

**Fairweather Pension Plan:  
Optimizing the Investment Portfolio  
Using MPT**

## *“Scene I”*

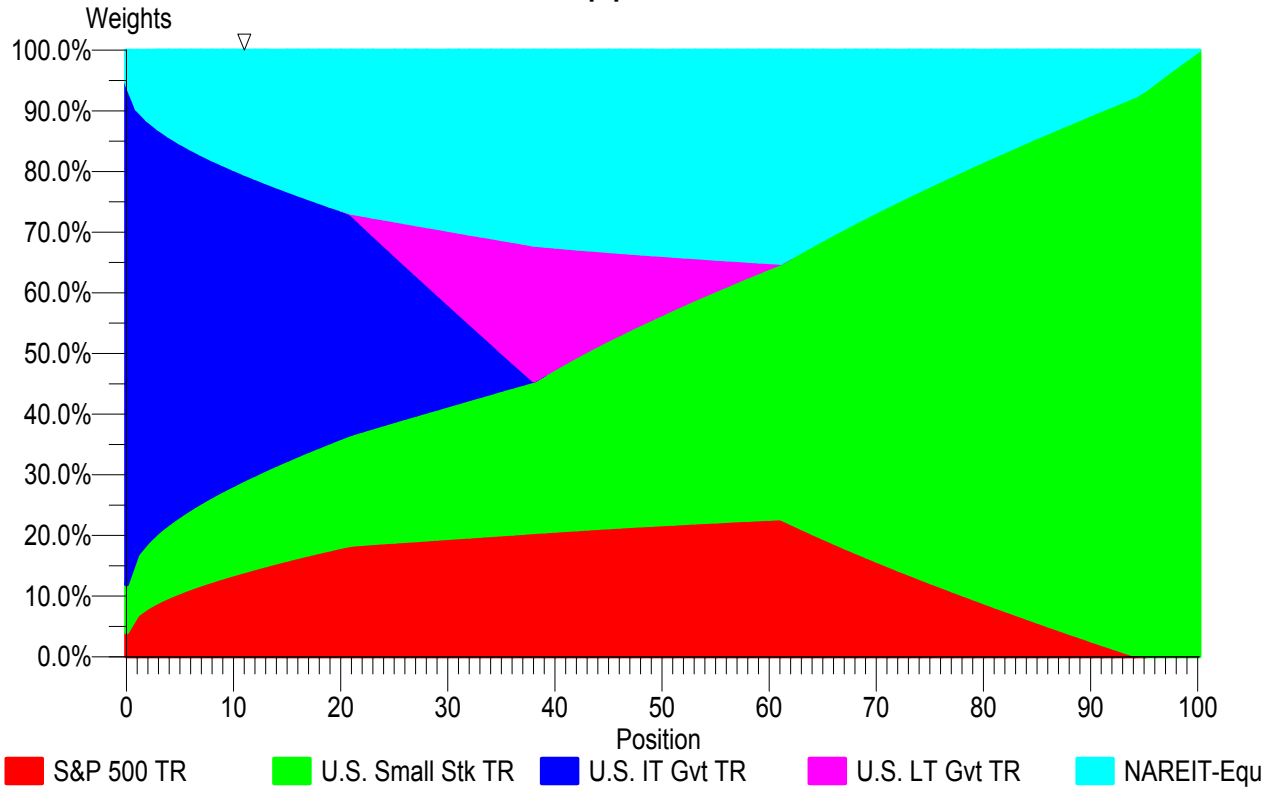
**First consider the possibilities presented by five major investment asset classes of all publicly-traded securities, based on historical risk & return performance during 1978-2002 . . .**

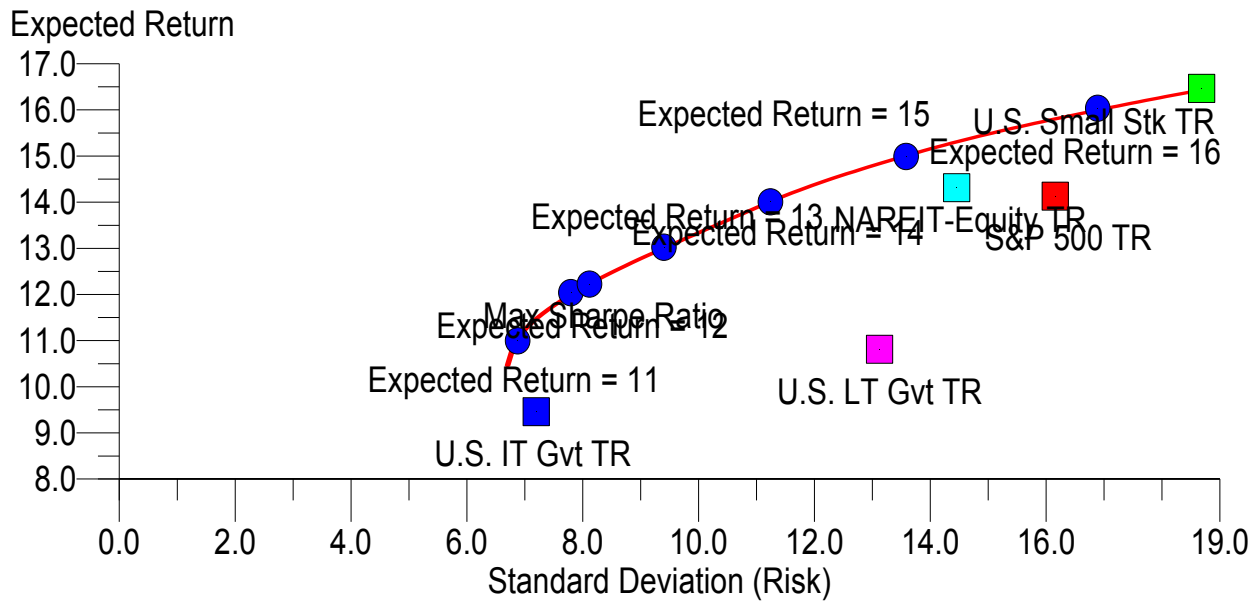
### **Portfolio Statistics**

	Expected Return = 11	Expected Return = 12	Expected Return = 13	Expected Return = 14	Expected Return = 15	Expected Return = 16	Max Sharpe Ratio
S&P 500 TR	7.17	13.12	18.61	20.44	22.46	5.53	14.42
U.S. Small Stk TR	10.18	14.44	18.90	24.91	39.75	80.37	15.37
U.S. IT Gvt TR	72.45	53.43	32.93	0.49	0.00	0.00	49.28
U.S. LT Gvt TR	0.00	0.00	2.21	22.09	3.07	0.00	0.00
NAREIT-Equity TR	10.20	19.01	27.35	32.06	34.73	14.11	20.93
Expected Return	11.00	12.00	13.00	14.00	15.00	16.00	12.22
Standard Deviation	6.85	7.80	9.40	11.22	13.57	16.90	8.11
Sharpe Ratio	0.63	0.68	0.67	0.65	0.61	0.55	0.69

