## **Reducing Earthquake Risk on the East Japan Railway through Investment in Safety**

**by**

Zhaopu Si

B.S. University of Science & Technology of China (1993)

Submitted to the Department of Civil and Environmental Engineering in partial fulfillment of the requirements for the degree of

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Author .... .. Department of Civil and Environmental Engineering June 12, 1996 Certified by *...* 'Professor Joseph M. Sussman JR East Professor Department of Civil and Environmental Engineering Thesis Supervisor Accepted by ... Professor Joseph M. Sussman JR East Professor Chairman, Departmental Committee on Graduate Studies

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## **Abstract**

Earthquakes can cause catastrophic accidents to the railroads in terms of serious facility damages and large number of passenger fatalities and injuries. To reduce earthquake risk, various safety counter-measures can be employed. Among them, performing an accelerated construction program of track strengthening, retrofitting rail viaducts, installing warning systems, and improving the braking systems are considered by JR East.

This thesis first modeled and estimated the collective earthquake risk which includes the physical damages due to track or viaduct failures and passenger fatalities and injuries due to the train derailment when running trains hit the track failures. Both the base collective risk and the perceived collective risk are calculated by employing the concepts of Safety Performance Index (SPI) and Perceived Safety Performance Index (PSPI). After assessing the earthquake risk, benefit-cost analyses are carried out for each of the four safety counter-measures. The design of the most cost efficient plans for both single and multiple track strengthening projects are studied here. Based on the benefit-cost analyses, the thesis uses a case study to develop a safety management plan in terms of allocating the investment among the counter-measures to reduce the earthquake risk. In the case study, the optimal investment allocation plans are obtained under different budget restrictions. When the risk level decreases, the marginal investment for reducing one additional unit of risk increases no matter which safety counter-measures are used as the optimal. In addition, if different risk measures in terms of different conversion factors to calculate the PSPI are used, the optimal investment plan may be different.

Thesis Supervisor: Joseph M. Sussman Title: JR East Professor, Civil and Environmental Engineering

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# **Chapter 1**

## **Introduction**

## **1.1 Background and motivation**

Being the largest regional passenger railroad in Japan, and, in fact, the largest passenger railroad in the world which carries about 15 million passengers per day, runs 12,000 trains per day and provides 100 billion passenger-km of service per year, the East Japan Railway Company (JR East) has given top priority to safety since it was founded in 1987. It has taken a very active role in the world of research and development with programs in the safety area. Among these is the JR East/MIT joint research program in rail risk assessment, which will be introduced in more detail in the following sections. The research presented in this thesis is sponsored by this program. It includes evaluation of various alternative strategies to improve railroad safety with respect to earthquakes.

Although seismic disasters have low probability, the damage consequences to the railroad facilities and passengers are very large. Also, because of this low probability, there are few historical data available for reference. All the above reasons make the seismic risk assessment research to be considered important by JR East, not only for the Shinkansen high speed passenger train system but also for the conventional railroad system. The requirement for such was highlighted by the Kobe earthquake.

On January 17, 1995, an earthquake of magnitude 7.2 in Richter Scale in the Kobe region caused severe damage to the railway infrastructure and facilities in that area, especially those of JR West and JR Central. The damages included the collapse of girder bridges and elevated sections of high-speed and conventional lines, and the derailment of many commuter trains. Fifteen trains that were running in the affected area crashed. The total cost of repairs to tracks and structures was assessed at 412 billion yen (US\$4.15 billion) by the Ministry of Transport of Japan. Although JR East did not suffer damages in the



Figure 1.1: Network map of **JR** East

earthquake, it is potentially vulnerable to moderate local seismicity and to the more frequent and more intense earthquakes that originate in the subduction zone off the eastern coast of Japan.

Therefore, improving the various facilities of the whole system to protect railroad property and passengers against earthquakes in the long run is a concern of JR East. But where to invest and how to allocate limited funds in order to achieve the best safety improvement, in turn, becomes an important topic worthy to be studied carefully. This is also the major task of this research.

There are various safety counter-measures that may be helpful to improve earthquakerelated safety. The followings are several counter-measures regarded by JR East:

- 1) Strengthening the tracks,
- 2) Retrofitting the rail viaducts,
- 3) Installing earthquake early warning systems,
- 4) Improving braking systems,
- 5) Reducing the operating speed of trains.

Before describing the individual counter-measures in a greater detail in the next section, we should first discuss the cost structures of these strategies briefly, especially the first four counter-measures because reducing operating speed actually does not need investment.

As for the earthquake warning and braking systems, the costs are different for different types of systems. Usually, the more advanced technology involved, the higher the cost will be. But for track strengthening projects, the situation is more complex. Since the costs for strengthening the tracks depend on not only the amount of strengthening, but also the construction speed of the projects which affects the risk reduction directly. For example, if we proceed all the strengthening projects with a high speed, the costs will likely be very high. But if we take a long time to finish the projects, then the chance for the unprotected railroad to be exposed to earthquakes will be increased. Thus, we need to balance the risk reduction with the costs.

Furthermore, there are interactive relationships between these counter-measures in terms of earthquake risk reduction. If the track condition is very good, we may not need substantial investment on warning and braking systems (In the extreme condition, the track cannot be broken by the earthquake shaking and we need no warning system). On the other hand, if we have very good earthquake warning systems, the requirement for the braking systems could be relatively lower. These interactive relationships make the optimization analysis more interesting, and, at the same time, more complicated.

## **1.2 Safety counter-measures to protect railroads against earthquakes**

As we mentioned in the above section, we will describe the safety counter-measures with respect to reducing earthquake risks in more detail.

#### **1.2.1 Track strengthening projects**

Track strengthening means enhancing the strength of tracks and their bases so as to reduce the track failure rate and degree of damage during severe earthquakes. Actually, track strengthening here includes not only the actions of strengthening the rails but also actions such as strengthening the track bases.

The advantages of the track strengthening include improving the overall safety condition of the rail lines. It will bring benefits for reducing risks not only of earthquakes but also of other natural disasters, such as rainfall and landslides.

Track strengthening also has its disadvantages. This includes the extremely high investment and effects on the traffic operations, such as reduction of traffic volume and hence revenue loss when the projects are in process.

#### **1.2.2 Retrofitting the rail viaducts**

Since a large percentage of the Shinkansen lines was built on concrete viaducts which are relatively more vulnerable to earthquakes compared with normal lines, then retrofitting the rail viaducts is also a concern for reducing the earthquake risks.

Retrofitting the rail viaducts means enhancing the strength of columns of the viaducts by increasing the cross-section area or improving the damage resistance of the columns, and therefore reducing the column failure rates as well as the track failure rates during severe earthquakes. In this research, retrofitting the rail viaducts is not only referred to the actions of strengthening the viaduct columns but also to the actions of strengthening the associated tracks of the viaduct lines.

Similar to the track strengthening counter-measure, retrofitting the rail viaducts also has its advantages and disadvantages. The advantages of the viaduct retrofitting include improving the overall safety condition of the rail lines. It will bring benefits for reducing risks not only of earthquakes but also of other natural disasters, such as rainfall and landslides.

Also similar to track strengthening, the disadvantages of retrofitting viaducts include the extremely high investment and effects on the traffic operations, such as reduction of traffic volume and hence cause revenue loss when the projects are in process.

Also similar to the track strengthening counter-measure, the disadvantages of retrofitting viaducts include the extremely high investment for proceeding with the projects and the effects on the traffic operations, which may reduce traffic volume and then cause revenue loss when the projects are in process.

#### **1.2.3 Early warning systems**

The early warning system is designed to provide early detection of arriving seismic waves, thus allowing early emergency braking of the trains. Through emergency braking, the early warning system reduces the distance traveled by trains on potentially damaged tracks and therefore reduces the risk of severe accidents like derailments, which under seismic conditions occur mainly when a running train encounters a damaged section of the track. Then, the costs are as high as track strengthening or viaduct retrofitting.

The main disadvantage of this counter-measure is that the effectiveness of the warning systems depends on the operation strategies. For example, the effectiveness is related to the earthquake intensity parameters used to determine the triggering of various actions, such as emergency braking, track inspection, and resumption of operation. If the operation strategies are not "perfect", there will be losses caused by unnecessary train cancellations and delays. Unfortunately, to have a "perfect" operation strategy of the early warning system is extremely difficult.

#### **1.2.4 Advanced emergency braking system**

As mentioned in the advantages of the early warning systems, advanced emergency braking systems can reduce the distance and time traveled by trains on potentially damaged tracks and therefore reduce the risk of severe accidents like derailments.

In addition to the advantage of reducing the earthquake risks the emergency braking systems can also be significantly effective when other hazard conditions occur.

#### **1.2.5 Reducing the operating speed of trains**

Another option to reduce the earthquake risk is to lower the operating speed of trains. This safety counter-measure can reduce train derailment risk caused by earthquakes because lower speed trains can be stopped in shorter distance by emergency braking.

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But there is an obvious disadvantage associated with this counter-measure. Lower operating speed may result in the loss of revenues and reduce the level of service. Assume that for a preliminary analysis, we ignore the loss caused by the reduction of level of service (which may induce the long-run demand decline and is difficult to measure) and only consider the possible revenue loss caused by the reduction of operating speed itself, we can think it in the following way:

To keep the revenue the same as before the operating speed is reduced without changing the total number of trains operated, the average headways of the trains in the whole system have to be decreased. But the decreased headways of trains could in turn increase the earthquake risk because more trains will be exposed in a certain area if an earthquake strikes there. If the risk increased by the shorter headways is dominated by the benefit by the lower train operating speed, then the reduction of the operating speed is worthwhile. Otherwise, it would not be recommended. Usually, the minimum headways are set by taking other railroad risks into account (not necessarily those consequences related to earthquakes). So we may not have much room to manipulate the train headways. However, we can still choose the possible alternative operating speeds and their corresponding headways and carry out the corresponding the risk benefit analysis.

As discussed above, all the strategies have their own advantages and disadvantages. Furthermore, these advantages and disadvantages are interdependent sometimes. For example, the effectiveness of a early warning system would depend on well implemented braking systems. Good track condition brought by track strengthening projects will make the operation of early warning system optimized (For example, the warning for minor earthquakes can be ignored and therefore many unnecessary train delays and cancellations can be avoided). So we should not only evaluate these counter-measures individually but also should consider their interdependencies when carrying out the risk-benefit analyses.

## **1.3 JR East/ MIT joint risk assessment program**

According to Sussman, 1995, the JR East/MIT joint Research Program in Risk Assessment began in January 1992. The project was conceived as a joint research program in the risk assessment area involving activities both at MIT and at JR East.

At MIT, many faculty and staff members along with students from various academic departments and centers such as Civil and Environmental Engineering, Aeronautics and Astronautics and Sloan School of Management, Center for Transportation Study and Operations Research Center participated in a variety of topics. In the aspect of JR East, the staff members of the Safety Research Laboratory at JR East and other JR East divisions such as Technical Research and Development Department, Technical Center and International Department also carried on parallel activities in the area.

As we have mentioned in the beginning, JR East has established safety as its top priority. Only through the risk assessment analysis can JR East determine an effective investment in improving safety. At the same time, MIT have carried out a lot of research in the area of risk assessment for nuclear and transportation safety. Therefore, a program in the risk assessment area was developed.

Since the beginning of this project, many topics have been carried out, according to Sussman, 1995, including:

- \* Development of safety indices for JR East operations
- \* Analysis of level-crossing safety from a risk assessment perspective
- \* Derailment analyses
- \* Risk assessment perspective on earthquake sensing and train operating policies
- \* Construction program design
- \* Risk assessment in the context of the 1995 Kobe earthquake
- \* Human factors analyses--rolling-stock maintenance
- Hazards due to rainfall
- \* Signal overruns

The program has proven to be a broadening experience for both MIT and JR East. A number of research documents have been produced and it is expected that some of the findings of the research will be reflected in JR East policies and practices in the future.



## **1.4 Research statement**

**Figure 1.2: Simplified framework of the thesis**

The research described here aims to provide JR East a set of methodologies to optimize the allocation of its earthquake safety investment **by** implementing modem risk assessment concepts such as safety performance indices, safety management plans and benefitcost analyses.

The research includes three major parts:

- 1) Review the risk assessment concepts and select the appropriate criteria for costbenefit comparison,
- 2) Model the earthquake risks and their effects on the railroad with and without the various safety improvement counter-measures,
- 3) Use case studies to explain the principles of developing safety management plans for allocating safety investment against earthquake threat.

First, the research reviews the concepts and methodologies of risk assessment technology and examines the cost-benefit criteria including NPV value, Benefit/Cost Ratio and Investment Return Rate which can be used in a safety management plan. Both NPV values and B/C Ratios are used for the unlimited and limited budget situations. The NPV values are more suitable in the situation when the budget is adequate while the B/C Ratios are more likely to be used in a tight budget situation.

Secondly, the risks to the railroad, trains and passengers in a damage region of an earthquake are determined by models using a variety of appropriate assumptions. The associated cost and benefit comparisons for the safety improvement counter-measures, such as track strengthening projects, retrofitting rail viaducts, installing earthquake early warning systems and improving braking systems, are therefore presented. The benefits of the counter-measures here are regarded as the reduction of the monetary measured risks caused by earthquakes compared to the condition before the corresponding safety countermeasures are implemented. The frameworks of the cost-benefit analyses for designing of the track strengthening program, viaduct retrofitting program, choosing early warning systems and braking systems are presented separately.

Finally, a safety management plan is provide in a simple network case. The plan can tell us how to invest for improving earthquake safety in the most efficient way if there is a certain budget for the total investment.

## **1.5 Outline of the thesis**

Chapter 1 was presented the motivation and background of this research, and then introduces the earthquake safety counter-measures which will be evaluated in the following chapters. The outline of the thesis is also provided in this chapter.

Chapter 2 introduces the basic concepts of risk assessment technology which are essential to the development of this research. First, the definition of "risk" is presented, followed by a review of the fundamental steps and methodologies of the modem risk assessment theory. Then, the definitions and applications of the Safety Performance Index and Perceived Safety Performance Index are introduced. After that, economic analysis methodologies which include the cost-benefit analyses and optimal investment allocation are presented in detail. Finally, the concept of a safety management plan is described.

Chapter 3 focuses on the earthquake risk assessment to railroads. At first, some earthquake assessment related concepts such as seismic Zoning and Microzoning maps are described briefly. Then follows the assessment of the earthquake consequences to the railroads. Models are presented to evaluate the earthquake risks to the railroad property and passengers, and calculate the monetary collective risks.

Chapter 4 presents the frameworks of cost-benefit analyses for the safety counter-measures corresponding to the earthquake threat. It starts with the frameworks of cost-benefit analysis for designing the track strengthening program. Then similar analysis for early warning system and advanced braking system are discussed. Optimal strategies of implementing these three counter-measures individually can be given by the models.

Chapter 5 uses two case studies of a simple railroad network to present the safety management plans for allocating the safety investment among the safety counter-measures to reduce earthquake risks. One case study is for a surface line, while the other is for a viaduct line.

Chapter 6 summarizes the thesis and provides the recommendation to JR East according to the results of this research.

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# **Chapter 2**

# **Basic Concepts of Risk Assessment Methodology**

In the first chapter, we presented the background and motivation of this research and gave a brief introduction of the content of the research is about. In this chapter, we will review the relevant literature on the risk assessment methodology which forms the basis for the research.

## **2.1 Overview of modern risk assessment**

Before presenting the research about the assessment of earthquake risks to the railroad and safety management plan to allocate the safety investment, we first introduce the fundamental concepts and methodologies in the field of modem risk assessment. By reviewing various literatures and previous works which have been done in the JR East/ MIT risk assessment project, this chapter will provide the readers with some basic knowledge of this field and help them to understand this research work.

### **2.1.1 'Risk' and 'risk assessment'**

First of all, the notion of risk is the most fundamental block for the risk assessment methodology. In general, risk can be defined as the average "cost" per unit of time due to the occurrence of unwanted events [Odoni, 1993]. This definition applies to any type of risk and to any environment. Specially, in the realm of transportation, risk can be defined as the product of the "average number of accidents per unit of time (in other words, the probability that an accident will occur)" multiplied by the "average cost per accident". Using an equation, it is:

$$
Risk (i) = P (i) \times C (i)
$$
 (2.1)

where  $p(i)$  is the probability of accident i,

C(i) is the cost of accident i.

The cost of an accident can be measured in terms of the number of fatalities or/and injuries that may result from the accidents, or it can be measured by the monetary amount of economic losses.

From the definition of risk, we can see that, independent of the units used to measure costs, risk can be reduced by either reducing the frequency of accidents or by reducing the consequences of an accident (in other words, the average cost per accident) or both. A large part of a risk assessment study is, in fact, concerned with identifying the most effective way to reduce the probability of accidents or the average cost of an accident when it occurs [Odoni, 1993].

#### **2.1.2 Fundamental steps and methodologies of modern risk assessment**

Modem risk assessment assists transportation professionals in deciding how much should be invested to reduce risk and how such investments should be allocated among the various potential alternatives. There are three steps to achieve this purpose:

- 1) Safety analysis
- 2) Risk appraisal
- 3) Economic analysis

The first step, safety analysis (or 'risk analysis') is the technical part of a risk assessment study. In this step, probabilities of various types of accidents and their consequences in terms of fatalities, injuries, property damages and interruption of services are identified and assessed. The effects of alternative investments for reducing risk on these probabilities and on the consequences of accidents should also be estimated.

To perform a good safety analysis, one must use a combination of methodologies. At first, one has to understand the physical properties, the technologies of each individual component and of the system as a whole. Secondly, one must use the probabilistic models, such as reliability theory, fault-tree and event-tree analysis, or statistical analysis of failure and accident data, to assess the risks.That is, (1) Identify adverse events, (2) Quantify their probability of occurrence, (3) Define and measure their expected consequences, (4) Sum over the system being analyzed.

The second step, risk appraisal, assesses and compares the relative importance of different types of failures or accidents. It also determines what is the appropriate level of safety that we should try to achieve.

To carry out the risk appraisal analysis, a measure of the overall risk of a system--the monetary collective risk need to be calculated first. To get this measurement, one needs to understand how individuals and society perceive risk. A good example is stated by Sussman and Roth, 1994; the deaths of 47 people in the 1993 Amtrak's derailment in Alabama received much more than 47 times the coverage and concern of the death of a single individual, say, in a grade crossing accident. That tells us when we try to calculate the monetary collective risk of an accident or that of a system, we must be careful about the societal valuation of risk. The calculation of monetary collective risk in this context will be explained in more detail in the following section when we introduce the safety performance indices.

