The Electric Vehicle Experiment:
Developing the Theoretical Model for 2.672

by

Matthew R. Zedler

SUBMITTED TO THE DEPARTMENT OF MECHANICAL ENGINEERING IN PARTIAL FULFULLMENT OF THE REQUIREMENTS FOR THE DEGREE OF

BACHELOR OF SCIENCE IN MECHANICAL ENGINEERING
AT THE
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

MAY 2007
[June 2007]

Signature of Author: ______________________
Department of Mechanical Engineering
May 11, 2007

Certified by: ______________________
C. Forbes Dewey
Professor of Mechanical Engineering
Thesis Supervisor

Accepted by: ______________________
John H. Lienhard V
Professor of Mechanical Engineering
Chairman, Undergraduate Thesis Committee

The Electric Vehicle Experiment: Developing the Theoretical Model for 2.672

by

Matthew Robert Zedler
Submitted to the Department of Mechanical Engineering on May 11, 2007 in partial fulfillment of the requirements for the Degree of Bachelor of Science in Mechanical Engineering

ABSTRACT

The purpose of this project was to develop a computer simulation of the proposed 2.672 electric vehicle experiment (EVE) to estimate the magnitudes of the powers required in different components of the drive train, piecewise component and system efficiencies, and the information that would need to be collected to construct different component power models. The EVE model had to incorporate both acceleration and deceleration of the vehicle, using regenerative braking as needed. The resulting model can be used to evaluate the safety and feasibility of the EVE and determine sizes of relevant testing equipment required to implement the EVE.

The model showed that the EVE could work safely and be modeled with a reasonable amount of effort by students in 2.672. The relatively low power flows (under 4kW) allow safe operation while the students are learning about the efficiencies of the individual components. The most inefficient component for the low speeds expected in the EVE was the motor/generator unit, though the efficiency increased as the torque increased. The gearbox and controller efficiencies were modeled as constants in the simplified model since the manufacturer's literature only quoted one value for the gearbox and since there was a lack of detailed information about the controller. The overall system energy recovery efficiency using regenerative braking was low, reaching a maximum of about 40% and falling as low as 10% when higher than expected power flows were used. The theoretical model was simplified by removing the effects of temperature and heat rise. Only with a built EVE can the actual performance of the system be characterized.

Thesis Supervisor: C. Forbes Dewey
Title: Professor of Mechanical Engineering and Bioengineering

TABLE OF CONTENTS

INTRODUCTION

The idea of fully electric vehicles has captured the headlines of newspapers and magazines as increased emphasis is once again placed on energy conservation and transportation alternatives. At the turn of the 20th century, hand-assembled electric vehicles became little more than a novelty item because of the mass-production of gasoline-powered internal combustion engine (ICE) vehicles [1]. While there was a slight resurgence of interest in the technology in the 1970's as a result of the oil crisis, the greatest potential for electric vehicles has arisen only in the past half decade with improvements in battery storage technology.

While there are no purely electric vehicles on the market yet, there are strong indicators of an upcoming emergence. General Motors (GM) has announced several concepts and test fleets in 2006, all using electric motors to power the wheels. The prime example of the recent resurgence is the Tesla Roadster, an electric sports car. With an 185kW electric motor and high energy density lithium-ion battery, this $100,000 vehicle has a 400km range and can be recharged in just 3.5 hours, though at a relatively high charging current. The car is going to first be delivered to the market in 2008, but over 250 cars have already been preordered. There is a demand for electric vehicles, and over two dozen companies have emerged to offer electric vehicles to interested consumers [2].

As an institution of higher learning that strives to teach about the latest technologies, the Massachusetts Institute of Technology has a history of fostering student learning through classroom experiments. The Project Laboratory class (2.672) in the mechanical engineering department is a prime example of a laboratory class that emphasizes "the interplay between analytical and experimental methods in solution of research and development problems" [3]. Students are given different problems and must find ways to utilize the experimental apparatus and tools to determine a solution. The majority of the experiments are concerned with heat and mass transfer rather than broadly covering significant portions of the engineering curriculum. The class instructors are always looking for ways to improve the class, including new experimental setups.

By designing a model of the transmission system of an electric vehicle in this lab space, it is hoped that students will get a better understanding of the performance of these complex systems. In addition, electric vehicle transmission modeling can encapsulate a whole series of inter-related areas, from fluid dynamics to mechanics and from electronics to chemistry. As such, an electric vehicle could provide more alternatives for different variations of the initial problem statement than some of the traditional experiments currently provide. The transmission of an electric vehicle could also be used as a model for the electric components of a hybrid vehicle drive train, thereby increasing the applications of this experiment to the world outside the classroom.

Before investment is made into such a complicated and therefore expensive lab setup, research needed to be done to determine the pedagogical value of an electric vehicle experiment (EVE). A possible physical layout of the proposed experiment had been developed [4], but little investigation into the power flows and details of the student-EVE interaction had been conducted. Without a more detailed knowledge of the modeling requirements for each component and the maximum powers possible in the machine, it was impossible to determine whether or not the EVE would be a safe and valuable experimental module for 2.672. The purpose of this thesis was therefore to develop a power flow model of the EVE and use those results to evaluate the pedagogical value of this experimental idea.

PREVIOUS WORK

The development of a stand-alone vehicle transmission in a laboratory setting that is safe, reliable, and educational for students requires several steps. Young [4] developed the setup and specifications for the physical components, determining that the same power generation system used in the electric vehicle could be used in the EVE. The inertial system was modified so the kinetic energy of the vehicle was stored in a flywheel (Fig. 1).

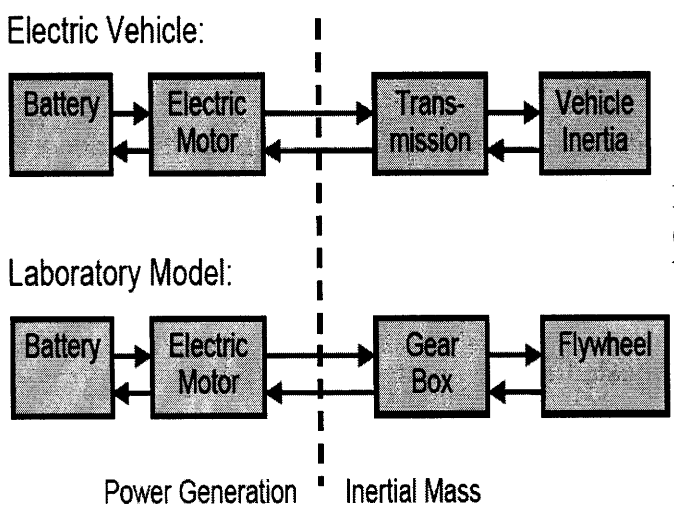


Fig. 1: Theoretical representation of electric vehicle and EVE. Note that the power generation systems are identical; the only modification is on the inertial mass side of the transmission.

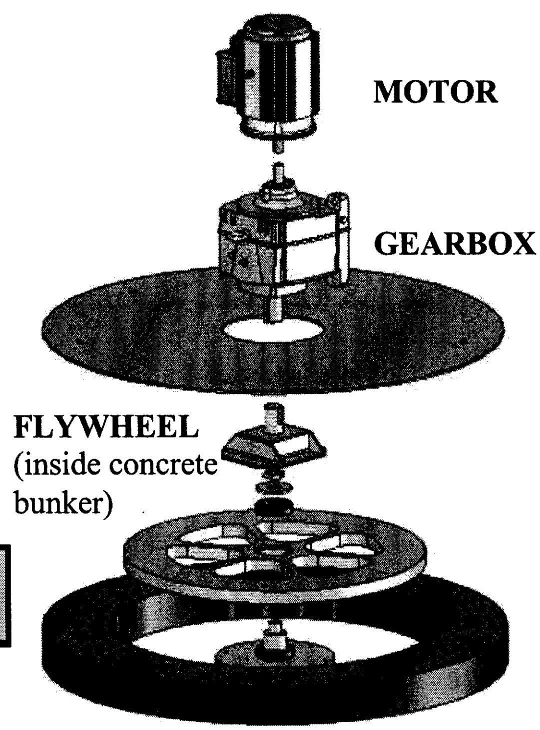


Fig. 2: Exploded view of EVE model. The motor, gearbox, and flywheel are connected in series, and the flywheel is housed in the steel and concrete containment bunker.

The EVE system (Fig. 2) was designed to have the same kinetic energy and rotational motor speed as an electric vehicle. By equating the kinetic energy of the electric vehicle and the EVE as well as the rotational speed of the electric vehicle and EVE motors, Young derived the following to find the moment of inertia I [kg/m^3] of the EVE's flywheel:

$$I = \frac{b^2}{n^2} m_{car} r_{car\ wheels}^2 \tag{1}$$

where m_{car} [kg] was the mass of the vehicle being modeled, b was the gearbox ratio of the EVE, n was the gearbox ratio of the electric vehicle being modeled, and $r_{car\ wheels}$ was the radius of the tires on the vehicle being modeled[1].

It was determined that the EVE must be capable of storing 8kJ in steady-state operation, the equivalent of having the electric vehicle operate at 10mph. Young determined the optimal size and design of the flywheel, balancing the safety need for a slow rotational speed and the design need for a relatively compact flywheel. Once the flywheel was designed, the containment bunker was designed and the other components were sized and selected (Fig. 3).

PART	DETAILS
Flywheel	15 kgm^2 steel disc (115kg, 0.45m radius, spoke-patterned design)
Bearings	Timken tapered bearings (2.75" diameter bore, 29675 inner & 29620 outer races)
Gearbox	Emerson Industrial CBN3000 gear reducer (5:1 gear ratio)
Motor	Azure Dynamics AC24 AC induction motor (14kW, 74Nm peak torque)
Controller	Solectrica DMOC445 DC-AC power electronics controller
Battery Bank	13 series-connected DieHard Gold North 33065 lead-acid batteries (875 CCA [2])

Fig. 3: Components selected for EVE model.

[1] Young developed the model for a 2006 Honda Insight coupe (m_{car} = 820kg, n = 10, $r_{car\ wheels}$ = 0.27m), but substitution could be made for another vehicle.

[2] Cold Cranking Amps (CCA) is a rating used in the battery industry to define a battery's ability to start an engine in cold temperatures. The rating is the number of amps a new, fully charged battery can deliver at 0° Farenheit for 30 seconds, while maintaining a voltage of at least 7.2 volts, for a 12 volt battery [9].

PROBLEM STATEMENT

The purpose of this project was to develop a computer simulation of the proposed 2.672 electric vehicle experiment to estimate the magnitudes of the powers required in different components of the drive train, piecewise component and system efficiencies, and the information that would need to be collected to construct different component models. The model had to incorporate both acceleration and deceleration of the vehicle, using regenerative braking as needed. The resulting model will be used to evaluate the safety and feasibility of the EVE and determine sizes of relevant testing equipment required to implement the EVE.

The overall goal of the project was to prove the pedagogical value of the EVE by determining the educational value that students could gain by performing this experiment. Therefore, a simplified benefit analysis of this project is included in the discussion section of this report.

MODEL DEVELOPMENT

The EVE model was developed as a series of separate functions that modeled each of the components, allowing the user to easily adjust to rearrangement of the physical layout or a change in the properties of the components used. MATLAB was chosen as the scripting language as it is the language used in the

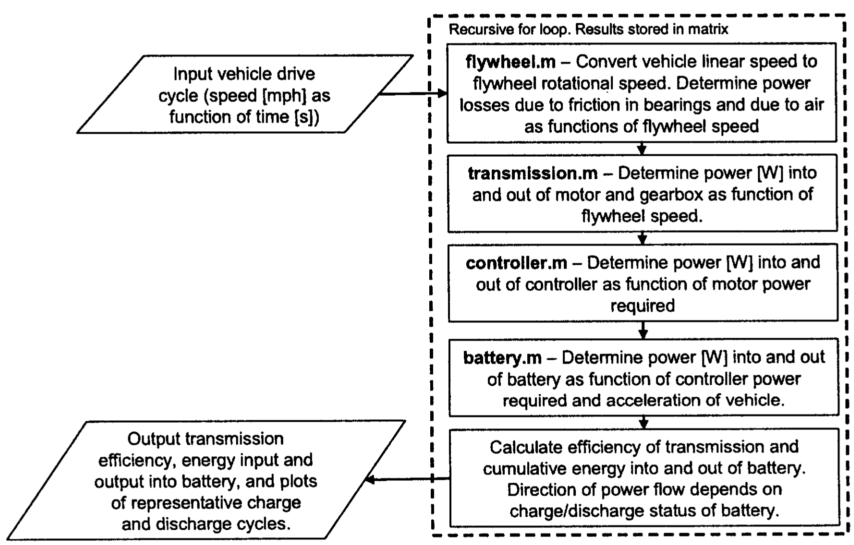


Fig. 4: Overall flow chart for the EVE efficiency model. Each of the components was modeled as a separate m-file to create a modular system that is highly adaptable.

experimental courses (2.671 and 2.672) in the Mechanical Engineering curriculum and would most likely be the language used by students to develop a similar model when conducting the EVE. The model used the drive cycle of a typical vehicle as its input (vehicle velocity [mph] as a function of time). This information was used to determine the corresponding speed of the flywheel in the EVE. The drive cycle could be custom-built or could be a standard drive cycle, such as the Federal Urban Drive Cycle or Rural Drive Cycle[3]. With the speed of the flywheel determined, the power required from the batteries as well as the power losses in the system could then be estimated. Once all the powers were calculated for a time interval, cumulative energy was found, and the drive-train efficiency was determined. Assumptions and limitations for each component model are discussed below, including simple flow diagrams for certain components.

