1 Bragg scattering in a periodic medium

Longshore sand bars are often found along gentle beaches. The number of bars can range from a few to dozens and the spacing from tens to hundreds of meters. The bar amplitudes can be as high as a meter. Figure 1 shows a sample profile in Chesapeake Bay, Maryland, USA, recorded by acoustic sounding (Dolan, 1982).

Scientifically it is natural to ask how these sand bars are generated and how the bars affect the propagation of waves. In fact these two questions are coupled through the complex dynamics of sediment transport. It is easier to just consider the second question.

If the bar amplitudes are small, \( D \ll h \), one might expect their effects on a train of progressive waves to be small and apply a straightforward second-order analysis so that the effect on waves will appear at the order \( kA(KD) \). The situation is different if however the incident waves are twice as long as the bar spacing, i.e., \( K = 2k \) then the phenomenon of Bragg resonance occurs and the reflection by many small and periodic bars can be very strong. The source of this resonance is due to constructive interference of incident and reflected waves and is well known in x-ray diffraction by crystalline materials. Referring to Fig. 2 where a number of bars of wavelength \( \lambda_b \) are fixed on a horizontal bed, we consider the propagation of a train of waves incident from the left. Every wave crest passing over a bar will be mostly transmitted toward the next bar ahead and sends a weak reflected wave towards the bar behind. At any given bar crest, say \( B \), the total amplitude of the left-going wave is the sum of all left-going wave crests each of which is the consequence of reflection by the \( n \)th bar on the right. Therefore each of these crests has traveled the distance of \( 2n\lambda_b \). When
$2\lambda_0$ equals the surface wavelength, $\lambda$, all these reflected crests are in phase upon arrival at $B$, and reinforce one another, resulting in strong (resonant) reflection. Thus many small bars can give rise to strong reflection if the Bragg resonance condition is met.

Bragg resonance is of interest in many branches of physics. In crystallography, the phenomenon is used to study the structure of a crystalline solid by x-rays.

Let us use a one-dimensional example to describe the phenomenon. First, since many scatters must be involved in order for this phenomenon to be appreciable, the total region of disturbances must be much greater than the typical wavelength. The perturbation method of multiple scales can be used. Second, since reflection is strong, incident and reflected wave must be allowed to be comparable in order.

Let us consider the one-dimensional scattering of elastic waves in a rod with a slightly periodic elasticity,

$$\rho = \text{constant}, \quad E = E_0(1 + \epsilon D \cos Kx),$$

where $D$ is of order unity, i.e.,

$$E_0 \frac{\partial}{\partial x} \left[(1 + \epsilon D \cos Kx) \frac{\partial u}{\partial x}\right] = \rho \frac{\partial^2 u}{\partial t^2}$$

(1.2)

We now assume that the spatial period of inhomogeneity $\ell \equiv 2\pi/K$ and the elastic wavelength $\ell' \equiv 2\pi/k = 2\pi \sqrt{E_0/\rho/\omega}$ are comparable. As a consequence, wave reflection can be significant.

Let us first try a naive expansion, $u = u_0 + \epsilon u_1 + \cdots$. The crudest solution is easily found to be

$$u_0 = \frac{A}{2} e^{ikx - i\omega t} + \text{c.c.},$$

(1.3)

where c.c. signifies the complex conjugate of the preceding term, and

$$\frac{2\pi}{k} \equiv \sqrt{\frac{E_0}{\rho} \frac{2\pi}{\omega}}, \quad \text{or} \quad \frac{\omega}{k} = C = \sqrt{\frac{E_0}{\rho}}.$$

(1.4)

At the next order the governing equation is

$$\frac{\partial}{\partial x} \left(E_0 \frac{\partial u_1}{\partial x}\right) - \rho \frac{\partial^2 u_1}{\partial t^2} = -E_0 D \frac{\partial}{\partial x} \left[(e^{iKx} + e^{-iKx}) \frac{\partial u_0}{\partial x}\right]$$

$$= -\frac{E_0 D}{2} \frac{\partial}{\partial x} \left[(e^{iKx} + e^{-iKx}) \left(\frac{ikA_0}{2} e^{ikx - i\omega t} - \frac{ikA_0}{2} e^{-ikx + i\omega t}\right)\right].$$

