

Dynamic Channel Allocation in Satellite and
Wireless Networks

by

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Submitted to the Department of Electrical Engineering and Computer
Science

in partial fulfillment of the requirements for the degree of

Doctor of Philosophy

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

May 2007

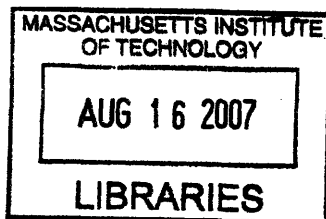
[June 2007]

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Abstract

The objective of this thesis is to understand how to utilize wireless channels in a fair and efficient manner within a multi-users communication environment. We start by exploring the allocation of a single wireless downlink fading channel among competing users. The allocation of a single uplink multiaccess fading channel is studied as well. With multiple parallel fading channels, a MAC protocol based on pricing is proposed to allocate network resource according to users' demand. We also investigate the use of parallel transmissions and redundant packets to reduce the file transmission delay.

Specifically, we develop a novel auction-based algorithm to allow users to fairly compete for a downlink wireless fading channel. We first use the second-price auction mechanism whereby user bids for the channel, during each time-slot, based on the fade state of the channel, and the user that makes the highest bid wins use of the channel by paying the second highest bid. Under the assumption that each user has a limited budget for bidding, we show the existence of a Nash equilibrium strategy. And the Nash equilibrium leads to a unique allocation for certain channel state distribution. We also show that the Nash equilibrium strategy leads to an allocation that is pareto optimal. We also investigate the use of another auction mechanism, the all-pay auction, in allocating a single downlink channel. A unique Nash equilibrium is shown to exist. We also show that the Nash equilibrium strategy achieves a throughput allocation for each user that is proportional to the user's budget

For the uplink of a wireless channel, we present a game-theoretical model of a wireless communication system with multiple competing users sharing a multiaccess fading channel. With a specified capture rule and a limited amount of energy available, a user opportunistically adjusts its transmission power based on its own channel state to maximize the user's own individual throughput. We derive an explicit form of the Nash equilibrium power allocation strategy. Furthermore, as the number of users in the system increases, the total system throughput obtained by using a Nash equilibrium strategy approaches the maximum attainable throughput.

In a communication scenario where multiple users sharing a set of multiple parallel channels to communicate with multiple satellites, we propose a novel MAC protocol

based on pricing that allocates network resources efficiently according to users' demand. We first characterize the Pareto efficient throughput region (i.e., the achievable throughput region). The equilibrium price, where satellite achieve its objective and users maximize their payoffs, is shown to exist and is unique. The resulting throughput at the equilibrium is shown to be Pareto efficient.

Finally, we explore how a user can best utilize the available parallel channels to reduce the delay in sending a file to the base-station or satellite. We study the reduction of the file delay by adding redundant packets (i.e., coding). Our objective here is to characterize the delay and coding tradeoff in a single flow case. We also want to address the question whether coding will help to reduce delay if every user in the system decides to add redundancy for its file transmission.

Thesis Supervisor: Eytan Modiano

Title: Associate Professor

To my dad

Acknowledgments

I'd like to express my deepest gratitude first and foremost to my advisor Professor Eytan Modiano. His guidance, encouragement, and support throughout my study here at MIT had been invaluable to me. I want to thank him for teaching me to see a problem in its simplest form and guiding me patiently in searching for a fundamental problem.

I am also grateful to Professor Lizhong Zheng for his support and counsel. His intuitive elucidation of the wireless communications made the subject much easy for me to grasp. Working with him, not only did I learn what constitutes an interesting research problem, but also the brilliant technique he demonstrated in solving these problems. I thank Professor Asuman Ozdaglar for her insightful comments and suggestions on the thesis. Her expertise in game theory helped me in making many of the proofs more rigorous. I also thank Professor Eitan Altman at INRIA and Professor Randall Berry at Northwestern University for their helpful discussions in the area of wireless networks.

I thank my office-mates, Andrew Brzezinski, Li-Wei Chen, Chunmei Liu, Krishna Jagannathan, Anand Srinivas, and Murtaza Zafer, for their camaraderie. The stimulating and often hilarious conversation with them makes my years at MIT much more enjoyable. I'd like to also thank my friends at MIT, Lik Mui, Shaohua Yang, Cheng Luo, and Ji-Jon Sit, for their constant support .

Finally, I thank my family dad, mom, sister and Wenzheng for their love and support.

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Chapter 1

Introduction

The objective of this thesis is to understand how to utilize a wireless channel in a fair and efficient manner within a multi-users communication environment. In wireless and satellite networks, network resources such as bandwidth and power are usually limited. A systematic procedure for fair and efficient resource allocation among competing individuals, therefore, is desirable.

Recently, numerous centralized channel allocation schemes have been proposed and studied in the context of wireless networks [9], [19], [25]. There, the fair channel allocation problem is usually formulated as an optimization problem with objectives such as maximizing the total system throughput and constraint that takes into account individual user's performance guarantee. While the objective of maximizing throughput can be a reasonable one for both the network manager and the individuals users, coming up with fairness constraints for the optimization problem can be rather arbitrary.

Our approach is to investigate interactions among users with conflicting interest and the resulting allocation as a consequence of users' selfish behaviors. This channel allocation result which takes the user's selfish behaviors into account will serve as a reference point for comparing other centralized allocation scheme. More importantly, it provides fundamental insight into the understanding and the design of a centralized channel allocation scheme that makes sense.

Specifically in this thesis, we consider a communication scenario where base-

stations or satellites communicate with multiple users. The case where multiple users are sharing a single channel to communicate with a single base-station or satellite is considered in Section 1.1, for both the downlink and the uplink transmission. We want to explore the use of an auction algorithm as a channel allocation mechanism to achieve a fair and efficient use of this single channel. In Section 1.2.1, we consider the case that multiple users sharing a set of multiple parallel channels to communicate with multiple satellites. There, we define the pareto optimal throughput region and investigate a possible random-access scheme that achieves the pareto optimality. Lastly, in Section 1.2.2 we introduce how to efficiently utilize the multiple parallel channels available to reduce file transmission delay when a user need to send a file through the satellite or base-station.

1.1 Allocation of A Single Channel

1.1.1 Downlink Channel Allocation

A fundamental characteristic of a wireless network is that the channel over which communication takes place is often time-varying. This variation of the channel quality is due to constructive and destructive interference between multipaths and shadowing effects (fading). For a single wireless channel over which a transmitter communicates with multiple users, the transmitter can send data at higher rates to users with better channels. However, the potential to exploit higher data throughput for users with good channel states may introduce a trade-off between system efficiency and fairness among users. In a time slotted system where only one user can be served during each time slot, the objective of maximizing the total system throughput may result in very low throughput for some users whose channel states are consistently poor. Hence, an allocation scheme that balances the conflicting objective of maximizing total system throughput and maximizing individual user's throughput is needed.

The following simple example illustrates different allocations that may result from different notions of fairness. We consider the communication system with one trans-

mitter and two users, A and B, and the allocations that use different notions of fairness such as the maxmin fairness and time fraction fairness (i.e., assigning a certain percentage of time slots to each user). We assume that the throughput is proportional to the the channel condition. The channel coefficient, which is a quantitative measure of the channel condition, takes value in the interval $[0, 1]$ with 1 as the best channel condition. In this example, the channel coefficients for user A and user B in the two time slots are $(0.1, 0.2)$ and $(0.3, 0.9)$ respectively. The throughput result for each individual user and for total system under different notions of fairness constraint are given in Table I. When there is no fairness constraint, to maximize the total system throughput would require the transmitter to allocate both time slots to user B. To achieve maxmin fair allocation, the transmitter would allocate slot one to user B and slot two to user A, thus resulting in a total throughput of 0.5. If the transmitter wants to maximize the total throughput subject to the constraint that each user gets one time slot (i.e., the approach of [9]), the resulting allocation, denoted as time fraction fair, is to give user A slot one and user B slot two. As a result, the total throughput is 1.0. In the above example, the transmitter selects an allocation to

	Throughput for A	Throughput for B	Total throughput
No fair constraint	0	1.2	1.2
Maxmin fair	0.2	0.3	0.5
Time fraction	0.1	0.9	1.0

Table 1.1: Throughput results using different notions of fairness.

ensure an artificially chosen notion of fairness. From Table I, we can see that from the user's perspective, no notion is truly fair as both users want slot two. In order to resolve this conflict, we use a new approach that allows users to compete for time slots. In this way, each user is responsible for its own action and its resulting throughput. We call the fraction of bandwidth received by each user *competitive fair*. Using this notion of competitive fairness, the resulting throughput obtained for each user can serve as a reference point for comparing various other allocations. Moreover, the competitive fair allocation scheme can provide fundamental insight into the design of

a fair scheduler that make sense.

In our model, users compete for time-slots. For each time-slot, each user has a different valuation (i.e., its own channel condition), and each user is only interested in maximizing its own throughput. Naturally, these characteristics give rise to an auction. Here, we consider the second-price and all-pay auction mechanisms. Using the auction mechanism, users submit a “bid” for the time-slot and the transmitter allocates the slot to the user that made the highest bid. In the second-price auction mechanism, the winner only pays the second highest bid [20]. However, in the all-pay auction mechanism, all users have to pay their bids. Each user is assumed to have an initial budget. The money possessed by each user can be viewed as fictitious money that serves as a mechanism to differentiate the QoS given to the various users. This fictitious money, in fact, could correspond to a certain QoS for which the user paid in real money. As for the solution of the slot auction game, we use the concept of Nash equilibrium, which is a set of strategies (one for each player) from which there are no profitable unilateral deviation.

In the downlink communication system with one transmitter and multiple receiving users, we assume the channel states for each user are independent and identically distributed with known probability distribution for each time slot. The channel states for different users are also independent. Given that each user wants to maximize its own *expected* throughput subject to an average budget constraint, we have the following results: We find the Nash equilibrium strategy for general channel state distribution. This Nash equilibrium strategy pair is shown to lead to a unique allocation for certain channel state distributions, such as the exponential distribution and the uniform distribution over a bounded interval. We then show that the Nash equilibrium strategy of this auction leads to an allocations at which total throughput is no worse than $3/4$ of the throughput obtained by an algorithm that attempts to maximize total system throughput without a fairness constraint under the uniform distribution. The throughput for each user, resulting from the use of the Nash equilibrium strategy, is shown to be pareto optimal (i.e., it is impossible to make some users better off without making some other users worse off). Lastly, based on the Nash equilibrium

strategies of the second price auction with money constraint, we also propose a *centralized* opportunistic scheduler that does not suffer the shortcomings associated with the proportional fair and the time fraction fair scheduler.

When the all-pay auction is used, we obtain Nash equilibrium strategy for each user for uniformly distributed channel state. We also show that the Nash equilibrium strategy pair provides an allocation scheme that is fair in the sense that the price per unit of throughput is the same for both users.

1.1.2 Uplink Random Access

For the uplink transmission, we present a distributed uplink Aloha based multiple access scheme. Specifically, we consider a communication system consisting of multiple users competing to access a satellite, or a base-station. Each user has an average power constraint, and time is slotted. During each time slot, each user chooses a power level for transmission based on the channel state of current slot, which is only known to itself. Depending on the capture model and the received power of that user's signal, a transmitted packet may be captured even if multiple users are transmitting at the same slot. If the objective of each user in the system is to find a power allocation strategy that maximizes its probability of getting captured based its average power constraint, we have a power allocation game that resembles the all-pay auction. Comparing with the all-pay auction, the average power constraint in the power allocation game corresponds to the average money constraint, and the transmission power corresponds to the money. Both power and money is taken away once a bidding or a transmission has taken place. In this uplink scenario, using the technique to solve for Nash equilibrium in the all-pay auction, we get a similar Nash equilibrium strategy in the uplink multiple access power allocation problem.

The game theoretical formulation of the uplink power allocation problem stems from the desire for a distributive algorithm in a wireless uplink. Due to the variation of channel quality in a fading channel, one can exploit the channel variation opportunistically by allowing the user with best channel condition to transmit, which require the presence of a centralized scheduler that knows each user's channel condi-

tion. As the number of users in the network increases, the delay in conveying user's channel conditions to the scheduler will limit the system's performance. Hence, a distributed multi-access scheme with no centralized scheduler becomes an attractive alternative. However, in a distributive environment, users may want to change their communication protocols in order to improve their own performance, making it impossible to ensure a particular algorithm will be adopted by all users in the network. Rather than following some mandated algorithm, in this work users are assumed to act selfishly (i.e., choose their own power allocation strategies) to further their own individual interests.

When each user wants to maximize its own expected throughput, we obtain a Nash equilibrium power allocation strategy which determines the optimal transmission power control strategy for each user. The obtained optimal power control strategy specifies how much power a user needs to use to maximize its own throughput for any possible channel state. Users get different average throughput based on their average power constraint. Hence, this transmission scheme can be viewed as mechanism for providing quality of service (QoS) differentiation; whereby users are given different energy for transmission. The obtained Nash equilibrium power allocation strategy is unique under certain capture rules. When all users have the same energy constraint, we obtained a symmetric Nash equilibrium. Moreover, as the number of users in the system increases, the total system throughput obtained by using a Nash equilibrium strategy approaches the maximum attainable throughput using a centralized scheduler.

1.2 Allocation of Multiple Parallel Channels

1.2.1 Multiple Parallel Channel Allocation Using Pricing

Having studied the single channel communication scenario, we now consider the case that multiple users sharing a set of multiple parallel channels to communicate with multiple satellites (one channel for each satellite). Specifically, we consider a commu-

nication network with multiple satellites, collectively acting as a network manager, that wishes to allocate network uplink capacity efficiently among a set of users, each with a utility function depending on their allocated data rate. We assume that each satellite uses a separate channel for communication, such as using different frequency band for receiving. Each user has data that needs to be sent to the satellite network, and there may be multiple satellites that a user on the ground can communicate with, or switched diversity termed in [1]. Therefore, the data rate for each user here is the rate at which each user can access the satellite network by sending its data to any satellite within its view.

Slotted aloha is used here as the multi-access scheme for its simplicity. Other multi-access schemes can be used in conjunction with the pricing scheme to provide QoS as well. Due to different path loss and fading, the channel gain from a user to different satellite within its view can be different. Therefore, during a single time slot, a user has to decide not only whether it should transmit but also to which satellite it will transmit. To control users' transmission rates, each satellite will set a price (may differ from satellite to satellite) for each *successfully received* packet. Based on the price set by each satellite, a user determines its targeted satellite and the transmission probability to maximize its net payoff, which is the utility of its received rate minus the cost.

It is well-known that the throughput of a slotted aloha system is low. Therefore, a reasonable objective for the network manager is to efficiently utilize the available resource. In this chapter, we want to explore the use of pricing as a control mechanism to achieve efficiency. To do so, we need to define the meaning of efficiency in the context of a slotted aloha system. With a wire-line, such as optical fiber, of capacity R , an allocation is efficient as long as the sum of the bandwidth allocated to each individual user is equal to R , i.e., no waste of bandwidth. With a collision channel in the aloha system, no simple extension of the wire-line case exists. We therefore use a concept called *Pareto efficient* for allocating resource in a collision channel. By definition, a feasible allocation (s_1, s_2, \dots, s_n) is Pareto efficient if there is no other feasible allocation $(s'_1, s'_2, \dots, s'_n)$ such that $s'_i \geq s_i$ for all $i = 1, \dots, n$ and $s'_i > s_i$ for

some i .

The multiple satellites communication networks considered here differ from the multichannel aloha networks in one key aspect—the channel quality associated with the path from a user to the satellite. This difference gives us insight on how to best utilize the multiple channels available to users. A multichannel aloha network consists of M parallel, equal capacity channels for transmission to one base station or satellite shared by a set of users [30] [31]. The M channels can be implemented based on either Frequency Division Multiplexing or Time Division Multiplexing approaches. When a user has a packet to send, it will *randomly* select one channel to transmit. This random selection of the channel is largely due to the lack of coordination among competing users. Intuitively, we would expect that the throughput of the system will be higher if the coordination in channel selection among users was available. In the multiple satellite networks, different price and channel state are two mechanisms that enable the coordination in channel selection among the competing users.

Specifically, we propose a novel MAC protocol based on pricing that allocates network resources efficiently according to users' demand. We then characterize the Pareto efficient throughput region in a single satellite network. The existence of an equilibrium price is presented. Furthermore, we show that such equilibrium price is unique. In the multiple satellites case, the Pareto efficient throughput region is also described. We then show that the equilibrium price exists and is unique. The resulting throughput at the equilibrium is shown to be Pareto efficient as well.

1.2.2 Multipath Routing over Wireless Networks

In this section, we consider the case when there are multiple parallel paths available for transmission between the source and the destination node [36], [42]. File transmission delay is studied instead of the average packet delay in networks where a file consists of multiple packets. The file transmission delay is defined to be the time interval between the time that the file was generated and the time at which the file can be reconstructed at the destination node. File transmission delay resembles more closely the delay experience of an average user. In a wireless transmission scenario,

the transmission delay of each packet can sometimes be modelled as i.i.d. random variable. The average file transmission delay not only depends on the mean of the packet delay but also its distribution especially the tail. Here, we focus on the problem of how to minimize the average file transmission delay in a wireless or satellite network.

For a file with a fixed number of packets, one can assign a certain fraction of these packets to each path and transmit them simultaneously. We assume that each packet will experience an independent and identically distributed transmission delay on a particular path, which we argue to hold for certain communication scenario. File transmission delay can be very different from the packet transmission delay especially when the distribution of the packet transmission delay has a heavy tail. After a source distributed the packets of a file among the available paths, the destination can reconstruct the file when all the packets of that file have arrived. The problem of how to distribute a file with finite number of packets among a finite set of parallel paths, each with different channel statistics, is studied in [46]. There, an optimal packets allocation scheme to minimize the average file transmission delay is presented. Reconstruction of the original file at the destination node require the arrivals of all packets of that file. This may take a long time due to the heavy tail of the packet delay distribution. Hence, it prompts us to code the original file at the packet level. Specifically, for a file with k packets originally, the source transmit $n > k$ packets by adding some redundant packets to the original file. At the destination node, upon receiving the first k packets out of the n transmitted packets, the destination node can reconstruct the original file. This kind coding at the packet level exists such as the digital fountain code or tornado code [45], [44].

Our objective here is to obtain an intuitive understanding of the tradeoff between the code rate and delay reduction in a communication setting with a single or multiple source destination pairs that share a set of parallel paths. In the single source-destination case, given a file size, we provide a practical guideline in determining the code rate to achieve a good file transmission delay. We show that only a few redundant packets are required for achieving a significant reduction in file transmission delay. Next, we consider the trade-off between the file transmission delay and code rate in a

multiple users environment. There, the redundant packets will increase the network congestion level, hence the packet's queueing delay. We will investigate whether adding redundant packet can still reduce the file transmission delay. The coding and delay tradeoff in this case is characterized in terms of the traffic load of the network. Depending on the load, a unique code rate that minimizes the transmission delay is obtained.

Chapter 2

Fair Allocation of A Wireless Fading Channel: An Auction Approach

The limited bandwidth and high demand in a communication network necessitate a systematic procedure in place for fair allocation. This is where the economic theory of pricing and auction can be applied in the field of communications and networks research, for pricing and auction are natural ways to allocate resources with limited supply. Recently, in the networks area, much work is done to address the allocation of a limited resource in a complex, large scaled system such as the internet [6], [16]. They approach the problem from a classical economic perspective where users have utility functions and cost functions, both measured in the same monetary unit. Pricing is used as a tool to balance users' demand for bandwidth.

Here, we are interested in solving a specific engineering problem of scheduling transmission among a set of users while achieving fairness in a specific wireless environment. We use game theoretical concepts such as Nash equilibrium as a tool for modelling the interaction among users. Both the objective and the constraint of the optimization problem that each user faces have physical meanings based on underlying system. Our focus in this chapter will be on the use of the second-price and the all-pay auction in allocating a downlink wireless fading channel.

2.1 Introduction

A fundamental characteristic of a wireless network is that the channel over which communication takes place is often time-varying. This variation of the channel quality is due to constructive and destructive interference between multipaths and shadowing effects (fading). In a single cell with one transmitter (base station or satellite) and multiple users communicating through fading channels, the transmitter can send data at higher rates to users with better channels. In a time slotted system, time slots are allocated among users according to their channel qualities.

The problem of resource allocation in wireless networks has received much attention in recent years. In [30] the authors try to maximize the data throughput of an energy and time constrained transmitter communicating over a fading channel. A dynamic programming formulation that leads to an optimal transmission schedule is presented. Other works address the similar problem, without consideration of fairness, include [7] and [8]. In [5], the authors consider scheduling policies for *maxmin fairness* allocation of bandwidth, which maximizes the allocation for the most poorly treated sessions while not wasting any network resources, in wireless ad-hoc networks. In [25], the authors designed a scheduling algorithm that achieves *proportional fairness*, a notion of fairness originally proposed by Kelly [6]. In [9], the authors present a slot allocation that maximizes expected system performance subject to the constraint that each user gets a fixed fraction of time slots. The authors did not use a formal notion of fairness, but argue that their system can explicitly set the fraction of time assigned to each user. Hence, while each user may get to use the channel an equal fraction of the time, the resulting throughput obtained by each user may be vastly different.

The following simple example illustrates the different allocations that may result from the different notions of fairness. We consider the communication system with one transmitter and two users, A and B, and the allocations that use different notions of fairness discussed in the previous paragraph. We assume that the throughput is proportional to the the channel condition. The channel coefficient, which is a quan-

titative measure of the channel condition ranging from 0 to 1 with 1 as the best channel condition, for user A and user B in the two time slots are (0.1, 0.2) and (0.3, 0.9) respectively. The throughput result for each individual user and for total system under different notions of fairness constraint are given in Table I. When there is no fairness constraint, to maximize the total system throughput would require the transmitter to allocate both time slots to user B. To achieve maxmin fair allocation, the transmitter would allocate slot one to user B and slot two to user A, thus resulting in a total throughput of 0.5. If the transmitter wants to maximize the total throughput subject to the constraint that each user gets one time slot (i.e., the approach of [9]), the resulting allocation, denoted as time fraction fair, is to give user A slot one and user B slot two. As a result, the total throughput is 1.0. In the above example, the

	Throughput for A	Throughput for B	Total throughput
No fair constraint	0	1.2	1.2
Maxmin fair	0.2	0.3	0.5
Time fraction	0.1	0.9	1.0

Table 2.1: Throughput results using different notions of fairness.

transmitter selects an allocation to ensure an artificially chosen notion of fairness. From Table I, we can see that from the user’s perspective, no notion is truly fair as both users want slot two. In order to resolve this conflict, we use a new approach which allows users to compete for time slots. In this way, each user is responsible for its own action and its resulting throughput. We call the fraction of bandwidth received by each user *competitive fair*. Using this notion of competitive fairness, the resulting throughput obtained for each user can serve as a reference point for comparing various other allocations. Moreover, the competitive fair allocation scheme can provide fundamental insight into the design of a fair scheduler that make sense.

In our model, users compete for time-slots. For each time-slot, each user has a different valuation (i.e., its own channel condition). And each user is only interested in getting a higher throughput for itself. Naturally, these characteristics give rise to an auction. In this chapter we consider the second-price auction and all-pay auction

mechanism. Using the second-price auction mechanism, users submit a “bid” for the time-slot and the transmitter allocates the slot to the user that made the highest bid. Moreover, in the second-price auction mechanism, the winner only pays the second highest bid [20]. The second-price auction mechanism is used here due to its “truth telling” nature (i.e., it is optimal for a user to bid its true valuation of a particular object). Using the all-pay auction mechanism, users submit a “bid” for the time-slot and the transmitter allocates the slot to the user that made the highest bid. Furthermore, in the all-pay auction mechanism, the transmitter gets to keep the bids of all users (regardless of whether or not they win the auction). The all-pay auction is explored here because it leads an intuitive throughput allocation (i.e., users’ throughput ratio is the same as users’ money ratio). Initially, each user is assumed to have a certain amount of money. The money possessed by each user can be viewed as fictitious money that serves as a mechanism to differentiate the QoS given to the various users. This fictitious money, in fact, could correspond to a certain QoS for which the user paid in real money. As for the solution of the slot auction game, we use the concept of Nash equilibrium, which is a set of strategies (one for each player) from which there are no profitable unilateral deviation.

In this chapter, we consider a communication system with one transmitter and multiple users. For each time slot, channel states are independent and identically distributed with known probability distribution. Each user wants to maximize its own *expected* throughput subject to an average money constraint. Our major results for the second-price auction include:

- We find the Nash equilibrium strategy for general channel state distribution.
- We show that the Nash equilibrium strategy pair leads to a unique allocation for certain channel state distribution, such as the exponential distribution and the uniform distribution over $[0, 1]$.
- We show that the Nash equilibrium strategy of this auction leads to an allocations at which total throughput is no worse than $3/4$ of the throughput obtained by an algorithm that attempts to maximize total system throughput without a

fairness constraint under uniform distribution.

- We show that the Nash equilibrium strategy leads to an allocation that is pareto optimal (i.e., it is impossible to make some users better off without making some other users worse off).
- Based on the Nash equilibrium strategies of the second price auction with money constraint, we also propose a *centralized* opportunistic scheduler that does not suffer the shortcomings associated with the proportional fair and the time fraction fair scheduler.

The results for the all-pay auction is given as follows:

- We find a unique Nash equilibrium when both channel states are uniformly distributed over $[0, 1]$.
- We show that the Nash equilibrium strategy pair provides an allocation scheme that is fair in the sense that the price per unit of throughput is the same for both users.
- We show that the Nash equilibrium strategy of this auction leads to an allocations at which total throughput is no worse than $3/4$ of the throughput obtained by an algorithm that attempts to maximize total system throughput without a fairness constraint.
- We provide an estimation algorithm that enables users to accurately estimate the amount of money possessed by their opponent so that users do not need prior knowledge of each other's money.

Game theoretical approaches to resource allocation problems have been explored by many researchers recently (e.g., [2][16]). In [2], the authors consider a resource allocation problem for a wireless channel, without fading, where users have different utility values for the channel. They show the existence of an equilibrium pricing scheme where the transmitter attempts to maximize its revenue and the users attempt to maximize their individual utilities. In [16], the authors explore the properties of

a congestion game where users of a congested resource anticipate the effect of their action on the price of the resource. Again, the work of [16] focuses on a wireline channel without the notion of wireless fading. Our work attempts to apply game theory to the allocation of a wireless fading channel. In particular, we show that auction algorithms are well suited for achieving fair allocation in this environment. Other papers dealing with the application of game theory to resource allocation problems include [3][23][24].

This chapter is organized as follows. Section 2 analyzes user's Nash equilibrium bidding strategy for the second-price auction. Specifically, in Section 2.1, we describe the communication system and the auction mechanism. In Section 2.2, we start by presenting the Nash equilibrium strategy pair for the two users game with general channel distribution. The uniqueness of the allocation scheme derived from the Nash equilibrium is shown when the channel state has the exponential or the uniform $[0, 1]$ distribution. We then derive the Nash equilibrium for the N-users game. In section 2.3, we show the unique Nash equilibrium strategy for the case that each user can use multiple bidding functions. The Pareto optimality of the allocation resulting from the Nash equilibrium strategies is established in Section 2.4. In Section 2.5, we compare the throughput results of the Nash equilibrium strategy with other centralized allocation algorithms. The analysis for user's Nash equilibrium bidding strategy is presented in Section 3. Section 3.1 presents the problem formulation for the all-pay auction. In Section 3.2, the unique Nash equilibrium strategy pair and the resulting throughput for each user are provided for the case that each user can use only one bidding function. In Section 3.3, we show the unique Nash equilibrium strategy pair for the case that each user can use multiple bidding functions. In Section 3.4, we compare the throughput results of the Nash equilibrium strategy with two other centralized allocation algorithms. In Section 3.5, an estimation algorithm that enables the users to estimate the amount of money possessed by their opponent is developed. Finally, Section 4 concludes the chapter.

2.2 Second-price Auction

2.2.1 Problem Formulation

We consider a communication scenario where a single transmitter sends data to N users over independent fading channels. We assume that there is always data to be sent to the users. Time is assumed to be discrete, and the channel state for a given channel changes according to a known probabilistic model independently over time. The transmitter can serve only one user during a particular slot with a constant power P . The channel fade state thus determines the throughput that can be obtained.

For a given power level, we assume for simplicity that the throughput is a linear function of the channel state. This can be justified by the Shannon capacity at low signal-to-noise ratio [30]. However, for general throughput function, it can be shown that the method used in this paper applies as well. Let X_i be a random variable denote the channel state for the channel between the transmitter and user i , $i = 1, \dots, N$. When transmitting to user i , the throughput will then be $P \cdot X_i$. Without loss of generality, we assume $P = 1$ throughout this paper.

We now describe the second-price auction rule used in this paper. Let α_i be the *average* amount of money available to user i during each time slot. We assume that the values of α_i 's are known to all users. Moreover, users know the distribution of X_i for all i . We also assume that the exact value of the channel state X_i is revealed to user i only at the beginning of each time slot. During each time slot, the following actions take place:

1. Each user submits a bid according to the channel condition revealed to it.
2. The transmitter chooses the one with the highest bid to transmit.
3. The price that the winning user pays is the second-highest bidder's bid. Users who lose the bid do not pay. In case of a tie, the winner is chosen among the equal bidders with equal probability.

Formally, this N -players game can be written as $\Gamma = [N, \{S_i\}, \{g_i(\cdot)\}]$ which specifies for each player i a set of strategies, or bidding functions, S_i (with $s_i \in S_i$)

and a payoff function $g_i(s_1, \dots, s_N)$ giving the throughput associated with outcome of the auction arising from strategies (s_1, \dots, s_N) .

The formulation of our auction is different from the type of auction used in economic theory in several ways. First, we look at a case where the number of object (time slots) in the auction goes to infinity (average cost criteria). While in the current auction research, the number of object is finite [20][21][22]. Second, in our auction formulation, the money used for bidding does not have a direct connection with the value of the time slot. Money is merely a tool for users to compete for time slots, and it has no value after the auction. Therefore, it is desirable for each user to spend all of its money. However, in the traditional auction theory, an object's value is measured in the same unit as the money used in the bidding process, hence their objective is to maximize the difference between the object's value and its cost. We choose to use the second-price and all-pay auction in this chapter to illustrate the auction approach to resource allocation in wireless networks. As we will see later, second price auction results in an allocation that is efficient. More specifically, it is pareto optimal.

The objective for each user is to design a bidding strategy, which specifies how a user will act in every possible distinguishable circumstance, to maximize its *own* expected throughput per time slot subject to the expected or average money constraint. Once a user, say user 1, chooses a function, say f_1 , to be its strategy, it bids an amount of money equal to $f_1(x)$ when it sees its channel condition is $X_1 = x$.

2.2.2 Nash Equilibrium under Second-Price Auction

We begin our analysis of the second-price auction with an average money constraint by looking at a 2-users case for simplicity. Specifically, we present here a Nash equilibrium strategy pair (f_1^*, f_2^*) for the second-price auction under general channel distribution. We consider here the case where users choose their strategies from the set F_1 and F_2 respectively. Each user's strategy is a function of its own channel state X_i . Thus, F_i is defined to be the set of continuous real-valued, and square integrable functions over the support of X_i . Without loss of generality, we further assume functions in F_i to be increasing. We define $A : (x_1, x_2) \rightarrow \{1, 2\}$ to be an

allocation that maps the possible channel state realization, (x_1, x_2) , to either user 1 or user 2. Here we are interested in the allocation that result from the Nash equilibrium strategies.

We first consider a channel state X_i that is continuously distributed over a finite interval $[l_i, u_i]$ where l_i and u_i are nonnegative real number with $u_i > l_i$. Later we will consider the case that u_i is infinite (e.g., when X_i is exponentially distributed).

To find the Nash equilibrium strategy pair, we use the following approach. Given user 1's strategy $f_1 \in F_1$ with its range from $f_1(l_1) = a$ to $f_1(u_1) = b$, user 2 wants to maximize its own expected throughput while satisfying its expected money constraint. For a given f_1 , if user 2 chooses a bidding function f_2 , the expected throughput or payoff function for user 2 is given by:

$$g_2(f_1, f_2) = E_{X_1, X_2}[X_2 \cdot 1_{f_2(X_2) \geq f_1(X_1)}] \quad (2.1)$$

where

$$1_{f_2(X_2) \geq f_1(X_1)} = \begin{cases} 1 & \text{if } f_2(X_2) \geq f_1(X_1) \\ 0 & \text{otherwise} \end{cases}$$

Recall that in the second-price auction rule, the price that the winner pays is actually the second highest bid. Therefore, the set of feasible bidding functions for user 2, denoted as $S_2(f_1)$, is given by:

$$S_2(f_1) = \{f_2 \in F_2 \mid E_{X_1, X_2}[f_1(X_1) \cdot 1_{f_2(X_2) \geq f_1(X_1)}] \leq \alpha_2\} \quad (2.2)$$

Note that the inverse function $f_1^{-1}(y)$ may not be well defined for $y \in [a, b]$ since f_1 may not be strictly increasing over $[a, b]$. Therefore, to avoid such problem, we define the following function:

$$h(y) = \begin{cases} l_1 & \text{if } y \leq a \\ \max \{x \mid f_1(x) \leq y\} & \text{if } a < y < b \\ u_1 & \text{if } y \geq b \end{cases} \quad (2.3)$$

In the special case that f_1 is strictly increasing, $h(y)$ is reduced to the following:

$$h(y) = \begin{cases} l_1 & \text{if } y \leq a \\ f_1^{-1}(y) & \text{if } a < y < b \\ u_1 & \text{if } y \geq b \end{cases} \quad (2.4)$$

For the rest of the paper, it is convenient to consider the definition of $h(y)$ given in Eq.(2.4).

We say the strategy f_2 is a *best response* for player 2 to his rival's strategy f_1 if $g_2(f_1, f_2) \geq g_2(f_1, f'_2)$ for all $f'_2 \in S_2(f_1)$. A strategy pair (f_1^*, f_2^*) is said to be in Nash equilibrium if f_1^* is the best response for user 1 to user 2's strategy f_2^* , and f_2^* is the best response for user 2 to user 1's strategy f_1^* . The following theorem characterizes the best response of user 2 to a fixed user 1's strategy.

Theorem 1. *Given user 1's bidding strategy $f_1 \in F_1$ with its range from $f_1(l_1) = a$ to $f_1(u_1) = b$, user 2's best response has the following form:*

$$\begin{aligned} f_2(x_2) &\leq a && \text{for } x_2 \in [l_2, \theta_1] \\ f_2(x_2) &= c_2 \cdot x_2 && \text{for } x_2 \in [\theta_1, \theta_2] \\ f_2(x_2) &\geq b && \text{for } x_2 \in [\theta_2, u_2] \end{aligned} \quad (2.5)$$

where $\theta_1, \theta_2 \in [l_2, u_2]$ and $c_2 \cdot \theta_1 = a$, $c_2 \cdot \theta_2 = b$.