Flywheel (flywheel.m)

The EVE consisted of a battery bank connected to a controller, motor, gearbox, and flywheel in series. Since it was desirable to have the EVE's relation to a road vehicle clearly visible, it was decided to have the user input the speeds of the modeled road vehicle, a 2006 Honda Insight coupe, in comprehensible units [mph]. These speeds were then converted to flywheel rotational speeds [rad/s]. Young [4] related the rotational speed of the flywheel to the linear speed of the vehicle by using a kinetic energy balance:

$$\omega_{in} = \sqrt{\frac{m_{car} v_{car}^{2}}{I_{flywheel}}} \quad (2)$$

where m_{car} [kg] was the mass of the vehicle being modeled, v_{car} [m/s] was taken from the inputted drive cycle, and $I_{flywheel}$ [kg/m2] was the moment of inertia for the flywheel being used in the EVE.

Once the rotational speed of the flywheel was known, the amount of energy stored in the flywheel was calculated using the angular velocity and the moment of inertia of the flywheel. The instantaneous power lost to friction and drag was determined. Using this information, the

[3] Used in the National Renewable Energy Laboratory's ADVISOR analytical software package [5].

theoretical maximum temperature rise inside the flywheel's containment or "bunker" could be found, but the non-ideal nature of the containment bunker meant that the actual temperature rise could not be found.

Bearing friction power losses

The power losses from the frictional forces between the bearing races and the rollers were calculated using the normal force and the coefficient of friction for the bearing lubrication. A pair of tapered roller bearings were used to allow the flywheel to rotate around a central vertical axis, and a preload of 200lbf (271N) had been applied to ensure longer life for the bearings. An industrial lubricant was applied to the roller surfaces before the bearings were preloaded. To calculate the frictional power losses [W], the following was derived from first principles:

$$P_{\text{Loss, Bearing}} = k\mu F_{\text{total}} \times \frac{1}{2}(r_{\text{Outer}} - r_{\text{Inner}}) \times \omega_{\text{flywheel}} \qquad (3)$$

where k is an orientation factor for angled bearing surfaces such as the 45° tapered bearings used in the model, μ is the dimensionless frictional coefficient between the lubricated bearing rollers and the races, F_{total} [N] is the total force of all bearing loads including the preload and any other weight force, and r_{Outer} and r_{Inner} [m] are the outer and inner radii of the bearing assembly respectively.

For both bearings, the coefficient of friction μ was set at a constant 0.005 [6] and the orientation factor k was set at 0.707 because of the 45° angle of the race contact. For the bottom bearing, the total force consisted of the preload plus the weight of the flywheel; for the top bearing, only the preload force was included as the gearbox and motor masses were supported on a separate frame. While an increasing temperature would lower the viscosity of the bearing grease and therefore reduce the power loss, the coefficient of friction changes by a negligible amount for the temperature range determined when modeling the containment as adiabatic. Since the actual temperature rise will be much less than that found adiabatically, the coefficient of friction can be assumed constant.

Air friction power losses

The flow regimes within the bunker encasing the flywheel and air pocket were complex and interrelated, making the calculation of the friction due to air's viscosity difficult. It was initially assumed that flow in the housing would be laminar around all three faces of the flywheel. Using the Navier-Stokes relationship in cylindrical coordinates for the air gap h_3 and in Cartesian coordinates for h_1 and h_2, the shear stresses, frictional forces, and power losses were found[4].

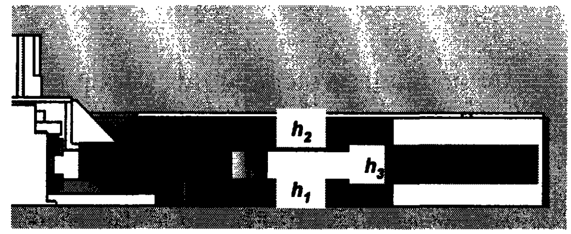


Fig. 5: A cross-section of the bunker containing the flywheel is shown. The relevant air gaps for calculating friction losses are marked.

Upon further investigation, it was decided that there was a certainty of turbulence inside the housing given the non-solid composition of the flywheel and the small size of the air gap between the stationary housing walls and the rotating flywheel's surface. Turbulent drag on a flywheel is a function of the Reynolds number *Re* [10], calculated based on the tip speed of the rotating disc. Once *Re* was known, the following relationships [11] could be used to determine the dimensionless moment coefficient C_M:

$$C_M = 2.67\,\mathrm{Re}^{-1/2}\ (\mathrm{Laminar}, \mathrm{Re} < 3\times10^5) \tag{4}$$

$$C_M = 0.0622\,\mathrm{Re}^{-1/5}\ (\mathrm{Turbulent}, \mathrm{Re} \geq 3\times10^5) \tag{5}$$

$$C_M = \frac{2M}{\frac{1}{2}\rho_{air}\,\omega_{flywheel}^{\ 2}\,R_{flywheel}^{\ 5}} \tag{6}$$

where M [Nm] is the frictional torque exerted by the air boundary layer on the flywheel.
The total power lost to drag was found using the frictional drag torque and the flywheel speed.

This power loss theoretically resulted in a heat rise in the containment housing. To find the

[4] The angular velocity was taken to be constant for any given time t, the solid surfaces of the flywheel, bunker, floor, and lid were all considered impermeable, the flow was considered circumferentially symmetrical, and the air was modeled as incompressible since the flywheel speed was well below Mach 0.8 (approximately 330m/s). The velocity profile was found by assuming the air would be stationary at the lid, floor, and wall of the bunker while it would be moving at the speed of the flywheel at all points on the flywheel.

maximum theoretical heat rise, it was assumed that all of the lost power went into heating the air and steel flywheel. Both air and steel were treated as thermal masses, and the volume-averaged temperature change was found using the specific heats of the substances for each time interval. In reality, the steel flywheel would act as a sink for the heat generated in the bearings and air flow would cool the air inside the containment, so the calculated adiabatic temperature rise was higher than should be expected.

The Reynolds number *Re* depends strongly on the kinematic viscosity of the fluid. Since viscosity and density are strong functions of temperature, the model should ideally take account of the temperature rise once empirical results are obtained. The Sutherland equation (7) [7] can be used to determine air's viscosity, and the density can be found using the ideal gas relation.

$$\mu_{\text{air}} = \frac{A\sqrt{T}}{1+S/T} \tag{7}$$

where A and S are coefficients for air and T is the temperature [K].

It was suspected that the actual temperature rise would be rather low given the leakage from the containment and the ability of the steel flywheel to act as a thermal sink.

Gearbox & Motor (transmission.m)

When the EVE system was designed, the main concerns were speed and weight of the flywheel. The rotational speed of the flywheel was designed to be a maximum of just over 300rpm by using a 5:1 gearbox speed reducer between the AC induction motor and the flywheel connection. The model determined the powers of both components given the desired speed of the flywheel.

Gearbox power losses

The gearbox used in the EVE model was a CBN3000 oil-filled gear reducer from Emerson Industrial Automation. The losses present in any gearbox include tooth friction, oil shearing on the journal bearings, and oil churning in partially-filled cases. Windage, or air friction losses, can be considered negligible because of the low speeds involved.

For the EVE model, the gearbox was assumed to operate with a constant efficiency of 98% as the manufacturer's literature quoted this efficiency for gearboxes using a helical inline reducer for gear ratios under 8:1 [8]. It was assumed that the speeds into and out of the gearbox would be simply related by the gearing ratio. Therefore, all power loss was considered to be a result of loss in torque transmission rather than speed.

Motor power losses

The motor efficiency (Fig. 6) was calculated using performance tests and torque-speed curves[5] provided by the manufacturer, Azure Dynamics (Please see Appendix A2). The required speed at the flywheel was multiplied by the gear ratio (5:1) to find the motor output speed. From the manufacturer's torque-speed curve, the torque at maximum power was found for the given motor speed. The fraction of the max torque actually used in the EVE was found to determine the degree of utilization of the electric motor. The efficiency of the system was then determined as a function of speed from the data given by Azure Dynamics. The torque-efficiency tests showed a relatively constant efficiency as the torque at constant speed increased for speeds ranging from 2000rpm to 9000rpm, with a drop in

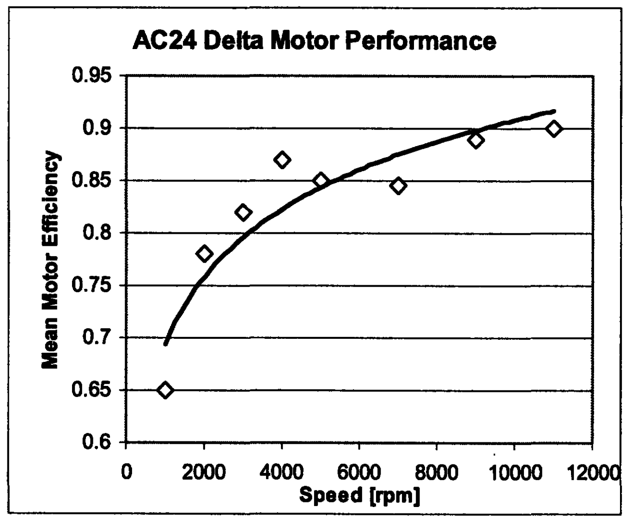


Fig. 6: Quadratic regression relating efficiency and speed. Note that the operational speeds of the flywheel are below 2000rpm, limiting the efficiency.

[5] The provided curves gave efficiencies for speeds between 2,000rpm and 11,000rpm, much higher than the expected operating range of the EVE. As a result, the actual efficiency performance of the motor below 2,000rpm may not be accurately predicted by this model.

efficiency only at very low torques. Since this linear relationship was present, an average efficiency was found for each torque-speed, and a quadratic regression was fitted to this curve (R^2-value of 0.857).

$$\eta_{MotorOut} = 0.093\ln(omega_{MotorOut}) + 0.0517 \qquad (8)$$

where $omega_{MotorOut}$ is the motor output speed or gearbox input speed [rpm].

The motor efficiency increased as the speed increased, with a maximum operational efficiency of around 90% at a speed of 9000rpm, far outside the operating range of the flywheel. As the speeds dropped to the ranges seen in the EVE, the efficiency dropped to a minimum of approximately 51%. The efficiency was used to calculate the power input required from the controller and battery bank.

Power Controller (controller.m)

The Solectrica DMOC445 converted a direct current (DC), unregulated voltage input from the battery bank to an alternating current (AC), regulated voltage output into the motor. Little information was available on the actual specifics of the controller operation, but it was assumed that that power loss would increase with current draw as the DC to AC conversion necessitated the use of a transformer somewhere in the circuit. For this model, the efficiency was modeled as static at 96%, but further investigation should be conducted into the following recommended model or another model formulation:

$$P_{in} = P_{out} + \left(C_1 + C_2 \times P_{out}\right) \qquad (9)$$

where C_1 and C_2 are constants determined empirically

Battery bank (battery.m)

The regenerative nature of the EVE model meant that the battery module had to handle both charging and discharging of the lead-acid battery bank. For the EVE model, thirteen lead-acid DieHard Gold North batteries were modeled as seventy-eight identical series-connected cells (six cells per battery). Specific data could not be acquired for this exact battery type, so the DieHard batteries were generalized as typical lead-acid batteries. The battery model (Fig. 8) could be applied when the battery was either charging or discharging, with the acceleration of the flywheel used to determine the status of the battery. If the flywheel was accelerating or being driven at a constant speed, the battery was being discharged. Alternately, if the flywheel had a negative acceleration (deceleration), the battery was being recharged. Both operational modes were modeled so that the chemical power in the battery could be calculated and used for battery efficiency calculations. A generic battery vector was used to store dynamic information about the voltage [V], current [A], resistance [Ω], capacity [Ah], chemical power [W], chemical energy [kJ], state of charge [%], maximum theoretical temperature [°C], and operating mode as well as static information about the battery's internal volume [m^3], current usage cycle, and maximum battery capacity [Ah]. The model was developed as an idealization of typical battery behavior, accounting for the electrical components rather than the chemical interactions.

Condition	State of Charge [%]	Voltage (Cell) [VDC]
Full battery	100	163.8 (2.1)
Empty battery	0	136.5 (1.75)

Fig. 7: Battery states. The states of charge and corresponding voltages are given for the battery bank and individual battery cells used in the EVE.

