Design Modeling and Fabrication of Experimental Apparatus for
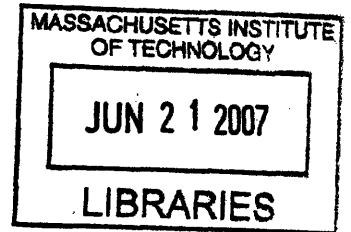Compliant Mechanism Education Kit

by

Sarah E. Shivers

SUBMITTED TO THE DEPARTMENT OF MECHANICAL ENGINEERING IN
PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF

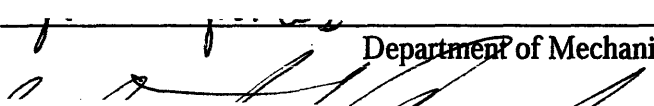BACHELOR OF SCIENCE
AT THE
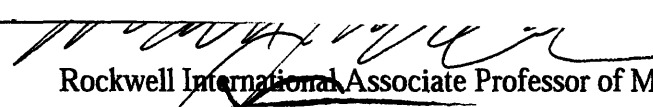MASSACHUSETTS INSTITUTE OF TECHNOLOGY

JUNE 2007

©2007 Sarah E. Shivers. All rights reserved.
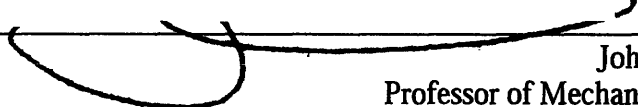
Signature of Author:_____
Department of Mechanical Engineering
May 9, 2007

Certified by:_____
Martin L. Culpepper
Rockwell International Associate Professor of Mechanical Engineering
Thesis Supervisor

Accepted by:_____
John H. Lienhard V
Professor of Mechanical Engineering
Chairman, Undergraduate Thesis Committee

Design Modeling and Fabrication of Experimental Apparatus for
Compliant Mechanism Education Kit

by

Sarah E. Shivers

Submitted to the Department of Mechanical Engineering on May 9, 2007 in Partial
Fulfillment of the Requirements for the Degree of Bachelor of Science in Mechanical
Engineering

ABSTRACT

The purpose of this thesis is to design an educational kit to be used to teach practicing
engineers about recent developments in the study and design of flexures. Flexure theory
can be difficult to explain. This kit is a physical example of the FACT method for
designing flexures. The first flexure is a linear motion flexure, which is a familiar design
to practicing precision engineers. The second design is a flexure which moves in a screw
motion, which has never been built before. The design of the screw flexure uses the
FACT method to combine constraints to create a linked linear and rotational motion. The
screw flexure is also designed to have a variable pitch, such that it ranges from pure
rotational motion to linear motion.

This thesis contains the modeling, design, and fabrication process for both the linear and
screw flexure. Two working prototypes were manufactured of each flexure. They are
assembled on a baseplate and include sensors to measure the motion of each flexure. One
kit was used to explain the concepts behind the design of the flexures to two students.
They were then able to answer a few questions about the concepts after experimenting
with the flexures.

Thesis Supervisor: Martin L. Culpepper
Title: Rockwell International Associate Professor of Mechanical Engineering

ACKNOWLEDGEMENTS

I would like to thank the people who helped make this thesis possible. I would first like to thank Professor Martin Culpepper for overseeing the project and providing many suggestions.

I would also like to thank Holly Laird for keeping me on track and ordering all the parts. She is a great thesis partner and I really could not have done it without her.

Thank you also to Jon Hopkins for answering both Holly and my questions and e-mails. He explained everything from the beginning and helped us all the way through.

Finally thank you to Gerald Wentworth and Patrick McAtamney in the LMP shop for all their help with manufacturing the flexure systems.

Table of contents

Table of Figures

Chapter 1
Introduction

The purpose of this thesis is to generate the knowledge required to design and fabricate a flexure with linear motion and a flexure with linked linear and rotational, or screw, motion to be used in educational kits. The theory behind such flexures can be difficult to explain. These kits will be used in presentations to practicing engineers involving advances in the study and design of flexures. The educational kits will give the engineers the opportunity to explore and perform experiments related to concepts that they are unfamiliar with and ultimately give them a better understanding. They can then take this knowledge back to their respective careers to use it in designing flexures. Ideally, they will be able to design flexures that are better suited for various purposes at a cheaper cost. This saving can then be felt by the consumers. Many consumer products use flexures in their design, such as products that contain switches, engines, gear drives, and circuit boards to name a few. The design cars, toys, computers, and countless other products that use these design features could benefit from advanced knowledge in the design of flexures.