Proof. Given user 1's bidding strategy f_1 and user 2's bid at a particular time slot is y , the probability that user 2 wins this slot, denoted as $P_2^{win}(y)$, is given by

$$\begin{aligned} P_2^{win}(y) &= P(f_1(X_1) \leq y) = P(X_1 \leq h(y)) \\ &= \int_{l_1}^{h(y)} p_{X_1}(x_1) dx_1 \end{aligned}$$

Therefore, the optimization problem that user 2 faces is to find a strategy f_2 that

maximize its expected throughput, which can be written as the following:

$$\begin{aligned}
& \max_{f_2} \int_{l_2}^{u_2} x_2 p_{X_2}(x_2) P_2^{win}(f_2(x_2)) dx_2 \\
& = \max_{f_2} \int_{l_2}^{u_2} x_2 p_{X_2}(x_2) \int_{l_1}^{h(f_2(x_2))} p_{X_1}(x_1) dx_1 dx_2 \\
& \text{subj. to } \int_{l_2}^{u_2} \int_{l_1}^{h(f_2(x_2))} f_1(x_1) p_{X_1}(x_1) p_{X_2}(x_2) dx_1 dx_2 \leq \alpha_2
\end{aligned} \tag{2.6}$$

where the integration is over the region that user 2's bid is higher than user 1's bid. The constraint term denotes the expected money that user 2 has to pay over the region which it has a higher bid than user 1. To solve the above optimization problem, we use the optimality condition in [15]. First, we write the Lagrangian function below:

$$\begin{aligned}
& \int_{l_2}^{u_2} \int_{l_1}^{h(f_2(x_2))} x_2 p_{X_1}(x_1) p_{X_2}(x_2) dx_1 dx_2 - \\
& \lambda_2 \left(\int_{l_2}^{u_2} \int_{l_1}^{h(f_2(x_2))} f_1(x_1) p_{X_1}(x_1) p_{X_2}(x_2) dx_1 dx_2 - \alpha_2 \right) = \\
& \int_{l_2}^{u_2} \left[\int_{l_1}^{h(f_2(x_2))} (x_2 - \lambda_2 f_1(x_1)) p_{X_1}(x_1) dx_1 \right] p_{X_2}(x_2) dx_2 \\
& - \lambda_2 \alpha_2
\end{aligned} \tag{2.7}$$

We then choose a function f_2 to maximize the above equation. Also, a positive λ_2 , which depends on f_1 , is chosen such that the inequality constraint is met with equality. Specifically, for each value x_2 , we solve for the optimal $f_2(x_2)$:

$$\max_{f_2(x_2)} \int_{l_1}^{h(f_2(x_2))} (x_2 - \lambda_2 f_1(x_1)) p_{X_1}(x_1) dx_1 \tag{2.8}$$

For convenience, we let $z = f_2(x_2)$. Then, Eq. (2.8) becomes

$$\max_z L_1(z) = \int_{l_1}^{h(z)} (x_2 - \lambda_2 f_1(x_1)) p_{X_1}(x_1) dx_1. \tag{2.9}$$

For a fixed x_2 , the term $x_2 - \lambda_2 f_1(x_1)$ is a decreasing function in x_1 since $f_1(x_1)$ is increasing. To maximize $L_1(z)$, it is equivalent to choosing a value for $h(z^*)$ that includes all value of x_1 such that $x_2 - \lambda_2 f_1(x_1)$ is positive, or maximizes the area

under the curve $x_2 - \lambda_2 f_1(x_1)$. It is apparent that the optimal value z^* should be chosen such that $x_2 - \lambda_2 f_1(h(z^*)) = 0$ or $z^* = \frac{x_2}{\lambda_2}$. However, if $x_2 - \lambda_2 f_1(h(z)) > 0$ for all $z \in [a, b]$, we let $z^* \geq b$. Similarly, if $x_2 - \lambda_2 f_1(h(z)) < 0$ for all $z \in [a, b]$, we let $z^* \leq a$. Thus, from Eq.(2.3), we see that the optimal bidding function has the following form

$$\begin{aligned} f_2(x_2) &\leq a && \text{for } x_2 \in [l_2, \theta_1] \\ f_2(x_2) &= c_2 \cdot x_2 && \text{for } x_2 \in [\theta_1, \theta_2] \\ f_2(x_2) &\geq b && \text{for } x_2 \in [\theta_2, u_2] \end{aligned}$$

where $\theta_1, \theta_2 \in [l_2, u_2]$ and $c_2 \cdot \theta_1 = a$, $c_2 \cdot \theta_2 = b$. □

The above theorem indicates that for user 2 to maximize its throughput given user 1's strategy f_1 , the optimal strategy may not be unique following the definition of the Nash equilibrium. For $x_2 \in [l_2, \theta_1]$, as long as $f_2(x_2) \leq a$, user 2 always loses the bid, and the throughput for user 2 does not change. However, from second-price bidding rule, user 2's strategy affects user 1's strategy through the expected budget constraint that user 1 must satisfy. This way, user 2 will choose $f_2(x_2) = a$ for $x_2 \in [l_2, \theta_1]$. Intuitively, even if user 2 knows that it will not win a particular time slot, it will still choose to maximize its bid in order to force user 1 to pay more. Hence, user 2's best response is in this sense unique. Therefore, although the second price auction with average money constraint does not in general have an unique Nash equilibrium, it does have an unique outcome. We will elaborate on this more in section III.B where the users' channel distributions are different.

Similarly, given user 2's bidding function f_2 , we can carry out the same analysis to find that the best response for user 1 has the form $f_1(x_1) = c_1 \cdot x_1$. The next theorem shows that indeed we can always find a pair c_1 and c_2 such that both users' money constraints are satisfied simultaneously, and thus show the existence of a Nash equilibrium strategy pair.

Theorem 2. *A Nash equilibrium exists in the second-price auction game $\Gamma = [2, \{S_i\}, \{g_i(\cdot)\}]$ with $\{S_i\}$ and $\{g_i(\cdot)\}$ defined in Eq.(2.2) and Eq.(2.1) respectively .*

Proof. For the channel state X_1 distributed over the interval $[l_1, u_1]$, the best response

given in Eq.(2.5) indicates that $f_1(x_1) = c_1 \cdot x_1$ for all x_1 in $[l_1, u_1]$ is a valid best response.

Without loss of generality, we consider only linear bidding functions (i.e., $f_1(x_1) = c_1 \cdot x_1 \forall x_1 \in [l_1, u_1]$, $f_2(x_2) = c_2 \cdot x_2 \forall x_2 \in [l_2, u_2]$ and $c_1, c_2 \in [0, \infty)$) for the purpose of showing the existence of a Nash equilibrium strategy pair. A Nash equilibrium exists if we can find a pair of c_1 and c_2 which satisfy the following two constraints:

$$E_{X_1, X_2}[f_2(X_2) \cdot 1_{f_1(X_1) \geq f_2(X_2)}] \leq \alpha_1 \quad (2.10)$$

$$E_{X_1, X_2}[f_1(X_1) \cdot 1_{f_2(X_2) \geq f_1(X_1)}] \leq \alpha_2 \quad (2.11)$$

Given user 2's strategy $f_2(x_2) = c_2 \cdot x_2$, we define the set $S_1(c_2)$ to be the set of feasible strategy for user 1. Specifically, $S_1(c_2) = \{c_1 \in [0, \infty) \mid E_{X_1, X_2}[c_2 X_2 \cdot 1_{c_1 X_1 \geq c_2 X_2}] \leq \alpha_1\}$. The best response for user 1 when user 2 chooses c_2 , $b_1(c_2)$, is given by:

$$b_1(c_2) = \arg \max_{y \in S_1(c_2)} E_{X_1, X_2}[X_1 \cdot 1_{y X_1 \geq c_2 X_2}]$$

To show that Nash equilibrium exists, we need to show that the best response correspondence $b_1(\cdot)$ is nonempty, convex-valued, and upper hemicontinuous [17]. Note first that $b_1(c_2)$ is the set of maximizer of a continuous function, here the function $E_{X_1, X_2}[X_1 \cdot 1_{y X_1 \geq c_2 X_2}]$, on a compact set $S_1(c_2)$. Hence, it is nonempty. The convexity of $b_1(c_2)$ follow because the set of maximizers of a quasiconcave function, i.e., $E_{X_1, X_2}[X_1 \cdot 1_{y X_1 \geq c_2 X_2}]$, on a convex set (here $S_1(c_2)$) is convex. $E_{X_1, X_2}[X_1 \cdot 1_{y X_1 \geq c_2 X_2}]$ is quasiconcave because it is non-decreasing in y . Finally, since the set $S_1(c_2)$ is compact for all $c_2 \in [0, \infty)$, following the Berge Maximum Theorem [18], we have $b_1(c_2)$ is upper hemicontinuous. Now, all the conditions of the Kalkutani fixed point theorem are satisfied [17]. Hence, there exists a Nash equilibrium for this game. \square

The Nash equilibrium strategy discussed above is in general not unique. However, under a continuous channel state distribution that starts with zero, such as the uniform distribution over $[0, 1]$ or the exponential distribution, the Nash equilibrium bidding strategies are unique and lead to an unique allocation. Next, we will discuss

the Nash equilibrium strategy pair of these two distribution.

Uniform channel distribution

In this section, we examine the two users game with the channel state X_i uniformly distributed over $[0, 1]$. Following the approach discussed in the previous section, we find the unique allocation resulting from the Nash equilibrium strategy. Given a strategy pair (f_1^*, f_2^*) to be in Nash equilibrium, we first investigate the bids that users submit when the channel state X_i is equal to 0 (i.e., the value of $f_1^*(0)$ and $f_2^*(0)$). The result is stated in the following lemma.

Lemma 1. *For a strategy pair (f_1^*, f_2^*) to be a Nash equilibrium strategy pair, we must have $f_1^*(0) = f_2^*(0) = 0$ when the channels are uniformly distributed over $[0, 1]$.*

Proof. We consider the following three cases regarding the bidding strategy when the channel state is at 0:

- Case 1: $f_1^*(0) = 0$ and $f_2^*(0) = 0$.
- Case 2: $f_1^*(0) = a$ with $a > 0$ and $f_2^*(0) = 0$, or $f_1^*(0) = 0$ and $f_2^*(0) = a$ with $a > 0$.
- Case 3: $f_1^*(0) = a$ and $f_2^*(0) = b$ with $a > 0$ and $b > 0$.

Case 2 cannot be true from the discussion in the previous section. To see this, consider $f_1^*(0) = a$ with $a > 0$ and $f_2^*(0) = 0$. Given a time slot with user 1's channel states $x_1 = 0$, the *expected* money user 1 has to pay is positive since user 2's bidding function is continuous and $f_2^*(0) = 0$. But the expected throughput rewarded for that time slot is zero for user 1. Hence, user 1 should bid zero when its channel state is zero. Similar idea can be used to show that case 3 is also impossible. Given a time slot with user 1's channel state $x_1 = 0$ and $f_1^*(0) = a \geq b$, the expected money user 1 has to pay is positive since user 2's bidding function is continuous and $f_2^*(0) = b$. However, the expected

throughput for that time slot is zero for user 1. So user 1 would rather bid zero in this time slot.

□

With the above lemma, we can get the exact form of the Nash equilibrium strategy pair.

Theorem 3. *With the channel states, X_1 and X_2 , uniformly and independently distributed over $[0, 1]$, the unique Nash equilibrium pair (f_1^*, f_2^*) has the following form: $f_1^*(x_1) = c_1 \cdot x_1$ and $f_2^*(x_2) = c_2 \cdot x_2$ where c_1 and c_2 are chosen such that the expected money constraints are satisfied.*

Proof. Combine Lemma 1 and the linear form of the bidding function shown previously, we have the above theorem. □

We now calculate the exact value of c_1 and c_2 . Without loss of generality, we assume that user 2 has more money than user 1 (i.e., $\alpha_1 < \alpha_2$). Since the form of the optimal bidding strategy for both users is known, we need to get the exact value of c_1 and c_2 from the money constraint that users must satisfy. Thus, from Eq.(2.10) and Eq. (2.11), the constraint for user 1 is given by:

$$\begin{aligned} \int_0^1 \int_0^{f_2^{-1}(f_1(x_1))} f_2(x_2) dx_2 dx_1 &= \alpha_1 \\ \Rightarrow \int_0^1 \int_0^{\frac{c_1}{c_2} x_1} c_2 \cdot x_2 dx_2 dx_1 &= \alpha_1 \end{aligned} \tag{2.12}$$

Note that the function $f_1^{-1}(f_2(x_2))$ is well defined for $f_2(x_2) \in [0, c_1]$. Therefore, the

constraint for user 1 is given by:

$$\begin{aligned}
& \int_0^{\frac{c_1}{c_2}} \int_0^{f_1^{-1}(f_2(x_2))} f_1(x_1) dx_1 dx_2 \\
& + \int_{\frac{c_1}{c_2}}^1 \int_0^1 f_1(x_1) dx_1 dx_2 = \alpha_2 \\
& \int_0^{\frac{c_1}{c_2}} \int_0^{\frac{c_2}{c_1} x_2} c_1 \cdot x_1 dx_1 dx_2 \\
& + \int_{\frac{c_1}{c_2}}^1 \int_0^1 c_1 \cdot x_1 dx_1 dx_2 = \alpha_2
\end{aligned} \tag{2.13}$$

Solving the two equations, we get

$$c_1 = 2(2\alpha_1 + \alpha_2) \tag{2.14}$$

$$c_2 = \frac{2(2\alpha_1 + \alpha_2)^2}{3\alpha_1} \tag{2.15}$$

The throughput of each user is then given by

$$G_1 = \frac{\alpha_1}{\alpha_2 + 2\alpha_1} \tag{2.16}$$

$$G_2 = \frac{1}{2} - \frac{3\alpha_1^2}{2(\alpha_2 + 2\alpha_1)^2} \tag{2.17}$$

Note that the linear bidding function leads to the following allocation: Given that the channel states are x_1 and x_2 during a time slot, the transmitter assigns the slot to user 1 if $x_1 \geq c \cdot x_2$, where $c = \frac{c_2}{c_1}$, and to user 2 otherwise. We will see later that this form of allocation leads to the Pareto optimality.

The Unique Outcome of the Game

As we mentioned previously, the Nash equilibrium is not unique in general (although unique for the cases where channel states are exponentially distributed or uniformly distributed over $[0, 1]$); however, the outcome of this second price auction with money constraint is unique. To see this, consider an example where X_1 is uniformly distributed over the interval $[0, 10]$, and X_2 is uniformly distributed over the interval

$[5 - \epsilon, 5 + \epsilon]$ with ϵ arbitrarily small. If both users have the same average money constraint, two strategy pairs are given in Fig. 2-1(a)(b) and (d)(e). They are both Nash equilibrium strategy pairs *by definition*. Given user 2's strategy shown in Fig. 2-1(b), user 1 can bid anything less than τ , which is the lowest bid of user 2, during the interval $[0, \sigma]$ since its throughput will be unaffected (this is the reason that multiple Nash equilibriums exist). In Fig. 2-1(a), we show the case that user 1 implements a strict linear bidding function, resulting in an expected throughput of 2.78 for user 1 and 3.33 for user 2. Although user 1's bid during interval $[0, \sigma]$ will not change its own throughput, it will affect the amount of money user 2 has to pay (i.e., user 2 has to pay more to win a slot if user 1's bid is close to τ instead of 0 during $[0, \sigma]$; consequently, user 2 will have less money to bid in other slots). Thus, a rational decision for user 1 is not to bid anything less than the smallest bid of user 2. Therefore, the Nash equilibrium strategy pair shown in Fig. 2-1(d)(e) is a more reasonable equilibrium strategy pair for this game. The outcome of the game is in this sense unique.

Exponential distribution

When the channel state X_i is exponentially distributed with rate μ_i , the analysis in the general distribution section is still valid. The unique Nash equilibrium strategy pair has the same form as the uniform case: $f_1^*(x_1) = c_1 \cdot x_1$ and $f_2^*(x_2) = c_2 \cdot x_2$. Using Eq.(2.10) and Eq.(2.11), we get a relationship between c_1 and c_2 to be $\frac{c_1}{c_2} = \frac{\alpha_1 \cdot \mu_1}{\alpha_2 \cdot \mu_2}$. Thus, the optimal allocation is given by:

$$A^*(x_1, x_2) = \begin{cases} 2 & \text{if } x_2 > (c_1/c_2)x_1 \\ 1 & \text{otherwise} \end{cases}$$

Write the decision in another form $\mu_2 X_2 > \frac{\alpha_1}{\alpha_2} \mu_1 X_1$. We see that only the normalized channel state distribution (i.e., $\frac{X_2}{E[X_2]}$ and $\frac{X_1}{E[X_1]}$ where $E[X_2] = \frac{1}{\mu_2}$, $E[X_1] = \frac{1}{\mu_1}$) are used in the comparison. This result corroborates the Score-Based scheduler proposed by [19], which selects a user when its transmission rate is high relative to its own *rate*

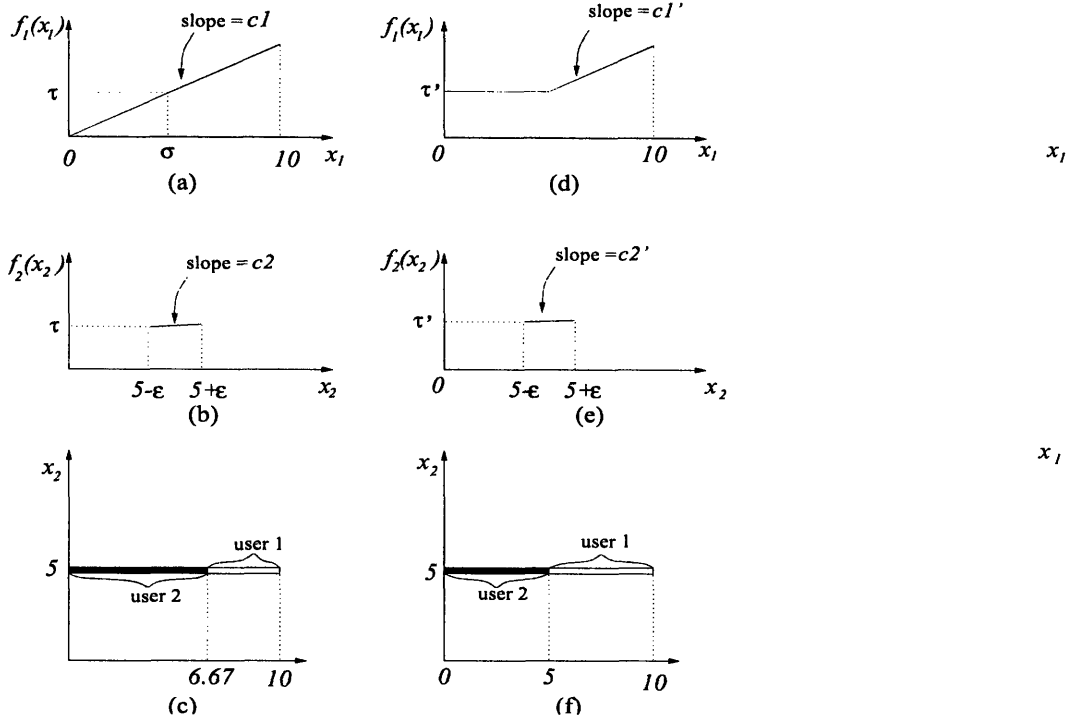


Figure 2-1: (a) Bidding function for user 1 when using linear bidding function. (b) Bidding function for user 2. (c) Resulting allocation shown in the support of X_1 and X_2 . (d) Bidding function for user 1 when it tries to make user 2 to pay more. (e) User 2's bidding function. (f) Resulting allocation when users using bidding function shown in (d) and (e).

statistics. The expected throughput for each user is given by:

$$G_1 = \frac{1}{\mu_1} \left[1 - \frac{\alpha_2^2}{(\alpha_1 + \alpha_2)^2} \right]$$

$$G_2 = \frac{1}{\mu_2} \left[1 - \frac{\alpha_1^2}{(\alpha_1 + \alpha_2)^2} \right]$$

The N-users Game

In this section, we explore the Nash equilibrium of the second price auction in which N users, each with an average money constraint α_i , compete for time slots. Given user i 's strategy $f_i \in F_i$ with range from $f_i(l_i) = a_i$ to $f_i(u_i) = b_i$ for $i = 2, \dots, N$, user 1 wants to maximize its own expected throughput while satisfying its expected money constraint. For a given $\{f_2, \dots, f_N\}$, if user 1 chooses a bidding function f_1 ,

the expected throughput or payoff function for user 1 is given by:

$$g_1 = E_{X_1, X_2, \dots, X_N} [X_1 \cdot \mathbf{1}_{f_1(X_1) \geq \max\{f_2(X_2), \dots, f_N(x_N)\}}] \quad (2.18)$$

The set of feasible bidding functions for user 1, denoted here as $S_1(f_2, \dots, f_N)$, can be written as:

$$\begin{aligned} S_1(f_2, \dots, f_N) = \\ \{f_1 \in F_1 \mid E_{X_2, \dots, X_N} [\max\{f_2(X_2), \dots, f_N(x_N)\}] \\ \cdot \mathbf{1}_{f_1(X_1) \geq \max\{f_2(X_2), \dots, f_N(x_N)\}}] \leq \alpha_1\} \end{aligned} \quad (2.19)$$

Similar to the 2-users case, we define the inverse function as the following for $i = 2, \dots, N$:

$$h_i(y) = \begin{cases} l_i & \text{if } y \leq a_i \\ f_i^{-1}(y) & \text{if } a_i < y < b_i \\ u_i & \text{if } y \geq b_i \end{cases} \quad (2.20)$$

The following theorem characterizes the best response of user 1 for fixed $\{f_2, \dots, f_N\}$ in this N -user game.

Theorem 4. *Given fixed bidding functions $\{f_2, \dots, f_N\}$ for user 2 to user N , and $a = \min\{f_2(x_2), \dots, f_N(x_N)\}$ and $b = \max\{f_2(x_2), \dots, f_N(x_N)\}$ for $\{x_2, \dots, x_N\} \in X_2 \times \dots \times X_N$, user 1's best response has the following form:*

$$\begin{aligned} f_1(x_1) &\leq a && \text{for } x_1 \in [l_2, \theta_1] \\ f_1(x_1) &= c_1 \cdot x_1 && \text{for } x_1 \in [\theta_1, \theta_2] \\ f_1(x_1) &\geq b && \text{for } x_1 \in [\theta_2, u_2] \end{aligned} \quad (2.21)$$

where $\theta_1, \theta_2 \in [l_2, u_2]$ and $c_1 \cdot \theta_1 = a$, $c_1 \cdot \theta_2 = b$.

Proof. See Appendix. □

Theorem 5. *A Nash equilibrium exists in the second-price auction game $\Gamma = [N, \{S_i\}, \{g_i(\cdot)\}]$ with $\{S_i\}$ and $\{g_i(\cdot)\}$ defined in Eq.(2.19) and Eq.(2.18) respectively .*

Proof. We can then follow the steps in the two users case to show the existence of a Nash equilibrium. The analysis is omitted for brevity. \square

2.2.3 Nash Equilibrium Strategy with Multiple Bidding Functions

In the previous section, we restricted the strategy space of each user to be a single bidding function. Specifically, once a user, say user 1, chooses a function, say $f_1(\cdot)$, for its strategy, it bids an amount of money equal to $f_1(x_1)$ when it sees its channel condition is $X_1 = x_1$. In other words, user 1 uses the same bidding function $f_1(\cdot)$ for all time slots. In this section, we will relax this single bidding function assumption, and investigate whether users have incentive to use different bidding function for different time slot (i.e., user 1 employs the bidding function $f_1^{(1)}(\cdot)$ for time slot 1, and $f_1^{(2)}(\cdot)$ for time slot 2) as long as their average constraint is not violated. And, given that users can choose multiple bidding functions, we explore whether the Nash equilibrium exists.

Again, for simplicity, we consider a 2-users game where the user's channel state is uniformly distributed over $[0, 1]$. Let F_1 and F_2 be, as before, the set of continuous, increasing, square integrable real-valued functions over the support of X_1 and X_2 respectively. Then, the strategy space for user 1, say S_1 , and user 2, say S_2 , are defined as follows:

$$\begin{aligned} S_1 &= \left\{ f_1^{(1)}, \dots, f_1^{(n)} \in F_1 \mid \frac{1}{n} \sum_{i=1}^n E[f_1^{(i)}(X_1)] = \alpha_1 \right\} \\ S_2 &= \left\{ f_2^{(1)}, \dots, f_2^{(n)} \in F_2 \mid \frac{1}{n} \sum_{i=1}^n E[f_2^{(i)}(X_2)] = \alpha_2 \right\} \end{aligned} \tag{2.22}$$

For each user, a strategy is a sequence of bidding functions $f^{(1)}, \dots, f^{(n)}$. Without loss of generality, we restrict each user to have n different bidding functions, where n can be chosen as an arbitrarily large number. Note that users now choose a strategy for a block of n time slots instead of just for a single time slot, one bidding function for each slot. In order to maximize the overall throughput (over infinite horizon), each

user chooses bidding functions to maximize the expected total throughput over this block of n slots. The term $E[f_1^{(i)}(X_1)]$ denotes the expected amount of money spent by user 1 if it uses bidding function $f_1^{(i)}$ for the i th slot in the block. The strategy space discussed in the previous section can be considered to be a special class of strategies of S_1 and S_2 in which each user can use only a single bidding function. More specifically, set $f_1 = f_1^{(1)} = \dots = f_1^{(n)}$ and $f_2 = f_2^{(1)} = \dots = f_2^{(n)}$.

To choose a strategy (i.e., a sequence of bidding functions) from the strategy space S_1 or S_2 , a user encounters two problems. First, it must decide how to allocate its money among these n bidding functions so that the average money constraint is still satisfied. Second, once the money allocated to the i th bidding function is specified, a user has to choose a bidding function for the i th slot. The second problem is already solved in the previous section (see Theorem 3). In this section, we will focus on the first problem that a user encounters, specifically, the problem of how to allocate money between bidding functions while satisfying the following condition: The total expected amount of money for the *sequence* of n bidding functions is $n \cdot \alpha_1$ for user 1 and $n \cdot \alpha_2$ for user 2. For convenience, we let $\sigma = \alpha_1$, $\beta = \alpha_2$, and further denote σ_i, β_i to be the average money allocated in slot i for user 1 and user 2 respectively. The strategy space or possible actions that can be taken by users are given by the following:

$$\begin{aligned}\hat{S}_1 &= \{\sigma_1, \dots, \sigma_n \mid \sigma_1 + \dots + \sigma_n = n \cdot \sigma\} \\ \hat{S}_2 &= \{\beta_1, \dots, \beta_n \mid \beta_1 + \dots + \beta_n = n \cdot \beta\}\end{aligned}$$

The objective of each user is still to maximize its own throughput. When user 1 and user 2 allocate σ_i and β_i for their i th bidding function which is given in Theorem 3, the payoff functions are $G_1(\sigma_i, \beta_i)$ for user 1 and $G_2(\sigma_i, \beta_i)$ for user 2.

The following lemma gives us a Nash equilibrium strategy pair for the auction game described in this section.

Lemma 2. *Given that user 2's strategy is to allocate its money evenly among its bidding functions (i.e., $\beta_i = \beta, i = 1 \dots n$), user 1's best response is to allocate its*

money evenly as well (i.e., $\sigma_i = \sigma, i = 1 \cdots n$); and vice versa. Therefore, a Nash equilibrium strategy pair for this auction is for both users to allocate their money evenly.

Proof. Without loss of generality, we consider the case that $n = 2$ where each user's strategy can consist of two different bidding functions. Suppose that user 2 allocates β for both bidding functions $f_2^{(1)}$ and $f_2^{(2)}$, and user 1 allocates σ_1 for bidding function $f_1^{(1)}$ and σ_2 for bidding function $f_1^{(2)}$ where $\sigma_1 + \sigma_2 = 2\sigma$ and $\sigma_1 \neq \sigma_2$. We will show that the throughput for user 1, $G_1(\sigma_1, \beta) + G_1(\sigma_2, \beta)$, is maximized when $\sigma_1 = \sigma_2 = \sigma$. Assume $\beta/2 < \sigma < \beta$. First, we consider the case that $\sigma_1 \leq \beta$ and $\sigma_2 \leq \beta$. The equation $G_1(\sigma_1, \beta)$ with β fixed

$$G_1(\sigma_1, \beta) = \frac{\sigma_1}{\beta + 2\sigma_1}$$

becomes

$$F(t) = \frac{t}{1+t}$$

where $t = \frac{\sigma_1}{\beta}$. $F(t)$ is concave for $t \geq 0$. Thus, we have $G_1(\sigma_1, \beta) + G_1(\sigma_2, \beta)$ maximized when $\sigma_1 = \sigma_2 = \sigma$. For the case that $\sigma_1 \geq \beta$ and $\sigma_2 = 2\sigma - \sigma_1 \leq \beta$, we have from Eq.(2.39) and Eq.(2.40)

$$G_1(\sigma_1, \beta) + G_1(2\sigma - \sigma_1, \beta) = \frac{1}{2} - \frac{3\beta^2}{2(\sigma_1 + 2\beta)^2} + \frac{2\sigma - \sigma_1}{\beta + 2(2\sigma - \sigma_1)}$$

The above function can be shown to be strictly decreasing for $\sigma_1 \in [\beta, 2\sigma]$. Hence, it is optimal to choose $\sigma_1 = \beta$ for σ_1 in the interval $[\beta, 2\sigma]$. We also know that in the case $\sigma_1 \leq \beta$ and $\sigma_2 \leq \beta$, $G_1(\sigma_1, \beta) + G_1(\sigma_2, \beta)$ is maximized when $\sigma_1 = \sigma_2 = \sigma$. Therefore, given user 2 allocates its money evenly among its bidding functions (i.e., $\beta_i = \beta, i = 1 \cdots n$), user 1's best response is to allocate its money evenly as well. \square

We have already obtained a Nash equilibrium strategy pair from the above lemma. The following theorem states that this Nash equilibrium strategy pair is in fact unique within the strategy space considered.

Theorem 6. *For the second price auction with user's strategy space defined in (2.22), a unique Nash equilibrium strategy for both users is to allocate their money evenly among the bidding functions.*

Proof. The complete proof is in the Appendix. □

In this section, users are given more freedom in choosing their strategies (i.e., they can choose n different bidding functions). However, as Theorem 6 shows, the unique Nash equilibrium strategy pair is for each user to use a single bidding function from its strategy space. Thus, the throughput result obtained in this broader strategy space- S_1 and S_2 -is the same as the throughput result from previous section. Therefore, there is no incentive for a user to use different bidding functions.

2.2.4 Pareto Optimality of the Nash Equilibrium Strategies

Thus far, we have a Nash equilibrium strategy pair and the resulting throughput when both players choose to use the Nash equilibrium strategy. In this section, we want to address the question whether the allocation resulting from the Nash equilibrium strategy is efficient, or Pareto optimal. An allocation is said to be Pareto optimal if it is impossible to make some individuals better off without making some other individuals worse off. This concept is a formalization of the idea that there is no waste in the allocation process.

We start by investigating an allocation with a fairness constraint that requires the resulting throughput of the users to be kept at a constant ratio. Specifically, let G_1 and G_2 denote the expected throughput for user 1 and user 2 respectively. We have the following optimization problem: for some nonnegative a ,

$$\begin{aligned} \max \quad & G_1 + G_2 \\ \text{subj.} \quad & \frac{G_1}{G_2} = a \end{aligned} \tag{2.23}$$

The optimal allocation is to divide the possible channel state realizations, (x_1, x_2) , into two regions by the separation line $x_2 = c \cdot x_1$, where c is some positive real number.

Above the line (i.e., $x_2 > c \cdot x_1$), the transmitter will assign the slot to user 2. Below the line (i.e., $x_2 < c \cdot x_1$), the transmitter will assign the slot to user 1.

To prove the above, we use a method that is similar to the one in [9]. By using an allocation A , the resulting throughput for user 1 and user 2 are $G_1^A = E[X_1 \cdot 1_{A(X_1, X_2)=1}]$ and $G_2^A = E[X_2 \cdot 1_{A(X_1, X_2)=2}]$ respectively. Now, we define an allocation as follows:

$$A^*(x_1, x_2) = \begin{cases} 1 & \text{if } x_1(1 + \lambda^*) \geq x_2(1 - a \cdot \lambda^*) \\ 2 & \text{otherwise} \end{cases}$$

where λ^* is chosen such that $G_1^{A^*}/G_2^{A^*} = a$ is satisfied.

Consider an arbitrary allocation A that satisfies $G_1^A/G_2^A = a$. We have

$$\begin{aligned} & E[X_1 \cdot 1_{A(X_1, X_2)=1}] + E[X_2 \cdot 1_{A(X_1, X_2)=2}] \\ &= E[X_1 \cdot 1_{A(X_1, X_2)=1}] + E[X_2 \cdot 1_{A(X_1, X_2)=2}] \\ &\quad + \lambda^*(E[X_1 \cdot 1_{A(X_1, X_2)=1}] - aE[X_2 \cdot 1_{A(X_1, X_2)=2}]) \\ &= E[(X_1 + \lambda^* X_1) \cdot 1_{A(X_1, X_2)=1}] + E[(X_2 - a\lambda^* X_2) \cdot 1_{A(X_1, X_2)=2}] \\ &\leq E[(X_1 + \lambda^* X_1) \cdot 1_{A^*(X_1, X_2)=1}] + E[(X_2 - a\lambda^* X_2) \cdot 1_{A^*(X_1, X_2)=2}] \\ &= E[X_1 \cdot 1_{A^*(X_1, X_2)=1}] + E[X_2 \cdot 1_{A^*(X_1, X_2)=2}] \\ &\quad + \lambda^*(E[X_1 \cdot 1_{A^*(X_1, X_2)=1}] - aE[X_2 \cdot 1_{A^*(X_1, X_2)=2}]) \\ &= E[X_1 \cdot 1_{A^*(X_1, X_2)=1}] + E[X_2 \cdot 1_{A^*(X_1, X_2)=2}] \end{aligned}$$

The inequality in the middle is from the definition of A^* . Specifically, if we were asked to choose an allocation A to maximize $E[(X_1 + \lambda^* X_1) \cdot 1_{A(X_1, X_2)=1}] + E[(X_2 - a\lambda^* X_2) \cdot 1_{A(X_1, X_2)=2}]$. Then, A^* will be an optimal scheme from its definition. Thus, $A^*(X_1, X_2)$ is an optimal solution to the optimization problem in (2.23).

So far, we have shown that the optimal allocation for the problem in (2.23) has the same form as the allocation scheme resulted from the Nash equilibrium strategy of second price auction (i.e., both allocation schemes compare channel state realization x_1 with $c \cdot x_2$ where c is a constant). Examining the optimization problem in (2.23), we see that the resulting throughput obtained is Pareto optimal. To show this, suppose G_1^p and G_2^p are the throughput of a Pareto optimal allocation, and $G_1^p/G_2^p = c_p$. If

the optimal solution of the problem maximizing $G_1 + G_2$ subject to the constraint $G_1/G_2 = c_p$ are G_1^* and G_2^* , we must have $G_1^* + G_2^* \geq G_1^p + G_2^p$ which implies $G_1^* \geq G_1^p$ and $G_2^* \geq G_2^p$ since $G_1^p/G_2^p = c_p$ and $G_1^*/G_2^* = c_p$. From the assumption that G_1^p and G_2^p are the throughput of a Pareto optimal allocation, we must have $G_1^* = G_1^p$ and $G_2^* = G_2^p$. Therefore, the solution to the optimization problem (2.23) is Pareto optimal which also implies the Pareto optimality of the allocation resulting from equilibrium strategy since they have the same form.

2.2.5 Comparison with Other Allocation Schemes

Based on our previous analysis on the Nash equilibrium strategy of the second price auction with average money constraint, we can implement a centralized opportunistic scheduler that is fair and efficient. Instead of allowing users to actually bid for each time slot, the centralized scheduler will assign time slots according to the Nash equilibrium strategy based on users' average money amount. If users are assumed to have equal priority (as in the cases of maxmin fairness and proportion fairness), the scheduler simply let each user have an equal money constraint, and assigns time slots according to the equilibrium strategy. Later in this section, we will compare our centralized scheduling scheme with proportional fair scheduling scheme. But first, we need to quantify the loss of efficiency by using Nash equilibrium strategies. Due to the fairness constraint, total system throughput will decrease as compared to the maximum throughput attainable without any fairness constraint. Hence we would like to compare the total throughput of the Nash equilibrium strategy to that of an unconstrained strategy. We address this question by first considering an allocation that maximizes total throughput subject to no constraint.

