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HOW PEOPLE EXECUTE HANDWRITING

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ABSTRACT

Handwriting is shown to be composed mainly of cup-shaped strokes lasting approximately 200 msec. The strokes are based on a hexagonal pattern, with quantized slopes and lengths. Each side of the hexagon is produced by a 40 msec acceleration burst. Smooth writing is produced by merging and rounding these bursts.

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1. Why Study Handwriting

Handwriting is an accessible physical skill for studying the human motor system. The muscles and joints involved in handwriting are localized and are relatively few in number. They can be separated into two orthogonal sets: one that executes the horizontal movement, the other the vertical movement.

Writing tablets hooked up to a computer make it practical to record displacement, velocity, and acceleration profiles for handwritten words. The acceleration profiles directly indicate muscular force, and allow one to make conjectures about the patterns of muscular activity that produce the written strokes. By varying the conditions under which handwriting takes place, such as varying the load or the writing surface, one can also observe how the muscular patterns adapt to new conditions. This should provide more insight into the functioning of the motor system.

Handwriting can also be studied synthetically with a mechanical arm. The virtue of a synthetic approach is in illuminating issues that cannot be conveniently studied any other way. A mechanical arm also allows one to test conjectures about muscular patterns derived from human handwriting.

Handwriting can be studied from the point of view of learning a physical skill. The slow, guarded strokes of the beginner eventually give way to the rapid, smooth curves of the accomplished writer. The acceleration curves may provide a useful indicator for tracing this development. It should be possible to develop better training strategies that are derived from simplifying and decomposing the expert's acceleration profiles. These strategies might also be useful for other physical skills.

Finally, knowing how people execute handwriting may lead to ideas about how people read handwriting. The acceleration profiles indicate how the letters were formed, and hence how they should be segmented. From the acceleration profiles I have seen, people segment words more or less in the same way. Personal style manifests itself within each of the segmented strokes as a particular pattern of acceleration profile. Readers may possibly perceive the essence of this pattern in a writer's writing, and adapt their perception of letters accordingly.

2. The Nature of Handwriting

Handwriting is essentially a matter of cup-shaped connectors. For a few letters, such as w in fig. 1a, the cup-like nature of the strokes is obvious. For most other letters, however, the cup strokes are evident, not in isolation, but in sequence with other letters. The cup in fig. 1b begins at the top of the first e and ends at the top of the next one.

From the top of the second e, another cup stroke has been started to join the next letter, but since no other letter has been added, the cup is stopped short. All letters end in the same way: in a partial cup stroke that prepares connection with other letters. Similarly, most letters begin with the right portion of a cup, such as the first e in fig. 1b. If another letter had preceded this e, the beginning portion would have been part of a complete cup.

When written in isolation, some letters often do not exhibit the partial connecting cups at the beginning and end. For example, the o in fig. 1c has no preceding connector and practically no postceding one. Both connectors are present, however, when the o is embedded in a word such as foo in fig. 1d. Studying letters in isolation is thus potentially misleading. Handwriting strokes belong to two letters as often as they belong to just one. When developing a vocabulary of handwriting strokes, therefore, one has to study the strokes in the context of words.

3. Strokes Last 200 msec.

Acceleration and velocity profiles of handwritten words reveal that writing is composed of roughly 200 msec strokes joined together. The word fell in fig. 1e, for example, is decomposed into four such strokes in fig. 1f. The time durations in milliseconds are indicated beneath each stroke. The beginning and end strokes are of shorter duration, as would be expected of partial cups. Why some strokes require 200 msec and others 240 is explained later.

4. 40 msec Bursts Quantize Slope

A closer inspection of acceleration profiles shows that the 200 msec strokes are composed of 40 msec spikes. The cup stroke joining the 2 l's in fig. 1e, for example, is composed of 6 spikes, as the acceleration profiles in fig. 2a reveal. The v" profile represents vertical acceleration, the h" profile the horizontal acceleration. These profiles were obtained by McDonald [1966], on whose data I base my results.

Each v"-h" spike pair represents an applied force that changes direction. An analogy can be made to a rolling ball that is poked with a finger at regular intervals to change its path. The path the ball follows is a chain of straight line segments connected at sharp angles. The directions are evidently quantized in a hexagonal manner (fig. 2b). The hexagon is not necessarily regular, but is skinny or fat according to an individual's writing style.

