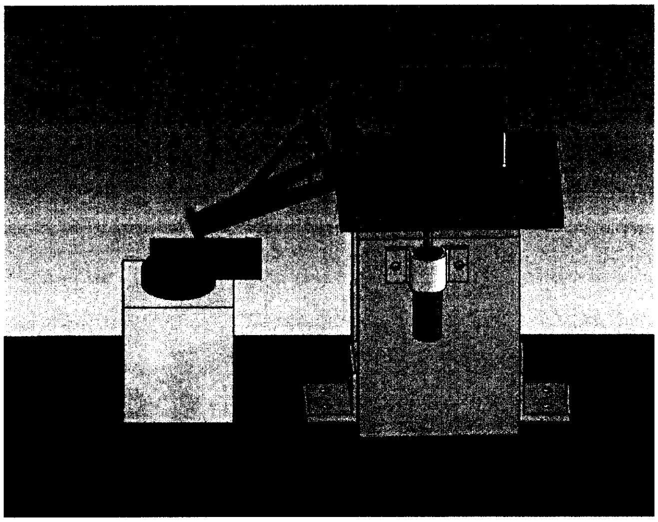

Figure 4: The Vehicle Suspension Setup viewed from the front

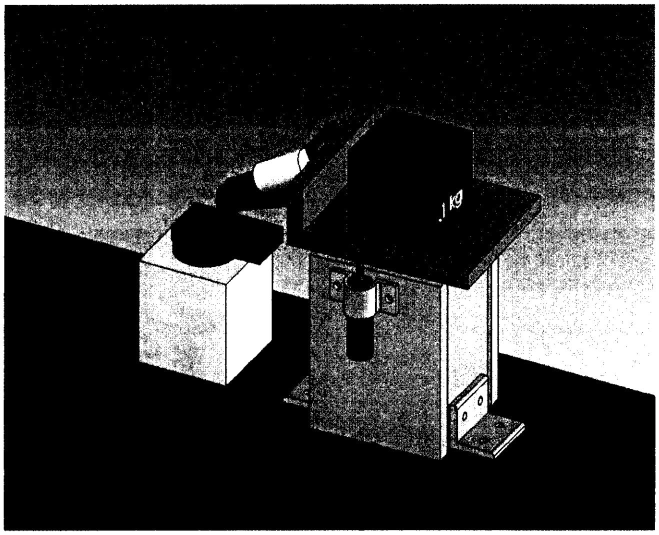

Figure 5: The Vehicle Suspension Setup with the Damper installed

After the system is fully prepared for testing, the system should undergo a step response with the oscillations and final deflection of the mass platform measured and recorded. With this completed, the data can be saved for further analysis. This input is sufficient to determine the stiffness K, of the system given the mass and the damping constant B. Using the theoretical equations, one can obtain reasonable values for the Gain, natural frequency, and damping ratio of the system. Using a variety of different software programs, one can tweak the damping parameter B and observe the effect on the system response. The goal is to find an ideal response that would maximize the comfort level of a passenger riding in a vehicle modeled by the laboratory setup. Observe how the damping parameter changes the response of a system, for example from a very under-damped harmonic response to a near critically-damped response. Examples of an under-damped response, a near critically-damped response, and an over-damped response are shown below:

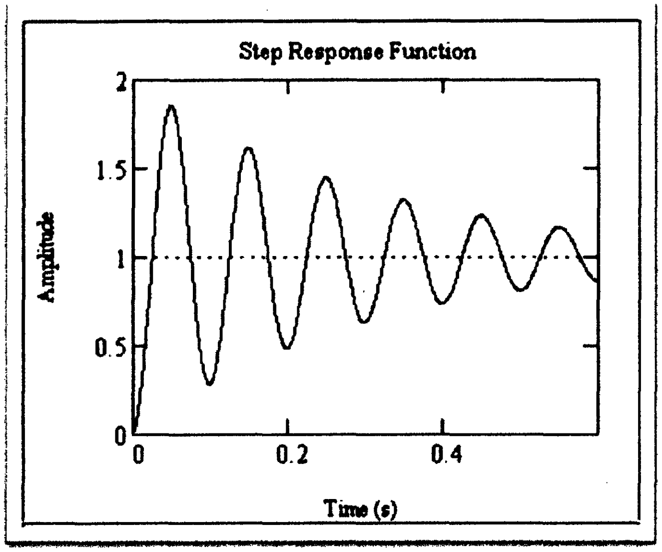

**Figure 6: An under-damped step response with unity Gain = 1, a natural frequency of $\omega_n$ = 10Hz, and a damping ratio of 0.05**