Finally, the economic analysis will lead to an optimal allocation of resources. In this step, we need to understand the relationship between investment and safety; in other words, what level of safety can be achieved at each different possible level of investment. After this relationship is understood, we can decide how much to invest on safety improvements and where to invest.

The methodologies for economic analysis include cost-benefit analyses and optimal allocation of resources. One thing which needs to be noted here is that the successful selection of the optimal combination of investment can take place only after the appropriate cost-

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benefit analyses has been completed. These methodologies will also be examined and explained in the following sections.

### **2.2 Safety performance indices**

The Safety Performance Index **(SPI)** and the Perceived Safety Performance Index (PSPI) were developed to monitor the global real and perceived safety performance of a system. The concepts of the Safety Performance Index and the Perceived Safety Performance Index are essential since only after the safety performance of a system is monitored, can we measure the efficiency and effectiveness of the safety investment.

#### **2.2.1 Safety Performance Index (SPI)**

According to Nasser, 1995, the Safety Performance Index, also identified as the real safety performance index, is the sum of the frequencies of safety outcome, such as material and human casualties, weighted by their costs. It is defined as:

$$
SPI = \sum_{i=1}^{I} p_i \cdot x_i
$$
 (2.2)

where SPI is the Safety Performance Index of System S,

 $x_i$  is the consequence associated with adverse event i,

 $p_i$  is the probability of adverse event i.

As we have mentioned in the previous section, when conducting the risk appraisal analysis, we need to calculate the monetary collective risk; SPI provides such a tool. To describe this calculation more deeply, some issues about value of life need to be presented at first.

In the calculation of SPI (quantitatively monetary collective risk) of a transportation system, we often need to quantify the human damage consequences like injury and fatality, which requires us to estimate the value of life. There are currently two coexisting methodologies in the realm of life valuation: the first one is based on human capital theory, while the second is based on the willingness-to-pay approach.

The human capital theory is constructed upon the argument that the value that society should attach to the loss of one life is precisely the cost to society of the loss of this life such as damage to property, medical costs, administrative costs, net present value of the net reduction in output, pain and suffering imposed upon the relatives [Nasser, 1995]. By estimating and quantifying these cost portions, one can then get the value of life.

The second approach to determine the value of life is the willingness-to-pay theory. This approach assumes the risk to life can be treated as any other economic good, using the appropriate tools of expected utility theory. The value of life is certainly not uniform across all segments of the population. Obviously, affluent people are willing to pay higher sums for a decrease in their probability of death. A comprehensive survey of this approach can also be found in Nasser, 1995.

The willingness-to-pay approach has more economic and behavioral meanings compared to the human capital theory; hence it is more popular recently than the latter. We will adopt this approach in our research.

#### **2.2.2 Perceived Safety Performance Index (PSPI)**

The Perceived Safety Performance Index considers the distortion of individual perception of risks which induce non-linearity and require conversion factors in the valuation of risk. The distortions include: (1) mis-perception of the likelihood of events; and (2) heterogeneity in the valuation of risk across the attributes of risk. In other words, the Perceived Safety Performance Index is the sum of the frequencies of safety outcomes weighted by their costs along with risk conversion factors that account for the non-linearity in the perception of risk. It calculate the perceived monetary collective risk of an accident or that of a system. The PSPI can be defined as:

$$
PSPI = \sum_{i=1}^{I} (p_i x_i)^{\alpha}
$$
 (2.3)

where PSPI<sup>1</sup> is the Perceived Safety Performance Index of System S,

 $\alpha$  is the risk conversion power factor ( $\alpha$ >=1).

Usually, for perceived risk,  $\alpha$  is always greater than one. If a is equal to one, then the PSPI equals to SPI. The risk conversion power factor relates the magnitude of accident consequences to the measure of risks. It actually reflects the distortion to the real individual risks by the society. For example, the public have different perceived risk between catastrophic accidents and ordinary accidents, in this situation,  $\alpha$  is greater than one. In additional, different  $\alpha$  indicates the different way of how people perceive the risk; the greater  $\alpha$  is, the more different people perceive the risk with different consequences.

In this research, we consider the conversion factors pertaining to the difference between catastrophic vs. ordinary accidents, since the train accidents caused by earthquakes, especially those involving high speed trains can lead to catastrophic accidents which may cause hundreds of deaths. For example, the perceived risk for a fatality in a catastrophic accident that causes 100 fatalities will be much larger than that of a fatality in an accident that cause only one fatality because people will pay more attention to the 100-fatality accident than the total of the 100 1-fatality accidents. This is demonstrated by the immense and intense 'non-linear' media coverage.

Actually, not only the users of transportation systems and the general public perceive the catastrophic accidents conversely but also the owner of the transportation systems like JR

$$
PSPI = \sum_{i=1}^{I} \alpha_i p_i x_i
$$

<sup>1.</sup> In Nasser, 1995, the definition of PSPI is slightly different. That is:

where  $\alpha_i$  is called the conversion factor, which is associated with each outcome. In this definition, the conversion factors for different type of accidents are different. For example, the conversion factors are different between catastrophic accidents and ordinary accidents.

East. The safety reputation of the company can impact its revenue a lot. Sometimes a catastrophic accident can even lead to the bankruptcy of a company or cause stricter regulations which may affect future business of the companies. So transportation companies also perceive the catastrophic accidents very negatively.

There are some difference between the general public and the transportation companies in the way they perceive the accidents. Usually, general public perceive no difference between the deaths while the transportation companies are concerned the deaths that they are responsible for. For example, in a 20-fatality level crossing accident, all the deaths are third-party people but not passengers and employees and the accident is not the train's or signal's fault. The railroad company is not as seriously concerned about the accident as the general public.

From the definitions, it is clear that SPI measures the real cost of safety from the perspective of the operator while PSPI represents the cost of safety as perceived by the users of the transportation system and the general public. These two indices provide us the measures for both real and perceived safety performance.

### **2.3 Economic analysis**

Economic analysis is the third and also the key stage in risk assessment. Since one of the major purposes of risk assessment is to allocate the safety investment to maximize the risk reduction in the most efficient way. As mentioned before, the methodologies for economic analysis include cost-benefit analyses and optimal allocation of investment. Each of them will be described in detail in the following section.

### **2.3.1 Cost-benefit analysis**

According to Zerbe and Dively, 1994, the cost-benefit analysis is a set of procedures used for decision-making by defining and comparing benefits and costs. Generally, the types of choice facing the decision-maker can be classified as follows:

*1) Accept-reject.* Facing a set of independent projects and no constraint on the number which can be undertaken, the decision-maker must decide which projects, if any, are worthwhile.

2) *Ranking.* If there exits some constraints, such as budget limits, then all "acceptable" projects cannot be undertaken. In this case, projects must be ranked in terms of objective function.

*3) Choosing between exclusive projects.* Frequently, projects are not independent of each other. One form of interdependence exists when one project can only be under taken to the exclusion of another project, e.g. two different ways to achieve the same objective. Another special case of exclusion exists when any given project can be undertaken now or in a later period. There is a problem of choosing the optimal point in time to start the project.

The steps of cost-benefit analysis include:

- 1) Define the problem to be analyzed and identify the objective of this problem.
- 2) Find as many feasible alternatives as possible.
- 3) Choose a technique and appropriate criteria to value the uncertainty and the consequences.

4) Draw together the results of the cost-benefit analysis, recommend the choices and make decisions.

Among the four steps, step 2) is crucial to a successful analysis while step 3) is the heart of the cost-benefit analysis. Obviously, if the alternative set is not complete, some superior choices would not even be considered. In this sense, the cost-benefit analysis would lose its strength. The criteria and techniques for the cost-benefit analysis are introduced below:. Generally, there are three criteria of cost-benefit analyses, which will be then described in the following sections:

- 1) Net Present Value (NPV);
- 2) Benefit cost ratio (B/C);
- 3) Investment return ratio (IRR).

#### **1.** Net **Present Value (NPV) criterion**

At first, we need to clarify the idea that benefits and costs accrue at different point in time. This is easily understood from the economic principle that investment in capital projects involves the sacrifice of present benefits in favor of future benefits. The sacrifice of present consumption would not be worth while unless the gains in future consumption are greater. This principle acknowledges the existence of "social time preference" --a preference which society supposedly exhibits for present benefit over future benefits.

Then, to compare the benefits and costs, we need to discount all the benefits and costs in the value at a same time by the social rate of discount, usually the present time is chosen as the base time. This leads to the definition of Present Value:

$$
PV = \sum_{t=0}^{T} \frac{C_t}{(1+r)^t}
$$
 (2.4)

where r is the social discount (interest) rate.

The Net Present Value of Cost and Benefit of Project A is defined as following:

NPV (A) = 
$$
\sum_{t=0}^{T} \frac{B(A)_t - C(A)_t}{(1+r)^t}
$$
 (2.5)

where  $B(A)_t$  and  $C(A)_t$  are the benefit and cost of project A at time t, respectively.

The decision rule for accepting project A is only when the Net Present Value of A is greater than zero, that is, the discounted benefits should exceed discounted costs.

#### **2. Benefit/ Cost Ratio criterion:**

The definition of Benefit/Cost Ratio is another criterion. The definition of the Benefit/Cost Ratio for project A can be expressed as the ratio of the present value of benefits and costs of project A:

$$
B/C(A) = \sum_{t=0}^{T} \frac{B(A)_t}{C(A)_t}
$$
 (2.6)

Usually, the project will be accepted as long as the ratio exceeds one.

#### **3. Internal Rate of Return criterion:**

The present value rule requires the use of some predetermined social discount rate to discount future benefits and costs. An alternative rule is to calculate the discount rate which would give the project an **NPV** of zero and then to compare this "solution rate" with the predetermined social discounted rate. This "solution rate" is called the "Internal Rate of Return", also called the "marginal efficiency of investment".

In other words, the Internal Rate of Return of project **A** is the solution of the equation:

$$
\sum_{t=0}^{T} \frac{B(A)_t - C(A)_t}{(1+r)} = 0
$$
\n(2.7)

Where i is the Internal Rate of Return of project A.

The decision rule for accepting project A by using the Internal Rate of Return is only when the Internal Rate of Return is greater than the predetermined social discounted rate.

### **2.3.2 Optimal allocation of investment**

The optimal allocation of investment is another methodology which can be used in economic analysis in risk assessment besides cost-benefit analysis. It takes care of society's interest in an efficient allocation of its resources.

The problem is a classical optimization problem and can be solved by the marginal-costcriterion. This criterion allocates resources in such a way that the marginal cost for risk reduction is equal for all subsystems [Bohnenblust, 1995]. It is always possible to reduce the risk by an additional unit of investment. But the incremental costs needed for reducing risk by an additional unit increases as the risk becomes smaller. With resources always being limited, the money spent at one place will be lacking at another. Hence, the limited funds for safety counter-measures must be used in such a way that a maximum level of safety is achieved. That is, we have to stop our efforts at a certain slope of the risk /cost diagram for any system we interested as shown in Figure 2.1.

For every level of investment, we can determine the optimal risk reduction given by the lowest level of risk that can be achieved for this investment. But at first, we need to choose the proper level of investment. The slope which is the optimal safety criterion from the society's point of view is actually the price that one pays for a marginal increase in safety. If we derive the optimal level of investment in terms of maximizing net benefits, the solution to the benefit maximization problem is to set the marginal risk reduction per unit of investment equal to  $-1<sup>1</sup>$ . In Figure 2.1, the optimal point is A and the corresponding investment level is **C\*.**

In reality, our objective for safety improvement is often constrained by the budget limitation. Then we have to focus on the suboptimal resulting of investment. Assume a certain budget exists, then in Figure 2.1, the suboptimal investment point is B. There are three situations: 1) Budget $\langle C^*$ , we cannot invest enough on safety to achieve the optimal risk reduction; 2)Budget= $C^*$ , the investment is optimal; 3) Budget $>C^*$ , we overspend on safety.

<sup>1. [</sup>Nasser, 1995] gives the derivation of this problem in pg 57.



**Figure 2.1: Optimal Risk Reduction Investment**

## **2.4 The Safety management plan**

### **2.4.1 What is a safety management plan?**

**A** safety management plan is an action-oriented decision-aiding tool based on risk assessment technics. It takes a holistic view in the sense that it requires all safety issues to be evaluated by the same rules and that these rules are embedded in the organization or company's overall objectives [Bohnenblust, 1995]. The concept of safety management plan is important for risk assessment analysis since it is actually the purpose of the risk assessment. Once the safety management plan is given, the decision maker will know where and how to invest in safety.

The general characteristics of a safety management plan are described below:

First and foremost, it focuses on the optimal allocation of resources. Cost effectiveness is of most concern. It ensures obtaining maximum safety for the money spent.

Secondly, it allows linking facts and values. Safety planning requires knowledge about the technical and scientific facts of the processes related to a company's activities. It ensures consistent and transparent decisions. Numerous decisions have to be taken in the rail system. The decisions range from the daily decisions made to the strategic decision. Many of these decisions affect safety issues. The safety management plan helps to ensure that these decisions are made in a consistent way with respect to safety.

At last, it eases communication about safety among specialists, and with the public and authorities and allows controlling the effectiveness of action taken. By using a systematic, analytic framework, the effectiveness of safety counter-measures can be shown.

### **2.4.2 Elements of a safety management plan**

Bohnenblust, Sussman and Odoni, 1995 gave the definition of the elements of a safety management plan: A safety management plan consists not only of an analytic framework, but includes process-oriented elements as well, since providing safety is not a one time action, but a continuing, dynamic process. There are three elements included which are safety policy statement, organizational structure and risk-based safety analysis. Figure 2.2 indicates the relationships between the three elements of a safety management plan.



**Figure 2.2: The three elements of a safety management plan [Bohnenblust, Sussman and Odoni, 1995]**

Among these three elements, in this thesis, we focus on the risk-based safety analysis, since no change or improvement can be achieved without being able to rely on appropriate tools and methods. **A** risk-based safety model is an appropriate tool to pursue an actionoriented course which is focused on the optimal allocation of the scarce resources

The safety policy statement describes the company's vision with respect to safety. It includes the guiding ideas which lead the company and its employees while acting on safety issues. It allows top management to delegate safety decisions to the appropriate management level and to ensure that all decisions will be consistent.

In this chapter, we reviewed and introduced the basic concepts of the risk assessment methodology. In the next chapter, we will employ these concepts and principles to present the earthquake risk assessment models for the railroads, which is the basis of the risk-benefit analysis for allocating the investment in earthquake safety.

# **Chapter 3**

## **Earthquake Risk Assessment of Railroads**

## **3.1 Evaluating the earthquake risk of railroads**

The serious damages caused by earthquakes are well known. Their extensive devastation affecting the economy of countries prone to high seismicity shows that vigorous measures should be undertaken to reduce the loss of life and property during catastrophic earthquakes. Japan is near the high seismic risk zone. In addition, railroads, especially high speed railroads, are high investment and utilization infrastructures. So it is important to evaluate the earthquake risks to the railroads and protect them against earthquakes.

In this chapter, we will present the model for assessing the earthquake risk of railroads. Before describe the probabilistic model in more detail, we first introduce some earthquake assessment concepts such as seismic zoning and micro zoning, which are important for implementing our risk assessment of railroads related to earthquakes.

### **3.1.1 Seismic zoning and microzoning**

According to Walker, 1982, seismic zoning is a procedure to provide knowledge of the characteristics of probable future earthquakes. It must be stated at the outset that it neither involves the influence of local soil conditions, nor engineering problems of soil-structure interaction. The main product of seismic zoning is a seismic zoning map.

A zoning map of a region should be capable of predicting the future possible earthquake sequences in the region. Each sequence is described at least by the times (frequency), locations and magnitudes of future events.
While seismic zoning takes into account the distribution of earthquake hazard over the entire country or region, seismic microzoning defines the detailed distribution of earthquake risk in each seismic zone.

Most studies show that the distribution of damage caused by earthquakes indicate that the areas of severe damage are highly localized, and that the degree of damage may change abruptly over distances as short as 0.5 to 1 km. Many geoscientists have been led to believe that it is the local subsoil conditions that are of primary importance in the assessment of damages to structures.

Japanese investigators began to develop the seismic microzoning methodology in 1950 based on their experience with past earthquake damage. Many cities in Japan published seismic microzoning maps. All these maps present subdivisions of the city areas according to soil conditions.

The methodology and criteria to develop the seismic zoning and microzoning maps are too complicated and diverse and also not necessary to permit detailed discussion and documentation. But the basic idea is that, if the seismic zoning and microzoning maps of the whole JR East area are available and could provide us with the information about the frequency and location of major future earthquakes as well as the local soil conditions for every microregion, it would be quite useful for us to assess the earthquake risks to the whole system. In other words, the risk assessment of earthquake safety has to begin with the seismic zoning and microzoning maps. That is why we give a brief introduction to the seismic zoning and microzoning maps.

#### **3.1.2 Consequences of earthquakes**

Before evaluating and predicting the probabilistic earthquake risks to the railroads, it is needed at first to assess all the possible consequences of earthquakes to the railroads. The basic purpose of this work is to obtain a preliminary idea of damage in the event of severe earthquakes. This kind of assessment should be made for JR East.

Catastrophic earthquakes and their secondary effects such as landslides and tsunamis can cause severe damages to railroad facilities and properties such as rails, ballasts, bridges tunnels, trains or even the communication and signal systems. Also the train derailments and collisions caused by earthquakes can cause numerous passenger fatalities and injuries especially when this happens to high speed trains.

Furthermore, the train delays and cancellations due to the earthquake strikes and the repairs of the damaged facilities can also cause significant revenue losses to the railroads.

### **3.2 Earthquake risk assessment of railroads**

As described in Section 3.1.1, we can obtain the information about potential earthquake probabilities in the future for any possible small area in the whole railroad regions from the seismic zoning and microzoning maps. Then we can always divide the whole system into many small exclusive subareas each of which has the same probability of severe earthquakes (To be simple but not lose the significance, here, we only consider severe earthquakes with a certain range of magnitudes, e.g, M>=6.5) and roughly same soil conditions. For example, there may exist a subarea with a track length of K kilometers.

Also we could know all the possible adverse consequences caused by a severe earthquake; then we could model the risks.

#### **3.2.1 Assumptions and descriptions**

We first set out some notions and describe several assumptions that we will invoke throughout our work.