The design of each flexure is explained within this thesis, with more emphasis put on the flexure with screw-like motion, which is the more complicated of the two. The linear flexure, seen in Figure 1-1, serves as an example of a flexure that is somewhat common, and the flexure seen in Figure 1-2 has never been built before. The educational kits will include sensors to measure the linear and rotational motion of the flexures as well as the force required to move them.
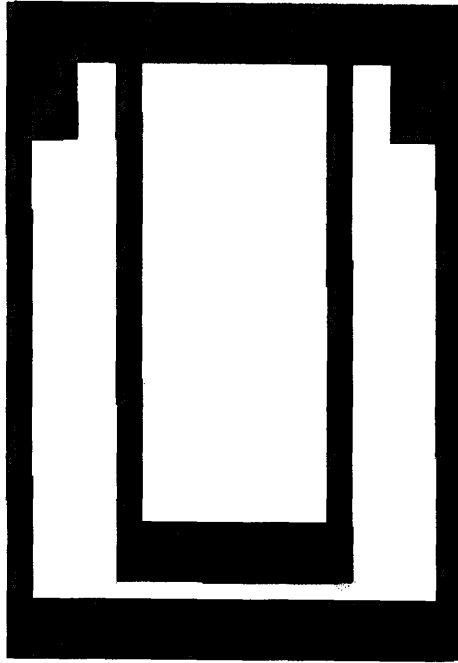
Figure 1-1: Linear Motion Flexure
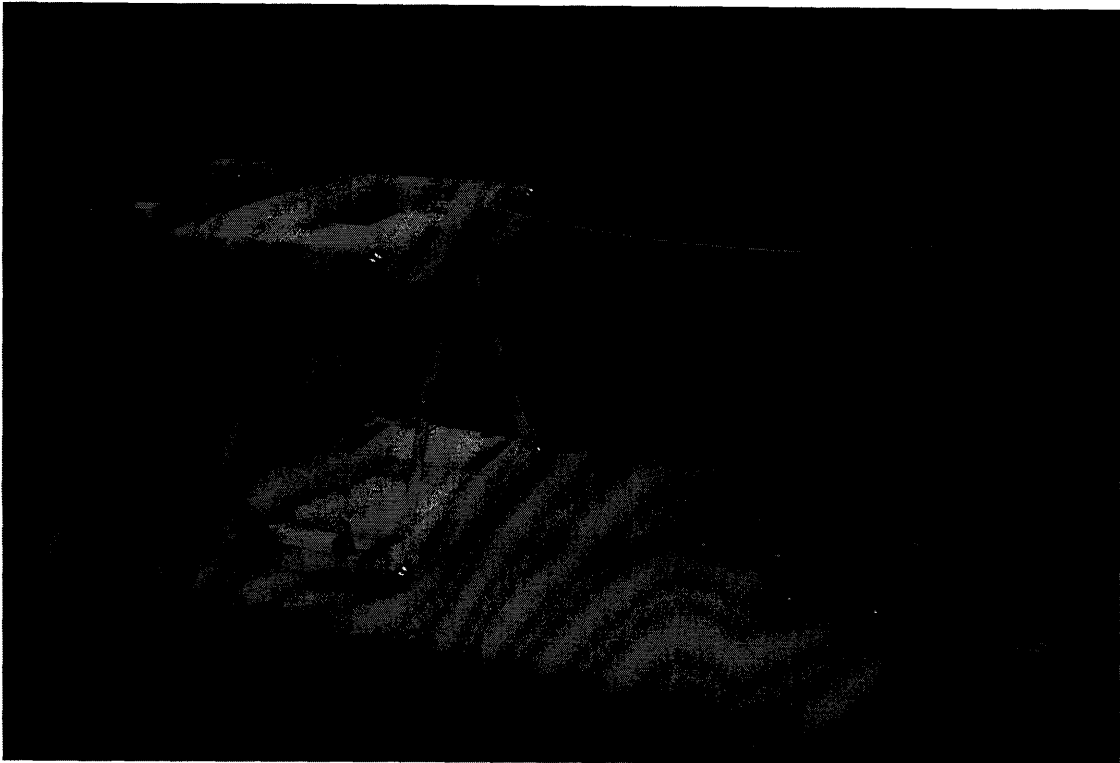
Figure 1-2: Screw Motion Flexure

12

The theory used in this thesis regarding the design and study of flexures is based on the work of Douglass L. Blanding. In his work, he uses the following equation showing that the number of degrees of freedom in a flexure is related to the number of non-redundant constraints such that

$$6 - C = R$$

where C is the number of non-redundant constraints and R is the number of degrees of freedom. Furthermore, each degree of freedom must intersect with each constraint (Blanding, 37). Blanding explains that

> An intersecting pair of Cs or Rs defines a disk of radial lines any two of which (provided the angle between them is not too small) may be selected to equivalently replace the original pair (for small motions). [1]

In his book, Blanding uses cubes to illustrate this. Figure 1-3 shows that the degrees of freedom are the same for two separate sets of constraints. However, the constraints are linked, as each constraint is still in the same plane as before. In the illustration, the solid lines are the constraint lines and the freedom lines are illustrated with dashed lines and arrows.
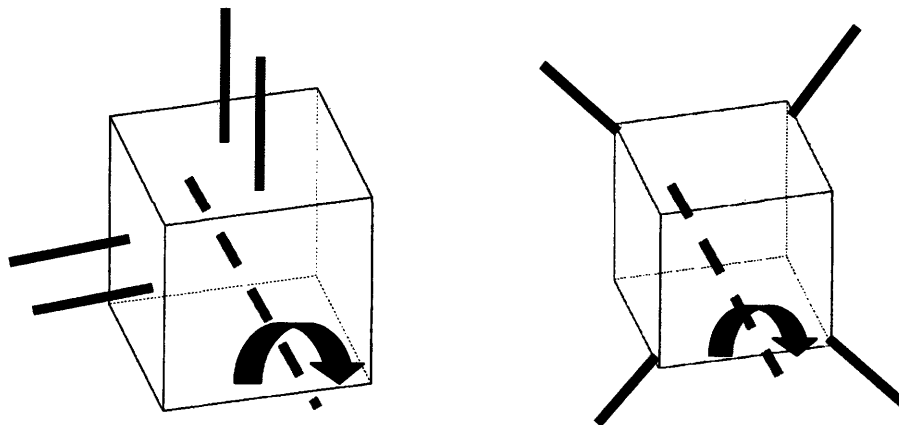
Figure 1-3: Two constraint models with the same degrees of freedom

There have been new developments in this field. MIT graduate student Jonathan Hopkins' research has found that the constraints need not only be in the same plane to produce the same degrees of freedom, but other topologies as well. His work with Freedom and Constraint Topologies, or FACT, shows there are other shapes beyond the disks that Blanding uses. Degrees of freedom can occur in disks, planes, spheres, hyperbolic paraboloids and hyperboloids. Constraints can occur in disks, planes, boxes, spheres, or lines. There are six different cases for the different numbers of degrees of freedom and constraints, beginning with no constraints and complete free motion ranging to six constraints and no motion. Each set of constraint shapes is unique to a set of freedom shapes. There are twenty-six cases total. In some cases, there is a screw motion, which is demonstrated in the screw flexure.