# Frontier Area Graph

Scene1optportf.aax





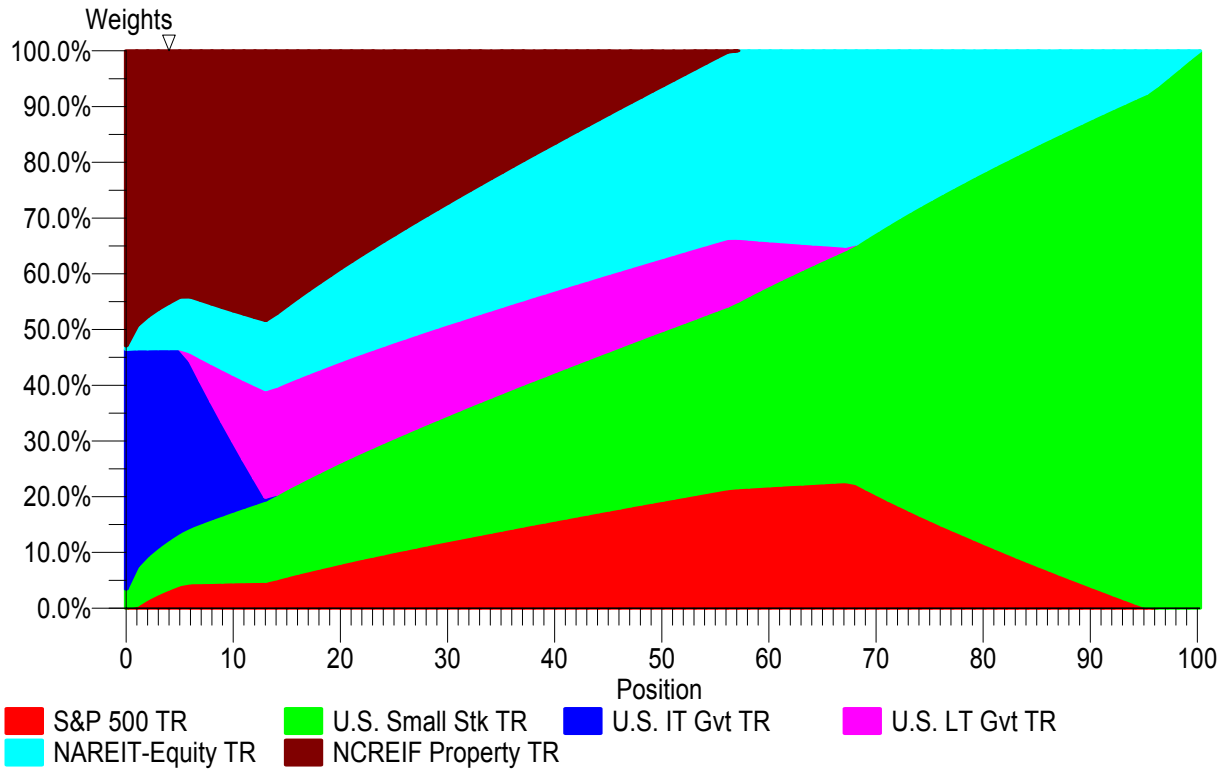
## *“Scene II”*

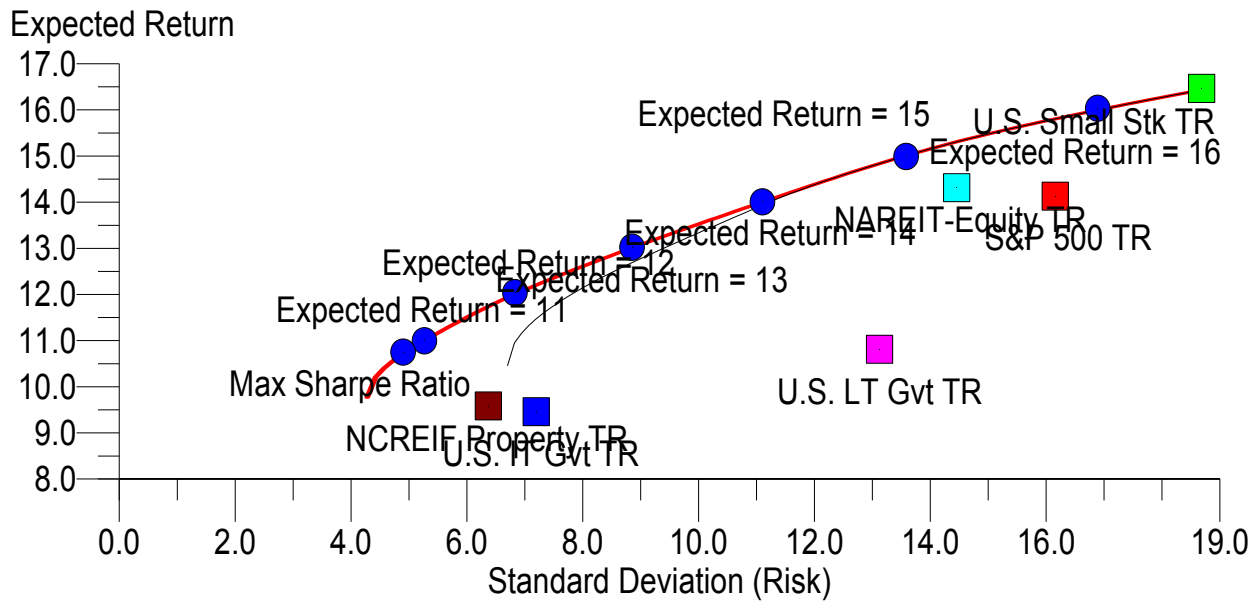
**Now consider the possibilities if we add a sixth major investment asset classes: private direct real estate, as represented by the NCREIF Property Index (NPI).**

### **Portfolio Statistics: With NCREIF**

	Expected Return = 11	Expected Return = 12	Expected Return = 13	Expected Return = 14	Expected Return = 15	Expected Return = 16	Max Sharpe Ratio
S&P 500 TR	4.59	7.06	12.76	18.46	22.46	5.53	3.80
U.S. Small Stk TR	10.67	17.08	23.28	29.49	39.75	80.37	9.17
U.S. IT Gvt TR	25.86	0.00	0.00	0.00	0.00	0.00	33.60
U.S. LT Gvt TR	3.95	18.59	16.05	13.51	3.07	0.00	0.00
NAREIT-Equity TR	10.22	15.25	22.44	29.64	34.73	14.11	8.66
NCREIF Property	44.72	42.03	25.47	8.91	0.00	0.00	44.77
Expected Return	11.00	12.00	13.00	14.00	15.00	16.00	10.73
Standard Deviation	5.27	6.82	8.83	11.10	13.57	16.90	4.91
Sharpe Ratio	0.82	0.78	0.72	0.66	0.61	0.55	0.83

