

**Chapter 19:**  
**Commercial Mortgage Economics & Investment**

## *Chapter Outline*

### **19.1 Basic Characteristics of Bonds**

**19.1.1 Duration & Maturity**

**19.1.2 Interest-rate risk, “habitat”**

**19.1.3 The yield curve**

**19.1.4 Maturity matching & mgt of fixed-income portfolios**

### **19.2 Commercial Mortgage Returns: Ex Ante & Ex Post**

**19.2.1 Mortgage yield: Components of ex ante returns**

**19.2.2 Performance: The historical record**

## 19.1 Basic characteristics of bonds

**Mortgages are like bonds:**

- *Contractually-fixed cash flows,*
- *Finite-lived assets.*

**- Trade in the debt market**

**- Supply of capital is from investors in debt market**

## ***19.1.1 Duration & Maturity***

**“Maturity”**

**↔ Pre-specified time in which the asset “*expires*” – provides no further cash flows or value.**

**Useful for some investment purposes (e.g., servicing a finite-maturity obligation).**

**Maturity → Sensitivity of asset present value to changes in market interest rates.**

## Example: Two Zero-coupon bonds...

<b>Bond A:</b> 6% coupon rate 5-yr maturity Current value = \$100 Fut. pmt = \$133.82 in 5 yrs $\$133.82 = \$100 * (1.06^5)$	<b>Bond B:</b> 6% coupon rate 10-yr maturity Current value = \$100 Fut. pmt = \$179.08 in 10 yrs $\$179.08 = \$100 * (1.06^{10})$
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Relevant mkt yield suddenly changes from 6% to 7%.

New present values are (*Remember: Future CFs are contractually fixed*) :

<b>Bond A:</b> $PV = \frac{\$133.82}{1.07^5} = \$95.41$	<b>Bond B:</b> $PV = \frac{\$179.08}{1.07^{10}} = \$91.04$
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**1 point (1%) change in market yield (from 6% to 7%) causes:**

- 5-yr bond to lose approx. 5% of its value,
- 10-yr bond to lose approx. 9% of its value.

## Duration

“Duration” is a more accurate way to measure sensitivity of bond values to changes in market interest rates.

Duration is related to maturity, but not exactly the same.

Duration is weighted average time until receipt of cash flows by the bond investor.

Weight is the pmt’s proportion of the bond PV.

For zero-coupon bonds: *Duration = Maturity*.

Otherwise, duration is less than maturity.

$$\text{Macaulay Duration} = \sum_{j=1}^N t_j \left[ \frac{CF_j / (1+YTM)^{t_j}}{\sum_{i=1}^N CF_i / (1+YTM)^{t_i}} \right] / 12$$

(Division by 12 is because duration is measured in *years*, and mortgages usually have monthly payments.)

## Example:

Exhibit 19-1:  
Computation of duration at par value for two interest-only, 6% coupon, annual-payment mortgages...

<b>Bond A:</b> <b>5-yr Maturity, 4.47-yr Duration:</b>					<b>Bond B:</b> <b>10-yr Maturity, 7.80-yr Duration:</b>			
Year = $t_j$	CF <sub>j</sub>	$\frac{PV(CF_j)=CF_j/(1.06)^{t_j}}$	$w = \frac{PV(CF_j)}{PV(Bond)}$	$w * t$	CF	$\frac{PV(CF_j)=CF_j/(1.06)^{t_j}}$	$w = \frac{PV(CF_j)}{PV(Bond)}$	$w*t$
1	\$6.00	\$5.66	0.0566	0.0566	\$6.00	\$5.66	0.0566	0.0566
2	\$6.00	\$5.34	0.0534	0.1068	\$6.00	\$5.34	0.0534	0.1068
3	\$6.00	\$5.04	0.0504	0.1511	\$6.00	\$5.04	0.0504	0.1511
4	\$6.00	\$4.75	0.0475	0.1901	\$6.00	\$4.75	0.0475	0.1901
5	\$106.00	\$79.21	0.7921	3.9605	\$6.00	\$4.48	0.0448	0.2242
6					\$6.00	\$4.23	0.0423	0.2538
7					\$6.00	\$3.99	0.0399	0.2793
8					\$6.00	\$3.76	0.0376	0.3012
9					\$6.00	\$3.55	0.0355	0.3196
10					\$106.00	\$59.19	0.5919	5.9190
Sum		\$100.00	1.0000	4.4651		\$100.00	1.0000	7.8017



$$\text{"Modified Duration"} = \frac{\text{Macaulay Duration}}{1 + YTM}$$

**Modified duration is a more accurate measure of sensitivity of bond value to interest rate changes. Using modified duration:**

$$\frac{\Delta D}{D} \approx -(\text{duration})(\Delta YTM)$$

where  $\Delta D$  is the change in bond (debt) value, and  $\Delta YTM$  is the change in market yield.

**Example:** Previous 6%, 10-yr, interest-only, annual-pmt mortgage, whose Macaulay duration was 7.8 yrs. Thus, its modified duration is  $7.8/1.06 = 7.36$  yrs. Suppose interest rates rise from 6% to 6.5% ( $\Delta YTM = +0.5\%$ ). Then prediction from above:

$$\frac{\Delta D}{D} \approx -(7.36)(0.5\%) = -3.68\%$$

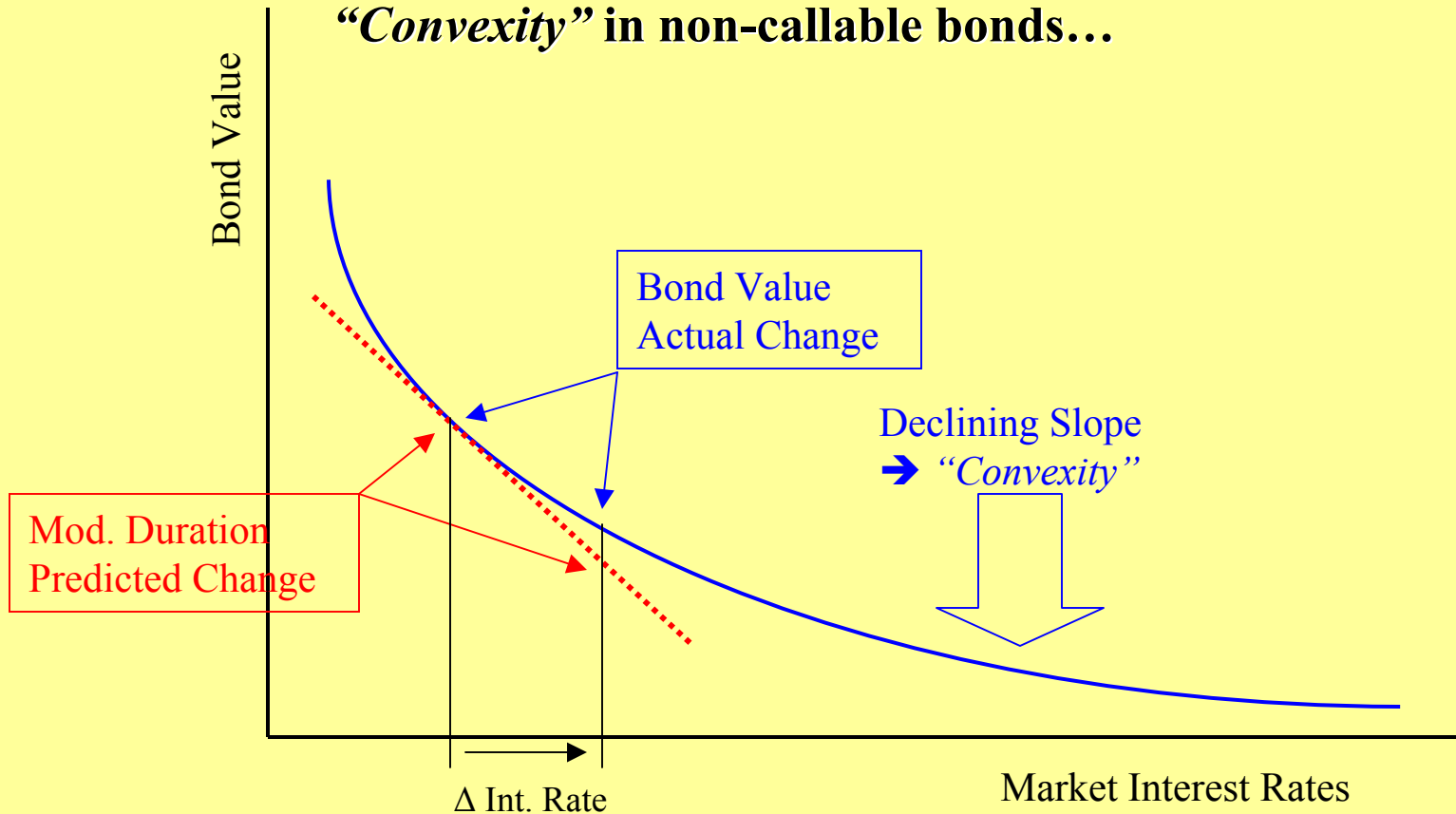
**Exact change in bond value is  $-3.59\%$ , calculated as:**

$$\begin{aligned} \$96.41 &= \sum_{j=1}^{10} \frac{\$6}{(1.065)^j} + \frac{\$100}{(1.065)^{10}} \\ \frac{\Delta D}{D} &= \frac{\$96.41 - \$100}{\$100} = -3.59\% \end{aligned}$$