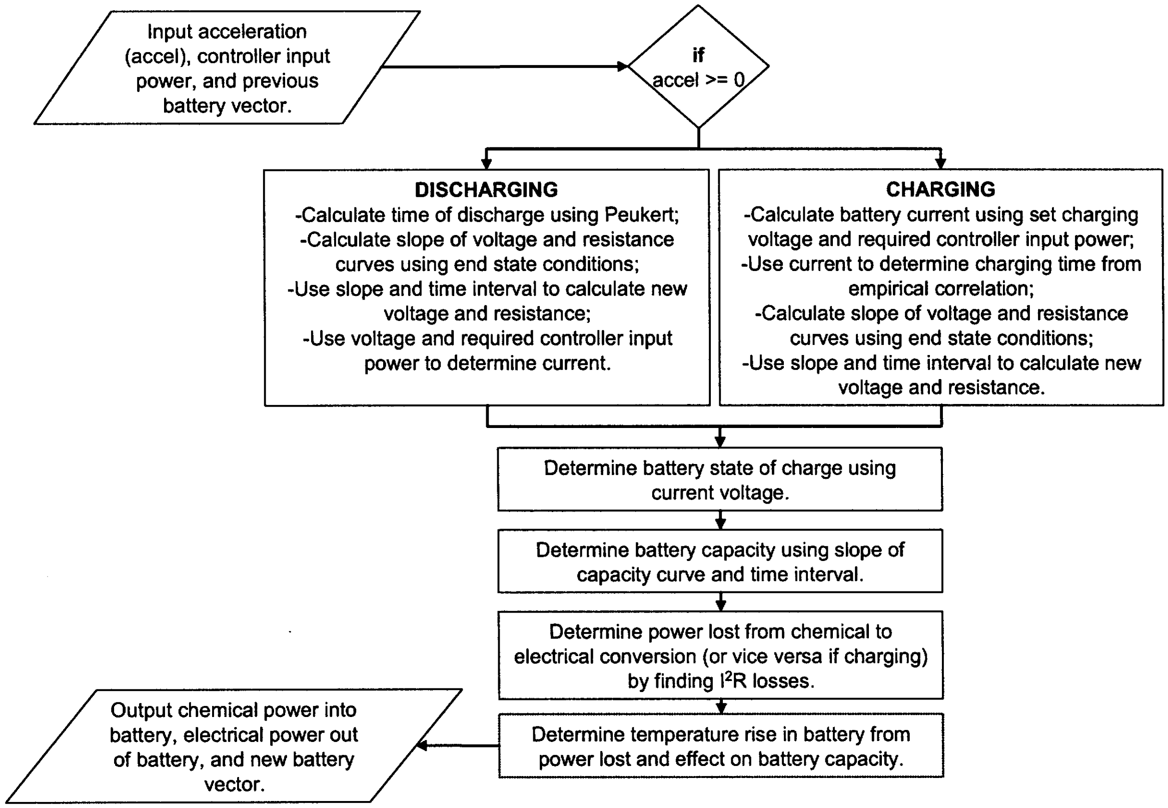


Fig. 8: Battery bank model flowchart. Note that temperature/heat rise box (shown in the dashed box above) was not used in the current work for determining results of model.

For discharging intervals, the battery module calculated the time until complete discharge was reached, the slope of the discharging voltage curve, and electrical current required to power the controller. Linear discharge curves are empirically valid for lead-acid batteries operating between 20% and 80%, with the voltage deviating slightly from the linear model at either end of the battery cycle [12]. For this simplified model, a linear voltage curve was used for the entire battery cycle. When considering the battery bank's internal resistance, there were two main timeframes – the short term discharge or charge of the battery during one usage cycle and the long term number of times the battery had been cycled from 100% to 0% state of charge. Empirical evidence has found battery resistance to increase as the battery is discharged as a result of "plating" of the anode and cathode with lead sulfate, reducing the effective area available for reaction during the short-term [13]. In the long-term, permanent degradation of the electrodes occurs, increasing overall resistance until the battery no longer functions at a reasonable level. It was assumed that the internal resistance increase in the short-term would be linear, and Ohm's law was used to find the electrical current given the discharge voltage and internal resistance. As the number of cycles on the battery increased, this linear curve was shifted up, increasing the overall internal resistance to account for long-term degradation. As the voltage fell and the battery was discharged, the current required to supply the required power to the controller increased. The total time of discharge $t_{discharge}$ was used to determine the rate of change of the voltage and resistance curves. This time was calculated using the modified version of Peukert's empirical relation [14], as shown:

$$t_{\text{discharge}} = C(C/R)^{n-1}/I^{n} \qquad (10)$$

where C is the battery capacity [Ah] at the R rating [h], I is the current [A], n is the Peukert coefficient, and $t_{discharge}$ is the time [h] the battery can be discharged until reaching 0% state of charge.

The Peukert equation was used to determine the remaining life of the battery given the current[6] and the capacity of the previous cycle. Essentially, the battery had a fixed capacity, and increasing the current draw removed progressively more power. This relationship was not linear; the capacity reduces much more quickly at higher current draw rates because of the Peukert coefficient n. For lead acid batteries, the Peukert coefficient falls between 1.1 and 1.3 [14]. A value of 1.3 was used for the model to give the most conservative estimate of capacity lifetime. The modified version of Peukert's original equation was used to account for variance in the battery current draw from the assumed 1A used in Peukert's original empirical relation.

[6] If the initial current was zero, such as at the beginning of the usage cycle, the lifetime of the battery was found using the current flow from the controller since the power and voltage of the controller were determined before the battery module was called.

For charging, the battery model assumed a constant input voltage, calculated the current, determined a charging time based on empirical correlations, and determined battery charging voltage and resistance in a linear fashion similar to that used for the discharging cycle. In the assumed constant potential regime, the input voltage from the controller was set at 185VDC, equivalent to an individual cell charging voltage of 2.4VDC. The relationship between charging time t_{charge} and current was found from standardized lead-acid empirical data for recharging a fully-drained battery[7] (Fig. 9) [15], giving the following power-law relationship (R^2-value of 0.998):

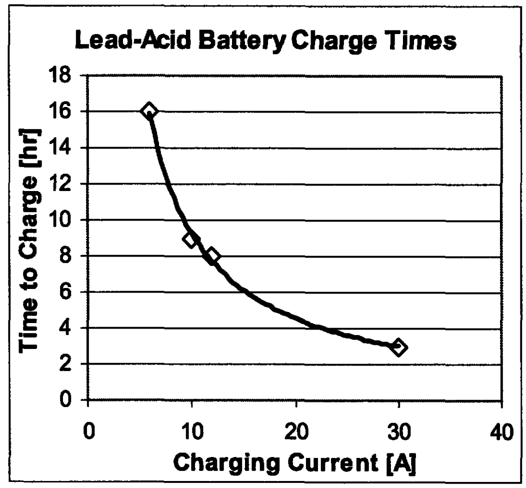


Fig. 9: Effect of charging current on charge time. While this data was found for fully depleted batteries, applying the curve to a battery with a positive state of charge would result in a conservative estimate of total time required to charge.

$$t_{\text{charge}} = 101.19|I|^{-1.034} \times 3600 \qquad (11)$$

While discharging, the resistance of the battery should decrease linearly as the lead sulfate is separated and becomes lead metal and lead oxide at the electrodes. As the battery is cycled through an increasing number of charge and discharge cycles, the overall resistance should increase due to permanent corrosion and degradation of the electrode material.

Battery power losses

Once the voltage, resistance, and current were found for either operational cycle, the state of charge and apparent electrical power were computed. The resistive losses (I^2R losses) were computed, and the chemical power of the battery was computed by adding the resistive losses to

[7] In most circumstances, the battery bank in the EVE was nowhere near fully drained when the recharge was started. Even so, the power law relationship (ampere-hour relationship) has been shown to be empirically valid for lead-acid batteries [15]. In reality, the leading coefficient would change to reflect the capacity previously discharged from the battery, but the relationship between this coefficient and the state of charge was ignored in this simplified model.

the apparent electrical power if the battery was discharging and by subtracting the resistive losses from the controller input power if the battery was charging. As a result, the chemical power of the battery would be higher than the output electrical power in discharge mode, with the opposite being true when the battery was charging. To find the maximum theoretical temperature rise, it was assumed that the resistive losses were captured as heat in an adiabatic battery case[8]. Therefore, the maximum temperature rise of the sulfuric acid electrolyte in the battery could be calculated using the specific capacity and density (assumed to be 10% of the battery volume) of sulfuric acid. Theoretically, this temperature rise should affect battery capacity; as the temperature increases, the battery reactions occur more quickly, and the capacity increases.

The calculation of battery capacity was also dependent on whether the battery was being charged or discharged. The capacity change per second was calculated by finding the difference between current battery capacity and end state battery capacity (maximum capacity if battery was being recharged or zero if battery was being discharged) and dividing by the calculated time of charge or discharge. The times of charge or discharge had been calculated previously using the Peukert relationship (10). While not accounted for in this simple model because of a lack of empirical data, the positive effect of an increase in battery temperature on capacity could theoretically be found using the following:

$$C_{t,actual} = C_t(1 + 0.008\Delta T) \qquad (12)$$

where ΔT is the temperature rise [°C] and C_t is capacity found using either the Peukert or linear relation. Finally, the battery total stored energy [kJ] was found by multiplying the battery voltage and capacity (in [As]). The battery model was highly recursive, using the data from previous time

[8] Technically, the heat will escape from the electrolyte to the lead electrodes and plastic casing which essentially act as heat dissipaters in the same fashion as the flywheel in the containment bunker.

intervals and the change in time to calculate the current voltage, current, resistance, and capacity of the battery bank.

Main program (ev_model.m)

The main program was used to gather user input data and then iterate through the drive cycle. All the battery parameters could be manipulated within this module, allowing the user to change the battery modeled if desired[9]. The user could also decide whether or not to include air drag in the flywheel module by changing a Boolean variable in this module. Once the time and speed data of the drive cycle had been entered, the model needed to run each component module for each data point of the drive cycle to determine the component power losses for the given vehicle speed. Once the power losses were known, first law efficiencies could be found for each component by dividing the output power by the input power. For charging cycles, the power flowed towards the battery from the motor, while in discharging cycles the power flow was reversed. The ***transmission efficiency***, defined as the power out of the gearbox and into the flywheel over the power into the controller from the battery, was found for each given data point of the drive cycle. The energy of each component was then found by determining the area under the power-time curve, using the right-hand bound of each time interval as shown in Fig. 10. The total energy put into the battery during charging periods and the total energy removed from the battery during discharging

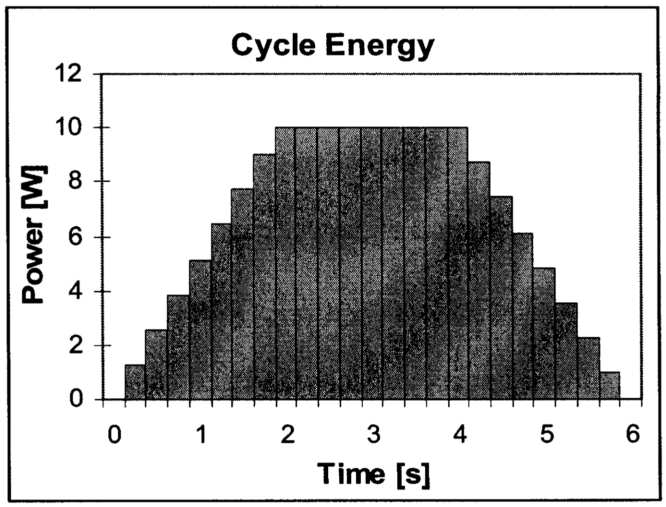


Fig. 10: Cycle energy determination. The cycle energy was found by approximating the area under the power-time curve using a right-hand bound approximation. The cycle energy is shown in gray in the figure.

[9] The user would need to change the battery.m module if using any battery type other than lead-acid as the chemical properties of these batteries are used in this module to calculate theoretical temperature rise.

periods were determined by summing the areas under the power-time curve for these respective areas. The ***recovery efficiency***, or the amount of energy put back into the battery over the amount removed from the battery, could then be found after the cycle analysis was completed.

RESULTS & ANALYSIS

Once the model was developed, several different cycles were tested and analyzed. Simple cycles were used to verify the model's general accuracy, though a more detailed empirical comparison must be undertaken once the system is physically installed. Since the model took as input a matrix of vehicle speed [mph] as a function of time [s], it required little effort to generate several different drive cycles to test various performance aspects.

Test Cycle 1 – Ramp up / Steady / Ramp down

The most basic cycle used was typical of inner city driving or driving on heavily congested roads. This cycle for the electric vehicle drive train involved ramping speeds up to the maximum operating speed of 10mph, maintaining a constant speed of 10mph, and then ramping down speeds until the vehicle was again stationary. The acceleration rate used was as close to the maximum acceleration of the Honda Civic coupe (2.31 m/s^2) as possible given the upper and lower speed constraints.

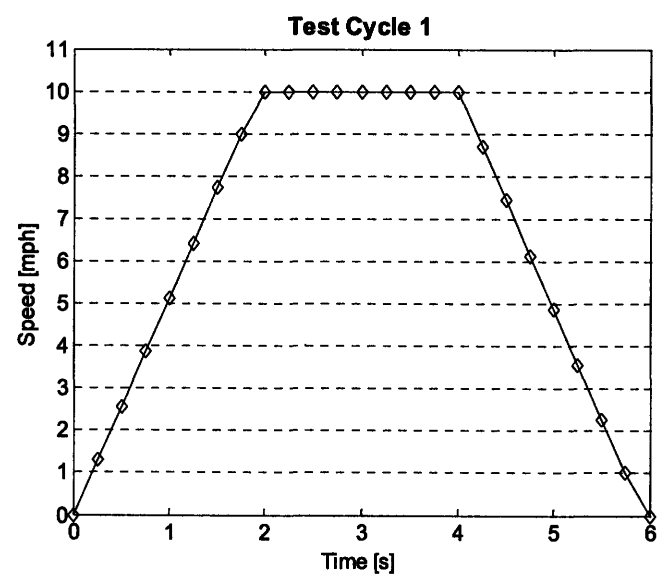


Fig. 11: Test Cycle 1. Note the acceleration is slightly less than the maximum possible at the ends of the ramps because of the need to stay within the 0mph to 10mph operating envelope.