Maximizing Throughput with No Constraint

To maximize throughput without any constraints, the transmitter serves the user with a better channel state during each time slot. Then the expected throughput is $E[\max\{X_1, X_2\}]$. For X_1 and X_2 independent uniformly distributed in $[0, 1]$, we

have $E[\max\{X_1, X_2\}] = \frac{2}{3}$. Using the Nash equilibrium strategy, the total expected system throughput, $G_1 + G_2$, is $\frac{1}{2}$ in the worst case (i.e., one user gets all of the time slots while the other user is starving). *Thus, the channel allocation proposed here can achieve at least 75 percent of the maximum attainable throughput.* This gives us a lower bound of the throughput performance of the allocation derived from the Nash equilibrium pair.

Proportional fairness

In this section, we examine the well-known proportional fairness allocation. Let G_1, G_2, A be defined similarly as in the previous section. The objective of proportional fairness is to maximize the term $(\log G_1 + \log G_2)$ [25]. Specifically, the optimization problem is given by:

$$\begin{aligned} & \max_A \log E[X_1 \cdot 1_{A(X_1, X_2)=1}] + \log E[X_2 \cdot 1_{A(X_1, X_2)=2}] \\ &= \max_A \log \left[\int_{(x_1, x_2) | A(x_1, x_2)=1} x_1 p_{X_1}(x_1) p_{X_2}(x_2) dx_1 dx_2 \right] \\ & \quad + \log \left[\int_{(x_1, x_2) | A(x_1, x_2)=2} x_2 p_{X_1}(x_1) p_{X_2}(x_2) dx_1 dx_2 \right] \end{aligned} \quad (2.24)$$

It is straightforward to see that the optimal allocation policy has to be a threshold rule. That is, for given (\bar{x}_1, \bar{x}_2) on the threshold and a particular time slot with channel state (x_1, x_2) , the scheduler will assign the time slot to user 1 if the channel state pair $(x_1, x_2) \in \{(a, b) \mid a > \bar{x}_1 \text{ and } b < \bar{x}_2\}$, and to user 2 if $(x_1, x_2) \in \{(a, b) \mid a < \bar{x}_1 \text{ and } b > \bar{x}_2\}$. To get the optimal allocation policy, we consider again a point (\bar{x}_1, \bar{x}_2) on the threshold and a small region with probability Δ around that point. Intuitively, since this region is on the threshold, an optimal scheduler can allocate it to either user 1 or user 2. If allocating the small region to user 1 will result in more gain than allocating it to user 2, this region will not be on threshold anymore but belongs to user 1. Thus, for A^* to be an optimal allocation rule, we have the

following first order approximation:

$$\begin{aligned}
& \log\left[\int_{(x_1, x_2)|A^*(x_1, x_2)=1} x_1 p_{X_1}(x_1) p_{X_2}(x_2) dx_1 dx_2 + \bar{x}_1 \cdot \Delta\right] \\
& - \log\left[\int_{(x_1, x_2)|A^*(x_1, x_2)=1} x_1 p_{X_1}(x_1) p_{X_2}(x_2) dx_1 dx_2\right] \\
& = \frac{\bar{x}_1 \cdot \Delta}{\int_{(x_1, x_2)|A^*(x_1, x_2)=1} x_1 p_{X_1}(x_1) p_{X_2}(x_2) dx_1 dx_2}
\end{aligned} \tag{2.25}$$

Similar equation can be written for \bar{x}_2 . Combine both equations, we have the following that describes the threshold of A^* :

$$\frac{\bar{x}_1}{G_1^{A^*}} = \frac{\bar{x}_2}{G_2^{A^*}}$$

The optimal allocation can then be stated as:

$$A^*(x_1, x_2) = \begin{cases} 1 & \text{if } x_1 \geq c \cdot x_2 \\ 2 & \text{otherwise} \end{cases}$$

where the constant $c = G_1^{A^*}/G_2^{A^*}$. We find the allocation with proportional fairness criteria has the same form as the allocation that resulted from the Nash equilibrium strategy (i.e., both of them are straight lines). Therefore, it is interesting to compare the performance of the proportional fairness algorithm to that of the auction algorithm. Consider an example where X_1 is uniformly distributed over the interval $[0, 10]$ and X_2 is uniformly distributed over the interval $[5 - \epsilon, 5 + \epsilon]$ (consistent with our previous example). Assuming ϵ is small, we can treat X_2 as a constant. Using the proportional fairness scheme, we need to find a threshold c such that $x_2 = c \cdot x_1$ and $c = G_1/G_2$. From Fig. 2-2(a), we see that $G_2 = \frac{5}{10c} \cdot 5$ and $G_1 = \frac{10+5/c}{2}(10 - 5/c) \cdot \frac{1}{10}$. Setting $\frac{G_2}{G_1} = c$, we have $c = \frac{1}{\sqrt{2}} = 0.707$. As a result of the proportional fair algorithm, the scheduler will assign almost 71 percent of the time slots to user 2. The user with a constant channel states obviously benefits more from the proportional fairness algorithm. For comparison, we use the centralized scheduler (based on the auction algorithm) described in the early part of this section (i.e., we let each user

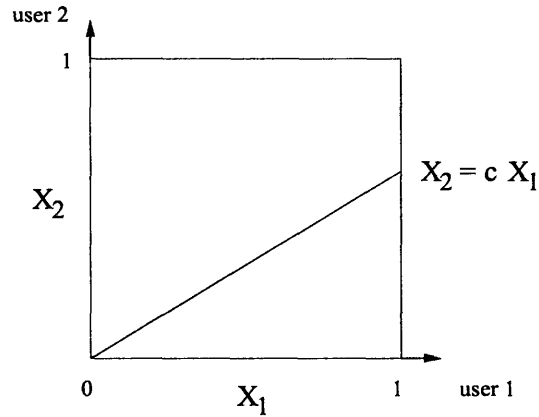


Figure 2-2: (a) The proportional fair allocation scheme. (b) The second price auction scheme with equal money constraint.

have the same average money constraint when employing the second price auction algorithm). From Fig. 2-2(b), we see that both users get almost half of the time slots (it does not bias towards user with a constant channel state). Furthermore, it also results in a higher total system throughput than that of the proportion fairness scheme. Specifically, the auction scheme yields a total throughput of 6.25 while the proportional fairness scheme yields a total throughput of 6.0.

2.3 All-pay Auction

2.3.1 All-pay Auction Problem Formulation

The formulation of the all-pay auction is similar to the formulation of the second-price auction. Again, for a given power level, we assume for simplicity that the throughput is a linear function of the channel state. Let X_i be a random variable denoting the channel state for the channel between the transmitter and user i , $i = 1, 2$. When transmitting to user i , the throughput will then be $P \cdot X_i$ and $P = 1$.

We now describe the all-pay auction rule used in this chapter. Let α and β be the *average* amount of money available to user 1 and user 2 respectively during each time slot. We assume that the values of α and β are known to both users. Both

users know the distribution of X_1 and X_2 . We also assume that the exact value of the channel state X_i is revealed to user i only at the beginning of each time slot. During each time slot, the following actions take place:

1. Each user submits a bid according to the channel condition revealed to it.
2. The transmitter chooses the one with higher bid to transmit.
3. Once a bid is submitted by the user, it is taken by the transmitter regardless of whether the user gets the slot or not, i.e., no refund for the one who loses the bid.

The objective for each user is again to design a bidding strategy, which specifies how a user will act in every possible distinguishable circumstance, to maximize its *own* expected throughput per time slot subject to the expected or average money constraint. Once a user, say user 1, chooses a function, say $f_1^{(i)}$, for its strategy in the i th slot, it bids an amount of money equal to $f_1^{(i)}(x)$ when it sees its channel condition in the i th slot is $X_1 = x$.

Formally, let F_1 and F_2 be the set of continuous and bounded real-valued functions with finite first and second derivative over the support of X_1 and X_2 respectively. Then, the strategy space for user 1, say S_1 , and user 2, say S_2 , are defined as follows:

$$\begin{aligned}
 S_1 &= \left\{ f_1^{(1)}, \dots, f_1^{(n)} \in F_1 \mid \frac{1}{n} \sum_{i=1}^n E[f_1^{(i)}(X_1)] = \alpha \right\} \\
 S_2 &= \left\{ f_2^{(1)}, \dots, f_2^{(n)} \in F_2 \mid \frac{1}{n} \sum_{i=1}^n E[f_2^{(i)}(X_2)] = \beta \right\}
 \end{aligned} \tag{2.26}$$

Note that the set of feasible bidding strategies of user 1 does not depend on the bidding strategy of user 2 in the all-pay auction, while the bidding strategies of user 1 does depend on the bidding strategy of user 2 in the all-pay auction. For each user, a strategy is a sequence of bidding functions $f^{(1)}, \dots, f^{(n)}$. Without loss of generality, we restrict each user to have n different bidding functions, where n can be chosen as an arbitrarily large number. Note that users choose a strategy for a block of n time

slots instead of just for a single time slot, one bidding function for each slot. In order to maximize the overall throughput (over infinite horizon), each user chooses bidding functions to maximize the expected total throughput over this block of n slots. The term $E[f_1^{(i)}(X_1)]$ denotes the expected amount of money spent by user 1 if it uses bidding function $f_1^{(i)}$ for the i th slot in the block.

We first consider a special class of strategies in which each user can use only a single bidding function. More specifically, by setting $f_1 = f_1^{(1)} = \dots = f_1^{(n)}$ and $f_2 = f_2^{(1)} = \dots = f_2^{(n)}$, we have the following:

$$\begin{aligned}\bar{S}_1 &= \left\{ f_1 \in F_1 \mid E[f_1(X_1)] = \alpha \right\} \\ \bar{S}_2 &= \left\{ f_2 \in F_2 \mid E[f_2(X_2)] = \beta \right\}\end{aligned}\tag{2.27}$$

By considering first the set of strategies in \bar{S}_1 and \bar{S}_2 , we are able to find the Nash equilibrium strategy pair within the set S_1 and S_2 .

Given a strategy pair (f_1, f_2) , where $f_1 \in \bar{S}_1$ and $f_2 \in \bar{S}_2$, the expected throughput or payoff function for user 1 is defined as the following assuming the constant power $P = 1$:

$$G_1(\alpha, \beta) = E_{X_1, X_2}[X_1 \cdot 1_{f_1(X_1) \geq f_2(X_2)}]\tag{2.28}$$

where

$$1_{f_1(X_1) \geq f_2(X_2)} = \begin{cases} 1 & \text{if } f_1(X_1) \geq f_2(X_2) \\ 0 & \text{otherwise} \end{cases}$$

Similarly, the throughput function for user 2 assuming $P = 1$:

$$G_2(\alpha, \beta) = E_{X_1, X_2}[X_2 \cdot 1_{f_2(X_2) > f_1(X_1)}]\tag{2.29}$$

Throughout this section, for simplicity, we let the channel state X_i be uniformly distributed over $[0, 1]$. However, our approach can be extended to the case where the channel state has a general distribution. Due to space limitations, we omit the more complex analysis for general channel state distribution.

2.3.2 Unique Nash equilibrium strategy with a single bidding function

We present in this section a unique Nash equilibrium strategy pair (f_1^*, f_2^*) . A strategy pair (f_1^*, f_2^*) is said to be in Nash equilibrium if f_1^* is the best response for user 1 to user 2's strategy f_2^* , and f_2^* is the best response for user 2 to user 1's strategy f_1^* . We consider here the case where both users choose their strategies from the strategy space \bar{S}_1 and \bar{S}_2 (i.e., the single bidding function strategy) and the value of α and β are known to both users.

To get the Nash equilibrium strategy pair, we first argue that an equilibrium bidding function must be nondecreasing. To see this, consider an arbitrary bidding function f such that $f(a) > f(b)$ for some $a < b$. If user 1 chooses f as its bidding function, user 1 will be better off if it bids $f(b)$ when the channel state is a and $f(a)$ when the channel state is b . This way, its odds of winning the slot when the channel state is b , which is more valuable to it, will be higher than before, and it has an incentive to change its strategy (i.e., f is not an equilibrium strategy). Hence, we conclude that, for each user, an equilibrium bidding function must be nondecreasing.

We further restrict users' bidding functions to be *strictly* increasing for technical reason which will be explained later. There is no loss of generality in this assumption because any continuous, bounded, nondecreasing function can be approximated by a *strictly* increasing function arbitrarily closely.

Next, we show some useful properties associated with the equilibrium strategy pair (f_1^*, f_2^*) .

Lemma 3. *If (f_1^*, f_2^*) is a Nash equilibrium strategy pair, $f_1^*(1) = f_2^*(1)$.*

Proof. Suppose $f_1^*(1) \neq f_2^*(1)$. Without loss of generality, let assume that $f_1^*(1) > f_2^*(1)$. Since both f_1^* and f_2^* are continuous, there exists $\delta > 0$ such that $f_1^*(x) > f_2^*(1) + \frac{f_1^*(1) - f_2^*(1)}{2} \quad \forall x \in [1 - \delta, 1]$. User 1 can devise a new bidding strategy, say \bar{f}_1 , by moving a small amount of money, say $\delta \cdot \frac{f_1^*(1) - f_2^*(1)}{2}$, away from the interval $[1 - \delta, 1]$ to some other interval, thus resulting in an increase in user 1's throughput. Therefore, when $f_1^*(1) > f_2^*(1)$, the bidding strategy pair (f_1^*, f_2^*) cannot be in equilibrium since

the strategy pair (\bar{f}_1, f_2^*) gives a higher throughput for user 1. Similar result holds for the case $f_2^*(1) > f_1^*(1)$. Thus, we must have $f_1^*(1) = f_2^*(1)$ if (f_1^*, f_2^*) is an equilibrium strategy pair. \square

We have just established that $f_1^*(1) = f_2^*(1)$ is a necessary condition for (f_1^*, f_2^*) to be an equilibrium strategy pair. We also find that $f_1^*(0) = f_2^*(0) = 0$ since it does not make sense to bid for a slot with zero channel state. Thus, from now on, to find the Nash equilibrium strategy pair (f_1^*, f_2^*) , we will consider only the function pair $f_1 \in \bar{S}_1$ and $f_2 \in \bar{S}_2$ that are strictly increasing and satisfying the above two boundary conditions (i.e., $f_1(1) = f_2(1)$ and $f_1(0) = f_2(0) = 0$).

These two boundary conditions, together with strictly increasing property of $f_1 \in \bar{S}_1$ and $f_2 \in \bar{S}_2$, make the inverse of f_1 and f_2 well defined. Thus, we are able to define the following terms. With user 2's strategy f_2 fixed, let $g_{f_2}^{(1)} : (x_1, b) \rightarrow \mathcal{R}$ denote user 1's expected throughput of a slot conditioning on the following events:

- User 1's channel state is $X_1 = x_1$.
- User 1's bid is b .

Specifically, we can write the equation:

$$g_{f_2}^{(1)}(x_1, b) = x_1 \cdot P(f_2(X_2) \leq b) \quad (2.30)$$

where $P(f_2(X_2) \leq b)$ is the probability that user 1 wins the time slot. Consequently, using a strategy f_1 , user 1's throughput is given by:

$$G_1(\alpha, \beta) = \int_0^1 g_{f_2}^{(1)}(x_1, f_1(x_1)) \cdot p_{X_1}(x_1) dx_1 = \int_0^1 g_{f_2}^{(1)}(x_1, f_1(x_1)) dx_1. \quad (2.31)$$

where the last equality results from the uniform distribution assumption.

With user 1's strategy f_1 fixed, similar terms for user 2 can be defined.

$$g_{f_1}^{(2)}(x_2, b) = x_2 \cdot P(f_1(X_1) \leq b)$$

Then, user 2's throughput is given by:

$$G_2(\alpha, \beta) = \int_0^1 g_{f_1}^{(2)}(x_2, f_2(x_2)) \cdot p_{X_2}(x_2) dx_2 = \int_0^1 g_{f_1}^{(2)}(x_2, f_2(x_2)) dx_2. \quad (2.32)$$

Due to the uniformly distributed channel state, $P(f_2(X_2) \leq b)$ is given by

$$P(f_2(X_2) \leq b) = P(X_2 \leq f_2^{-1}(b)) = f_2^{-1}(b)$$

where f_2^{-1} is well defined. Thus, we can rewrite Eq. (3.4) as

$$g_{f_2}^{(1)}(x_1, b) = x_1 \cdot f_2^{-1}(b).$$

Hence we have,

$$G_1(\alpha, \beta) = \int_0^1 x_1 \cdot f_2^{-1}(f_1(x_1)) dx_1 \quad (2.33)$$

$$G_2(\alpha, \beta) = \int_0^1 x_2 \cdot f_1^{-1}(f_2(x_2)) dx_2 \quad (2.34)$$

The following lemma gives a necessary and sufficient condition of a Nash equilibrium strategy pair. For convenience, we denote $\frac{\partial g_{f_2}^{(1)}(x_1, b)}{\partial b} \Big|_{b=b^*}$ (i.e., the marginal gain at $b = b^*$) as $Dg_{f_2}^{(1)}(x_1, b^*)$.

Lemma 4. *A strategy pair (f_1^*, f_2^*) is a Nash equilibrium strategy pair if and only if $Dg_{f_2}^{(1)}(x_1, f_1^*(x_1)) = c_1$ and $Dg_{f_1}^{(2)}(x_2, f_2^*(x_2)) = c_2$, for some constants c_1 and c_2 , for all $x_1 \in [0, 1]$ and all $x_2 \in [0, 1]$.*

To understand the lemma intuitively, suppose there exists $x \neq \tilde{x}$ such that $Dg_{f_2}^{(1)}(x, f_1^*(x)) > Dg_{f_2}^{(1)}(\tilde{x}, f_1^*(\tilde{x}))$. Reducing the bid at \tilde{x} to $f_1^*(\tilde{x}) - \delta$ and increasing the bid at x to $f_1^*(x) + \delta$ will result in an increase in the throughput by $(Dg_{f_2}^{(1)}(x, f_1^*(x)) - Dg_{f_2}^{(1)}(\tilde{x}, f_1^*(\tilde{x}))) \cdot \delta$. Thus, user 1 has an incentive to change its bidding function, and (f_1^*, f_2^*) cannot be a Nash equilibrium strategy pair in this case.

Proof. The complete proof is given in the Appendix. □

With Lemma 4, we are able to find the unique Nash equilibrium strategy pair. The exact form of the equilibrium bidding strategies are presented in the following Theorem.

Theorem 7. *Under the assumption of a single bidding function, the following is a unique Nash equilibrium strategy pair for the auction:*

$$f_1^*(x) = c \cdot x^{\gamma+1} \quad (2.35)$$

$$f_2^*(x) = c \cdot x^{\frac{1}{\gamma}+1} \quad (2.36)$$

where the constant γ and c are chosen such that

$$\int_0^1 c \cdot x^{\gamma+1} dx = \alpha \quad (2.37)$$

$$\int_0^1 c \cdot x^{\frac{1}{\gamma}+1} dx = \beta \quad (2.38)$$

Equations (3.11) and (3.12) impose the average money constraints. Fig. 2-3 shows an example of the Nash equilibrium bidding strategy pair when $\alpha = 1$ and $\beta = 2$. Since user 1 has less money than user 2, user 1 concentrates its bidding on time slots with very good channel state.

Proof. We show here that $f_1^*(x) = c \cdot x^{\gamma+1}$ and $f_2^*(x) = c \cdot x^{\frac{1}{\gamma}+1}$ is indeed a Nash equilibrium strategy pair by using the sufficiency condition of Lemma 4, and we leave the uniqueness part to the appendix. It is easy to check that both the condition $f_1^*(1) = f_2^*(1)$ and $f_1^*(0) = f_2^*(0)$ are satisfied. Since both functions are strictly increasing, we can write $g_{f_2^*}^{(1)}(x, b) = x \cdot f_2^{*-1}(b)$ and $g_{f_1^*}^{(2)}(x, b) = x \cdot f_1^{*-1}(b)$. Also, since both f_1^* and f_2^* are differentiable, we have $g_{f_2^*}^{(1)}(x, b)$ and $g_{f_1^*}^{(2)}(x, b)$ both differentiable with respect to b . Therefore,

$$\left. \frac{\partial g_{f_2^*}^{(1)}(x, b)}{\partial b} \right|_{b=f_1^*(x)} = \frac{x}{f_2^{*'}(f_2^{*-1}(f_1^*(x)))} = \frac{x}{f_2^{*'}(x^\gamma)} = \frac{\gamma}{c(1+\gamma)}.$$

Similarly,

$$\left. \frac{\partial g_{f_1^*}^{(2)}(x, b)}{\partial b} \right|_{b=f_2^*(x)} = \frac{x}{f_1^{*\prime}(f_1^{*-1}(f_2^*(x)))} = \frac{x}{f_1^{*\prime}(x^{1/\gamma})} = \frac{1}{c(1+\gamma)}.$$

From Lemma 4, we see that (f_1^*, f_2^*) is indeed a Nash equilibrium strategy pair because both $Dg_{f_2^*}^{(1)}(x, f_1^*(x))$ and $Dg_{f_1^*}^{(2)}(x, f_2^*(x))$ are constants.

The proof of uniqueness of (f_1^*, f_2^*) is given in the appendix. \square

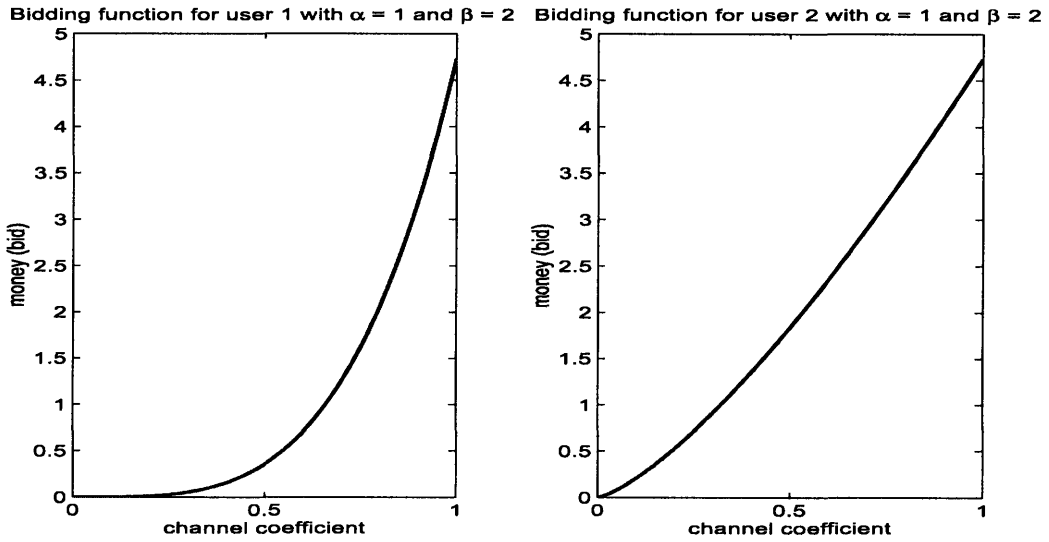


Figure 2-3: An example of Nash equilibrium strategy pair for $\alpha = 1$ and $\beta = 2$.

Fig. 2-4 shows the resulting allocation scheme when both users employ the Nash equilibrium strategy shown in Fig. 2-3. Above the curve, time slots will be allocated to user 2 since user 2's bid is higher than user 1's in this region. Similarly, user 1 gets the slots below the curve. Here, user 2 is allocated more slots than user 1 since it has more money.

If both players use Nash equilibrium strategies, the expected throughput obtained are given by:

$$G_1(\alpha, \beta) = \frac{\alpha}{\alpha + \beta + \sqrt{(\alpha - \beta)^2 + \alpha\beta}} \quad (2.39)$$

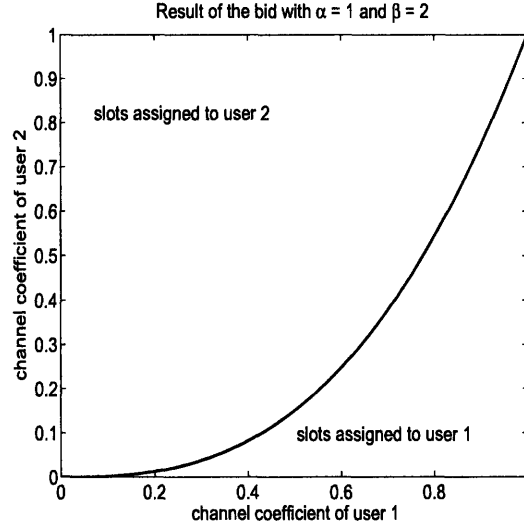


Figure 2-4: Allocation scheme from Nash equilibrium strategy pair for $\alpha = 1$ and $\beta = 2$.

$$G_2(\alpha, \beta) = \frac{\beta}{\alpha + \beta + \sqrt{(\alpha - \beta)^2 + \alpha\beta}} \quad (2.40)$$

As can be seen, the ratio of the throughput obtained $\frac{G_1(\alpha, \beta)}{G_2(\alpha, \beta)}$ is equal to $\frac{\alpha}{\beta}$ which is the ratio of the money each user had initially. Thus, the Nash equilibrium strategy pair provides an allocation scheme that is fair in the sense that the price per unit of throughput is the same for both users.

2.3.3 Unique Nash Equilibrium Strategy with multiple bidding functions

In the previous section, we restricted the strategy space of each user to be a single bidding function (i.e., \bar{S}_1 and \bar{S}_2) instead of a sequence of bidding functions (i.e., S_1 and S_2). However, the money constraint imposed upon each user is a long term average money constraint. A natural question to ask is the following: Is it profitable for an individual user to change its bidding functions over time while satisfying the long term average money constraint? Therefore, in this section, we allow the users to use a strategy within a broader class of strategy space, S_1 and S_2 , and explore

whether there is an incentive for a user to do so (i.e., whether there exists a Nash equilibrium strategy so that it can increase its throughput).

To choose a strategy (i.e., a sequence of bidding functions) from the strategy space S_1 or S_2 , a user encounters two problems. First, it must decide how to allocate its money among these n bidding functions so that the average money constraint is still satisfied. Second, once the money allocated to the i th bidding function is specified, a user has to choose a bidding function for the i th slot. The second problem is already solved in the previous section (see Theorem 7). In this section, we will focus on the first problem that a user encounters, specifically, the problem of how to allocate money between the bidding functions while satisfying the following condition: The total expected amount of money for the *sequence* of n bidding functions is $n \cdot \alpha$ for user 1 and $n \cdot \beta$ for user 2.

More precisely, the strategy space or possible actions that can be taken by users are the following:

$$\hat{S}_1 = \{\alpha_1, \dots, \alpha_n \mid \alpha_1 + \dots + \alpha_n = n \cdot \alpha\}$$

$$\hat{S}_2 = \{\beta_1, \dots, \beta_n \mid \beta_1 + \dots + \beta_n = n \cdot \beta\}$$

The objective of each user is still to maximize its own throughput. When user 1 and user 2 allocate α_i and β_i for their i th bidding function which is given in Theorem 7, the payoff functions are $G_1(\alpha_i, \beta_i)$ for user 1 and $G_2(\alpha_i, \beta_i)$ for user 2.

The following lemma gives us a Nash equilibrium strategy pair for the auction game described in this section.

Lemma 5. *Given that user 2's strategy is to allocate its money evenly among its bidding functions (i.e., $\beta_i = \beta, i = 1 \dots n$), user 1's best response is to allocate its money evenly as well (i.e., $\alpha_i = \alpha, i = 1 \dots n$); and vice versa. Therefore, a Nash equilibrium strategy pair for this auction is for both users to allocate their money evenly.*

Proof. Without loss of generality, we consider the case that $n = 2$ where each user's strategy can consist of two different bidding functions. Suppose that user 2 allocates

β for both bidding functions $f_2^{(1)}$ and $f_2^{(2)}$, and user 1 allocates α_1 for bidding function $f_1^{(1)}$ and α_2 for bidding function $f_1^{(2)}$ where $\alpha_1 + \alpha_2 = 2\alpha$ and $\alpha_1 \neq \alpha_2$. We now show that the throughput for user 1, $G_1(\alpha_1, \beta) + G_1(\alpha_2, \beta)$, is maximized when $\alpha_1 = \alpha_2 = \alpha$. Consider the function $G_1(\alpha_1, \beta)$ with β fixed. The equation

$$G_1(\alpha_1, \beta) = \frac{\alpha_1}{\alpha_1 + \beta + \sqrt{(\alpha_1 - \beta)^2 + \alpha_1\beta}}$$

becomes

$$F(t) = \frac{t}{1 + t + \sqrt{(1 - t)^2 + t}}$$

where $t = \frac{\alpha_1}{\beta}$. $F(t)$ is concave for $t \geq 0$. Thus, we have $G_1(\alpha_1, \beta) + G_1(\alpha_2, \beta)$ maximized when $\alpha_1 = \alpha_2 = \alpha$. \square

We have already obtained a Nash equilibrium strategy pair from the above Lemma. The following theorem states that this Nash equilibrium strategy pair is in fact unique within the strategy space considered.

Theorem 8. *For the auction in this section, a unique Nash equilibrium strategy for both users is to allocate their money evenly among the bidding functions.*

Proof. The complete proof is in the Appendix. \square

In this section, users are given more freedom in choosing their strategies (i.e., they can choose n different bidding functions). However, as Theorem 8 shows, the unique Nash equilibrium strategy pair is for each user to use a single bidding function from its strategy space. Thus, the throughput result obtained in this broader strategy space- S_1 and S_2 -is the same as the throughput result from previous section. Therefore, there is no incentive for a user to use different bidding functions.

2.3.4 Comparison with Other Allocation Schemes

To this end, we have a unique Nash equilibrium strategy pair and the resulting throughput when both players choose to use the Nash equilibrium strategy. Inevitably, due to the fairness constraint, total system throughput will decrease as

compared to the maximum throughput attainable without any fairness constraint. We compare the throughput of the centralized allocation scheme that maximize the total throughput subject to the constraint that the resulting throughput of individual user is kept at certain ratio (throughput ration constraint). The result of the throughput ratio constraint problem was given in the analysis of the second-price auction.

Using the Nash equilibrium strategy pair, the ratio of the resulting throughput pair $\frac{G_1(\alpha, \beta)}{G_2(\alpha, \beta)}$ is the same as the ratio of money individual user possess ($\frac{\alpha}{\beta}$). For the optimization problem described in (2.23), by setting $a = \alpha/\beta$, we compare the resulting throughput with the throughput obtained when both users employ the Nash equilibrium strategy. Fig. 2-5 shows the comparison. For both users, the Nash equilibrium throughput result is very close to the throughput obtained by solving the constrained optimization problem (within 97 percent to be precise).

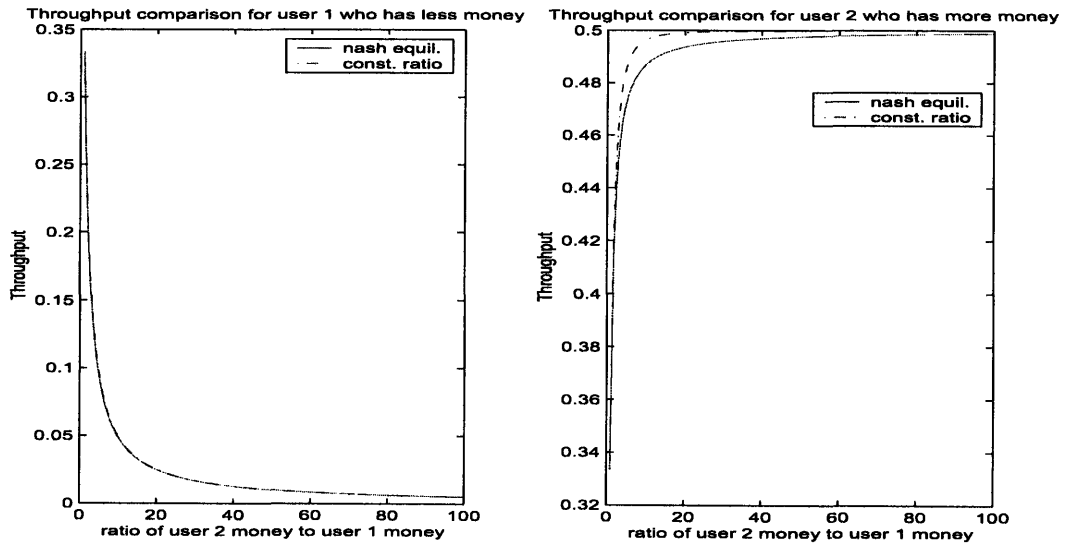


Figure 2-5: Throughput result comparison for both users.

2.4 Estimation of unknown α and β

For the auction algorithm discussed so far, we assume that the initial amounts of money, α and β , are known to both users. In this section, we present an estimation algorithm that estimates the opponent's money when this prior knowledge of α and β is not available. Assuming that both users use the Nash equilibrium strategy, by observing the bidding outcome for each time slot (i.e., whether user gets the time slot) one can estimate the other user's initial money amount.

To illustrate the main idea of our algorithm, we show the estimation process of user 1. Initially, user 1 knows only its own money amount α and guesses user 2's money to be β_{est} . From the pair (α, β_{est}) , it is able to calculate the constants c_1 and γ_1 from equations (3.11) and (3.12). Based on the channel state of that slots, say x_1 , it then bids $f_1^*(x_1) = c_1 \cdot x_1^{\gamma_1+1}$. If it wins the slot, it possibly overestimated its opponent's money, thus bidding too high. Therefore, user 1 may want to decrease β_{est} by a step whose size depends on user 1's channel condition. If the channel condition of user 1 is good (e.g., $x_1 = 0.99$), its probability of winning the slot, say P_{win} , is very high regardless of β_{est} . In other words, its winning of the slot is more likely due to the good channel condition than to an overestimate of its opponent's money. Thus, user 1 may not want to decrease β_{est} too much. In our algorithm, user 1 decreases β_{est} by $(1 - P_{win}) \cdot step$ to take the channel condition into account where *step* is the step size used. Similarly, if user 1 loses a bid, it may have underestimated its opponent's money. If its channel condition happens to be very good also (P_{win} is high), it may have *severely* underestimated user 2's money. Therefore, user 1 wants to increase β_{est} by a bigger step, $P_{win} \cdot step$.

In Fig 2-6, we use two thousand time slots to demonstrate the algorithm. Initially, $\alpha = 1$ and $\beta = 4$. To ensure fast convergence, variable step size was used at each iteration. Specifically, the step size was multiplied by a constant factor, which is less than one, after each iteration. From Fig 2-6, we see that the estimated value converges in about one thousand slots. Of course, this estimation procedure is merely for the purpose of demonstrating the possibility of operating without knowledge of

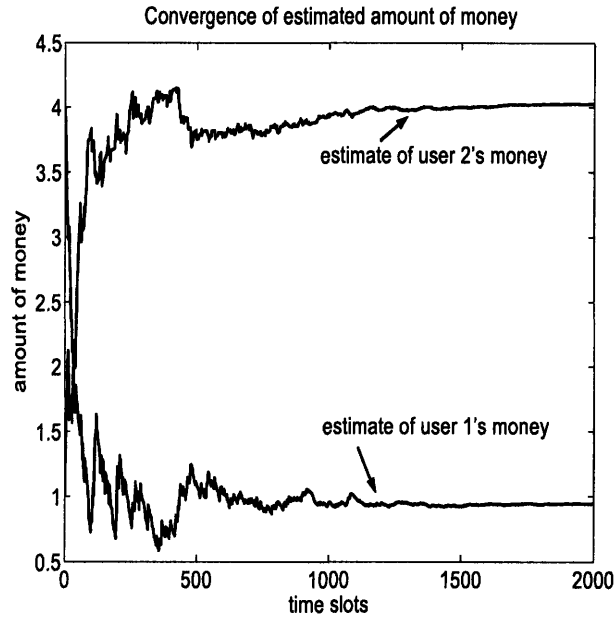


Figure 2-6: Convergence of estimated money possessed by opponent.

each user's budget. A more sophisticated estimation procedure, with faster convergence times, is a direction for future work.