# Frontier Area Graph With NCREIF





- **Preceding mean-variance optimal portfolios trace out the “efficient frontier” (non-dominated allocations), based on ex post historical total return performance of 6 asset classes (2 stocks, 2 bonds, and 2 real estate).**

- **These input assumptions are shown in the table below:**

	Expeted Return	Standard Devn	Correl with S&P 500	Correl with U.S. Small Stk	Correl with U.S. IT Gvt	Correlw ith U.S. LT Gvt	Correlat ion with Equity REIT	Correl with NPI	Correl with U.S. 30 Day TBill TR	Correl with U.S. Inflation
S&P 500 TR	14.14	16.15	1.0000	0.5739	0.1800	0.2362	0.2859	0.1272	0.1916	0.0212
U.S. Small Stk TR	16.43	18.67	0.5739	1.0000	0.0102	-0.0202	0.6433	0.0750	0.2235	0.2872
U.S. IT Gvt TR	9.46	7.20	0.1800	0.0102	1.0000	0.9389	0.2005	-0.1715	0.2072	-0.2303
U.S. LT Gvt TR	10.81	13.11	0.2362	-0.0202	0.9389	1.0000	0.1959	-0.2389	-0.0233	-0.3970
NAREIT-Equity TR	14.29	14.44	0.2859	0.6433	0.2005	0.1959	1.0000	0.0435	0.2008	0.1785
NCREIF Property TR	9.54	6.35	0.1272	0.0750	-0.1715	-0.2389	0.0435	1.0000	0.5482	0.5177
U.S. 30 Day TBill TR	6.70	3.00	0.1916	0.2235	0.2072	-0.0233	0.2008	0.5482	1.0000	0.6856
U.S. Inflation	4.43	3.19	0.0212	0.2872	-0.2303	-0.3970	0.1785	0.5177	0.6856	1.0000



- **The preceding efficient portfolios are superior to Fairweather's all-bond portfolio from a mean-variance perspective. E.g., If Fairweather has been in intermediate-term bonds, then the portfolio would have achieved:**
  - **9.46% average return; 7.20% volatility.**
  - **In contrast, an efficient 5-class portfolio of only publicly-traded securities could have achieved:**
    - **11.00% average return; 6.85% volatility;**
    - **By investing in: 72% intermediate-term bonds, 10% small stocks, 10% REITs, and 7% large stocks.**
    - **Or, including also private (direct) real estate (as represented by the NCREIF Index), the portfolio could have achieved, for example:**
      - **11.00% average return; 5.27% volatility;**
      - **By investing in: 45% private real estate, 26% intermediate-term bonds, 11% small stocks, 10% REITs, 5% large stocks, and 4% long-term bonds.**

## ***“Scene III”***

**Preceding analysis has two problems:**

- **Historical risk/return patterns not necessarily completely representative of reasonable or typical current investor expectations looking forward in time;**
- **The private real estate return statistics, particularly the “*second moments*” (volatility & correlations) probably reflect a “*smoothing and lagging bias*” that tends to lower these statistics (volatility biased toward zero, correlations biased downward).**

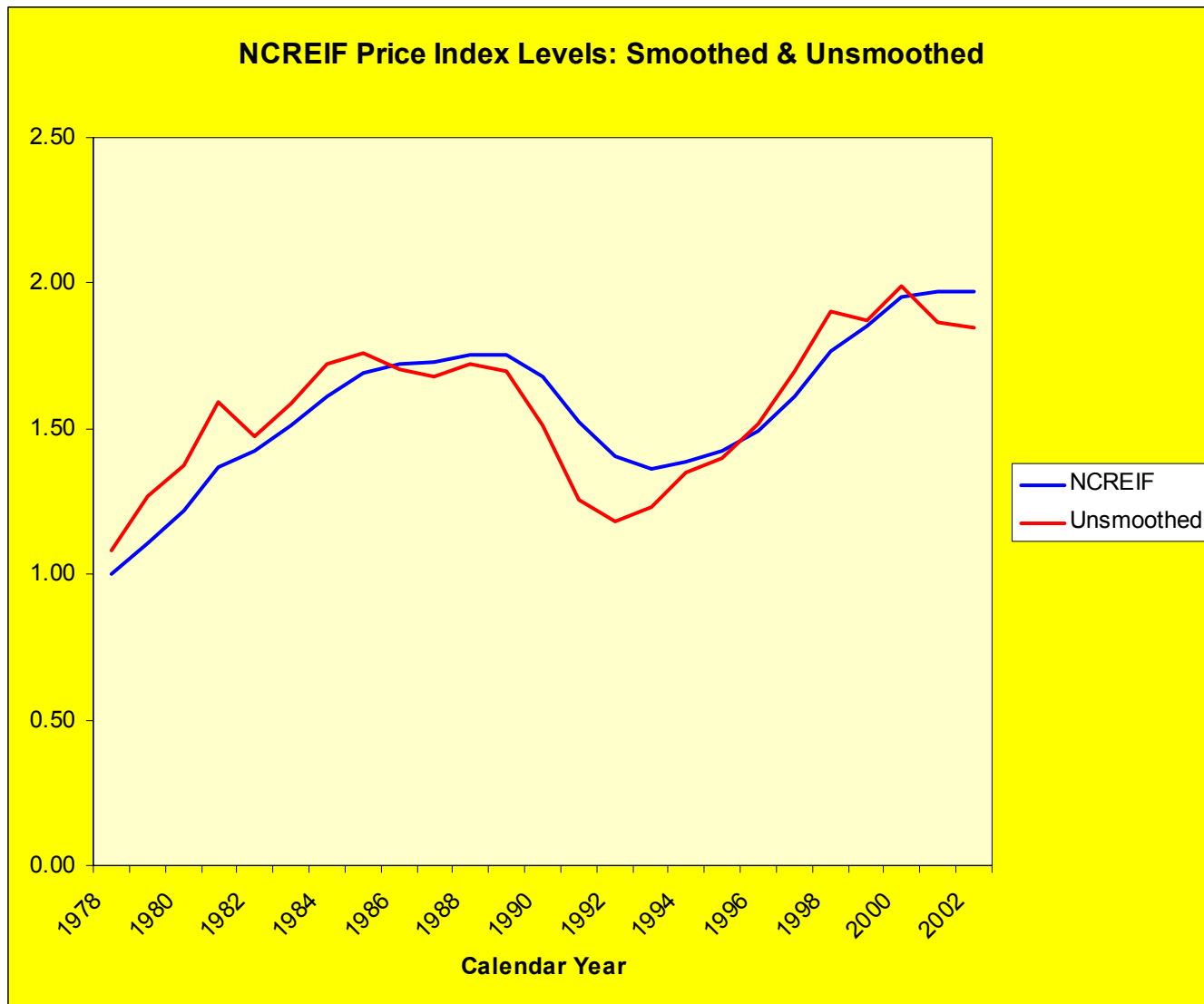
**In addressing these problems, we suggest three major considerations...**