**Aside:**

**“Convexity” in non-callable bonds...**

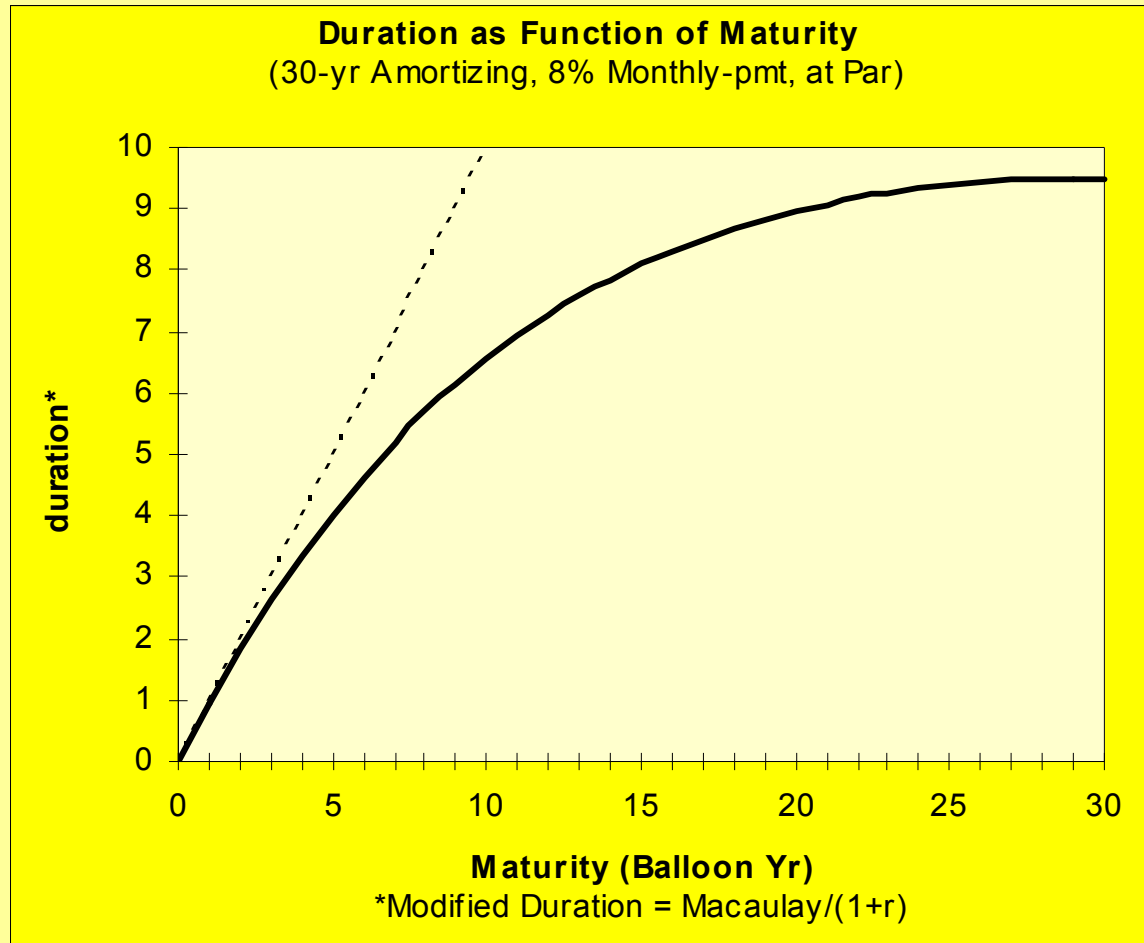


In previous example:

Why the exact value change was -3.59% when the modified duration was -3.68%.

## Exhibit 19-2:

Dashed line is for zero-coupon bond (Duration = Maturity).



## **Duration is determined by:**

- **Maturity:**
  - **Greater maturity → Greater duration;**
  - **Duration  $\leq$  Maturity.**
- **Coupon interest rate of the loan:**
  - **Greater coupon rate → Lower duration.**
- **Amortization rate of the loan:**
  - **Greater amort. (faster pay-down) → Lower duration.**
- **Current market yield (mkt interest rates):**
  - **Higher interest rates → Lower duration;**
  - **This is known as “Convexity”. For non-callable bonds:**
    - **Bond values fall less than duration when interest rates rise;**
    - **Bond values increase more than duration when interest rates fall.**

## ***Why is duration important ? . . .***

- **Determines the sensitivity of the bond value to changes in market interest rates.**
- **To the extent that changes in mkt int rates are *unpredictable*, such changes are a major source of *risk* for bond investors.**
- **Hence, *duration* → *risk* in the bond investment.**  
(Including “*maturity matching*” question for assets & liabilities.)
- **To the extent that changes in mkt int rates *are* predictable, investors speculate on them:**
- **Hence, *duration* → investment tactical *policy*.**

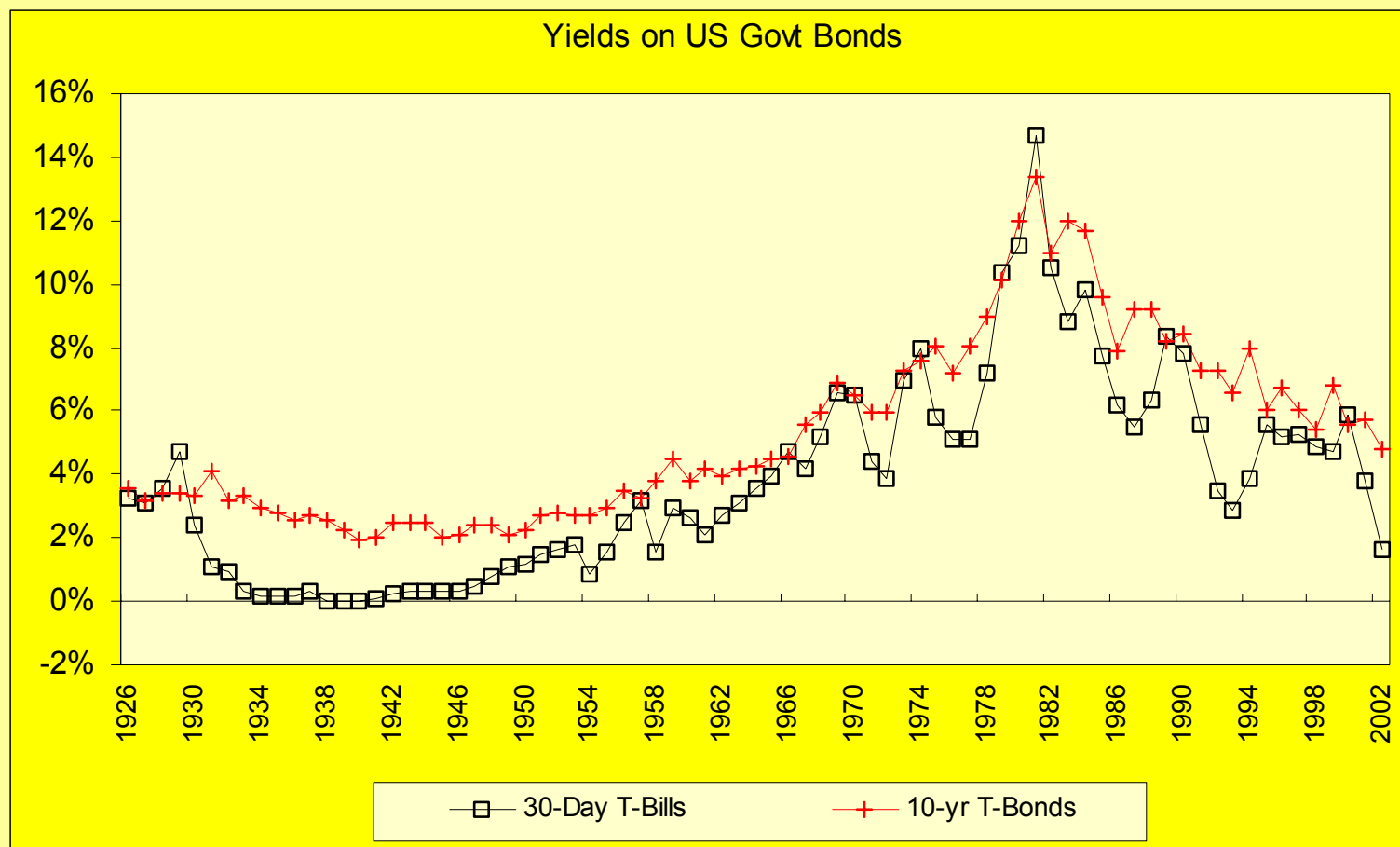
## **19.1.2 Interest Rate Risk & Preferred Habitat**

### **Interest Rate Risk...**

- Bonds are not riskless investments (even Govt bonds that cannot default).**
- They are subject to “interest rate risk”.**
- The value of a bond changes as market interest rates change.**
- More so, the greater the duration.**