2.5 Conclusion

We apply an auction algorithm to the problem of fair allocation of a wireless fading channel. Using the second price auction mechanism, we are able to obtain the Nash equilibrium strategies for general channel state distribution. Our strategy allocates bandwidth to the users in accordance with the amount of money that they possess. Hence, this scheme can be viewed as a mechanism for providing quality of service (QoS) differentiation; whereby users are given fictitious money that they can use to bid for the channel. By allocating users different amounts of money, the resulting QoS differentiation can be achieved.

In this chapter, we find the unique Nash equilibrium strategy for certain commonly used channel state distribution. We also show that the Nash equilibrium strategy for the second-price auction leads to an allocation at which total throughput is no

worse than $3/4$ the maximum possible throughput when fairness constraints are not imposed (i.e., slots are allocated to the user with the better channel) under uniform distribution. Moreover, the equilibrium strategies leads to an allocation that is pareto optimal. Based on the Nash equilibrium strategies of the second price auction with money constraint, we also propose a centralized opportunistic scheduler that does not suffer the shortcomings associated with the proportional fair and the time fraction fair scheduler. Using the all-pay auction mechanism, we are able to obtain a unique Nash equilibrium strategy. Our strategy allocated bandwidth to the users in accordance with the amount of money that they possess. Hence, this scheme can be viewed as a mechanism for providing quality of service (QoS) differentiation; whereby users are given fictitious money that they can use to bid for the channel.

Nevertheless, in the second price auction, the problem of how to obtain the multiplicative constant in user's equilibrium bidding strategy using a computational efficient way has yet to be explored. Also, to make our proposed centralized scheduler (based on the Nash equilibrium strategy) suitable for real time implementation, an algorithm that does not require the prior knowledge of channel distribution but still results in the Nash equilibrium allocation for each user will be an important topic for the future research.

Chapter Appendix: Proof of Theorem 4

Proof. If user 1 bids y for a particular time slot, the probability that it win, denoted as $P_1^{win}(y)$, is given by:

$$\begin{aligned} P_1^{win}(y) &= Pr(\max\{f_2(X_2), \dots, f_N(x_N)\} \leq y) \\ &= Pr(f_2(X_2) \leq y, \dots, f_N(X_N) \leq y) \end{aligned}$$

The optimization for user 1 can be written as follows:

$$\begin{aligned} \max_{f_1} \int_0^1 \int_{l_2}^{h_2(f_1(x_1))} \dots \int_{l_N}^{h_N(f_1(x_1))} x_1 \\ \cdot p_{X_1}(x_1) \dots p_{X_N}(x_N) dx_1 \dots dx_N \end{aligned} \quad (2.41)$$

subject to

$$\begin{aligned} \int_0^1 \int_{l_2}^{h_2(f_1(x_1))} \dots \int_{l_N}^{h_N(f_1(x_1))} \max\{f_2(x_2), \dots, f_N(x_N)\} \\ \cdot p_{X_1}(x_1) \dots p_{X_N}(x_N) dx_1 \dots dx_N \leq \alpha_1 \end{aligned} \quad (2.42)$$

After writing the Lagrangian function, we then solve the following optimization problem:

$$\begin{aligned} \max_{f_1(x_1)} \int_{l_2}^{h_2(f_1(x_1))} \dots \int_{l_N}^{h_N(f_1(x_1))} \\ (x_1 - \lambda_1 \max\{f_2(x_2), \dots, f_N(x_N)\}) \\ \cdot p_{X_2}(x_2) \dots p_{X_N}(x_N) dx_2 \dots dx_N \end{aligned} \quad (2.43)$$

Writing $y = f_1(x_1)$ for convenience, we have the following:

$$\begin{aligned} \max_y \int_{l_2}^{h_2(y)} \dots \int_{l_N}^{h_N(y)} (x_1 - \lambda_1 \max\{f_2(x_2), \dots, f_N(x_N)\}) \\ \cdot p_{X_2}(x_2) \dots p_{X_N}(x_N) dx_2 \dots dx_N \end{aligned}$$

The term $(x_1 - \lambda_1 \max\{f_2(x_2), \dots, f_N(x_N)\})$ is decreasing since $(\lambda_1 \max\{f_2(x_2), \dots, f_N(x_N)\})$ is increasing. Therefore, it is desirable to choose y as large as possible while keeping $(x_1 - \lambda_1 \max\{f_2(x_2), \dots, f_N(x_N)\}) \geq 0$.

For a fixed x_1 , if the term $(x_1 - \lambda_1 \max\{f_2(x_2), \dots, f_N(x_N)\})$ is positive for all $x_i \in [l_i, u_i]$ for $i = 2, \dots, N$, the optimal y^* can be chosen such that $y^* \geq \max\{b_2, \dots, b_N\}$. Likewise, if the term $x_1 - \lambda_1 \max\{f_2(x_2), \dots, f_N(x_N)\}$ is negative for all $x_i \in [l_i, u_i]$ for $i = 2, \dots, N$, the optimal y^* can be chosen such that $y^* \leq \min\{a_2, \dots, a_N\}$. In the case that $(x_1 - \lambda_1 \max\{f_2(x_2), \dots, f_N(x_N)\}) = 0$ for some $(x_2, \dots, x_N) \in [l_2, u_2] \times \dots \times [l_N, u_N]$, we can choose y^* such that $(x_1 - \lambda_1 \max\{f_2(h_2(y^*)), \dots, f_N(h_N(y^*))\}) = 0$. From the definition of $h_i(\cdot)$ in Eq.(2.20), each term $f_i(h_i(y))$ equals to $\min\{y^*, b_i\}$ for $i = 2, \dots, N$. Hence, we have the following:

$$\max\{f_2(h_2(y^*)), \dots, f_N(h_N(y^*))\} = y^*.$$

Consequently, the optimal bid for user 1, y^* , is $\frac{1}{\lambda_1}x_1$. Again, the optimal bidding strategy for a user in the N-user game is a linear function of the user's channel state. The constant coefficient, i.e., $\frac{1}{\lambda_1}$, is chosen such that the average money constraint is met with equality.

□

Chapter Appendix: Proof of Theorem 6

Proof. Again, we consider $n = 2$ case for simplicity. For $\sigma_1 + \sigma_2 = 2\sigma$ and $\beta_1 + \beta_2 = 2\beta$, this theorem stated that the pair (σ_1, β_1) and (σ_2, β_2) cannot be in equilibrium if $\sigma_1 \neq \sigma_2$ and $\beta_1 \neq \beta_2$. We will show this by contradiction. Here, we present the proof for the case that $\sigma_1 \leq \beta_1$ and $\sigma_1 \leq \beta_1$. Other cases can be shown similarly. Now, suppose the pair (σ_1, β_1) and (σ_2, β_2) are in equilibrium for $\sigma_1 \neq \sigma_2$ and $\beta_1 \neq \beta_2$. That is, for given β_1 and β_2 , σ_1 and σ_2 are chosen such that user 1's throughput $G_1(\sigma_1, \beta_1) + G_1(\sigma_2, \beta_2)$ is the maximum. This implies the following:

$$\left. \frac{\partial G_1(\sigma, \beta_1)}{\partial \sigma} \right|_{\sigma=\sigma_1} = \left. \frac{\partial G_1(\sigma, \beta_2)}{\partial \sigma} \right|_{\sigma=\sigma_2}. \quad (2.44)$$

To see this, if $\left. \frac{\partial G_1(\sigma, \beta_1)}{\partial \sigma} \right|_{\sigma=\sigma_1} > \left. \frac{\partial G_1(\sigma, \beta_2)}{\partial \sigma} \right|_{\sigma=\sigma_2}$, we will have $G_1(\sigma_1 + \delta, \beta_1) + G_1(\sigma_2 - \delta, \beta_2) > G_1(\sigma_1, \beta_1) + G_1(\sigma_2, \beta_2)$ by first order expansion, thus contradicting the statement that $G_1(\sigma_1, \beta_1) + G_1(\sigma_2, \beta_2)$ is the maximum throughput for user 1 for given β_1 and β_2 .

Similarly, for given σ_1 and σ_2 , if β_1 and β_2 maximize $G_2(\sigma_1, \beta_1) + G_2(\sigma_2, \beta_2)$ then,

$$\left. \frac{\partial G_2(\sigma_1, \beta)}{\partial \beta} \right|_{\beta=\beta_1} = \left. \frac{\partial G_2(\sigma_2, \beta)}{\partial \beta} \right|_{\beta=\beta_2}. \quad (2.45)$$

By taking the derivative of equations (2.39) and (2.40), we get the following:

$$\left. \frac{\partial G_1(\sigma, \beta_1)}{\partial \sigma} \right|_{\sigma=\sigma_1} = \frac{\beta_1}{(\beta_1 + 2\sigma_1)^2} \quad (2.46)$$

$$\left. \frac{\partial G_2(\sigma_1, \beta)}{\partial \beta} \right|_{\beta=\beta_1} = \frac{3\sigma_1^2}{(\beta_1 + 2\sigma_1)^3} \quad (2.47)$$

Substituting Eq.(2.46) into Eq.(2.62) and Eq.(2.65) into Eq.(2.63), we then have the following after combining Eq.(2.62) and Eq.(2.63):

$$\frac{\beta_1}{\beta_2} = \frac{(\beta_1 + 2\sigma_1)^2}{(\beta_2 + 2\sigma_2)^2} \quad (2.48)$$

$$\frac{3\sigma_1^2}{3\sigma_2^2} = \frac{(\beta_1 + 2\sigma_1)^3}{(\beta_2 + 2\sigma_2)^3} \quad (2.49)$$

$$\Rightarrow \frac{\sigma_1}{\beta_1^{\frac{3}{4}}} = \frac{\sigma_2}{\beta_2^{\frac{3}{4}}} \quad (2.50)$$

Now, we have $\frac{\sigma_1}{\beta_1^{\frac{3}{4}}} = \frac{\sigma_2}{\beta_2^{\frac{3}{4}}}$. We further show that $\sigma_1 = \sigma_2$ and $\beta_1 = \beta_2$. Observe that for fixed β_1 and β_2 , we can write

$$G_1(\sigma, \beta_1) = \frac{\sigma}{\beta_1 + 2\sigma} = \frac{\frac{\sigma}{\beta_1^{\frac{3}{4}}}}{\frac{\beta_1^{\frac{3}{4}}}{\beta_1} + \frac{2\sigma}{\beta_1^{\frac{3}{4}}}} \triangleq F\left(\frac{\sigma}{\beta_1}\right) \quad (2.51)$$

where

$$F(\gamma) = \frac{\gamma}{\beta_1^{\frac{1}{4}} + 2\gamma}.$$

Thus, we have

$$\frac{\partial G_1(\sigma, \beta_1)}{\partial \sigma} \Big|_{\sigma=\sigma_1} = \frac{1}{\beta_1^{\frac{3}{4}}} \frac{\partial F(\gamma)}{\partial \gamma} \Big|_{\gamma=\frac{\sigma_1}{\beta_1^{\frac{3}{4}}}} \quad (2.52)$$

$$\frac{\partial G_1(\sigma, \beta_2)}{\partial \sigma} \Big|_{\sigma=\sigma_2} = \frac{1}{\beta_2^{\frac{3}{4}}} \frac{\partial F(\gamma)}{\partial \gamma} \Big|_{\gamma=\frac{\sigma_2}{\beta_2^{\frac{3}{4}}}} \quad (2.53)$$

From Eq.(2.62), we have

$$\frac{1}{\beta_1^{\frac{3}{4}}} \frac{\partial F(\gamma)}{\partial \gamma} \Big|_{\gamma=\frac{\sigma_1}{\beta_1^{\frac{3}{4}}}} = \frac{1}{\beta_2^{\frac{3}{4}}} \frac{\partial F(\gamma)}{\partial \gamma} \Big|_{\gamma=\frac{\sigma_2}{\beta_2^{\frac{3}{4}}}} \quad (2.54)$$

It is easy to verify that $\frac{\partial F(\gamma)}{\partial \gamma} \neq 0 \forall \gamma \geq 0$. Therefore, since $\frac{\sigma_1}{\beta_1^{\frac{3}{4}}} = \frac{\sigma_2}{\beta_2^{\frac{3}{4}}}$, the above equation implies that $\beta_1 = \beta_2$ which contradicts our original assumption of $\beta_1 \neq \beta_2$. Therefore, the pair (σ_1, β_1) and (σ_2, β_2) cannot be in equilibrium if $\sigma_1 \neq \sigma_2$ and $\beta_1 \neq \beta_2$.

□

Chapter Appendix: Proof of Lemma 4

Proof: We first show that if (f_1^*, f_2^*) is a Nash equilibrium strategy pair, $Dg_{f_2^*}^{(1)}(x_1, f_1^*(x_1))$ and $Dg_{f_1^*}^{(2)}(x_2, f_2^*(x_2))$ must be constants for all $x_1 \in [0, 1]$ and $x_2 \in [0, 1]$. From user 1's perspective with f_2^* fixed, consider a small variation of the function f_1^* . Specifically, let $f_\delta = f_1^* + \delta(\hat{f} - f_1^*)$ where \hat{f} is an arbitrary function in \bar{S}_1 . Since both \hat{f} and f_1^* are in \bar{S}_1 , they are both bounded (i.e., $|\hat{f}(x_1)| \leq B$ and $|f_1^*(x_1)| \leq B$ for all $x_1 \in [0, 1]$). Therefore, we have $|f_\delta(x_1) - f_1^*(x_1)| \leq 2B\delta$ for all $x_1 \in [0, 1]$. Using the Lagrange's form of Taylor's theorem, we get for any $x_1 \in [0, 1]$, there exists a real number $c_{[x_1]} \in [f_1^*(x_1), f_\delta(x_1)]$ such that

$$\begin{aligned} g_{f_2^*}^{(1)}(x_1, f_\delta(x_1)) &= g_{f_2^*}^{(1)}(x_1, f_1^*(x_1)) \\ &+ \delta(\hat{f}(x_1) - f_1^*(x_1)) \frac{\partial g_{f_2^*}^{(1)}(x_1, b)}{\partial b} \Big|_{b=f_1^*(x_1)} \\ &+ \frac{1}{2} \delta^2 (\hat{f}(x_1) - f_1^*(x_1))^2 \frac{\partial^2 g_{f_2^*}^{(1)}(x_1, b)}{\partial b^2} \Big|_{b=c_{[x_1]}} \end{aligned} \quad (2.55)$$

The last term is bounded by $K \cdot \delta^2$ for some K since both \hat{f} and f_1^* are bounded, and $g_{f_2^*}^{(1)}(x_1, b)$ has finite second derivative. Therefore, for small enough δ , it is negligible comparing with the other terms.

Now we show that if $Dg_{f_2^*}^{(1)}(x_1, f_1^*(x_1))$ is not a constant for all $x_1 \in [0, 1]$, we can find a strategy f_δ which gives user 1 a higher throughput than f_1^* . To do that, we can write the following equations:

$$\begin{aligned} &\int_0^1 g_{f_2^*}^{(1)}(x_1, f_\delta(x_1)) dx_1 - \int_0^1 g_{f_2^*}^{(1)}(x_1, f_1^*(x_1)) dx_1 \\ &= \delta \int_0^1 (\hat{f}(x_1) - f_1^*(x_1)) \frac{\partial g_{f_2^*}^{(1)}(x_1, b)}{\partial b} \Big|_{b=f_1^*(x_1)} dx_1 + o(\delta) \end{aligned} \quad (2.56)$$

Now, since $Dg_{f_2^*}^{(1)}(x_1, f_1^*(x_1))$ is not a constant for all $x_1 \in [0, 1]$, we can find a \hat{f} such that the above equation is positive which implies that there is an incentive for user 1 to use f_δ . Hence, (f_1^*, f_2^*) is not a Nash equilibrium strategy pair. Similarly, we can show that $Dg_{f_1^*}^{(2)}(x_2, f_2^*(x_2))$ is a constant for all $x_2 \in [0, 1]$ if (f_1^*, f_2^*) is a Nash

equilibrium strategy pair.

For the converse, consider again Eq.(3.24). Since $Dg_{f_2^*}^{(1)}(x_1, f_1^*(x_1)) = \frac{\partial g_{f_2^*}^{(1)}(x_1, b)}{\partial b} \Big|_{b=f_1^*(x_1)}$ equals to a constant c_1 for all $x_1 \in [0, 1]$. We have

$$\begin{aligned} & \delta \int_0^1 (\hat{f}(x_1) - f_1^*(x_1)) \frac{\partial g_{f_2^*}^{(1)}(x_1, b)}{\partial b} \Big|_{b=f_1^*(x_1)} dx_1 \\ &= \delta c_1 \int_0^1 (\hat{f}(x_1) - f_1^*(x_1)) dx_1 = 0 \end{aligned} \tag{2.57}$$

for all $\hat{f} \in \bar{S}_1$ (i.e., $\int_0^1 \hat{f}(x_1) dx_1 = \alpha$). Thus, there is no incentive for user 1 to use strategy \hat{f} . Therefore, (f_1^*, f_2^*) is a Nash equilibrium strategy pair.

Chapter Appendix: Proof of Theorem 7 (the Uniqueness)

Consider any Nash equilibrium strategy pair (f_1, f_2) under the all-pay auction rule. From previous discussion, we know that the inverse functions, f_2^{-1} and f_1^{-1} , are well defined. With user 2's strategy f_2 fixed, we have

$$g_{f_2}^{(1)}(x_1, b) = x_1 \cdot P(f_2(X_2) \leq b) = x_1 \cdot f_2^{-1}(b)$$

Similarly, with user1's strategy f_1 fixed, we get

$$g_{f_1}^{(2)}(x_2, b) = x_2 \cdot P(f_1(X_1) \leq b) = x_2 \cdot f_1^{-1}(b)$$

From Lemma 4, we know that $Dg_{f_2}^{(1)}(x_1, f_1(x_1))$ and $Dg_{f_1}^{(2)}(x_2, f_2(x_2))$ are two constants for all $x_1 \in [0, 1]$ and $x_2 \in [0, 1]$ since (f_1, f_2) is a Nash equilibrium strategy pair.

Now, consider the set of channel state pair (x_1, x_2) such that $f_1(x_1) = f_2(x_2)$ (i.e., two users' bids are equal). It forms a separation line in space span by X_1 and X_2 . Mathematically, this line can be defined as $h : [0, 1] \rightarrow [0, 1]$ such that $x_2 = h(x_1) = f_2^{-1}(f_1(x_1))$. By the *all-pay* auction rule, a slot with channel state (x_1, x'_2) will be assigned to user 2 if (x_1, x'_2) is above the line $x_2 = h(x_1)$ and to user 1 if (x_1, x'_2) is below the separation line. Fig.2-4 shows an example of $h(x_1)$. The following lemma shows the uniqueness of $h(x_1)$. We then derive the uniqueness of the strategy pair (f_1, f_2) from the lemma.

Lemma 6. *If $Dg_{f_2}^{(1)}(x_1, f_1(x_1))$ and $Dg_{f_1}^{(2)}(x_2, f_2(x_2))$ are two constants, c_1 and c_2 respectively, for all $x_1 \in [0, 1]$ and $x_2 \in [0, 1]$, then $h(x_1) = x_1^{c_1/c_2}$.*

Proof. Since $Dg_{f_2}^{(1)}(x_1, f_1(x_1)) = c_1$, from $g_{f_2}^{(1)}(x_1, b) = x_1 \cdot f_2^{-1}(b)$, we have

$$Dg_{f_2}^{(1)}(x_1, f_1(x_1)) = \left. \frac{\partial g_{f_2}^{(1)}(x_1, b)}{\partial b} \right|_{b=f_1(x_1)} = \frac{x_1}{f_2'(f_2^{-1}(f_1(x_1)))} = c_1$$

$$f_2'(h(x_1)) = \frac{x_1}{c_1} \quad (2.58)$$

Similarly, for user 2, we get

$$\begin{aligned} Dg_{f_1}^{(2)}(x_2, f_2(x_2)) &= \left. \frac{\partial g_{f_1}^{(2)}(x_2, b)}{\partial b} \right|_{b=f_2(x_2)} = \frac{x_2}{f_1'(f_1^{-1}(f_2(x_2)))} = c_2 \\ f_1'(h^{-1}(x_2)) &= \frac{x_2}{c_2} \end{aligned} \quad (2.59)$$

We also know that $f_1(x_1) = f_2(h(x_1))$ and $f_1'(x_1) = f_2'(h(x_1)) \cdot h'(x_1)$. Thus, we have

$$\begin{aligned} f_1'(h^{-1}(x_2)) &= f_2'(h(h^{-1}(x_2))) \cdot h'(h^{-1}(x_2)) \\ &= f_2'(x_2) \cdot h'(x_1) = f_2'(h(x_1)) \cdot h'(x_1) \end{aligned} \quad (2.60)$$

By combining the equations $f_1'(h^{-1}(x_2)) = \frac{x_2}{c_2}$ and $f_1'(h^{-1}(x_2)) = f_2'(h(x_1)) \cdot h'(x_1)$, we get

$$\frac{x_2}{c_2} = f_2'(h(x_1)) \cdot h'(x_1).$$

Next we substitute Eq.(3.26) and $x_2 = h(x_1)$ in the above equation to obtain,

$$\begin{aligned} x_1 \cdot \frac{dh(x_1)}{dx_1} &= \frac{c_1}{c_2} h(x_1) \Rightarrow \frac{dh(x_1)}{h(x_1)} = \frac{c_1}{c_2} \frac{dx_1}{x_1} \\ \ln |h(x_1)| &= \frac{c_1}{c_2} \ln |x_1| + c_3 \Rightarrow h(x_1) = e^{c_3} \cdot x_1^{\frac{c_1}{c_2}} \end{aligned}$$

Combined with fact that $h(1) = 1$, we get $h(x_1) = x_1^{\frac{c_1}{c_2}}$. □

Now, we are in a position to derive the exact form of the Nash equilibrium strategy pair. From the equations $f_1'(h^{-1}(x_2)) = \frac{x_2}{c_2}$ and $x_2 = h(x_1)$, we get $f_1'(x_1) = \frac{h(x_1)}{c_2} = x_1^{\frac{c_1}{c_2}}/c_2$. Combined with the condition that $f_1(0) = 0$, we have $f_1(x) = \frac{1}{c_1+c_2} x^{\frac{c_1}{c_2}+1}$. Following the similar method, we get $f_2(x) = \frac{1}{c_1+c_2} x^{\frac{c_2}{c_1}+1}$. Let $\gamma = \frac{c_1}{c_2}$ and $c = \frac{1}{c_1+c_2}$, the Nash equilibrium strategy pair for the all-pay auction must have the following form:

$$f_1^*(x_1) = c \cdot x_1^{\gamma+1}, \quad f_2^*(x_2) = c \cdot x_1^{\frac{1}{\gamma}+1} \quad (2.61)$$

The constant γ and c are chosen such that equations (3.11) and (3.12) are satisfied. The uniqueness of the above Nash equilibrium strategy comes from the fact that there is a unique pair of c and γ that satisfy equations (3.11) and (3.12).

Chapter Appendix: Proof of Theorem 8

Proof. Again, we consider $n = 2$ case for simplicity. For $\alpha_1 + \alpha_2 = 2\alpha$ and $\beta_1 + \beta_2 = 2\beta$, this theorem stated that the pair (α_1, β_1) and (α_2, β_2) cannot be in equilibrium if $\alpha_1 \neq \alpha_2$ and $\beta_1 \neq \beta_2$. We will show this by contradiction. Suppose the pair (α_1, β_1) and (α_2, β_2) are in equilibrium for $\alpha_1 \neq \alpha_2$ and $\beta_1 \neq \beta_2$. That is, for given β_1 and β_2 , α_1 and α_2 are chosen such that user 1's throughput $G_1(\alpha_1, \beta_1) + G_1(\alpha_2, \beta_2)$ is the maximum. This implies the following:

$$\left. \frac{\partial G_1(\alpha, \beta_1)}{\partial \alpha} \right|_{\alpha=\alpha_1} = \left. \frac{\partial G_1(\alpha, \beta_2)}{\partial \alpha} \right|_{\alpha=\alpha_2}. \quad (2.62)$$

To see this, if $\left. \frac{\partial G_1(\alpha, \beta_1)}{\partial \alpha} \right|_{\alpha=\alpha_1} > \left. \frac{\partial G_1(\alpha, \beta_2)}{\partial \alpha} \right|_{\alpha=\alpha_2}$, we will have $G_1(\alpha_1 + \delta, \beta_1) + G_1(\alpha_2 - \delta, \beta_2) > G_1(\alpha_1, \beta_1) + G_1(\alpha_2, \beta_2)$ by first order expansion, thus contradicting the statement that $G_1(\alpha_1, \beta_1) + G_1(\alpha_2, \beta_2)$ is the maximum throughput for user 1 for given β_1 and β_2 .

Similarly, for given α_1 and α_2 , if β_1 and β_2 maximize $G_2(\alpha_1, \beta_1) + G_2(\alpha_2, \beta_2)$ then,

$$\left. \frac{\partial G_2(\alpha_1, \beta)}{\partial \beta} \right|_{\beta=\beta_1} = \left. \frac{\partial G_2(\alpha_2, \beta)}{\partial \beta} \right|_{\beta=\beta_2}. \quad (2.63)$$

By taking the derivative of equations (2.39) and (2.40), we get the following:

$$\left. \frac{\partial G_1(\alpha, \beta_1)}{\partial \alpha} \right|_{\alpha=\alpha_1} = -\frac{\beta_1(-2\sqrt{\alpha_1^2 - \alpha_1\beta_1 + \beta_1^2} + \alpha_1 - 2\beta_1)}{2(\alpha_1 + \beta_1 + \sqrt{\alpha_1^2 - \alpha_1\beta_1 + \beta_1^2})^2 \sqrt{\alpha_1^2 - \alpha_1\beta_1 + \beta_1^2}} \quad (2.64)$$

$$\left. \frac{\partial G_2(\alpha_1, \beta)}{\partial \beta} \right|_{\beta=\beta_1} = -\frac{\alpha_1(-2\sqrt{\alpha_1^2 - \alpha_1\beta_1 + \beta_1^2} + \beta_1 - 2\alpha_1)}{2(\alpha_1 + \beta_1 + \sqrt{\alpha_1^2 - \alpha_1\beta_1 + \beta_1^2})^2 \sqrt{\alpha_1^2 - \alpha_1\beta_1 + \beta_1^2}} \quad (2.65)$$

Substituting Eq.(2.64) into Eq.(2.62) and Eq.(2.65) into Eq.(2.63), we then have

the following after combining Eq.(2.62) and Eq.(2.63):

$$\begin{aligned} & \frac{\beta_1(-2\sqrt{\alpha_1^2 - \alpha_1\beta_1 + \beta_1^2} + \alpha_1 - 2\beta_1)}{\beta_2(-2\sqrt{\alpha_2^2 - \alpha_2\beta_2 + \beta_2^2} + \alpha_2 - 2\beta_2)} \\ &= \frac{\alpha_1(-2\sqrt{\alpha_1^2 - \alpha_1\beta_1 + \beta_1^2} + \beta_1 - 2\alpha_1)}{\alpha_2(-2\sqrt{\alpha_2^2 - \alpha_2\beta_2 + \beta_2^2} + \beta_2 - 2\alpha_2)} \end{aligned} \quad (2.66)$$

To simplify the above equation, we multiply $\frac{\alpha_2^2}{\alpha_1^2}$ on both sides, and let $\gamma_1 = \frac{\beta_1}{\alpha_1}$, $\gamma_2 = \frac{\beta_2}{\alpha_2}$. We get

$$\frac{\gamma_1(-2\sqrt{1 - \gamma_1 + \gamma_1^2} + 1 - 2\gamma_1)}{\gamma_2(-2\sqrt{1 - \gamma_2 + \gamma_2^2} + 1 - 2\gamma_2)} = \frac{-2\sqrt{1 - \gamma_1 + \gamma_1^2} + \gamma_1 - 2}{-2\sqrt{1 - \gamma_2 + \gamma_2^2} + \gamma_2 - 2} \quad (2.67)$$

or, after rearranging terms, the following:

$$\frac{\gamma_1(-2\sqrt{1 - \gamma_1 + \gamma_1^2} + 1 - 2\gamma_1)}{-2\sqrt{1 - \gamma_1 + \gamma_1^2} + \gamma_1 - 2} = \frac{\gamma_2(-2\sqrt{1 - \gamma_2 + \gamma_2^2} + 1 - 2\gamma_2)}{-2\sqrt{1 - \gamma_2 + \gamma_2^2} + \gamma_2 - 2} \quad (2.68)$$

We define

$$f(\gamma) = \frac{\gamma(-2\sqrt{1 - \gamma + \gamma^2} + 1 - 2\gamma)}{-2\sqrt{1 - \gamma + \gamma^2} + \gamma - 2}.$$

Then Eq.(2.68) becomes $f(\gamma_1) = f(\gamma_2)$. Now we show that this implies $\gamma_1 = \gamma_2$ by observing that

$$\frac{\partial f(\gamma)}{\partial \gamma} = -\frac{(\gamma + 1)(2\sqrt{1 - \gamma + \gamma^2} - 1 + 2\gamma)}{\sqrt{1 - \gamma + \gamma^2}(-2\sqrt{1 - \gamma + \gamma^2} + \gamma - 2)},$$

and it is easy to check that $\frac{\partial f}{\partial \gamma} > 0 \forall \gamma \geq 0$. Now, we have $\frac{\beta_1}{\alpha_1} = \frac{\beta_2}{\alpha_2}$. We further show that $\alpha_1 = \alpha_2$ and $\beta_1 = \beta_2$. Observe that for fixed β_1 and β_2 , we can write

$$\begin{aligned} G_1(\alpha, \beta_1) &= \frac{\alpha}{\alpha + \beta_1 + \sqrt{(\alpha - \beta_1)^2 + \alpha\beta_1}} \\ &= \frac{\frac{\alpha}{\beta_1}}{1 + \frac{\alpha}{\beta_1} + \sqrt{(1 - \frac{\alpha}{\beta_1})^2 + \frac{\alpha}{\beta_1}}} \triangleq F\left(\frac{\alpha}{\beta_1}\right) \end{aligned} \quad (2.69)$$

where

$$F(\sigma) = \frac{\sigma}{1 + \sigma + \sqrt{(1 - \sigma)^2 + \sigma}}.$$

Thus, we have

$$\left. \frac{\partial G_1(\alpha, \beta_1)}{\partial \alpha} \right|_{\alpha=\alpha_1} = \frac{1}{\beta_1} \left. \frac{\partial F(\sigma)}{\partial \sigma} \right|_{\sigma=\frac{\alpha_1}{\beta_1}} \quad (2.70)$$

$$\left. \frac{\partial G_1(\alpha, \beta_2)}{\partial \alpha} \right|_{\alpha=\alpha_2} = \frac{1}{\beta_2} \left. \frac{\partial F(\sigma)}{\partial \sigma} \right|_{\sigma=\frac{\alpha_2}{\beta_2}} \quad (2.71)$$

From Eq.(2.62), we have

$$\left. \frac{1}{\beta_1} \frac{\partial F(\sigma)}{\partial \sigma} \right|_{\sigma=\frac{\alpha_1}{\beta_1}} = \left. \frac{1}{\beta_2} \frac{\partial F(\sigma)}{\partial \sigma} \right|_{\sigma=\frac{\alpha_2}{\beta_2}} \quad (2.72)$$

It is easy to verify that $\frac{\partial F(\sigma)}{\partial \sigma} \neq 0 \forall \sigma \geq 0$. Therefore, since $\frac{\beta_1}{\alpha_1} = \frac{\beta_2}{\alpha_2}$, the above equation implies that $\beta_1 = \beta_2$ which contradicts our original assumption of $\beta_1 \neq \beta_2$. Therefore, the pair (α_1, β_1) and (α_2, β_2) cannot be in equilibrium if $\alpha_1 \neq \alpha_2$ and $\beta_1 \neq \beta_2$.

□

Acknowledgment

We acknowledge gratefully the helpful discussions with Prof. Ramesh Johari, Prof. Eitan Altman, and Prof. Asuman Ozdaglar.

Chapter 3

Opportunistic Power Allocation for Fading Channels with Non-cooperative Users and Random Access

In the previous chapter, we have studied the use of auction theory in allocating a downlink wireless fading channel. There, users are assumed to have a fictitious amount of money that serves as a mechanism to differentiate QoS given to each user. For the uplink with random access, a natural physical constraint—the energy constraint—exists to serve as the mechanism for differentiating QoS. In this chapter, we are going to present a game-theoretical model of a wireless communication system with multiple competing users sharing a multiaccess fading channel. With a specified capture rule and a limited amount of energy available, a user opportunistically adjusts its transmission power based on its own channel state to maximize the user's own individual throughput.

3.1 Introduction

In a wireless or satellite network, the channel over which communication takes place is often time-varying. When multiple users try to communicate with a satellite, one can exploit the channel variation opportunistically by allowing the user with the best channel condition to transmit. This transmission scheme implies the performance of the network is dictated by the best channel state rather than the average one. Hence, the total throughput of such a network tends to increase with number of users.

An important assumption in using this kind scheme is that there is a centralized scheduler who knows each user's channel condition. To get information about user's channel condition, the scheduler will require each user to estimate its channel fading and transmit this information back. As the number of users in the network increases, the delay in conveying user's channel conditions to the scheduler will limit the system's performance. Hence, a distributive multi-access scheme with no centralized scheduler becomes an attractive alternative.

Distributed multi-access schemes such as the aloha random access protocol have long been studied. Recently, a variation on the aloha scheme that takes user's channel state into consideration (channel-aware aloha) has been proposed by [11]. In their formulation, each user only has knowledge of its own channel condition, but no knowledge of the channel fading of the other users sharing the communication link. Capture was not considered in their chapter. In [27], the authors studies multiple power level aloha with the objective of maximizing total system throughput when channels are time invariant. In this chapter, we allow the satellite to capture packet depending on the received power and assume the channel is a time-varying fading channel. To maximize their own individual throughput subject to the available energy, users opportunistically adjust their transmission power based on their own channel condition. Also, all of the aforementioned work assume that users will implement the same mandated algorithm and behave in a predictable manner. However, in a distributive environment, users may want to change their communication protocols in order to improve their own performance, making it impossible to ensure a particular algorithm

will be adopted by all users in the network. Rather than following some mandated algorithm, in this chapter users are assumed to act selfishly (i.e., choose their own power allocation strategies) to further their own individual interests.

The communication system considered in this chapter consists of multiple users competing to access a satellite, or a base-station. Each user has an average power constraint. Time is slotted. During each time slot, each user chooses a power level for transmission based on the channel state of current slot, which is only known to itself. Depending on the capture model and the received power of that user's signal, a transmitted packet may be captured even if multiple users are transmitting at the same slot.

With each user wants to maximize its own expected throughput, we obtain a Nash equilibrium power allocation strategy which determines the optimal transmission power control strategy for each user. Nash equilibrium of a game is a set of strategies (one for each user) from which there is no profitable unilateral deviation. The obtained optimal power control strategy specifies how much power a user needs to use to maximize its own throughput for any possible channel state. Users get different average throughput based on their average power constraint. Hence, this transmission scheme can be viewed as mechanism for providing quality of service (QoS) differentiation; whereby users are given different energy for transmission. The obtained Nash equilibrium power allocation strategy is unique under certain capture rule. When all users have the same energy constraint, we obtained a symmetric Nash equilibrium.