The stroke begins from zero velocity. The first 40 msec burst starts the hand angling at about 245 degrees. Combining the v" and h" components into a force vector applied to a stationary hand, one can compute from the acceleration profiles a starting angle of 210 degrees (fig. 2c-d). The next v" burst is accompanied by a smaller h" spike, which causes the hand to angle downwards. The measured angular change is 20 degrees. Computing from the acceleration profiles once again, vector addition of the second force vector to the first yields a new vector that has changed direction 15 degrees from the first. Thus the computed and measured angular change agree fairly well.

The rest of the 40 msec bursts also show rough agreement between measured and computed direction change. The acceleration profiles are probably not accurate enough to hope for better agreement. The spikes are also broad and do not represent an instantaneous application of force that would make simple vector addition completely valid.

The last burst represents a force that is more or less collinear to the resultant vector at the end of the 5th burst, but is opposite in direction. Thus the last stroke acts merely to brake the speed of the hand to zero without changing direction.

5. Adapting the Hexagon to Different Strokes

The size of a cup stroke is determined by the amplitude of the 40 msec bursts. Thus the firing pattern for the cup in <u>ee</u> is identical to that for <u>ll</u> except for amplitude. Fig. 3a and 3b shows that the cups are composed of the same number of straight lines differing only in length. Corroborating this claim for amplitude coding rather that duration coding is the observation that stroke time does not change with handwriting size [Denier van der Gon and Thuring 1965].

For cup strokes such as <u>el</u> or <u>le</u> in figs. 3c and 3d, the patterns are once again the same but the line lengths are mixed. One portion contains line lengths appropriate for a small <u>ee</u> cup, the other portion contains line lengths appropriate for a large <u>ll</u> cup. Evidently amplitudes are quantized as well as slopes, and the quantization is basically to two levels. At the transition point from large to small line length and vice versa, the velocity must be brought close to zero to avoid distortion. This I have observed in the velocity profiles. It is particularly important when going from large to small line length.

Strokes vary in the degrees of arc they compose. An <u>ee</u> cup composes more degrees than a <u>w</u> cup (figs. 3e and 3f). The <u>w</u> cup has vertical sides to begin and end with, whereas an ee cup must begin with a line that rounds the top of the e. Thus the ee cup requires an extra 40 msec burst, which makes that stroke 240 msec long as compared to 200 msec for the w cup.

The small and relatively flat connective cups such as from b to r (fig. 3g) require even fewer 40 msec bursts. The total duration for this br connective cup is 160 msec.

6. Connecting Bursts and Connecting Strokes

The lines corresponding to the 40 msec bursts would be straight and joined at sharp angles if the bursts were narrow, isolated spikes. Figure 4a shows lines from two such bursts and the corresponding vertical acceleration profile. The transition between two bursts can be made smoother by merging the spikes a little (fig. 4b). Rounding the bursts also makes the line segments curved instead of straight, yielding an even smoother curve (fig. 4c).

People with very smooth and flowing handwriting have carried out this rounding and merging of bursts to such an extent that the individual 40 msec bursts are hardly discernible. What one sees instead is an acceleration plateau that might have slight vestiges of the original bursts (fig. 4d).

The 200 msec strokes are fairly isolated from each other in the acceleration profiles. The end of one stroke finds the hand at zero velocity, and the next stroke begins from a rest position. Thus the transition between strokes is angular. There is, however, practically no elapsed time between strokes.

That is not to say that the end of one stroke does not prepare the hand for the beginning of the next stroke. I have observed that the braking action at the end of a stroke can cause a slight hook to occur (fig. 4e left curve). The braking action is applied a little more strongly than needed to stop the stroke, so that the hand actually reverses direction for a short time, producing a hook. This hook is in the proper direction to start the next stroke (fig. 4e right curve). The data, however, are not good enough to substantiate this hypothesis.

7. Other Types of Strokes

The counterclockwise drawn cup stroke is the most important one in (English) handwriting. Other types of strokes are needed to produce all letters of the alphabet and all the ways letters might be drawn. The number of examples in [McDonald 1966] is unfortunately rather limited, and as a result I can make only a few comments about such other strokes.

The _J_ in fig. 5a exhibits a clockwise cup. The acceleration profiles show that the v" component is exactly the same as for similar counterclockwise cups. It is the difference in sign of the h" component that brings about the clockwise action. Physiologically separating the movement into h and v directions, therefore, has its benefits in terms of modularity.