The screw flexure is designed to have a pitch that can be varied throughout experimentation. Pitch is the distance that a screw moves linearly. In this case, the pitch is dependant upon the distance that the z-shaped constraint is from the middle of the flexure and the angle of the Z from the base. The specific relationship is

$$p = d \tan \theta$$

This is illustrated on the flexure system in Figure 1-4.
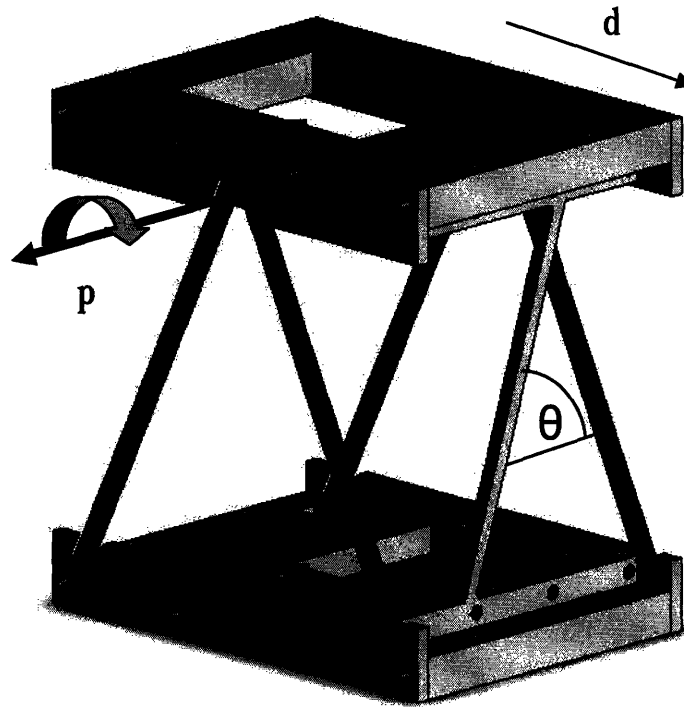
Figure 1-4: Illustration of pitch on flexure system with screw motion

The angle of the Z, θ, is set, so only the distance can be varied to vary the pitch. Those

using the kit can use this relationship when adjusting the flexure to obtain data for

different pitches, from pure rotation when the z-piece is set in the middle of the flexure,

to near pure linear motion. Pure linear motion occurs when the distance is equal to

infinity, but this is obviously impossible. However, at the farthest setting, the pitch is

high enough that, given the relatively small motion of the screw flexure, the motion may

be considered to be just linear.

This thesis will detail the design and manufacture of both the linear flexure

system and the flexure with linked rotational and linear motion, as well as the entire

experimental setup and the testing of the educational kits on fellow students to see what

they learned. The second chapter will begin with the design and manufacturing of the

linear flexure. The third chapter will contain details involving the design and

manufacture of the flexure system with screw-like motion. This flexure system will be covered in greater depth as it is much more complex than the linear flexure system. The third chapter will discuss the experimental setup and the various sensors involved, as well as the design of the baseplate and support system for these sensors relative to the flexures. The fourth chapter will describe the testing of the educational kit with the screw flexure. Finally, chapters five and six will cover the results, analysis and conclusion.

Chapter 2
Design and Manufacturing of Linear Flexure

The linear flexure system is a common design. The key is having two separate set of thin flexures to oppose any arc motion. With only one set of flexures, the flexure system will move in an arc downward as it moves linearly. However, with two sets of thin flexures, the middle block is restrained and the force is applied to the bottom block not connected to the middle block. Then, the top block moves in a downward arc as the bottom block moves in an upward arc. The two motions oppose each other and the flexure moves linearly. The two sets of flexures need to be identical to accurately oppose the motion.
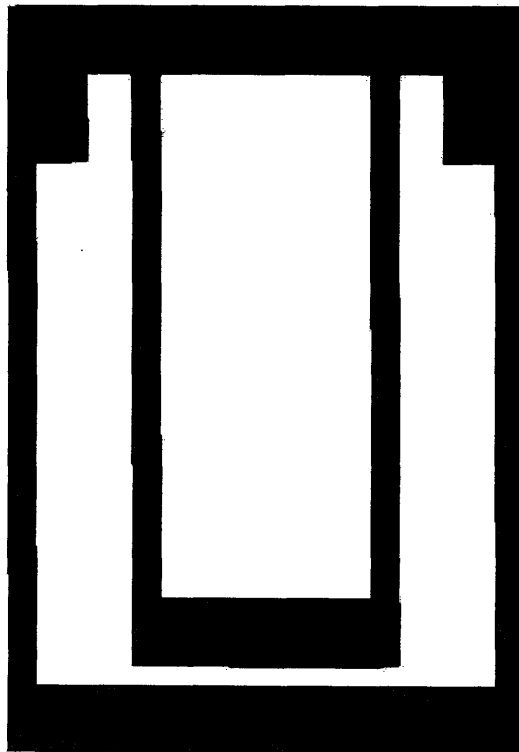
Figure 2-1: Linear Motion Flexure Solid Model

2.1 Design of linear flexure

To determine the dimensions of the flexure, the thin flexures are modeled as cantilever beams. Therefore, the stiffness, k is dependent upon the thickness of the flexure in the direction of intended motion cubed. They need to be thin in the direction of motion. However, the thickness in the direction perpendicular to the intended motion, but in the same plane, is needed to prevent buckling. ComosWorks finite element analysis is then used to test the numbers determined by the cantilever beam model. One of the functional requirements of the linear flexure system is that the linear motion is clearly visible, or approximately one inch of linear motion for one pound of force. CosmosWorks FEA is also used to test for buckling.
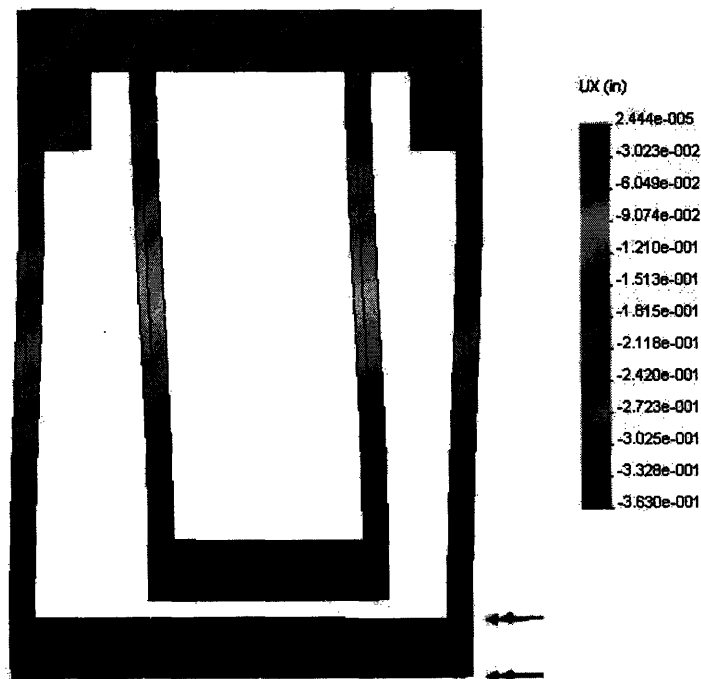
Figure 2-2: CosmosWorks FEA displacement analysis on linear flexure