# Mkt interest rates fluctuate over time, in ways that are difficult to predict...

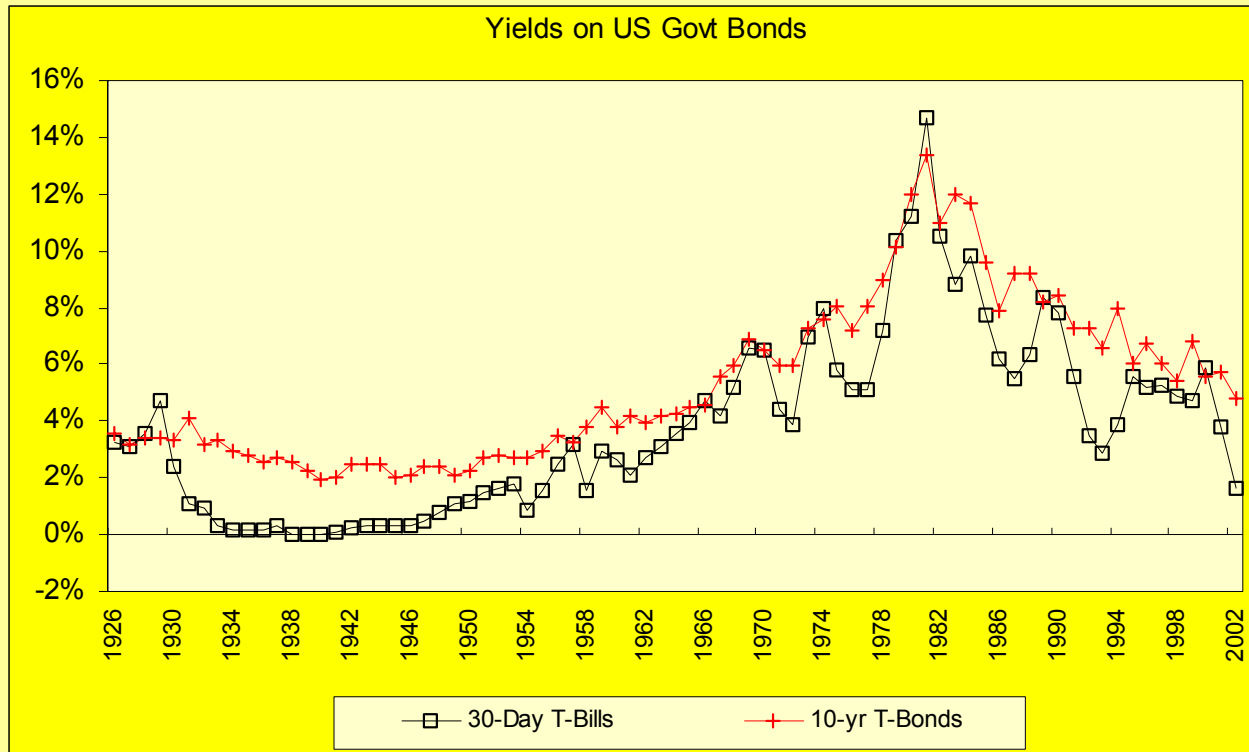
Exhibit 19-3: Yields on U.S. Government Bonds, 1926-2002:



**Causes volatility in bond values: more so in long-duration bonds than in short-duration bonds (also note ST rates are themselves more variable, & more subject to Govt control).**

**Both short-term & long-term interest rates change over time, and they often do not change together in the same way. But:**

- **Short-term yields usually (not always) below long-term yields.**
- **Difference betw ST vs LT yields is reflected in the “yield curve”.**
- **“Yield Curve” is level of bond yields (interest rates) as a function of the duration of the bonds.**
- **The shape, as well as the level, of the yield curve changes over time.**



## The Yield Curve

*Two major “theories” on the source and cause of the yield curve...*

**Theory 1:** *“Expectations Theory” (aka: Why the yield curve changes shape):*

**Mkt expectations about what future short-run interest rates will be. Long-term yields must equal average of expected short-run yields over the horizon of the long-term bond maturity. Otherwise, arbitrage opportunities would be available (Buy/Sell across maturities). *Relates to:***

- a) *Inflation expectations,*
- b) *Expectations about capital supply & demand (macro-economy),*
- c) *Expectations theory explains why the slope of the yield curve is positively correlated with future economic changes: Increasing LT-ST yield spread → Likely improving GDP growth; Decreasing LT-ST yield spread → Likely reduced GDP growth;*
- d) *But expectations theory alone cannot explain tendency of yield curve to usually be positively sloped (higher yields for greater maturity bonds)...*



## The Yield Curve

*Two major “theories” on the source and cause of the yield curve...*

**Theory 2: Liquidity Preference Theory (aka: Why the yield curve is usually upward-sloping):**

**Interest rate risk & investors’ preferred duration “habitat”:**

a) **Longer duration bonds are more volatile due to interest rate fluctuations, hence require higher expected return for investors (due to risk aversion);**

b) **Liquidity preference:**

i) **Bond investors (lenders) often have a preference for shorter-maturity bonds (get their cash par value back quicker and with less exposure to interest rate risk), hence require higher return to invest in long-term bonds;**

ii) **Borrowers often have a preference for longer-maturity loans (gives them greater “liquidity”, less need to come up with cash in the short run to pay off maturing loans, allows matching of investments in long-duration assets with financing by long-duration debt), hence borrowers (bond issuers) willing to pay higher interest on longer-term loans (bonds).**

## Example of “preferred habitat” . . .

- Sue has an obligation to pay \$1,000,000 in 5 years.
- She wants to invest now to be able to cover that obligation when it comes due.

*Aside: What kind of financial institutions face situations like this? . . .*

### **Sue faces three choices:**

1) Buy a zero-coupon bond that matures in 5 years.

e.g., if int.rates are 6%, Sue could invest \$748,258 today and obtain \$1,000,000 in 5 yrs with certainty:  $\$748,258 = \$1,000,000 / 1.06^5$ .

2) Buy shorter-term bonds, then reinvest when they mature.

- But then she would be subject to “reinvestment risk”:
- The mkt interest rates at which she will have to reinvest might be lower when the short-term bonds mature.

3) Buy longer-term bonds (>5yrs maturity), and sell them in 5 yrs.

- But then she would be subject to “interest rate risk”:
- The mkt interest rates that determine the price at which she can sell her bond may increase between now and 5 yrs, causing her bond to be worth less at that time than she might expect given today’s interest rates.

Alternative (1) has the least risk. Thus, Sue’s “preferred habitat” in the bond market is to invest in bonds with a duration of 5 years.

**Bond investors (lenders) and bond issuers (borrowers) have “*preferred habitats*”.**

**Usually, more *investors* tend to have *shorter* preferred habitats, and more *borrowers* tend to have *longer-term* preferred habitats.**

**Thus, there is:**

- **More supply of capital (more demand to invest in bonds) at the short end of the maturity range;**
- **More demand for capital (more borrowers) at the long end of the maturity range.**

**Thus,**

**At the short end of the maturity range, bond prices are driven up by investors (yields driven down).**

**At the long end of the maturity range, bond yields are bid up by borrowers (prices, or bond issue proceeds, driven down, by competition among borrowers to sell bonds).**

***Result is positively-sloped yield curve.***

**But sometimes not,**

**due to expectations about future short-term interest rates, due to:**

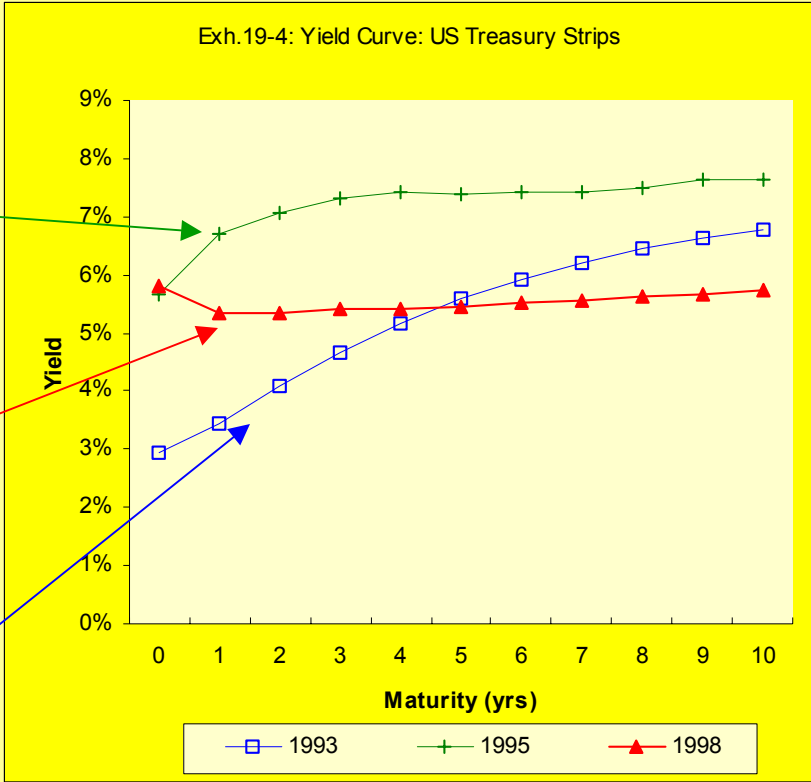
- **Federal Reserve Board actions,**
- **Inflation expectations,**
- **Economic expectations.**

**Therefore, the yield curve changes over time as economic and capital market conditions change . . .**

1995: Stable economy

1998: Boom but worried about recession

1993: Recession looking at recovery



Average (“typical”) yield curve is “*slightly upward sloping*” (100-200 bps) because:

- **Interest Rate Risk:**
  - **Greater volatility in LT bond values and periodic returns (simple HPRs) than in ST bond values and returns:**
  - **→ LT bonds require greater ex ante risk premium ( $E[RP]$ ).**
- **“Preferred Habitat”:**
  - **More borrowers would rather have LT debt,**
  - **More lenders would rather make ST loans:**
  - **→ Equilibrium requires higher interest rates for LT debt.**