- 1. Adjust for the difference between current inflation expectations (say, 2.5%/yr) versus historical avg inflation in our 1978-2002 history (4.35%).**
- 2. “*Unsmooth*” the private real estate second moments using a simple model that approximately corrects for the lag bias in the NCREIF Index (the “*Simple 1-Step Model*”).**
- 3. Increase expected private real estate correlation with bonds to zero: The unsmoothed returns still show negative correlation with bonds, which may reflect the particular historical period including the late 1970s & 80s in which inflation was particularly volatile and of concern to investors (real estate benefited from inflation, while bonds were hurt).**

## Adjusting the mean return expectations for inflation...

<b>Arithmetic Average Time-Weighted Total Returns:</b>		
	<b>Actual 1978-2002</b>	<b>Less Infla Difference (4.35%-2.5%)</b>
<b>Large Stocks</b>	<b>14.14%</b>	<b>12.29%</b>
<b>Small Stocks</b>	<b>16.43%</b>	<b>14.58%</b>
<b>IT Bonds</b>	<b>9.46%</b>	<b>7.61%</b>
<b>LT Bonds</b>	<b>10.81%</b>	<b>8.96%</b>
<b>REITs</b>	<b>14.29%</b>	<b>12.44%</b>
<b>NCREIF</b>	<b>9.25%</b>	<b>7.40%</b>

# Adjusting the private real estate returns for smoothing bias...



## Revised Inputs Summary\*

	Exp. Ret (79-02)	St. Dev	Corr. w/ S&P 500	Corr. w/ Small Stk	Corr. w/ IT Gvt	Corr. w/ LT Gvt	Corr. w/ NARE IT	Corr. w/ NCRE IF
<b>S&amp;P 500</b>	<b>12.29</b>	<b>16.15</b>	<b>1.00</b>	<b>0.57</b>	<b>0.18</b>	<b>0.24</b>	<b>0.29</b>	<b>0.09</b>
<b>U.S. Small Stk</b>	<b>14.58</b>	<b>18.67</b>	<b>0.57</b>	<b>1.00</b>	<b>0.01</b>	<b>-0.02</b>	<b>0.64</b>	<b>0.04</b>
<b>U.S. IT Gvt</b>	<b>7.61</b>	<b>7.20</b>	<b>0.18</b>	<b>0.01</b>	<b>1.00</b>	<b>0.94</b>	<b>0.20</b>	<b>0.00</b>
<b>U.S. LT Gvt</b>	<b>8.96</b>	<b>13.11</b>	<b>0.24</b>	<b>-0.02</b>	<b>0.94</b>	<b>1.00</b>	<b>0.20</b>	<b>0.00</b>
<b>NAREIT-Equity</b>	<b>12.44</b>	<b>14.44</b>	<b>0.29</b>	<b>0.64</b>	<b>0.20</b>	<b>0.20</b>	<b>1.00</b>	<b>0.12</b>
<b>NCREIF Property (79-02)</b>	<b>7.40</b>	<b>8.54</b>	<b>0.09</b>	<b>0.04</b>	<b>0.00</b>	<b>0.00</b>	<b>0.12</b>	<b>1.00</b>

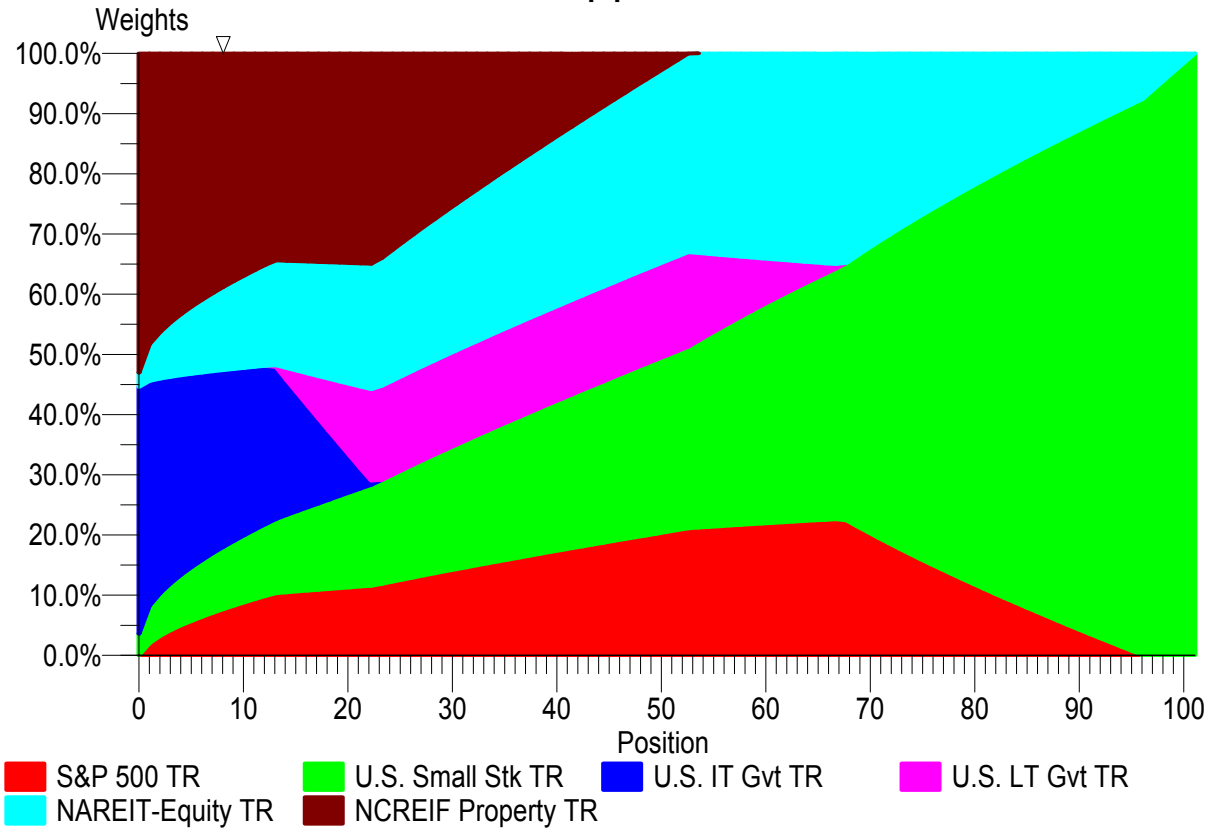
**\* Based on 1978-2002 historical returns (except 79-02 for private real estate), modified for changed inflation expectations.**