2.2 Fabrication of linear flexure

The linear flexure system is made of 19.05 mm [0.75 in] Delrin. The thickness is determined by the thickness perpendicular to the intended direction of motion. The waterjet is then used to cut out the design of the flexure. The waterjet works well for this design, as all the cutting is in one direction. To transfer the design to the waterjet computer, the solid model is made into a drawing which is then opened in OMAX, where starting and ending points are determined and a cutting path is drawn. The file can then be opened on the waterjet computer and the part can be made.

Chapter 3
Design and Manufacturing

The flexure system, shown in Figure 3-1, is made of seven unique parts. The only pieces that are symmetric are the triangular support pieces. The design of each piece will be described in this chapter, as well as manufacturing details. The entire flexure is made of Delrin, but the thicknesses vary with the parts. The inner structure with the Z-shaped support is independent of the outer structure with the triangular supports. The Z-structure determines the pitch of the screw-like motion of the flexure system. The pitch may be adjusted by moving the Z-structure.
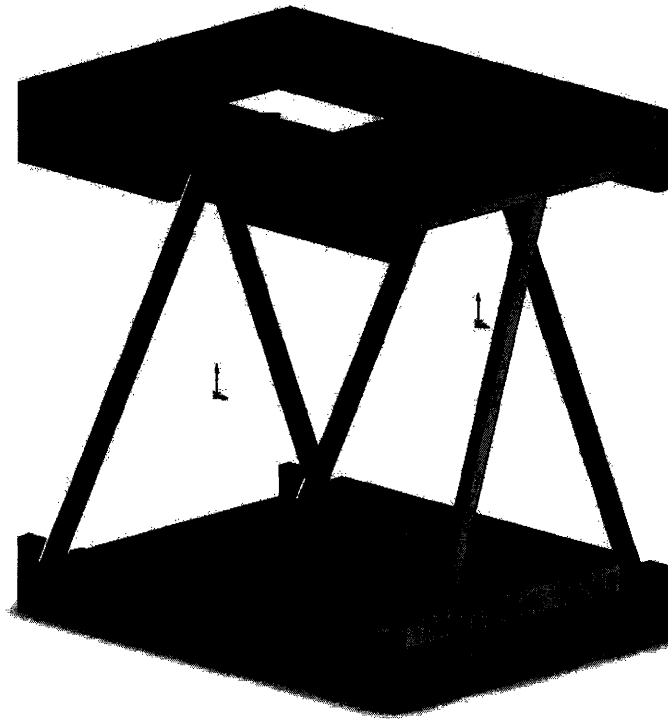
Figure 3-1: Flexure system with screw-like motion solid model solid model
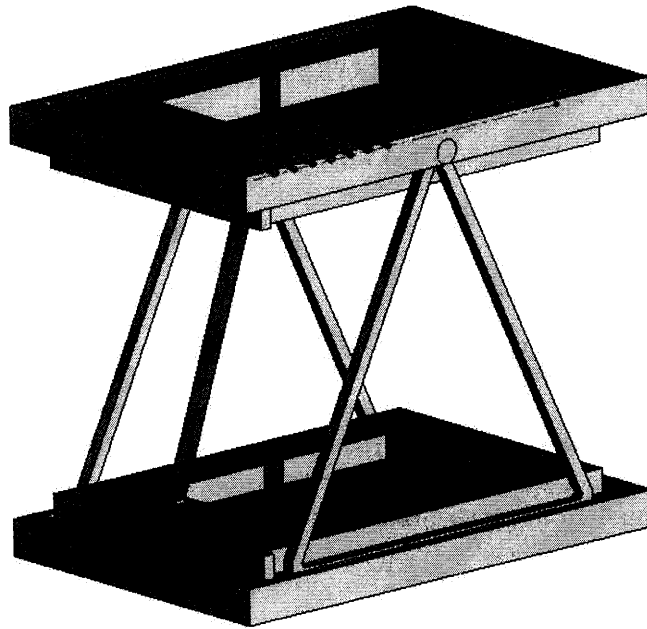
Figure 3-2: First design iteration of Flexure system

In the first design, shown in Figure 3-2, the different parts were essentially the

same as those shown in Figure 3-1. The main difference was that the parts were

connected with tabs fitting in to slots. The triangular and Z-shaped supports had tabs at

either end that fit into slots into their respective base and top. This was determined to be

too difficult to manufacture given the level of accuracy needed in the design of the slots.

For the tabs to stay in the slots, they would need to be press-fit. However, one of the

design requirements was that the flexure be easily assembled and disassembled. Press-fit

tabs and slots would prevent this. Another problem with the design is the extended base

and top of the Z-shaped support. For most settings, part of them would be hanging over

the edge of the base of the triangular shaped supports, which could cause the flexure to be

off-balance. There is also a stop at one end to prevent the Z-shaped support from going