(1.5)
Clearly, when
\[ K = 2k + \delta, \quad \delta \ll k, \]  
(1.6)
some of the forcing terms on the right will be close to a natural mode \( \exp(\pm i(kx + \omega t)) \). Resonance of the reflected waves must be expected. It suffices to illustrate the response to one of these terms,

\[ E_o \frac{\partial^2 u_1}{\partial x^2} - \rho \frac{\partial^2 u_1}{\partial t^2} = Ae^{i\phi_o} e^{i\delta x}, \quad \text{with} \quad \phi_o = kx + \omega t. \]

Combining homogeneous and inhomogeneous solutions and requiring that \( u_1(0, t) = 0 \), we find

\[ u_1 = \frac{Ae^{i\phi_o} \left( 1 - e^{i\delta x} \right)}{E_o((k + \delta)^2 - k^2)}. \]

Clearly if \( \delta = O(\epsilon) \), \( \epsilon u_1 \sim O(\epsilon/\delta) \) and is not small compared to \( u_0 \) except for \( \delta x \ll 1 \). Furthermore as \( x \) increases, \( u_1 \) grows as \( \epsilon x \). This implies that the reflected waves are resonated and is no longer much smaller that the incident waves in the distance \( \epsilon x = O(1) \). The relation \( 2K = k \) (cf. (1.6) is the well-known condition for Bragg resonance.

Let us now focus attention on the case of Bragg resonance. To render the solution uniformly valid for all \( x \), we introduce fast and slow variables in space

\[ x, \bar{x} = \epsilon x \]  
(1.7)

To allow slight detuning from exact resonance, we assume that the incident wave frequency is \( \omega + \epsilon \omega' \), where \( \epsilon \omega' \) represents the small detuning and gives rise to a very slow variation in time. Therefore two time variables are needed,

\[ t, \bar{t} = \epsilon t \]  
(1.8)

The following multiple scale expansion is then proposed,

\[ u = u_0(x, \bar{x}; t, \bar{t}) + \epsilon u_1(x, \bar{x}; t, \bar{t}) + \cdots. \]  
(1.9)

After making the changes

\[ \frac{\partial}{\partial x} \rightarrow \frac{\partial}{\partial x} + \epsilon \frac{\partial}{\partial \bar{x}}, \quad \frac{\partial}{\partial t} \rightarrow \frac{\partial}{\partial t} + \epsilon \frac{\partial}{\partial \bar{t}} \]  
(1.10)
and substituting (1.9), (1.10) into (1.2), we get
\[
\frac{\partial}{\partial x} \left( E_o \frac{\partial u_0}{\partial x} \right) - \rho \frac{\partial^2 u_0}{\partial t^2} = 0
\] (1.11)
at \mathcal{O}(1). Anticipating strong but finite reflection, we take the solution to be
\[
u_0 = \frac{A}{2} e^{ikx-i\omega t} + * + \frac{B}{2} e^{-ikx-i\omega t} + \text{c.c.}.
\] (1.12)
where \(A(x_1, t_1)\) and \(B(x_1, t_1)\) vary slowly in space and time. At the order \(\mathcal{O}(\varepsilon)\) we have
\[
\frac{\partial}{\partial x} \left( E_o \frac{\partial u_1}{\partial x} \right) - \rho \frac{\partial^2 u_1}{\partial t^2} = -2E_o \frac{\partial^2 u_0}{\partial x \partial x} + 2\rho \frac{\partial^2 u_0}{\partial t \partial t} \\
- \frac{E_oD}{2} \frac{\partial}{\partial x} \left[ \left( e^{2ikx} + e^{-2ikx} \right) \frac{\partial u_0}{\partial x} \right] \\
= -E_o \left[ \frac{\partial A}{\partial x} (ik)e^{ikx-i\omega t} + \text{c.c.} + \frac{\partial B}{\partial x} (-ik)e^{-ikx-i\omega t} + \text{c.c.} \right] \\
+ \rho \left[ \frac{\partial A}{\partial t} (-i\omega)e^{ikx-i\omega t} + \text{c.c.} + \frac{\partial B}{\partial t} (-i\omega)e^{-ikx-i\omega t} + \text{c.c.} \right] \\
- \frac{E_oD}{4} \frac{\partial}{\partial x} \left\{ \left( e^{2ikx} + \text{c.c.} \right) \frac{\partial}{\partial x} \left[ Ae^{ikx-i\omega t} + \text{c.c.} + Be^{-ikx-i\omega t} + \text{c.c.} \right] \right\}
\] (1.13)
The last line can be reduced to
\[
- \frac{E_oD}{4} \left( k^2 B e^{ikx-i\omega t} + \text{c.c.} + k^2 A e^{-ikx-i\omega t} + \text{c.c.} \\
- 3k^2 A e^{3ikx-i\omega t} + \text{c.c.} - 3k^2 B e^{-3ikx-i\omega t} + \text{c.c.} \right)
\]
To avoid unbounded resonance of \(u_1\), i.e., to ensure the solvability of \(u_1\), we equate to zero the coefficients of terms \(e^{\pm i(kx-\omega t)}\) and \(e^{\pm i(kx+\omega t)}\) on the right of (1.13). The following equations are then obtained:
\[
\frac{\partial A}{\partial t} + c \frac{\partial A}{\partial x} = \frac{i \varepsilon D}{4} B
\] (1.14)
\[
\frac{\partial B}{\partial t} - c \frac{\partial B}{\partial x} = \frac{i \varepsilon D}{4} A
\] (1.15)
where \(\sqrt{E_o/\rho_o} = C = \omega/k\) denotes the phase speed. These equations govern the macroscale variation of the envelopes of the incident and reflected waves, and can be combined to give the Klein-Gordon equation
\[
\frac{\partial^2 A}{\partial t^2} - C^2 \frac{\partial^2 A}{\partial x^2} + \left( \frac{k \varepsilon D}{4} \right)^2 A = 0.
\] (1.16)
Note that
\[ \frac{kCD}{4} = \frac{\omega D}{4} \equiv \Omega_0 \] (1.17)
has the dimension of frequency.