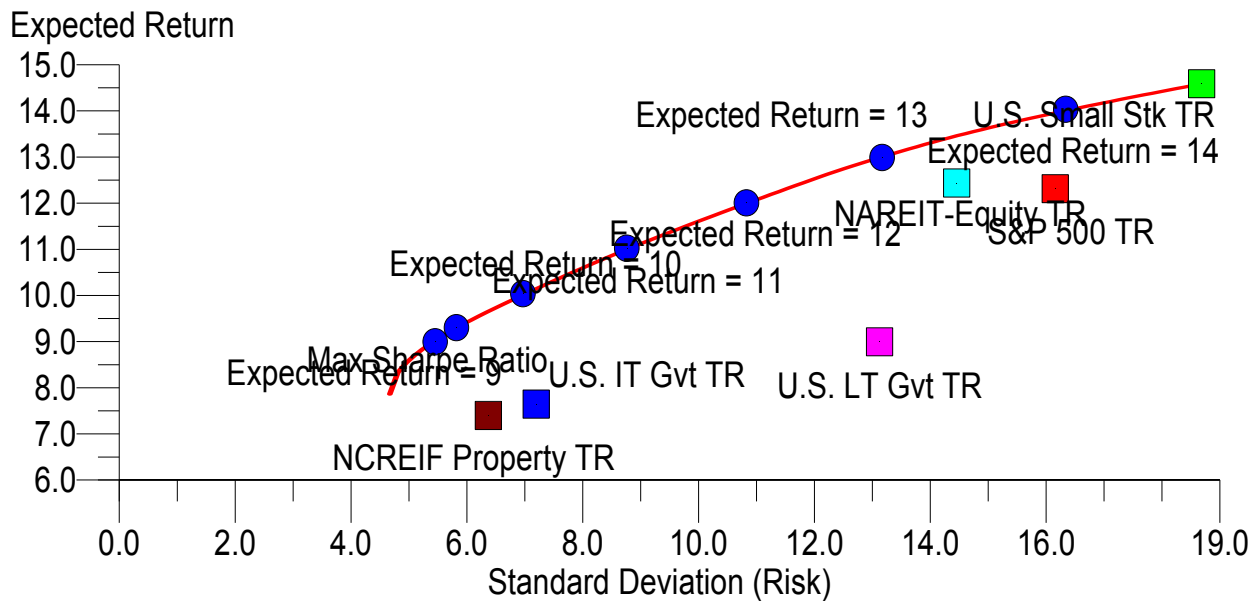
# Efficient Frontier Portfolios With Revised Input Assumptions

	Expected Return = 9	Expected Return = 10	Expected Return = 11	Expected Return = 12	Expected Return = 13	Expected Return = 14	Max Sharpe Ratio
<b>S&amp;P 500 TR</b>	6.01	10.77	14.02	18.70	22.05	8.53	7.67
<b>U.S. Small Stk TR</b>	9.14	14.00	20.31	26.86	37.58	73.50	10.36
<b>U.S. IT Gvt TR</b>	31.69	16.03	0.00	0.00	0.00	0.00	29.34
<b>U.S. LT Gvt TR</b>	0.00	5.87	15.65	15.70	6.06	0.00	0.00
<b>NAREIT-Equity TR</b>	11.57	18.72	24.04	30.00	34.32	17.97	13.84
<b>NCREIF Property</b>	41.59	34.62	25.97	8.74	0.00	0.00	38.79
<b>Expected Return</b>	9.00	10.00	11.00	12.00	13.00	14.00	9.28
<b>Standard Deviation</b>	5.43	6.95	8.77	10.84	13.16	16.32	5.79
<b>Sharpe Ratio</b>	0.70	0.69	0.66	0.63	0.59	0.54	0.70

# Frontier Area Graph

Scene3optportf.aax



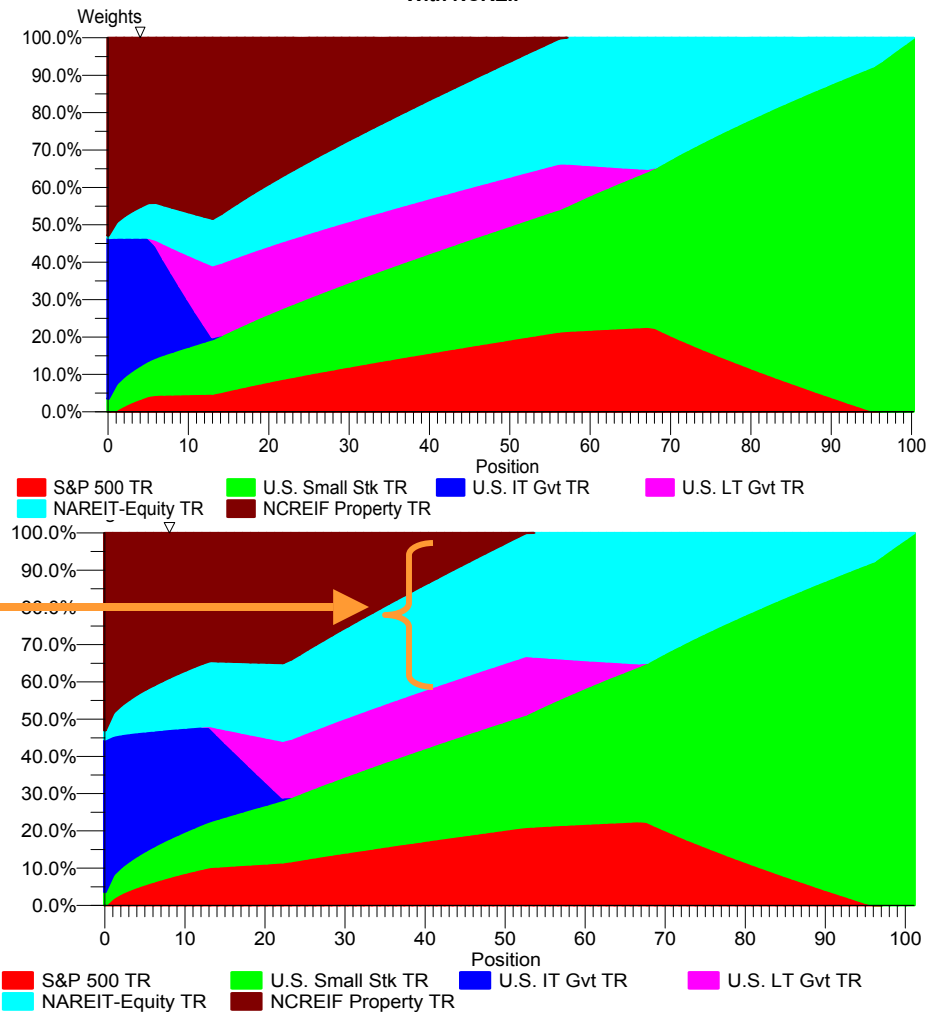




As seen below, the revised assumptions do not much change the composition of the optimal portfolio...