#### **1.** Probability **issues:**

1)  $P_0$  (times/year): Probability that an earthquake (greater than M6.5 in magnitude) will strike in the project area per year.

2) The track failures caused by an earthquake follow a homogeneous spatial Poisson process with rate **X** (failures/km). Tracks with different qualities as well as with different local soil conditions are associated with different values of  $\lambda$ . So,  $\lambda$  can indicate the safety conditions of the tracks. Construction program can be implemented which lower the value of **X.** This is one of the counter-measures we study in this thesis.

#### **2. Discount and inflation issues:**

**i: JR** East's discount rate,

**j:** real expected inflation of labor and construction materials.

#### **3. Risk issues:**

1) Physical damages: Including the direct loss (repair cost) of the track and train damage and the indirect loss (revenue loss caused **by** repair, which increases with the degree of infrastructure damage.)

2) Fatalities and Injuries: The number of fatalities and injuries is very difficult to estimate precisely. But the number should be closely related to the number of trains which would possibly crash in the earthquake. Also it is reasonable to assume that the number of passenger fatalities and injuries are proportional to the speed of the running train when it crashes.

#### **3.2.2 Modeling the earthquake risk of railroads**

Here, we consider two kinds of track lines. One is the normal track lines whose failures in an earthquake follow a homogeneous spatial Poisson process with rate  $\lambda$  (failures/km). The other is the viaduct track line whose failures are corresponding to the failures of the columns which directly support them. Let us study them in turn.

Figure 3.1 and Figure 3.2 explain the two kind of track failures caused by earthquakes, respectively.



**Figure 3.1: Track failures of** surface lines



Figure **3.2:** Track failures of viaduct lines

As stated above, the damages to the railways include physical damages and passenger fatalities and injuries. We can evaluate them separately at first and then calculate the monetary collective risks using the concepts of SPI and PSPI.

#### **3.2.3 Risks associated with physical damages**

At first, we consider the normal track lines (surface lines). We have assumed that the track fails in a homogeneous spatial Poisson process with rate  $\lambda$ . Then, the expected number of failures of a K-kilometer-long normal track line in an earthquake is  $\lambda$ K, for track with a safety condition of **X.**

Then the consequence involved with track damages can be expressed as:

$$
R(\lambda)_{1} = \lambda K \cdot d_{n} + r(\lambda) \tag{3.1}
$$

where  $d_n$  is the average repair cost of one normal track failure,

 $r(\lambda)$  is the revenue loss caused by the track repair after the earthquake.

As for the viaduct lines, if we assume that the columns as well as the distances of columns are same for any viaduct and we also will strengthen the columns to the same standard, then the failure situation of the viaducts are very similar to that of the surface lines, since we can assume the viaduct lines also as homogeneous lines. Then, to measure the risk of viaduct lines, we can simply introduce a weighting factor  $w(s)$  (normally,  $w(s) > 1$ ), which describes the fragility of the track structures on the viaducts across the region relative to the strength of surface track. JR East engineers would have the specific expertise to construct such a weighting function for each viaduct. So the track failure rate of lines on viaducts is:

$$
\lambda_{\mathbf{V}}(s) = \mathbf{w}(s) \cdot \lambda(s) \tag{3.2}
$$

where  $\lambda(s)$  is the track failure rate of normal lines at location s,

 $\lambda_V(s)$  is the track failure rate of lines on viaducts at location s.

By now, we can use the same models to evaluate the earthquake risks by the surface line failures and the line failures caused by column failures of viaducts.

#### **3.2.4 Risks associated with passenger fatality and injury**

Here, we use a simple model to estimate the number of passenger fatalities and injuries. At first, assume that a train will derail and crash when it enters a track failure with a speed not equal to zero. The distance to stop a train with emergency braking, x, can be calculated by:

$$
x = \frac{V_0^2}{2a} \tag{3.3}
$$

where,  $V_0$  is normal operating speeds,

a is the emergency deceleration rate.

Then, if an earthquake strikes, the probability of a train crash when it is running on the track with a failure rate of  $\lambda$  is:

$$
p_d = p \left( \text{Derail} | P_0 \right) = 1 - p \left( 0 \right) = 1 - \frac{e^{-\lambda x} \left( \lambda x \right)^0}{0!} = 1 - e^{-\lambda x} \tag{3.4}
$$

Where  $P_0$  is the probability of severe earthquakes which will occur per year, p(O) is the probability that a running train will not enter one track failure before it stops.

Now, let us consider what will happen in the K kilometer-long-track project area as shown in Figure 3.1, when an earthquake strikes. Assume the average headway of the normally operated trains is H, then about [K/H] (which means the largest integer less than or equal to K/H) trains will be running on the tracks of the project area.

Since the number of passenger fatalities and injuries associated with a train derailment is directly related to the train speed when the train crashes--the higher the speed, the more fatalities and injuries will be involved, and vice verse. Train vehicle survivability in a given crash scenario is a function of the kinematic behavior of the entire consist, the integrity and collapse characteristics of the structure of each vehicle and the overall interior configuration of a compartment and occupant/surface contact characteristics [Arthur D. Little and Calspan Corporation, 1993]. It is reasonable to assume that the numbers of passenger fatalities and injuries associated with one train derailment, say F and I, are proportional to the kinetic energy at the moment before it derails. This means:

$$
F = k_1 \cdot \left(V_0^2 - 2as\right) \tag{3.5}
$$

$$
I = k_2 \cdot \left( V_0^2 - 2as \right) \tag{3.6}
$$

where,  $k_1$ ,  $k_1$  are parameters which are functions of the vehicle mass and structure characteristics, etc.,

s is the train braking stop distance before it hits a track failure.

Then the expected number of fatalities and injuries of a train in an earthquake are:

$$
F = \int_{0}^{x} Ff(s) ds
$$
 (3.7)  

$$
I = \int_{0}^{x} If(s) ds
$$
 (3.8)

where,  $x = V_0^2/2a$ , which is the distance to stop a train with emergency braking,  $f(s)=\lambda e^{-\lambda s}$ , which is the probability density function of exponential distribution of s.

Now, the total expected number of fatalities and injuries by trains in the K kilometer-long tracks in an earthquake are:

$$
E(F) = \sum_{i=0}^{N} (N-i) \cdot F \cdot {N \choose i} \cdot p_d^{(N-i)} \cdot (1-p_d)^i
$$
 (3.9)

Where N=[K/H] is the no. of trains running on tracks when an earthquake occurs.

$$
E(I) = \sum_{i=0}^{N} (N-i) \cdot I \cdot {N \choose i} \cdot p_d^{(N-i)} \cdot (1-p_d)^i
$$
 (3.10)

Where  $E(I)$  is the expected number of passenger injury.

Also, the possible physical damages to the trains is:

$$
R(\lambda)_{2} = DT \cdot \sum_{i=0}^{N} \left[ (N-i) \cdot {N \choose i} \cdot p_{d}^{(N-i)} \cdot (1-p_{d})^{i} \right]
$$
(3.11)

Where DT is the average monetary damage to a crashed train.

#### **3.2.5 Calculating the monetary collective risks**

Now, we need to quantify the total risks involved in an earthquake. That is, the monetary collective risks need to be calculated.

As we have stated in Chapter 2, we can calculate the monetary collective risks and perceived monetary collective risks by using the concept of Safety Performance Index (SPI) and Perceived Performance Index (PSPI). These two kinds of monetary collective risks are from different points of view. While the monetary collective risk measures the real cost of safety from the perspective of the operator (here, the railroad), the perceived monetary collective risk measures the cost of safety as perceived by the users of the transportation system (here, the passengers) and the general public. Also we will use the willingness-to-pay theory to determine the value of life here.

According to the definition of SPI, the monetary collective risk which will be involved in K km long tracks when a severe earthquake strikes is:

$$
CR(\lambda) = R(\lambda)_{1} + R(\lambda)_{2} + W_{F} \cdot E(F) + W_{I} \cdot E(I)
$$
\n(3.12)

Where  $CR(\lambda)$  indicates the monetary collective risk,

 $W_F$  is the willingness-to-pay for a death,

 $W_I$  is the willingness-to-pay for a injury.

Similarly, according to the definition of PSPI, the perceived monetary collective risk which will be involved in K km long tracks when a severe earthquake strikes is:

$$
PCR (\lambda) = R (\lambda)_{1}^{\alpha} + R (\lambda)_{2}^{\alpha} + (W_{F} \cdot E(F))^{\alpha} + (W_{I} \cdot E(I))^{\alpha}
$$
 (3.13)

Where  $PCR(\lambda)$  indicates the perceived monetary collective risk,

 $\alpha$  indicates the risk conversion power factor.

In this chapter, we developed probabilistic models to evaluate the earthquake risk

This chapter actually covers the first two steps of the risk assessment methodology, which are safety analysis and risk appraisal. In the following chapter, we will employ the benefitcost analyses for alternative earthquake safety counter-measures. This is economic analysis, which the third step of the risk assessment methodology.

# **Chapter 4**

# **Risk-Benefit Analyses for the Safety Counter-Measures**

In this chapter, we will provide the frameworks to determine the optimal investment in terms of risk reduction for every earthquake safety counter-measure such as track strengthening project design, constructing early warning systems and installing advanced braking systems, **by** appropriate risk-benefit analyses.

# **4.1 Track strengthening program design**

As mentioned earlier, the **1995** Kobe earthquake caused severe damage to the railways in that area. In the wake of the Kobe tragedy and in view of what was learned from it, acceleration of track strengthening projects on the JR East system seems to be a policy worthy of consideration. This study takes initial steps on developing a framework **by** using a riskbenefit analysis to determine how fast a set of construction projects should be completed and the *degree of strengthening* that is appropriate.

The advantages of accelerating the track-strengthening projects are straightforward. Strengthening rail tracks reduces the probability and consequences of damage to infrastructure and trains, and passenger fatality and injury due to train crashes in future earthquakes. **If** a track-strengthening project is completed more quickly, that portion of the railroad is exposed to earthquake damage for a shorter time.

Damage from an earthquake to the track where a project has been completed would be less than if the track-strengthening project has not been started or is in the process of completion. More importantly, the probability of train accidents caused **by** failure of the track infrastructure during an earthquake is lowered as well.

However, there are disadvantages to accelerating a multiple-project construction program aimed at strengthening the railroad in anticipation of earthquakes---the construction cost may be significantly greater because of the rushed construction and the interactions among these multiple projects. The details will be discussed later when we are doing the cost analysis and assuming the construction cost vs. project duration functions.

This section is aimed at developing a modeling framework that JR East could use to 1) evaluate the trade-offs between track-strengthening programs involving multiple projects and 2) search for optimal project schedules. The model uses a risk-benefit analysis framework which balances the construction costs of the various ways of organizing the multiple project program with the benefits that accrue to JR East from the safety improvements that are derived from these construction programs. The model is necessarily probabilistic since earthquakes, the prime motivation for this construction program, are by nature probabilistic in magnitude and location.

In short, performing an accelerated construction program by starting track-strengthening projects earlier and performing each project more quickly, has clear benefits because the railroad is exposed to catastrophic earthquake damage and risk for a shorter period of time. At the same time the construction costs of such an accelerated program can be greater than a program extended over a longer period of time with less project simultaneity. The purpose of this model is to balance those costs and benefits.

#### **4.1.1 Single project at one location:**

In this section, we first consider a simple situation where only one track strengthening project for surface lines is processed in the network and the length of the project is K km with a homogeneous track failure rate  $\lambda$ . Similar analysis will be carried out for lines on viaducts. The risk-benefit modeling framework for single track strengthening project case is shown in Figure 4.1



Figure 4.1: Risk-Benefit analysis framework for single project

From the flowchart in Figure 4.1, it is clear that to carry out the cost-benefit analysis, we need to first identify the costs and benefits associated with the events and then compare them.

#### **1. Cost analysis:**

For the cost issues, we make the following assumption which will be valid throughout the thesis. That is, for a given safety improvement requirement  $(\lambda - \lambda_1)$ , the average cost of strengthening one kilometer's track (the direct cost of construction) changes only with the construction rate of the project, which is defined as the kilometers strengthened per unit time.

Furthermore, let us focus on the variable cost of construction. Considering an individual project, construction costs are likely to be greater when this project is performed on an accelerated basis. Completing the same amount of work in, for example, six months rather than one year will lead to higher construction costs. Rush work (higher construction rate) will lead to higher variable cost.

Levis (1967) gave the time(rate)-cost curves for construction activities and pointed out that the total variable cost will increase or at least stay the same as project duration is decreased because the constructor must employ additional resources to finish the activity in less time as the economics law of diminishing returns takes effect. Lessard (1972) indicated that the direct cost shows a minimum for 'normal' direct construction duration, when project activities are performed at minimum cost with maximum efficiency. When project duration is shorter, some activities have to be "crashed" (done more quickly) and direct cost increases because of higher overtime wages, reduced crew productivity, etc. If a project has a duration longer than its normal duration, crew productivity is below maximum efficiency and extra costs cover for idle manpower and for equipment. Figure 4.2 gives the cost-time diagram according to Lessard.



**Figure 4.2: Cost-time diagram by Lessard (1972)**

If we know the direct cost, and according to the Poisson process assumption we made for the track failure rates in Chapter 3, the annual cost of the track strengthening project can be expressed as:

$$
AC = C (K/T, \lambda - \lambda_1) \cdot \frac{K}{T} + C_1
$$
\n(4.1)

where, AC is the annual cost of the project,

T is the total construction time of the project,

K is the total length of tracks in the project,

 $\lambda$ ,  $\lambda$ <sub>1</sub> are the Poisson process rate of the track failures in an earthquake before and after the project, respectively,

 $C(K/T, \lambda-\lambda_1)$  is the variable cost function of unit (per kilometer) track,  $C_1$  is the fixed cost per year and the corresponding annual revenue loss caused by the process of the project (due to train cancellations and delays).

Considering the time-value discounting of money, which means the cost coming sooner will have a greater impact on total project cost, we can get the total completed project cost  $(in constant money value at present t=0):$ 

$$
TC = \sum_{t=1}^{T} \left[ AC \cdot \left( \frac{1+j}{1+i} \right)^t \right] - \frac{S \cdot K}{\left( 1+i \right)^{T+L}} + C_F \tag{4.2}
$$

where, TC is the total completed project cost  $(t=0)$ ,

L is the useful lifetime of the project,

**S** is the salvage value of per km track after the useful lifetime of the project is over,

 $C_F$  is the fixed cost for setting up the project (assumed to occur at t=0). Here, we assume  $C_F=0$  for simplicity.

#### **2. Benefit analysis:**

To estimate the benefit of the project, we measure the expected reduction of physical (track) damages to the railway in an earthquake with and without the project. This is a measure of the reduction of earthquake risks by the safety counter-measures. If a earthquake happens, there will be two situations: one is that the earthquake strikes during the project and the other is that it strikes after the project is completed (remember that we have assumed that the project begins at  $t=0$ ). We can consider the benefit in each of the two situations under the assumption that an earthquake actually strikes.

In Chapter 3, the models of assessing the earthquake risks have already been developed. The collective and perceived monetary collective risks are given in Equations 3.12 and 3.13. It is easy to show that if an earthquake strikes after the project is over, the benefit from the project which improves the track safety condition from  $\lambda$  to  $\lambda_1$  is:

$$
B = CR (\lambda) - CR (\lambda_1)
$$
 (4.3)

where  $CR(\lambda)$ ,  $CR(\lambda_1)$  are the monetary collective risks with and without the project, respectively.

Or, if according to the perceived monetary collective risks, the benefit is:

$$
PB = PCR (\lambda) - PCR (\lambda_1)
$$
 (4.4)

where  $PCR(\lambda)$ ,  $PCR(\lambda_1)$  are the perceived monetary collective risks with and without the project, respectively.

But the earthquakes can also strike during the construction period. In this situation, the benefit as well as the cost are assumed proportional to the completed portion of the project.

#### **3. Optimization analyses:**

Here, we carry out the cost-benefit analysis for one project (subsequent work will consider multiple projects); it is reasonable to assume that the project will begin at  $t=0$  because there is no reason to delay unless there are no funds available (in which situation, the project won't be considered).

If we assume that AC occurs at the end of the construction year (here, we assume the annual costs are the same for each year) and use B to indicate the benefit associated with the project, the cost-benefit stream diagram of a 3-year project (i.e., a project planned for a three year duration) is as shown in Figure 4.3, where  $p(t=n)$  means the probability of the first earthquake strikes in year t=n.





The probability tree of the time that an earthquake will happen is as shown in Figure 4.4, L indicates the economic lifetime of the project. So, if we let  $P_0$  indicates the earthquake probability per year, the probability that an earthquake will happen by year t is:

$$
p(t=1)=P_0
$$
  
\n
$$
p(t=2)=P_0(1-P_0)
$$
  
\n...  
\n
$$
p(t=k)=P_0(1-P_0)^{k-1}
$$
  
\n
$$
p(t=L+T)=P_0(1-P_0)^{L+T-1}
$$



**Figure 4.4: Probability tree of earthquakes**

Then, the total expected real and perceived benefits of the project can be computed as:

TB = 
$$
\sum_{t_0=1}^{T} \left[ B \cdot \frac{t_0}{T} \cdot p(t = t_0) \right] + \sum_{t_0=T+1}^{T+L} [B \cdot p(t = t_0)]
$$
 (4.5)

$$
PTB = \sum_{t_0 = 1}^{T} \left[ PB \cdot \frac{t_0}{T} \cdot p(t = t_0) \right] + \sum_{t_0 = T + 1}^{T + L} [PB \cdot p(t = t_0)] \tag{4.6}
$$

where,  $p(t=t_0)$  is the probability that the first earthquake will strike in year  $t_0$ .

So, the Net Present Value of cost and benefit of the project is:

NPV = (P) TB – 
$$
\sum_{t_0=1}^{T}
$$
 [(t<sub>0</sub> - 1) · AC · p(t = t<sub>0</sub>)] +  $\frac{S \cdot K}{(1 + i)^{T+L}}$  (4.7)

where, AC is the annual cost of the project,

(P)TB means using PTB or TB.

To get the optimal solution for a given project, the following programming problem should be solved:

MAX (NPV)  
T, 
$$
\lambda_1
$$
  
s.t. AC< Annual Budget

The above programming problem is a nonlinear programming problem (NLP). To solve it, some simplifications should be made with it or we could calculate the NPV values by enumeration and then select the best one.

Similar calculation can be done for the optimal Benefit/Cost Ratio of the project if we use Benefit/Cost Ratios criterion.