With suitable initial and boundary conditions on the macro scale, one finds the slow variation of these wave envelopes, hence the global behaviour of wave motion.

Let the inhomogeneity of wavenumber $2k$ be confined in $0 < \bar{x} < L$ and the incident wave train be slightly detuned from resonance, so that the wave frequency is $\omega + \epsilon \Omega$ and the wavenumber is $k + \epsilon K$, where $\Omega = O(\omega)$ and $K = O(k)$. Since $\omega + \epsilon \Omega$ and $k + \epsilon K$ must be related by the dispersion relation (1.4),
\[ \Omega = KC . \] (1.18)
The detuned incident wave
\[ \zeta = A_o \exp[i(k + \epsilon K)x - (\omega + \epsilon \Omega)t] + *, \quad \bar{x} < 0 , \] (1.19)
can be alternatively written as
\[ \zeta = A(\bar{x}, \bar{t}) e^{ikx - i\omega t} , \quad \bar{x} < 0 , \] (1.20)
where
\[ A(\bar{x}, \bar{t}) = A_o e^{iK(\bar{x} - C\bar{t})} , \quad \bar{x} < 0 . \] (1.21)
When such a wavetrain passes a patch of periodic bars, $A$ and $B$ must vary with $\bar{x}$ and $\bar{t}$ according to (1.14) and (1.15).

To the left and to the right of the bars, the governing equations are simply
\[ A_t + CA_x = 0 , \quad B_t - CB_x = 0 , \quad \bar{x} < 0 , \quad \text{and} \quad \bar{x} > L . \] (1.22)
We shall assume further that $B = 0$ for $\bar{x} > L$. Over the bars (1.14) and (1.15), or (1.16) hold. In order that displacement and stress and horizontal velocity be continuous at $x = 0, L$, $A$ and $B$ must be continuous at $x = 0, L$. Since the solutions must be of the form,
\[ (A, B) = A_0(T(\bar{x}), R(\bar{x}))e^{-i\Omega \bar{t}} , \quad 0 < \bar{x} < L . \]
$T$ and $R$ are governed by
\[ T_{\bar{x}\bar{x}} + \frac{(\Omega^2 - \Omega_0^2)}{C}T = 0 , \quad 0 < \bar{x} < L . \]
Several cases can be distinguished according to the sign of $\Omega^2 - \Omega_0^2$:

**Subcritical detuning:** $0 < \Omega < \Omega_0$.

Let

$$Q_c = (\Omega_0^2 - \Omega^2)^{1/2}$$

then

$$T(x) = \frac{iQC \cosh Q(L - \bar{x}) + \Omega \sinh Q(L - \bar{x})}{iQC \cosh QL + \Omega \sinh QL}$$

and

$$R(x) = \frac{Q \sinh Q(L - \bar{x})}{iQC \cosh QL + \Omega \sinh QL}.$$  (1.25)

On the incidence side the reflection coefficient is just $R(0)$ and on the transmission side the transmission coefficient is $T(L)$. Clearly the dependence on $L$ and $\bar{x}$ is monotonic.