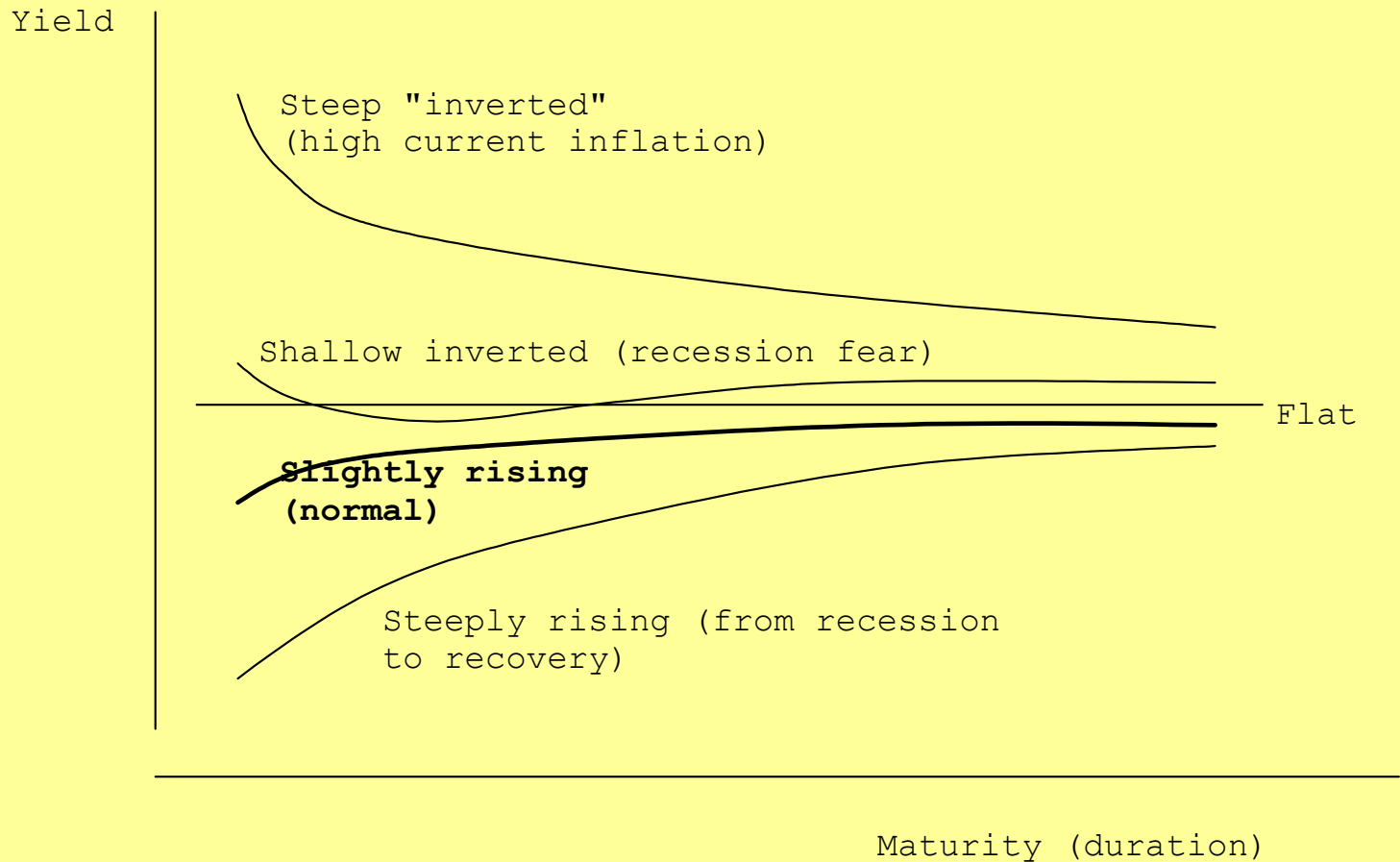
**This is the main fundamental reason why ARMs tend to have slightly lower lifetime average interest rates than otherwise similar FRMs, yet not every borrower wants an ARM. Compared to similar FRM:**

- ***ARM borrower takes on more interest rate risk,***
- ***ARM lender takes on less interest rate risk.***

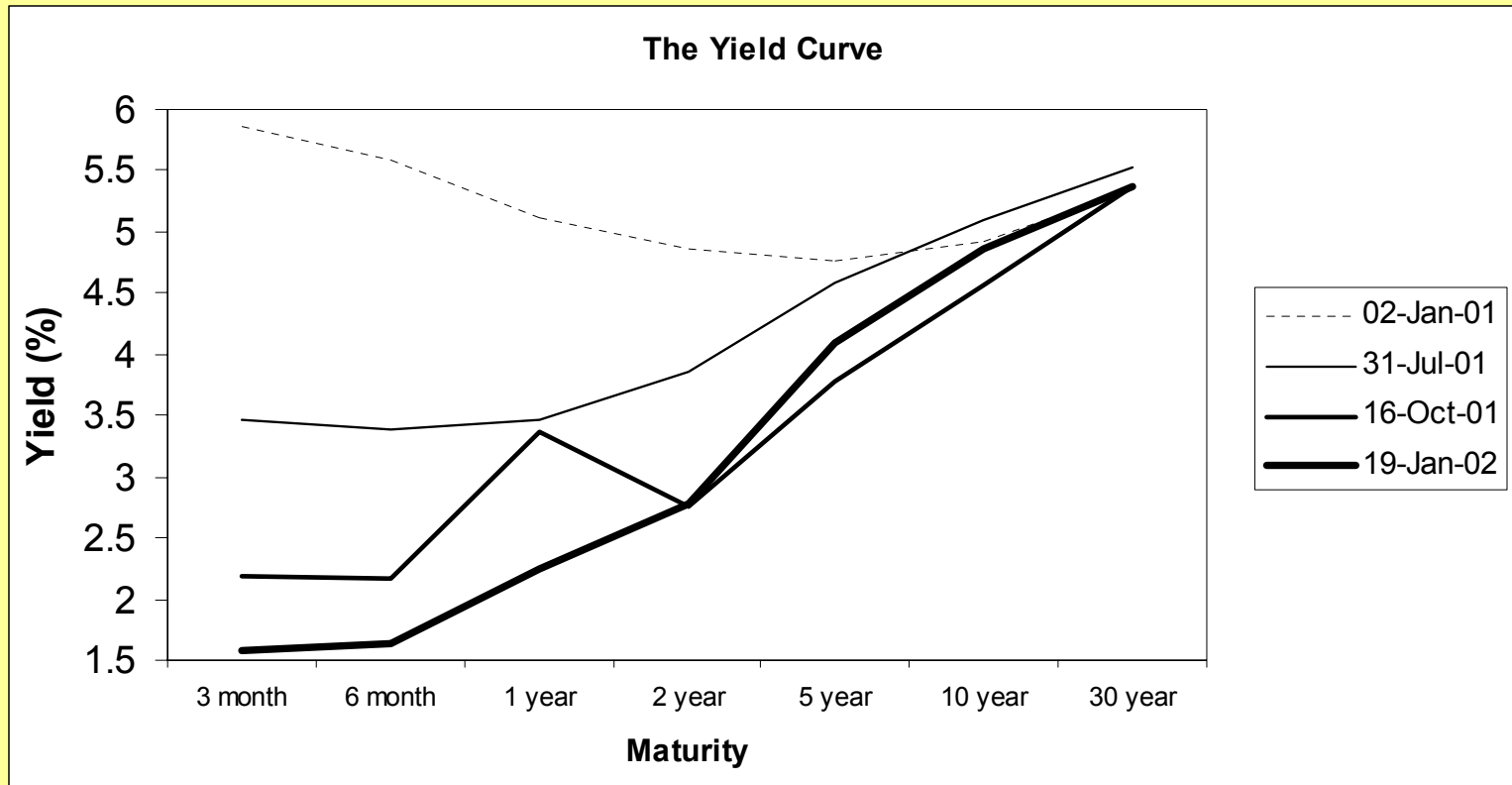
# Typical yield curve shapes and the economic conditions they are associated with:

Exhibit 19-5:

**Typical yield curve shapes . . .**



**Here is a more recent & dramatic example of yield curve changes:**



**Check out “*The Living Yield Curve*” at:**

**<http://www.smartmoney.com/onebond/index.cfm?story=yieldcurve>**

## *Recall from Chapter 17 . . .*

When the yield curve is ***steeply rising*** (e.g., 200-400 bps from ST to LT yields), ARM rates may appear *particularly favorable* (for borrowers) relative to FRM rates.

But what do borrowers need to watch out for during such times? . . .

For a long-term borrower, the FRM-ARM differential may be somewhat misleading (ex ante) during such times:

The steeply rising yield curve reflects the “***Expectations Theory***” of the determination of the yield curve:

- LT yields reflect current *expectations* about *future short-term yields*.

Thus, ARM borrowers in such circumstances face greater than average risk that their rates will go up in the future.





**Using the *Expectations Theory* (purely) to derive bond mkt predictions of future short-term interest rates...**

Maturity	02-Jan-01	31-Jul-01	16-Oct-01	19-Jan-02
3 month	5.87	3.46	2.18	1.58
6 month	5.58	3.39	2.17	1.63
1 year	5.11	3.46	3.36	2.25
2 year	4.87	3.87	2.75	2.78
5 year	4.76	4.59	3.77	4.10
10 year	4.92	5.10	4.57	4.86
30 year	5.35	5.52	5.37	5.37

Source: Data taken from Smart Money web site at [www.smartmoney.com/bonds/](http://www.smartmoney.com/bonds/)

*Actual* 1-yr yield in Jan.01 = 5.11%.

*Expectations Theory* yield curve prediction as of Jan.01 of future 1-yr yield in Jan.02. . .

$$(1.0511)(1 + r_2) = (1.0487)^2$$

$$r_2 = \frac{(1.0487)^2}{(1.0511)} - 1 = \frac{1.09977}{1.0511} - 1 = 1.0463 - 1 = 4.63\%$$

*Actual* 1-yr yield in Jan.02 = 2.25%.

Jan.01 yield curve predicted a fall in short-term interest rates (from 5.11% to 4.63% by Jan.02), but not as steep a fall as what actually occurred (to 2.25%). The recession that was only “*feared*” as of Jan.01 had fully materialized by Jan.02. ← Also, *preferred habitat* caused the earlier 2-yr rate to overstate the mkt’s actual predicted future 1-yr rate one year in the future.