A complete vocabulary of strokes would also have to include inverted cups drawn both clockwise and counterclockwise. A clockwise inverted stroke would be a part of such letters as h and n; a counterclockwise inverted stroke would form the top portion of the loop in such letters as a and o (fig. 5b). Unfortunately McDonald's examples did not exhibit such strokes, and I am only surmising their existence.

McDonald's examples did have some instances of o's, n's, and h's, but these letters were drawn with the standard counterclockwise cup. One might think that, ideally, an o is composed of two cups: an inverted counterclockwise cup followed by a standard counterclockwise cup (fig. 5c). McDonald had his subjects engage in a rotary motion of the hand, and the strokes had precisely this form. The acceleration profiles showed that one complete circle was composed of two 200 msec strokes corresponding to fig. 5c. When subjects drew o's in handwriting, however, the o was drawn as a standard cup closed at the top (fig. 5d). Drawing the o in this manner has the advantage of being 200 msec faster than the o in fig. 5c.

The letters hn in another example were also drawn with standard cups (fig. 5e). Drawing hn with inverted cups would have required an additional 400 msec. Speed is not the only reason for eliminating inverted cups from one's writing; an unwillingness to change directions of a stroke from clockwise to counterclockwise may also be a factor.

A final type of stroke that would be included in a stroke vocabulary is a straight or gently arching stroke. Letters are often begun with straight strokes when not preceded by other letters; for example, the isolated l and r in fig. 5f. The top flat portion of the r is another example of a straight stroke. Straight strokes that I have seen vary in duration from 80 to 200 msec, depending on length and on curvature. The simplest stroke has two acceleration peaks: a starting peak and a braking peak. This gives a fairly straight line (fig. 6a). To add curvature, a third acceleration peak may be included to yield a 120 msec stroke (fig. 6b).

Straight strokes of 200 msec are illustrated in fig. 1e-f. Note that the standard method of writing an f is a little faster (fig. 6c): 440 msec vs. 520 for the fig. 2a f. It is easier, however, to obtain a fuller, longer bottom loop with the 520 msec method. I have observed that the 440 msec method often causes me to rush the bottom loop, making it shorter and skinnier.

8. Slow vs. Fast Writing

Normal fast handwriting is a ballistic motion executed without feedback. Slow handwriting is done in an entirely different manner: the hand is advanced a little at a time by 100 msec bursts (fig. 6d). Missing is the acceleration- deflection- deceleration character of a ballistic motion.

This disparity calls into question teaching handwriting with slow strokes. It is not obvious to me how learning slow strokes prepares one at all for fast handwriting. What I see happening is that youngsters on their own eventually abandon slow writing in favor of fast strokes. What I don't see happening is that slow strokes become faster and faster (i.e., fewer 100 msec bursts per letter) until one is executing fast writing.

The magic number 200 msec also appears in slow strokes. McDonald asked his subjects to execute a horizontal back-forth motion of the wrist at different speeds. The times required to move the wrist from one extreme point to the other were multiples of 200: 800, 600, 400, and 200 msec. The subjects chose these frequencies without prompting. Why slow movements are also executed modulo 200 msec is not clear at present.

9. Related Work

McDonald [1966], whose data I reexamined, approximated the acceleration profiles by trapezoidal peaks. He interpreted the muscular action as a bang-bang servo: the muscular activity rose linearly to a plateau level, then decayed linearly. His fitting of trapezoidal approximations to the acceleration profiles was rather arbitrary. These approximations, moreover, averaged out the important features.

Koster and Vredenbregt [1971] segmented letters at points of reversal of vertical direction. They proposed that ratios of the corresponding time intervals were characteristic of individual letters, and that different people wrote a given letter with the same interval ratios. My analysis of handwriting shows why their proposal is often true. Yet their proposal in itself does not really illuminate the nature of handwriting.

Eden [1962] developed a stroke vocabulary and enumerated all letters of the alphabet in terms of them. These strokes were represented mathematically as sine curves. His strokes, however, do not correspond to the way people actually piece together a letter. The sinusoidal approximation, moreover, misses the pulselike nature of muscle movement.

10. Acknowledgment

David Marr first introduced me to the notion of 200 msec motor programs, and suggested handwriting as a suitable motor skill for study. Without his encouragement, this study would not have taken place.