Due to the selfish behavior of individual users, the overall system throughput will be less than that of a system where users employ the same mandated algorithm. This loss in efficiency is also quantified. In the multiple users' case, as the number of user in the system increases, the symmetric Nash equilibrium strategy approaches the optimal algorithm specified by a system designer (i.e., algorithm that results in the largest total system throughput). In this case, there is no loss of efficiency when users employ the symmetric Nash equilibrium.

Game theoretical approaches to Aloha random access problems have been explored

by a number of researchers recently (e.g., [10][28]). In [10], the authors characterized the stability region for a slotted Aloha system with multipacket reception and selfish users for the case of perfect information. In [28], the authors considered the problem of a node computing its own optimal channel access rate in a random access network with two-way traffic. In their setting, a node is interested in both receiving as well as transmitting packets. The existence of Nash equilibrium is shown for node without power constraint as well as with battery power constraint. Our work attempts to apply game theory to the access of a wireless fading channel. In particular, we show that the Nash equilibrium strategy derived is well suited to be used as a power control scheme when there is a large number of users in the system. Other papers dealing with the application of game theory to the random access and resource allocation problems in wireless network include [16][23][24].

This chapter is organized as follows. In Section 2, we describe the communication system. In Section 3, we start by presenting the Nash equilibrium strategy pair for the two users game when the channel state is uniformly distributed over $[0, 1]$. The uniqueness of the Nash equilibrium strategy is shown under certain capture rule. A symmetric Nash equilibrium is also obtained when users have the same average power constraint. We then explore the Nash equilibrium strategy for general channel state distribution. In section 4, a symmetric Nash equilibrium strategy is derived for the multiple users case. The throughput obtained by using the Nash equilibrium strategy is shown to approach the maximum attainable throughput. Finally, Section 5 concludes the chapter.

3.2 Problem Formulation

We consider a communication environment with multiple users sending data to a single base station or satellite over multiple fading channels. We assume that each user always has data to be sent to the base station. Time is assumed to be discrete, and the channel state for a given user changes according to a known probabilistic model independently over time. The channels between the users and the base station

are assumed to be independent of each other. Let X_i be a random variable denoting the channel state for the channel between user i and the base station.

When multiple users are transmitting during the same time slot, it is still possible for the receiver to capture one (or more) user's data. The capture model can be described as a mapping from the received power of the transmitting users to the set $\{1, \dots, n, 0\}$, where 0 indicates no packet is successfully received. In this chapter, we are going to investigate two capture models which will be presented in the later sections.

We assume that each individual user is energy constrained. Specifically, each user i has an *average* amount of energy e_i available to itself during each time slot. We assume that the e_i values are known to all users, and that users know the distribution of X_i 's. However, the exact value of the channel state X_i is known to user i only at the beginning of each time slot.

With a given capture model and the energy constraint, the objective for each user is to design a power allocation strategy to maximize its *own* expected throughput (or probability of success) per time slot subject to the expected or average power constraint. The power allocation strategy will specify how a user will allocate power in every time slot upon observing its channel state. Under power allocation strategy $g_i(\cdot)$, user i transmits a packet with power equal to $g_i(x)$ when it sees its channel condition in this time slot is $X_i = x$. The received power at the base station is denoted as $f_i(x) = x \cdot g_i(x)$.

Formally, let F_i be the set of continuous and bounded real-valued functions with finite first and second derivative over the support of X_i . Then, the strategy space for user i (the set of all possible power allocation schemes), say S_i , is defined as follows:

$$S_i = \left\{ g_i \in F_i \mid E[g_i(X_i)] \leq e_i \right\} \quad (3.1)$$

3.3 Two Users Case

We start by investigating users' strategies in a communication system consisting of exactly two users and one base station. The analytical method used in this section will help us in obtaining equilibrium power allocation scheme in the multiple users case. We begin our analysis with the assumption that channel state X_i is uniformly distributed over $[0, 1]$ for all i . The Nash equilibrium power allocation strategy with general channel state distribution is presented in the subsequent section.

Suppose user 1 and user 2 choose their power allocation strategies to be g_1 and g_2 respectively. Given a time slot with channel state realization (x_1, x_2) , user 1 and user 2 will transmit their packets using power levels $g_1(x_1)$ and $g_2(x_2)$ respectively. The corresponding received power at the base station are $f_1(x_1) = x_1 \cdot g_1(x_1)$ and $f_2(x_2) = x_2 \cdot g_2(x_2)$. As in [12] and [13], the capture model used in this section is the following: if $[x_1 \cdot g_1(x_1)]/[x_2 \cdot g_2(x_2)] \geq K$ where $K \geq 1$, user 1's packet will be captured. Likewise, user 2's packet will be captured if $[x_2 \cdot g_2(x_2)]/[x_1 \cdot g_1(x_1)] \geq K$. Thus, given a power allocation strategy pair (g_1, g_2) , where $g_1 \in S_1$ and $g_2 \in S_2$, the expected throughput for user 1 is defined as the following:

$$G_1(e_1, e_2) = E_{X_1, X_2}[1_{f_1(X_1) \geq K \cdot f_2(X_2)}] \quad (3.2)$$

where

$$1_{f_1(X_1) \geq K \cdot f_2(X_2)} = \begin{cases} 1 & \text{if } f_1(X_1) \geq K \cdot f_2(X_2) \\ 0 & \text{otherwise} \end{cases}$$

Similarly, the throughput function for user 2:

$$G_2(e_1, e_2) = E_{X_1, X_2}[1_{f_2(X_2) > K \cdot f_1(X_1)}] \quad (3.3)$$

3.3.1 Nash equilibrium strategy

In this part, we present a Nash equilibrium power allocation strategy pair (g_1^*, g_2^*) . A strategy pair (g_1^*, g_2^*) is said to be in Nash equilibrium if g_1^* is the best response

for user 1 to user 2's strategy g_2^* , and g_2^* is the best response for user 2 to user 1's strategy g_1^* . We consider here the case where both users choose their strategies from the strategy space S_1 and S_2 and the value of e_1 and e_2 are known to both users.

To get the Nash equilibrium strategy pair, we first argue that at equilibrium the received power function $f_i^*(x_i)$ must be strictly increasing in x_i .

Lemma 7. *Given a Nash equilibrium power allocation strategy pair (g_1^*, g_2^*) and its corresponding received power function (f_1^*, f_2^*) , the received power function $f_1^*(x_1)$ must be strictly increasing in x_1 . Similarly, $f_2^*(x_2)$ must be strictly increasing in x_2 .*

Proof. For an arbitrary received power function f which is not strictly increasing, we can always find another received power function that will result in a larger throughput gain. To see this, consider time slots with channel state in the small intervals $(a - \delta, a + \delta)$ and $(b - \delta, b + \delta)$ where $a < b$. When δ is small, the received power function is close to $f(a)$ for time slots in the interval $(a - \delta, a + \delta)$. Likewise, the received power function is close to $f(b)$ for time slots in the interval $(b - \delta, b + \delta)$.

For received power function f such that $f(a) = a \cdot g(a) > f(b) = b \cdot g(b)$ for some $a < b$. The total amount of transmission power used in time slots with channel state in the two intervals is given by:

$$[g(a) + g(b)]2\delta = \left[\frac{f(a)}{a} + \frac{f(b)}{b}\right]2\delta.$$

Now, if user 1 employs a new power allocation strategy \bar{g} such that $\bar{g}(b) = \frac{f(a)}{b}$ and $\bar{g}(a) = \frac{f(b)}{a}$, user 1 will achieve the same expected throughput as before. However, the amount of power used $[\bar{g}(b) + \bar{g}(a)]2\delta$ is less than $[g(a) + g(b)]2\delta$, and the extra power can be used to get higher throughput. Hence, both equilibrium received power function $f_1^*(x_1)$ and $f_2^*(x_2)$ must be strictly increasing in x_1 and x_2 respectively. \square

With one user's power allocation strategy, say g_2 , fixed, we now seek the optimal power allocation scheme for user 1. From Lemma 7, we see that the inverse of f_1 and f_2 are well defined. With user 2's strategy g_2 fixed, let $u_{g_2}^{(1)} : (x_1, b) \rightarrow \mathcal{R}$ denote user 1's expected throughput of a slot conditioning on the following events:

- User 1's channel state is $X_1 = x_1$.
- User 1's allocated power is b .

For convenience, we will drop the term g_2 in the expression $u_{g_2}^{(1)}(x_1, b)$, and simply write it as $u_1(x_1, b)$. Specifically, we can write the equation:

$$u_1(x_1, b) = P(f_2(X_2) \cdot K \leq x_1 \cdot b) \quad (3.4)$$

where $P(f_2(X_2) \cdot K \leq x_1 \cdot b)$ is the probability that user 1's packet gets captured in a time slot. Consequently, using a strategy g_1 , user 1's throughput is given by:

$$\begin{aligned} G_1(e_1, e_2) &= \int_0^1 u_1(x_1, g_1(x_1)) \cdot p_{X_1}(x_1) dx_1 \\ &= \int_0^1 u_1(x_1, g_1(x_1)) dx_1. \end{aligned} \quad (3.5)$$

where the last equality results from the uniform distribution assumption.

With user 1's strategy g_1 fixed, similar terms for user 2 can be defined.

$$u_2(x_2, b) = u_{g_1}^{(2)}(x_2, b) = P(f_1(X_1) \cdot K \leq x_2 \cdot b)$$

Then, user 2's throughput is given by:

$$\begin{aligned} G_2(e_1, e_2) &= \int_0^1 u_2(x_2, g_2(x_2)) \cdot p_{X_2}(x_2) dx_2 \\ &= \int_0^1 u_2(x_2, g_2(x_2)) dx_2. \end{aligned} \quad (3.6)$$

Due to the uniformly distributed channel state, $P(f_2(X_2) \cdot K \leq x_1 \cdot b)$ is given by

$$\begin{aligned} P(f_2(X_2) \cdot K \leq x_1 \cdot b) &= P(X_2 \leq f_2^{-1}(\frac{1}{K}x_1 \cdot b)) \\ &= f_2^{-1}(\frac{1}{K}x_1 \cdot b) \end{aligned}$$

where f_2^{-1} is well defined. Thus, we can rewrite Eq. (3.4) as

$$u_1(x_1, b) = f_2^{-1}\left(\frac{1}{K}x_1 \cdot b\right).$$

Hence we have,

$$G_1(e_1, e_2) = \int_0^1 f_2^{-1}\left(\frac{1}{K}x_1 \cdot g_1(x_1)\right) dx_1 \quad (3.7)$$

$$G_2(e_1, e_2) = \int_0^1 f_1^{-1}\left(\frac{1}{K}x_2 \cdot g_2(x_2)\right) dx_2 \quad (3.8)$$

We begin our analysis of the Nash equilibrium strategy pair by first considering the power allocation on the boundary points 0 and 1. For a pair of power allocation functions (g_1^*, g_2^*) to be a Nash equilibrium, it is straightforward to see that $g_1^*(0) = g_2^*(0) = 0$ since it does not make sense to allocate power for a slot with zero channel state. Likewise, we must have $g_1^*(1) \leq K \cdot g_2^*(1)$ and $g_2^*(1) \leq K \cdot g_1^*(1)$ since allocating power $g_1(1) = K g_2(1)$ or $g_1(1) = K g_2(1) + \epsilon$, where $\epsilon > 0$, will result in the same throughput for user 1. We call these properties the boundary conditions of a Nash equilibrium strategy pair.

With the boundary conditions satisfied, the following lemma gives a necessary and sufficient condition for a pair of power allocation strategies to be a Nash equilibrium strategy pair. For convenience, we denote the marginal gain for user 1 when $X_1 = x_1$ and the allocated power $b = b^*$ as

$$\left. \frac{\partial u_1(x_1, b)}{\partial b} \right|_{b=b^*} \triangleq Du_1(x_1, b^*).$$

Lemma 8. *Given a power allocation strategy pair (g_1^*, g_2^*) that satisfies the boundary conditions, (g_1^*, g_2^*) is a Nash equilibrium strategy pair if and only if $Du_1(x_1, g_1^*(x_1)) = c_1$ and $Du_2(x_2, g_2^*(x_2)) = c_2$, for some constants c_1 and c_2 , for all $x_1 \in [0, 1]$ and all $x_2 \in [0, 1]$.*

Note that the above lemma does not depend on the assumption of the uniformly distributed channel state. Thus, it is quite general and will be used in the subsequent

section where channel states are not uniformly distributed. To understand the lemma intuitively, suppose there exists $x \neq \tilde{x}$ such that $Du_1(x, g_1^*(x)) > Du_1(\tilde{x}, g_1^*(\tilde{x}))$. Reducing the power allocated at \tilde{x} to $g_1^*(\tilde{x}) - \delta$ and increasing the power at x to $g_1^*(x) + \delta$ will result in an increase in the throughput by $(Du_1(x, g_1^*(x)) - Du_1(\tilde{x}, g_1^*(\tilde{x}))) \cdot \delta$. Thus, user 1 has an incentive to change its allocation function, and (g_1^*, g_2^*) cannot be a Nash equilibrium strategy pair in this case.

Proof. The complete proof is given in the Appendix. □

With Lemma 8, we are able to find the Nash equilibrium strategy pair. The exact form of the equilibrium power allocation strategies are presented in the following Theorem.

Theorem 9. *Given the average power constraint e_1 and e_2 , the Nash equilibrium power allocation strategy pair has the following form:*

$$g_1^*(x) = c_1 \cdot x^\gamma \tag{3.9}$$

$$g_2^*(x) = c_2 \cdot x^{\frac{1}{\gamma}} \tag{3.10}$$

where the constants c_1 , c_2 and γ are chosen such that

$$\int_0^1 c_1 \cdot x^\gamma dx = e_1 \tag{3.11}$$

$$\int_0^1 c_2 \cdot x^{\frac{1}{\gamma}} dx = e_2 \tag{3.12}$$

Equations (3.11) and (3.12) impose the average power constraints.

Proof. We show here that $g_1^*(x) = c_1 \cdot x^\gamma$ and $g_2^*(x) = c_2 \cdot x^{\frac{1}{\gamma}}$ is indeed a Nash equilibrium strategy pair by using the sufficiency condition of Lemma 8. Since both functions are strictly increasing, we can write $u_1(x, b) = f_2^{*-1}(\frac{1}{K}x \cdot b)$ and $u_2(x, b) = f_1^{*-1}(\frac{1}{K}x \cdot b)$ where $f_i^*(x) = x \cdot g_i^*(x)$. Also, since both f_1^* and f_2^* are differentiable,

we have $u_1(x, b)$ and $u_2(x, b)$ both differentiable with respect to b . Therefore, using

$$\begin{aligned} f_2^{*'}(x) &= c_2 \left(1 + \frac{1}{\gamma}\right) x^{\frac{1}{\gamma}}, & f_1^{*'}(x) &= c_1 (1 + \gamma) x^\gamma \\ f_2^{*-1}(x) &= \left(\frac{1}{c_2} x\right)^{\frac{\gamma}{1+\gamma}}, & f_1^{*-1}(x) &= \left(\frac{1}{c_1} x\right)^{\frac{1}{1+\gamma}} \end{aligned}$$

we have

$$\begin{aligned} \frac{\partial u_1(x, b)}{\partial b} \Big|_{b=g_1^*(x)} &= \frac{\frac{1}{K} x}{f_2^{*'}(f_2^{*-1}(\frac{1}{K} x g_1^*(x)))} \\ &= \frac{\frac{1}{K} x}{f_2^{*'}([\frac{c_1}{K c_2}]^{\frac{\gamma}{1+\gamma}} x^\gamma)} = \frac{\frac{1}{K}}{c_2 (1 + \frac{1}{\gamma}) (\frac{c_1}{K c_2})^{\frac{1}{1+\gamma}}}. \end{aligned}$$

Similarly,

$$\begin{aligned} \frac{\partial u_2(x, b)}{\partial b} \Big|_{b=g_2^*(x)} &= \frac{\frac{1}{K} x}{f_1^{*'}(f_1^{*-1}(\frac{1}{K} x g_2^*(x)))} \\ &= \frac{\frac{1}{K} x}{f_1^{*'}([\frac{c_2}{K c_1}]^{\frac{1}{1+\gamma}} x^{1/\gamma})} = \frac{\frac{1}{K}}{c_1 (1 + \gamma) (\frac{c_2}{K c_1})^{\frac{\gamma}{1+\gamma}}}. \end{aligned}$$

From Lemma 8, we see that (f_1^*, f_2^*) is indeed a Nash equilibrium strategy pair because both $Du_1(x, g_1^*(x))$ and $Du_2(x, g_2^*(x))$ are constants. \square

From the above theorem, we see that equations (3.9) and (3.10) specify the Nash equilibrium power allocation strategy pair. Since there are two equations with three unknowns, the resulting Nash equilibrium may not be unique in general. However, if a packet with stronger received power can always be captured (i.e., $K = 1$), the Nash equilibrium power allocation strategy is unique.

Corollary 1. *For $K = 1$, the unique Nash equilibrium power allocation pair has the following form:*

$$g_1^*(x) = c \cdot x^\gamma \tag{3.13}$$

$$g_2^*(x) = c \cdot x^{\frac{1}{\gamma}} \tag{3.14}$$

where the constants c and γ are chosen such that the average power constraints are

satisfied.

To show the corollary, we first present the following Lemma.

Lemma 9. *If (g_1^*, g_2^*) is a Nash equilibrium strategy pair, $g_1^*(1) = g_2^*(1)$.*

Proof. Suppose $g_1^*(1) \neq g_2^*(1)$. Without loss of generality, let assume that $g_1^*(1) > g_2^*(1)$. Since both g_1^* and g_2^* are continuous, there exists $\delta > 0$ such that $g_1^*(x) > g_2^*(1) + \frac{g_1^*(1) - g_2^*(1)}{2} \quad \forall x \in [1 - \delta, 1]$. User 1 can devise a new allocation strategy, say \bar{g}_1 , by moving a small amount of power, say $\delta \cdot \frac{g_1^*(1) - g_2^*(1)}{2}$, away from the interval $[1 - \delta, 1]$ to some other interval, thus resulting in an increase in user 1's throughput. Therefore, when $g_1^*(1) > g_2^*(1)$, the power allocation strategy pair (g_1^*, g_2^*) cannot be in equilibrium since the strategy pair (\bar{g}_1, g_2^*) gives a higher throughput for user 1. Similar result holds for the case $g_2^*(1) > g_1^*(1)$. Thus, we must have $g_1^*(1) = g_2^*(1)$ if (g_1^*, g_2^*) is an equilibrium strategy pair. \square

The condition that $g_1^*(1) = g_2^*(1)$ will be useful in proving the uniqueness of the Nash equilibrium. The complete proof of the corollary is shown in the appendix.

Fig. ?? shows an example of the Nash equilibrium power allocation strategy pair when $e_1 = 1$ and $e_2 = 2$. Since user 1 has less average power than user 2, user 1 concentrates its power on time slots with very good channel state. Fig. 3-2 shows the capture result when both users employ the Nash equilibrium strategy shown in Fig. ?. For a time slot with channel state realization that fall into the region above the curve, user 2's packet will be successfully captured since user 2's received power is higher than that of user 1 in this region. Here, user 2 has more successful transmissions than user 1 since it has more power.

3.3.2 General Channel State Distribution

In this section, we specify the conditions that a general channel state distribution has to satisfy in order for a Nash equilibrium strategy pair to exist.

From Lemma 7, one can see that f_1 and f_2 have to be increasing functions regardless of the distribution of the X_i 's. Let $p_{X_i}(\cdot)$ denote the probability density

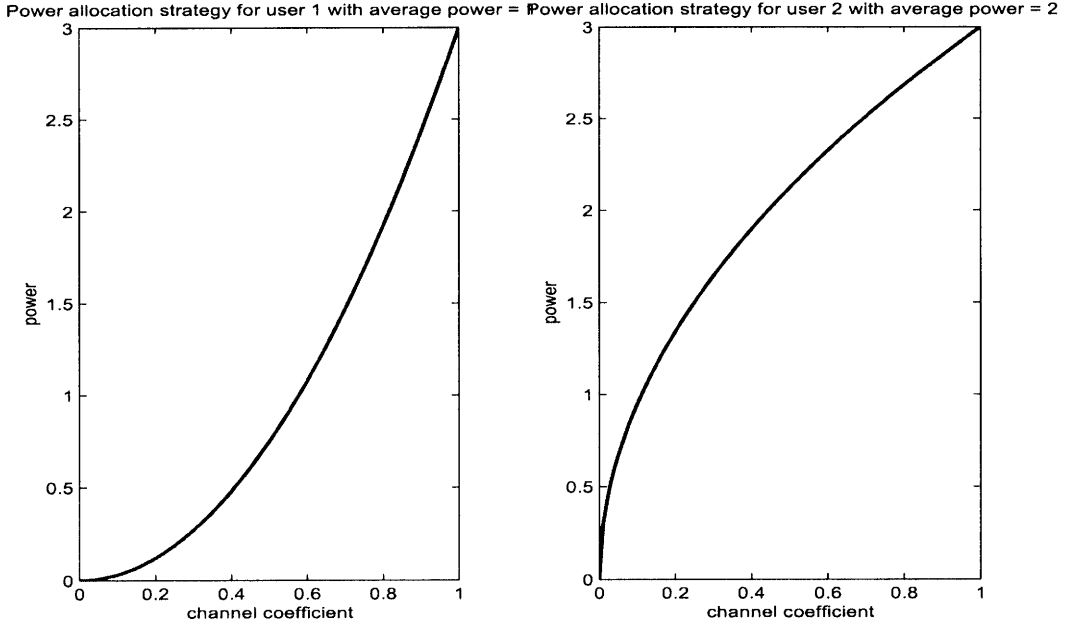


Figure 3-1: An example of Nash equilibrium strategy pair for $e_1 = 1$ and $e_2 = 2$.

function of X_i with the support over an interval starting at zero. Assuming $K = 1$, the probability that user 1's packet will be captured in a time slot with $X_1 = x_1$ and $g_1(x_1) = b$ can be written as the following:

$$\begin{aligned}
 u_1(x_1, b) &= P(f_2(X_2) \leq x_1 \cdot b) \\
 &= P(X_2 \leq f_2^{-1}(x_1 \cdot b)) \\
 &= \int_0^{f_2^{-1}(x_1 \cdot b)} p_{X_2}(x_2) dx_2
 \end{aligned} \tag{3.15}$$

From the optimality condition stated in Lemma 8, we must have $Du_1(x_1, b) = c_1$ where c_1 is some constant. This condition can be expanded as follows:

$$\frac{\partial u_1(x_1, b)}{\partial b} = p_{X_2}(f_2^{-1}(x_1 \cdot b)) \frac{x_1}{f_2'(f_2^{-1}(x_1 \cdot b))} = c_1 \tag{3.16}$$

Now, let's focus on finding a symmetric Nash equilibrium power allocation strategy. Substituting $b = g_1(x_1)$, the term $f_2^{-1}(x_1 \cdot b)$ is equal to $f_2^{-1}(f_1(x_1)) = x_1$ since $f_1 = f_2$.

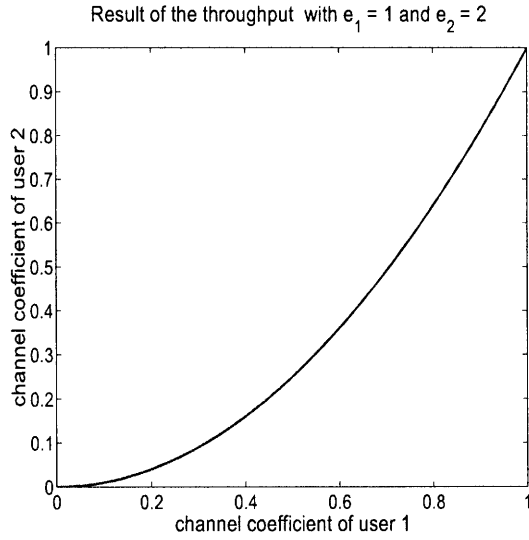


Figure 3-2: Results obtained when using the Nash equilibrium strategy pair for $e_1 = 1$ and $e_2 = 2$.

Thus, Eq.(3.16) can be reduced to the following:

$$\begin{aligned}
 p_{X_2}(x_1) \frac{x_1}{f_2'(x_1)} &= c_1 \\
 \Rightarrow f_2'(x_1) &= \frac{1}{c_1} x_1 \cdot p_{X_2}(x_1)
 \end{aligned} \tag{3.17}$$

The above equation provides a condition on the distribution of the X_i such that there exists a Nash equilibrium power allocation scheme. The condition can be restated as the following:

$$x_1 \cdot g_1(x_1) = \int \frac{1}{c_1} x_1 \cdot p_{X_2}(x_1) dx_1 \tag{3.18}$$

From the above condition, for example, we see that if $p_{X_2}(\cdot)$ is a strictly increasing polynomial, there exist a Nash equilibrium power allocation strategy.

3.4 Multiple Users Equilibrium Strategies

In this section, we explore the Nash equilibrium power allocation strategies when n users are competing to access the single base station. User i 's power alloca-

tion function is denoted as $g_i(\cdot)$. Given a time slot with channel state realization $\vec{x} = (x_1, \dots, x_n)$, the transmitting power for each user is $g_i(x_i)$. The corresponding received power at the base station is again denoted as $f_i(x_i) = x_i \cdot g_i(x_i)$. The new capture rule used in this section is given as the following: a packet from user 1 will be successfully received if the following holds:

$$f_1(x_1) \geq (1 + \Delta) \max(f_2(x_2), \dots, f_n(x_n))$$

Similar capture model can be found in [26] (i.e., protocol model). The quantity Δ models situations where a guard zone is specified to prevent interference. Note also that the capture rule used in the two users' case can be viewed as a special case the above capture rule.

We start with each user facing the same average power constraint (i.e., $e_1 = e_2 = \dots = e_n$). Since users are identical, it is reasonable to seek a symmetric Nash equilibrium power allocation strategy. Specifically, the set of strategies ($g_1 = g, \dots, g_n = g$) is said to be a symmetric Nash equilibrium strategies if $g_i = g$ is the best power allocation strategy for user i when all other users are also employing the power allocation strategy g . For a power allocation function g to be a symmetric Nash equilibrium strategy, $f(x) = xg(x)$ must be a strictly increasing function using a similar argument as in the two users case. The following theorem shows the existence and the form of a symmetric Nash equilibrium power allocation strategy.

Theorem 10. *Given that each user has the same average power constraint, there exists a symmetric Nash equilibrium power allocation strategy with the following form:*

$$g_i(x_i) = c \cdot x_i^{n-1} \quad \forall i \in \{1, \dots, n\} \quad (3.19)$$

where c is chosen such that the average power constraint is satisfied.

Proof. The complete proof is given in the Appendix. □

With the symmetric Nash equilibrium power allocation strategy given in Eq.(3.19),

the expected throughput for each user is given by:

$$\begin{aligned}
& P(f(X_1) \geq (1 + \Delta) \max(f(X_2), \dots, f(X_n))) \\
&= P(X_1^n \geq (1 + \Delta) \max(X_2^n, \dots, X_n^n)) \\
&= P(X_1 \geq (1 + \Delta)^{\frac{1}{n}} \max(X_2, \dots, X_n))
\end{aligned} \tag{3.20}$$

To quantify the loss of efficiency due to users' selfish behavior, we consider a system where all users implement the same power allocation policy provided by a system designer such that the overall system throughput is maximized. To find such scheme, we solve the following optimization problem as in the two users' case:

$$\max_{v \in S_1} P(X_1 v(X_1) \geq (1 + \Delta) \cdot \max(X_2 v(X_2), \dots, X_n v(X_n)))$$

By symmetry, we have the following upper bound for the above probability:

$$P(X_1 v(X_1) \geq (1 + \Delta) \cdot \max(X_2 v(X_2), \dots, X_n v(X_n))) < \frac{1}{n}$$

As in the two users' case, we consider a series of functions, $v_m(x) = x^m$ for $m \geq 1$. As $m \rightarrow \infty$, we have

$$\begin{aligned}
& P(X_1^{m+1} \geq (1 + \Delta) \cdot \max(X_2^{m+1}, \dots, X_n^{m+1})) \\
&= P(X_1 \geq (1 + \Delta)^{\frac{1}{m+1}} \max(X_2, \dots, X_n)) \rightarrow \frac{1}{n}
\end{aligned}$$

Thus, there indeed exists a power allocation scheme that will achieve the maximum possible throughput. In other words, it is possible to have a packet successfully captured in every time slot. Now, when users behave selfishly, the expected throughput for each user is given as follows from Eq.(3.20):

$$P(X_1 \geq (1 + \Delta)^{\frac{1}{n}} \max(X_2, \dots, X_n)) \tag{3.21}$$

As n increases, the above equation goes to $1/n$ which is the maximum attainable throughput. Therefore, as the number of users becomes large, the symmetric Nash

equilibrium power allocation scheme is optimal in the sense that the throughput obtained approaches the maximum attainable throughput.

For the special case where $\Delta = 0$, the capture rule becomes that the user with the largest received power get captured. With this simple rule, a Nash equilibrium strategy can be derived with general channel state distribution (i.e., X_i has probability density function $p_{X_i}(\cdot)$). From Eq.(3.37), we have

$$\begin{aligned} p_Z(f^{-1}(x_1 \cdot b)) \frac{x_1}{f'(f^{-1}(x_1 \cdot b))} &= c \\ f'(x_1) &= \frac{1}{c} x_1 p_Z(x_1) \end{aligned} \tag{3.22}$$

where

$$p_Z(z) = (n-1)p_{X_1}(z) \left[\int_0^z p_{X_1}(x) dx \right]^{n-2}.$$

Hence, we can write the received power function as the following:

$$f(x) = \frac{1}{c} \int x p_Z(x) dx$$

From the above equation, one can get the optimal power allocation function by using $g(x) = \frac{f(x)}{x}$.

3.5 Conclusion

In this chapter, we consider a communication system with multiple users competing, in a non-cooperative manner, for the access of a single satellite, or base station. With a specified capture rule and an average power constraint, users opportunistically adjust their transmission power based on their channel state to maximize their throughput. A Nash equilibrium power allocation strategy is characterized, and the resulting throughput efficiency loss, due to selfish behavior, is quantified. As the number of users increases, the Nash equilibrium power allocation strategy approaches the optimal power allocation strategy that can be achieved in a cooperative environment.

Chapter Appendix: Proof of Theorem 8

Proof: We first show that if (g_1^*, g_2^*) is a Nash equilibrium strategy pair then $Du_1(x_1, g_1^*(x_1))$ and $Du_2(x_2, g_2^*(x_2))$ must be constants for all $x_1 \in [0, 1]$ and $x_2 \in [0, 1]$. From user 1's perspective with g_2^* fixed, consider a small variation of the function g_1^* . Specifically, let $g_\delta = g_1^* + \delta(\hat{g} - g_1^*)$ where \hat{g} is an arbitrary function in S_1 . Since both \hat{g} and g_1^* are in S_1 , they are both bounded (i.e., $|\hat{g}(x_1)| \leq B$ and $|g_1^*(x_1)| \leq B$ for all $x_1 \in [0, 1]$). Therefore, we have $|g_\delta(x_1) - g_1^*(x_1)| \leq 2B\delta$ for all $x_1 \in [0, 1]$. Using the Lagrange's form of Taylor's theorem, we get for any $x_1 \in [0, 1]$, there exists a real number $c_{[x_1]} \in [g_1^*(x_1), g_\delta(x_1)]$ such that

$$\begin{aligned} u_1(x_1, g_\delta(x_1)) &= u_1(x_1, g_1^*(x_1)) \\ &+ \delta(\hat{g}(x_1) - g_1^*(x_1)) \frac{\partial u_1(x_1, b)}{\partial b} \Big|_{b=g_1^*(x_1)} \\ &+ \frac{1}{2} \delta^2 (\hat{g}(x_1) - g_1^*(x_1))^2 \frac{\partial^2 u_1(x_1, b)}{\partial b^2} \Big|_{b=c_{[x_1]}} \end{aligned} \quad (3.23)$$

The last term is bounded by $K \cdot \delta^2$ for some K since both \hat{g} and g_1^* are bounded, and $u_1(x_1, b)$ has finite second derivative. Therefore, for small enough δ , it is negligible comparing with the other terms.

Now we show that if $Du_1(x_1, g_1^*(x_1))$ is not a constant for all $x_1 \in [0, 1]$, we can find a strategy g_δ which gives user 1 a higher throughput than g_1^* . To do that, we can write the following equations:

$$\begin{aligned} &\int_0^1 u_1(x_1, f_\delta(x_1)) dx_1 - \int_0^1 u_1(x_1, f_1^*(x_1)) dx_1 \\ &= \delta \int_0^1 (\hat{g}(x_1) - g_1^*(x_1)) \frac{\partial u_1(x_1, b)}{\partial b} \Big|_{b=g_1^*(x_1)} dx_1 + o(\delta) \end{aligned} \quad (3.24)$$

Now, since $Du_1(x_1, g_1^*(x_1))$ is not a constant for all $x_1 \in [0, 1]$, we can find a \hat{g} such that the above equation is positive which implies that there is an incentive for user 1 to use g_δ . Hence, (g_1^*, g_2^*) is not a Nash equilibrium strategy pair. Similarly, we can show that $Du_2(x_2, g_2^*(x_2))$ is a constant for all $x_2 \in [0, 1]$ if (g_1^*, g_2^*) is a Nash equilibrium strategy pair.

For the converse, consider again Eq.(3.24). Since $Du_1(x_1, g_1^*(x_1)) = \frac{\partial u_1(x_1, b)}{\partial b} \Big|_{b=g_1^*(x_1)}$ equals to a constant c_1 for all $x_1 \in [0, 1]$. We have

$$\begin{aligned} & \delta \int_0^1 (\hat{g}(x_1) - g_1^*(x_1)) \frac{\partial u_1(x_1, b)}{\partial b} \Big|_{b=g_1^*(x_1)} dx_1 \\ & = \delta c_1 \int_0^1 (\hat{g}(x_1) - g_1^*(x_1)) dx_1 = 0 \end{aligned} \tag{3.25}$$

for all $\hat{g} \in S_1$ (i.e., $\int_0^1 \hat{g}(x_1) dx_1 = e_1$). Thus, there is no incentive for user 1 to use strategy \hat{g} . Therefore, (g_1^*, g_2^*) is a Nash equilibrium strategy pair.

Chapter Appendix: Proof of Corollary

We have established that $g_1^*(1) = g_2^*(1)$ is a necessary condition for (g_1^*, g_2^*) to be an equilibrium strategy pair from lemma 9. Combining with $g_1^*(0) = g_2^*(0) = 0$, we will consider only the function pair $g_1 \in S_1$ and $g_2 \in S_2$ that satisfy the above two boundary conditions (i.e., $g_1(1) = g_2(1)$ and $g_1(0) = g_2(0) = 0$).

Consider any Nash equilibrium strategy pair (g_1, g_2) under the capture rule described in the two users' case. From previous discussion, we know that the inverse functions, f_2^{-1} and f_1^{-1} where $f_1 = xg_1(x)$ and $f_2 = xg_2(x)$, are well defined. With user 2's strategy g_2 fixed, we have

$$u_1(x_1, b) = P(f_2(X_2) \leq x_1 \cdot b) = f_2^{-1}(x_1 \cdot b)$$

Similarly, with user1's strategy f_1 fixed, we get

$$u_2(x_2, b) = P(f_1(X_1) \leq x_2 \cdot b) = f_1^{-1}(x_2 \cdot b)$$

From Lemma 8, we know that $Du_1(x_1, g_1(x_1))$ and $Du_2(x_2, g_2(x_2))$ are two constants for all $x_1 \in [0, 1]$ and $x_2 \in [0, 1]$ since (g_1, g_2) is a Nash equilibrium strategy pair.