The median transmission efficiency, defined previously as the ratio of the power out of the gearbox to the power into the controller from the battery when in discharge mode or the ratio of power into the battery from the controller to the power into the gearbox when in recharge mode, was 0.62. Fig. 12 shows where the energy put into the system (from the battery when the EVE was accelerating or the flywheel when the EVE was decelerating) is lost. The largest portion of the losses was found in the motor/generator. Losses were greater when operating as a generator (nearly 50% of the input energy) than when operating as a motor (slightly over 20% of the input energy). It was also interesting to note that there was little loss in the battery conversion of chemical to electrical energy when discharging or electrical to chemical energy when charging. Upon further investigation, it was found that the low currents in the system resulted in low resistive losses and therefore a high conversion efficiency.

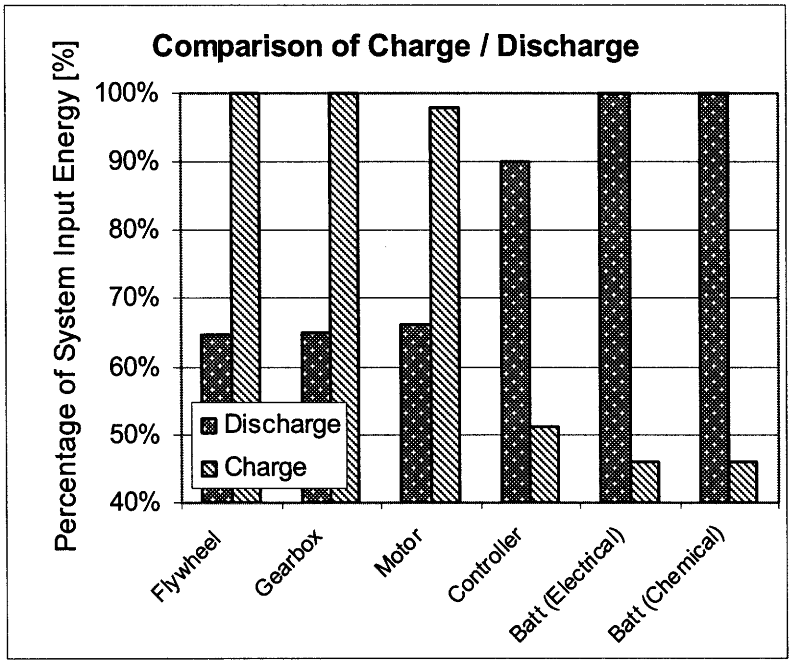


Fig. 12: Component energy comparison. For discharge, the battery (chemical) energy is the system input energy. The flywheel energy is the system input energy for charging. Note that greatest losses are in the motor/generator for both cycles.

For this cycle, the overall recovery efficiency, defined as the cumulative energy returned to the battery over the cumulative energy taken from the battery, was 0.28. This result was not surprising given the greater amount of discharging time relative to charging time in the cycle.

Test Cycle 2 – Ramp up / Steady / Ramp down repeated ten times

To determine the performance of the cycle over repeated discharge and charge cycles, the second test cycle repeated the first test cycle ten times in quick succession. The speeds and accelerations were identical for each of the discharge-charge "humps" shown in Fig. 13. Since the total number of cycles was less than the number needed to start changing the battery bank's internal resistance, the overall energy recovery efficiency of this system was identical to that of Test Cycle 1 at 0.30. If the number of cycles increased above one hundred, permanent deterioration of the battery cells would have caused the overall battery capacity and performance to decline, lowering the recovery efficiency.

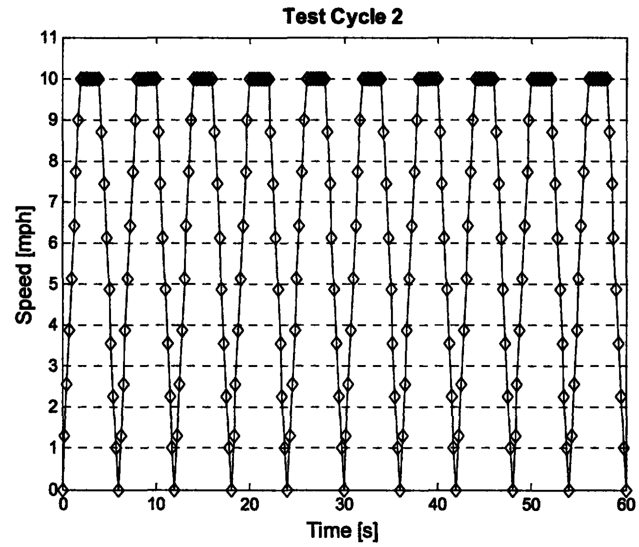


Fig. 13: Test Cycle 2. Test Cycle 1 is repeated ten times to see the effects of repeated cycling on system performance.

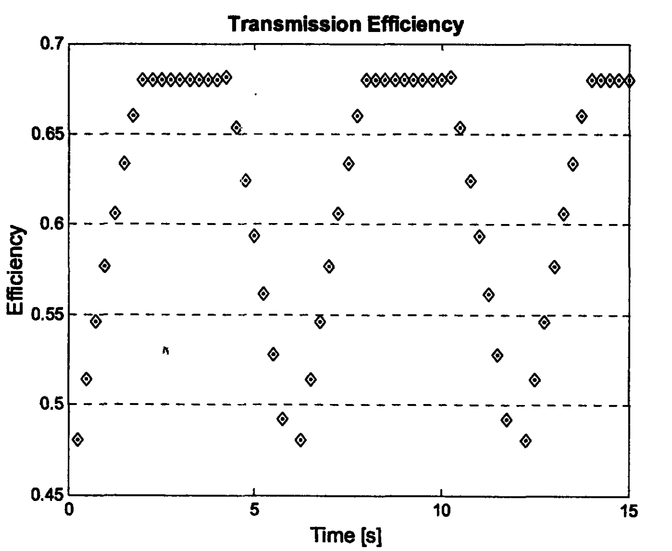


Fig. 14: Transmission efficiencies. Note the average efficiency of around 0.6. Only 2.5 cycles are shown for reference, though all ten cycles show identical results.

Since the motor, gearbox, and controller component efficiencies were only affected by the speed of the flywheel, the transmission efficiency is the same for each of the repeated discharge-charge cycles. Increased speed improved the transmission efficiency because of increased motor efficiency since the gearbox and controller efficiencies were constants.

The most interesting result of the multiple cycle test was the slow decline of the state of charge of the battery, shown in Fig. 15. The overall decline in state of charge was rather small as a result of the small currents resulting from lower power demands than anticipated in the different components. The results also clearly show that discharging has a much greater effect on state of charge when compared to charging at the same speeds; in other words, the recovery efficiency of the system is rather low.

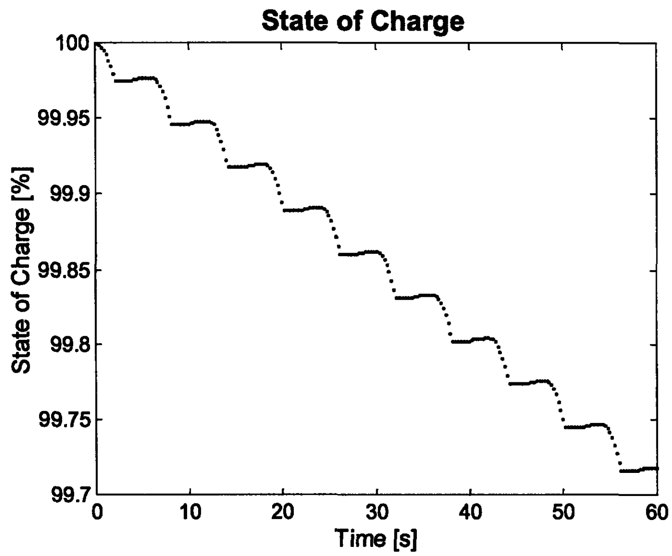


Fig. 15: Battery bank state of charge. Calculated from battery voltage and representative of battery capacity.

Test Cycle 3 – Large speeds and accelerations

While the EVE model will be limited to an operational envelope for safety purposes, testing the model outside of the speed "limits" of 0mph to 10mph proved to be informative for discussing operation of electric vehicles. The profile of the third cycle tested was similar to that used for the Test Cycle 1 - ramping up to a maximum speed of 40mph rather than 10mph, maintaining that speed of 40mph, and then ramping back down to the starting speed of 0mph (scaling Fig. 11). The acceleration of this cycle was over 7.5 times the maximum acceleration of the modeled Honda Civic coupe and had a steady-state speed of 40mph, resulting in power flows that would be much too high for use in the laboratory setting. High current flows reached values of over 600A and lowered the capacity discharge and charging times to minutes. Even so, the 0.1s time interval between each speed input meant that the total capacity and state of charge changes were low (state of charge declined from 100% to 98.9%).

Comparing the energy lost in different parts of the system (Fig. 16), one could see a visible decrease in the amount of energy lost in the motor when the battery was discharging while the resistive losses in the battery became more prominent. The motor's efficiency increased at higher speeds, therefore reducing the amount of energy lost as chemical energy from the battery was transformed into mechanical energy at the flywheel. On the other hand, the battery's ability to convert chemical to electrical energy was reduced because of the high currents and high resistive losses (I^2R losses) present in this higher speed cycle.

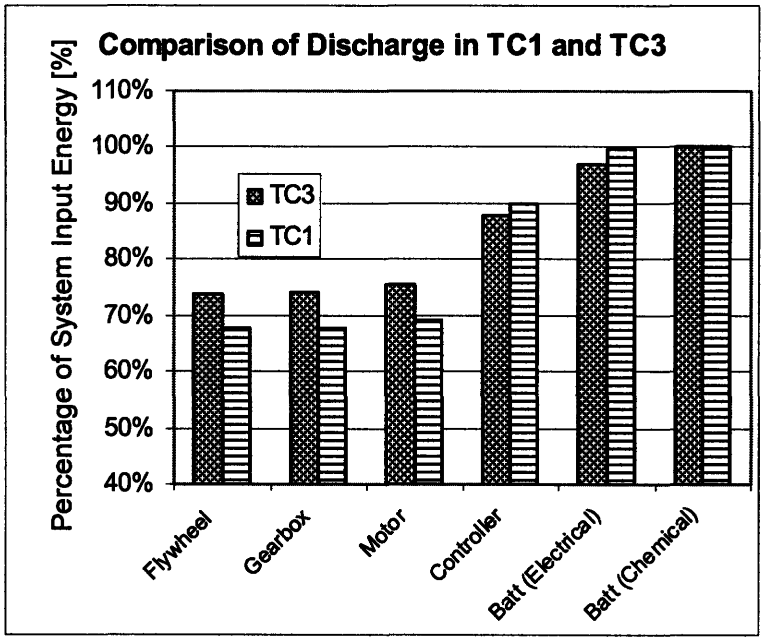


Fig. 16: Comparison of high and low speed discharge cycles. Note that the motor performance improves discharge in TC3 while the resistive losses in the battery lower the system conversion.

It was also noteworthy that higher speeds increased the transmission efficiency and also increased the energy recovery efficiency of the system, raising the fraction of energy returned over energy removed from the battery to 0.41. Since the slopes of the ramps were steeper, the areas under the curves were increased, and the higher efficiency of the transmission coupled with the higher energy available at the flywheel meant the recovery of energy could be improved.

Test Cycle 4 – Long time cycles

The final investigation undertaken using the EVE model examined the effect of time on vehicle performance. Because all of the previous cycles had used relatively short time periods, the possible changes in states of charge, voltages, currents, and capacities were restricted. By expanding Test Cycle 3 (40mph steady-state speed) to a cycle that had time intervals of ten minutes rather than intervals of seconds, the effects of time could be examined. The increased times reduced accelerations and therefore reduced power flows over the time intervals, increasing the time required to discharge or charge the battery. As can be seen in Fig. 17, the state of charge and therefore battery voltage were seen to decrease tremendously when the time intervals were increased. The only caveat to this statement was that air friction must be present; in the case of no air friction, the bearing friction losses in the flywheel only lowered the state of charge of the batteries by 3%. The higher speeds of the flywheel led to higher drag torque, increasing the power draw on the battery.