11. References

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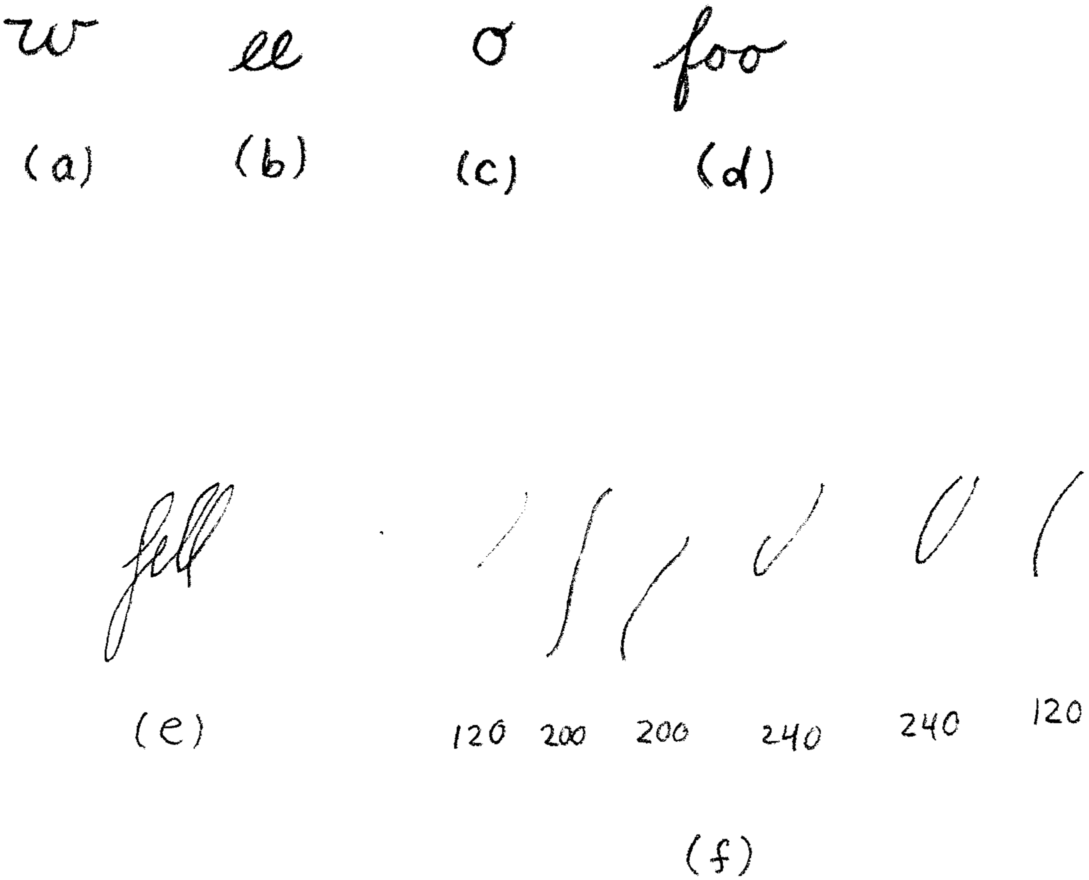
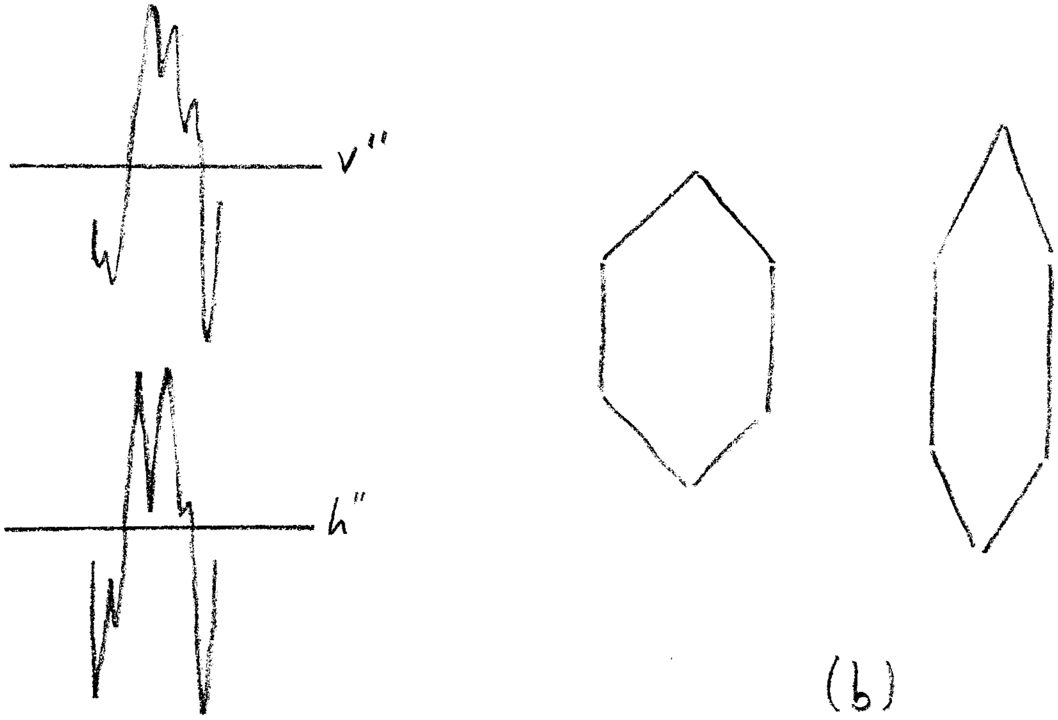