## 19.1.4: Maturity Matching & the Management of Fixed-Income Portfolios...

### The “*Maturity Matching*” Problem (*Simplified National Bank*):

Exhibit 19-6a: Simplified National Bank Balance Sheet as of 6/30/2000			
<b>Assets:</b>		<b>Liabilities &amp; Equity:</b>	
Mortgages	\$100,000,000	Deposits	\$90,000,000
		Equity	\$10,000,000
Total Assets	\$100,000,000	Total L&E	\$100,000,000

*Simplified's* assets are all **10-yr**, zero-coupon mortgages @ 6% fixed rate.

***What is the duration of *Simplified's* assets?... Answer: 10 years.***

*Simplified's* deposits are all **1-yr**, CDs @ 5% fixed rate.

***What is the duration of *Simplified's* liabilities?... Answer: 1 year.***

Now suppose the bond market suddenly changes and market interest rates all across the yield curve increase by one point:

- LT ylds  $\Delta+1\%$ , from 6%  $\rightarrow$  7%.
- ST ylds  $\Delta+1\%$ , from 5%  $\rightarrow$  6%.

***What will happen to *Simplified's* balance sheet?...***

## Mkt val (PV) of assets:

Changes from:  $\$179,084,770 / 1.06^{10} = \$100,000,000$ ;

to:  $\$179,084,770 / 1.07^{10} = \$91,037,616$ . ← 9% fall

## Mkt val (PV) of liabilities:

Changes from:  $\$94,500,000 / 1.05 = \$90,000,000$ ;

to:  $\$94,500,000 / 1.06 = \$89,150,943$ . ← 1% fall

## Balance Sheet changes from:

Exhibit 19-6a: Simplified National Bank Balance Sheet as of 6/30/2000			
Assets:		Liabilities & Equity:	
Mortgages	\$100,000,000	Deposits	\$90,000,000
		Equity	\$10,000,000
Total Assets	\$100,000,000	Total L&E	\$100,000,000

To:

Exhibit 19-6b: Simplified National Bank Balance Sheet as of 7/01/2000			
Assets:		Liabilities & Equity:	
Mortgages	\$91,037,616	Deposits	\$89,150,943
		Equity	\$1,886,673
Total Assets	\$91,037,616	Total L&E	\$91,037,616

**Assets** must always equal **Liabilities + Owners' Equity**. Thus:

Equity falls from \$10,000,000 to \$1,886,673, **an 80% fall!**

☰ Previous is example of problem caused by a classical ***“Maturity Gap”***...

### *Maturity Gap*

↔ Investor's assets have wtd avg duration > Investor's liabilities.

**Maturity gap problem typically exaggerated in financial institutions due to use of *leverage* (huge liabilities, small equity):**

- **Financial institutions are usually highly levered.**

(That's how they make money. It's *“the nature of the business.”* \*)

**Interaction of Maturity Gap and Leverage:**

$$\text{Combining defn of "Duration": } dur = \frac{-\Delta V/V}{\Delta YTM}$$

with accounting identity:  $A = L + E$ , we obtain:

$$dur_E = \frac{-\Delta E/E}{\Delta YTM} = \frac{[(dur_A)A - (dur_L)L]}{E}$$

$$= (dur_A - dur_L) \frac{A}{E} + dur_L$$

$$\approx (dur_A - dur_L) \frac{A}{E}, \text{ if } dur_L \text{ is small.}$$

$$(dur_A - dur_L) \equiv \text{"Maturity Gap"}, \frac{A}{E} \equiv \text{"Levg.Ratio"}$$

- **Leverage ( $A/E$ ) must be great due to narrow spreads betw borrowing & lending rates.**
- **For “depository institutions” (e.g., banks), the duration of liabilities ( $dur_L$ ) is usually small (e.g., demand deposits, CDs).**
- **Equity duration (int.rate sensitivity)  $\approx$  Maturity Gap X Leverage Ratio [e.g., Simplified:  $(10-1) \times 10 = 90 \rightarrow \Delta 1\% \text{ int.} \approx \Delta 90\% \text{ equity}$ ].**
- **→ Major problem if assets are long duration (large  $dur_A$ ).**

Thus, banks (depository institutions) seek *short term assets* (e.g., *construction loans, consumer debt, S.T. business loans, lines of credit, comm.paper, etc.*), or:

- Assets that behave like S.T. assets regarding interest-rate risk (e.g., *ARMs, long-term loans with adjustable rates such that  $\Delta PV/\Delta YTM$  is small, effectively short duration, even though loan is long maturity*).
- To sell off their long-term assets and make money from *fee services* (e.g., *loan origination &/or servicing, investment mgt, consumer information mgt*).

On the other hand, financial institutions with long duration liabilities (life insurance companies, pension funds) seek *long term assets* (e.g., mortgages, bonds).

*Why? (i.e., what is wrong with a negative maturity gap?)*

**Answer:**

*Negative maturity gap → Negative profits! (Equity return < 0).*

*Why?...*

**Answer:**

*Yield curve is usually upward-sloping (recall int.rate risk & prefrd.habitat).*

# Fixed-Income Investment: *The Big Picture*:

Why do investors invest in bonds (& mortgages)? . . .

Recall from Chapter 7 that investors are *heterogeneous*.

There are many different types of investors, with different objectives and concerns and constraints, different time horizons, and different risk preferences.

**Major bond investors in the U.S. include:**

- Life insurance companies
- Pension funds
- Endowment funds
- Mutual funds
- Wealthy individuals
- Foreign investors

**Broadly, bonds are useful investments for three major purposes:**

1. Conservative element in the portfolio.
2. Diversification effect (with stocks).
3. Matching asset/liability maturity.

**Major concerns include:**

- Interest rate risk
- Default risk
- Inflation vulnerability
- Low average returns

## Fixed-Income Investment Strategies:

Two major types of investment strategies for bonds:

1. Trading-oriented strategies;
2. Immunization-oriented strategies.

### Trading-oriented Strategies:

- Involve regular buying & selling of bonds *prior to maturity*;
- Are therefore *exposed to interest rate risk*.
- Include “*active*” and “*passive*” approaches:
  - Active strategies seek to “*beat the market*” (earn superior risk-adjusted returns) using research & expertise to *buy low & sell high*, by:
    - Obtaining superior interest rate forecasts;
    - Finding bonds that are *mispriced*.
  - Passive approach seeks to *minimize investmt mgt expenses* by replicating a target bond index:
    - Still requires some trading (e.g., preserve duration).
- May or may not employ leverage (depending on risk tolerance).
- Oriented toward: (i) *Return maximization*; (ii) *Diversification*.

## Fixed-Income Investment Strategies:

Two major types of investment strategies for bonds:

1. Trading-oriented strategies;
2. Immunization-oriented strategies.

### Immunization-oriented Strategies:

- Involve buying and *holding bonds to maturity*;
- Therefore *avoiding interest rate risk*.
- Include “*cash flow matching*” and “*duration matching*” approaches:
  - Cash flow matching technique seeks to match specific future cash outflow obligations to specific bond maturities, to completely “immunize” the investor from interest rate risk. (Generally not possible with complex liability structure.)
  - Duration matching technique matches (as close as possible) the weighted average duration of the investor’s assets and liabilities, thereby greatly reducing (but not completely eliminating) int.rate risk.\*
- Returns are low; Strategy oriented toward *risk minimization*.
- Not as popular as trading-oriented strategies.