Thus, even adjusting return expectations to be more realistic, the role of real estate (including both private and public) is substantial from a classical MPT perspective.

Frontier Area Graph  
With NCREIF



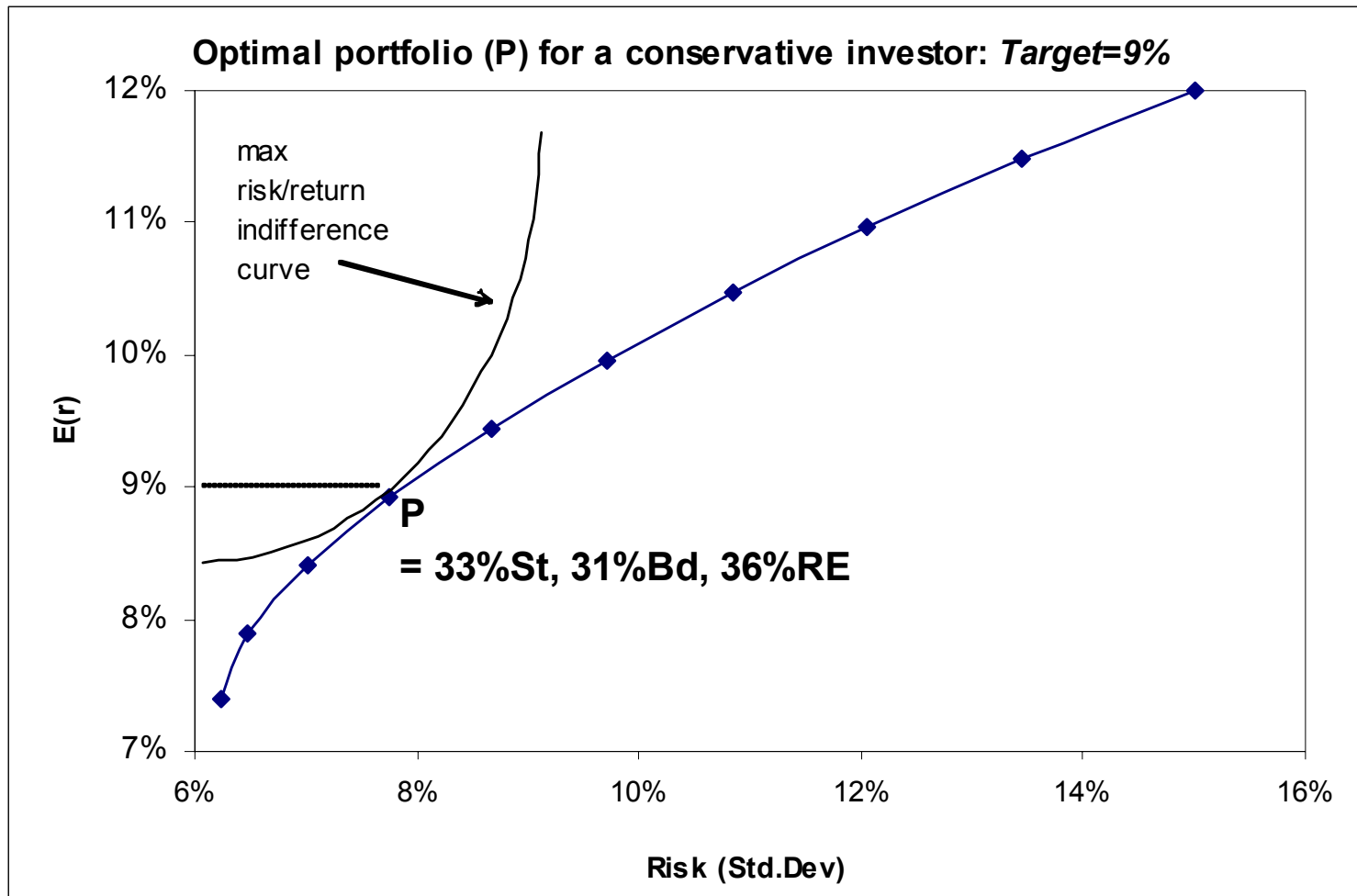
## ***“Scene IV”***

### **Extending the Analysis:**

- ***Where should Fairweather be on the frontier (risk tolerance)?...***
- ***Broader considerations (beyond MPT)...***