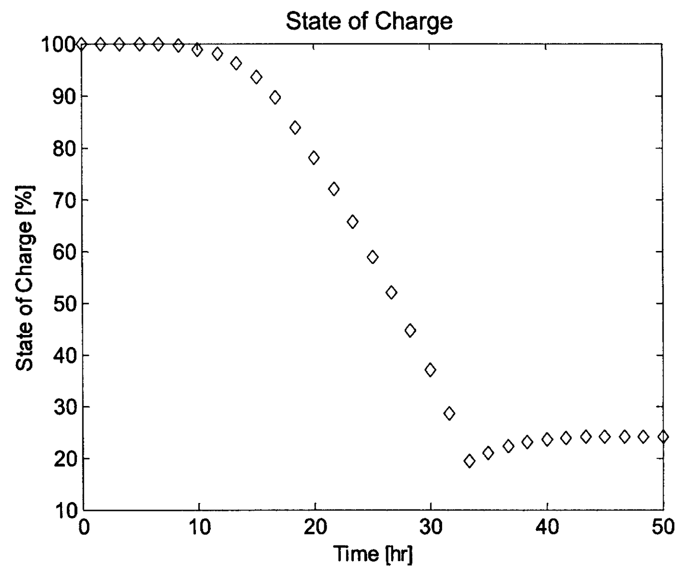


Fig. 17: Effect of time on state of charge. Note that time is given in hours rather the seconds.

The energy recovery also decreased over time, falling to 0.08, as a result of the lower motor performance and the increased times required for charging. Long running times drained the system of energy more rapidly than short running times even when the charging and discharging intervals were equal.

DISCUSSION

The model presented in this thesis has been derived from theory and some generalized empirical data; it can only be truly verified once the system has been assembled and specific empirical data has been gathered. Even so, there are several important results that can be derived from the theoretical model presented. The largest amounts of energy were put into or taken out of the EVE system when the speeds were high and the time intervals were large. The transmission efficiency increased as the speed of the flywheel increased, mainly as a result of increases in motor efficiencies. The transmission efficiency reached a maximum of around 80% when operating outside the operational envelope of the EVE experiment; in reality, the maximum transmission efficiency for the system used in the laboratory was between 60% and 70%. Conversely, the efficiency of battery conversion from chemical to electrical energy decreased from nearly 100% to 96% as the flywheel speed increased. As the powers and currents increased, the electrical resistive losses increased, lowering this conversion efficiency by a few percentage points. The state of charge and capacity of the battery bank were highly dependent on the electrical current flowing into the battery, with an increased rate of battery draw down as the current increased. Finally, the amount of energy put back into the system through regenerative braking and charging of the battery was always less than 50%, usually closer to 30%. The main difficulties in recovering the energy were in converting the flywheel energy to electrical energy through the motor/generator.

The two main motivations for modeling the EVE system were to prove that this system is safe for usage in a lab course and to determine what information or tools would need to be given to the students to gain some knowledge from this exercise. Both questions are addressed below:

Safety

Test Cycle 1 gave the maximum rate of energy change in the system as this cycle had the highest acceptable vehicle acceleration. Therefore, the maximum power flow through the components found during this cycle can be used to judge the safety of the system. The maximum mechanical power delivered to the flywheel was just under 2.5kW, occurring when the speed was nearly 10mph. The flywheel energy at this speed was just under 8kJ, the maximum design energy. Analysis had been done previously to show that this energy load could be absorbed by the containment, therefore avoiding the danger of a catastrophic accident [4]. In the electrical subsystem, the required power of slightly over 3.5kW resulted in a current flow of nearly 22A. Current flow at this level is rather low, demonstrating that this system would not be extremely dangerous to students. The power flow through the motor was only 20% of the motor's maximum power. While operation outside of the safety envelope would result in large speeds and currents that would decrease the safety of the system, the preliminary results of the model suggest that the power and energy flows through the system will be small enough to allow safe operation.

Component	Output Power [W]
Flywheel	2,270
Gearbox	2,275
Motor	2,321
Controller	3,193
Batt (Electrical)	3,549
Batt (Chemical)	3,552

Fig. 18: Maximum power flows. Power flows given for acceleration near model maximum. Battery chemical power is nearly 3.5kW while flywheel mechanical power is less than 2.5kW.

Operating the system outside of the 10mph and 2.3m/s^2 maximum acceleration envelope would be dangerous because of the high powers, currents, and energy flows. In designing the control system, the operational limits must be hard-wired into the system so that these limits cannot be exceeded. Even with the operational systems in place, utmost care must be taken to keep the user away from components that could be potentially hazardous, such as the battery bank and flywheel.

Pedagogical value

The final question to be answered by modeling this system concerns the pedagogical value of the EVE. While modeling the system, it became clear that the system is rather complex, containing mechanical, electrical, thermal, and fluidic systems. Most of the experiments currently in the 2.672 portfolio focus on thermal and fluidic systems, but this experiment has the potential to expand the learning opportunities for mechanical engineering students. Several simplifying assumptions had to be incorporated to arrive at a satisfactory model, but the process of making these assumptions is part of the learning process. Modeling a system as complex as an electric vehicle drive train is quite a learning experience, and seeing how the different systems interact is a valuable activity.

Looking at individual components, one can see that certain information must be presented to the students to help them model the EVE, especially information related to the battery. The empirical data for charging of the battery, the torque-speed curve of the motor, and the speed-efficiency curve of the motor would all need to be provided beforehand, though the battery charging data could be made more accurate by finding empirical results from the charging of a battery that had not been fully drained. If the battery model was to be redeveloped by each student, the modified Peukert equation would need to be provided along with an explanation of how it works and how to use it. Manufacturer's data should also be given for the gearbox, and correlations from the final controller model may also need to be given to the students. Looking at the flywheel, the specific correlations between speed and drag torque coefficient could be provided or could be found through experimentation. Of course, to accurately do this experiment would require disconnecting the transmission from the flywheel, spinning the flywheel, and determining the relationship between stopping time and rotational speed. The losses slowing the flywheel would be a combination of the air drag losses and the bearing losses. Given the vertical layout of the system and the low rotational speeds achievable when manually spinning the flywheel, finding this correlation may be more difficult than anticpated. It may be necessary to give the students the relationship between the speed (or Reynolds number) and the drag coefficient, depending on the amount of time given to conduct the experiment.

Practical issues

There are several outstanding issues that must be resolved before this EVE becomes a reality at MIT. One of the largest is how to instrument the system so that the students can gather the required data. To determine the efficiencies, the students must know the power flows through the different components. While the majority of the information would be relatively easy to find using simple equipment, there are a few areas of uncertainty, as outlined below:

Component	Power in measurement method
Flywheel	Speed can be measured with an optical tachometer, and torque can be measured by determining the relationship between shear strain on the frame holding the equipment and the applied torque.
Gearbox	Speed can be measured with an optical tachometer, and torque can be measured by determining the relationship between shear strain on the frame holding the equipment and the applied torque.
Motor	Current (AC) and voltage can be measured with multimeter.
Controller	Current (DC) and voltage can be measured with multimeter.
Battery (Electrical)	Resistive losses can be determined by measuring the specific gravity of the battery and the state of charge.

Fig. 19: Proposed methodology for collecting required power flows data on EVE system.

The final large issue of practical concern is the charging of the system. It appears that the state of charge of the batteries would stay relatively high given the low loads on the flywheel when operating within the stated operational envelope. As a result, the system should be operational over a day with no degradation in performance. The system could be recharged in the evenings using a trickle charger. Several different charging methods are possible, but keeping the current low is a priority to lower resistive power losses and extend the battery life.

CONCLUSION

The 2.672 EVE had been designed and now has been modeled to get a rough approximation for the power flows and efficiencies of the different components. The results of the modeling analysis showed the greatest energy losses were in the motor/generator unit, with small losses in the battery conversion, controller, and gearbox. The overall recovery energy efficiency of the system for the desired operational envelope is less than 30%, but, with the expected low power flows, the experiment could be used several times before being recharged. The modeling of the system is complex, but several simplifying assumptions can be made to allow students to generate a relatively robust model given some coefficients and empirical relations. To fully characterize the system and account for all the inter-related dynamics, the system must now be built. The work plan moving forward is appended to this thesis to give an overview of the remaining steps to completion.

Once the EVE is built, students will have the opportunity to learn about electric vehicles and their efficiencies and feasibility as a transportation solution through the 2.672 lab class. The model can be modified to reflect the empirical results, and this lab setup could be used to test the performance of other vehicles as energy storage devices for electric vehicles. While these batteries can be modeled in the lab, discharging them and charging them in a setup such as the EVE allows better characterization and knowledge of their performance. The need for a better understanding of these technologies is high as the global energy situation is becoming a topic of increasing importance in the 21st century.

ACKNOWLEDGEMENTS

As I get ready to submit this thesis to the Mechanical Engineering department in completion of my undergraduate degree, I would like to acknowledge the assistance of the following people:

Prof. C. Forbes Dewey – For his unending support of this project and thesis even when the department cut funding and changed the scope of the project from a preferred building project to a less preferred Matlab coding exercise.

Prof. J. Brisson – While only a small supporter, Prof. Brisson helped me to consider the implications of operating such a complex system as the EVE and has provided support to me for the past three years as my undergraduate advisor.

Richard Fenner – Dick's support during the fall term was critical in redesigning and building the frame for holding the gearbox and motor. This frame will be mounted to the containment as the EVE is built. Dick was always positive and always had a solution, and his assistance along with the help of all the Pappalardo staff, was much appreciated.

David Gandy – For helping to develop the EVE model of the controller and for providing assistance in presenting and disseminating knowledge about this idea to the Mechanical Engineering community.

For further information about this thesis or the EVE experiment, please contact Matt Zedler at

mzedler@alum.mit.edu.

REFERENCES

[1] "Electric Vehicle History." Electric Auto Association. Accessed on 3 May 2007 on the World Wide Web at http://www.eaaev.org/Flyers/eaaflyer-evhistory.pdf.

[2] Voelcker, J. "Top 10 Tech Cars." *IEEE Spectrum* (April 2007). pp. 34-35.

[3] "2.672 – ME Lab." Massachussets Institute of Technology. Accessed on 9 May 2007 on the World Wide Web at http://web.mit.edu/2.672/www.

[4] Young, B. "Physical Model of a Hybrid Electric Drive Train." Massachusetts Institute of Technology. 12 May 2006.

[5] "Vehicle System Analysis." National Renewable Energy Laboratory. 4 April 2007. Accessed on 9 May 2007 on the World Wide Web at http://www.nrel.gov/vehiclesandfuels/vsa.

[6] "Needle Thrust and Roller Thrust Bearings." NRB Bearings. Accessed on 9 May 2007 on the World Wide Web at http://www.nrbbearings.com/product/nrbproduct_catalogue/BEARINGS.

[7] Crane. "Viscosity." Answer.com (1988). Accessed on 9 May 2007 on the World Wide Web at http://www.answers.com/topic/viscosity.

[8] "CBN3000 Product Overview." Emerson Industrial Automation. Accessed on 30 April 2007 on the World Wide Web at http://www.emerson-ept.com.

[9] "Frequently Asked Questions." AutoBatteries.com. Accessed on 28 April 2007 on the World Wide Web at http://www.autobatteries.com/faq/index.asp.

[10] Genta, G. "4.2 Aerodynamic drag on flywheels." *Kinetic energy storage: Theory and practice of advanced flywheel systems*. Butterworth & Co. (Publishers) Ltd. : London, UK (1985). pp. 170-177.

[11] Schlichting, H. "The rotating disc." *Boundary-Layer Theory* (6th ed). McGraw-Hill Book Company : New York, US (1968). pp. 606-610.

[12] Linden, D. "Chapter 24. Lead-Acid Batteries." *Handbook of Batteries* (2nd Edition). McGraw-Hill, Inc. : New York, US (1995). pp. 24.1-24.8.

[13] Rand, D., Woods, R., & Dell, R. *Batteries for Electric Vehicles.* Research Studies Press : Somerset, England (1998).

[14] "An in depth analysis of the maths behind Peukert's Equation (Peukert's Law)." SmartGauge Electronics (January 2007). Accessed on 26 April 2007 on the World Wide Web at http://www.smartgauge.co.uk/peukert_depth.html.

[15] Chaiken, J. "Charging Time: Time to Charge Fully Discharged Battery." Japlar/Schauer (June 2006). Accessed on 26 April 2007 on the World Wide Web at http://www.battery-chargers.com/charging_instructions.htm.