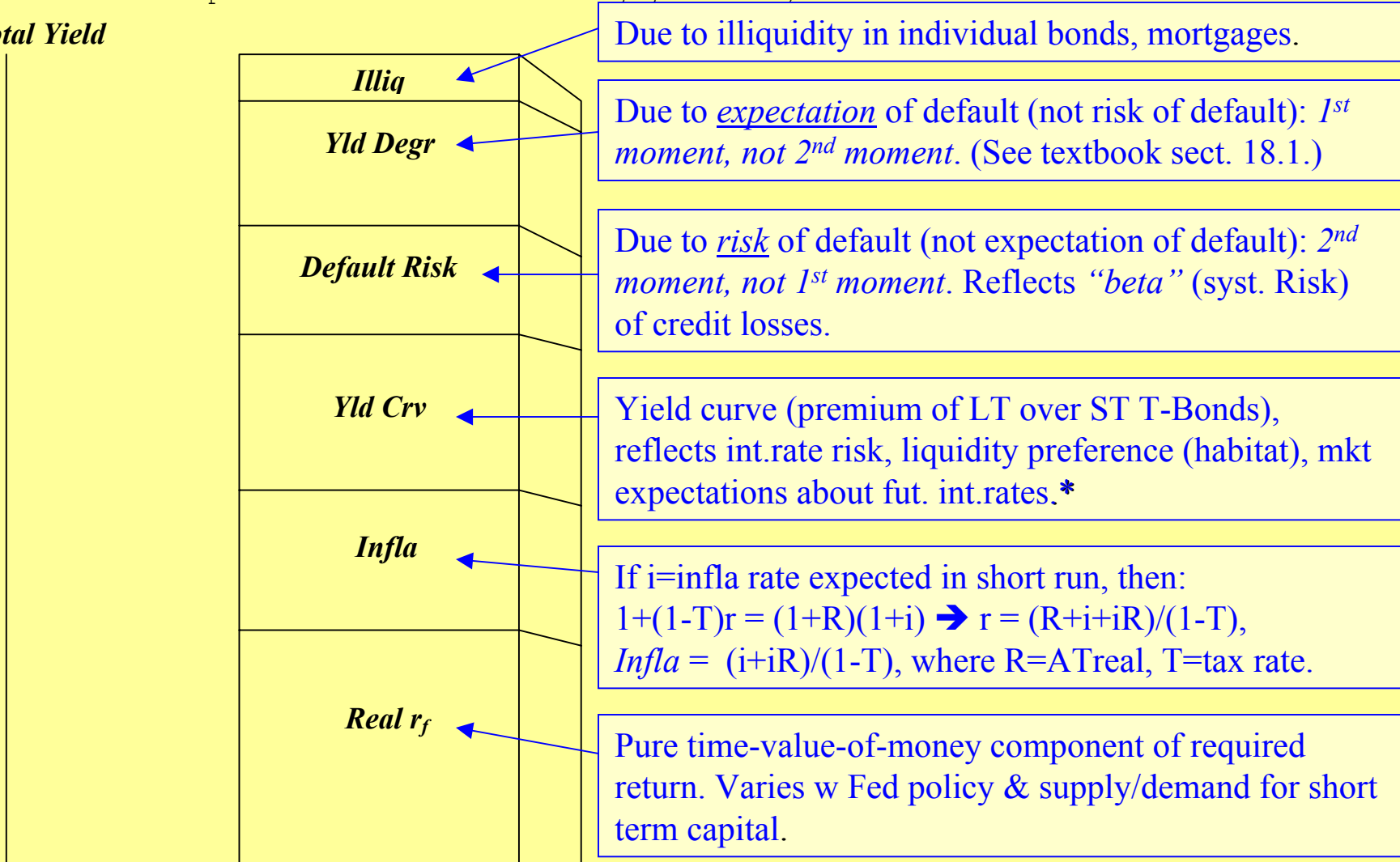




# 19.2.1 Mortgage Yield: The Components of Ex Ante Returns...

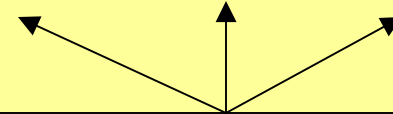
Exhibit 19-7: Components of commercial mortgage total yield:

**Total Yield**



**Typical magnitudes of components of the “*yield stack*” :**

<b>Exh.19-8: Approx. Comm.Mortg. Stated Yld Components at 3 Historical Periods</b>	<b>2003</b>	<b>Late 1990s</b>	<b>1993</b>
<b>Real Riskfree Rate:</b>	<b>-0.50%</b>	<b>2.50%</b>	<b>0.00%</b>
<b>Inflation Premium:</b>	<b>2.50%</b>	<b>2.50%</b>	<b>3.50%</b>
<b>Yield Curve:</b>	<b>2.50%</b>	<b>1.00%</b>	<b>3.50%</b>
<b>Default Risk:</b>	<b>0.75%</b>	<b>0.75%</b>	<b>1.00%</b>
<b>Yield Degradation:</b>	<b>0.75%</b>	<b>0.75%</b>	<b>0.50%</b>
<b>Illiquidity:</b>	<b>0.25%</b>	<b>0.25%</b>	<b>0.50%</b>
<b>Total stated yield:</b>	<b>6.25%</b>	<b>7.75%</b>	<b>9.00%</b>
<b><i>Implied E[RP] = E[r] - r<sub>f</sub> = Yld-YDEGR-r<sub>f</sub></i></b>	<b>3.50%</b>	<b>2.00%</b>	<b>5.00%</b>



**What do you think underlies these shifts in the *expected return risk premium*?**

## 19.2.2 Ex Post Performance: The Historical Record

Recall that the yields we've just been discussing (in 19.2.1) are *ex ante* stated yields (contractual YTM's).

As *market rates*, they reflect the before-tax *expectations* of marginal participants (supply & demand) in the mortgage market branch of the capital markets.

They will also equal ex post realized multi-period yields (IRR's), but only:

- For investors *holding the loans to maturity*.
- In the *absence of default* by the borrower\*.

Otherwise, interest rate movements, or defaults, will cause the realized ex post return to differ from the contractual rate and from the ex ante expectation.

**Recall also that mortgages (and bonds) are used in various ways by investors:**

**Many investors adopt a *trading-oriented* investment strategy,**

**Seeking higher returns through active investment, or to obtain *diversification benefits* in a portfolio (as mortgages are not perfectly correlated with other asset classes).**

**For such investors, *periodic returns* (aka “period-by-period returns”, as defined in Chapter 9), are more relevant than multi-period yields-to-maturity (YTM) measured over the lifetime of the loan.**

**Recall that periodic returns are based on simple “*holding period returns*” (HPRs) in each consecutive (short) period of time (typically annual frequency or shorter periods, quarterly, monthly, daily).**

**HPRs are computed as if the asset were bought and sold at the beginning and end of each period, requiring “*marking to market*” at frequent intervals.**

**HPRs are particularly relevant for *portfolios* (or indexes) composed of many individual mortgages (or bonds).**

## **Computing HPRs for mortgages . . .**

**Recall that the simple periodic total return includes any net cash generated by the investment during the period, plus any change in asset value, all divided by the asset value as of the beginning of the period. For mortgages, this is defined as:**

$$r_t = \frac{CF_t + (V_t - V_{t-1})}{V_{t-1}} = \frac{(PMT_t + REC_t) + (D_t - D_{t-1})}{D_{t-1}}$$

**Where:**

**$PMT_t$  = Regular debt service during period t,**

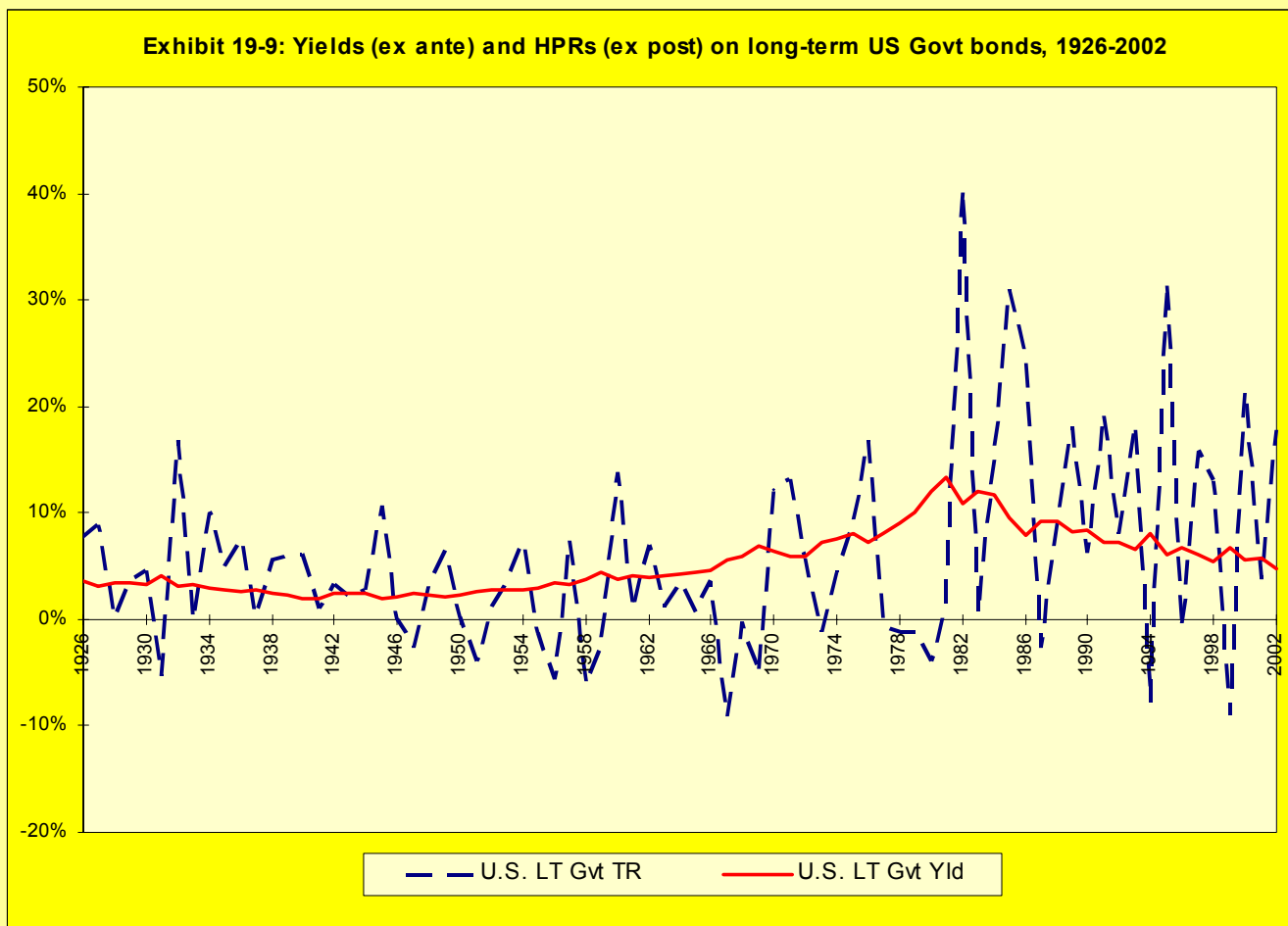
**$REC_t$  = Value of any prepayments, or the net recovery in any foreclosures during period t,**

**$D_t$  = Market value of any remaining debt as of the end of period t.**

**In a mortgage index, this formula is aggregated across a large number of individual loans composing the index portfolio.**

**The most widely-used index of commercial mortgage whole loans is the *Giliberto-Levy Commercial Mortgage Price Index (GLCMPI\*)*.**

- In general, realized ex post periodic returns are much more volatile than ex ante return expectations.
- This is no more true in the bond market than in other branches of the capital markets.
- But in the bond market we can more easily empirically observe the difference, because the ex ante return expectations are objectively observable in the market YTM's at which the bonds trade.