#### **4.1.2 Multiple projects in a railway network**

Now let us consider a construction program of a set of projects. Here, costs are functions of the total construction activities in a given time period (because of constraints on equipment, labor, etc.). We are to devise an optimal plan for the set of projects, where each project can start at any time t, and be of some duration with some level of strengthening. For example, a 4-project program can be depicted as in Figure 4.5:



**Figure 4.5: Time Diagram of a 4-project program**

For each project we have three decision variables. First, we consider the degree of strengthening that should take place  $(\lambda/\lambda_1)$ . Second, we consider the start date for the project. Third, we consider the project duration. Our problem is to select these parameters for each project in a multiple project set.

The different beginning times of different projects and the cost interaction among the multiple projects make the analyses more complicated than the single project case and we cannot simply sum the results of the single project case as the multiple project case result. But the basic idea of the modeling is similar to the single project case.

#### **1. Cost** analyses:

As mentioned in the above section, the interaction among the multiple projects complicates the cost analysis. These are several factors to be considered here. First, each construction project causes some disruption of service on the system (for example, train delays). If multiple projects are proceeding simultaneously, one could argue that the disruptions would be greater collectively than the simple sum of the disruptions caused by the projects individually.

Also, one can hypothesize that the construction costs accruing to JR East will be greater in a circumstance where multiple projects are going on simultaneously because of limitations in the construction capacity of the company. For example, performing a number of projects at one time might require the railroad to rent additional construction equipment. On the other hand, if the construction schedule did not have overlap in projects, the railroad would use only its own equipment at a lower overall construction cost. This situation might require the railroad to hire additional, less efficient workers on a temporary basis since their own work force may not have the capacity to perform a number of projects at the same time.

So, we cannot simply give the variable cost function as we have done in the single project case without associating with it a certain period of time t. What we do here is give the annual cost function directly by using the idea of the cost function in the single project case. The cost associated with the number of projects which are being constructed at the same time period and the corresponding revenue loss are also included. Now, we can use the following cost function to express the annual cost of year t:

$$
AC(t) = \beta_1 \sum_{n=1}^{N} \left[ \left( \frac{\lambda_n}{\lambda_{1n}} - 1 \right)^2 k(n, t) \right] + \beta_2 \sum_{n=1}^{N} k(n, t) \sqrt{\sum_{n=1}^{N} k(n, t) + \beta_3 \sqrt{n(t)}} \qquad (4.8)
$$

where, N is the total number of projects,

 $K(n)$  is the length of project n,

T(n) is construction time of project n,

 $k(n,t)$  is the length of tracks being constructed in year t of project n,

$$
= \begin{cases} \frac{K(n)}{T(n)}, & \text{if project n is being constructed at t,} \\ 0, & \text{if project n is not being constructed at t.} \end{cases}
$$

n(t) is the number of projects being constructed at year t.

In this function, the first term indicates the cost portion corresponding to the track strengthening; the second term indicates the cost corresponding to the overall construction rate. The third term indicates the cost corresponding to the number of the projects being constructed at the same time due to the constraint of equipment and labor source and the revenue loss (Here, we measure the monotonic relationship between annual cost and the number of projects which are being constructed at the same time period using a square root function). The fourth item adds the effects of the fixed cost and the corresponding annual revenue loss caused by the construction of each project.

Similar to the single project case, the total cost (if all projects are finished) is:

$$
TC = \sum_{t=1}^{T} AC(t) \cdot \left(\frac{1+j}{1+i}\right)^{t} - \sum_{n=1}^{N} \frac{S \cdot K(n)}{(1+i)^{ET(n)+L}} + \sum_{n=1}^{N} C_{Fn}(t) \cdot \left(\frac{1+i}{1+j}\right)^{t} \tag{4.9}
$$

where,  $ET(n)$  is the ending construction time of project n,

 $T= max$  ( $ET(n)$ ),

S is the average salvage value of one kilometer track,

 $C_{Fn}(t)$  is the fixed cost of project n which is begun at year t. Same as the situation in single project, we assume  $C_{Fn}(t)=0$  for simplification.

#### 2. Benefit **analyses:**

As in the single project case, the benefit is the reduction of the physical damages and the reduction in fatalities and injuries of passengers. So the benefit associated with the multiple projects case is the sum of the benefit of each project.

The maximum possible benefit (i.e., if the first earthquake strikes after all the projects) is the sum of the possible benefits associated with all projects in the multiple-project program. That is:

$$
B = \sum_{n=1}^{N} B(n)
$$
 (4.10)

where,  $B(n)$  is the total possible benefit of project n.

#### **3. Benefit-cost comparison: An example**

If we do not have any information about the plans for the multiple-project construction, all the possible combinations of the beginning time and construction rate of the projects should be considered. In this example, for simplicity, we can assume that all the projects in the system should be constructed to a same safety level  $(\lambda)$ . Further, we assume that the probability of a serious earthquake striking is the same everywhere in the JR East system. Then we can determine the optimal beginning time and the construction rate combination of all projects by calculating the NPV of benefits and costs of every possible combination.

For example, if we want to find the optimal beginning time and construction rate combination of the four-project case as shown in Figure 4.5, we can calculate the NPV of benefits and costs of all the combinations. One combination of the four-project case can be shown in table 4.1.

In this example, for a given safety improvement level  $\lambda_1$ , if we assume that everything should be done within four years, then the total number of the possible combinations is:  $(8+7+6+5+4+3+2+1)^4$ =36<sup>4</sup>=1679616.

We can see that the number of possibilities in a real-world case can be very large, especially if the number of projects is large.

	Project 1	Project 2	Project 3	Project 4
$\sqrt{T}=[0, 0.5]$				
$T=[0.5, 1.0]$				
$T=[1.0, 1.5]$				
$T=[1.5, 2.0]$				
$T=[2.0, 2.5]$				
$T=[2.5, 3.0]$				
$T=[3.0, 3.5]$				
$\ $ T=[3.5, 4.0]				

**Table 4.1: Alternative plan of multiple projects**

In some situations, it is not necessary to consider all the possible plans for the multipleprojects. What we can do is select an economical plan among several plans submitted by experienced experts. Then, the following calculations should be done for each plan to obtain the NPV values, we then choose the one with the largest NPV and with costs meeting the budget constraint as the optimal plan.

If the first earthquakes strike during the projects, then the expected Net Present Value of Benefits and costs is:

$$
NPV_1 = \sum_{t=1}^{T} \left[ \sum_{n=1}^{N} \left( \frac{\max \{ (t - BT(n)), 0 \} \cdot ak(n)}{K(n)} \right) \cdot B(n) \cdot p(n, t) - AC(t) \right] \quad (4.11)
$$

where,  $BT(n)$  is the beginning construction time of project n,

 $ET(n)$  is the ending construction time of project n,  $ak(n)=\frac{K(n)}{T(n)}$ , is annual construction length of project n, p(n, t) is the probability of the first earthquake strikes in the area of project n,  $T=Max(ET(n)).$ 

If the first earthquakes strike after all the projects, then the expected Net Present Value of benefits and costs is:

$$
NPV_2 = \sum_{t = T + 1}^{T + L} \sum_{n = 1}^{N} \left[ B(n) \cdot p(n, t) + \frac{S \cdot K(n)}{(1 + i)^{L + ET(n)}} \right]
$$
(4.12)

So, the total expected NPV value of the N project is:

$$
\max NPV = NPV_1 + NPV_2 \tag{4.13}
$$
  
s.t. TC  $\leq$  budget

To select the optimal plan, what we should do is select the maximum NPV value computed above with costs meeting the budget constraint to obtain the final optimal plan among all the plans. Similar calculation can be done for the optimal Benefit/Cost Ratios of the projects if we use Benefit/Cost Ratios criterion.

#### **4. Numerical example of comparing alternative plans: Four-project case**

Here, we use a simple four-project case example as illustration to show how to compare the alternative multiple project plans in terms of **NPV** values.

#### **General conditions:**

1) Cost of track damage of one track failure: **d= \$600,000,**

- 2) Cost of damage to a crashed train: DT= **\$50,000,**
- **3)** Value of a human life: V=\$2,000,000 **(by** willingness-to-pay theory).
- 4) Useful lifetime of the all track strengthening project: tL=20 year,
- **5)** Average train headways: Headway=25 km,

6) Average fatalities and injuries when a train crashes on surface lines at a speed of 50km/ hour: **F=30,** I=90 (F and I will vary with train crashing speed according to **Eq. 3.5,** 3.6),

**7)** Annual cost function for multiple track strengthening projects:

$$
AC(t) = 20000 \sum_{n=1}^{N} \left[ \left( \frac{\lambda_n}{\lambda_{1n}} - 1 \right)^2 k(n, t) \right] + 200 \sum_{n=1}^{N} k(n, t) \cdot \sqrt{\sum_{n=1}^{N} k(n, t) + 500 \sqrt{n(t)}} \qquad (4.14)
$$

**Conditions of the four projects:**

Project #	Prob. of Earthquake	λ	length (km)
	$\overline{0.01}$	0.2	
	0.005	0.2	30
o	0.005	0.2	25
	0.005	0.2	25

**Table 4.2: Condition of the four projects in the example.**

## **Alternative project schedule plans:**



**Table 4.3: Nine alternative project schedule plans**



**Figure 4.6: NPV values of the nine alternative plans**

Figure 4.6 shows the **NPV** values of the nine alternative plans. In this example, according to **NPV** value criterion, the best plan is Plan **#6,** which is proceeding the four projects one **by** one and the duration of each of them is one year.

## **4.2 Viaduct retrofitting program design**

As we have pointed out in Chapter 3, the lines on viaducts can also be considered as homogeneous lines in terms of the uniform characteristics of existing viaduct columns and this enables us to use a weighting factor  $w(s)$  to measure the failure rate of viaduct line,  $\lambda_{\rm v}$ (s). Also since the cost structure of viaduct retrofitting program is very similar to that of the track strengthening projects, we can use the same form of direct cost function here as we used for track strengthening project program by giving a weighting factor for it. On the other hand, since a train derailment accident on the viaducts will usually cause a train to roll down from the viaducts, so it is more likely to be a severe derailment than a normal derailment accident on the surface lines, that is, it will probably result in more fatalities and injuries. Now we can use the same models that we presented in section 4.1 to design the viaduct retrofitting program by using the failure rate  $\lambda_{\nu}(s)$  instead of  $\lambda(s)$  by making some adjustments.

## **4.3 Risk-benefit analysis of early warning systems**

As we have discussed in Chapter 3, the early warning system reduces the distance traveled by trains on potentially damaged tracks and therefore reduces the risk of severe accidents like derailments, which under seismic conditions occur mainly when a running train encounters a damaged section of the track.

Here, it is assumed that the each early warning systems take effect independently of others. That means we assume each location along the lines is served by exactly one warning system. Hence, we will consider the effect of one early warning system when considering a specified geographic area.

Unlike the track strengthening strategy which can reduce the track damage by the earthquakes, the only benefit of installing early warning system comes from not hitting a broken rail. To be more clearly stated, the early warning system could only reduce the earthquake risks related to train derailment since people can do nothing to protect the rails except stopping trains by emergency braking and reducing the probability of train derailment when they are warned by the early warning system.

First, the emergency braking stop distance under warning time  $T_0$  can be calculated as:

$$
X_{\rm W} = \frac{(V_0 - a \cdot wT_0)^2}{2a} \tag{4.15}
$$

where  $X_W$  is the emergency braking stop distance with a warning system,

 $V_0$  is the average operating speed of trains,

 $wT_0$  is the warning time of the early warning system served for the area,

a is the emergency deceleration rate.

From Equation 4.15, it is clear that different early warning system cause different emergency braking distance for a certain kind of braking system. Recall that using Equation 3.4 the probability of train derailment can be calculated. Furthermore, the earthquake risks in terms of the passenger fatalities and injuries, train damages with and without the early warning system can be obtained by using Equations 3.5, 3.6, 3.7 and 3.8. Then the benefit associated with the early warning system can be calculated as (if using the real monetary collective risk):

$$
B_W = R_2(X) - R_2(X_w) + W_F \cdot [E(F) - E_W(F)] + W_I \cdot [E(I) - E_W(I)] \tag{4.16}
$$

where  $R_2(X_w)$ ,  $E_w(F)$ ,  $E_w(I)$  and  $R_2(X)$ ,  $E(F)$ ,  $E(I)$  are earthquake risks in terms of train damage, passenger fatalities, injuries with and without warning system, respectively.

We assume that the cost of the warning systems are related to their maximum warning time because of the different level of technology and system complexity associated with them. For simplicity, we assume that when all warning systems are operated optimally, that is no unnecessary train delays and cancellations are caused by the warning systems, so the maximum warning time can be achieved. Also considering the basic equipment and technology necessary for a warning system, we assume a nonlinear relationship between

the cost of a earthquake early warning system and its maximum warning time, as shown in Equation 4.17:

$$
C(wT_0) = \alpha + \beta \cdot wT_0^2 \tag{4.17}
$$

where  $wT_0$  is the maximum early warning time of a system,  $\alpha$  and  $\beta$  are coefficients.

Comparing the benefit and certain cost for installing the early warning system, it is easy to find which kind of warning system is most worthwhile from the investment point of view by selecting the optimal NPV value(s) or B/C ratio(s).

## **4.4 Risk-benefit analysis of advanced braking systems**

As we have also discussed in Chapter 3, the earthquake risks can be reduced if the braking system is more advanced, since if the earthquake strikes, the risk for a train to encounter a track failure can be reduced by decreasing the emergency braking stop distances.

The risk-benefit analysis for the strategy of installing advance braking system is similar to that for constructing early warning system. the difference is only that instead of using Equation 4.15 to calculate the emergency stop distance under a warning time  $wT_0$ , the emergency stop distance by changing the braking system can be calculated by the following equation:

$$
X = \frac{(V_0)^2}{2a}
$$

(4.18)

where a is the is the deceleration rate of the braking system.

Also by using Equations 3.5, 3.6, 3.7 and 3.8, the earthquake risks in terms of passenger fatalities and injuries, train damages with and without changing to an advanced emergency braking system can be calculated. Then the benefit associated with the advanced braking system can be calculated as (if using the real monetary collective risk):

$$
B_b = R_2(X) - R_2(X_b) + W_F \cdot [E(F) - E_b(F)] + W_I \cdot [E(I) - E_b(I)] \tag{4.19}
$$

where  $R_2(X_b)$ ,  $E_b(F)$ ,  $E_b(I)$  and  $R_2(X)$ ,  $E(F)$ ,  $E(I)$  are earthquake risks in terms of train damages, passenger fatality, injury with and without changing to an advanced braking system, respectively.

We assume that the cost of the braking systems are related to their minimum stop distance because of the different level of technology. Similar to early warning systems, we assume a nonlinear relationship between the cost of brakes for one car and the reduction of emergency braking stop distance considering the necessities of the basic technology and equipment for a early warning system. The more advanced warning systems are often achieved by updating the previous warning system with shorter warning time. On the other hand, it will cost much more when reducing the braking distance for one more unit from the more advanced level than that from the relatively lower level, so we assume a cost function of square form for the braking systems:

$$
C(X) = \alpha \cdot (X_0 - X)^2 + \beta \tag{4.20}
$$

where  $C(X)$  is the investment for replacing a braking system with minimum stop distance  $X_0$  with that of X for one car,  $\alpha$  and  $\beta$  are the coefficients.

By comparing the benefit and certain cost for installing the advanced emergency system, it is easy to find which kind of braking system is most worthwhile from the investment point of view by selecting the optimal NPV value(s) or B/C ratio(s).

# **4.5 A safety management plan for allocation of safety investment among the safety counter-measures**

A safety management plan is an action-oriented decision aid based on risk assessment techniques. It focuses on the optimal allocation of resources and aims at ensuring achieving maximum safety for the money spent. The purpose of the risk assessment analysis is to develop a safety management plan.

So after presenting the methods for finding the optimal investment for individual safety counter-measures in terms of getting the largest benefit-cost ratios, we need next to compare these alternative earthquake safety counter-measures and determine the optimal way of allocating the resources among them.

A potential problem when we try to compare two safety counter-measures or plans is the following: Should we regard the best safety plan only according to the benefit-cost ratios or should we try to use the resource as much as possible to improve the safety? For example, say A and B as shown in Figure 4.6 are two safety plans composed of the individual safety counter-measures such as track strengthening, installing warning systems, and improving braking systems, etc. or their combinations. The benefit-cost ratio of Plan A is better than that of Plan B, but the total risk reduction of Plan B is larger than Plan A. Should we choose A or B?



**Figure 4.7: Risk Reduction vs. budget.**

To answer this problem, we can view it from the point of the company's head who need to consider all the investment problems for the company in addition to safety investment. He also needs to consider the optimal allocation among safety investment and other investment such as adding new trains, recruiting more employees, etc. So at least theoretically, when we consider the optimal allocation among the safety counter-measures, we can always try to find the optimal allocations under hypothetical budget scenarios and then determine the optimal level of investment (if it exists) according to the discussion of Section 2.3.2.

Then we can focus on the allocation problem under different budget scenarios. In the case of our earthquake safety improvement problem, since there are interactive relationships between the counter-measures (For example, Section 4.3 shows that the risk reduction of a early warning system  $B_w$  is a function of not only its early warning time  $wT_0$  but also the deceleration rate of the emergency braking systems  $a$  as well as the track failure rate  $\lambda_1$ which is determined by track strengthening projects, section 3.4 shows similar interactive relationships), Under a certain budget, we should consider all possible safety plans (here, we use the word "plan" which means single strategy or the combination of strategies, such as track strengthening, installing warning systems, and improving braking systems etc., or their combinations), we cannot easily add up the optimal risk reductions of two strategies as the total optimal risk reduction of the two strategies. That means, under a certain investment level, we should consider all possible safety plans (here, we use the word "plan" which means individual counter-measures or the possible combination of more than one counter-measures). This adds complexity to this problem. We can see the application in the case studies in the next chapter.

Also, when we consider the allocation problem under a certain investment budget, the decisions are affected by the way how the company perceives the risk (i.e., use different risk measures). As we stated in 2.2.2, different risk measures refer to the fact that an accident is perceived to be different compared to its direct consequences. So if we perceive the risk using different risk measures (i.e., different risk conversion factors), the results of the optimal allocation problem may be very different. This is shown in the sensitivity analyses in the next chapter.