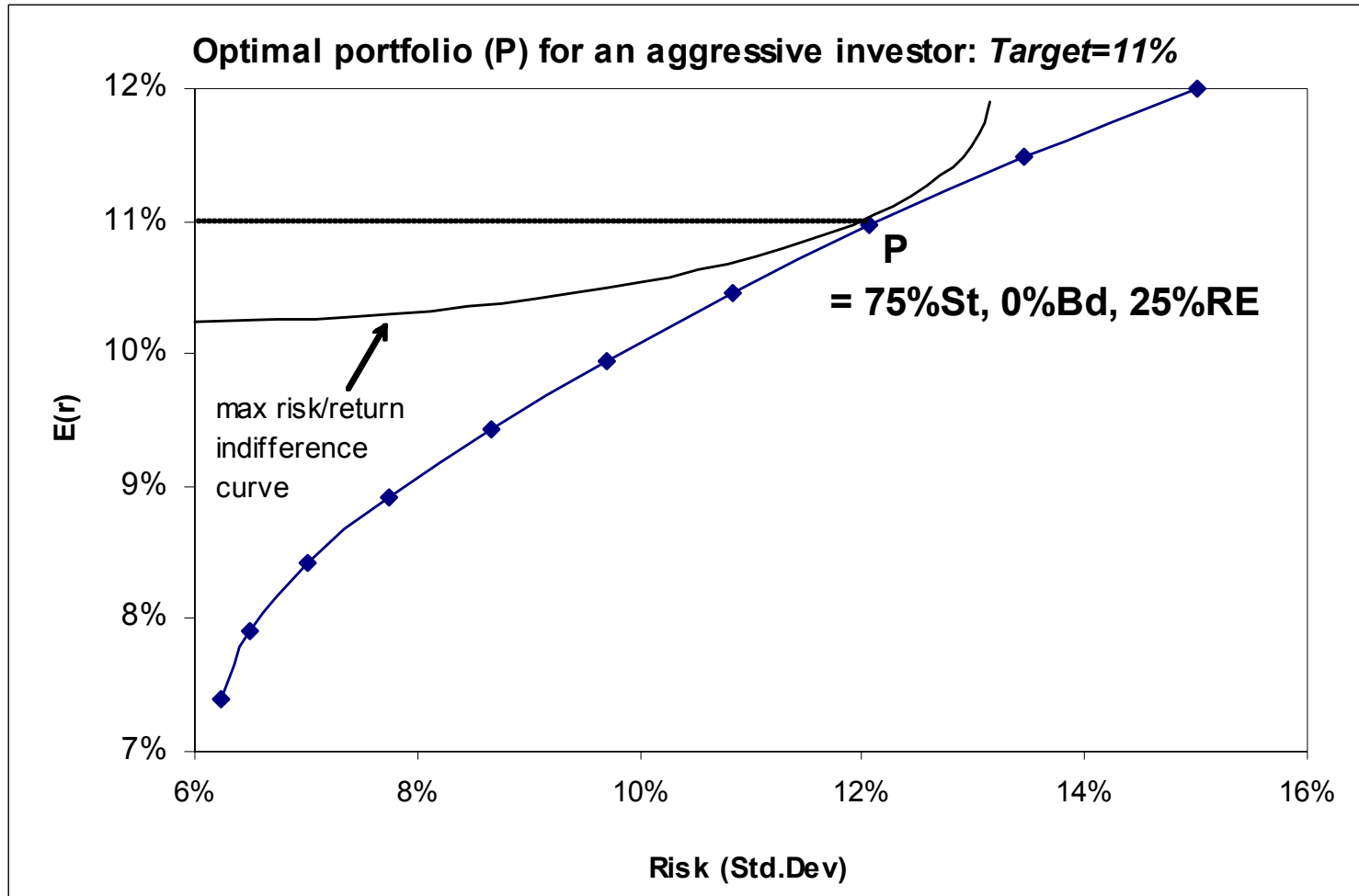
# Where should Fairweather be on the frontier? . . .

***E.G., ARE YOU HERE (9%)?...***





## ***OR ARE YOU HERE (11%)?...***



- **Pension funds by their fundamental nature (and by law) must be managed relatively conservatively. However, within the generally conservative perspective,...**
- **Fairweather's relatively young pension member age profile, and Fairweather's track record as a relatively stable, growing company, suggests that Fairweather might consider a relatively aggressive (high return) target within the range typical of pension funds.**
- **For example, a target roughly in the mid-range of the classical MPT frontier. Say, an 11% or 12% nominal target (8%-10% real)?...**

	Expected Return = 9	Expected Return = 10	Expected Return = 11	Expected Return = 12	Expected Return = 13	Expected Return = 14	Max Sharpe Ratio
S&P 500 TR	6.01	10.77	14.02	18.70	22.05	8.53	7.67
U.S. Small Stk TR	9.14	14.00	20.31	26.86	37.58	73.50	10.36
U.S. IT Gvt TR	31.69	16.03	0.00	0.00	0.00	0.00	29.34
U.S. LT Gvt TR	0.00	5.87	15.65	15.70	6.06	0.00	0.00
NAREIT-Equity TR	11.57	18.72	24.04	30.00	34.32	17.97	13.84
NCREIF Property	41.59	34.62	25.97	8.74	0.00	0.00	38.79
Expected Return	9.00	10.00	11.00	12.00	13.00	14.00	9.28
Standard Deviation	5.43	6.95	8.77	10.84	13.16	16.32	5.79
Sharpe Ratio	0.70	0.69	0.66	0.63	0.59	0.54	0.70

Another perspective on the target question would be to consider the implications of assuming the existence of a “*riskless asset*”.

This theoretical construct makes some sense as an approximation of reality, in that short-term Govt bonds (T-Bills) have very little risk, and highly liquid investors such as pension funds can borrow or lend short-term at interest rates not much different from T-Bills (“cash management”).

If T-Bills are riskless, then classical MPT implies that *no matter what your risk preferences*, the optimal combination of risky assets is that which maximizes the “*Sharpe Ratio*” (the portfolio excess expected return over T-Bills, divided by the volatility of the portfolio).



	Expected Return = 9	Expected Return = 10	Expected Return = 11	Expected Return = 12	Expected Return = 13	Expected Return = 14	Max Sharpe Ratio
S&P 500 TR	6.01	10.77	14.02	18.70	22.05	8.53	7.67
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Sharpe Ratio	0.70	0.69	0.66	0.63	0.59	0.54	0.70

**Either of the preceding perspectives suggests that the role of real estate (both REITs and private property) should be considerable in the optimal pension portfolio.**

	Expected Return = 9	Expected Return = 10	Expected Return = 11	Expected Return = 12	Expected Return = 13	Expected Return = 14	Max Sharpe Ratio
S&P 500 TR	6.01	10.77	14.02	18.70	22.05	8.53	7.67
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U.S. LT Gvt TR	0.00	5.87	15.65	15.70	6.06	0.00	0.00
NAREIT-Equity TR	11.57	18.72	24.04	30.00	34.32	17.97	13.84
NCREIF Property	41.59	34.62	25.97	8.74	0.00	0.00	38.79
Expected Return	9.00	10.00	11.00	12.00	13.00	14.00	9.28
Standard Deviation	5.43	6.95	8.77	10.84	13.16	16.32	5.79
Sharpe Ratio	0.70	0.69	0.66	0.63	0.59	0.54	0.70

**However, some important considerations relevant especially to private real estate are left out of the classical MPT model, such as:**

- **Illiquidity of real estate.**
- **Transaction cost differentials (&/or related holding period and rebalancing constraints).**
- **Asset operational management requirements for direct real estate investment.**
- **Lack of informational efficiency in private asset markets.**

***These considerations suggest caution in allocating as much to private real estate as suggested by MPT. Some major large P.F.s have set private R.E. targets around 10%.***