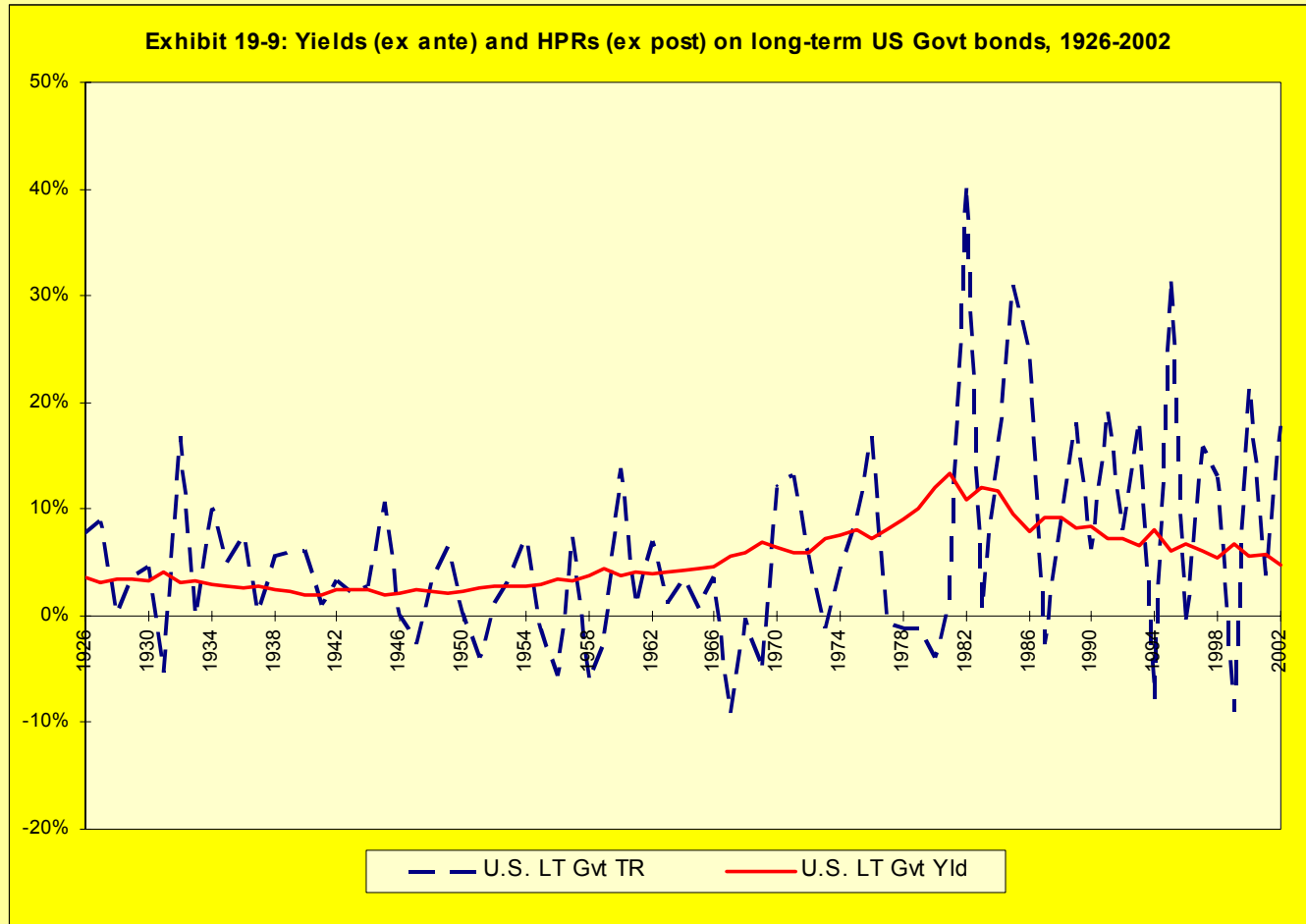
• What is the relationship between the *average ex post* realized return and the *average ex ante* yield? (*Why?*)

GMean Ann 1926-2002: T-Bond Yld = 5.27%  $\approx$  T-Bond HPR = 5.45%. “Rational Expectations”.

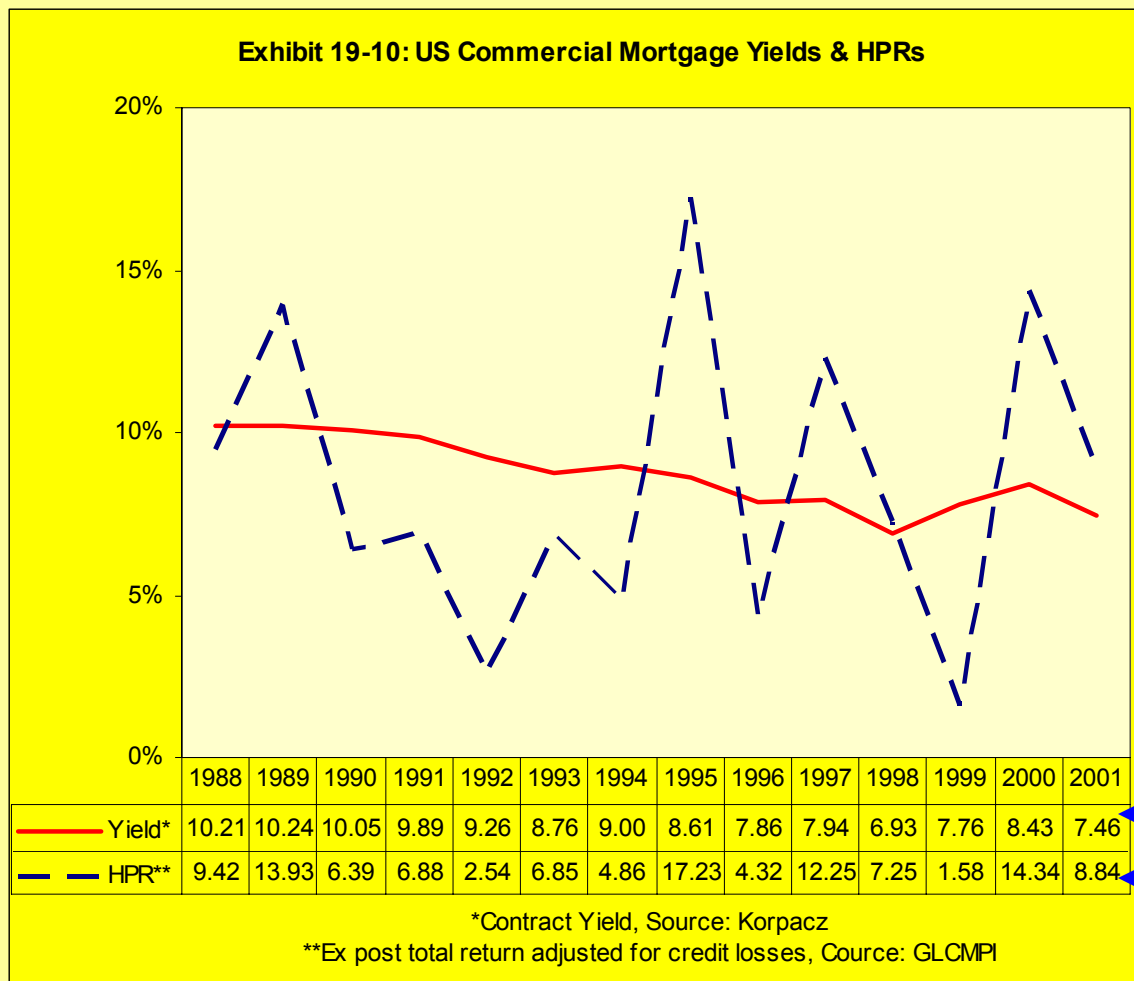
• What is the one major source of the differences seen below between the *ex ante* yields and the *ex post* realized periodic returns for long-term Government bonds? ← Int. Rate Risk

• Is there another potential source of difference for corporate or municipal bonds, and for mortgages?

Default Risk  
(Credit Losses)



 The picture is similar for commercial mortgages . . .



**Credit Losses take a toll in Commercial Mortgages:**

Avg Contract: 8.74%  
 Avg HPR net losses: 8.33% < 8.74% despite downward int. rate trend.