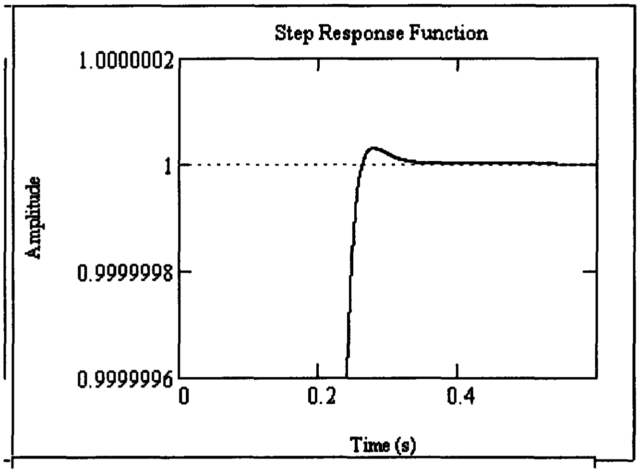

**Figure 7: A near critically- step response with unity Gain = 1, a natural frequency of $\omega_n$ = 10Hz, and a damping ratio of 0.98**

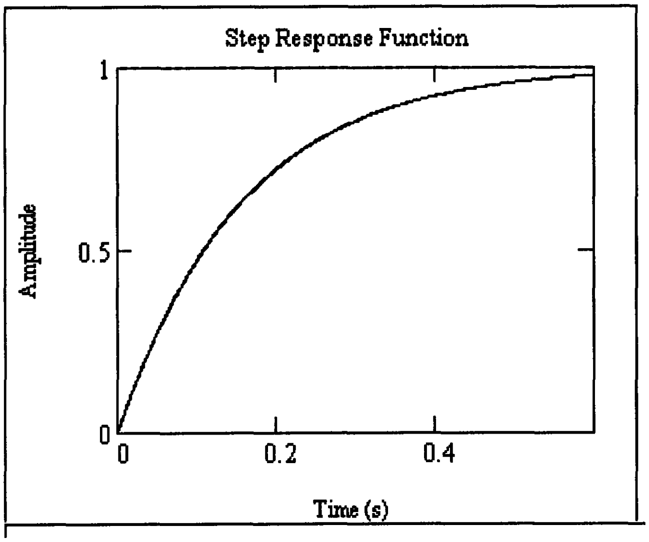

**Figure 8: An over-damped step response with unity Gain = 1, a natural frequency of $\omega_n$ = 10Hz, and a damping ratio of 5**

With consideration as to what the ideal type of response the passenger of a vehicle would experience, the students are able to adjust the damping parameter of the variable damper to achieve the desired damping ratio $\zeta$.

### *TESTING THE IDEAL SYSTEM*

After a clear understanding of the system is obtained, the variable damper should be reset to the damping parameter that will give the "ideal" damping ratio. The system can be run again with the new damping parameter and the results again observed and recorded. The system response should match the predicted results used to determine the new damping parameter.

## DISCUSSION

The damping calibration allows for the setting of the variable damper to be specific enough to make detailed and desired changes to the response system.

The driving force coil on the overall vehicle suspension system can be easily changed to represent a variety of input force functions in time. A straightforward investigation of the system allows for the identification of the system parameters and later the confirmation of predicted results. In the case of this lab, a typical damper can easily over-damp a system, with a damping ratio over 1.0, or on lower damping settings allow for a very under-damped system, with a damping ratio under 0.5. In both cases the frequency response is determined by the mass and the stiffness, and a mass of 1kg is recommended to result in frequency responses in the tens of hertz.

## CONCLUSIONS

This lab is very feasible and has a few unique aspects that make it a worthwhile teaching device. The basic approach allows for a student to gain the proper understanding of the desired material without necessarily requiring a great depth of enthusiasm for modeling, 2nd-order damped harmonic response systems, or vehicle suspension systems. However, a major advantage of this laboratory procedure is the fact that students with a high interest for any aspect of the lab are given an opportunity to explore their interest. The connecting structure's stiffness and inertia is an area that may or may not be left open for students to analyze. As well, the system can be tested with a variety of different damping parameters and masses, and one lofty option is to offer different structures to allow for complete control over all the parameters of the system.

## FUTURE CONSIDERATIONS

While the efforts of this paper are mostly aimed toward design and planning, thought toward the physical implementation of the laboratory procedure can and should start without delay.

### HARDWARE

All of the design elements need to be purchased, potentially machined, and assembled before a true analysis of the system can be undertaken. The hardware should be selected both with the long-term function of the laboratory procedure and the aesthetic quality of the laboratory setup taken into account. Robust, durable materials like steel should be used for the construction of stabilizing structures, as well as the components of the model if possible. Ideally the laboratory setup will last over two decades without a need for major repair or hardware replacement. In the short term, the variable damper used may be one of the several kinds of dampers available for model vehicles, or the construction of more interesting damping options is available.

### MR FLUID DAMPER

One of the more interesting options to use as a variable damper is non-Newtonian magnetorheological (MR) fluid. MR Fluids are a type of smart fluids with a viscosity that changes depending on the strength of the magnetic field around it. Given proper control over a surrounding magnetic field, one could use a plunger damper immersed in a MR fluid as a variable damper. A small amount of the MR fluid would be used for each run of the experiment. Using a MR fluid opens the door to a new area of learning and exploration for the students, since they are rarely seen in undergraduate laboratory procedures. Studying the characteristics of a damping MR fluid would be both challenging and engaging. Some of the leading ideas about using a MR fluid damper for a laboratory setup like this have come from MIT graduate student Stephen Samouhos.