# Excel template (“Portfo1”) results for no riskless asset, 11% target:

OPT PORTF FINDER NO RISKLESS ASSET (Input Target Mean= 11.00%)	
7-Asset Portfolio Optimizer: (Check to make sure target mean is sufficiently high to be above bottom edge of feasible frontier, that is, the bottom side of "the parabola".)	
Based on Variance Stats:	
Inputs:	Outputs: Use Solver in Tools menu to find optimal portfolio.
Asset #: Definition:	Shares Target cell is portf Variance in cell i50 which should be MINimized.
	By varying portfolio weights in cells b39:h39.
	Subject to constraints:
	Each weight (B39 through H39) >=0;
	Portf Mean equal Target Mean (i41=G1);
	Sum of weights equal 1 (i39=1).
	(For less than 7 assets, make "junk" assets with very high variance and correlation, and very low means, then save spreadsheet under new name.)
1 Large Stocks	13.92%
2 Small Stocks	21.78%
3 IT Bonds	0.00%
4 LT Bonds	16.00%
5 NAREIT	21.96%
6 NCREIF	26.33%
7 Junk	0.00%
Portf Mean=	11.00%
Portf STD=	8.76%
Inputs...	Asset #:
Stat	1 2 3 4 5 6 7
Mean	0.1229 0.1458 0.0761 0.0896 0.1244 0.0740 -10.0000
Std.Dev	0.1615 0.1867 0.0720 0.1311 0.1444 0.0620 10.0000
Corr.Tbl:	
1	1.0000 0.5739 0.1800 0.2362 0.2859 0.0912 1.0000
2	1.0000 0.0102 -0.0202 0.6433 0.0389 1.0000
3	1.0000 0.9389 0.2005 0.0000 1.0000
4	1.0000 0.1959 0.0000 1.0000
5	1.0000 0.1174 1.0000
6	1.0000 1.0000
7	1.0000
Mechanics...	
Covariance Table:	
1	0.0261 0.0173 0.0021 0.0050 0.0067 0.0009 1.6151
2	0.0173 0.0348 0.0001 -0.0005 0.0173 0.0004 1.8667
3	0.0021 0.0001 0.0052 0.0089 0.0021 0.0000 0.7200
4	0.0050 -0.0005 0.0089 0.0172 0.0037 0.0000 1.3113
5	0.0067 0.0173 0.0021 0.0037 0.0209 0.0011 1.4441
6	0.0009 0.0004 0.0000 0.0000 0.0011 0.0038 0.6200
7	1.6151 1.8667 0.7200 1.3113 1.4441 0.6200 100.0000
Opt. Share	0.1392 0.2178 0.0000 0.1600 0.2196 0.2633 0.0000 1
	Sum w*R
w*R	0.0171 0.0317 0.0000 0.0143 0.0273 0.0195 0.0000 0.1100 =Port Mean
Weighted Pairwise Covariance Matrix (wiwjCOVij):	
	0.0005 0.0005 0.0000 0.0001 0.0002 0.0000 0.0000
	0.0005 0.0017 0.0000 0.0000 0.0008 0.0000 0.0000
	0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
	0.0001 0.0000 0.0000 0.0004 0.0001 0.0000 0.0000
	0.0002 0.0008 0.0000 0.0001 0.0010 0.0001 0.0000
	0.0000 0.0000 0.0000 0.0000 0.0001 0.0003 0.0000
	0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
	Sum cells:
	0.0077 =Portf Var



**Results should not be expected to match exactly due to round-offs in both inputs and outputs, and due to numerical iteration procedures used to find optima.**

OPT PORTF FINDER NO RISKLESS ASSET (Input Target Mean= 11.00%  
 7-Asset Portfolio Optimizer: (Check to make sure target mean i  
 Based on Variance Stats:

Asset #:	Inputs: Definition:	Outputs: Shares
1	Large Stocks	13.92%
2	Small Stocks	21.78%
3	IT Bonds	0.00%
4	LT Bonds	16.00%
5	NAREIT	21.96%
6	NCREIF	26.33%
7	Junk	0.00%

Portf Mean= 11.00%  
 Portf STD= 8.76%

Use Solver in Tools me  
 Target cell is portf Vari:  
 By varying portfolio wei  
 Subject to constraints:  
 Each weight (B39 thro  
 Portf Mean equal Targ  
 Sum of weights equal  
 (For less than 7 assets

	Expected Return = 9	Expected Return = 10	Expected Return = 11	Expected Return = 12	Expected Return = 13	Expected Return = 14	Max Sharpe Ratio
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Expected Return	9.00	10.00	11.00	12.00	13.00	14.00	9.28
Standard Deviation	5.43	6.95	8.77	10.84	13.16	16.32	5.79
Sharpe Ratio	0.70	0.69	0.66	0.63	0.59	0.54	0.70



**Results should not be expected to match exactly due to round-offs in both inputs and outputs, and due to numerical iteration procedures used to find optima.**

OPTIMAL PORTFOLIO FINDER WITH RISKI Input Target Return= 9.28%  
 7-Asset Risky Portfolio Optimizer: Input Riskfree Rate= 5.20%  
 Based on Variance Stats, assuming riskless asset exists (optimal portfolio is Sharpe Ratio = 0.7069)

Inputs:	Outputs:	
Asset #:	Definition:	Opt.Shares
0	Riskless	-1.40%
1	Large Stocks	7.46%
2	Small Stocks	12.34%
3	IT Bonds	30.32%
4	LT Bonds	0.00%
5	NAREIT	10.60%
6	NCREIF	40.67%
7	Junk	0.00%

Portf Mean= 9.28%  
 Portf STD= 5.77%  
 Portf Sharpe= 0.7069

Use Solver in Tools menu  
 Target cell is portf Sharpe Ratio  
 By varying risky portfolio weights  
 Subject to constraints:  
 Each weight (B39 through B45) >= 0  
 Sum of risky weights <= 1  
 (For less than 7 risky assets, set the weight of the riskless asset to 0)  
 (To achieve target mean return, set the target cell to the target return)

	Expected Return = 9	Expected Return = 10	Expected Return = 11	Expected Return = 12	Expected Return = 13	Expected Return = 14	Max Sharpe Ratio
S&P 500 TR	6.01	10.77	14.02	18.70	22.05	8.53	7.67
U.S. Small Stk TR	9.14	14.00	20.31	26.86	37.58	73.50	10.36
U.S. IT Gvt TR	31.69	16.03	0.00	0.00	0.00	0.00	29.34
U.S. LT Gvt TR	0.00	5.87	15.65	15.70	6.06	0.00	0.00
NAREIT-Equity TR	11.57	18.72	24.04	30.00	34.32	17.97	13.84
NCREIF Property	41.59	34.62	25.97	8.74	0.00	0.00	38.79
Expected Return	9.00	10.00	11.00	12.00	13.00	14.00	9.28
Standard Deviation	5.43	6.95	8.77	10.84	13.16	16.32	5.79
Sharpe Ratio	0.70	0.69	0.66	0.63	0.59	0.54	0.70