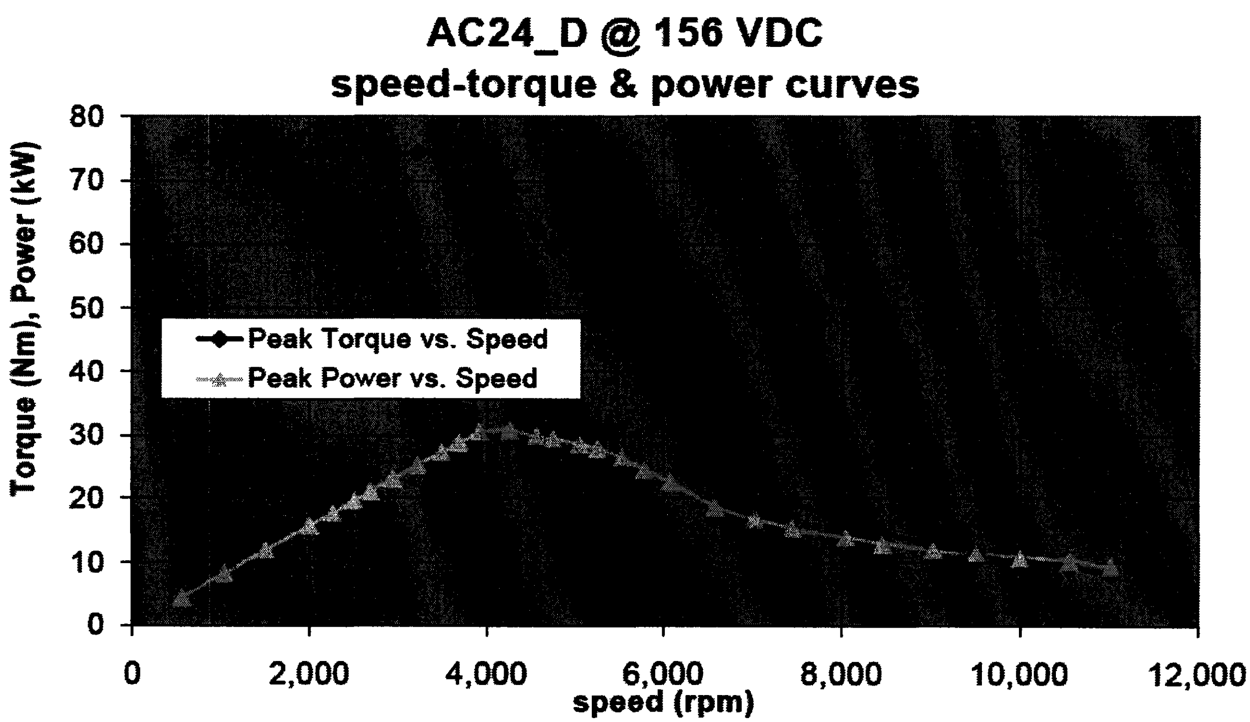
APPENDICES

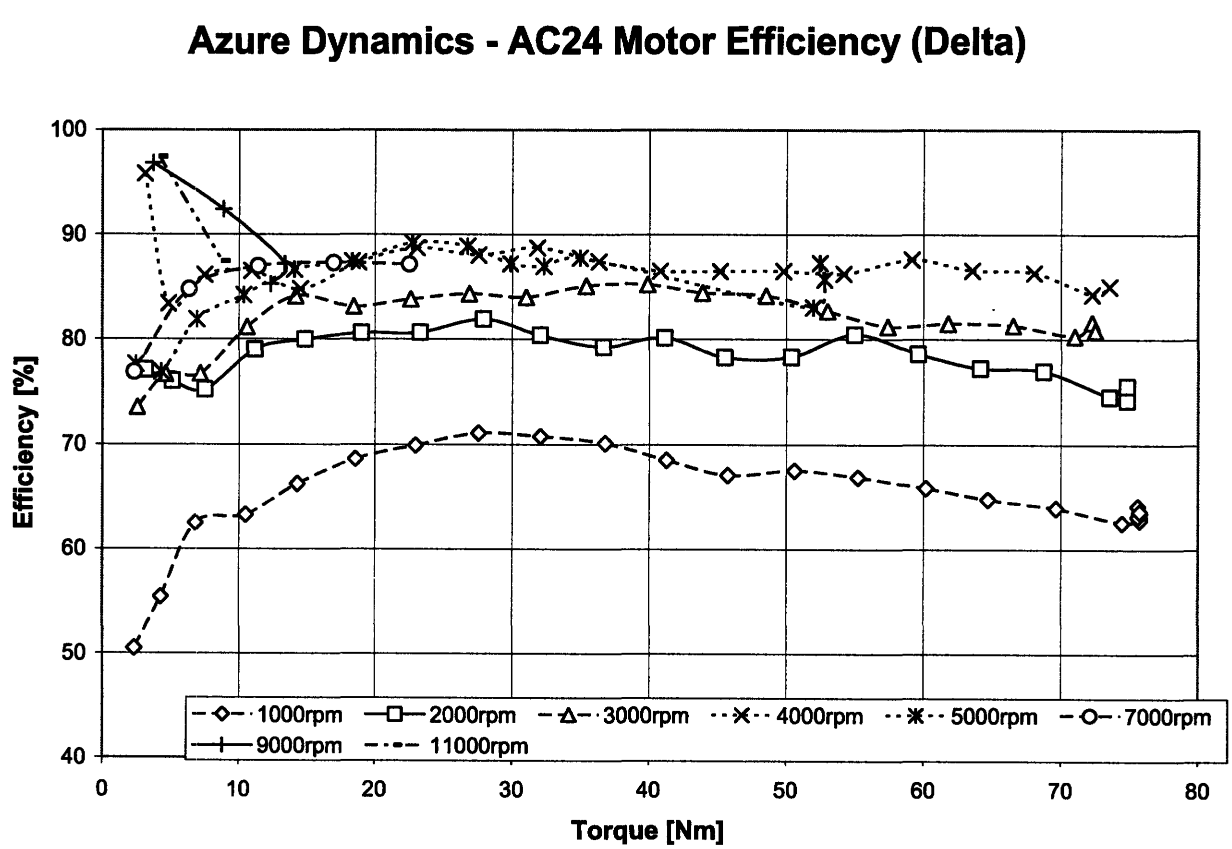
A1) Continuing Work Plan

While Brady Young and I have both sought to construct the EVE in the hopes of getting empirical results to validate Young's design and the experimental model developed in this thesis, time and departmental support constraints have postponed this activity. To complete the construction of the EVE, the following rough guidelines are provided:

1) Mount the motor-flywheel shaft connection to the flywheel using the guide hole in the centre of the 2" steel shaft for alignment purposes (Two bolts per side).
2) Collect the welded steel frame from MIT's Central Machine Shop.
3) Drill connection holes into frame for attaching to flywheel containment cover.
4) Drill connection holes into frame for gearbox.
5) Using assistance, mount gearbox to frame and lift frame into position on cover. USE ASSISTANCE WHEN MOVING THE FRAME AND GEARBOX.
6) Place coupling between shaft of gearbox and shaft of flywheel. Ensure alignment.
7) Mount frame and cover to concrete base using long bolts. NOTE: May need to drill out holes in concrete as they are currently over-constraining the system.
8) Mill centre hole in top plate for motor. Mill holes for motor connection to plate and for plate attachment to frame.
9) Mount motor to plate. Mount plate to frame, making sure to place gearbox-motor coupling in place before securing.
10) Spin flywheel to assure no grinding or undue friction is occurring.
11) Purchase battery connection cables to wire batteries in series. NOTE: In-line fuses and some sort of rack for the batteries should be developed at this time.
12) Connect batteries to controller. Connect controller to motor.
13) After ensuring all connections are secure and all bolts are tightened, run tests on EVE. Calibrate controller to given input equipment.

A2) Manufacturer's Data





Data provided by Azure Dynamics for AC24 motor wired in DELTA connection. For more information on Azure Dynamics, please visit: http://www.azuredynamics.com/

A3) Matlab Code Modules

All code modules are available as *.m modules from Prof. C. Forbes Dewey for future work.