In the limit of $L \to \infty$, it is easy to find that

$$T(x) = e^{-Q\bar{x}}, \quad R(x) = \frac{Q}{iQC + \Omega} e^{-Q\bar{x}}.$$  (1.26)

Thus all waves are localized in the range $\bar{x} < O(1/Q)$.

**Supercritical detuning:** $\Omega > \Omega_0$.

Let

$$P_c = (\Omega^2 - \Omega_0^2)^{1/2}$$

then the transmission and reflection coefficients are:

$$T(x) = \frac{PC \cos P(L - \bar{x}) - i\Omega \sin P(L - \bar{x})}{PC \cos PL - i\Omega \sin PL}$$

and

$$R(x) = \frac{-iQ_0 \sin P(L - \bar{x})}{PC \cos PL - i\Omega \sin PL}.$$  (1.29)

The dependence on $L$ and $\bar{x}$ is clearly oscillatory. Thus $\Omega_0$ is the cut-off frequency marking the transition of the spatial variation. For subcritical detuning complete reflection can occur for sufficiently large $L$. For super-critical detuning there can be windows of strong transission.

In the special case of perfect resonance, we get from (1.24) and (1.25) that

$$T(\bar{x}) = \frac{A}{A_o} = \frac{\cosh \frac{\Omega_o(L - \bar{x})}{c}}{\cosh \frac{\Omega_o}{c}} \quad R(\bar{x}) = \frac{B}{A_o} = -\frac{i \sinh \frac{\Omega_o(L - \bar{x})}{c}}{\cosh \frac{\Omega_o}{c}}.$$  (1.30)
In a laboratory experiment for water waves, Heathershaw (1982) installed 10 sinusoidal bars of amplitude $D = 5$ cm and wavelength 100 cm on the bottom of a long wave flume. Incident waves of length $2\pi/k = 200$ cm were sent from one side of the bar patch. On the transmission side, waves are essentially absorbed by breaking on a gentle beach. Sizable reflection coefficients were measured along many stations over the bar patch. This experiment gives the first observed evidence of strong reflection by periodic bars. Let us apply the present theory to a more general case where the normally incident wave is slightly detuned from perfect resonance.

Clearly they both decrease monotonically from $\bar{x} = 0$ to $\bar{x} = L$. These results agree quite well with the experiments of Heathershaw, as shown in Fig. 3, therefore confirm that enough small bars can generate strong reflection, especially in very shallow water.

**Exercise 5.1: Bragg resonance by a corrugated river bank.**

An infinitely long river has constant depth $h$ and constant averaged width $2a$. In the stretch $0 < x < L$, the banks are slightly sinusoidal about the mean so that

$$y = \pm a \pm B \sin Kx, \quad KB \equiv \epsilon \ll 1.$$  \hspace{1cm} (1.31)

See Fig. 4. Let a train of monochromatic waves be incident from $x \sim -\infty$,

$$\zeta = \frac{A}{2} e^{i(kx - \omega t)}$$  \hspace{1cm} (1.32)

where $kh, ka = O(1)$. Develop a uniformly valid linearized theory to predict Bragg resonance. Can the corrugated boundary be used to reflect waves as a breakwater? Discuss your results for various parameters that can affect the function as a breakwater.