Now, consider the set of channel state pair (x_1, x_2) such that $f_1(x_1) = f_2(x_2)$ (i.e., two users' received power are equal). It forms a separation line in space span by X_1 and X_2 . Mathematically, this line can be defined as $h : [0, 1] \rightarrow [0, 1]$ such that $x_2 = h(x_1) = f_2^{-1}(f_1(x_1))$. By the capture rule, a slot with channel state (x_1, x_2') will be successfully used by user 2 if (x_1, x_2') is above the line $x_2 = h(x_1)$ and by user 1 if (x_1, x_2') is below the separation line. Fig.3-2 shows an example of $h(x_1)$. The following lemma shows the uniqueness of $h(x_1)$. We then derive the uniqueness of the strategy pair (g_1, g_2) from the lemma.

Lemma 10. *If $Du_1(x_1, g_1(x_1))$ and $Du_2(x_2, g_2(x_2))$ are two constants, c_1 and c_2 respectively, for all $x_1 \in [0, 1]$ and $x_2 \in [0, 1]$, then $h(x_1) = x_1^{c_1/c_2}$.*

Proof. Since $Du_1(x_1, g_1(x_1)) = c_1$, from $u_1(x_1, b) = f_2^{-1}(x_1, b)$, we have

$$\begin{aligned}
Du_1(x_1, g_1(x_1)) &= \left. \frac{\partial u_1(x_1, b)}{\partial b} \right|_{b=g_1(x_1)} \\
&= \frac{x_1}{f_2'(f_2^{-1}(x_1 \cdot g_1(x_1)))} \\
&= \frac{x_1}{f_2'(f_2^{-1}(f_1(x_1)))} = c_1 \\
f_2'(h(x_1)) &= \frac{x_1}{c_1}
\end{aligned} \tag{3.26}$$

Similarly, for user 2, we get

$$\begin{aligned}
Du_2(x_2, f_2(x_2)) &= \left. \frac{\partial u_2(x_2, b)}{\partial b} \right|_{b=f_2(x_2)} \\
&= \frac{x_2}{f_1'(f_1^{-1}(f_2(x_2)))} = c_2 \\
f_1'(h^{-1}(x_2)) &= \frac{x_2}{c_2}
\end{aligned} \tag{3.27}$$

We also know that $f_1(x_1) = f_2(h(x_1))$ and $f_1'(x_1) = f_2'(h(x_1)) \cdot h'(x_1)$. Thus, we have

$$\begin{aligned}
f_1'(h^{-1}(x_2)) &= f_2'(h(h^{-1}(x_2))) \cdot h'(h^{-1}(x_2)) \\
&= f_2'(x_2) \cdot h'(x_1) \\
&= f_2'(h(x_1)) \cdot h'(x_1)
\end{aligned} \tag{3.28}$$

By combining the equations $f_1'(h^{-1}(x_2)) = \frac{x_2}{c_2}$ and $f_1'(h^{-1}(x_2)) = f_2'(h(x_1)) \cdot h'(x_1)$, we get

$$\frac{x_2}{c_2} = f_2'(h(x_1)) \cdot h'(x_1).$$

Next we substitute Eq.(3.26) and $x_2 = h(x_1)$ in the above equation to obtain,

$$\begin{aligned}
x_1 \cdot \frac{dh(x_1)}{dx_1} &= \frac{c_1}{c_2} h(x_1) \\
\frac{dh(x_1)}{h(x_1)} &= \frac{c_1}{c_2} \frac{dx_1}{x_1} \\
\ln |h(x_1)| &= \frac{c_1}{c_2} \ln |x_1| + c_3
\end{aligned}$$

$$h(x_1) = e^{c_3} \cdot x_1^{\frac{c_1}{c_2}}$$

Combined with fact that $h(1) = 1$, we get $h(x_1) = x_1^{\frac{c_1}{c_2}}$. □

Now, we are in a position to derive the exact form of the Nash equilibrium strategy pair. From the equations $f_1'(h^{-1}(x_2)) = \frac{x_2}{c_2}$ and $x_2 = h(x_1)$, we get $f_1'(x_1) = \frac{h(x_1)}{c_2} = x_1^{\frac{c_1}{c_2}}/c_2$. Combined with the condition that $f_1(0) = 0$, we have $f_1(x) = \frac{1}{c_1+c_2}x^{\frac{c_1}{c_2}+1}$. Following the similar method, we get $f_2(x) = \frac{1}{c_1+c_2}x^{\frac{c_2}{c_1}+1}$. Let $\gamma = \frac{c_1}{c_2}$ and $c = \frac{1}{c_1+c_2}$, the received power of a Nash equilibrium strategy pair must have the following form:

$$f_1^*(x_1) = c \cdot x_1^{\gamma+1} \tag{3.29}$$

$$f_2^*(x_2) = c \cdot x_1^{\frac{1}{\gamma}+1} \tag{3.30}$$

Consequently, we have the Nash equilibrium power allocation strategy to be the form:

$$g_1^*(x_1) = c \cdot x_1^\gamma \tag{3.31}$$

$$g_2^*(x_2) = c \cdot x_1^{\frac{1}{\gamma}} \tag{3.32}$$

The constant γ and c are chosen such that equations (3.11) and (3.12) are satisfied. The uniqueness of the above Nash equilibrium strategy comes from the fact that there is a unique pair of c and γ that satisfy equations (3.11) and (3.12).

Chapter Appendix: Proof of Theorem 10

Proof. With all users $i \neq 1$ using a fixed power allocation strategy g , we now explore the optimal power allocation strategy for user 1 which is denoted by g_1^* . Let $u_g^{(1)} : (x_1, b) \rightarrow \mathcal{R}$ denote user 1's expected throughput during a slot conditioning on the following events:

- User 1's channel state is $X_1 = x_1$.
- User 1's allocated power is b .

As before, we will drop the term g in the expression $u_g^{(1)}(x_1, b)$, and simply write it as $u_1(x_1, b)$. Specifically, we can write the equation:

$$\begin{aligned} u_1(x_1, b) &= P((1 + \Delta) \max(f_2(X_2), \dots, f_n(X_n)) \leq x_1 \cdot b) \\ &= P((1 + \Delta)Y \leq x_1 \cdot b) \end{aligned}$$

where $Y = \max(f_2(X_2), \dots, f_n(X_n))$. Since all users $i \neq 1$ use the same strategy g , we have $Y = \max(f(X_2), \dots, f(X_n))$ where $f(X_i) = X_i \cdot g(X_i)$ for all $i \neq 1$. Moreover, since f is strictly increasing, we can write:

$$Y = \max(f(X_2), \dots, f(X_n)) = f(\max(X_2, \dots, X_n))$$

Denoting $Z = \max(X_2, \dots, X_n)$, we have the following:

$$\begin{aligned} u_1(x_1, b) &= P((1 + \Delta)Y \leq x_1 \cdot b) \\ &= P(Z \leq f^{-1}\left(\frac{1}{1 + \Delta} x_1 \cdot b\right)) \\ &= \int_0^{f^{-1}\left(\frac{1}{1 + \Delta} x_1 \cdot b\right)} p_Z(z) dz \end{aligned} \tag{3.33}$$

where $p_Z(\cdot)$ denote the probability density function of the random variable Z . The optimization problem that user 1 faces can be written as the following:

$$\begin{aligned} \max G_1(e) &= \int_0^1 u_1(x_1, g_1(x_1)) \cdot p_{X_1}(x_1) dx_1 \\ &= \int_0^1 u_1(x_1, g_1(x_1)) dx_1 \\ \text{subj. } &\int_0^1 g_1(x_1) dx_1 \leq e \end{aligned} \quad (3.34)$$

Writing the Lagrangian function, we have

$$\begin{aligned} &\int_0^1 u_1(x_1, g_1(x_1)) dx_1 - \lambda \left(\int_0^1 g_1(x_1) dx_1 - e \right) \\ &= \int_0^1 [u_1(x_1, g_1(x_1)) - \lambda g_1(x_1)] dx_1 + \lambda e \end{aligned} \quad (3.35)$$

Therefore, for each fixed x_1 , we want to choose a $g_1(x_1)$ to maximize the term $u_1(x_1, g_1(x_1)) - \lambda g_1(x_1)$. For convenience, let $b = g_1(x_1)$. Then, we have

$$\begin{aligned} \max_b L(b) &= u_1(x_1, b) - \lambda b \\ &= \max_b \int_0^{f^{-1}(\frac{1}{1+\Delta}x_1 \cdot b)} p_Z(z) dz - \lambda b \end{aligned} \quad (3.36)$$

Maximizing $L(b)$ with respect to b yields the first order condition:

$$\frac{\partial L(b)}{\partial b} = p_Z\left(f^{-1}\left(\frac{1}{1+\Delta}x_1 \cdot b\right)\right) \frac{\frac{x_1}{1+\Delta}}{f'\left(f^{-1}\left(\frac{1}{1+\Delta}x_1 \cdot b\right)\right)} - \lambda = 0 \quad (3.37)$$

Since $Z = \max(X_2, \dots, X_n)$ and X_i 's are i.i.d, we have

$$p_Z(z) = (n-1)z^{n-2}.$$

Now, consider $b = g_1(x_1) = cx_1^m$. Since we are seeking a symmetric Nash equilibrium power allocation strategy, user $i \neq 1$ will adopt the same strategy as user 1. Thus, we have $f(x) = x \cdot g(x) = x \cdot cx^m = cx^{m+1}$. The second term in Eq.(3.37) can be

written as the following:

$$\begin{aligned}
& f'(f^{-1}(\frac{1}{1+\Delta}x_1 \cdot b)) \\
&= f'(f^{-1}(\frac{c}{1+\Delta}x_1 \cdot x_1^m)) \\
&= f'((\frac{1}{1+\Delta}x_1 \cdot x_1^m)^{\frac{1}{m+1}}) \\
&= c(m+1)(\frac{1}{1+\Delta})^{\frac{m}{m+1}}x_1^m
\end{aligned} \tag{3.38}$$

Similarly,

$$\begin{aligned}
& p_Z(f^{-1}(\frac{1}{1+\Delta}x_1 \cdot b)) \\
&= p_Z((\frac{1}{1+\Delta})^{\frac{1}{m+1}}x_1) \\
&= (n-1)(\frac{1}{1+\Delta})^{\frac{n-2}{m+1}}x_1^{n-2}
\end{aligned} \tag{3.39}$$

Eq.(3.37) can be re-written in the following form:

$$(n-1)(\frac{1}{1+\Delta})^{\frac{n-2}{m+1}}x_1^{n-2} \frac{\frac{x_1}{1+\Delta}}{c(m+1)(\frac{1}{1+\Delta})^{\frac{m}{m+1}}x_1^m} - \lambda = 0 \tag{3.40}$$

Since the above equality has to hold for all $x_1 \in [0, 1]$, the following must be true

$$x_1^{n-2} \cdot x_1 \cdot x_1^{-m} = 1$$

Thus, we have $m = n - 1$ and $g_i(x) = cx^{n-1}$ for all $i = 1, \dots, n$. □

Chapter 4

Channel Allocation Using Pricing in Satellite Networks

Having studied the single channel allocation problem in the previous two chapters, we now explore the case that there are multiple channels between a source and destination node. Future satellite communication networks are envisioned to provide diverse quality of service based on user's demand. Hence, it is vital to have a Medium Access Control (MAC) protocol that provides fair and efficient channel access for each user. In this chapter, we propose a novel MAC protocol based on pricing that allocates network resources efficiently in response to users' demand.

4.1 Introduction

We consider here a communication network with multiple satellites, collectively acting as a network manager, who wish to allocate network uplink capacity efficiently among a set of users, each endowed with a utility function depending on their data rate. We assume that each satellite uses a separate channel for communication, such as using different frequency band for receiving. Each user has data that needs to be sent to the satellite network, and there may be multiple satellites that a user on the ground can communicate with, or switched diversity termed in [1]. Therefore, the data rate for each user here is the rate at which each user can access the satellite network by

sending its data to any satellite within its view.

Slotted aloha is used here as the multi-access scheme for its simplicity. Other multi-access schemes can be used in conjunction with our pricing scheme to provide QoS as well. Due to different path loss and fading, the channel gain from a user to different satellites within its view can be different. Therefore, during a single time slot, a user has to decide not only whether it should transmit but also to which satellite it will transmit. To control users' transmission rates, each satellite will set a price that may differ from satellite to satellite for each *successfully received* packet. Based on the price set by each satellite, a user determines its target satellite and the transmission probability to maximize its net payoff, which is the utility of its received rate minus the cost.

It is well-known that the throughput of a slotted aloha system is low. Therefore, to efficiently utilize the available resource is a reasonable objective for the network manager. In this chapter, we want to explore the use of pricing as a control mechanism to achieve efficiency. To do so, we need to define the meaning of efficiency in the context of a slotted aloha system. With a wire-line, such as optical fiber, of capacity R , an allocation is efficient as long as the sum of the bandwidth allocated to each individual user is equal to R , i.e., no waste of bandwidth. With a collision channel in the aloha system, no simple extension of the wire-line case exists. We therefore use a concept called *Pareto efficiency* for allocating resource in a collision channel. By definition, a feasible allocation (s_1, s_2, \dots, s_n) is Pareto efficient if there is no other feasible allocation $(s'_1, s'_2, \dots, s'_n)$ such that $s'_i \geq s_i$ for all $i = 1, \dots, n$ and $s'_i > s_i$ for some i .

The multiple satellites communication networks considered here differ from the multichannel aloha networks in only one aspect—the channel quality associated with the path from user to the satellite is different in the multiple satellites case. This difference gives us insight on how to best utilize the multiple channels available to users. A multichannel aloha network consists of M parallel, equal capacity channels for transmission to one receiver shared by a set of users. The M channels can be implemented based on either Frequency Division Multiplexing or Time Division Mul-

time-sharing approaches. When a user has a packet to send, it will *randomly* select one channel to transmit. This random selection of the channel is largely due to the lack of coordination among competing users. Intuitively, we would expect that the throughput of the system will be higher if the coordination in channel selection among users was available. As we show in this chapter, in multiple satellite networks, different prices and channel states are two mechanisms that enable the coordination in channel selection among the competing users.

The multi-channel slotted aloha problem has been studied by numerous researchers. In [32], the authors develop a distributed approach for power allocation and scheduling in a wireless network where users communicate over a set of parallel multi-access fading channels, as in an OFDM or multi-carrier system. In [33], the authors show how to improve the classic multichannel slotted aloha protocols by judiciously using redundant transmissions. The use of pricing strategy to control the behavior of users who are sharing a single channel using aloha medium access protocol was investigated in [34]. A game theoretical model for users competing for the limited resources is provided. Multiple channels and the associated channel states for the users are not considered in their work.

The organization of this chapter is as follows. In section 2, we characterize the Pareto efficient throughput region in a single satellite network. The existence of an equilibrium price is presented. Furthermore, we show that such equilibrium price is unique. In section 3, we describe the Pareto efficient throughput region in a multi-satellites environment given that coordination of satellite selection was allowed among users. We then show that the equilibrium price exists and is unique. The resulting throughput at the equilibrium is shown to be Pareto efficient also. An example with multiple satellites, where the competitive equilibrium is explicitly calculated, is given in Section 4. Section 5 concludes the chapter.

4.2 A Single Satellite Network

We consider an uplink communication scenario in a single satellite network with n users. Each user has unlimited number of packets in its buffer need to be transmitted. As in standard slotted ALOHA model, if two or more packets are transmitted during the same time slot, we assume no packets will be received at the satellite. Now, let z_i denote the transmission probability of user i . The probability that user i 's packet is successfully received by the satellite is then denoted as $s_i = z_i \prod_{j \neq i} (1 - z_j)$. We further denote the constant channel state coefficient from user i to the satellite as c_i . Assume all users transmitting at a constant power P . Given user i 's transmission during a particular time slot was successful, the throughput of that time slot for user i can be written as $q_i = g_i(c_i, P)$, where g_i is a concave function (e.g., Shannon capacity equation). Thus, the data rate that user i received can be written as $q_i \cdot s_i$. User i , therefore, receives a *utility* equal to $U_i(q_i \cdot s_i)$, where the utility is measured in monetary units. The utility function $U_i(\cdot)$ is assumed to be concave, strictly increasing, and continuously differentiable. As mentioned in most literature, concavity corresponds to the assumption of elastic traffic.

We therefore use a concept called *Pareto efficient* for allocating resource in a collision channel. By definition, a feasible allocation (s_1, s_2, \dots, s_n) is Pareto efficient if there is no other feasible allocation $(s'_1, s'_2, \dots, s'_n)$ such that $s'_i \geq s_i$ for all $i = 1, \dots, n$ and $s'_i > s_i$ for some i . Here, the allocation is in terms of the success probability of each packet instead of the actual data rate $q_i \cdot s_i$. As we will mention later, it is sufficient for us to consider s_i 's only. The following theorem gives the capacity region (i.e., the pareto efficient allocation) of the aloha system considered here.

Theorem 11. *Given a set of transmission probabilities (z_1, z_2, \dots, z_n) , the resulting allocation (s_1, s_2, \dots, s_n) is Pareto efficient if and only if $z_1 + z_2 + \dots + z_n = 1$.*

Proof. First, we will find the capacity region or the Pareto efficient (s_1, s_2, \dots, s_n) .

We begin by considering the following optimization problem:

$$\begin{aligned} & \max_{z_1, z_2, \dots, z_n} s_1 + s_2 + \dots + s_n \\ & \text{subj. } \frac{s_2}{s_1} = \alpha_2, \dots, \frac{s_n}{s_1} = \alpha_n \end{aligned} \quad (4.1)$$

The Lagrangian is given by:

$$\begin{aligned} & L(s_1, \dots, s_n, \lambda_2, \dots, \lambda_n) \\ & = (1 - \sum_{i=2}^n \lambda_i \alpha_i) s_1 + \sum_{i=2}^n (1 + \lambda_i) s_i \\ & = \beta_1 s_1 + \beta_2 s_2 + \dots + \beta_n s_n \end{aligned} \quad (4.2)$$

where $\beta_1 = (1 - \sum_{i=2}^n \lambda_i \alpha_i)$ and $\beta_i = 1 + \lambda_i$ for $i = 2, \dots, n$. Substituting $s_i = z_i \prod_{j \neq i} (1 - z_j)$ and differentiating $L(s_1, \dots, s_n, \lambda_2, \dots, \lambda_n)$ with respect to z_i 's, we have

$$\frac{\partial L}{\partial z_i} = \beta_i \prod_{j \neq i} (1 - z_j) - \sum_{k \neq i} \beta_k z_k \prod_{j \neq k, j \neq i} (1 - z_j) \quad (4.3)$$

Next, we claim that the solution to the system of equations ($\frac{\partial L}{\partial z_1} = 0, \dots, \frac{\partial L}{\partial z_n} = 0$) has the following form:

$$z_i = \frac{\sum_{j \neq i} \beta_j - (n-2)\beta_i}{\sum_{i=1}^n \beta_i} \quad (4.4)$$

We will now show that the above solution form indeed solves the system of equations. Substituting Eq.(4.4) into Eq.(4.3), the first term of Eq.(4.3) is given by:

$$\begin{aligned} \beta_i \prod_{j \neq i} (1 - z_j) & = \beta_i \prod_{j \neq i} \frac{(n-1)\beta_j}{\sum_{k=1}^n \beta_k} \\ & = \frac{(n-1)^{n-1}}{(\sum_{k=1}^n \beta_k)^{n-1}} \beta_1 \cdot \beta_2 \cdot \dots \cdot \beta_n \end{aligned} \quad (4.5)$$

Similarly, the second term of Eq.(4.3) is given by:

$$\begin{aligned}
\sum_{k \neq i} \beta_k z_k \prod_{j \neq k, j \neq i} (1 - z_j) &= \sum_{k \neq i} z_k \frac{(n-1)^{(n-2)} \prod_{j \neq i} \beta_j}{(\sum_{j=1}^n \beta_j)^{(n-2)}} \\
&= \frac{(n-1)^{(n-2)}}{(\sum_{j=1}^n \beta_j)^{(n-1)}} \left(\prod_{j \neq i} \beta_j \right) \left[\sum_{k \neq i} \left(\sum_{j \neq k} \beta_j - (n-2)\beta_k \right) \right] \\
&= \frac{(n-1)^{(n-2)}}{(\sum_{j=1}^n \beta_j)^{(n-1)}} \left(\prod_{j \neq i} \beta_j \right) (n-1)\beta_i
\end{aligned} \tag{4.6}$$

Comparing the two terms, we see that Eq.(4.4) is indeed the solution to Eq.(4.3). $\sum_{i=1}^n z_i = 1$ follows trivially. Also, note that the set z_i 's given in Eq. (4.4) is a stationary point for the function $L(\cdot)$. It is straight forward to see that the set z_i 's given in Eq.(4.4) cannot be a minimum point of the function $L(\cdot)$. Hence, the set of z_i 's given in Eq.(4.4) must maximize $L(\cdot)$.

We have shown that for an Pareto efficient allocation, the sum of individual transmission probability has to be one. Conversely, if the sum of individual transmission probability is one, we know it is a solution to the optimization problem defined above for appropriately chosen α_i 's. Therefore, the resulted s_i 's must be Pareto efficient. \square

The utility function of each user, $U_i(\cdot)$, is not available to the satellite in general. Therefore, we consider a pricing scheme for controlling the transmission probability. We assume that the satellite, or network manager, treats all users the same (i.e., the satellite does not *price discriminate*). In our case, the price per successfully received packet charged by the satellite is the same for all users.

Given a price p per successfully received packet and other users' transmission probability z_j for $j \neq i$, user i acts to maximize the following payoff function over $0 \leq z_i \leq 1$:

$$\begin{aligned}
U_i(z_i \prod_{j \neq i} (1 - z_j) \cdot q_i) - z_i \prod_{j \neq i} (1 - z_j) \cdot p \\
= U_i(s_i \cdot q_i) - s_i \cdot p
\end{aligned} \tag{4.7}$$

The first term represents the utility to user i if it receives a data rate of $s_i \cdot q_i$, and the

second term represents the price that user i pays to the network manager. We say a set (s_1, \dots, s_n, p) with $s_i = z_i \prod_{j \neq i} (1 - z_j)$ for $i = 1, \dots, n$ and $p \geq 0$ is a *competitive equilibrium* if users maximize their payoff as defined in (4.7), and the satellite sets a price p so as to make $\sum_{i=1}^n z_i = 1$ (i.e., network is efficiently utilized).

The following theorem shows the existence of a unique competitive equilibrium for the pricing scheme considered here.

Theorem 12. *Assume for each user i , the utility function U_i is concave, strictly increasing, and continuously differentiable. Then there exists a unique competitive equilibrium.*

Proof. We first provide the condition for users to be in the equilibrium. At an equilibrium point, user i chooses a transmission probability z_i to maximize its payoff, $U_i(s_i \cdot q_i) - s_i \cdot p$ which is equivalent to the following conditions:

$$U'_i(q_i \cdot z_i \prod_{j \neq i} (1 - z_j)) = \frac{p}{q_i}, \quad \text{if } 0 < z_i < 1 \quad (4.8)$$

$$U'_i(0) \leq \frac{p}{q_i}, \quad \text{if } z_i = 0 \quad (4.9)$$

$$U'_i(q_i \prod_{j \neq i} (1 - z_j)) \geq \frac{p}{q_i}, \quad \text{if } z_i = 1 \quad (4.10)$$

Eq.(4.9) represents the case that the price set by the satellite is too high; therefore, user i will not transmit anything. Similarly, Eq.(4.10) indicates that the price per successfully received packet is too low; hence, user i will always transmit. We consider the case that each user's utility function is strictly concave. Since the utility U_i is strictly concave, strictly increasing, and continuously differentiable, U'_i is a continuous, strictly decreasing function with its domain $[0, q_i]$ and range over the interval $[a, b]$ where b could be infinity. Consequently, the inverse U'_i , say V_i , is also well defined over the interval $[a, b]$, and it is continuous and strictly decreasing. We can write Eq.(4.8) as the following:

$$s_i = \frac{1}{q_i} V_i\left(\frac{p}{q_i}\right) \quad (4.11)$$

We can think of the s_i 's as the *desired* throughput for user i given the price p ,

even though the set (s_1, \dots, s_n) may not be feasible (i.e., there does not exist a set (z_1, \dots, z_n) and $0 \leq z_i \leq 1$ such that $s_i = z_i \prod_{j \neq i} (1 - z_j)$). The set (s_1, s_2, \dots, s_n) forms a strictly decreasing continuous trajectory in \mathcal{R}^n from $(1, 1, \dots, 1)$ to $(0, 0, \dots, 0)$ as p increases. The continuity property of the trajectory is due to the continuity of V_i .

For any Pareto optimal allocation (s_1, s_2, \dots, s_n) , we must have $\sum_{i=1}^n z_i = 1$. For convenience, we write $\vec{z} = (z_1, z_2, \dots, z_n)$ and $\vec{s} = (s_1, s_2, \dots, s_n)$. Then, let $A = \{ \vec{z} \mid z_i \geq 0 \text{ for } i = 1, \dots, n \text{ and } \sum_{i=1}^n z_i = 1 \}$. Moreover, let $f_i(\vec{z}) = z_i \prod_{j \neq i} (1 - z_j)$. Thus, the mapping \mathbf{f} of A into \mathcal{R}^n is defined by:

$$\mathbf{f}(\vec{z}) = (f_1(\vec{z}), \dots, f_n(\vec{z}))$$

Since each of the functions f_1, \dots, f_n is continuous, \mathbf{f} is continuous as well. We then have the set $B = \{ \mathbf{f}(\vec{z}) \mid \vec{z} \in A \}$ is compact because A is compact. Thus, the set B forms a surface in \mathcal{R}^n that separates the point $(1, 1, \dots, 1)$ from the origin. To see this, we use induction. In the two dimensional case, this is obviously true. Now, suppose this statement is true for the n -dimensional case. For the $n + 1$ dimensional case, let's look at the boundary points of the simplex $\sum_{i=1}^{n+1} z_i = 1$. The boundary points has dimension n . Thus, the resulting mapping is closed surface from induction hypothesis. The following figure illustrate the idea by going from two dimension to three dimension.

Therefore, for the continuous trajectory (s_1, s_2, \dots, s_n) parameterized by the price p to go from $(1, 1, \dots, 1)$ to the origin, it must intersect with the set B at a unique point. That point is the unique competitive equilibrium in our pricing scheme. \square

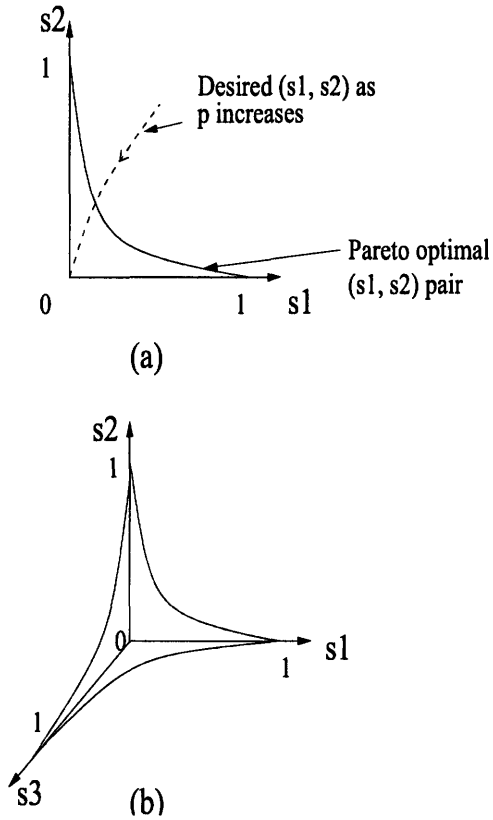


Figure 4-1: (a) The relationship between a Pareto optimal (s_1, s_2) . (b) The relationship of a Pareto optimal (s_1, s_2, s_3) .

4.3 Multi-satellite System

4.3.1 The Pareto Optimal Throughput Region of A Multi-satellite System

In a network with multiple satellites, we assume that users can simultaneously transmit to different satellites using different frequencies during the same time slot. The case that a user can transmit to only one single satellite during a time slot is a special case of the model where simultaneous transmissions are allowed. We let $z_{(i,j)}$ denote the transmission probability from user i to satellite j . Similarly, let $q_{(i,j)}$ denote the quality of the channel from user i to satellite j . The set of users that transmits to satellite j is denoted as A_j . The set of satellites that user i transmits to is denoted

as B_i . The probability of a success transmission from user i to satellite j is denoted as $s_{(i,j)}$. We also assume that users are backlogged. A graph $G = (V, E)$ can be used to represent the connections between users and satellites, where V is a set of nodes representing the users and the satellites; the edge (i, j) is in E if $z_{(i,j)}$ is positive. We first consider the case where the channels from the users to the satellites are all identical. The Pareto optimal throughput region of this multiple satellites system with identical channel quality is given by the following theorem:

Theorem 13. *Given a multi-satellite system represented by a connected graph $G = (V, E)$, the resulting throughput is Pareto optimal if and only if the following two conditions are satisfied:*

1. *there is no cycle in the graph G*
2. $\sum_i z_{(i,j)} = 1 \forall j$

For a multi-satellite system that cannot be represented by a *connected* graph, we can consider each disconnected part of that graph separately. The following figure is a graphical representation of a possible communication scenario.

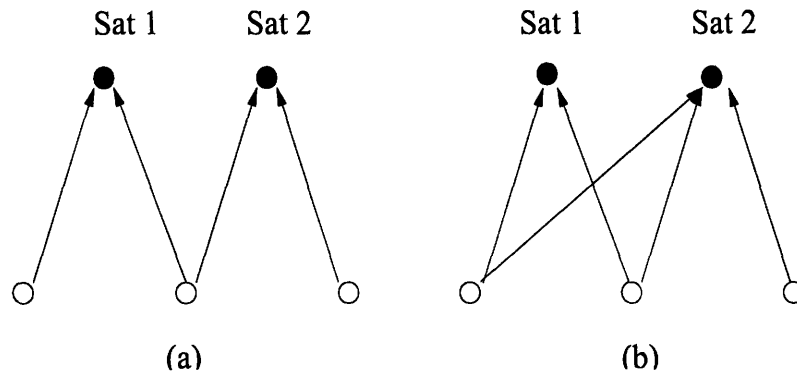


Figure 4-2: (a) A graphical representation containing no cycle. (b) A graphical representation containing cycle.

Proof. Condition (2) is straightforward from Theorem 11. We will prove condition (1) here. Suppose that we have m satellites and n users. The probability of success

for user i can be written as follows:

$$s_i = \sum_{k \in B_i} z_{(i,k)} \cdot \prod_{j \in A_k} (1 - z_{(j,k)}) \quad (4.12)$$

A set of transmission probabilities $z_{(i,j)}$ achieving pareto optimality implies that we cannot find a set of small variation $\delta_{z_{(i,j)}}$ on $z_{(i,j)}$ such that the throughput can be improved for all users. Hence, given a set of transmission probability $z_{(i,j)}$, to see whether such transmission probabilities achieves pareto optimality, we need to check whether we can find a set of $\delta_{z_{(i,j)}}$ to improve the throughput performance for some users without decreasing the throughput for other users. For satellite j , if there are k users transmitting to this satellite, we can freely vary the transmission probability by a small amount to only $k - 1$ users in order to satisfy the condition $\sum_i z_{(i,j)} = 1$ (If we change the transmission probability of all k users by a small amount, the condition $\sum_i z_{(i,j)} = 1$ may be violated). In this case, we say that we have $k - 1$ degree of freedom in varying the transmission probabilities. Therefore, for a system with m satellites, the degree of freedom in varying the transmission probabilities is $\sum_{i=1}^n |B_i| - m$. For a connected graph, we must have

$$\sum_{i=1}^n |B_i| \geq n + m - 1.$$

Similarly, for the connected graph to contain a cycle, we must have

$$\sum_{i=1}^n |B_i| \geq n + m.$$

Therefore, for a connected graph contains no cycle, we have

$$\sum_{i=1}^n |B_i| = n + m - 1.$$

To satisfy the pareto optimality, from the first order condition, we need to check

whether we can find a set of $\delta_{z(i,j)}$ such that

$$\sum_{i,j} \frac{\partial s_i}{\partial z(i,j)} \cdot \delta_{z(i,j)} \geq 0 \quad \forall i \quad (4.13)$$

and

$$\sum_{i,j} \frac{\partial s_i}{\partial z(i,j)} \cdot \delta_{z(i,j)} > 0 \quad \text{for some } i. \quad (4.14)$$

If we can find a set of $\delta_{z(i,j)}$ satisfying the above equation, the set of transmission probability $z(i,j)$ cannot be pareto optimal transmission probabilities. Since there is a total of n users, we will have n linear equations. The variables in these linear equations are the small variation $\delta_{z(i,j)}$. The number of variable is the degree of freedom in varying the transmission probabilities, which is $\sum_{i=1}^n |B_i| - m$. For a graph with cycle, we have $\sum_{i=1}^n |B_i| - m \geq n$. In this case, since we have n positive linear equations and $k \geq n$ variables, we can certainly find a set of $\delta_{z(i,j)}$ of dimension k that satisfies Eq.(4.13) and Eq.(4.14).

Now suppose that a connected graph G satisfies both conditions of this theorem. If we increase the transmission probability of one link, we must also decrease the transmission probability of some other link due to the constraint that $\sum_i z(i,j) = 1 \forall j$ and the fact that there is no cycle in the graph. Hence, the resulting throughput is pareto optimal. \square

In the case that there is a channel state $q(i,j)$ associated with each channel, the above theorem provides a necessary condition for obtaining the Pareto optimal throughput region.

Now, let's consider a network consisting of only two satellites for simplicity. We investigate how these two satellites can each set their own prices, p_1 and p_2 respectively, to achieve Pareto optimal throughput region. The objective for user i is to maximize the following function:

$$U_i(s(i,1) \cdot q(i,1) + s(i,2) \cdot q(i,2)) - s(i,1)p_1 - s(i,2)p_2 \quad (4.15)$$

where

$$s_{(i,k)} = z_{(i,k)} \cdot \prod_{j \in A_k} (1 - z_{(j,k)}) \quad (4.16)$$

The term $s_{(i,1)} \cdot q_{(i,1)} + s_{(i,2)} \cdot q_{(i,2)}$ denotes the throughput of user i . We first assume that the transmission probability $z_{(i,1)}$ is independent of the transmission probability $z_{(i,2)}$ for user i . That is, user i can transmit to both satellites during the same time slot. The case that user i can send to only one satellite during a time slot is the same as the case which allows simultaneous transmission when $z_{(i,1)} + z_{(i,2)} \leq 1$. To increase the utility function in Eq.(4.15) by a small amount, user i can increase either $s_{(i,1)}$ or $s_{(i,2)}$. The marginal costs are $p_1/q_{(i,1)}$ and $p_2/q_{(i,2)}$ respectively. Thus, if $p_1/q_{(i,1)}$ is strictly less than $p_2/q_{(i,2)}$, user i will transmit to satellite 1 only. To maximize Eq.(4.15), the following equation must be satisfied:

$$\frac{\partial}{\partial s_{(i,1)}} U_i(s_{(i,1)} \cdot q_{(i,1)}) = \frac{p_1}{q_{(i,1)}} \quad (4.17)$$

Likewise, if $p_2/q_{(i,2)}$ is strictly less than $p_1/q_{(i,1)}$, user i will transmit to satellite 2 only. The following equations must be satisfied to maximize Eq.(4.15):

$$\frac{\partial}{\partial s_{(i,2)}} U_i(s_{(i,2)} \cdot q_{(i,2)}) = \frac{p_2}{q_{(i,2)}} \quad (4.18)$$

In the case that $p_1/q_{(i,1)} = p_2/q_{(i,2)}$, user i can transmit to either satellite, and the following equation holds:

$$U'_i(s_{(i,1)} \cdot q_{(i,1)} + s_{(i,2)} \cdot q_{(i,2)}) = \frac{p_1}{q_{(i,1)}} = \frac{p_2}{q_{(i,2)}} \quad (4.19)$$

Following the single satellite case, in the m satellites case we say a set $(s_{(1,1)}, s_{(1,2)}, \dots, s_{(n,1)}, s_{(n,2)}, p_1, \dots, p_m)$ with $s_{(i,k)}$ defined in Eq.(4.16) and $p_j \geq 0$ for $j = 1, \dots, m$ is a *competitive equilibrium* if users maximize their payoff, and satellites set a price vector (p_1, \dots, p_m) so as to make $\sum_{i=1}^n z_{(i,j)} = 1$ for $j \in \{1, \dots, m\}$. To test for equilibrium, given the price set by the satellite, we ask whether a particular user has the desire to change its transmission strategy. That is, a user will take the price as

fixed, and decide the optimal action based on this price. We also make the following channel diversity assumptions:

1. There does not exist i and j such that $q_{(i,k)} = q_{(j,k)}$ for all k .
2. $q_{(i,k_1)} \neq q_{(i,k_2)}$ for all k_1 and k_2 .