out of one end. This was also attached with a tab and slot in the triangular shaped base and top.
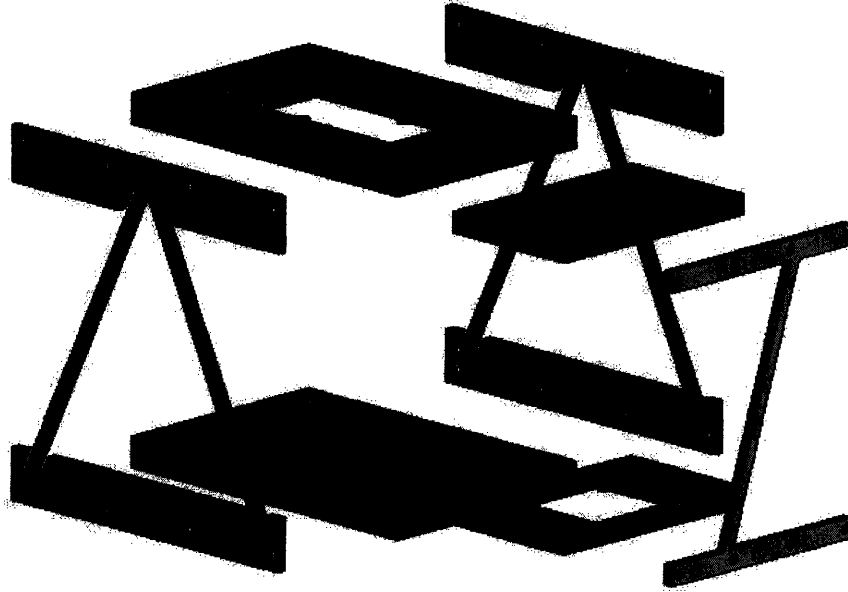
Figure 3-3: Flexure system solid model, exploded view

These design problems were adjusted in the next design, shown in exploded view in Figure 3-3. The triangular and Z-shaped supports are bolted to the outside of their respective base and top, eliminating the tabs and slots. This allows the flexure to be easily assembled and disassembled for transportation. Another adjustment on the second design is the base and top for the Z-shaped support is half the size of those in the original design. This way, it stays contained within the triangle support structure. A third change to this design is that the mechanical stop is removed. In the second design, the base of the triangular supports is bolted to a larger base plate. The heads of the screws serve as a mechanical stop in themselves, so there is no need an entirely different part. The individual pieces of the second design are pictured below with descriptions as to their individual functions.

3.1 Outer structure with triangular supports

The outer structure needs to be self-supporting, but the triangular shaped supports need to be thin enough to serve as flexures. Most of the modeling surrounds the triangular supports, shown in Figure 3-6. The design of the base and top were then designed base on the dimensions of the triangular supports.
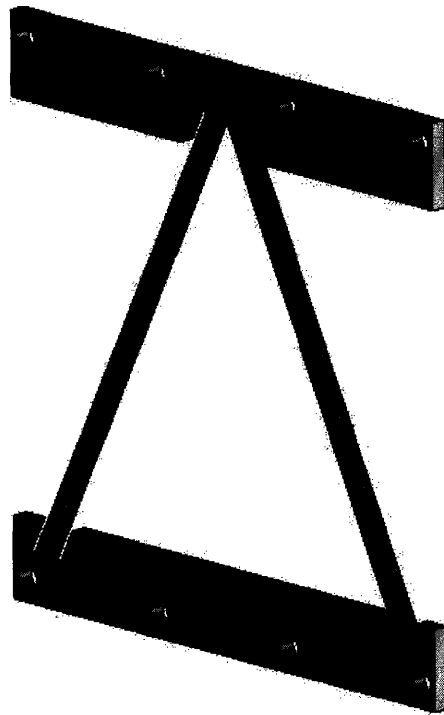
Figure 3-4: Triangle support for flexure system

Before modeling the triangular supports, it was decided to simplify the model. The width and thickness of the thin parts of the support would be the same. This way, only two dimensions needed to be determined independent of each other, the thickness and the length. It was difficult to account for both the linear and rotational motion in a model, so FEA analysis in CosmosWorks was used to determine the stress and displacement for various values of thickness and length, shown in Figure 3-5.

24

URES (in)

8.394e-001
7.694e-001
6.995e-001
6.295e-001
5.596e-001
4.896e-001
4.197e-001
3.497e-001
2.798e-001
2.098e-001
1.399e-001
6.995e-002
3.937e-032
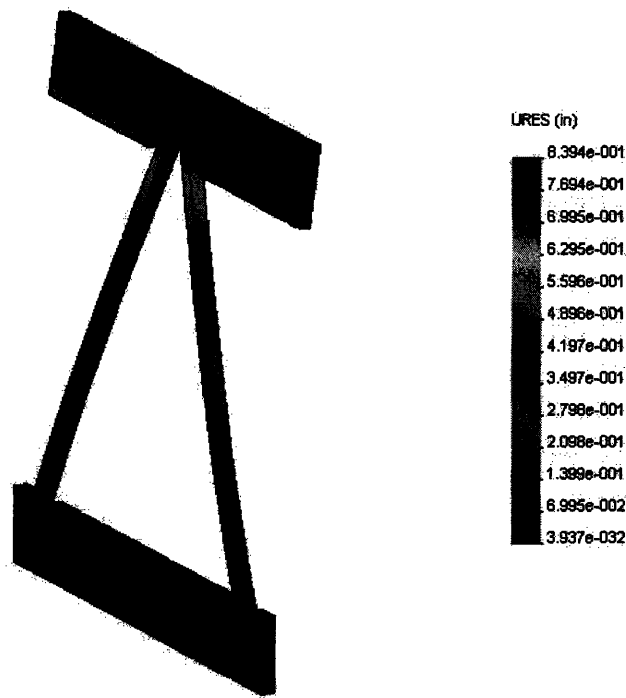
Figure 3-5: CosmosWorks FEA displacement analysis on triangle support