## 2 Wave localization in a random medium


There are numerous situations where one needs to know how waves propagate through a medium with random impurities: light through sky with dust particles, sound through water with bubbles, elastic waves through a solid with cracks, fibers, cavities, hard or soft grains. Sea waves over a irregular topography, etc. It is known that, for one-dimensional
propagation, multiple scattering yields a change in the wavenumber (or phase velocity) as well as an amplitude attenuation, if the inhomogeneities extend over a large distance. These changes amount to a shift of the complex propagation constant with the real part corresponding to the wavenumber and the imaginary part to attenuation. In particular, the spatial attenuation is a distinctive feature of randomness and is effective for a broad range of incident wave frequencies. This is in sharp contrast to periodic inhomogeneities which cause strong scattering only for certain frequency bands (Bragg scattering, see e.g., Chapter 1). Phillip W. Anderson (1958) was the first to show, in the context of solid-state physics, that a metal conductor can behave like an insulator, if the microstructure has is disordered. This phenomenon, now called Anderson localization, is now known to be important in classical systems also. A survey of localization in many types of classical waves based on linearized theories can be found in Sheng (1998).

For weak inhomogeneities, the shift of propagation constant amounts to slow spatial modulations with a length scale much longer than the wavelength by a factor inversely proportional to the correlation of the fluctuations. In this section we apply the method of multiple scales to introduce the theory for the simplest example of one-dimensional sound.

We begin with the Helmholtz equation for sinusoidal waves,

\[
\frac{d^2U}{dx^2} + k^2(1 + \epsilon V(x))^2U = 0, \quad \infty x < \infty. \tag{2.1}
\]

Let \(V(x)\) be a random function of \(x\) with zero mean and \(V(x) \to 0\), for \(x \sim -\infty\). An incident wave train

\[
U_{inc} = A_0 e^{ikx} \tag{2.2}
\]

arrives from the left-infinity where there is no disorder. What will happen, on the average, to waves after they enter the region of disorder?

Consider an ensemble of random media. For each realization, the wave number now fluctuates about the mean \(k\) by the amount order \(O(\epsilon)\). Since \(\langle V \rangle = 0\), we expect that, on the average, the wave phase is affected only by the root-mean-square, which is of the order \(O(\epsilon^2)\). With this guess, it is natural that slow variations described by \(x_2 = \epsilon^2 x\) will be relevant. We assume that the disorder has two characteristic scales so that

\[
V = V(x, x_2) \tag{2.3}
\]
For simplicity we shall further assume that $V$ is stationary with respect to the short scale
\[ \langle V(x, x_2)V(x', x_2) \rangle = C_{vv}(|x - x'|, x_2) \] (2.4)
where $\langle f \rangle$ denotes the ensemble average of $f$.

Let us try the following expansion,
\[ U = U_0(x, x_2) + \epsilon U_1(x, x_2) + \epsilon^2 U_2(x, x_2) + \cdots \] (2.5)
Substituting (2.5) into (2.1), the following perturbation equations are found,
\[ \frac{\partial^2 U_0}{\partial x^2} + k^2 U_0 = 0, \] (2.6)
\[ \frac{\partial^2 U_1}{\partial x^2} + k^2 U_1 = -2k^2 V U_0, \] (2.7)
\[ \frac{\partial^2 U_2}{\partial x^2} + k^2 U_2 = -2 \frac{\partial U_0}{\partial x \partial x_2} - k^2 \left( 2V U_1 + V^2 U_0 \right), \] (2.8)
The solution at the leading order is
\[ U_0 = A(x_2)e^{ikx} \quad \text{where} \quad A(0) = A_0. \] (2.9)

At the next order the inhomogeneous equation is solved by Green’s function $G(x, x')$ defined by
\[ \frac{\partial^2 G}{\partial x^2} + k^2 G = \delta(x - x'), \] (2.10)
where $G$ is outgoing at infinities. The solution is easily found to be
\[ G = -\frac{i}{2k}e^{ik|x-x'|} \] (2.11)
\[ G = -\frac{i}{2k}e^{ik|x-x'|} \] (2.12)
The solution for $U_1$ is
\[ U_1 = -\int_{-\infty}^{\infty} dx' G(x, x') \left[ 2k^2 V(x', x_2)U(x', x_2) \right] \]
\[ = ik \int_{-\infty}^{\infty} dx' V(x', x_2)e^{ikx'}e^{ik|x-x'|} \] (2.13)
which is random with zero mean. For the $O(\epsilon^2)$ problem, we note that
\[ 2 \frac{\partial^2 U_0}{\partial x \partial x'} = 2ik e^{ikx} \frac{\partial A}{\partial x_2}, \]
\[ 2k^2 U_1 = 2i k A(x_2) e^{ikx} \int V(x, x_2) V(x', x_2) e^{ik|x-x'|} e^{-ik(x-x')} dx', \]
\[ k^2 V^2 U_0 = k^2 e^{ikx} V(x, x_2) V(x, x_2) A(x_2). \]