Figure 1.

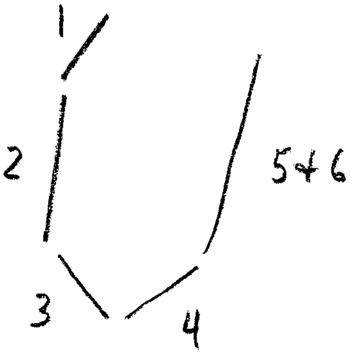


(a)

(b)

stroke	measured	calculated
1	245°	210°
2	20	15
3	30	20
4	295	155
5	10	10
6		

(c)



(d)

Figure 2.

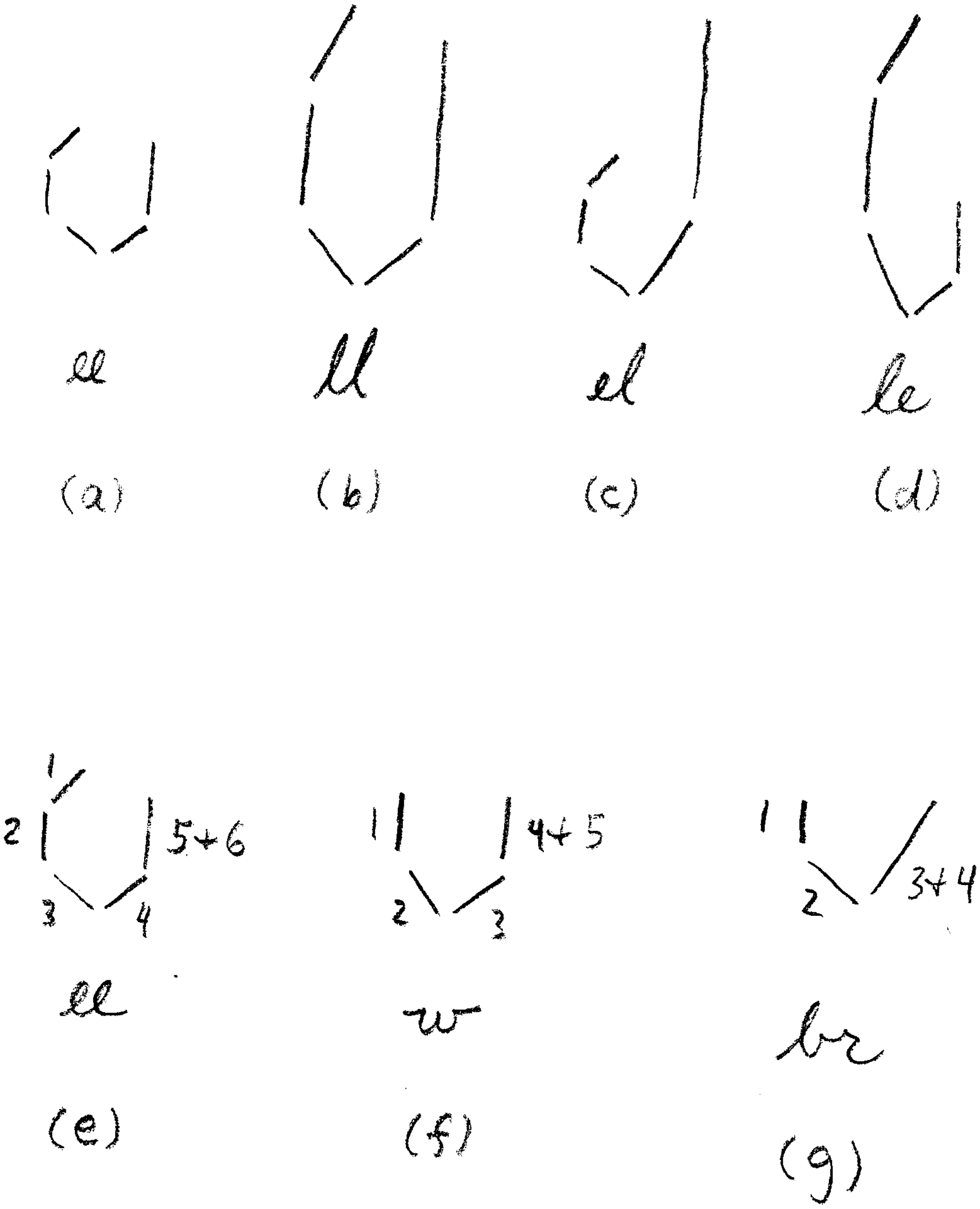


Figure 3.