Assumption 1 implies that no two users have identical channel to both satellites. Assumption 2 implies that, for each user, the channel states to different satellites are different. The following theorem shows the existence of a competitive equilibrium in a multi-satellites environment.

Theorem 14. *With the channel diversity assumption, given that each user's utility function U_i is concave, strictly increasing, and continuously differentiable, there exists a unique competitive equilibrium in this network with n users and m satellites.*

Proof. We first consider the case that $m = 2$ and $n = 3$ for illustration. Because $V_i = U'_i$ is strictly decreasing, as the price p_j increases, the desired throughput for each user also decreases, moving closer to a feasible point (i.e., $\sum_{i \in A_j} z_{(i,j)} = 1$). Eventually, the desired throughput meet a feasible point. This part is the same as the single satellite part. However, as one satellite decreases or increases the price p_1 , it may cause a user, say user 1, to start transmitting to the other satellite. This happens when $p_1/q_{(1,1)} = p_2/q_{(1,2)}$. If the user's desired throughput is r , it can choose $s_{(1,1)}$ and $s_{(1,2)}$ such that $s_{(1,1)} \cdot q_{(1,1)} + s_{(1,2)} \cdot q_{(1,2)} = r$. For fixed r , $s_{(1,1)}$ is a continuous function of $s_{(1,2)}$. If p_1 is too high, user 1 could start transmitting to satellite 2, thus forcing satellite 2 to change its price to meet the Pareto operating point. In case that two prices are decoupled, we have two desired operating points, one for each satellite, with two control parameters. In case that prices are coupled, we can control one price and one transmitting probability to get the two desired operating points. In both cases, we have two control parameters, thus are able to get to the equilibrium point.

For the general n -users case, we know that user i should send to satellite 2 if the

following holds:

$$\frac{q(i,1)}{q(i,2)} < \frac{p_1}{p_2}.$$

Also, from our channel diversity assumption, there can be only one user such that

$$\frac{q(i,1)}{q(i,2)} = \frac{p_1}{p_2}.$$

This implies that at most one user can transmit to both satellites.

Now, we show that the equilibrium is indeed unique. Assuming there exists two equilibrium points: $(s_{(1,1)}, \dots, s_{(n,2)}, p_1, p_2)$ and $(s'_{(1,1)}, \dots, s'_{(n,2)}, p'_1, p'_2)$, we will show that there is a contradiction. At a equilibrium point, we know that two scenarios are possible: 1) no user transmits to both satellites; 2) there is exactly one user transmitting to both satellites. First, we consider the case that no user transmits to both satellites at both equilibriums. Without loss of generality, we number users such that the following order holds:

$$\frac{q(1,1)}{q(1,2)} > \frac{q(2,1)}{q(2,2)} > \dots > \frac{q(n,1)}{q(n,2)}.$$

If both equilibrium points have the same graphical representation (i.e., user transmits to the same satellite in both equilibrium), the two equilibrium points have to be identical from the derivation in the single satellite case. Let's now consider the case that two equilibriums points have different graphical representations. Specifically, users 1 to k transmit to satellite one, and users $k + 1$ to n transmit to satellite two for the equilibrium $(s_{(1,1)}, \dots, s_{(k,1)}, s_{(k+1,2)}, \dots, s_{(n,2)}, p_1, p_2)$. For the equilibrium $(s'_{(1,1)}, \dots, s'_{(l,1)}, s'_{(l+1,2)}, \dots, s'_{(n,2)}, p'_1, p'_2)$, users 1 to l transmit to satellite one and users $l + 1$ to n transmit to the second satellite, where $l > k$. Since $l > k$, we have

$$\frac{p_1}{p_2} > \frac{p'_1}{p'_2}.$$

If $p_1 < p'_1$, we have $p_2 < p'_2$ from the above equation. With price p_2 , the desired throughput at satellite two is $(s_{(k+1,2)}, \dots, s_{(n,2)})$. Similarly, with price p'_2 , the desired

throughput at satellite two is $(s'_{(l+1,2)}, \dots, s'_{(n,2)})$. Since $p_2 < p'_2$, we have the desired throughput $s_{(i,2)} > s'_{(i,2)}$ for all $i \in \{l+1, \dots, n\}$. We know that $(s'_{(l+1,2)}, \dots, s'_{(n,2)}, p'_2)$ is at equilibrium in satellite two. Therefore, $(s_{(k+1,2)}, \dots, s_{(l,2)}, \dots, s_{(n,2)}, p_2)$ cannot be in equilibrium. That is, there does not exist $(z_{(k+1,2)}, \dots, z_{(n,2)})$ such that

$$s_{(i,2)} = z_{(i,2)} \prod_{j \neq i, j \in A_2} (1 - z_{(j,2)}) \quad \forall i = \{k+1, \dots, n\}$$

and $\sum_{i=k+1}^n z_{(i,2)} = 1$. Hence, we have a contradiction here. If $p_1 > p'_1$, we get a similar contradiction.

Thus far, we have discussed the case that no user transmits to both satellites for both equilibrium points. If there is exactly one user transmits to both satellites for the two equilibrium points, a similar contradiction can be derived. For the other cases (i.e., one user transmits to both satellites in one equilibrium while no user transmits to both satellites in the other equilibrium), we can get similar contradiction. Therefore, the equilibrium is unique. □

Corollary 2. *The equilibrium throughput obtained using the pricing scheme is Pareto optimal.*

Proof. From the proof of the Theorem 13, we see there cannot be any cycle in the graph even when users having different channel qualities. Let the set of users transmitting to satellite one and satellite two be denoted as A_1 and A_2 respectively. Since $q_{(i,1)}/q_{(i,2)} \geq q_{(j,1)}/q_{(j,2)}$ for all $i \in A_1$ and $j \in A_2$, thus switching the receiving satellite cannot expand the throughput region. Hence, the equilibrium throughput is Pareto optimal. □

4.4 Example

In this example, we consider a communication scenario with two satellites and three users and try to obtain an exact expression of the equilibrium point. The channel

conditions are given as follows:

$$\begin{aligned} q_{(1,1)} &= 0.8, & q_{(2,1)} &= 0.5, & q_{(3,1)} &= 0.5 \\ q_{(1,2)} &= 0.3, & q_{(2,2)} &= 0.4, & q_{(3,2)} &= 0.7 \end{aligned}$$

The utility function for user i is given by the following:

$$U_i(x) = a_i \cdot x^{b_i} \quad (4.20)$$

where $a_1 = 1, a_2 = 2, a_3 = 1.5$ and $b_1 = b_2 = b_3 = 0.5$. We first make the assumption that user 2 transmits to both satellites; user 1 only transmits to satellite 1 while user 3 transmits to satellite 2 only. If we can find an equilibrium, we know that our assumption is correct. Therefore, the following equations must hold:

$$\begin{aligned} U_1'(s_{(1,1)} \cdot q_{(1,1)}) &= \frac{p_1}{q_{(1,1)}} \\ U_2'(s_{(2,1)} \cdot q_{(2,1)} + s_{(2,2)} \cdot q_{(2,2)}) &= \frac{p_1}{q_{(2,1)}} = \frac{p_2}{q_{(2,2)}} \\ U_3'(s_{(3,2)} \cdot q_{(3,2)}) &= \frac{p_2}{q_{(3,2)}} \end{aligned} \quad (4.21)$$

We have the following after simplification:

$$\begin{aligned} s_{(1,1)} \cdot 0.8 &= \alpha_1 \cdot p_1^{\frac{1}{b_1-1}} \\ s_{(2,1)} \cdot 0.5 + s_{(2,2)} \cdot 0.4 &= \alpha_2 \cdot p_2^{\frac{1}{b_2-1}} \\ s_{(3,2)} \cdot 0.7 &= \alpha_3 \cdot p_2^{\frac{1}{b_3-1}} \end{aligned} \quad (4.22)$$

where

$$\begin{aligned} \alpha_1 &= \left(\frac{1}{q_{(1,1)} a_1 b_1} \right)^{\frac{1}{b_1-1}} \\ \alpha_2 &= \left(\frac{1}{q_{(2,2)} a_2 b_2} \right)^{\frac{1}{b_2-1}} \\ \alpha_3 &= \left(\frac{1}{q_{(3,2)} a_3 b_3} \right)^{\frac{1}{b_3-1}} \end{aligned}$$

The set of $s_{(1,1)}$ and $s_{(2,1)}$ such that $z_{(1,1)} + z_{(2,1)} = 1$ are related by the following equation:

$$s_{(1,1)} = (1 - \sqrt{s_{(2,1)}})^2. \quad (4.23)$$

Similar relation holds for $s_{(2,2)}$ and $s_{(3,2)}$. Hence, we have the following equation:

$$(1 - \sqrt{s_{(1,1)}})^2 \cdot 0.5 + (1 - \sqrt{s_{(3,2)}})^2 \cdot 0.4 = \alpha_2 \cdot p_2^{\frac{1}{b_2-1}} \quad (4.24)$$

Since user 2 is transmitting to both satellites, the equation $p_1/q_{(2,1)} = p_2/q_{(2,2)}$ holds. We can write $s_{(1,1)}$ and $s_{(3,2)}$ as a function of p_2 only. Substituting $s_{(1,1)}$ and $s_{(3,2)}$ into Eq. (4.24), we can solve for p_2 . From p_2 , we can get the unique competitive equilibrium for this example, which is given below:

$$\begin{aligned} p_1 &= 1.097, & s_{(1,1)} &= 0.166, & s_{(2,2)} &= 0.081 \\ p_2 &= 0.877, & s_{(2,1)} &= 0.351, & s_{(3,2)} &= 0.511 \end{aligned}$$

The transmission probability is given as the following:

$$\begin{aligned} z_{(1,1)} &= 0.407, & z_{(2,2)} &= 0.285 \\ z_{(2,1)} &= 0.593, & z_{(3,2)} &= 0.715 \end{aligned}$$

4.5 Conclusion

In this chapter, we investigate how to better utilize the multiple channels available in a satellite network. Specifically, we use pricing as a mechanism to control users' transmission probabilities and exploit different channel qualities to coordinate transmission among users. Hence, the throughput performance of the system is improved. We also characterize the Pareto optimal throughput region for both single satellite network and multiple satellites network. We show that users' throughput is Pareto optimal at the equilibrium price. The characterization of the Pareto optimal throughput region for multiple channels with time varying channel states can be a possible direction for the future research.

Chapter 5

Multipath Routing over Wireless Networks: Coding and Delay Tradeoff¹

With multiple parallel channels existing between a source and a destination node, one can utilize these parallel channels to improve the quality of service such as the delay performance by using parallel transmission. Indeed, the deployment of various wireline and wireless networks make it possible for multiple alternative routing path to exist between a source and destination pair. In this chapter, we propose to use digital fountain code to transmit data file with redundancy. Given that a file with k packets is encoded into n packets for transmission, the use of digital fountain code allows the file to be received when only k out of n packets are received. By adding the redundant packets, the destination node does not have to wait for packet arrive late, hence reducing the delay of the file transmission. We characterize the tradeoff between the code rate (i.e., ratio between the number of transmitted packet and the number of the original packets) and the file delay reduction. As a rule of thumb, we provide a practical guideline in determining an appropriate code rate for a fixed file to achieve a reasonable transmission delay. We show that only a few redundant packets

¹This chapter is the result of the collaboration with Yonggang Wen. This joint research has also benefited from discussions with Professor Vincent Chan and his insightful feedbacks.

are needed to achieve a significant reduction in file transmission delay. In the second part of this chapter, we consider the reduction of file delay when there are multiple users sharing the set of parallel paths. Adding redundant packets for transmission will increase the congestion level of the network, consequently the queuing delay of an individual packet. Hence, the file transmission delay may under some coding rate. We also show that there exists a unique coding rate that minimizes the file transmission delay.

5.1 Introduction

Network delay is an important quality of service requirement to support various real time applications. In today's network, packet delay is usually specified by the service provider to demonstrate the performance level that it can guarantee consumers. However, with increasing volume of the data traffic such as the electronic files in today's data networks, packet delay is not always a good indication of the performance that a typical user will experience. On the other hand, file transmission delay, which is the time interval that the destination node has to wait before it can reconstruct the original file, resembles more closely to the delay experience of an average user. In this paper, we focus on the problem of how to minimize the file transmission delay in a wireless or satellite network.

We consider the case when there are multiple parallel paths available for transmission between the source and the destination node [36], [42]. That is, for a file with a fixed number of packets, one can assign a certain fraction of these packets on each path and transmit them simultaneously. We assume that each packet will experience an independent and identically distributed transmission delay on a particular path, which we argue to hold for certain communication scenario. File transmission delay can be very different from the packet transmission delay especially when the distribution of the packet transmission delay has a heavy tail. After a source distributed the packets of a file among the available paths, the destination can reconstruct the file when all the packets of that file have arrived. The problem of how to distribute a file

with finite amount of packets among a finite set of parallel paths, each with different channel statistics, is studied in [46]. There, an optimal packets allocation scheme to minimize the average file transmission delay is presented. To reconstruct the original file at the destination node require the arrivals of *all* packets of that file. This may take a long time due to the heavy tail of the packet delay distribution. Hence, it prompts us to code the original file at the packet level. Specifically, for a file with k packets originally, the source transmit $n > k$ packets by adding some redundant packets to the original file. At the destination node, upon receiving the first k packets out of the n transmitted packets, the destination node can reconstruct the original file. This kind coding at the packet level exists such as the digital fountain code or tornado code [45], [44].

The problem of two nodes communicating using multiple paths has received considerable attention in various contexts (for example, traffic balancing, higher throughput and path redundancy for higher reliability) for wired networks [35], [36], [40]. Recently, with advent of wireless networks such as the Roofnet, multi-beams satellite networks [43], and ad-hoc networks, there is a resurging interest in the multi-path routing research [41], [42]. In wireless networks, channels are often unreliable due to fading and interference. Multi-path routing, due to its diverse routing path, becomes an effective method in mitigating unreliable channels and providing a good delay performance. In [38], the authors propose models to analyze and compare single-path and multi-path routing protocols in terms of overheads, traffic distribution and connection throughput in a mobile ad-hoc network. In [39], the authors developed a framework for optimal rate allocation and multi-path routing in multi-hop wireless networks. Analytical results for optimal rate allocation for Poisson arrivals at each node are derived. More recently, in [41], the authors show how to split, replicate, and erasure code message fragments over multiple delivery paths to optimize the probability of successful message delivery in a delay tolerant network. Simulations that covers wide range of delay tolerant application are provided.

The work that bears the closest resemblance to this chapter is the seminal work presented in [36]. There, the author proposed the dispersity routing scheme which

sub-divides the message and disperses it through the maze of paths comprising the network. The author also considered adding redundant message to the original file to reduce the file delay, and the average delay was plotted numerically. We know that the file transmission delay will be reduced by adding more redundant packets. In this chapter, our aim is to obtain an intuitive understanding of the tradeoff between the code rate and delay reduction in a communication setting with a single or multiple source-destination pairs that sharing a set of parallel paths. In the single source-destination case, given a file size, we then provided a practical guideline in determining the code rate to achieve a good reasonable file transmission delay. We show that only a few redundant packet is required for achieving a significant reduction in file transmission delay. Next, we consider the trade-off between the file transmission delay and code rate in a multiple users environment. There, the redundant packets will increase the network congestion level, hence the packet's queuing delay. We will investigate whether adding redundant packet can still reduce the file transmission delay. The coding and delay tradeoff in this case is characterized in terms of the traffic load of the network. Depending on the load, a unique coding rate that minimizes the transmission delay is obtained.

The rest of this chapter is organized as follows: Section 2 describes the detailed formulation of this file transmission delay minimization problem. In section 3, the coding and delay tradeoff in the case of a single source and destination pair is presented. Section 4 describe the coding and delay relationship in the case where multiple users are sharing the same set of parallel paths. Finally, section 5 concludes this chapter.

5.2 Problem Formulation

In this chapter, we consider a communication network with a rich set of disjoint paths between a source and destination pair of interest. Given this set of disjoint paths, we focus on the problem of how to best utilize these paths to minimize file transfer delay from the source node to the destination node. We assume there are n_p paths between the source and destination node. The transmission delay distribution of a

single packet is the same for all paths. Here, the transmission delay denotes the time interval that begins with a packet is being transmitted at the source node to the time that this packet reaches its destination node. For a particular path, the transmission delay of a single packet is an identical and independently distributed random variable. The time takes to transmit two packets is then the sum of two i.i.d random variables. The assumption of a random transmission time for each packet is reasonable in the wireless communication scenario such as the multi-beam satellite downlinks, or wireless mesh networks. In a multi-beam satellite downlink, due to the time-varying nature of the channel quality, the transmission time of each packet can be different. For example, when the channel is bad, the satellite may take longer time to transmit a packet in order to save energy because of the concavity of the rate-power curve. Similarly, in a wireless mesh network, the channel fading of a wireless link and the Aloha type contention resolution in the MAC layer both contribute to the randomness of the transmission time of a single packet. Due to the possibility of a deep fade (i.e., no transmission is possible), it may take a long time for a packet to arrive its destination. Hence, we model the delay distribution of a packet as a distribution with a tail (i.e., the probability of having a very large delay is nonzero). In this chapter, for simplicity, we use exponential distribution with rate μ to model the delay distribution of a single packet.

The assumption that the transmission delay of each packet is identically distributed is reasonable since the communication channel is identical to each other statistically. To make sense of the assumption that delay of each packet is independently distribution, we need to focus on satellite or wireless network. For a multi-hop wire-line network, the transmission delay of one packet cannot be independent of each other since the transmission delay in a wireline system consists mainly the propagation delay which is highly correlated for each packet. Here, we are considering a satellite downlink with time-varying channel which can be described by an ergodic process. Therefore, it is reasonable to assume that the transmission delay of each packet is independent. In the case of a heterogeneous network, we consider the case where a wireless link connecting wireline links in each path. The delay in transmitting

a packet is going to be dominated by the transmission of the wireless link. Hence, the independence assumption of the packet transmission delay is still a reasonable one.

At the source node, a file generated consists of k packets. The source can then encode this file into n packets such that the destination node can decode the whole file as soon as it received k packets (i.e., Digital Fountain code). Note that Digital Fountain code actually require the destination node to receive $k(1 + \epsilon)$ packets in order to decode the original file, where ϵ is small. For the first part of the chapter, we assume that the source is the only node that has packets to send to the destination node. Assume that a file is generated at the source node at time zero. The file transmission delay, denoted here as \mathcal{D} , is the time at which the destination node receive k packets. The code rate is define to be n/k .

5.3 Delay-coding tradeoff for a single source destination pair

We start this section by giving the following motivating example. Consider sending a file with k packets, numbered (P_1, \dots, P_k) , from the source to the destination. Let $n_p = 2$, and these two paths be identical. The time required for sending a packet P_i , denoted here as τ_i , is an i.i.d. exponential random variable with mean $\frac{1}{\mu}$. To transmit the file using both paths, a simple way would be to allocate packets $(P_1, \dots, P_{k/2})$ on the first path and packets $(P_{k/2+1}, \dots, P_k)$ on the second path, assuming that k is even. Let T_j , $j \in \{1, \dots, n_p\}$, represents the total time needed for a path to clear all packets assigned to it. Here, we have

$$T_1 = \tau_1 + \dots + \tau_{k/2}$$

$$T_2 = \tau_{k/2+1} + \dots + \tau_k.$$

The file transmission delay \mathcal{D} is given by:

$$\mathcal{D} = \max\{T_1, T_2\}.$$

For the case where k is much larger than n_p , we have

$$\frac{\mathcal{D}}{k/2} \approx \frac{T_1}{k/2} \approx \frac{T_2}{k/2}.$$

Since k is large, both paths will be busy in serving the packets. There is not much we can do to further reduce the file transmission delay. However, in this chapter, we focus on the case where k is *not* much larger than n_p .

Now, consider the case where a file consists of only six packets and $n_p = 2$. We assign packets (P_1, P_2, P_3) to the first path and (P_4, P_5, P_6) to the second path. If any one of the two paths clear its packets first, it will remain idle while the other path is transmitting its packets. We see that there is a non-negligible fraction of system time wasted in idle instead of serving packets. A natural way to resolve the above problem and reduce the file transmission delay is to do the following: assigning packets (P_1, P_2, \dots, P_6) on path one and transmitting these packets in this order; similarly, assigning packets (P_6, P_5, \dots, P_1) on path two and sending them in this order. This way, whenever the destination received a total of six packets, the original file can be reconstructed. Since both paths are transmitting packets (i.e., a faster path can serve more packets than the slower path instead of waiting idly), the file transmission delay will be reduced. In fact, the arrangement will minimize the file transmission delay. Therefore, as we added redundant packets on each path, the file transmission delay can be reduced. Now, we come to the first points that we want to illustrate in this chapter: there is a relationship between the redundancy and the file transmission delay.

With only two paths, the above transmission strategy achieves the minimum file transmission delay. With $n_p \geq 3$, it is not clear how to allocate packet on each path so as to reduce the file transmission delay. In that case, we use digital fountain code for transmission so that we do not have to worry about the assignment of each packet on a particular path. All we need to concern is the total number of packets assigned to a particular path. In the following section, we are going to present the trade-off between coding and file transmission delay for two different cases: $n \leq n_p$ and $n > n_p$.

5.3.1 Case I: $n > n_p$

First, we consider the case where the number of packets in the file is greater than the number of parallel paths. For convenience, we let $k = l \cdot n_p$ and $n = m \cdot n_p$. As we mentioned before, sending a file using parallel transmission through multiple paths requires the destination node to wait for *all* packets of the file to arrive. To reduce the waiting time of the last few packets, we can add redundant packets to the original file. In this section, we study the tradeoff between the file delay and the amount of redundant packets added to the file. Specifically, we consider the file delay of a single source and destination pair with n_p identical and disjoint paths between them. We also define a transmission strategy to be a packet allocation vector $\vec{a} = \{a_1, a_2, \dots, a_{n_p}\}$, where a_i denotes the number of packets that needs to be transmitted on path i . The following lemma provides the optimal transmission strategy for using the multiple paths.

Lemma 11. *Given a set of identical paths between a source and destination pair, the expected file delay is minimized when allocating packets evenly on each path.*

Proof. See Appendix. □

For a file with $l \cdot n_p$ packets, the source can encoded the file to $m \cdot n_p$ packets. From the above lemma, we know that allocating packets evenly on each path will result in the minimum expected file transmission delay. Now, consider all of the n_p paths. With each path assigned m packets, we define $N_i(t)$ to be the number of packets that had arrived the destination node by time t . To reconstruct the original file at the destination node at time t , the following condition has to be satisfied:

$$\sum_{i=1}^{n_p} N_i(t) \geq l \cdot n_p \quad (5.1)$$

or

$$\frac{1}{n_p} \sum_{i=1}^{n_p} N_i(t) \geq l \quad (5.2)$$

The file transmission delay is given by:

$$\mathcal{D} = \inf\{t : \sum_{i=1}^{n_p} N_i(t) \geq l \cdot n_p\} \quad (5.3)$$

As the number of path gets large, from the law of large number, we have

$$\lim_{n_p \rightarrow \infty} \frac{1}{n_p} \sum_{i=1}^{n_p} N_i(t) \rightarrow E[N(t)], \quad (5.4)$$

and the file transmission delay can be written as:

$$\mathcal{D} = \inf\{t : E[N(t)] \geq l\} \quad (5.5)$$

Now, consider a single path with m packets that need to be transmitted. Let $N(t)$ denote the total number of packets had arrived at the destination node by time t for this path with a total of m packets. Similarly, let $\hat{N}(t)$ denote the number of arrivals by the time t for a poisson process with rate μ . For the case where the packet transmission is exponentially distributed with rate μ , note that the first m arrivals of the process $N(t)$ and $\hat{N}(t)$ are statistically identical. Hence, we can write the expected number of packet arrived by time t as the following:

$$E[N(t)] = \sum_{i=1}^m i \cdot P(\hat{N}(t) = i) + m \cdot \sum_{i=m+1}^{\infty} P(\hat{N}(t) = i) \quad (5.6)$$

To get the file transmission delay, we need to first evaluate $E[N(t)]$. Expand Eq.

(5.6), we have:

$$\begin{aligned}
\sum_{i=1}^m iP(\hat{N}(t) = i) &= \sum_{i=1}^m i \frac{(\mu t)^i e^{-\mu t}}{i!} \\
&= \frac{(\mu t)\Gamma(m, \mu t)}{\Gamma(m)} \\
m \sum_{i=m+1}^{\infty} P(\hat{N}(t) = i) &= m \sum_{i=m+1}^{\infty} \frac{(\mu t)^i e^{-\mu t}}{i!} \\
&= \frac{\Gamma(m+1) - \Gamma(m+1, \mu t)}{\Gamma(m)}
\end{aligned} \tag{5.7}$$

where

$$\Gamma(m) = \frac{1}{(m-1)!}.$$

Thus, we have

$$E[N(t)] = \frac{(\mu t)\Gamma(m, \mu t) + \Gamma(m+1) - \Gamma(m+1, \mu t)}{\Gamma(m)} \tag{5.8}$$

where

$$\Gamma(m, \mu t) = \Gamma(m)e^{-\mu t} \sum_{i=0}^{m-1} \frac{(\mu t)^i}{i!}.$$

Since $E[N(t)]$ is a continuous function in t , the file transmission delay \mathcal{D} satisfies $E[N(\mathcal{D})] = l$. The above equation for $E[N(t)]$ is hard to solve in general. However, when $\mu \cdot t = m$, Eq. (5.8) will have a simpler form. In this case, we have

$$(\mu t)\Gamma(m, \mu t) - \Gamma(m+1, \mu t) = -(\mu t)^m e^{-\mu t} = -m^m e^{-m}$$

and

$$E[N(t)] = m - \frac{m^m e^{-m}}{(m-1)!} \tag{5.9}$$

By using the Stirling approximation, we can further simplify the above equation as

follows:

$$\begin{aligned}
E[N(t)] &= m - \frac{m^m e^{-m}}{(m-1)!} = m - m \cdot \frac{m^m e^{-m}}{m!} \\
&\approx m - m \cdot \frac{m^m e^{-m}}{\sqrt{2\pi m} m^m e^{-m}} \\
&= m - \sqrt{\frac{m}{2\pi}}
\end{aligned} \tag{5.10}$$

At this point, one may think that $\mu t = m$ is merely an equation that simplifies $E[N(t)]$. However, as we will show later, the equation $\mu t = m$ provides important insight in obtaining the best coding and file delay tradeoff. First, let's examine the implication of $\mu t = m$. This equation give rise to the following communication scenario: Given a file that contains $(m - \sqrt{m/2\pi})n_p$ packets, the source first encodes the file into $m \cdot n_p$ packets and transmits m packets on each path. To reconstruct the file at the destination node, the destination node has to wait for $(m - \sqrt{m/2\pi})n_p$ packets to arrive. The time takes for $(m - \sqrt{m/2\pi})n_p$ packets to arrive the destination node, or the file delay, is simply $t = m/\mu$.

For a file with a fixed size, it is intuitive to see that the file delay will decrease as more redundant packets are added during the actual transmission of the file. However, the above communication scenario only provides the delay for one specific code rate (i.e., $m/(m - \sqrt{m/2\pi})$). It does not give us the file delay for other code rates. As the source adds redundant packets for transmission, at the beginning a few redundant packets may reduce the delay significantly while each additional redundant packet may not reduce the file delay much. Without a complete code rate and delay curve, it seems that we do not know how to achieve an appropriate balance between code rate and delay. Nevertheless, we are going to show next a coding strategy that achieve a good balance between the code rate and the file delay by using the equation $\mu t = m$. First, we derive the minimum file transmission delay, denoted here as \mathcal{D}_{min} , for a file with fixed size $l \cdot n_p$ in the following lemma.

Lemma 12. *Given a file with $l \cdot n_p$ packets, the minimum achievable file transmission delay $\mathcal{D}_{min} = \frac{l}{\mu}$.*

Proof. The minimum file transmission delay is achieved by sending infinity number of packets on each path. In this case, we have the expected number of arrival by time t given by:

$$E[N(t)] = \mu t \quad (5.11)$$

Because $E[N(\mathcal{D}_{min})] = l$, we have $\mathcal{D}_{min} = l/\mu$. \square

The following theorem presents a coding scheme that provides a good tradeoff between the code rate and the file transmission delay.

Theorem 15. For a file with $k = l \cdot n_p$ packets, coding the file with $n = m \cdot n_p$ packets, where

$$m = \left(\frac{1/\sqrt{2\pi} + \sqrt{1/(2\pi) + 4l}}{2} \right)^2,$$

will result in a file delay that is $O(\sqrt{l})/\mu$ away from \mathcal{D}_{min} .

Proof. By letting $\mu t = m$, the expected number of packet arrived destination node by time t is given by:

$$E[N(t)] = m - \sqrt{\frac{m}{2\pi}}$$

In order to reconstruct the file at the destination node, we must have $E[N(t)] = l$ also. Combining the previous two equations, we have

$$m = \left(\frac{1/\sqrt{2\pi} + \sqrt{1/(2\pi) + 4l}}{2} \right)^2.$$

The file transmission delay with m packets on each path will result in a delay

$$\mathcal{D} = \frac{m}{\mu}.$$

The difference of the above delay and \mathcal{D}_{min} is given by:

$$\begin{aligned} \mathcal{D} - \mathcal{D}_{min} &= \frac{m - l}{\mu} = \frac{1}{\mu} \sqrt{\frac{m}{2\pi}} \\ &= \frac{1}{\sqrt{2\pi}} \frac{1/\sqrt{2\pi} + \sqrt{1/(2\pi) + 4l}}{2\mu}. \end{aligned}$$

□

The above theorem states that using coding rate

$$\left(\frac{1/\sqrt{2\pi} + \sqrt{1/(2\pi) + 4l}}{2}\right)^2 \frac{1}{l}$$

we can achieve a file transfer delay that is asymptotically optimal. That is, let

$$\epsilon = \frac{\mathcal{D} - \mathcal{D}_{min}}{\mathcal{D}_{min}},$$

we then have the difference between the delay obtained using our simple coding scheme and the minimum delay goes to zero as l gets large (i.e., $\epsilon = 1/O(\sqrt{l})$). This idea is illustrated in the figure below. Here, for simplicity, we let $\mu = 1$. The x-axis of the plot indicates the file transmission delay. The y-axis denotes the number of packet (per path) in the original file. Each curve in the plot represents a coding and file delay tradeoff for a fixed number of packets assigned to each path. For example, the top curve represents the coding and file delay tradeoff if we assign six packets to each path. Let the x-axis and y-axis of the points A and B be represented by (A_x, A_y) and (B_x, B_y) respectively. Thus, if the original file contains B_y packets per path, the file transmission delay will be B_x if we encoded the original file to six packets per path. As for point A, if the original file size is still $B_y = A_y$, the delay will be A_x if we encoded the original file with infinity number of packets on each path. The benefit of using a code rate of infinity rather than $6/B_y$ is the reduction of the file transmission delay by $B_x - A_x$. The reduction of a file delay for using a code rate of $5/B_y$ instead of infinity is also shown in the plot. For the same file, the code rate of $6/B_y$ and $5/B_y$ will result in a different file transmission delay. Obviously, depending on the source's preference, one rate may be more suitable than the other. Using the transmission strategy stated in Theorem 15, the resulting coding and file delay tradeoff is plotted with a dashed line. As l gets large, this transmission strategy achieves a code rate of almost one (i.e., the redundant packets is negligible comparing with the original file size), and a file delay that is within $O(\sqrt{l})/\mu$ of the minimum file delay. Hence,

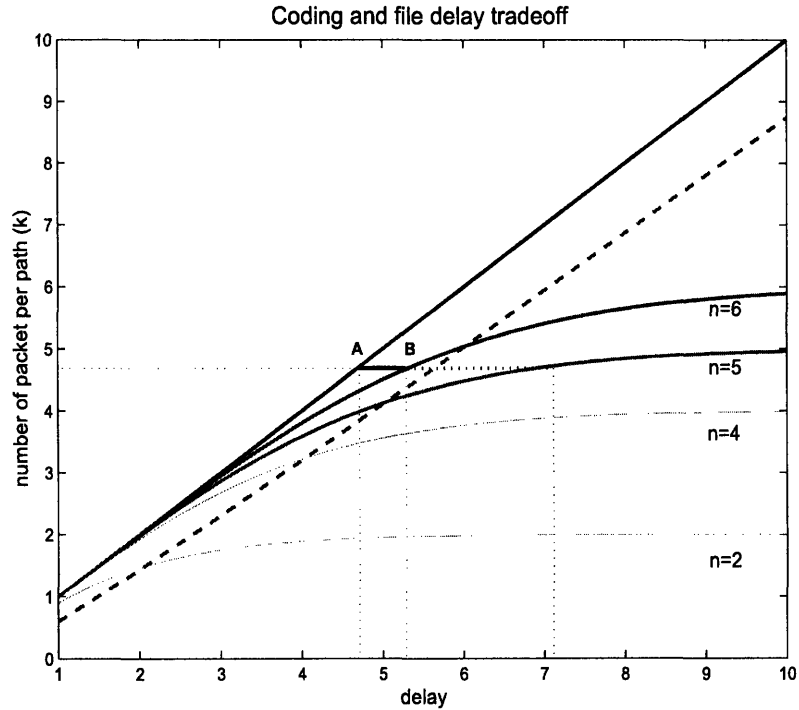


Figure 5-1: Coding and file delay tradeoff.

Theorem 1 can serve as a practical guideline for adding redundant packets to the original file in order to reduce the file transmission delay.