# **Chapter 5**

# **Case Studies**

In this section, we use two simple examples as illustrations of the principles discussed earlier in Chapter 3 and Chapter 4. This is a very simple situation for analyzing resource allocation to improve safety; our intension is to show how the models and principles work.

# **5.1 Case study I: Surface line**

Suppose there is a small passenger railroad company which has a very simple railroad network. In the network, there are two stations connected by a homogeneous single line (with sidings). The line is 50 km long. The company has a fleet with 4 trains (each with 15 cars and a locomotive). The average operating speed of the trains is 50 km/hour. One train is sent every hour in each direction which means there are always 2 trains running on the line. Assume that no early warning systems are currently available in the railroad network.



#### **Figure 5.1: Simple homogeneous line in Case Study I.**

In this case, three safety counter-measures are employed to improve the earthquake safety: track strengthening, installing warning systems, and improving the braking systems.

As stated in Chapter 4, to find the optimal investment plan which is the main purpose of risk assessment, we need always consider the available safety investment budgets First, we can show the results of cost-benefit analyses for individual safety counter-measures. Then, we will see how the safety investment should be allocated among these safety counter-measures in the optimal way under different budget scenarios. Different risk measures (associated with different risk conversion power factor  $\alpha$  in Equation 2.3) are used here to evaluate the real and perceived risk which may affect how we allocate resources among the counter-measures.

#### **The numerical assumptions related to this case study are following:**

- 1) The probability of earthquake:  $P_0$ =0.005 /year (once per 200 years), for sensitivity analyses (presented in Appendix B), we also consider  $P_0=0.01$ /year,  $P_0=0.02$ /year and  $P_0 = 0.05$ /year.
- 2) Failure rate of the current condition tracks in case of earthquake:  $\lambda = 0.2$ (failures/km),
- 3) The minimum track failure rate achievable through strengthening:  $\lambda_1$ =0.05(failures/km),
- 4) Average train operating speed:  $V_0=50$  km/hour, for sensitivity analyses (presented in Appendix B), we also consider  $V_0$ =40 km/hour,  $V_0$ =60 km/hour and  $V_0$ =70 km/hour.
- 5) Useful lifetime of the track strengthening project: tL=20 year,
- 6) Cost of track damage of one track failure: d=\$60,000,
- 7) Cost of damage to a crashed train: DT=\$50,000,
- 8) Value of a human life: V=\$2,000,000,
- 9) Current emergency braking stop distance is 1.0 km,
- 10) Useful lifetime of the braking systems: **bL=10** year,
- 11) The minimum braking stop distance of all braking systems for conventional trains (with operation speed of 50km/hour as assumed): X=0.4 km,
- 12) Useful lifetime of the warning systems: **wL=10** year,
- 13) The early warning time of all early warning systems is up to:  $wT_0 = 30$  second,
- 14) Risk conversion power factors for calculating real and perceived risks:  $\alpha=1.0$ ,  $\alpha=1.5$ ,  $\alpha=2.0$ ,
- 15) Average fatality and injury numbers when a train crashes on surface lines at a speed

of 50km/hour: F=30, I=90 (F and I will vary with train crashing speed according to Equation 3.5 and 3.6,

16) Direct construction cost function for track strengthening projects:

$$
C\left(\frac{K}{T}, \frac{\lambda}{\lambda_1}\right) = 20000\left(\frac{\lambda}{\lambda_1} - 1\right)^2 + 2000\sqrt{\frac{K}{100T}}
$$
\n(5.1)

for sensitivity analyses (presented in Appendix B), we also consider the cost functions:

$$
C\left(\frac{K}{T}, \frac{\lambda}{\lambda_1}\right) = 30000\left(\frac{\lambda}{\lambda_1} - 1\right)^2 + 3000\sqrt{\frac{K}{100T}} \text{ (high construction costs)}\tag{5.2}
$$

$$
C\left(\frac{K}{T}, \frac{\lambda}{\lambda_1}\right) = 10000\left(\frac{\lambda}{\lambda_1} - 1\right)^2 + 1000\sqrt{\frac{K}{100T}} \text{ (low construction costs)}\tag{5.3}
$$

**17)** Cost function of warning systems:

$$
C (wT_0) = 8000 + 20 \cdot T_0^2
$$
 (5.4)

for sensitivity analyses (presented in Appendix B), we also consider the cost functions:

$$
C(wT_0) = 12000 + 30T_0^2
$$
 (high cost warning system) (5.5)

$$
C (wT0) = 4000 + 10T02 (low cost warning system)
$$
 (5.6)

18) Cost function of braking systems:

$$
C(X_0 - X) = 1200 \cdot (X_0 - X)^2 + 160
$$
 (5.7)

for sensitivity analyses (presented in Appendix B), we also consider the cost functions:

$$
C (X_0 - X) = 1800 \cdot (X_0 - X)^2 + 240
$$
 (high cost braking system) (5.8)

$$
C (X_0 - X) = 600 \cdot (X_0 - X)^2 + 80
$$
 (low cost braking system) (5.9)

Here, Equation 5.1, 5.4, and 5.7 represent the medium cost level for track strengthening, warning systems, and braking systems, respectively. Equation 5.2, 5.5, and 5.8 represent the high cost level. Equation 5.3, 5.6, and 5.9 represent the low cost level.

#### **5.1.1 Risk-benefit analyses for the earthquake safety counter-measures**

**If** we assume that the three safety counter-measures are implemented individually, then under any investment level, we can find the optimal solution for each of the counter-measures **by** calculating the B/C Ratios. Table **5.1** and Table **5.2** shows an example of the results by real risk measure  $(\alpha=1.0)$ .

TL Cost (\$K)	Track strengthening	Warning systems	<b>Braking</b> systems
10		9.14	7.55
20		8.78	6.37
30			5.18
40	4.32		
$\overline{50}$	4.07		
60	3.82		
70	3.60		
80	3.39		
90	3.21	--	
100	3.05		

**Table 5.1: Summary of B/C Ratios of the three safety counter-measures (Real risk: a=1.0)**

TL Cost (\$K)	T(yr.)	$\lambda_1$	wT(sec.)	X(km)
$10\,$		--	$\overline{10}$	$\overline{0.9}$
20		--	25	0.6
30		--		0.4
40	3.0	0.16		
50	2.5	0.15	--	--
60	2.5	0.14		
70	2.0	0.14	--	--
80	2.0	0.13	--	--
90	1.5	0.13		--
100	1.5	0.12		--

Table 5.2: Implement of the three safety counter-measures (Real risk:  $\alpha=1.0$ )
From Table 5.1, we can see clearly that the three safety counter-measures have different cost characteristics. When the budget is tight, the track strengthening project cannot be performed because of its high set-up costs. The counter-measures of installing warning systems and improving braking systems, on the other hand, do not need as much investment. But when the budget is not very tight, for example, \$50K, installing warning systems and improving braking systems cannot provide further safety improvement due to the assumed technology limit (i.e., we assume the braking distance cannot be less than 0.4km and the early warning time can not be longer than 30 seconds in this example).

#### **5.1.2 Safety investment allocation among the safety counter-measures**

As we stated earlier, the main purpose of risk assessment is to make investment decisions. Now, we try to use this example and show how to allocate the safety investment among the different counter-measures.

We have made the risk-benefit analyses for the three single safety counter-measure individually in Section 5.1.1. Since there are interactive relationships among these earthquake safety counter-measures, we can not simply sum up the optimal benefits and costs of the three counter-measures as the optimal benefit and cost of the combination safety plan of these three counter-measures.

It is much easier when we consider investment for, say, level crossing safety and track strengthening projects, since the investment in level crossing safety has nothing to do with the earthquake safety improvement except the budget constraint. So we can just consider these two independent safety investments in the following way: First, carry out the benefit-cost analyses for these two safety counter-measures separately, then balance the budget between them to get the optimal investment allocation (in the case of optimal allocation, the B/C Ratios for the two investments should be equivalent).

However, in this case, the situation is more complicated. Because of the interactive relationship among these earthquake safety counter-measures which we mentioned many times in the previous chapters, the optimal investment for one of the counter-measures is also a function of the optimal investment for other counter-measures.

So for comprehensive analysis and comparisons, all the possible options have to be considered. For the case stated above, there are seven safety options in total for consideration:

- 1. Track strengthening projects,
- 2. Warning systems,
- 3. Braking systems,
- 4. Track strengthening +Warning systems,
- 5. Track strengthening + Braking systems,
- 6. Braking systems + Warning systems,
- 7. Track strengthening +Warning systems + Braking systems.

The strategy for finding the optimal investment allocation is as follows: Under a certain investment level, we can first consider all possible allocation alternatives among the safety counter-measures and get the optimal plan with the biggest B/C value for each option, then compare the seven optimal plans and choose the best one as the final optimal solution for this investment level.

The optimal investment allocation to the safety options under various budgets are summarized in Tables 5.3 for real risk measure ( $\alpha=1.0$ ). In Section 2.3.2, we have discussed that we can get the optimal level of safety investment when the marginal risk reduction by per unit of investment equal to -1 (approximately, we can use  $\Delta TB = \Delta TC$  to get the optimal investment level). In this case, the optimal level investment is \$130K.

Similarly, results based on two different risk measures (associated with two different conversion power factors  $\alpha = 1.5$  and  $\alpha = 2.0$ ) of perceived risk are summarized in Tables 5.4 and 5.5. When  $\alpha$ =1.5, the optimal investment level is \$340K, while when  $\alpha$ =2.0, the theoretical optimal investment level can not be achieved due to the assumed technology restriction. In this situation we regard the biggest possible investment level as the optimal investment level, say, in this case, it is \$340K.

Figure 5.2 and 5.3 shows the optimal real and perceived risk reduction under different budget scenarios based on different risk measures. From the figure, it is clear that the when the risk levels becomes lower, the marginal investment for one unit of risk reduction become larger.

TL Cost (SK)	$\text{Cs}^1$	$Cb^2$	$Cw^3$	Option	B/C Ratio	<b>Risk</b> Reduction (SK)
$\overline{10}$	0.0	0.0	$\overline{10.0}$	$\overline{2}$	9.14	91.3
20	0.0	0.0	20.0	$\overline{2}$	8.78	175.6
30	0.0	15.5	14.5	6	7.29	218.8
40	0.0	23.6	16.4	6	5.78	231.3
50	36.5	13.5	0.0	5	5.20	260.0
60	44.5	15.5	0.0	$\overline{5}$	4.76	285.5
70	52.2	17.8	0.0	$\overline{5}$	4.38	306.4
80	62.2	17.8	0.0	5	4.05	323.7
90	72.2	17.8	0.0	$\overline{5}$	3.76	338.4
100	74.0	13.5	12.5	$\overline{\tau}$	3.52	352.0
110	82.0	15.5	12.5	$\overline{7}$	3.31	364.4
120	92.0	15.5	12.5	$\overline{\tau}$	3.13	375.3
130	102.0	15.5	12.5	$\overline{\tau}$	2.96	385.1
140	112.0	15.5	12.5	7	2.81	393.9
150	122.0	15.5	12.5	$\overline{7}$	2.68	402.0

**Table 5.3: Optimal investment allocation of the safety options for surface lines under various budgets (Real risk:**  $\alpha=1.0$ **)** 

- 1. Budget allocated to track strengthening,
- 2. Budget allocated to installing advanced braking systems,
- 3. Budget allocated to setting up early warning systems.

TL Cost (SK)	Cs <sup>1</sup>	$Cb^2$	$\text{Cw}^3$	Options	<b>B/C</b> Ratio	<b>Risk</b> Reduction (SK)
$\overline{10}$	$\overline{0.0}$	0.0	$\overline{10.0}$	$\overline{2}$	57.66	376
20	0.0	0.0	20.0	$\overline{2}$	53.12	1062
30	0.0	15.5	14.5	6	41.88	1256
40	0.0	23.5	16.5	6	32.62	1304
50	36.5	13.5	0.0	5	29.60	1479
60	44.5	15.5	0.0	$\overline{5}$	26.80	1607
70	54.5	15.5	0.0	5	24.44	1710
80	64.5	15.5	0.0	5	22.43	1794
90	72.2	17.8	0.0	5	20.78	1869
100	74.0	13.5	12.5	7	19.07	1907
110	84.0	13.5	12.5	7	17.87	1965
120	94.0	13.5	12.5	7	16.81	2017
.						
330	305.6	11.9	12.5	7	7.65	2526
340	315.6	11.9	12.5	7	7.46	2536

**Table 5.4: Optimal investment allocation of the safety options for surface lines under** various budgets (Perceived risk:  $\alpha$ =1.5)

- 1. Budget allocated to track strengthening,
- 2. Budget allocated to installing advanced braking systems,
- 3. Budget allocated to setting up early warning systems.

TL Cost (SK)	Cs <sup>1</sup>	$Cb^2$	$\text{Cw}^3$	Options	<b>B/C</b> Ratio	<b>Risk</b> Reduction (SK)
$\overline{10}$	0.0	$\overline{0.0}$	$\overline{10.0}$	$\overline{2}$	362.0	3620.2
20	0.0	0.0	20.0	$\overline{2}$	320.7	6413.9
30	0.0	15.5	14.5	6	241.7	7250.5
40	0.0	23.5	16.5	6	186.2	7449.0
50	36.5	13.5	0.0	5	167.3	8363.1
60	44.5	15.5	0.0	5	150.8	9051.9
70	54.5	15.5	0.0	5	137.0	9591.2
80	64.5	15.5	0.0	5	125.5	10039.6
90	74.5	15.5	0.0	5	116.1	10444.6
100	74.5	13.5	12.5	7	108.2	10818.6
						.
340	315.6	11.9	12.5	7	7.46	2536

**Table 5.5: Optimal investment allocation of the safety options for surface lines under various budgets (Perceived risk:**  $\alpha=2.0$ **)** 

- 1. Budget allocated to track strengthening,
- 2. Budget allocated to installing advanced braking systems,
- 3. Budget allocated to setting up early warning systems.

As assumed and illustrated, the results of Case Study I show that under different budget scenarios, the optimal safety options are usually different. When the budget is small, say, \$10K or \$20K, choosing a single inexpensive safety counter-measure such as installing advanced braking systems is the most cost-efficient way to invest. But when the budget becomes larger and larger, then more than one safety counter-measures can be invested at the same time.



Figure **5.2:** Optimal investment level and optimal risk reduction under different budget scenarios by real risk measure  $(\alpha=1.0)$ 



Figure **5.3:** Optimal investment level and optimal risk reduction under different budget scenarios by perceived risk measure  $(\alpha=2.0)$ 

The reason is that as shown in Table 5.1, the safety counter-measures such as installing advanced braking systems and warning systems are more cost efficient compared to track strengthening projects under lower investment levels. On the other hand, the benefits of these two counter-measures are limited by the technology and physical constraint since for example, we can never stop the train within zero time or within zero stop distance by any braking systems. Under this consideration for the parameters selected, if there is extra money available, we also need to invest in track strengthening projects and allocate the investment among these three safety counter-measures. For this system, the case study shows such optimal allocations when the budget is greater than \$20K.

One thing should be noted here is that when we use different risk measures (conversion factors) to calculate the real and perceived risk reduction, 1) The optimal level of investment are very different, which is closely related to the different magnitudes of B/C Ratios. 2) Optimal options may change for the same investment level.

However, there are only slight differences on the investment allocation among these safety counter-measures. For example, under the budget of \$80K, the optimal safety investment option for both real risk measure ( $\alpha$ =1.0) and perceived risk measures ( $\alpha$ =1.5 and  $\alpha$ =2.0) is option 5, which is the combination of the counter-measures of track strengthening and improving the braking systems. For  $\alpha=1.0$ , the investment allocations among these two safety counter-measures are \$62.2K, and \$17.8K, respectively. On the other hand, with  $\alpha$ equal to 1.5, the allocations among them are \$64.5K, and \$15.5K, respectively.

Why are not these changes very significant? The answer is that all these safety countermeasures are intended to reduce the earthquake risk. That means, although these safety measures can reduce earthquake risk in different ways, either by reducing the consequences or by reducing the probabilities of derailment accidents, or both, the category of the accidents involved with the benefit-cost analyses of these earthquake counter-measures is the same, which is the catastrophic train derailment accidents caused by earthquakes. In this situation, the effects of the risk conversion power factors (which reflects the way how the public and transportation companies perceive the risk) on the risk reductions by the different counter-measures are not significantly different.

But if we consider resource allocation between the earthquake counter-measures and the safety counter-measures for improving level-crossing safety, the situation will be much different. As we described in Section 2.3.2, the marginal-cost-criterion for solving the allocation optimization problem allocates resources in such a way that the marginal cost for risk reduction is equal for all subsystems. Although level crossing accidents are relatively frequent, they normally have very few fatalities in each accident. The effects of the conversion factors  $(\alpha)$  on the level crossing safety investment and earthquake safety investment are very different, and this can dramatically affect the B/C Ratios of these two different categories of investment. Therefore, the allocation between the investment on earthquake safety and level crossing safety will be very different under different assumption of risk conversion power factors. The larger the risk conversion power factor we use, the larger portion of budget will be used for improving earthquake safety.

Even the investment allocations under different measures for risk (or perceived risk) are slightly different, as shown in Figure 5.2 and 5.3, in each case, the Benefit/Cost Ratios decreases when the investment level increases. This means that an additional risk reduction by one unit of safety investment decreases when the risk level of the system decreases.

Now, we perform sensitivity analysis. If we change the earthquake probabilities, train operation speed and the cost functions of the three earthquake safety counter-measures, what will happen to the optimal investment levels and allocation? Appendix B shows the detailed results. Figure 5.4-5.6 can provide a good sense of the change of optimal investment levels with respect to the change of earthquake probabilities, train operation speed and cost structures of the three counter-measures.

From Figure 5.4 shows that the optimal investment levels change with respect to the change of earthquake probabilities (other conditions remain unchanged) in the project area. The higher the earthquake probabilities are, the higher the investment level is needed to achieve the optimal. This is very intuitive; higher earthquake probabilities mean higher earthquake risk, therefore, more safety investment is needed.