Main program (ev_model.m)

```
% M. Zedler (mzedler@MIT.edu), 9 May 2007
% 2.ThU - Undergraduate Thesis, Prof. C. Forbes Dewey
% Model of transmission system used in lab-based electric vehicle (EV)

% Main program. Loads external file with driving cycle and graphs desired
efficiency curves
% Requires following m-files to properly function:
%   battery.m - Models behavior of battery bank (Thirteen series-connected
DieHard Gold North 33065 lead acid batteries)
%   controller.m - Models behavior of controller (Solectrica DMOC445)
%   transmission.m - Models behavior of motor (Azure Dynamics AC24 AC
induction motor) and gearbox (5 : 1 speed reduction, oil-filled)
%   flywheel.m - Models behavior of flywheel in housing, calculating
frictional losses
%   speedConvert.m - Converts vehicle speed in mph to flywheel speed in rad/s

% *********************************************************

% Empty all storage and clear all plots
clear all;

% Define global variables (constants)
global g
g = 9.81; % [m/s^2], Acceleration due to gravity on the surface of the earth.
% Set initial conditions for system
%    Initial temperature of system
InitTemp = 25; % [degC], ambient temperature
bunker_temp = InitTemp;

%    Battery characteristics (Diehard Gold North 33065 lead acid)
BattLen = 0.324; % [m], length of battery
BattWid = 0.175; % [m], width of battery
BattHht = 0.225; % [m], height of battery
BattNum = 13; % number of batteries in bank
BattVol = BattNum * BattLen * BattWid * BattHht; % [m^3], internal volume of
battery

BattCap = 45; % [Ah], capacity of single battery
BattInitV = 163.8; % [V], initial voltage of battery bank
BattInitSOC = 100; % [%], initial state of charge of battery bank
BattCycle = 1; % cycle number of usage (1 is new battery, 500 is fully
exhausted battery)

% Load driving cycle
%   File should contain title followed by times [s] and vehicle speeds [mph]
in columns 1 and 2 respectively.
%   Vehicle speeds should not exceed 10mph.
filename = input('Enter the filename of the driving cycle: ','s');
fid = fopen(filename,'r');
    if(fid < 0)
        error('Could not open file!');
```

```
    end;
driveCycleName = fgetl(fid);
[driveCycle,count] = fscanf(fid, '%f %f\n', [2 inf]);
driveCycle = driveCycle';
fclose(fid);

% Reformat driving cycle as time vector and speed vector
time = driveCycle(:,1)';
speed = driveCycle(:,2)';
totalTime = time(1, count/2);

% Determine if user wants to plot driving cycle
boolPlot = input('View drive cycle plot (time [s] vs. speed [mph]) now? (y /
n): ','s');
if(boolPlot == 'y') | (boolPlot == 'Y')
   plot(time,speed);
    title(driveCycleName);
    xlabel('Time [s]');
    ylabel('Speed [mph]');
end

% Determine if user wants to limit speeds and accelerations
boolConstraint = input('Constrain system to speed and acceleration limits? (y
/ n): ','s');
if(boolConstraint == 'y') | (boolConstraint == 'Y')
    limitSet = 1;
else
    limitSet = 0;
end

% Determine if user wants to include air drag or run the flywheel in a
vaccuum
boolAirDrag = input('Include air drag in the flywheel housing? (y / n):
','s');
if(boolAirDrag == 'y') | (boolAirDrag == 'Y')
    airDrag = 1;
else
    airDrag = 0;
end

% Set initial battery parameters, with resistance depending on cycle
BATTERY = [BattInitV 0 0 BattCap 0 0 BattInitSOC InitTemp 0 BattVol BattCycle
BattCap];
if(BATTERY(11) < 100)
    BATTERY(3) = 0.0078;
elseif((BATTERY(11) >= 100) & (BATTERY(11) < 250))
    BATTERY(3) = 0.0312;
elseif((BATTERY(11) >= 250) & (BATTERY(11) < 400))
    BATTERY(3) = 0.0546;
elseif((BATTERY(11) >= 400) & (BATTERY(11) < 500))
    BATTERY(3) = 0.0858;
else
    BATTERY(3) = 0.117;
end

j = 1;
batt_energy_out = 0;
```

```
batt_energy_in = 0;
% Calculate omega, power losses, and efficiencies for driving cycles
for i = 1 : count/2
    % results Matrix
    %   1 - Time [s]
    %   2 - Linear Velocity [mph]
    %   3 - Radial Velocity [rad/s]
    %   4 - Current Acceleration [m/s^2]
    %   5 - Flywheel Output Power [W]
    %   6 - Gearbox Output Power [W]
    %   7 - Motor Output Power [W]
    %   8 - Controller Output Power [W]
    %   9 - Battery Output Power [W]
    %  10 - Battery Chemical Power [W]
    %  11 - Battery State of Charge [%]
    %  12 - Energy Stored in Flywheel [kJ]
    %  13 - Energy Stored in Battery [kJ]
    %  14 - Power fraction (used power in electric motor over max power
available)

    % Store time [s] and velocity of car [mph] in first two columns of
matrix.
    results(i,1) = time(i);
    if(limitSet == 1)
        if(speed(i) > 10)
            error('Speed out of range - reduced to maximum speed (10mph)');
            speed(i) = 10;
        elseif(speed(i) < 0)
            error('Speed out of range - increased to minimum speed (0mph)');
            speed(i) = 0;
        end
    end
    results(i,2) = speed(i);

    if (i == 1)
        if(time(i) ~= 0)
            inc = time(i);
        else
            inc = 1;
        end
        speed_inc = speed(i);
        BATTERY_OLD = BATTERY;
        charge_start_0 = 0;
    else
        inc = time(i) - time(i-1);
        speed_inc = speed(i) - speed(i-1);
    end

    % Store gearbox output power [W], flywheel output power / end power [W],
and energy stored in flywheel [kJ] in sixth, fifth, and twelfth columns
respectively
    [results(i,6) results(i,5) results(i,12) bunker_temp_new, results(i,3)
results(i,4) k] = flywheel(results(i,2), inc, speed_inc, bunker_temp,
airDrag, limitSet);
    bunker_temp = bunker_temp_new;
```

```
    % Store controller output power [W], motor output power [W], and max
power fraction in the eighth, seventh, and fourteenth columns of matrix
respectively
    [results(i,8) results(i,7) results(i,14)] = transmission(results(i,3),
results(i,6), results(i,4));

    % Store battery outupt power [W] in the ninth column of the matrix
    if(speed(i) ~= 0)
        if(i == 1)
            results(i,9) = controller(time(i), 0, results(i,4),
results(i,8));
            [BATTERY_NEW charge_start_1 time_special] = battery(BATTERY_OLD,
results(i,9), time(i), 0, time(count/2), results(i,4), charge_start_0);
        else
            results(i,9) = controller(time(i), time(i-1), results(i,4),
results(i,8));
            [BATTERY_NEW charge_start_1 time_special] = battery(BATTERY_OLD,
results(i,9), time(i), time(i-1), time(count/2), results(i,4),
charge_start_0);
        end
    else
        BATTERY_NEW = BATTERY_OLD;
        charge_start_1 = charge_start_0;
        time_special = 0;
    end

    % Allow recursion in electrical module
    BATTERY_OLD = BATTERY_NEW;
    charge_start_0 = charge_start_1;

    % Store battery chemical power [W], stored energy [kJ], and state of
charge in tenth, thirteenth, and eleventh columns respectively
    results(i,10) = BATTERY_NEW(5);
    results(i,13) = BATTERY_NEW(6);
    results(i,11) = BATTERY_NEW(7);

    % energy Conversion Matrix - Used for determining how much available
energy is stored during a time interval
    %    DISCHARGING - Conversion efficiency is from chemical energy in
battery to mechanical energy in flywheel
    %    CHARGING - Conversion efficiency is from mechanical energy in
motor/generator to chemical energy in battery
    %    1 - Time [s]
    %    2 - Speed [mph]
    %    3 - Flywheel mechanical energy [kJ]
    %    4 - Gearbox mechanical energy [kJ]
    %    5 - Motor mechanical energy [kJ]
    %    6 - Controller electrical energy [kJ]
    %    7 - Battery electrical energy [kJ]
    %    8 - Battery chemical energy [kJ]
    %    9 - Conversion efficiency (defined above)

    if(i == 1)
        time_inc = time(i);
    else
        time_inc = time(i) - time(i-1);
    end
```

```
    energy(1,i) = results(i,1);
    energy(2,i) = results(i,2);
    energy(3,i) = results(i,5) * time_inc / 1000;
    energy(4,i) = results(i,6) * time_inc / 1000;
    energy(5,i) = results(i,7) * time_inc / 1000;
    energy(6,i) = results(i,8) * time_inc / 1000;
    energy(7,i) = results(i,9) * time_inc / 1000;
    energy(8,i) = results(i,10) * time_inc / 1000;
    if(results(i,4) >=0 ) & (results(i,2) > 0)
        energy(9,i) = energy(3,i) / energy(8,i);
        if(results(i,5) ~= 0)
            energy_plot_discharge = ([results(i,5) results(i,6) results(i,7)
results(i,8) results(i,9) results(i,10)] / results(i,10)) * 100;
        end
        batt_energy_out = batt_energy_out + energy(8,i);
        energy(10,i) = batt_energy_out;
    elseif(results(i,4) < 0) & (results(i,2) > 0)
        energy(9,i) = energy(8,i) / energy(5,i);
        energy_plot_charge = ([results(i,5) results(i,6) results(i,7)
results(i,8) results(i,9) results(i,10)] / results(i,5)) * 100;
        batt_energy_in = batt_energy_in + energy(8,i);
        energy(11,i) = batt_energy_in;
    end

    % Efficiency matrix
    %   1 - Time [s]
    %   2 - Acceleration [m/s^2]; model should recharge if (-) or discharge
if (+)
    %   3 - Battery SOC
    %   4 - Battery Charge / Discharge bool (0 if discharging)
    %   5 - Battery efficiency (Electrical / Chemical); should be (+) if
discharging and (-) if recharging
    %   6 - Controller efficiency (Electrical Out / Electrical In); should be
constant until model implemented
    %   7 - Motor efficiency (Mechanical Out / Electrical In)
    %   8 - Gearbox efficiency (Mechanical Out / Mechanical In); should be
constant until model implemented
    %   9 - Flywheel efficiency (Mechanical Out / Mechanical In)
    %  10 - Overall efficiency
    % 11 - Flywheel
    % 12 - Battery
    % 13 - Power frac energy conversion (Flywheel energy / Battery energy)

    eff(i,1) = results(i,2);
    eff(i,2) = results(i,4);
    eff(i,3) = results(i,11);
    eff(i,4) = BATTERY_NEW(9);

    % Battery efficiency
    if(results(i,2) ~= 0)
        if(eff(i,4) == 0)
            eff(i,5) = results(i,9) / results(i,10); % Battery discharging
efficiency (elec / chem)
            eff(i,6) = results(i,8) / results(i,9); % Controller efficiency
(elec / elec)*
```

```
            eff(i,7) = results(i,7) / results(i,8); % Motor efficiency (mech
/ elec)
            eff(i,8) = results(i,6) / results(i,7); % Gearbox efficiency
(mech / mech)*
            eff(i,9) = results(i,5) / results(i,6); % Flywheel efficiency
(mech / mech)
        else
            eff(i,5) = results(i,10) / results(i,9);
            eff(i,6) = results(i,9) / results(i,8); % Controller efficiency
(elec / elec)*
            eff(i,7) = results(i,8) / results(i,7); % Generator efficiency
(elec / mech)
            eff(i,8) = results(i,7) / results(i,6); % Gearbox efficiency
(mech / mech)*
            eff(i,9) = results(i,5) / results(i,6); % Flywheel efficiency
(mech / mech)
        end
        eff(i,10) = eff(i,6) * eff(i,7) * eff(i,8); % Transmission efficiency
    else
        eff(i,10) = 0;
    end
    subplot(2,2,1);
    plot(results(i,1),eff(i,10),'db');
    title('Transmission Efficiency');
    xlabel('Time [s]');
    ylabel('Efficiency');
    hold on;

    subplot(2,2,2);
    plot(results(i,1)/3600,BATTERY_NEW(7),'dk');
    title('State of Charge');
    xlabel('Time [hr]');
    ylabel('State of Charge [%]');
    hold on;

    if(eff(i,10) > 0.1)
        histData(1,j) = eff(i,10);
        j = j + 1;
    end
end
subplot(2,2,3);
bar(energy_plot_discharge);
title('Representative Discharge');
xlabel('Component');
ylabel('Energy [kJ]');
subplot(2,2,4);
bar(energy_plot_charge);
title('Representative Charge');
xlabel('Component');
ylabel('Energy [kJ]');

figure;
hist(histData);
title('Histogram, Efficiencies');

disp('Drive Train Efficiency Results');
mean = mean(histData)
```

```
median = median(histData)
standard_deviation = std(histData)

recovery_eff = batt_energy_in / batt_energy_out;
sprintf('Energy taken from battery: %g kJ     ///    Energy put into battery:
%g kJ', abs(batt_energy_out), abs(batt_energy_in))
sprintf('Recovery efficiency of system (Energy Returned / Energy Used):
%0.2f', abs(recovery_eff))
```

Flywheel (flywheel.m)

```
% M. Zedler (mzedler@MIT.edu), 16 Apr 2007
% 2.ThU - Undergraduate Thesis, Prof. C. Forbes Dewey
% Model of transmission system used in lab-based electric vehicle (EV)

% Models behavior of flywheel in housing, calculating frictional losses

function [PIn, POut, EStored, Temp_New, omegaIn, accel, k] =
flywheel(roadSpeedInput, timeInc, speedInc, Temp, airDrag_bool,
limitSet_bool);
% Model of the losses in the rotating flywheel for the electric vehicle
model.
% Based on experimental results for air friction losses and simple physics
analysis for bearings.
% INPUT: Speed of the vehicle (roadSpeedInput) [mph], the time length of the
current speed (timeInc) [s], current temperature of flywheel housing bunker
(Temp) [K],
%        a boolian to determine whether or not to consider air drag
(airDrag_bool) [0 / 1], and a boolian to determine if errors for excess
speeds should be reported.
% OUTPUT: Theoretical power of flywheel assuming ideal state (PIn) [W], power
out of flywheel given calculated losses (POut) [W], total energy stored
%         in the flywheel (EStored) [kJ], new temperature of the flywheel
housing bunker (Temp_New) [K], flywheel speed (omegaIn) [rad/s], and
acceleration.

    % Set initial dimensions
    global g
    %   Vehicle parameters (2006 Honda Insight Coupe)
    mCar = 820; % [kg], mass of car.
    aMax = 2.37; % [m/s^2], max acceleration of car (0-60 mph in 11.3 s)
    %    Flywheel dimensions.
    mWheel = 150; % [kg] Mass of flywheel.
    rWOuter = 0.45; % [m] Distance from centre of flywheel to outer edge of
flywheel.
    tWheel = 0.0381; % [m] Distance from top to bottom surface of flywheel.
    IWheel = 15; % [kg/m^2] Inertia to angular rotation.
    %    Housing gap dimensions.
    h1 = 0.03; % [m] Distance from bottom of flywheel to floor.
    h2 = 0.04; % [m] Distance from top of flywheel to lid.
    h3 = 0.04; % [m] Distance from outer edge of flywheel to the inside of
the steel cage.
    %    Material properties for steel.
    c_steel = 486; % [J/kgK]
    rho_steel = 7800; % [kg/m^3]
    %    Material properties for air pocket.
    c_air = 1005; % [J/kgK]
```

```
    P_air = 101325; % [N/m^2] Air standard pressure
    R_air = 287.05; % [J/kgK] Air specific gas constant
    rho_air = P_air / (R_air * Temp); % [kg/m^3]
    %    Bearing dimensions.
      rBouter = 0.056; % [m] Distance from centre of flywheel to outer edge
of outer race (Cone = 29675, Cup = 29620).
    rBinner = 0.035; % [m] Distance from centre of flywheel to inner edge of
inner race (Cone = 29675, Cup = 29620).
    FPreload = 271; % [N] Force used to keep bearings fully engaged (200
lbf).
    muBearing = 0.005; % [-] Relation between normal force and frictional
force.

    % Convert from mph to m/s
    roadSpeed = roadSpeedInput * 0.44704; % [m/s], linear speed of vehicle

    % Assume the kinetic energies of the car and flywheel are equal.
    omegaIn = sqrt(mCar * roadSpeed^2 / IWheel); % [rad/s], rotational speed
of flywheel

    % Determine the vehicle acceleration over the time period (assumed
linear) [m/s^2]
    % Store current acceleration in fourth column of matrix
    accel = speedInc * 0.44704 / timeInc; % [m/s^2], vehicle acceleration

    k = abs(accel / aMax); % fractional acceleration (1 means vehicle is at
max acceleration)
    if(k > 1) & (limitSet_bool == 1)
        error('Acceleration too high (Greater than aMax). Reduced to aMax.');
        k = 1;
        if(accel < 0)
            accel = -aMax;
        else
            accel = aMax;
        end
    end

    % Compute air's viscosity (etaAir) at given temperature (Sutherland's
formula).
    A = 1.462E-6; % [sqrt(K)]
    S = 110; % [K]
    etaAir = A * sqrt(Temp) / (1 + (S / Temp)); % [kg/m sec]

    % Compute bearing power losses, assuming only frictional wear.
    POutBbearing = 1/sqrt(2) * muBearing * (mWheel * g + FPreload) * (0.5 *
(rBouter - rBinner)) * omegaIn; % [N] Bottom bearing, include weight of wheel
and preload
    POutTbearing = 1/sqrt(2) * muBearing * (FPreload)*(0.5 * (rBouter -
rBinner)) * omegaIn; % [N] Top bearing, include preload

    % Compute frictional torque on wheel after determining whether flow is
turbulent or laminar
    nuAir = etaAir / rho_air;
    ReAir = omegaIn * rWOuter^2 / nuAir;

    if(omegaIn ~= 0)
```

```
        if(ReAir < 2 * 10^5) % Laminar flow; gap s/R > 0.02 (Larger than
boundary layer)
            Cm = 2.67 * ReAir^(-1/2);
        else  % Turbulent flow; gap s/R > 0.02 (Larger than boundary layer);
experimental results about 17% higher
            Cm = 0.0622 * ReAir^(-1/5);
        end
    else
        Cm = 0;
    end

    M_fric = 1/4 * Cm * rho_air * omegaIn^2 * rWOuter^5; % frictional torque
coefficient
    POutDrag = M_fric * omegaIn; % [W], total power loss due to drag

    % Determine whether or not to add air drag into the total lost power.
    if(airDrag_bool == 0)
        POutDrag = 0;
    end

    % Compute total power required for flywheel
    alpha = abs(accel) /  rWOuter; % [rad/s^2], acceleration of flywheel
    tauOut = IWheel * alpha; % [Nm], torque required for acceleration
    POut = tauOut * abs(omegaIn);

    % Compute total lost power
    PLoss = POutDrag + POutBbearing + POutTbearing;
    if(accel >= 0)
        PIn = POut + PLoss;
    else
        PIn = POut - PLoss;
    end

    % Compute kinetic energy of flywheel [kJ]
    EStored = 0.5 * IWheel * omegaIn^2 / 1000;

    % Calculate heat rise within bunker, assuming all heat goes into
temperature rise
    VolAir = 2 * rWOuter * pi * (tWheel + h1 + h2) * h3 + pi * rWOuter^2 *
(h1 + h2);
    VolSteel = pi * rWOuter^2 * tWheel;
    VolTot = VolAir + VolSteel;

    % Find temperature increase rates, given the thermal mass and power
(assumes adiabatic bunker)
    deltaT_air = POutDrag / (VolAir * rho_air * c_air);
    deltaT_steel = (POutTbearing + POutBbearing) / (VolSteel * rho_steel *
c_steel);

    % New temp is volume averaged value of the temp rise in steel and air
    Temp_New = Temp; %(0.33 * (VolAir / VolTot) * deltaT_air + (VolSteel /
VolTot) * deltaT_steel) + Temp;
```