5.3.2 Case II: $n \leq n_p$

In the case where the number of parallel paths, n_p , is larger than the number of encoded packets in a file, n , the expected file transmission delay can be obtained without the assumption that n_p is large. Assigning at most one packet to each path, the expected file transmission delay will be the expected value of the k th largest random variable out of the n random variables. Let X_i for $i = 1, \dots, n$ denote the arrival time of the i th packet. Then, let S_i for $i = 1, \dots, n$ denote the order statistics of X_i . The expected value of S_1 (i.e., the first arrival of n independent Poisson

processes) is given by the following:

$$E[S_1] = \frac{1}{n\mu}.$$

Due to the memoryless property of the exponential distribution, the expected inter-arrival time between the first packet and the second packet is given by:

$$E[S_2 - S_1] = \frac{1}{(n-1)\mu}.$$

Hence, the file transmission delay, which is S_k , has the following form:

$$E[S_k] = \sum_{i=0}^{k-1} \frac{1}{(n-i)\mu}. \quad (5.12)$$

In the figure below, we plot the file transmission delay using different code rate (i.e., different n) for $k = 10$. We see that the reduction of file transmission delay can be significant even with a moderate coding rate. From integration, we have the following:

$$E[S_k] - \frac{1}{n\mu} \geq \frac{1}{\mu} \int_{n-k+1}^n \frac{1}{x} dx \geq E[S_k] - \frac{1}{(n-k+1)\mu}$$

or

$$\begin{aligned} -\frac{1}{\mu} \ln\left(1 - \frac{k-1}{n}\right) + \frac{1}{n\mu} &\leq E[S_k] \\ &\leq -\frac{1}{\mu} \ln\left(1 - \frac{k-1}{n}\right) + \frac{1}{(n-k+1)\mu}. \end{aligned} \quad (5.13)$$

The term $-\ln(1 - \frac{k-1}{n})$ dictates the decrease rate of the delay.

Now, consider the case where the number of packets in a file to be large while still satisfying $k < n < n_p$. We use $N_i(t)$ as an indicator random variable to denote whether the i th coded packet has arrive its destination node by time t . At the destination node, to reconstruct the original file, we need the following to hold:

$$\sum_{i=1}^n N_i(t) \geq k \quad (5.14)$$

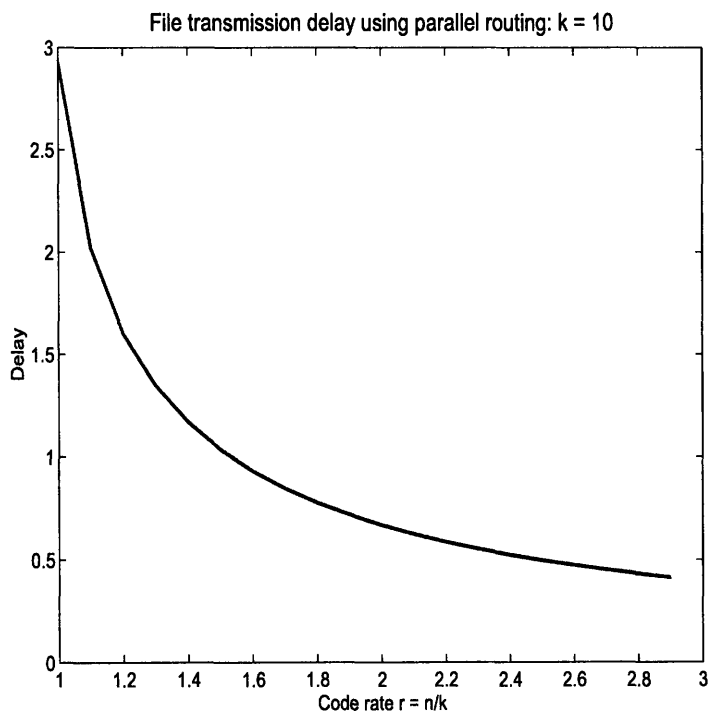


Figure 5-2: Coding and file delay tradeoff.

or

$$\frac{1}{n} \sum_{i=1}^n N_i(t) \geq \frac{k}{n} \quad (5.15)$$

As the number of coded packet gets large, from the law of large number, we have

$$\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i=1}^n N_i(t) \rightarrow E[N_1(t)]. \quad (5.16)$$

Since $N_1(t)$ is an indicator random variable, we can rewrite

$$E[N_1(t)] \geq \frac{k}{n}$$

as

$$P(N_1(t) = 1) \geq \frac{k}{n}.$$

Now, let \mathcal{D}_p denote the transmission delay of a single packet. We will then have the

following:

$$P(N_1(t) = 1) = P(\mathcal{D}_p \leq t). \quad (5.17)$$

Again, let \mathcal{D} denote the file transmission delay in this multi-users transmission scenario. Then, the file transmission delay can be written as

$$\mathcal{D} = \inf\{t : P(\mathcal{D}_p \leq t) \geq \frac{k}{n}\}.$$

With exponentially distributed packet transmission delay, we have the file transmission delay given by the following:

$$\mathcal{D} = -\frac{1}{\mu} \ln\left(1 - \frac{k}{n}\right) \quad (5.18)$$

Note the similarity between the above equation and Eq. (5.13).

5.4 Delay-coding tradeoff for multiple source destination pairs

In the previous section, we have considered adding redundancy to a file so as to reduce the file transmission delay by using the multiple paths between the source and destination pair. No other source and destination pair is using these multiple paths. File delay is being reduced since we do not have to wait for the last few packets to arrive. Now, if there are multiple users, or source destination pairs, are using these paths, adding redundancy will increase the system load and queueing delay for each packet. Consequently, the end-to-end delay (i.e., the transmission delay plus the queueing delay) for each *individual packet* will increase. Hence in this section, we are going to explore whether redundancy can still reduce a file transmission delay in a multiple users environment.

Again, we assume that there are n_p identical paths between the sources and the destinations. The packet transmission delay distribution is the same for all paths. Here, in the multiple users' environment, the packet transmission delay denotes the

time interval that begins with a packet is being served at the source node to the time that this packet reaches its destination node (i.e., not including the queuing delay). For a particular path, the transmission delay of a single packet is an identical and independently distributed random variable. Using the same argument as in the single user case, we assume that the packet transmission delay is an exponential random variable with rate of μ . We also assume that there are N users in the system, each generates file with a rate of λ_f files per second. Here, each file contains k packets. We let $n \geq k$ denote the encoded file size after adding the redundant packets. As we will explain later, we assume that $n \ll n_p$ also. Again, the destination can reconstruct a file as soon as the k th packet arrives at the destination. The transmission scenario in the multi-users case is illustrated in the following figure.

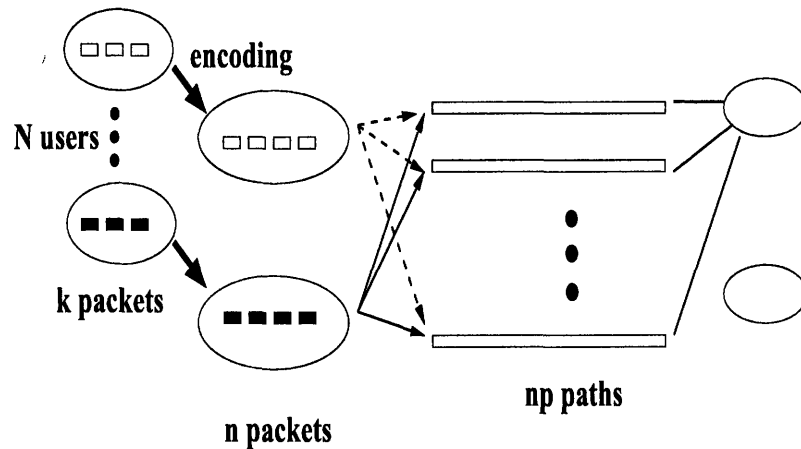


Figure 5-3: Parallel transmission with multiple users sharing a set of identical paths.

Given a file with n encoded packets, using parallel transmission implies that each path will be allocated at most one packet due to $n_p > n$. We now describe a random parallel routing scheme based on parallel transmission:

- Given a file with n encoded packets, the n packets will be randomly assigned to the n_p paths, and no path will contain more than one packet.

Next, we will explore the coding and delay tradeoff in the multiple users environment using the random parallel routing scheme.

Consider a file with n coded packets. Since each packet will be randomly assigned to a different path, we use $N_i(t)$ as an indicator random variable to denote whether the i th packet has arrive its destination node by time t . Note that $N_i(t)$ are i.i.d random variables since $n_p \gg n$. Hence, at the destination node, to reconstruct the original file, we need the following to hold:

$$\sum_{i=1}^n N_i(t) \geq k \quad (5.19)$$

or

$$\frac{1}{n} \sum_{i=1}^n N_i(t) \geq \frac{k}{n} \quad (5.20)$$

Similar to the previous section, as the number of coded packet gets large, from the law of large number, we have

$$\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i=1}^n N_i(t) \rightarrow E[N_1(t)]. \quad (5.21)$$

Since $N_1(t)$ is an indicator random variable, we can rewrite

$$E[N_1(t)] \geq \frac{k}{n}$$

as

$$P(N_1(t) = 1) \geq \frac{k}{n}.$$

Now, let \mathcal{D}_p denote the end-to-end delay of a single packet (i.e., transmission delay plus queueing delay). We will then have the following:

$$P(N_1(t) = 1) = P(\mathcal{D}_p \leq t). \quad (5.22)$$

Again, let \mathcal{D} denote the file transmission delay in this multi-users transmission scenario. Then, the file transmission delay can be written as

$$\mathcal{D} = \inf\{t : P(\mathcal{D}_p \leq t) \geq \frac{k}{n}\}.$$

Next, we want to obtain the distribution of \mathcal{D}_p . The arrivals of packets to a particular path is a poisson process with rate

$$\lambda = \frac{N\lambda_f n}{n_p}.$$

This can be seen from the following reasons:

- There can be at most one packet from a file assigned to a particular path.
- The packets of a file are assigned to the parallel paths randomly.
- The file are generate according to a poisson process with rate λ_f .

Since the service time of each packet, or the transmission time, is an exponentially distributed random variable with rate μ , the distribution of the end-to-end delay of a packet is the same as that of a M/M/1 queue. The total time of packet spent in a M/M/1 queue is well known. Hence, \mathcal{D}_p has the following cumulative distribution function:

$$F_{\mathcal{D}_p}(t) = 1 - e^{-\mu(1-\rho)t} \quad (5.23)$$

where

$$\rho = \frac{\lambda}{\mu} = \frac{N\lambda_f n}{n_p \mu}.$$

Since \mathcal{D} is defined to be $F_{\mathcal{D}_p}(\mathcal{D}) = k/n$, we have the file delay given by:

$$\mathcal{D} = \frac{-\ln(1 - \frac{k}{n})}{\mu(1 - \rho)}. \quad (5.24)$$

We define the original load of the network ρ_0 as the following:

$$\rho_0 = \frac{N\lambda_f k}{n_p \mu}.$$

Also writing the code rate $r = n/k$, the file delay can be rewritten as:

$$\mathcal{D} = \frac{-\ln(1 - \frac{k}{n})}{\mu(1 - \frac{n}{k}\rho_0)}$$

$$= \frac{-\ln(1 - \frac{1}{r})}{\mu(1 - r\rho_0)}. \quad (5.25)$$

Examining the above expression, we see that the code rate has to satisfy the following expression in order for the network to have a finite file transmission delay:

$$1 < r < \frac{1}{\rho_0}.$$

The following lemma shows that there exists a unique code rate which minimizes the file transmission delay.

Lemma 13. *Let*

$$f(r) = \frac{-\ln(1 - \frac{1}{r})}{\mu(1 - r\rho_0)}.$$

There exists a unique r^ such that*

$$f(r^*) = \inf_{1 < r < 1/\rho_0} f(r).$$

Proof. Taking the first derivative of $f(r)$, we have

$$f'(r) = \frac{1}{\mu(1 - r\rho_0)} \left(-\frac{1}{r^2(1 - \frac{1}{r})} - \frac{\ln(1 - \frac{1}{r})\rho_0}{1 - r\rho_0} \right)$$

Setting $f'(r) = 0$, the stationary point satisfy the following equation:

$$-\frac{\ln(1 - \frac{1}{r})\rho_0}{1 - r\rho_0} = \frac{1}{r^2(1 - \frac{1}{r})}$$

which implies

$$[-\ln(1 - \frac{1}{r})]r^2(1 - \frac{1}{r}) + r = \frac{1}{\rho_0} \quad (5.26)$$

We will first show that

$$g(r) = -[\ln(1 - \frac{1}{r})]r^2(1 - \frac{1}{r})$$

is a strictly increasing function of r . That is, we need to show that $g'(r) > 0$ for all

$1 < r < 1/\rho_0$, or

$$\begin{aligned} g'(r) &= -[\ln(1 - \frac{1}{r})](2r - 1) - 1 > 0 \\ \Rightarrow (1 - \frac{1}{r}) &< e^{-\frac{1}{2r-1}}. \end{aligned}$$

Let $y = 1 - \frac{1}{r}$, we then need to show that

$$y < e^{-\frac{1-y}{1+y}}$$

for $y \in (0, 1)$. From the following inequality

$$e^x > x + 1 \quad \text{for } x < 1,$$

we have

$$e^{-\frac{1-y}{1+y}} > -\frac{1-y}{1+y} + 1 = \frac{2y}{1+y}. \quad (5.27)$$

Indeed, we have

$$\frac{2y}{1+y} > y$$

since $y \in (0, 1)$. Thus, we have shown that $g(r)$ is an strictly increasing function for $1 < r < 1/\rho_0$. It is straightforward to see the left hand side of Eq. (5.26) is a continuous and strictly increasing function. Now, for an arbitrary ρ_0 , we want to show that there exists a unique r such that $g(r) + r = 1/\rho_0$. Since we already know that $g(r) + r$ is a strictly increasing function in $(1, 1/\rho_0)$, we only to show that there exists r_1 and r_2 such that $g(r_1) + r_1 < 1/\rho_0$ and $g(r_2) + r_2 > 1/\rho_0$. To see that there exists a $r_1 \in (1, 1/\rho_0)$ such that $g(r_1) + r_1 < 1/\rho_0$, we let r approaches 1. As r approaches 1, $g(r)$ will approach zero since

$$\lim_{x \rightarrow 0} x(\ln \frac{1}{x}) = 0.$$

Hence, we have $g(r) + r$ approaches one, which is strictly smaller than $1/\rho_0$, as r approaches one. Thus, we can always find r_1 such that $g(r_1) + r_1 < 1/\rho_0$.

To see that there exists an r_2 such that $g(r_2) + r_2 > 1/\rho_0$, we let r approaches $1/\rho_0$ from below. In this case, $g(r)$ approaches the following

$$-\ln(1 - \rho_0) \left(\frac{1}{\rho_0}\right)^2 (1 - \rho_0) > 0.$$

Hence, we can find r_2 such that $g(r_2) + r_2 > 1/\rho_0$ when r approaches $1/\rho_0$.

Let r^* satisfy the following expression

$$\left[-\ln\left(1 - \frac{1}{r^*}\right)\right] (r^*)^2 \left(1 - \frac{1}{r^*}\right) + r^* = \frac{1}{\rho_0} \quad (5.28)$$

for we have shown that r^* exists. Examine the following term in $f'(r)$:

$$\left(-\frac{1}{r^2\left(1 - \frac{1}{r}\right)} - \frac{\ln\left(1 - \frac{1}{r}\right)\rho_0}{1 - r\rho_0}\right).$$

For $r < r^*$, the above term is less than zero, and greater than zero for $r > r^*$. We see the continuous function $f(r)$ is decreasing for $r < r^*$ and increasing $r > r^*$. Hence, r^* is the unique minimum of the function $f(r)$. \square

Given the load of the network ρ_0 , the optimal code rate that minimizes the file transmission delay is the r^* satisfy Eq. (5.28). For various value of ρ_0 , we plot the file transmission delay as a function of code rate $r = n/k$. In the bottom plot of Fig. 5-4, we plot a file transmission delay under a light load (i.e., ρ_0 is small). The file transmission delay decreases sharply as a few redundant packets are added. However, the reduction of delay becomes small as more redundant packets are added. This behavior is similar the one exhibited in the single source and destination pair case. As we expected, when more and more redundant packets are added to the network, the file transmission delay starts to increase. On the top plot of Fig. 5-4, a plot of the file transmission with high load is shown. With a small code rate, the file transmission starts to increase.

Intuitively, using parallel transmission will reduce the file transmission delay in the single source and destination case. In the multi-users communication scenario, one can employ a routing scheme, which is termed the serial routing scheme, that assign

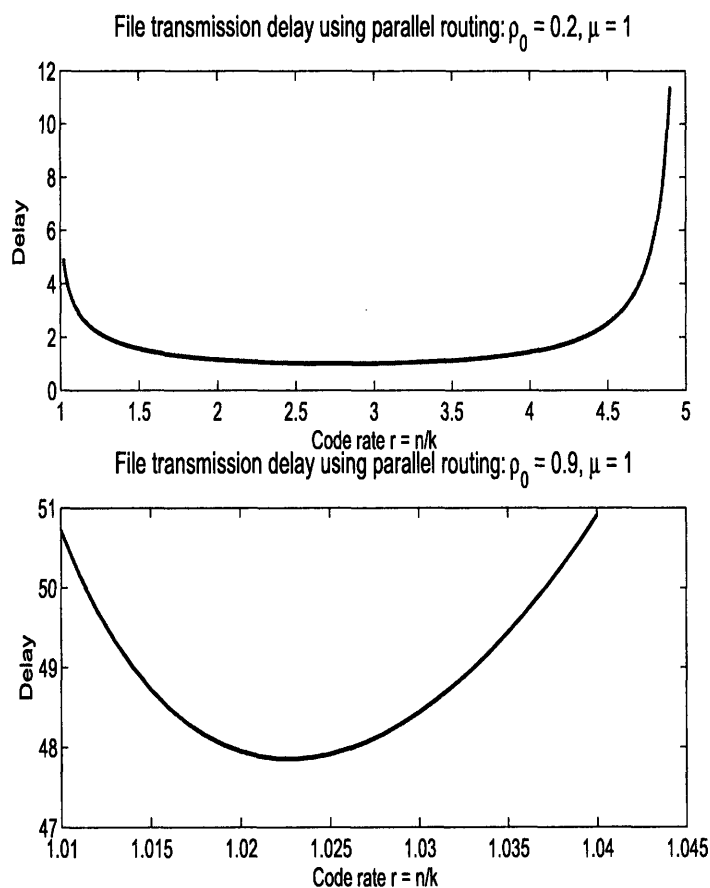


Figure 5-4: Comparison of the file transmission delay under different traffic load.

all n packets of a particular file to a randomly chosen path. At this point, it is not clear that using parallel transmission with coding will have a shorter file transmission delay than that of a serial routing scheme in the multiple users environment. We are going to compare these two transmission schemes now.

For the serial routing scheme, the file arrival rate to each path is the following:

$$\lambda_p = \frac{N\lambda_f}{n_p}.$$

Since the path is randomly chosen and the files are generated according to a Poisson process, the file arrival to a particular path is also a Poisson process. On a particular path, the service time for a file with k packets is a sum of k exponential random

variable. So the path can be thought of as a M/G/1 queue. Let \bar{X} and \bar{X}^2 denote average service time and the second moment of service time respectively. We then have

$$\begin{aligned}\bar{X} &= \frac{k}{\mu} \\ \bar{X}^2 &= \frac{k}{\mu^2} + \left(\frac{k}{\mu}\right)^2\end{aligned}$$

The total waiting time for file, in queue and in service, is

$$\begin{aligned}\mathcal{D}_s &= \bar{X} + \frac{\lambda_p \bar{X}^2}{2(1 - \lambda_p \bar{X})} \\ &= \frac{k}{\mu} + \frac{\frac{N\lambda_f}{n_p} \left(\frac{k}{\mu^2} + \left(\frac{k}{\mu}\right)^2\right)}{2\left(1 - \frac{N\lambda_f}{n_p} \left(\frac{k}{\mu}\right)\right)} \\ &= \frac{k}{\mu} + \frac{\rho_0(1+k)}{2\mu(1-\rho_0)}\end{aligned}$$

Given a file size of k packets and a load of ρ_0 , the minimum delay using the parallel transmission, denoted here as \mathcal{D}^* , is achieved by using a code rate of r^* given in Eq. (5.28). In Fig. 5-5, we plot the file transmission delay for both parallel routing scheme and serial routing scheme. Here, the service rate μ is equal to one, and the number of packets in a file $k = 6$. To get the minimum file transmission \mathcal{D}^* using the parallel routing scheme, we first get ρ_0 for a given code rate r according to Eq. (5.28). From this pair of r and ρ_0 , we can derive the minimum transmission delay according to Eq. (5.25). In the figure, the file transmission delay of the parallel routing scheme is much smaller than that of the serial routing scheme when the load of the network ρ_0 is small. As ρ_0 gets large, the delay of the parallel routing scheme eventually surpasses the delay of the serial routing scheme. Hence, we see that multipath parallel routing with coding helps to reduce the file transmission delay only when the system load ρ_0 is not too large. Note also that the number of packets in a file, k , will not affect the file transmission when using the parallel routing scheme due to the random assignment of packets on different paths. However, k will affect the file transmission delay of the serial routing scheme. In fact, keeping the load ρ_0 to be a constant, the file

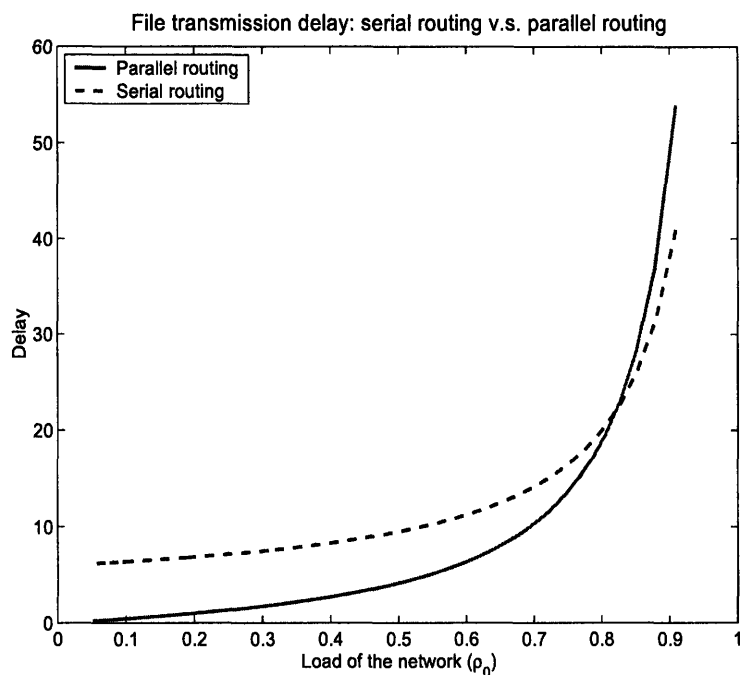


Figure 5-5: Comparison of the file transmission delay using the parallel routing scheme and the serial routing scheme.

transmission delay increases linearly as k increases.

5.5 Conclusion

In this chapter, we explore the use of multipath routing to reduce the file transmission delay in a wireless network. By avoiding the long tail in the distribution of a packet's transmission time, we show that the file transmission delay can be significantly reduced with only a few redundant packets in the single source destination case. For a given file, an encoding strategy is provided to obtain a good code and file transmission delay tradeoff. In the multiple users communication scenario, we show that there exists a unique code rate, which depends on the traffic load of network, that minimize the average file transmission delay. This optimal code is also presented for a given load.

As for future research, we are planning to investigate the file delay under a differ-

ent path model, where there are multiple hops on the path between the source and destination nodes. Similarly, the file transmission delay in the case where each path has a different average packet transmission time is also worth exploring.

Chapter Appendix: Proof of Lemma 11

We first show that the above lemma is true for the case that there is no redundant packets added to the original file (i.e., no coding). Consider the following packet allocation vector $\vec{a} = \{m_1 - 1, m_2 + 1, m_3, \dots, m_{n_p}\}$. We will show that the expected file transmission delay for the allocation vector \vec{a} , denoted here as $E[\mathcal{D}_a]$, is less than the expected file transmission delay for an allocation $\vec{b} = \{m_1, m_2, m_3, \dots, m_{n_p}\}$ which is denoted as $E[\mathcal{D}_b]$.

Now, we focus on the first two paths among these n_p paths. There are m_1 packets that needs to be transmitted on path one and m_2 packets that needs to be transmitted on path two, where $m_1 \leq m_2$. We denote the time takes for all m_1 packets on path one to arrive the destination node to be T_{m_1} . Likewise, the time takes for all m_2 packets on path two to arrive destination can be defined as T_{m_2} . We then let $T_{(m_1, m_2)} = \max\{T_{m_1}, T_{m_2}\}$. Hence, $T_{(m_1, m_2)}$ is the time takes for all packets in both paths to arrive the destination. Then, we consider another allocation of packets on these two paths, with $m_1 - 1$ packets on one path and $m_2 + 1$ packets on the other. The terms $T_{m_1 - 1}$, $T_{m_2 + 1}$, and $T_{(m_1 - 1, m_2 + 1)}$ can be similarly defined. The time takes for all packets on these two paths to arrive the destination is now denoted as $T_{(m_1 - 1, m_2 + 1)}$. The allocation (m_1, m_2) is more balanced than $(m_1 - 1, m_2 + 1)$ since $m_1 < m_2$. Lastly, we define the T_0 to be the time required for all packets on the other paths to arrive the destination node. Then, we can write $\mathcal{D}_a = \max\{T_{(m_1 - 1, m_2 + 1)}, T_0\}$ and $\mathcal{D}_b = \max\{T_{(m_1, m_2)}, T_0\}$. By showing $P(T_{(m_1, m_2)} < t) > P(T_{(m_1 - 1, m_2 + 1)} < t)$ for all t , we can prove $E[\mathcal{D}_a] \geq E[\mathcal{D}_b]$.

Now, we show that $P(T_{(m_1, m_2)} < t) > P(T_{(m_1 - 1, m_2 + 1)} < t)$ for all t . That is, the cumulative distribution function of $T_{(m_1, m_2)}$ is greater $T_{(m_1 - 1, m_2 + 1)}$ pointwisely, which implies that the more balanced allocation (i.e., (m_1, m_2)) tends to have a smaller transmission delay than the delay of the unbalanced allocation.

In order to get the cumulative distribution function of $T_{(m_1, m_2)}$, we need to get an expression for T_{m_1} . Consider path one with m_1 packets requires to be transmitted. The arrival time of the m_1 th packet at the destination is statistically the same with

the arrival time of the m_1 th arrival in a poisson process with rate μ . Hence, we can write the cumulative distribution of T_{m_1} as follows:

$$\begin{aligned} P(T_{m_1} \leq t) &= P(N(t) \geq m_1) \\ &= \sum_{k=m_1}^{\infty} \frac{(\mu t)^k e^{-\mu t}}{k!} \end{aligned}$$

The cumulative distribution function of $T_{(m_1, m_2)}$ and $T_{(m_1-1, m_2+1)}$ are given by the following:

$$\begin{aligned} P(T_{(m_1, m_2)} < t) &= P(T_{m_1} < t) \cdot P(T_{m_2} < t) \\ &= \left(\sum_{k=m_1}^{\infty} \frac{(\mu t)^k e^{-\mu t}}{k!} \right) \left(\sum_{k=m_2}^{\infty} \frac{(\mu t)^k e^{-\mu t}}{k!} \right) \end{aligned} \quad (5.29)$$

$$\begin{aligned} P(T_{(m_1-1, m_2+1)} < t) &= \left(\sum_{k=m_1-1}^{\infty} \frac{(\mu t)^k e^{-\mu t}}{k!} \right) \\ &\quad \cdot \left(\sum_{k=m_2+1}^{\infty} \frac{(\mu t)^k e^{-\mu t}}{k!} \right) \end{aligned} \quad (5.30)$$

To see that the cumulative distribution function of $T_{(m_1, m_2)}$ is greater $T_{(m_1-1, m_2+1)}$ pointwisely, we expand the following expression:

$$\begin{aligned} &P(T_{(m_1, m_2)} < t) - P(T_{(m_1-1, m_2+1)} < t) \\ &= \left(\sum_{k=m_1}^{\infty} \frac{(\mu t)^k e^{-\mu t}}{k!} \right) \left(\sum_{k=m_2}^{\infty} \frac{(\mu t)^k e^{-\mu t}}{k!} \right) \\ &\quad - \left(\sum_{k=m_1-1}^{\infty} \frac{(\mu t)^k e^{-\mu t}}{k!} \right) \left(\sum_{k=m_2+1}^{\infty} \frac{(\mu t)^k e^{-\mu t}}{k!} \right) \end{aligned} \quad (5.31)$$

$$\begin{aligned}
&= \left(\sum_{k=m_1}^{\infty} \frac{(\mu t)^k e^{-\mu t}}{k!} \right) \left(\frac{(\mu t)^{m_2} e^{-\mu t}}{m_2!} + \sum_{k=m_2+1}^{\infty} \frac{(\mu t)^k e^{-\mu t}}{k!} \right) \\
&\quad - \left(\frac{(\mu t)^{m_1-1} e^{-\mu t}}{(m_1-1)!} + \sum_{k=m_1}^{\infty} \frac{(\mu t)^k e^{-\mu t}}{k!} \right) \\
&\quad \cdot \left(\sum_{k=m_2+1}^{\infty} \frac{(\mu t)^k e^{-\mu t}}{k!} \right)
\end{aligned} \tag{5.32}$$

$$\begin{aligned}
&= \left(\sum_{k=m_1}^{\infty} \frac{(\mu t)^k e^{-\mu t}}{k!} \right) \left(\frac{(\mu t)^{m_2} e^{-\mu t}}{m_2!} \right) \\
&\quad - \left(\frac{(\mu t)^{m_1-1} e^{-\mu t}}{(m_1-1)!} \right) \left(\sum_{k=m_2+1}^{\infty} \frac{(\mu t)^k e^{-\mu t}}{k!} \right) \\
&= \left(\sum_{k=m_1}^{\infty} \frac{(\mu t)^{k+m_2}}{k! m_2!} \right) e^{-2\mu t} - \left(\sum_{k=m_2+1}^{\infty} \frac{(\mu t)^{k+m_1-1}}{k! (m_1-1)!} \right) e^{-2\mu t}
\end{aligned} \tag{5.33}$$

Examining the above expression, the first sum contains more summand than the second sum does since $m_1 < m_2$. We can thus rewrite the above expression in terms of the difference of summand in the summation. Specifically, we have the following:

$$\begin{aligned}
&\left(\sum_{k=m_1}^{\infty} \frac{(\mu t)^{k+m_2}}{k! m_2!} \right) e^{-2\mu t} - \left(\sum_{k=m_2+1}^{\infty} \frac{(\mu t)^{k+m_1-1}}{k! (m_1-1)!} \right) e^{-2\mu t} \\
&= \left(\frac{1}{m_1} - \frac{1}{m_2+1} \right) \frac{1}{(m_2)! (m_1-1)!} e^{-2\mu t} (\mu t)^{m_1+m_2} \\
&\quad + \left(\frac{1}{(m_1+1)m_1} - \frac{1}{(m_2+2)(m_2+1)} \right) \\
&\quad \cdot \frac{1}{(m_2)! (m_1-1)!} e^{-2\mu t} (\mu t)^{m_1+m_2+1} + \dots
\end{aligned} \tag{5.34}$$

Since $m_1 < m_2$, we will have $P(T_{(m_1, m_2)} < t) > P(T_{(m_1-1, m_2+1)} < t)$ for all t .

The expected packet transmission delay of allocation vector \vec{a} and \vec{b} are given by:

$$\begin{aligned}
E[\mathcal{D}_a] &= \int_0^{\infty} 1 - P(\mathcal{D}_a \leq t) dt \\
&= \int_0^{\infty} 1 - P(T_{(m_1-1, m_2+1)} \leq t) P(T_0 \leq t) dt
\end{aligned} \tag{5.35}$$

$$\begin{aligned}
E[\mathcal{D}_b] &= \int_0^\infty 1 - P(\mathcal{D}_b \leq t) dt \\
&= \int_0^\infty 1 - P(T_{(m_1, m_2)} \leq t)P(T_0 \leq t) dt
\end{aligned} \tag{5.36}$$

Since $P(T_{(m_1, m_2)} < t) > P(T_{(m_1-1, m_2+1)} < t)$ for all t , we have $E[\mathcal{D}_b] \leq E[\mathcal{D}_a]$. This implies that we can always obtain a smaller file transmission delay if we try to balance the packets allocated to any two paths. Hence, the optimal transmission strategy is to allocate packet evenly among the available paths. Because the transmission delay for each packet is independent, the expected file transmission delay will be minimized by allocating packets evenly on each path for the case the file is coded.

Chapter 6

Conclusion

In this thesis, we address the question of how to utilize a wireless channel in an efficient and fair manner. With finite bandwidth available, users in wireless system often have to compete for the access of the channel. By allowing users to behave selfishly, we try to obtain an allocation algorithm that is distributed and robust.

Specifically, in the downlink case, we apply an auction algorithm to the problem of fair allocation of a wireless fading channel. Using the second price auction mechanism, we are able to obtain the Nash equilibrium strategies for general channel state distribution. Our strategy allocates bandwidth to the users in accordance with the amount of money that they possess. Hence, this scheme can be viewed as a mechanism for providing quality of service (QoS) differentiation; whereby users are given fictitious money that they can use to bid for the channel. By allocating users different amounts of money, the resulting QoS differentiation can be achieved.

We find the unique Nash equilibrium strategy for certain commonly used channel state distribution. We also show that the Nash equilibrium strategy for the second-price auction leads to an allocation at which total throughput is no worse than $3/4$ the maximum possible throughput when fairness constraints are not imposed (i.e., slots are allocated to the user with the better channel) under uniform distribution. Moreover, the equilibrium strategies leads to an allocation that is pareto optimal. Based on the Nash equilibrium strategies of the second price auction with money constraint, we also propose a centralized opportunistic scheduler that does not suffer

the shortcomings associated with the proportional fair and the time fraction fair scheduler. Using the all-pay auction mechanism, we are able to obtain a unique Nash equilibrium strategy. Our strategy allocated bandwidth to the users in accordance with the amount of money that they possess. Hence, this scheme can be viewed as a mechanism for providing quality of service (QoS) differentiation; whereby users are given fictitious money that they can use to bid for the channel.

Nevertheless, in the second price auction, the problem of how to obtain the multiplicative constant in user's equilibrium bidding strategy using a computational efficient way has yet to be explored. Also, to make our proposed centralized scheduler (based on the Nash equilibrium strategy) suitable for real time implementation, an algorithm that does not require the prior knowledge of channel distribution but still results in the Nash equilibrium allocation for each user will be an important topic for the future research.

For the uplink, we studied the scenario where multiple users competing, in a non-cooperative manner, for the access of a single satellite, or base station. With a specified capture rule and an average power constraint, users opportunistically adjust their transmission power based on their channel state to maximize their throughput. We characterized the Nash equilibrium power allocation strategy and quantified the resulting throughput efficiency loss, due to selfish behavior. As the number of users increases, the Nash equilibrium power allocation strategy approaches the optimal power allocation strategy that can be achieved in a cooperative environment.

With multiple channels available for communication, we again investigate how to better utilize these multiple channels available in a satellite network. Specifically, we use pricing as a mechanism to control users' transmission probabilities and exploit different channel qualities to coordinate transmission among users. Hence, the throughput performance of the system is improved. We also characterize the Pareto optimal throughput region for both single satellite network and multiple satellites network. We show that users' throughput is Pareto optimal at the equilibrium price. The characterization of the Pareto optimal throughput region for multiple channels with time varying channel states can be a possible direction for the future research.

To further exploit the multiple channels available for transmission, we study the use of multipath routing to reduce the file transmission delay in a wireless network. By avoiding the long tail in the distribution of a packet's transmission time, we show that the file transmission delay can be significantly reduced with only a few redundant packets in the single source destination case. For a given file, an encoding strategy is provided to obtain a good code and file transmission delay tradeoff. In the multiple users communication scenario, we show that there exists a unique code rate, which depends on the traffic load of network, that minimize the average file transmission delay. This optimal code is also presented for a given load. As for future research, we are planning to investigate the file delay under a different path model, where there are multiple hops on the path between the source and destination nodes. Similarly, the file transmission delay in the case where each path has a different average packet transmission time is also worth exploring.

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