Figure 5.4: Sensitivity analysis: optimal investment levels vs. earthquake probabilities



Figure **5.5:** Sensitivity analysis: optimal investment levels vs. train operation speed



- \* 1. Containing high cost levels for both braking and warning
	- 2. Containing high cost for warning+medium cost for braking
	- 3. Containing high cost for warning+low cost for braking
	- 4. Containing medium cost for warning+high cost for braking
	- 5. Containing medium cost levels for both warning and braking
	- 6. Containing medium cost for warning+low cost for braking
	- 7. Containing low cost for warning+high cost for braking
	- 8. Containing low cost for warning+medium cost for braking
	- 9. Containing low cost levels for both warning and braking

**Figure 5.6: Sensitivity analysis: optimal investment levels vs. different cost function combinations**

Similarly, Figure 5.5 shows that the optimal investment levels change with respect to the change of train operation speed (other conditions remain unchanged), the higher the train operation speed is, the higher the investment level is needed to achieve the optimal.

Figure 5.6 shows the change of optimal investment levels with respect to the change of magnitude of cost functions of the three counter-measures.The change in this case is not very significant. We can explain this phenomena as that the optimal investment levels are affected by two factors which are different directions. One of them is that lower cost levels the counter-measures have, the more "productive" they can be in terms of safety improvement (higher B/C Ratios), so the optimal investment level should be higher (like the situations by real risk measure and perceived risk measures). But on the other hand, the lower cost levels the counter-measures have, the less safety investment are needed since the safety improvement is easier to achieve in this situation. This two factors determine that the optimal investment levels of the plans with different cost function combinations do not change very significantly.

### **5.2 Case Study II: Viaduct line**

In this section, we use the same railroad network example as we used in Case Study **I. All** other conditions are the same except that we assume the 50 km lines are all viaducts lines. Figure 5.7 is an illustration of this simple heterogenous lines. We still assume that no early warning systems are currently available in the railroad network.



**Figure 5.7: Viaduct line in Case Study II.**

In this case, three safety counter-measures can be employed to improve the earthquake safety, which are retrofitting the rail viaducts, installing warning systems, and improving the braking systems.

Similar to Case Study I, we have to consider all the safety counter-measures and their combinations. The options are below:

- 1. Viaduct retrofitting only,
- 2. Installing warning systems only,
- 3. Improving braking systems only,
- 4. Viaduct retrofitting+Warning systems,
- 5. Viaduct retrofitting +Braking systems,
- 6. Warning systems +Braking systems,
- 7. Viaduct retrofitting+Warning systems+Braking systems.

As discussed in Section 3.2.3, a weighting factor w(s) can be used to measure the failure rate of viaduct lines,  $\lambda_v(s)$ , which is given in Equation 3.2. Also according to the assumptions that the form of direct cost function of retrofitting viaduct projects is the same as that of track strengthening projects except that higher costs are involved with viaduct retrofitting projects.

Now, we can analyze this case study by using the same other numerical assumptions used in Case Study I except the following:

- 1) Current failure rate of viaduct lines in case of earthquake:  $\lambda_v = 1.5 * \lambda = 0.3$  (failures/km),
- 2) Average fatality and injury numbers when a train crashes on viaducts at a speed of 50km/hour: F=50, 1=150 (F and I will vary with train crashing speed according to Equation 3.5 and 3.6,
- 3) Direct construction cost function for viaduct retrofitting projects:

$$
C_v \left( \frac{K}{T}, \frac{\lambda_v}{\lambda_{v_1}} \right) = 40000 \left( \frac{\lambda_v}{\lambda_{v_1}} - 1 \right)^2 + 4000 \sqrt{\frac{K_v}{100T}}
$$
(5.10)

By carrying out the similar benefit-cost analyses in Case Study I and also, considering different risk measures (associated with different risk conversion power factor  $\alpha$  in Equation 2.3) for real and perceived risk calculation, the optimal safety options and the optimal investment allocations to the safety options under various budgets are summarized in Table 5.6 for real risk measure ( $\alpha$ =1.0), here, the optimal level of investment is \$350K. Similarly, the results based on two different risk measures (associated with different conversion power factors  $\alpha=1.5$  and  $\alpha=2.0$ ) of perceived risk are summarized in Tables 5.7-5.8, here, the optimal investment levels are all much beyond the optimal investment level by real risk measure, \$350K.

TL Cost (SK)	Cs <sup>1</sup>	$Cb^2$	$\text{Cw}^3$	Option	<b>B/C Ratio</b>	<b>Risk</b> Reduction (SK)
$\overline{10}$	$\overline{0.0}$	$\overline{0.0}$	$\overline{10.0}$	$\overline{2}$	32.7	327
20	0.0	0.0	20.0	$\overline{2}$	31.6	632
30	0.0	15.5	14.5	6	26.4	791
40	0.0	23.6	16.4	6	20.9	838
50	0.0	30.8	19.2	6	17.1	855
60	0.0	34.9	25.1	6	14.3	861
70	44.0	13.5	12.5	$\tau$	12.5	874
80	54.0	13.5	12.5	7	11.5	921
90	64.0	13.5	12.5	$\tau$	10.9	977
100	74.0	13.5	12.5	$\tau$	10.2	1024
340	303.5	20.5	16.0	$\overline{7}$	4.36	1482
350	313.5	20.5	16.0	$\overline{7}$	4.26	1492

**Table 5.6: Optimal investment allocation of the safety options for viaduct line under various budgets (Real risk:**  $\alpha=1.0$ **)** 

- 1. Budget allocated to track strengthening,
- 2. Budget allocated to installing advanced braking systems,
- 3. Budget allocated to setting up early warning systems

TL Cost (SK)	Cs <sup>1</sup>	$Cb^2$	$\text{Cw}^3$	Option	B/C Ratio	<b>Risk</b> Reduc. $(SK)$
10	0.0	0.0	$\overline{10.0}$	2	267	2670
20	0.0	0.0	20.0	$\overline{2}$	248	4952
30	0.0	15.5	14.5	6	196	5879
40	0.0	20.5	$\overline{19.5}$	6	153	6113
50	40.8	9.2	0.0	$\overline{5}$	127	6373
$\overline{60}$	50.8	9.2	0.0	$\overline{5}$	115	6889
70	60.8	9.2	0.0	5	104	7268
80	70.8	9.2	0.0	$\overline{5}$	$\overline{95}$	7579
90	80.8	9.2	0.0	$\overline{5}$	$\overline{87}$	7846
100	90.8	9.2	0.0	$\overline{5}$	81	8073
790	762.0	15.5	12.5		$\overline{15}$	11993

**Table 5.7: Optimal investment allocation of the safety options for viaduct line under** various budgets (Perceived risk:  $\alpha$ =1.5)

<b>TL</b> Cost(K)	Cs <sup>1</sup>	$Cb^2$	$\text{Cw}^3$	Option S	B/C Ratio	Risk Reduc. (SK)
$10\,$	0.0	0.0	10.0	2	2171	21710
$\overline{20}$	0.0	0.0	20.0	$\overline{2}$	1932	38647
30	0.0	15.5	14.5	6	1461	43830
40	0.0	23.6	16.4	6	1126	45074
50	40.8	9.2	0.0	5	928	46380
60	50.8	9.2	0.0	5	836	51155
70	60.8	9.2	0.0	5	756	52930
80	70.8	9.2	0.0	5	690	55206
90	80.8	9.2	0.0	$\overline{5}$	635	57158
100	90.8	$\overline{9.2}$	0.0	5	588	58822
790 :	764.0	13.5	12.5	7	111	87433

**Table 5.8: Optimal investment allocation of the safety options for viaduct line under** various budgets (Perceived risk:  $\alpha$ =2.0)

- 1. Budget allocated to track strengthening,
- 2. Budget allocated to installing advanced braking systems,
- 3. Budget allocated to setting up early warning systems.

The results of Case study II also show that under different budgets, the optimal safety strategies are usually different. Also if we use different risk measures (i.e., different conversion factor  $\alpha$ ) to calculate the perceived risk reduction, the optimal investment levels are very different (in this case, the optimal investment levels for perceived risk are also not the theoretical optimal but the biggest possible investment levels). The optimal allocation options also change under the same budget scenarios. For example, under the budget of \$70K-\$100K, the optimal safety option for investment is Option 7 under real risk, but is Option 5 under perceived risks. An additional risk reduction by one unit of safety investment decreases when the risk level of the system decreases. In other words, when the risk level becomes lower, the marginal investment for one unit of risk reduction become larger.

In this chapter, we use two case studies to explain the models and principles presented in the previous chapters. The next chapter will provide summaries and conclusions of the entire thesis.

# **Chapter 6**

# **Conclusions and Future Work**

### **6.1 Summary and conclusions**

The task of this thesis is to provide JR East with a framework to develop a safety management plan for reducing earthquake risk and to illustrate its use. In this thesis, we employ risk assessment methodology to assess the earthquake risk to the railroad and discuss the principle of optimally allocating the safety investment among the alternative safety counter-measures considered by JR East. This research also has its limits. The conclusions are based on the data which are not real data but represent reasonable approximations. Numerical sensitivity studies are performed to bracket the results.

First, various literatures in the field of risk assessment and previous work finished by the JR East/MIT program are reviewed. The important concepts of the risk assessment methodology are introduced, which include the notation of risk, steps of implementing the risk assessment technique, and the safety management plan.

Then, we present the models to assess the earthquake risk to the railroad facilities and passengers. The earthquake related risk to the railroad are of two major kinds: (i) physical damages to the railroad infrastructure and equipments such as viaduct and track failures, train damages, etc. and (ii) passenger fatalities and injuries caused by train derailment accidents when the running trains hit the track failures caused by earthquakes. By making an assumption that the track (viaduct) failures during earthquakes follow a homogenous spatial Poisson distribution, we can use probabilistic models to estimate both kinds of risk. The monetary collective risks (both by real risk measure  $(\alpha=1)$  according to SPI and perceived risk measure  $(\alpha > 1)$ according to PSPI) are obtained by combining these two kinds of risk.

After developing the models to assess the earthquake risks, we evaluate various safety measures which can be implemented to reduce the earthquake risks. Four counter-measures considered by JR East are: strengthening the tracks, retrofitting the rail viaducts, installing early warning systems, and improving braking systems.

Each of these counter-measures has its advantages and disadvantages. For example, the track strengthening and viaduct retrofitting can improve the overall safety condition of the rail lines. They will bring benefits for reducing risks not only of earthquakes but also of other natural disasters, such as rainfall and landslides. But they are high investment projects and can affect traffic operations, such as reduction of traffic volume and cause revenue loss when the projects are in process.

Since the cost structures of these safety counter-measures are complex, we carry out the benefit-cost analysis for each of them and try to find the optimal investment strategy in terms of cost efficiency (measured by the Benefit/Cost ratios) for each of the counter-measures. Due to the consideration of limited construction resources, the costs of track strengthening and viaduct retrofitting projects are not only related to their safety improvement but also related to the processing rates of the projects and number of projects which are processed at the same time. Therefore, the cost structures of these two counter-measures are complicated, especially when there is more than one project being processed at the same time. This study takes initial steps on developing a framework by using a riskbenefit analysis approach to determine how *fast* a set of construction projects for track strengthening or viaduct retrofitting should be completed and the *degree of strengthening* that is appropriate.

As for the braking systems and warning systems, the costs are only related to the minimum stopping distances (deceleration rates) and maximum early warning time they can provide, respectively, implying a more straightforward analysis structure.

Finally, since the purpose of the risk assessment analysis is to get a safety management plan which focuses on the optimal allocation of resources and ensures obtaining maximum safety for the money spent, we need to determine how to allocate the safety investment

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among these four safety counter-measures optimally. In the real world, the resources are always limited, so we develop methods which can work under different investment levels. Also, because there are interactive relationships among these safety counter-measures, we need to consider all the possible safety options (For a situation with three safety countermeasures, there are seven safety options in total). Carrying out the cost-benefit analysis for each of the options under each budget limitation, we can get the optimal plan for each option based on the B/C Ratios and select the one with the biggest B/C Ratio as the final optimal plan under such budget limits.

We use two case studies to show how the models work. The sensitivity analyses for earthquake probabilities, train operating speeds, and costs are shown in Appendix B.

Based on the selected data and parameters which we think are reasonable, the conclusions from the case studies and sensitivity analyses are that the optimal strategies for allocating safety investment among the various safety counter-measures are different under different investment levels. When the budget is smaller, the best strategies and the optimal investment levels are usually inexpensive single safety counter-measures such as installing warning systems. When the budget is larger, the optimal strategies are more likely to be the combinations of some counter-measures.

Furthermore, as shown in Figure 5.4-5.6 and Appendix B, the optimal investment levels as well as the optimal strategies under each investment level will change with the earthquake probabilities, train operation speed and the unit costs of the counter-measures. Remaining other situations the same, the optimal investment levels will increase as the earthquake probability increases. Similarly, the optimal investment levels will also increase as the train operation speed increases. And for both cases, the allocation among the counter-measures usually does not change significantly under each investment level. Oppositely, with respect to the unit costs of the counter-measures, the optimal investment level does not change very significantly, however, the investment allocation among the counter-measures under each investment level is sensitive to the change of the unit costs. This is just as expected: on one hand, for example, the cheaper counter-measure(s) are more likely to be used, but on the other hand, because of the lower unit cost, the total investments needed for the cheaper counter-measures will not be very high. These two different direction effects cause the sensitive relationship between the unit costs of the counter-measures and the investment allocation among them under the same investment levels. The change of investment allocations depends on which of the two direction effects is more significant.

Another observation that stands out from the case studies is that when we use different risk measures (different  $\alpha$ ) to calculate the monetary collective risks, the optimal strategy and the allocations among the counter-measures under each investment level may change. However, for the parameter selected, the change is also not very dramatic.

The reason why the investment allocations do not change significantly under the same investment levels (except by different unit cost combinations) is that all the earthquake safety counter-measures reduce the earthquake risk. Even though they reduce the earthquake risk in different ways, the category of the accidents involved with the benefit-cost analyses is the same, which is the catastrophic train derailment accidents caused by earthquakes. In this situation, the effect of the earthquake probabilities, train operation speed, and the risk conversion power factors (which reflects the way that the public and transportation companies perceive the risk) on the risk reduction by the different counter-measures are not significantly different although risk measure  $(\alpha)$  does affect optimal investment level.

On the other hand, if we look at the resource allocation between the earthquake countermeasures and the safety counter-measures for improving level-crossing safety, the situation we would expect to be significantly different. Since solving the allocation optimization problem requires allocating resources in such a way that the marginal cost for risk reduction is equal for all subsystems, and the level crossing accidents normally involve very few passenger fatalities, the effects of the conversion factors on the level crossing safety investment and earthquake safety investment are very different. This can dramatically affect the B/C Ratios of these two categories of investment. So, the allocation between the investment on earthquake safety and level crossing safety will be very different under different risk measures (different risk conversion power factors). The larger the risk conversion power factor we use, the larger portion of the whole budget will be used for improving earthquake safety

Another importation conclusion is that as the risk level becomes smaller and smaller, it is more and more difficult to improve safety since we need to invest more for one unit of additional risk reduction.

### **6.2 Future Works and recommendation to JR East**

#### **1. Data refinement**

The case studies in the thesis employed hypothetical but, we think, reasonable numbers. We need to refine them by using the real world data get more realistic and more meaningful results. Some of the data such as train operating speeds, current emergency braking distances, are easy to get while others like earthquake probabilities in the project areas and track failures rates and construction costs for multiple simultaneous projects are not very easy.

The most difficult part of task for getting these data is to get precise empirical direct cost function for track strengthening and viaduct retrofitting projects. To get data for a certain area of JR East, we may need to consult the Japanese construction engineers. Also a better way for getting the costs of different type of warning systems and braking systems is to estimate the costs of the alternative systems one by one instead of using continuous functions as we used in the case study for simplification. If JR East has a complete database for these aspects we mentioned above, it will be very helpful to get the safety management plan for reducing earthquake risks.

#### 2. Employ **the methods for complex networks**

We have carried out the benefit-cost analysis and investment allocation methods to a very simple railway network (two station and single line). This is suitable for a small regional analysis. But, to develop an earthquake safety management for a big railway network like JR East, it would be useful to employ the model to a more complex network. To achieve this, more complicated analysis need to be carried out because of the interactive relationships among the multi-projects of track strengthening (as well as the viaduct retrofitting multi-projects). In Section 4.1.2, we have shown some benefit-cost analysis for the multiple track strengthening projects. The analysis is preliminary, more study need to be done for more complex railway network.

In the thesis, we presented the models developed for assessing earthquake risk and improving earthquake safety through investment for railroads. We hope this will be helpful for the decision making of JR East in safety improvement and for other rail system researches.

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## **Appendix A**

# **Flow Charts of Computer Programs for Case Study I**

This appendix provides the flow chart of computer program for each of the seven safety improvement options in Case Study I. The computer programs can calculate the optimal investment allocations among the earthquake counter-measures (track strengthening, early warning systems, and braking systems) under alternative budget scenarios.

#### **1. Flow chart** for optioionl-Track strengthening program design



Figure **A.1:** Flow chart of track strengthening project design

#### 2. Flow **chart for option 2-Benefit-cost analysis for warning systems:**



Figure **A.2:** Flow chart of benefit-cost analysis warning systems

#### 2. Flow **chart for option** 3-Benefit-cost analysis **for** braking systems:



Figure **A.3:** Flow chart of benefit-cost analysis backing systems

4. Flow chart **for option** 4-benefit-cost analyses of warning systems +track strengthening project:



Figure A.4: Flow chart of benefit-cost analysis installing warning systems+track strengthening project design

**5.** Flow chart for option 5-benefit-cost analyses of braking systems +track strengthening project design:



#### Figure **A.5:** Flow chart of benefit-cost analysis improving braking systems+track strengthening project design

**6.** Flow chart **for** option 6-benefit-cost analyses of braking systems +warning systems:





**7.** Flow chart for option 7-benefit-cost analyses of braking systems +warning systems +track strengthening project design:



Figure **A.7:** Flow chart of benefit-cost analysis improving braking systems+installing warning systems+track strengthening project design

## **Appendix B**

## **Sensitivity Analysis of Optimal Investment Allocation in Case Study I (Real Risk:o=1.0)**

This appendix provides the sensitivity analyses of optimal investment allocations by earthquake probabilities, train operation speed, and unit costs of the safety counter-measures (track strengthening, early warning systems, and braking systems) in Case Study I. All the sensitivity analyses performed here use real risk measure (i.e.,  $\alpha=1.0$ ).

### **1 For earthquake probabilities:**



### (1)  $P_0 = 0.01$

Table B.1: Optimal investment allocation (P<sub>0</sub> =0.01)

- **1.** Budget allocated to track strengthening,
- 2. Budget allocated to installing advanced braking systems,
- 3. Budget allocated to setting up early warning systems.