Transmission (transmission.m)

```
% M. Zedler (mzedler@MIT.edu), 16 Apr 2007
```

```
% 2.ThU - Undergraduate Thesis, Prof. C. Forbes Dewey
% Model of transmission system used in lab-based electric vehicle (EV)

% Models behavior of motor (Azure Dynamics AC24 AC induction motor) and
gearbox (5 : 1 speed reduction, oil-filled)

function [PMotorIn, PMotorOut, powerFrac] = transmission(omegaGearboxOut,
PGearboxOut, accel);
% Takes rotational speed from flywheel and uses it to determine power into
transmission.
% Transmission consists of motor (AC24 Induction, Delta connection) and
gearbox (5:1 reduction)
% INPUT: Rotational speed of flywheel (omegaGearboxOut) [rad/s], power out of
gearbox into flywheel (PGearboxOut) [W], and acceleration.
% OUTPUT: Power out of controller / into motor (PMotorIn) [W], power out of
motor / into gearbox (PMotorOut) [W], and fraction of max motor power used
(powerFrac).

    % Define system
    effGearbox = 0.98;
    N = 5; % 5:1 gearbox

    % Determine motor ouput speed assuming no speed loss through gearbox
(simple reduction)
    omegaMotorOut = omegaGearboxOut * N; % [rad/s]
    rpmMotorOut = omegaMotorOut * 30 / pi; % [rpm]

    % Determine motor output power
    if(accel >= 0)
        PMotorOut = PGearboxOut / effGearbox; % [W]
    else
        PMotorOut = PGearboxOut * effGearbox; % [W]
    end

    if(omegaMotorOut == 0)
        tauMotorOut = 0;
    else
        tauMotorOut = PMotorOut / omegaMotorOut;
    end

    % Use speed-torque curve to determine the maximum power available from
the motor at given speed.
    if (rpmMotorOut >= 0) & (rpmMotorOut <= 4000)
        tauMotorOutMax = 74;
    elseif (rpmMotorOut > 4000) & (rpmMotorOut <= 6500)
        tauMotorOutMax = -0.0188 * rpmMotorOut + 149.2;
    elseif (rpmMotorOut > 6500) & (rpmMotorOut <= 10500)
        tauMotorOutMax = -0.00425 * rpmMotorOut + 54.4;
    else
        tauMotorOutMax = 5;
        %error('Motor speed outside of torque-speed curve');
    end

    % Calculate maximum power fraction (based solely on torque)
    powerFrac = tauMotorOut / tauMotorOutMax;

    % Use speed to determine efficiency for motor.
```

```
    if(rpmMotorOut > 0)
        effMotor = 0.093*log(rpmMotorOut)+0.0517;
    else
        effMotor = 0.5;
    end

    % Use motor efficiency to determine required input power.
    if(accel >= 0);
        PMotorIn = PMotorOut / effMotor; % [W]
    else
        PMotorIn = PMotorOut * effMotor; % [W]
    end
```

Controller (controller.m)

```
% M. Zedler (mzedler@MIT.edu), 27 Apr 2007
% 2.ThU - Undergraduate Thesis, Prof. C. Forbes Dewey
% Model of transmission system used in lab-based electric vehicle (EV)

% Models behavior of EV controller (Solectrica DMOC445)

function [PCont_In] = controller(t, t_old, accel, PCont_Out);
% Calculate controller input power (PCont_In) [W].
%   Assume controller output voltage is constant when vehicle is accelerating
%   When vehicle is slowing (regenerating), battery input voltage is
constant; CURRENT fluctuates to cover power requirement

% INPUT: Time (t) [s], time of last iteration (t_old) [s], acceleration of
flywheel (accel) [m/s^2], power output from controller to motor (PCont_out)
[W].
% RETURN: Power inputted from the battery into the controller (PCont_In) [W].

C_1 = 0;
C_2 = 0.9;
effController = C_1 * accel * (t - t_old) + C_2; % <<!!!!>> Should be fxn of
time (temp)

    if(accel >= 0)
        VCont_Out = 136.5; % [V], assume motor is driven at constant voltage
        PCont_In = PCont_Out / effController; % [W]
    else
        PCont_In = PCont_Out * effController; % [W]
    end
```

Battery Bank (battery.m)

```
% M. Zedler (mzedler@MIT.edu), 2 May 2007
% 2.ThU - Undergraduate Thesis, Prof. C. Forbes Dewey
% Model of transmission system used in lab-based electric vehicle (EV)

% Models behavior of battery bank (Thirteen series-connected DieHard Gold
North 33065 lead acid batteries
function [BATTERY_1, charge_start_1, time] = electrical(BATTERY_0, PCont_In,
t, t_old, totalTime, accel, charge_start_0);
```

```
% Model of the battery for the electric vehicle model. Assuming 13 Pb-acid
batteries connected in series, linear models for all battery parameters,
% resistance-free connections, and known initial battery inputs
% Based on models proposed in "Batteries for Electric Vehicles" [Rand,
D.A.J., Woods, R., & Dell, R.M.] and "Handbook of Batteries" [Linden, D.]

% INPUT: BATTERY vector (See definition below) at time t-1, power inputted
from the battery into the controller (PCont_In) [W], time (t) [s], time of
last iteration
%        (t_old) [s], acceleration of flywheel (accel) [m/s^2], and boolian
indicating whether or not battery is being charged (charge_start_0) [0 / 1].
% RETURN: BATTERY vector (See definition below) at time t, and boolian
indicating whether or not the battery is getting charged (charge_start_1).

% BATTERY MATRIX DESCRIPTION
%   1 = Voltage [V]
%   2 = Current [A]
%   3 = Internal Resistance [Ohms]
%   4 = Capacity [Ah]
%   5 = Power [W]
%   6 = Energy [kJ]
%   7 = State of Charge [%]
%   8 = Temperature [degC]
%   9 = Status Bool [0 if discharging, 1 if recharging]
%---------- CONSTANTS -----------------
%  10 = Internal Volume [m^3]
%  11 = Usage Cycle [#]
%  12 = Starting Battery Capacity [Ah]

% Standard voltages for 78 Pb-acid cells connected in series
    VBatt_Full = 163.8; % [V], voltage when fully charged (SOC = 1)
    VBatt_Empty = 136.5; % [V], voltage when fully discharged (SOC = 0)
    VBatt_Charge = 187.2; % [V], voltage when being recharged (SOC ~= 1.9)
    VCont_Out = 136.5;

% Standard resistances for 78 Pb-acid cells connected in series
% Resistance range varies depending on battery lifetime
    if(BATTERY_0(11) < 100)
        RBatt_Full = 0.0078;
        RBatt_Empty = 0.039;
    elseif((BATTERY_0(11) >= 100) & (BATTERY_0(11) < 250))
        RBatt_Full = 0.0312;
        RBatt_Empty = 0.0702;
    elseif((BATTERY_0(11) >= 250) & (BATTERY_0(11) < 400))
        RBatt_Full = 0.0546;
        RBatt_Empty = 0.0858;
    elseif((BATTERY_0(11) >= 400) & (BATTERY_0(11) < 500))
        RBatt_Full = 0.0858;
        RBatt_Empty = 0.117;
    else
        RBatt_Full = 0.117;
        RBatt_Empty = 0.117;
    end

% Set constants for Peukert curve-fit and battery recharging
    n = 1.3; % [--], should be between 1.1 and 1.4 for Pb-acid
    rating = 20; % [hr], rate at which the battery capacity is given
```

```
% Set battery composition
    C_H2SO4 = 1420; % [J/kgK], heat capacity of sulfuric acid
    rho_H2SO4 = 1840; % [kg/m^3], density of sulfuric acid

% Determine if battery is discharging or recharging
%   If vehicle is accelerating, battery will discharge until the battery
capacity is exhausted (t_discharge < t - t_old)
%   If vehicle is decelerating, battery will recharge until capacity is
returned to starting value + temp changes
    if(accel >= 0)

        % Battery is discharging
        BATTERY_1(9) = 0;
        charge_start_1 = 0;

        % Determine lifetime using Peukert curve-fit
        if(BATTERY_0(2) == 0)
            I_Peukert = PCont_In / BATTERY_0(1);
            C_Peukert = BATTERY_0(12);
        else
            I_Peukert = abs(BATTERY_0(2));
            C_Peukert = BATTERY_0(4);
        end
        t_discharge = C_Peukert * (C_Peukert / rating) ^ (n-1) / I_Peukert ^
n * 60 * 60;

        % Determine end voltages for linear regressions
        VBatt_End = VBatt_Empty;
        RBatt_End = RBatt_Empty;

        if(BATTERY_0(1) > VBatt_Full)
            BATTERY_0(1) = VBatt_Full;
        end

        % Determine slopes for linear regressions
        if(t_discharge <= (t - t_old))
            t_cycle = t_old + t_discharge;
        else
            t_cycle = t;
        end

        if(t_discharge > 0)
            mV = (VBatt_End - BATTERY_0(1)) / t_discharge;
            mR = (RBatt_End - BATTERY_0(3)) / t_discharge;
        else
            mV = 0;
            mR = 0;
        end

        % Determine battery voltage, resistance, and current
        BATTERY_1(1) = BATTERY_0(1) + mV * (t_cycle - t_old);
        BATTERY_1(3) = BATTERY_0(3) + mR * (t_cycle - t_old);
        BATTERY_1(2) = PCont_In / BATTERY_1(1);

    else
```

```
        % Battery is charging
        BATTERY_1(9) = 1;

        % Set the charging start time (charge_start_1) to t of the initial
negative acceleration
        if(charge_start_0 == 0)
            charge_start_1 = t;
        else
            charge_start_1 = charge_start_0;
        end

        % Determine battery current from given input power and constant
voltage
        BATTERY_1(2) = -PCont_In / VBatt_Charge;

        % Use current rate and a time constant to determine charging time
        %   If no current is flowing, battery charging time is nearing
infinity
        %   Otherwise, battery charging time is inversely proportional to
current
        if(BATTERY_1(2) == 0)
            t_charge = 100000000;
        else
            t_charge = 101.19 * abs(BATTERY_1(2)) ^ -1.034 * 60 * 60;
        end

        % Determine end voltages for linear regressions
        VBatt_End = VBatt_Charge;
        RBatt_End = RBatt_Full;

        % Determine slopes for linear regressions
        mV = (VBatt_End - BATTERY_0(1)) / t_charge;
        mR = (RBatt_End - BATTERY_0(3)) / t_charge;

        % Determine battery voltage and resistance
        BATTERY_1(1) = BATTERY_0(1) + mV * (t - t_old);
        BATTERY_1(3) = BATTERY_0(3) + mR * (t - t_old);
    end

    % Determine state of charge (BATTERY(7)) [%]
    BATTERY_1(7) = (BATTERY_1(1) - VBatt_Empty) / (VBatt_Full - VBatt_Empty)
* 100;
    if(BATTERY_1(7) ~= 0) & (BATTERY_0(7) <= 0)
        BATTERY_1(11) = BATTERY_0(11) + 1;
    end

% Determine battery power losses (PLoss) [W] due to internal resistance (I^2R
losses)
%   If accelerating, total battery power is sum of power losses and power
output to controller
%   If decelerating, total battery power is power output to controller less
the resistive power losses
    PLoss = BATTERY_1(2)^2 * BATTERY_1(3);
    if(accel >= 0)
        BATTERY_1(5) = PLoss + PCont_In;
    else
        BATTERY_1(5) = PCont_In - PLoss;
```

```
    end

% Determine temperature rise (deltaT) [degC] by assuming all power loss goes
into heat rise
    deltaT = PLoss / (BATTERY_0(10) * rho_H2SO4 * C_H2SO4);
    BATTERY_1(8) = BATTERY_0(8) + deltaT;

% Determine new capacity of battery (BATTERY(4)) [Ah] based on temperature
and current
%   If accelerating, new capacity is found by using the Peukert curve fit
with the remaining discharge time
%   If decelerating, new capacity grows from capacity whil accelerating to
maximum capacity due to temperature effects
    if(accel >= 0)
        CEnd = 0;
        time = t_discharge;
        t_charge = 0;
    else
        CEnd = BATTERY_0(12);
        time = t_charge;
    end

    if(time > 0)
        mC = (CEnd - BATTERY_0(4)) / time;
    else
        mC = 0;
    end

    BATTERY_1(4) = mC * (t - t_old) + BATTERY_0(4);
    if(BATTERY_1(4) < 0)
        BATTERY_1(4) = 0;
    end

% Determine stored energy [kJ] by multiplying potential (voltage) by energy
storage rate (capacity)
    BATTERY_1(6) = BATTERY_1(1) * BATTERY_1(4) * 60 * 60 / 1000; % [kJ];
instantaneous energy stored in battery

    BATTERY_1(10) = BATTERY_0(10);
    BATTERY_1(11) = BATTERY_0(11);
    BATTERY_1(12) = BATTERY_0(12);
```