*Why has the general ex ante trend been downward?...*

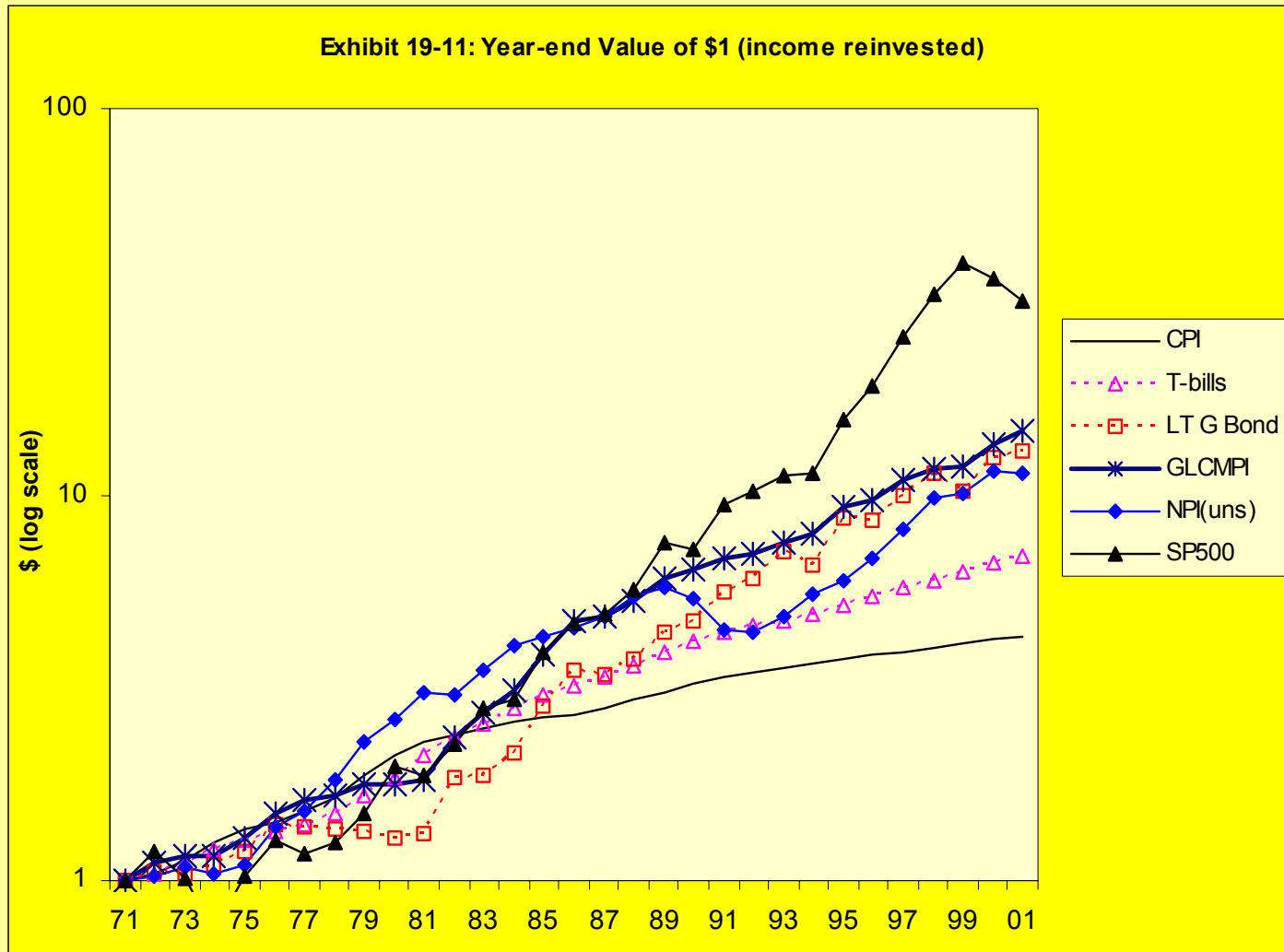
*Might this have an implication for the relation between average realized ex post periodic returns and prior ex ante expectations?...*

*Can this trend continue forever?...*

GMean Ann 1980-2002:  
 T-Bond Yld = 8.46% < T-Bond HPR = 11.18%.



# Historical cumulative returns 1971-2001: Five Asset Classes: *T-Bills, LT G Bonds, Comm.Mortgs, Property, Lg Cap Stocks:*





## 30-year Annual Total Return Historical Statistics

Exhibit 19-12: Historical annual periodic total return statistics, 1972-2001:						
	CPI	T-bills	LT G Bond	GLCMPI**	NCREIF***	SP500
Geom. Mean	4.98%	6.65%	8.91%	9.37%	8.46%	12.24%
Arith. Mean	5.03%	6.69%	9.55%	9.60%	8.89%	13.58%
Volatility	NA	2.67%	12.33%	7.35%	9.68%	17.14%
Ex Post Risk Prem (arith)	NA	NA	2.87%	2.92%	2.21%	6.89%
Sharpe*	NA	NA	0.23	0.40	0.23	0.40
Correlations:						
CPI	100%	61%	-41%	-43%	18%	-28%
T-bills	61%	100%	3%	7%	19%	-3%
LT G Bond	-41%	3%	100%	82%	-20%	37%
GLCMPI	-43%	7%	82%	100%	-1%	40%
NPI(uns)	18%	19%	-20%	-1%	100%	14%
SP500	-28%	-3%	37%	40%	14%	100%

\*The Sharpe Ratio is equal to: (Mean - Tbill Mean)/Volatility. It is a crude measure of return adjusted for risk (see Chapters 21 and 27).  
 \*\*Net of credit losses.  
 \*\*\*Unsmoothed (1-Step).  
 Sources: Ibbotson Assoc., John B.Levy & Co., NCREIF.

***Why does it make sense for average Commercial Mortgage returns to have exceeded average Govt. Bond returns?***      **Ans: Default Risk Premium**

***Why do you suppose the G-Bonds were more volatile than the Mortgages?***

**Ans: Duration Differences**

***What do you make of the relationship between the average mortgage return and the average return to the underlying property that backs the mortgages? Could this be an equilibrium relationship in the ex ante returns?***



## Is there “positive leverage” in borrowing to finance R.E. investment?...

Exhibit 19-12: Historical annual periodic total return statistics, 1972-2001:						
	CPI	T-bills	LT G Bond	GLCMPI**	NCREIF***	SP500
Geom. Mean	4.98%	6.65%	8.91%	9.37%	8.46%	12.24%
Arith. Mean	5.03%	6.69%	9.55%	9.60%	8.89%	13.58%
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\*The Sharpe Ratio is equal to: (Mean - Tbill Mean)/Volatility. It is a crude measure of return adjusted for risk (see Chapters 21 and 27).  
 \*\*Net of credit losses.  
 \*\*\*Unsmoothed (1-Step).  
 Sources: Ibbotson Assoc., John B.Levy & Co., NCREIF.

*Recall Chapters 21 & 22.*

- The risk that matters in asset prices and expected returns is the risk that matters to the *marginal investor* in the capital market.
- The marginal investors in the bond (mortg) mkt are probably not the conservative *immunization-oriented* investors who buy & hold, but the more aggressive *trading-oriented* investors who are subject to the mortgage ex post *periodic returns (HPRs)* whose statistics are summarized here.
- For such investors, what portfolio diversification and inflation-hedging considerations are indicated in the correlation statistics that could imply that such investors would view long-term fixed-rate mortgages as being *more risky* (i.e., requiring of a larger ex ante return risk premium) than underlying property (especially stabilized “institutional property”)?

☰ If there is no positive leverage in long-term fixed-rate mortgages financing institutional quality commercial property, then what does this imply about the effect of such leverage on the expected return of the leveraged equity, relative to that of the underlying (unlevered) property?

Now consider the effect of such leverage on the *volatility* of the levered equity...

The relationship between levered equity volatility, underlying property volatility, and the debt return volatility is given by the following equation:

$$S_E = \sqrt{\left((LR)S_P\right)^2 + \left((LR - 1)S_D\right)^2 - 2C_{PD} \left((LR)(LR - 1)S_P S_D\right)}$$

Where  $S_P$  and  $S_D$  are the volatilities of the underlying property and debt (respectively), and the correlation between these two is  $C_{PD}$ .

Thus, the volatility of real estate levered equity actually increases *more* than that of the typical corporate stock as a result of borrowing using long-term fixed rate debt, *e.g.:*

**+37% bond, stock correlation; -1% mortgage, property correlation.**

**How can borrowing *increase volatility* while actually *decreasing the risk* of the equity?...**

## *A Nagging Data Problem in Commercial Mortgage HPR Indices...*

Recall from Chapter 18 (Sect. 18.1) that yield degradation (and therefore realized ex post returns) in debt investments are a function not only of the *incidence* of default, but of the *loss severity* in the event of default:

$$YDEGR_t = YTM - YLD|DEF_t = YTM - IRR(\text{loss severity}_t)|DEF_t$$

For example, with 30% loss severity the realized IRR of a 3-yr, 10% loan that defaults in yr.3 is:

$$0 = -\$100 + \frac{\$10}{1 + (-0.0112)} + \frac{\$10}{(1 + (-0.0112))^2} + \frac{\$77}{(1 + (-0.0112))^3}$$

Realized IRR = -1.12%.

The same situation with 20% loss severity provides:

$$0 = -\$100 + \frac{\$10}{1 + (0.0287)} + \frac{\$10}{(1 + (0.0287))^2} + \frac{\$88}{(1 + (0.0287))^3}$$

Realized IRR = +2.87%.

Mortgage periodic return indices like the GLCMPI are calculated based on *estimates* of the loss severities experienced by the representative mortgages. There is a lack of good solid data on the magnitude of the actual loss severities (proprietary info). The GLCMPI has *assumed* loss severities around 30%. If actual loss severities were greater than the assumption in the GLCMPI, then the GLCMPI will *overstate* the actually achieved ex post mortgage returns.

## Summarizing the question of “positive leverage”:

- Ex post historical periodic returns statistics suggest there may be no positive leverage,
- That risk as the capital market cares about it may be as great or greater in long-term fixed-rate mortgages as in underlying (unlevered) property equity, at least for stabilized “institutional quality” commercial property. *But:*
- These *ex post* returns statistics may skew the picture due to:
  - The particular historical period covered includes a long secular reduction in interest rates (declining inflation), which *may* have caused average realized ex post returns to have substantially exceeded the corresponding ex ante expectations;
  - Data problems in indices of commercial mortgage HPRs (notably, difficulty accurately quantifying the *conditional loss severity* portion of the credit losses computation) *may* cause those indices to overstate actual average achieved mortgage returns.
- Most investors seem to generally believe (subjectively) that positive leverage exists *ex ante* in the total expected returns, e.g.:

$$E[RP]_{\text{mortg}} \approx 150\text{bp}-300\text{bp}; \quad E[RP]_{\text{prop}} \approx 300\text{bp}-400\text{bp}.$$