TL Cost B/C Risk  $(SK)$   $\begin{bmatrix} Cs \\ \hline \end{bmatrix}$   $\begin{bmatrix} Cy \\ \hline \end{bmatrix}$   $\begin{bmatrix} Cy \\ \hline \end{bmatrix}$   $\begin{bmatrix} P & \hline \end{bmatrix}$  Ratio  $\begin{bmatrix} Reduc.(SK) \\ \hline \end{bmatrix}$ 10 0.0 0.0 10.0 2 26.4 264 <sup>20</sup>**0.0 0.0** 20.0 2 **25.3 506 30 0.0** 15.5 14.5 **6 20.9 628** 40 **0.0** 23.6 16.4 6 16.6 664 *50 34.5 15.5* 0.0 *5* 14.4 721 60 44.2 15.5 0.0 *5* 13.2 791 70 52.2 17.8 0.0 *5* 12.1 847 80 62.2 17.8 0.0 *5* 11.2 893 90 64.0 13.5 12.5 7 10.4 *935* 100 72.0 15.5 12.5 7 9.7 973  $\cdots$ 260 232.0 *15.5* 12.5 7 4.9 1269 270 242.0 15.5 12.5 7 4.7 1279

 $(2)$   $P_0 = 0.015$ 



(3)  $P_0=0.02$ 

TL Cost (SK)	Cs	Cb	Cw	Option	B/C Ratio	<b>Risk</b> Reduc.(\$K)
$10\,$	0.0	0.0	Tσ	$\overline{2}$	34.5	345
20	$\overline{0.0}$	10.6	9.4	6	33.4	668
$\overline{30}$	0.0	15.3	14.7	6	27.3	819
40	0.0	23.6	16.4	6	21.6	866
$\overline{50}$	34.5	15.5	0.0	$\overline{5}$	18.5	926
60	44.5	15.5	0.0	$\overline{5}$	16.9	1015
$\overline{70}$	$\overline{52.2}$	17.8	0.0	$\overline{5}$	15.5	1086
80	$\overline{54.0}$	13.5	12.5	7	14.3	1144
$\overline{90}$	62.0	15.5	12.5	7	13.3	1199
100	$\overline{72.0}$	15.5	12.5	7	12.5	1247
290	262.0	15.5	12.5	7	$\overline{5.7}$	1654
300	$\overline{272.0}$	15.5	12.5	7	$\overline{5.6}$	1665

**Table B.3: Optimal investment allocation**  $(P_0 = 0.02)$ 

## **(2. For train operating speed:**

TL Cost (\$K)	$\mathbf{C}\mathbf{s}$	C <sub>b</sub>	Cw	Option	<b>B/C Ratio</b>	<b>Risk</b> Reduc.(\$K)
$\mathbf{I}$	0.0	0.0	10.0	σ	4.8	48.4
$\overline{20}$	0.0	0.0	20.0	$\overline{2}$	4.9	97.4
$\overline{30}$	0.0	12.2	17.8	6	$\overline{4.6}$	136
40	0.0	23.6	16.4	6	$\overline{3.7}$	$\overline{146}$
50	36.5	13.5	0.0	5	$\overline{3.4}$	170
60	44.5	15.5	0.0	5	$\overline{3.1}$	$\overline{187}$
70	54.5	15.5	0.0	5	$\overline{2.9}$	201
80	62.2	17.8	0.0	5	$\overline{2.7}$	$\overline{213}$
90	72.2	17.8	0.0	5	2.5	223
100	82.2	17.8	0.0	5	$\overline{2.3}$	232

(1)  $V_0$ =40(km/hour)







Table B.5: Optimal investment allocation  $(V_0=60)$ 

TL Cost (SK)	Cs	$\mathbf C\mathbf b$	Cw	Option	<b>B/C Ratio</b>	<b>Risk</b> Reduc.(\$K)
10	$0.0\,$	0.0	10.0	2	23.3	233
$\overline{20}$	$\overline{0.0}$	0.0	$\overline{20.0}$	$\overline{2}$	$\overline{20.5}$	410
$\overline{30}$	$\overline{0.0}$	15.5	14.5	6	14.8	444
40	$\overline{0.0}$	$\overline{20.5}$	19.5	6	11.5	461
50	36.5	13.5	$\overline{0.0}$	5	$\overline{10.0}$	499
60	35.6	11.9	12.5	7	$\overline{9.2}$	551
70	45.6	11.9	12.5	7	8.5	594
80	55.6	$\overline{11.9}$	12.5	7	7.8	627
90	64.0	13.5	12.5	7	$\overline{7.3}$	656
100	74.0	13.5	12.5	7	6.8	681
190	165.6	11.9	12.5	7	$\overline{4.3}$	816
200	$\sqrt{174.0}$	13.5	12.5	7	4.1	826

(3)  $V_0 = 70$ (km/hour)

Table B.6: Optimal investment allocation (V<sub>0</sub>=70)

#### **3. For cost functions:**

Before doing the numerical sensitivity analyses, we represent the cost functions in the following way for clarity:

(1) Cost functions for track strengthening:

Equation  $5.1$  ---> $T_m$ Equation  $5.2$  ---> $T_h$ Equation  $5.3$  --->T\_l

(1) Cost functions for track strengthening:

Equation 5.4 --->W\_m Equation  $5.5$  --->W\_h Equation  $5.6 \rightarrow W_l$ 

(1) Cost functions for track strengthening:

Equation  $5.7$  ---> $B$ <sub>-</sub>m Equation 5.8 --->B\_h Equation 5.9 --->B\_1
TL Cost (SK)	Cs	Cb	Cw	Option	<b>B/C</b> Ratio	<b>Risk</b> Reduc.(\$K)
10		-	$\ddot{\phantom{1}}$			
20	0.0	0.0	$\overline{20}$	2	6.7	$\overline{134}$
$\overline{30}$	$\overline{0.0}$	0.0	30	$\overline{2}$	$\overline{5.9}$	176
40	0.0	18.7	21.3	6	$\overline{5.1}$	203
$\overline{50}$	$\overline{0.0}$	21.2	28.8	6	4.4	218
$\overline{60}$	41.2	$\overline{0.0}$	18.8	$\overline{4}$	$\overline{3.8}$	230
$\overline{70}$	48.9	21.2	0.0	5	$\overline{3.6}$	$\overline{250}$
$\overline{80}$	55.9	24.1	0.0	5	$\overline{3.4}$	269
90	65.9	24.1	0.0	5	$\overline{3.2}$	285
100	75.9	24.1	0.0	5	$\overline{3.0}$	$\overline{300}$
$\overline{120}$	92.4	$\overline{27.6}$	$\overline{0.0}$	5	$\overline{2.7}$	324
130	102.4	27.6	0.0	$\overline{5}$	2.6	334

 $(1) T_h+W_h+B_h$ 

**Table B.7: Optimal investment allocation (T\_h+W\_h+B\_h)**



TL Cost (SK)	Cs	Cb	Cw	Option	<b>B/C</b> Ratio	<b>Risk</b> Reduc.(\$K)
10	$_{\odot}$	$\overline{\mathrm{10}}$	$\left($		7.6	76
20	0	0	$\overline{20}$	$\overline{2}$	6.7	$\overline{134}$
$\overline{30}$	$\theta$	$\overline{13.5}$	16.5	6	6.8	$\sqrt{204}$
40	$\overline{0}$	20.5	19.5	$\overline{6}$	$\overline{5.6}$	$\overline{224}$
50	$\Omega$	$\overline{27}$	23	6	$\overline{4.7}$	$\overline{233}$
60	36.8	23.2	$\overline{0}$	5	4.1	246
$\overline{70}$	46.8	23.2	$\overline{0}$	5	$\overline{3.8}$	$\sqrt{268}$
80	59.5	20.5	$\theta$	5	$\overline{3.6}$	$\overline{285}$
90	69.5	20.5	0	5	$\overline{3.3}$	300
100	76.6	23.4	0	$\overline{5}$	$\overline{3.1}$	$\overline{313}$
<b>110</b>	86.6	23.4	0	5	$\overline{3.0}$	$\overline{325}$
120	96.6	23.4	0	5	2.8	$\overline{335}$

**Table B.8: Optimal investment allocation (T\_h+W\_h+B\_m)**

TL Cost (SK)	$\mathbf{C}$ s	Cb	$\mathrm{Cw}$	Option	<b>B/C</b> Ratio	<b>Risk</b> $Reduce.(SK)$
10	0	Īσ	$\Omega$	3	12.5	125
$\overline{20}$	$\theta$	7.1	12.9	6	9.3	187
$\overline{30}$	$\Omega$	15.7	14.3	6	7.6	227
40	$\Omega$	17.8	22.2	6	$\overline{5.9}$	236
$\overline{50}$	37.9	12.1	0	5	4.9	246
60	46.2	13.8	$\Omega$	5	4.5	270
70	54.3	15.7	$\Omega$	5	4.1	288
80	62.2	17.8	$\Omega$	5	$\overline{3.8}$	304
90	72.2	17.8	0	$\overline{5}$	$\overline{3.5}$	318
100	82.2	17.8	0	5	$\overline{3.3}$	329
110	92.2	17.8	0	5	3.1	339

 $(3) T_h+W_h+B_l$ 



(4) T\_h+W\_m+B\_h

TL Cost (SK)	Cs	Cb	Cw	Option	<b>B/C Ratio</b>	<b>Risk</b> Reduc.(\$K)
$\overline{\mathrm{10}}$	0	7)	10	$\overline{2}$	9.1	91
20	$\boldsymbol{0}$	$\overline{0}$	$\overline{20}$	$\overline{2}$	$\overline{8.8}$	$\overline{176}$
$\overline{30}$	0	13.2	16.8	6	6.6	199
$\overline{40}$	$\overline{0}$	21.2	18.9	$\overline{6}$	$\overline{5.5}$	221
$\overline{50}$	$\Omega$	$\overline{27.6}$	22.4	6	4.6	$\overline{231}$
60	44	0	$\overline{16}$	$\overline{4}$	4.1	$\sqrt{243}$
$\overline{70}$	54	$\overline{0}$	$\overline{16}$	4	$\overline{3.7}$	$\overline{260}$
$\overline{80}$	64	$\overline{0}$	$\overline{16}$	$\overline{4}$	$\overline{3.4}$	$\overline{275}$
$\overline{90}$	58.8	18.7	12.5	7	$\overline{3.2}$	290
100	65.3	18.7	16	7	$\overline{3.1}$	$\overline{305}$
$\overline{120}$	85.3	18.7	16	7	2.7	330
130	82.5	21.5	16	7	$\overline{2.6}$	$\overline{340}$

**Table B.10: Optimal investment allocation (T\_h+W\_m+B\_h)**

TL Cost (SK)	Cs	Cb	Cw	Option	<b>B/C Ratio</b>	<b>Risk</b> $Reduce.($K)$
10	0.0	0.0	10.0	2	9.1	91
$\overline{20}$	0.0	0	20.0	$\overline{2}$	8.8	176
$\overline{30}$	0.0	15.5	14.5	6	7.3	219
40	0.0	23.6	16.4	6	$\overline{5.8}$	231
50	$\overline{0.0}$	30.8	19.2	6	$\overline{4.7}$	236
$\overline{60}$	42.2	17.8	0.0	5	$\overline{4.1}$	246
$\overline{70}$	52.2	17.8	0.0	5	$\overline{3.8}$	268
$\overline{80}$	66.5	13.5	0.0	5	$\overline{3.6}$	286
$\overline{90}$	62.0	15.5	12.5		$\overline{3.4}$	302
$\overline{100}$	72.0	15.5	12.5	7	$\overline{3.2}$	$\overline{316}$
$\overline{110}$	82.0	15.5	12.5	7	$\overline{3.0}$	327
120	89.7	17.8	12.5		$\overline{3.2}$	337

 $(5) T_h + W_m + B_m$ 

**Table B.11: Optimal investment allocation (T\_h+W\_m+B\_m)**

**(6)** T\_h+W\_m+B\_I

TL Cost (\$K)	Cs	$\mathbf C\mathbf b$	Cw	Option	<b>B/C Ratio</b>	<b>Risk</b> Reduc.(\$K)
10	0.0	10.0	ত.ত		12.5	125
$\overline{20}$	$\overline{0.0}$	9.2	$\overline{10.8}$	6	$\overline{10.7}$	$\sqrt{214}$
$\overline{30}$	$\overline{0.0}$	17.8	12.2	6	$\overline{7.8}$	234
40	0.0	17.8	22.2	6	5.9	237
$\overline{50}$	37.9	12.1	0.0	5	4.9	$\overline{246}$
60	46.2	13.8	0.0	5	$\overline{4.5}$	$\overline{270}$
70	54.3	$\overline{15.7}$	0.0	5	4.1	288
$\overline{80}$	62.2	17.8	0.0	5	$\overline{3.8}$	304
90	72.2	17.8	0.0	5	$\overline{3.5}$	$\overline{318}$
100	82.2	17.8	0.0	5	$\overline{3.3}$	329
$\overline{110}$	92.2	17.8	0.0	5	$\overline{3.1}$	339

Table B.12: Optimal investment allocation  $(T_h+W_m+B_l)$ 

TL Cost (\$K)	Cs	Cb	Cw	Option	<b>B/C Ratio</b>	<b>Risk</b> Reduc.(\$K)
$\overline{10}$	$0\overline{0}$	0.0	10.0		17.6	176
$\overline{20}$	0.0	14.7	$\overline{5.3}$	6	8.6	172
$\overline{30}$	0.0	18.7	11.3	6	7.4	$\overline{221}$
$\overline{40}$	0.0	27.6	12.4	6	$\overline{5.8}$	$\overline{232}$
$\overline{50}$	39.7	0.0	10.3	4	4.8	$\overline{242}$
$\overline{60}$	47.0	0.0	13.0	4	4.3	260
$\overline{70}$	57.0	0.0	13.0	4	$\overline{3.9}$	$\overline{275}$
80	67.0	$\overline{0.0}$	$\overline{13.0}$	4	$\overline{3.6}$	288
90	77.0	0.0	13.0	4	$\overline{3.3}$	299
100	87.0	0.0	13.0	4	$\overline{3.1}$	309

 $(7) T_h+W_l+B_h$ 



**(8) T\_h+W\_1+B\_1**

TL Cost (\$K)	Cs	Cb	Cw	Option	<b>B/C Ratio</b>	<b>Risk</b> $Reduce.(K)$
10	ত.ত	ত.ত	10.0	2	17.6	176
$\overline{20}$	$\overline{0.0}$	$\overline{12.1}$	7.7	6	11.5	231
$\overline{30}$	$\overline{0.0}$	17.8	$\overline{12.2}$	6	7.9	238
40	32.0	0.0	$\overline{8.0}$	4	5.4	$\overline{214}$
$\overline{50}$	35.7	$\overline{8.0}$	6.3	7	$\overline{5.0}$	250
60	42.8	9.2	$\overline{8.0}$	7	4.6	$\overline{275}$
70	52.8	9.2	8.0		$\overline{4.2}$	294
80	61.4	10.6	$\overline{8.0}$		$\overline{3.7}$	$\overline{310}$
90	71.4	10.6	$\overline{8.0}$		3.6	$\overline{323}$
100	81.4	10.6	$\overline{8.0}$	7	$\overline{3.3}$	334
$\overline{110}$	91.4	10.6	8.0	ר	$\overline{3.1}$	344

**Table B.14: Optimal investment allocation (T\_h+W\_I+B\_I)**

TL Cost (SK)	Cs	$\mathbf{C}$	Cw	Option	<b>B/C</b> Ratio	<b>Risk</b> $Reduce.($K)$
10	$0.0\,$	0.0	10.0	2	17.6	176
20	0.0	11.9	$\overline{8.1}$	6	10.6	213
$\overline{30}$	0.0	17.8	12.2	6	$\overline{7.7}$	232
$\overline{40}$	0.0	27.0	13.0	6	5.9	237
$\overline{50}$	0.0	39.8	10.2	4	4.8	241
60	0.0	47.0	$\overline{13.0}$	4	$\overline{4.3}$	260
$\overline{70}$	47.9	11.9	10.2	7	4.0	281
$\overline{80}$	57.9	$\overline{11.9}$	$\overline{10.2}$	7	$\overline{3.7}$	298
90	66.3	13.5	10.2	7	$\overline{3.5}$	$\overline{311}$
100	76.3	13.5	10.2	7	$\overline{3.3}$	$\overline{325}$
110	86.3	13.5	$\overline{10.2}$	7	$\overline{3.1}$	336
120	96.3	13.5	10.2	7	2.9	346

(9) T\_h+W\_1+B\_m



 $(10)$  T\_m+W\_h+B\_h

TL Cost (SK)	Cs	$\mathbf C\mathbf b$	Cw	Option	<b>B/C</b> Ratio	<b>Risk</b> $Reduce.($K)$
$\overline{\mathrm{10}}$	$\overline{\phantom{0}}$		$\overline{\phantom{a}}$	$\overline{\phantom{a}}$		
20	0.0	0.0	20.0	$\overline{2}$	$\overline{6.7}$	134
$\overline{30}$	0.0	0.0	30.0	$\overline{2}$	5.9	176
$\overline{40}$	0.0	15.5	$\overline{24.5}$	$\overline{6}$	$\overline{5.6}$	169
$\overline{50}$	31.3	18.7	0.0	5	4.7	237
60	41.3	18.7	0.0	5	4.5	$\overline{267}$
$\overline{70}$	48.8	21.2	0.0	$\overline{5}$	$\overline{4.1}$	290
$\overline{80}$	$\overline{58.8}$	21.2	0.0	$\overline{5}$	$\overline{3.9}$	310
$\overline{90}$	68.8	21.2	0.0	$\overline{5}$	$\overline{3.6}$	326
100	78.8	21.2	0.0	$\overline{5}$	$\overline{3.4}$	340
.						.
130	105.9	$\overline{24.1}$	0.0		2.9	375
140	115.9	24.1	0.0		$\overline{2.7}$	385