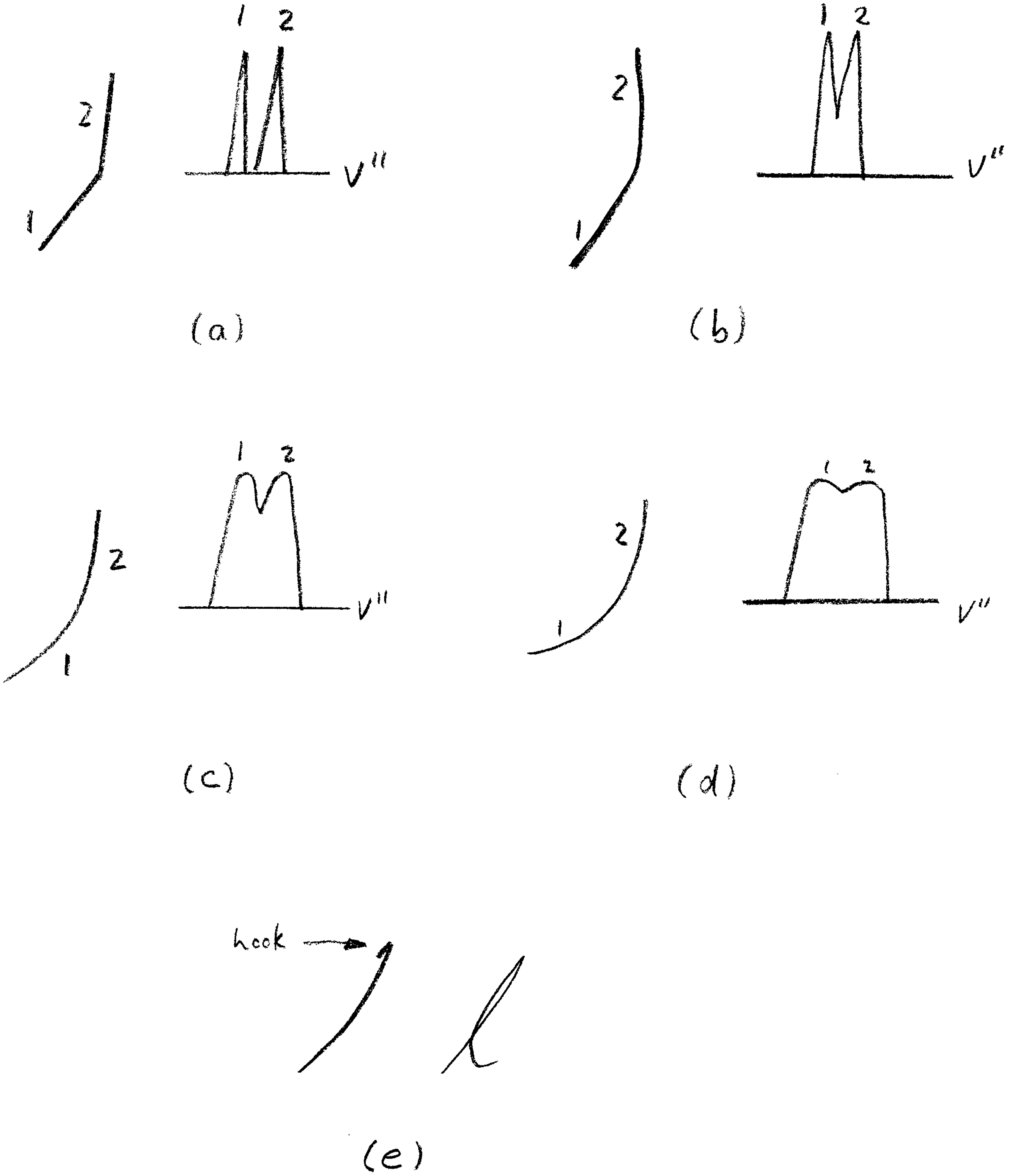