The entire flexure system was too large to run an FEA analysis, so the triangular and Z-shaped supports were tested independently. Initially, the results showed that a thickness of 3.175 mm [0.125 in] had the best displacement. However, there was concern over buckling, so that dimension was increased to 6.35 mm [0.25 in]. The length was then increased from 190.5 mm [7.5 in] to 215.9 mm [8.5 in] to regain some of the flexibility. There are four holes for 9.525 mm [0.375 in] bolts to attach the triangular supports to the base and top. The height of the top and bottom of the triangular flexures is designed to extend past the base and top so that they can serve as guides for the Z-support structure. For manufacturing, the parts were cut on the waterjet out of 6.35 mm [0.25 in] thick Delrin.
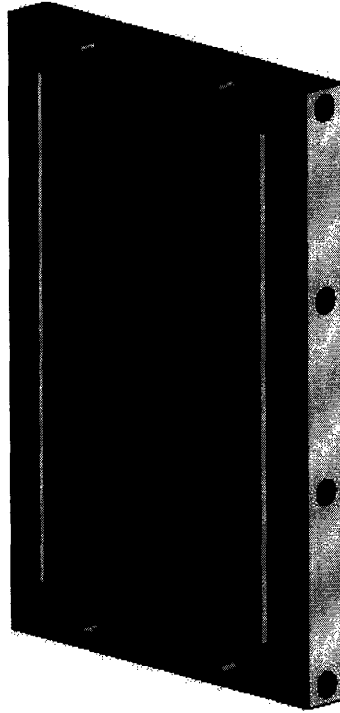
Figure 3-6: Base of triangle support for flexure system

The bottom of the triangular support structure has several functions besides

simply a base for the triangular supports. The slots in the middle serve as a measurement

when changing the pitch. The seven slots are each 12.7mm [0.5in] apart. They can be

used to determine the distance from the center, d, when calculating the pitch of the

flexure. There are also four 9.525mm [0.375in] holes to be used to attach the structure to

a larger base plate. The slots, holes and main shape of the base were cut on the waterjet

out of 19.05mm [0.75in] thick Delrin. The four 9.525mm [0.375in] holes on each side

are to attach the triangular supports. They were drilled on a mill and then tapped using

the same mill to power tap the holes. Finally, there are slots milled for 3.175mm

[0.125in] steel shim stock which will be used for the magnets in the Z-support structure

to slide on.
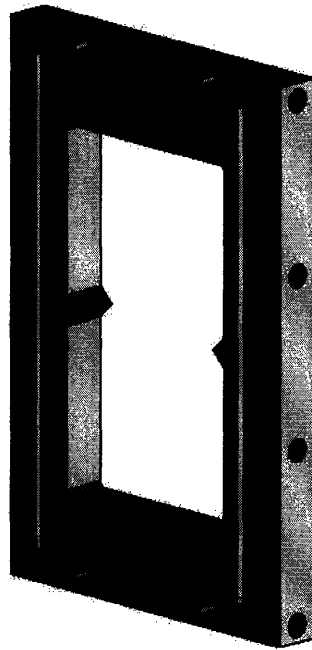
Figure 3-7: Top of triangle support for flexure system

The top piece of the triangular support structure has the same outer dimensions as the bottom piece. It also has the same 9.525mm [0.375in] holes on both the top and the sides and was manufactured in the same way. The only difference is the middle. Instead of slots to measure the distance from the center, the center is a hole with points in the center to keep track of what slot marker the Z-support structure is lined up with. Initially, the top and bottom pieces were to be the same, both having these holes and the bottom and top of the Z-support structure would have the measurement slots. However, then the design was changed such that the flexure system would be bolted to a larger base plate, so it would be impossible to flip the flexure over to line up both the top and bottom of the Z-support structure. For this reason, the design was changed so the bottom of the triangular support structure was changed to have the measurement slots. The top still has

the hole so that the user can see to adjust the Z-support flexure. The hole and basic shape of the top of the triangular support flexure were cut on the waterjet.

3.2 Inner structure with Z-shaped support

Without the inner structure, the outer structure would just be a linear flexure, for small movements. It is the added constraint in the Z-shaped support that causes the rotational motion.
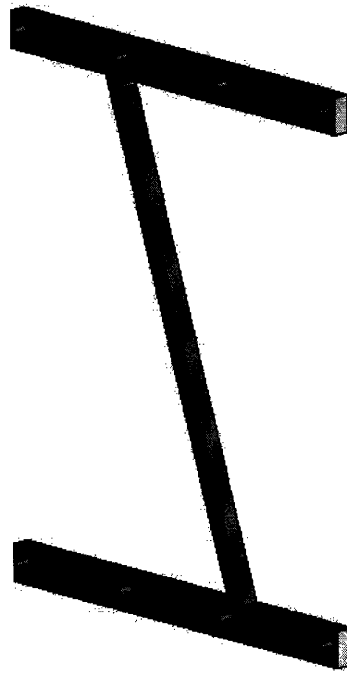
Figure 3-8: Z-shaped support for flexure system

The Z-shaped support was designed to have the same thickness and width as those in the triangular supports. The length of the Z-support is also determined by the length of the triangular flexures, as the Z-support flexure needs to fit within the triangular support structure. Like the triangular support, the Z-support was analyzed with CosmosWorks FEA software to test the displacement and stresses, as shown in Figure 3-9.

28
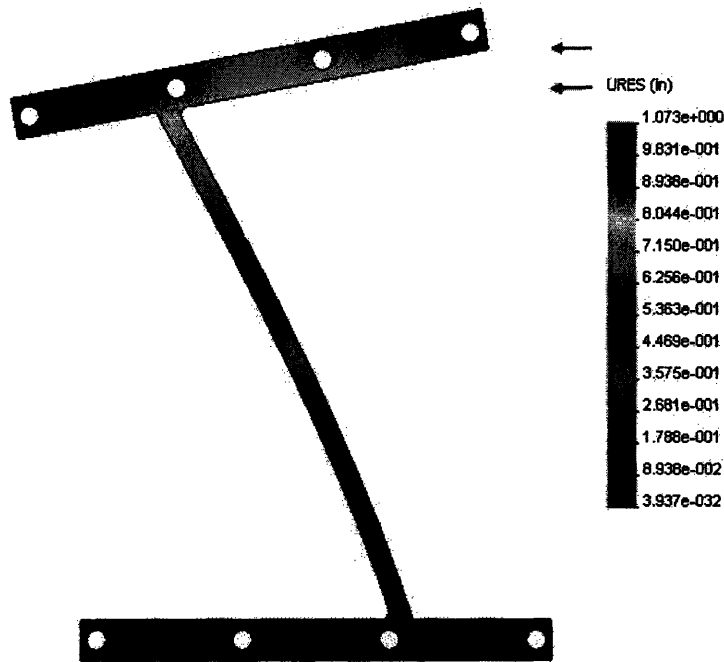
Figure 3-9: CosmosWorks FEA displacement analysis on Z-shaped support

The angle of the diagonal of the Z was set to approximately 71.57°, the tangent of

which is equal to three. Therefore, when determining the pitch, the equation for this

flexure is simplified to $p = 3d$. Finally, there are four 6.35mm [0.25in] holes in the top

and bottom of the Z-support to allow it to be bolted to the base and top. The entire Z-

support was cut on the waterjet from 6.35mm [0.25in] thick Delrin.