Table B.16: Optimal investment allocation  $(T_m+W_h+B_h)$ 

TL Cost (SK)	Cs	$\mathbf{C}$	Cw	Option	<b>B/C Ratio</b>	<b>Risk</b> $Reduce.(K)$
10	0.0	T0.0	0.0	3	7.6	76
$\overline{20}$	0.0	$\overline{20.0}$	0.0	3	6.6	134
$\overline{30}$	0.0	13.5	16.5	6	6.8	203
40	$\overline{0.0}$	20.5	19.5	6	$\overline{5.6}$	224
50	36.5	13.5	$\overline{0.0}$	5	$\overline{5.2}$	260
60	44.5	15.5	0.0	5	$\overline{4.8}$	286
70	52.2	17.8	0.0	5	4.4	306
80	62.2	17.8	0.0	5	$\overline{4.0}$	324
90	72.2	17.8	0.0	5	$\overline{3.8}$	338
100	82.2	17.8	$\overline{0.0}$	5	$\overline{3.5}$	352
120	99.5	20.5	0.0	5	$\overline{3.1}$	$\overline{374}$
$\overline{130}$	109.5	20.5	0.0	5	$\overline{3.0}$	384

(11) T\_m+W\_h+B\_m



TL Cost (\$K)	$\mathbf{C}\mathbf{s}$	C <sub>b</sub>	Cw	Option	<b>B/C Ratio</b>	<b>Risk</b> $Reduce.(K)$
10	Ծ.Ծ	10.0	0.0	3	12.5	125
$\overline{20}$	0.0	$\overline{7.1}$	12.9	6	9.3	187
$\overline{30}$	0.0	$\overline{15.7}$	14.3	6	7.6	227
40	$\overline{29.4}$	10.6	0.0	5	6.3	$\overline{254}$
$\overline{50}$	27.9	12.1	0.0	5	$\overline{5.7}$	283
60	46.2	13.8	0.0	5	$\overline{5.1}$	306
70	56.2	13.8	0.0	5	4.6	$\overline{325}$
$\overline{80}$	64.3	15.7	0.0	5	4.3	340
90	74.3	15.7	0.0	5	$\overline{3.9}$	354
$\overline{100}$	84.3	$\overline{15.7}$	0.0	5	$\overline{3.7}$	366
$\overline{110}$	94.3	15.7	0.0		$\overline{3.4}$	376

(12) T\_m+W\_h+B\_1

Table B.18: Optimal investment allocation  $(T_m+W_h+B_l)$ 

TL Cost (SK)	Cs	$\mathbf{C}$	Cw	Option	<b>B/C</b> Ratio	<b>Risk</b> $Reduce.(K)$
10	$0.0\,$	0.0	T0.0	2	9.1	91
$\overline{20}$	$\overline{0.0}$	0.0	20.0	$\overline{2}$	8.8	176
$\overline{30}$	0.0	16.8	$\overline{13.2}$	6	6.6	199
40	0.0	21.2	18.8	6	$\overline{5.5}$	221
50	$\overline{37.5}$	$\overline{0.0}$	$\overline{12.5}$	4	$\overline{5.1}$	254
60	$\overline{47.5}$	0.0	12.5	4	4.6	$\overline{275}$
$\overline{70}$	$\overline{54.0}$	0.0	16.0	$\overline{4}$	$\overline{4.2}$	292
80	$\overline{50.7}$	16.8	12.5	7	$\overline{3.9}$	310
$\overline{90}$	58.8	18.7	12.5	7	$\overline{3.6}$	327
100	68.8	18.7	12.5	7	$\overline{3.4}$	242
						.
120	88.8	$\overline{18.7}$	12.5	7	$\overline{3.1}$	376
130	98.8	$\overline{18.7}$	12.5	7	2.9	377

 $(13)$  T\_m+W\_m+B\_h



TL Cost (SK)	$\mathbf{C}$ s	Cb	Cw	Option	<b>B/C</b> Ratio	<b>Risk</b> $Reduce.($K)$
10	$0.0\,$	$10.0\,$	0.0	3	12.5	125
20	$\overline{0.0}$	9.2	10.8	6	10.7	214
30	0.0	17.8	$\overline{12.2}$	6	7.8	234
40	$\overline{29.4}$	10.6	0.0	5	$\overline{6.3}$	$\overline{254}$
50	37.9	12.1	0.0	5	$\overline{5.7}$	283
60	46.2	13.8	0.0	5	5.1	306
$\overline{70}$	56.2	13.8	0.0	$\overline{5}$	4.6	$\overline{325}$
80	64.3	15.7	0.0	5	4.3	340
90	74.3	15.7	0.0	5	$\overline{3.9}$	$\overline{354}$
100	84.3	15.7	0.0	5	$\overline{3.7}$	366
110	94.3	15.7	0.0	5	3.4	377

 $(14)$  T\_m+W\_m+B\_l

Table B.20: Optimal investment allocation  $(T_m+W_m+B_l)$ 

TL Cost (SK)	Cs	C <sub>b</sub>	Cw	Option	<b>B/C Ratio</b>	<b>Risk</b> Reduc.(\$K)
10	$0.0\,$	$0.0\,$	T0.0	2	17.6	176
20	0.0	$\overline{14.7}$	$\overline{5.3}$	6	8.6	172
$\overline{30}$	0.0	18.7	11.3	6	$\overline{7.4}$	221
40	32.0	0.0	$\overline{8.0}$	4	$\overline{6.2}$	247
$\overline{50}$	39.7	0.0	10.3	4	$\overline{5.4}$	271
60	49.7	0.0	10.3	4	$\overline{4.8}$	290
70	59.7	0.0	10.3	4	4.4	306
80	52.9	16.8	10.3	7	$\overline{4.0}$	324
90	62.9	16.8	10.3	7	$\overline{3.8}$	339
<b>100</b>	72.9	16.8	10.3	7	$\overline{3.5}$	353
$\overline{120}$	92.9	16.8	10.3	7	3.1	376
130	102.9	16.8	10.3		$\overline{3.0}$	386

(15) T\_m+W\_1+B\_h



TL Cost (SK)	$\mathbf{C}\mathbf{s}$	$\mathbf C\mathbf b$	Cw	Option	<b>B/C Ratio</b>	<b>Risk</b> Reduc.(\$K)
10	$0.0\,$	0.0	10.0	2	17.6	176
$\overline{20}$	0.0	11.9	$\overline{8.1}$	6	10.6	$\overline{213}$
$\overline{30}$	0.0	17.8	$\overline{12.2}$	6	7.7	232
40	32.0	0.0	$\overline{8.0}$	4	6.2	247
$\overline{50}$	39.7	0.0	10.3	4	5.4	$\overline{271}$
60	40.1	11.9	$\overline{8.0}$	7	4.9	296
70	50.1	11.9	8.0	7	$\overline{4.5}$	317
80	57.8	11.9	10.3	7	4.2	334
$\overline{90}$	67.8	11.9	10.3		3.9	349
100	77.8	11.9	10.3	7	$\overline{3.6}$	362
110	87.8	11.9	$\overline{10.3}$	7	$\overline{3.4}$	373
120	97.8	11.9	10.3		$\overline{3.2}$	383

(16) T\_m+W\_1+B\_m

**Table B.22: Optimal investment allocation (T\_m+W\_1+B\_m)**

TL Cost (SK)	$\mathbf{C}\mathbf{s}$	$\mathbf C\mathbf b$	Cw	Option	<b>B/C Ratio</b>	<b>Risk</b> Reduc.(\$K)
10	$0.0\,$	$0.0\,$	10.0	2	17.6	176
$\overline{20}$	0.0	12.1	7.9	6	11.5	231
$\overline{30}$	0.0	$\overline{17.8}$	$\overline{12.2}$	6	7.9	238
40	26.6	7.1	6.3		6.4	254
$\overline{50}$	35.7	8.0	6.3		$\overline{5.7}$	287
60	44.0	8.0	8.0		$\overline{5.2}$	311
70	52.8	$\overline{9.2}$	$\overline{8.0}$		$\overline{4.7}$	329
80	62.8	9.2	8.0		$\overline{4.3}$	344
90	72.8	9.2	8.0		4.0	358
100	82.8	9.2	8.0		$\overline{3.7}$	370
$\overline{110}$	92.8	9.2	$\overline{8.0}$		$\overline{3.5}$	380

(17) T\_m+W\_I+B\_1



(18)T\_I+W\_h+B\_h

TL Cost (SK)	Cs	$\mathbf C\mathbf b$	Cw	Option	<b>B/C Ratio</b>	<b>Risk</b> Reduc.(\$K)
$10\,$				$- -$		
$\overline{20}$	$\overline{0.0}$	$\overline{0.0}$	20.0	$\overline{2}$	6.7	134
$\overline{30}$	14.5	15.5	0.0	5	6.8	205
40	$\overline{24.5}$	15.5	$\overline{0.0}$	$\overline{5}$	6.7	270
$\overline{50}$	33.2	16.8	$\overline{0.0}$	5	$\overline{6.2}$	310
60	43.2	16.8	0.0	5	$\overline{5.7}$	$\overline{340}$
70	53.2	16.8	0.0	$\overline{5}$	$\overline{5.2}$	364
80	63.2	16.8	0.0	$\overline{5}$	$\overline{4.8}$	385
90	73.2	16.8	0.0	$\overline{5}$	$\overline{4.5}$	401
100	83.2	16.8	0.0	$\overline{5}$	$\overline{4.2}$	416
$\overline{120}$	103.2	16.8	0.0	5	$\overline{3.7}$	441
130	113.2	16.8	0.0	5	$\overline{3.5}$	451

Table B.24: Optimal investment allocation  $(T_l+W_h+B_h)$ 

TL Cost (SK)	Cs	Cb	Cw	Option	<b>B/C Ratio</b>	<b>Risk</b> $Reduce.(K)$
$\overline{\text{10}}$	0.0	$10.0\,$	$0.\overline{0}$	3	7.6	76
$\overline{20}$	$\overline{10.8}$	$\overline{9.2}$	0.0	5	$\overline{7.1}$	$\overline{143}$
$\overline{30}$	19.4	10.6	0.0	5	8.3	249
40	28.1	11.9	$\overline{0.0}$	5	7.4	297
$\overline{50}$	38.1	$\overline{11.9}$	0.0	5	6.6	330
60	48.1	11.9	0.0	5	5.9	357
70	56.5	13.5	0.0	5	$\overline{5.4}$	378
80	66.5	13.5	0.0	3	5.0	396
90	76.5	13.5	0.0	$\overline{5}$	4.6	411
100	86.5	13.5	0.0	5	$\overline{4.2}$	424
						.
120	108.1	$\overline{11.9}$	$\overline{0.0}$	$\overline{5}$	$\overline{3.7}$	448
130	109.1	11.9	0.0	5	$\overline{3.5}$	458

(19) T\_1+W\_h+B\_m

Table B.25: Optimal investment allocation  $(T_1+W_1+W_m)$ 

## (20) T\_1+W\_h+B\_I



Table B.26: Optimal investment allocation  $(T_l+W_h+B_l)$ 

TL Cost (SK)	Cs	Cb	Cw	Option	<b>B/C</b> Ratio	<b>Risk</b> Reduc.(\$K
10	$0.0\,$	0.0	10.0	$\overline{2}$	9.1	91
$\overline{20}$	$\overline{0.0}$	$\overline{0.0}$	20.0	$\overline{2}$	8.8	176
$\overline{30}$	$\overline{20.0}$	$\overline{0.0}$	10.0	4	$\overline{8.1}$	242
40	30.0	0.0	10.0	4	$\overline{7.1}$	283
$\overline{50}$	40.0	0.0	$\overline{10.0}$	$\overline{4}$	6.2	312
$\overline{60}$	50.0	0.0	10.0	$\overline{4}$	$\overline{5.6}$	334
$\overline{70}$	43.2	16.8	10.0	7	$\overline{5.1}$	355
$\overline{80}$	53.2	16.8	10.0	7	4.7	377
90	63.2	16.8	10.0	7	4.4	$\overline{395}$
100	73.2	16.8	$\overline{10.0}$	7	$\overline{4.1}$	$\overline{411}$
						.
130	103.2	16.8	10.0	7	$\overline{3.4}$	447
140	113.2	16.8	$\overline{10.0}$	7	$\overline{3.3}$	456

 $(21) T_l + W_m + B_h$ 

Table B.27: Optimal investment allocation  $(T_l+W_m+B_h)$ 

 $(22) T_l + W_m + B_m$ 

TL Cost (SK)	$\mathbf{C}$ s	Cb	Cw	Option	<b>B/C Ratio</b>	<b>Risk</b> Reduc.(\$K
$10\,$	0	0	ΤΟ	2	9.1	91
$\overline{20}$	0	0	20	$\overline{2}$	8.8	$\overline{176}$
$\overline{30}$	19.4	10.6	$\Omega$	$\overline{5}$	8.3	$\overline{249}$
40	28.1	11.9	$\theta$	$\overline{5}$	$\overline{7.4}$	297
$\overline{50}$	38.1	11.9	$\overline{0}$	$\overline{5}$	6.6	$\overline{330}$
60	48.1	11.9	$\Omega$	5	$\overline{5.9}$	357
$\overline{70}$	56.5	13.5	$\overline{0}$	5	$\overline{5.4}$	378
80	66.5	13.5	$\theta$	$\overline{5}$	$\overline{5.0}$	396
90	76.5	13.5	$\overline{0}$	$\overline{5}$	$\overline{4.6}$	$\overline{411}$
$\overline{100}$	86.5	13.5	$\overline{0}$	$\overline{5}$	$\overline{4.2}$	424
110	86.5	13.5	$\theta$	5	$\overline{4.2}$	437
120	86.5	13.5	$\overline{0}$	5	$\overline{4.2}$	448

Table B.28: Optimal investment allocation  $(T_l+W_m+B_m)$ 

TL Cost (SK)	$\mathbf{C}\mathbf{s}$	Cb	Cw	Option	<b>B/C Ratio</b>	<b>Risk</b> Reduc.(\$K)
10	0.0	$10.0\,$	$0.0\,$		12.5	125
$\overline{20}$	0.0	9.2	10.8	6	10.7	214
$\overline{30}$	23.8	6.2	0.0	5	$\overline{9.3}$	280
40	31.6	$\overline{8.4}$	0.0	5	$\overline{8.0}$	320
$\overline{50}$	40.8	9.2	0.0	$\overline{5}$	7.0	350
60	49.4	10.6	0.0	5	6.2	373
$\overline{70}$	$\overline{59.4}$	10.6	0.0	5	5.6	392
80	69.4	$\overline{10.6}$	0.0	5	$\overline{5.1}$	408
90	79.4	10.6	$\overline{0.0}$	5	4.7	421
100	89.4	10.6	0.0	5	$\overline{4.3}$	434
$\overline{110}$	99.4	10.6	0.0	5	4.0	445
$\overline{120}$	100.8	9.2	0.0	5	$\overline{3.8}$	455

(23) T\_I+W\_m+B\_I



(24) T\_1+W\_1+B\_h

TL Cost (SK)	Cs	$\mathbf C\mathbf b$	Cw	Option	<b>B/C Ratio</b>	<b>Risk</b> Reduc.(\$K)
IΌ	0.0	10.0	$0.0\,$	2	17.6	176
$\overline{20}$	15.0	$\overline{0.0}$	$\overline{5.0}$	4	10.5	210
$\overline{30}$	25.0	$\overline{0.0}$	$\overline{5.0}$	4	$\overline{8.9}$	267
40	35.0	$\overline{0.0}$	$\overline{5.0}$	4	7.5	301
$\overline{50}$	45.0	$\overline{0.0}$	$\overline{5.0}$	4	6.5	327
60	36.5	15.5	8.0	4	$\overline{5.8}$	348
70	46.5	15.5	$\overline{8.0}$	7	$\overline{5.3}$	370
80	56.5	15.5	$\overline{8.0}$	7	4.9	389
$\overline{90}$	66.5	15.5	$\overline{8.0}$	7	$\overline{4.5}$	406
100	76.5	15.5	$\overline{8.0}$	7	$\overline{4.2}$	419
120	96.5	15.5	$\overline{8.0}$	7	$\overline{3.7}$	443
130	106.5	15.5	$\overline{8.0}$		$\overline{3.5}$	453

**Table B.30: Optimal investment allocation (T\_I+W\_I+B\_h)**

TL Cost (SK)	Cs	Cb	Cw	Option	<b>B/C</b> Ratio	<b>Risk</b> Reduc.(\$K)
10	0.0	0.0	10.0	2	17.6	176
$\overline{20}$	0.0	11.9	$\overline{8.1}$	6	10.6	$\overline{213}$
30	$\overline{25.0}$	0.0	$\overline{5.0}$	4	$\overline{8.9}$	267
40	35.0	0.0	5.0	4	$\overline{7.5}$	301
$\overline{50}$	33.2	10.6	6.2	7	6.7	333
60	40.1	11.9	$\overline{8.0}$	7	6.0	$\overline{361}$
70	51.4	10.6	$\overline{8.0}$	7	$\overline{5.5}$	385
80	61.4	10.6	8.0	7	$\overline{5.0}$	400
90	71.4	10.6	8.0	7	4.6	414
100	81.4	10.6	$\overline{8.0}$	7	4.3	427
110	91.4	10.6	$\overline{8.0}$	7	4.0	439
120	101.4	10.6	$\overline{8.0}$	~	$\overline{3.7}$	449

(25) T\_I+W\_1+B\_m

**Table B.31: Optimal investment allocation (T\_I+W\_I+B\_m)**

 $(26)$  T\_l+W\_l+B\_l

TL Cost (SK)	$\mathbf{C}$ s	Cb	Cw	Option	<b>B/C</b> Ratio	<b>Risk</b> $Reduce.($K)$
I0	$0.0\,$	0.0	10.0	$\mathfrak{I}% _{T}=\mathfrak{I}_{T}\!\left( a,b\right) ,\ \mathfrak{I}_{T}% =\mathfrak{I}_{T}\!\left( a,b\right) ,$	17.6	176
$\overline{20}$	0.0	12.1	7.9	6	11.5	231
$\overline{30}$	0.0	8.0	$\overline{22.0}$	6	9.3	280
$\overline{40}$	$\overline{0.0}$	$\overline{9.2}$	30.8	6	$\overline{8.0}$	320
$\overline{50}$	36.3	$\overline{7.1}$	6.2	7	$\overline{7.0}$	350
60	45.8	8.0	$\overline{6.2}$	7	6.2	374
$\overline{70}$	55.8	$\overline{8.0}$	6.2	7	5.6	393
80	65.8	$\overline{8.0}$	6.2	7	$\overline{5.1}$	408
90	75.8	8.0	6.2	7	4.7	422
100	85.8	8.0	6.2	7	4.3	434
110	95.8	8.0	$\overline{6.2}$	7	4.0	445
120	105.8	8.0	6.2		$\overline{3.8}$	455

**Table B.32: Optimal investment allocation (T\_I+W\_I+B\_I)**