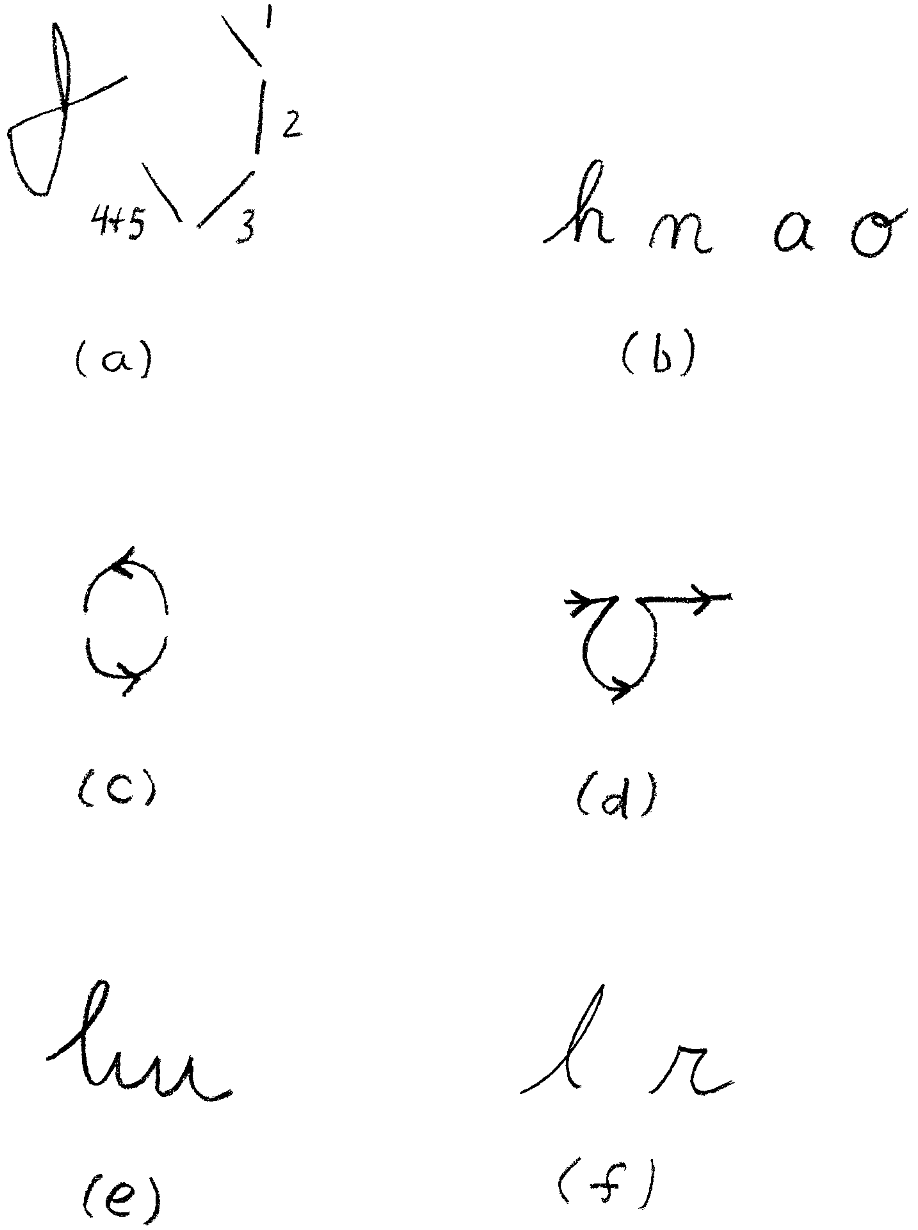


Figure 4.