We now take the ensemble average of \( (2.14) \), and get
\[
\frac{\partial^2 \langle U_2 \rangle}{\partial x^2} + k^2 \langle U_2 \rangle = -2i k e^{ikx} \frac{\partial A}{\partial x_2} - 2i k^2 A(x_2) e^{ikx} \int_{-\infty}^{\infty} \langle V(x, x_2) V(x', x_2) e^{ik|x-x'|} e^{-ik(x-x')} dx' - k^2 e^{ikx} A(x_2) \langle V^2(x, x_2) \rangle
\]

For \( \langle U_2 \rangle \) to be solvable, we set the right-hand-side to zero,
\[
\frac{\partial(A)}{\partial x_2} + \langle A \rangle \left\{ k^2 \int_{-\infty}^{\infty} dx' C_{vv}(|x-x'|, x_2) e^{ik|x-x'|} e^{-ik(x-x')} dx' - \frac{i k}{2} C_{vv}(0, x_2) \right\} = 0
\]

Clearly the integral above is just a known function of \( x_2 \) once the correlation function is prescribed. Denoting
\[
\beta(x_2) = \beta + i \beta = k^2 \int_{-\infty}^{\infty} dx' C_{vv}(|x-x'|, x_2) e^{ik|x-x'|} e^{-ik(x-x')} dx' - \frac{i k}{2} C_{vv}(0, x_2). \tag{2.14}
\]

If \( \beta = 0, x_2 < 0 \) and \( \beta = \text{constant}, x_2 > 0 \), then the solution is simply
\[
A = A(0) e^{-i \beta} e^{-\beta x_2} \tag{2.15}
\]

Thus, not only the phase is changed but the amplitude decays exponentially over the distance \( O(L) \) where
\[
L = 1/\beta \epsilon^2 \tag{2.16}
\]

In summary, due to scattering by disorder, an apparent damping is created. The distance \( L \) is called the localization distance.

For simple correlation functions, the integral for \( \beta \) can be explicity evaluated. For example let
\[
C_{vv}(|x-x'|, x_2) = \sigma^2 (x_2) e^{-\alpha|x-x'|} \tag{2.17}
\]
so that \( \sigma^2 \) is the RMS amplitude of the disorder. We leave it as an exercise to show that
\[
\beta = \beta + i \beta = 2k^2 \sigma^2 \frac{\alpha^2 + 2k^2}{\alpha(\alpha^2 + 4k^2)} - \frac{i k \sigma^2}{2} \frac{\alpha^2}{\alpha^2 + 4k^2} \tag{2.18}
\]
The leading order wave is

\[ U_0 = A_0 \exp \left\{ ik \left[ 1 + \frac{\varepsilon^2 \sigma^2}{2} \frac{\alpha^2}{\alpha^2 + 4k^2} \right] x \right\} \exp \left\{ -\frac{2\varepsilon^2 k^2 \sigma^2}{\alpha} \left( \frac{\alpha^2 + 2k^2}{\alpha^2 + 4k^2} \right) x \right\}, \quad x > 0 \]

(2.19)

As the RMS of the disorder increases, the wave number increases, hence the wave length decreases. A dimensionless localization distance can be defined as

\[ kL_{loc} = \frac{1}{\frac{2\varepsilon^2 \sigma^2}{k} \left( \frac{1 + 4k^2/\alpha^2}{1 + 2k^2/\alpha^2} \right)} \]

(2.20)

Note that the correlation length is \( O(\alpha^{-1}) \). If the waves are much longer than the correlation length, \( k/\alpha \ll 1 \); \( kL_{loc} \) increases without bound and localization is weak. If the waves are much shorter than the correlation length \( k/\alpha \gg 1 \); \( kL_{loc} \) decreases; waves cannot penetrate deeply into the disordered region.

**IAP (challenge) Project**: Scattering of elastic waves by random distribution of hard grains or cavities.

### References


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Image by MIT OCW.

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Can wavy banks serve as a breakwater?

Image by MIT OCW.