The bottom and top of the Z-support structure are similar in design to the bottom

and top of the triangular support structure, only they are inverted. For this structure, the

bottom, shown in Figure 3-10, has the whole with the arrows and the top, shown in

Figure 3-11, has the measuring slots. The parts were also cut on the waterjet as well. The

main difference, besides the size, which was explained earlier in this chapter, is that,

instead of slots for steel stock, there are holes for magnets. These holes were milled on a

CNC machine so that their placement was easily repeatable. Also, the depth of the holes

was important so that the magnets sat flush against the parts so that they would easily slide along the steel. Finally, both the top and bottom have four 6.35mm [0.25in] holes which were drilled and power tapped to allow easy assembly with the Z-shaped support.
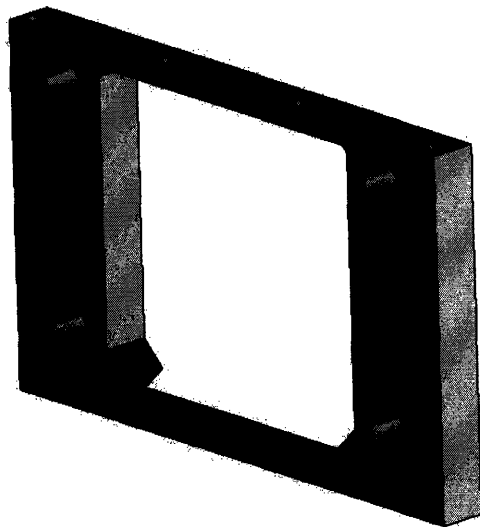

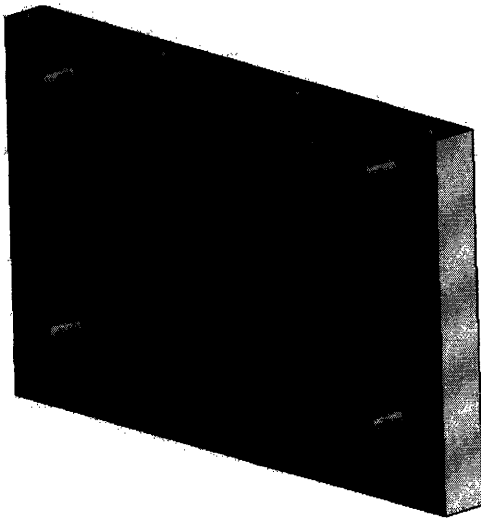Figure 3-10: Bottom of Z-shaped support for flexure system

Figure 3-11: Top of Z-shaped support for flexure system

Chapter 4
Experimental Setup and Sample Testing

Once the flexure systems were designed and built, there needed to be some way to measure the motion of each system. Holly Laird's thesis consists of selecting sensors to measure the motion of the flexures as well as the forces required to move them. A linear potentiometer was selected for the linear flexure and a setup was designed to support the flexure, linear potentiometer, and force gage, which was used to measure the force applied to the flexure. For the screw flexure, both linear and rotary potentiometers were needed to measure the two linked motions. A force gage was also used. The supports for these sensors may be seen in Figure 1-1.

The purpose of these flexure systems is to be used in educational kits. Therefore, a test was conducted to see how much someone with a given knowledge of flexures could learn from them. As the flexure system with screw-like motion is the more complicated and unfamiliar of the two, it was chosen to be tested. Two MIT seniors in mechanical engineering that are currently taking 2.72, Mechanical Design volunteered after an e-mail was sent to the class requesting participants. This class was selected as flexures and constraints were part of the curriculum, so the students' level of knowledge regarding the subject was known. They first listened to a presentation describing the flexure system with screw-like motion and how it worked. Next, they were given a worksheet with several questions to answer and told they were allowed to experiment with the flexure system to answer the questions. Some of the questions included determining the pitch for a given distance as well as determining the distance for a given pitch. For these questions, they were then asked to calculate the linear and rotational displacement when

the flexure was set for the given pitch and distance. Finally, they were given a survey regarding how much they learned from the presentation and experimenting with the flexure.

Chapter 5
Results and Analysis

5.1 Testing Results

Overall, the testing went well. The students asked several questions and seemed to enjoy working with the flexure system. All their answers were correct on the worksheet, although there were some technical difficulties with the sensors, such that they were unable to obtain actual data.

For the survey, they were asked three questions where the answers ranged from one to five, with five being the highest. The results are as follows.

1) Did you learn anything from the presentation?

   Average answer: 3

2) Did you learn anything from experimenting with the screw flexure?

   Average answer: 5

3) How well did the flexure demonstrate the concepts discussed?

   Average answer: 4

The last two questions asked if anything was unclear or should be presented in a different manner and left space for comments. Both mentioned that more time could be spent in the presentation on constraint and freedom lines specifically for flexure systems. In the comments section, both again requested more emphasis on flexure systems in general for the presentation.

5.2 Analysis

The results from the survey, specifically questions two and three show that the students learned from interacting with the flexure system and not just listening to a

lecture. They both circled five for the second question, asking if they learned anything from experimenting with the flexure system, and one student added "It was fun to play with." The resulting responses to question three, which asked how well the flexure demonstrated the concepts discussed, averaged out to four, because one student responded with a five, while the other circled three. The student who circled three added that the "screw flexure has 'weird' [degree of freedom] lines" and wished that that issue would be addressed more in the lecture.

The size of the flexure appears to be another advantage. While most flexures used in products are very small, the linear and screw flexures are on the order of about eight inches in height. This allows potential students to easily see what exactly is happening as the flexure moves. In general, the practicing engineers that the flexure will eventually be presented to are adults with larger hands, such that were smaller size flexures built, they would be difficult for them to move and adjust.