(a) (b) (c) (d) (e) (f)

Figure 5.

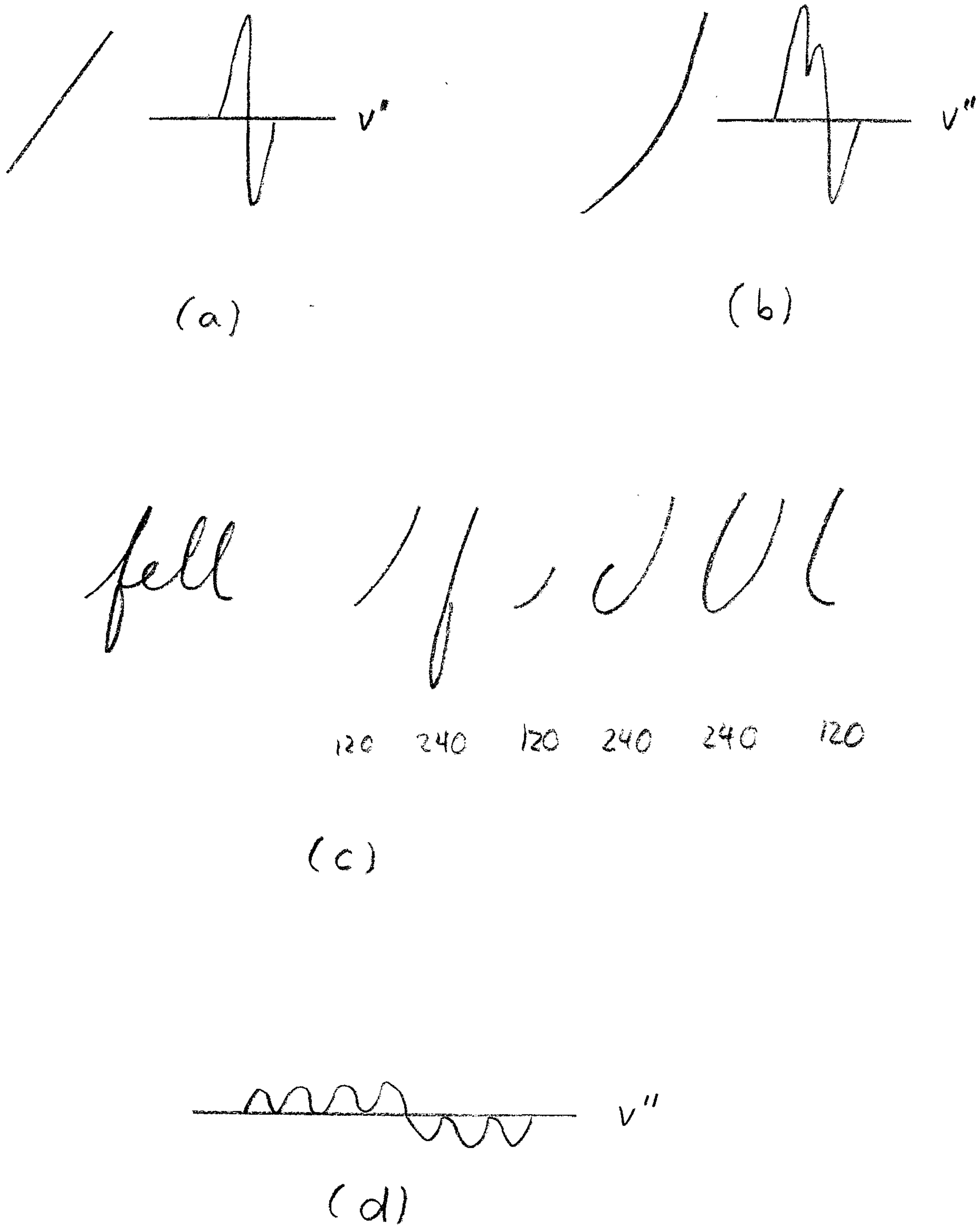


Figure 6.