Chapter 6
Conclusion

6.1 Summary

This thesis gives the design details for both a familiar linear motion flexure and a new flexure with screw motion. Both are to be used in educational kits given to practicing engineers in a class that introduces them to new developments in flexures. The linear flexure is designed as something they will have seen before and understand the concepts behind. The flexure system with linked linear and rotational motion is something that they will have never seen before. Therefore, it is designed in a manner such that they can experiment with it and adjust the pitch.

This thesis is based on the graduate work of Jonathan Hopkins in the area of Freedom and Constraint Topologies, which builds on the work of Douglas L. Blanding. Blanding introduced the idea that the number of degrees of freedom in a flexure is equal to six minus the number of non-redundant constraints.

A working prototype of each the linear motion and screw motion flexure were manufactured. Sensors were added to the kits to measure the motion and force. The kit for the flexure system with linked linear and rotational motion was the presented to two undergraduate students in mechanical engineering to find out what if they learned more from experimenting with the flexure. Results of this test indicate enough promise to further refine the kit.

6.2 Future Work

Although the second iteration of the flexure system works, there are still several changes that can be made to make it work better and ease the manufacturing and assembly process. First of all, it was somewhat stiff. In the next design iteration, the thickness could be decreased to 3.175mm [0.125in]. If buckling is still a concern, the length can be decreased. Also, it could be made out of ABS, which has a lower Young's modulus than Delrin, which is a factor in stiffness.

Another adjustment concerns the magnets. In the completed screw flexure, they were super-glued in. However, given the strength of the magnets, they frequently came loose when sliding along the steel tracks. In the next iteration, they should be press-fit into the holes.

There were several problems with the various holes in the flexure. First, it was difficult getting them to line up when assembling. Therefore, the holes on the triangular and Z-shaped supports should be slotted so that they can be better aligned with the tapped holes in their respective bottoms and tops. Second, the wholes are a variety of different sizes, which means many different size bolts are needed and can be confusing for assembly. Next time, perhaps the different types of bolts can be limited to one or two.

References

[1] Blanding, Douglass L., 1950-   Exact constraint : machine design using kinematic principles / Douglass L. Blanding.   New York :   ASME Press,   1999.

[2] Hopkins, J.B. and Culpepper, M.L., "Synthesis of Multi–degree of Freedom Flexure System Concepts via Freedom and Constraint Topologies (FACT),"

[3] All material properties referenced from http://www.matweb.com.

Appendix A

Worksheet

The following is the worksheet given to the students testing the flexure kit with screw motion. After listening to a brief presentation on the subject, they were asked to complete a worksheet to determine what they had learned while experimenting with the flexure system.
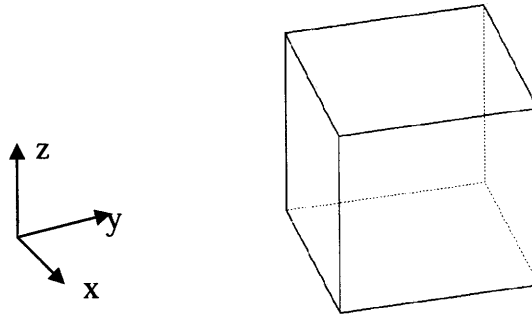
Name:_____

Part I: Qualitative

1) Draw freedom and constraint lines for the screw flexure on the picture. Please use blue for constraint lines and red for freedom lines.
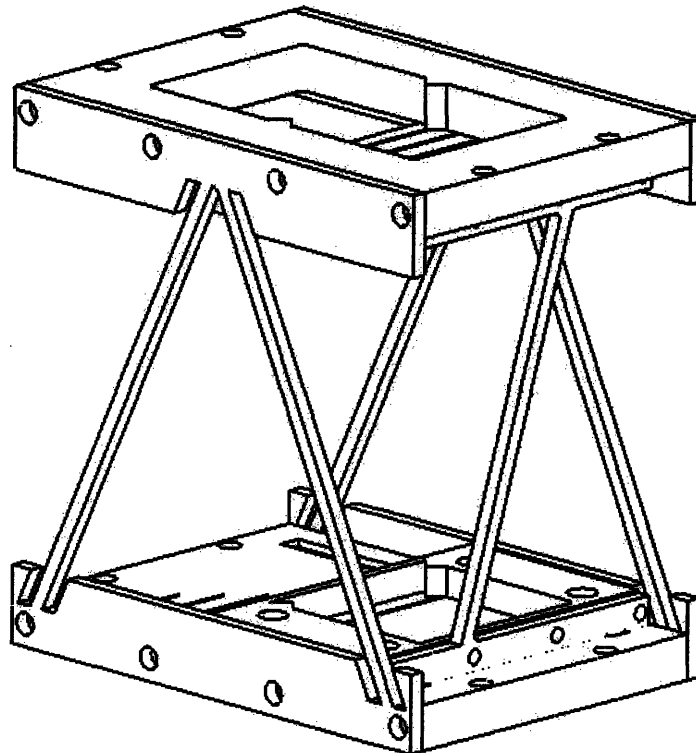
2) Please name at least one case where
   a. the pitch goes to 0

   b. the movement is purely rotational

3) Constrain the box such that it only moves in the z and theta z directions.

Part II: Quantitative

Fill in the table with results you get from the given loads.

Force	Linear Displacement	Angular Displacement
¼ Pound		
½ Pound		
1 Pound		
2 Pounds		
3 Pounds		

Based on the data above, calculate the following:

1) Please calculate the pitch for a distance d=2in.

2) Please calculate the distance for pitch p=4.5

Appendix B

Survey Questions

This appendix contains the survey questions asked of the students who tested the flexure system with screw motion. When they completed the worksheet shown in appendix A, they were asked to complete this survey for data gathering purposes.

Closing Survey

1. Did you learn anything from the presentation?
   Not really   1   2   3   4   5   A lot

2. Did you learn anything from experimenting with the screw flexure?
   Not really   1   2   3   4   5   A lot

3. How well did the flexure demonstrate the concepts discussed?
   Not at all   1   2   3   4   5   Very well

4. Was anything unclear or was there anything you like presented in a different manner?

5. Comments/Suggestions