MIT 14.04 Intermediate Microeconomic Theory

Fall 2003

Answer keys for problem set 7

Exercise 15.4

Expected payoff when value is v: (value-price)*prob of winning $= (v - \frac{v}{2})v = \frac{v^2}{2}$.

Exercise 15.6

Compute the best response BR to solve this game.

- If Column is mixing with probability p i.e. plays $pL + (1-p)R$

  by playing T Row gets $2 - 5p$

  by playing B he gets $1 - p$

  $BR_{row}(p) = T$ if $p < .25$, $= B$ if $p > .25$, and $= \{T, B\}$ if $p = .25$.

- If Row is mixing with probability q i.e. plays $qT + (1-q)B$

  by playing L Column gets $2 - 5q$

  by playing R he gets $1 - q$

  $BR_{col}(q) = L$ if $q < .25$, $= R$ if $q > .25$, and $= \{L, R\}$ if $q = .25$.

The two pure strategy equilibria are (B, L) and (T, R).
The mixed strategy equilibrium is $(.25T + .75B, .25L + .75R)$
Prob of both teenagers survive= $1 - .25^2 = \frac{15}{16}$

Exercise 16.4

Solve for the Bertrand and Cournot quantities:

$$\begin{aligned} y_i^b &= b_i \frac{2a_i b_j + c a_j}{4 b_1 b_2 - c^2} \\ y_i^c &= \frac{2\alpha_i \beta_j - \gamma \alpha_j}{4 \beta_1 \beta_2 - \gamma^2} \end{aligned}$$

Then find the relation between the Greek letters and the Roman ones

$$\begin{aligned} \alpha_i &= \frac{a_i b_j + c a_j}{b_1 b_2 - c^2} \\ \beta_i &= \frac{b_j}{b_1 b_2 - c^2} \\ \gamma &= \frac{c}{b_1 b_2 - c^2} \end{aligned}$$

Express the Cournot quantities with the Roman letters

$$y_i^c = \frac{2a_ib_ib_j + cb_ia_j - c^2a_i}{4b_1b_2 - c^2} = y_i^b - \frac{c^2a_i}{4b_1b_2 - c^2}$$

As $4b_1b_2 - c^2 > 0$ is the stability condition, the Cournot quantities are always lower than the Bertrand ones. Then use the demand $p_i = \alpha_i - \beta_i y_i - \gamma y_j$, to conclude that the Cournot prices are always higher than the Bertrand prices.

More intuitive approach The intuition here is that in a Bertrand equilibrium if a firm cuts its price the other firm will also cut its price, whereas in a Cournot equilibrium if a firm decreases its quantity in order to increase the price then its competitor will increase its own quantity and will enjoy part of the price increase.

One way to show the result in the case of linear demand is to prove first that if a firm chooses its Cournot price in the Bertrand competition then the other firm will choose a price lower than its Cournot one i.e. $p_i(p_j^c) < p_i^c$.

Solve the Cournot equilibrium, to find:

$$p_i^c = \beta_i y_i^c$$

The reaction functions in the Bertrand equilibrium are:

$$p_i(p_j) = \frac{a_i + cp_j}{2b_i}$$

Now show that firm i will choose a lower price than its Cournot one if firm j chooses its Cournot price.

$$\begin{aligned}
p_i(p_j^c) &= \frac{a_i + cp_j^c}{2b_i} \\
&= \frac{y_i^c + b_ip_i^c}{2b_i} \quad \text{using the demand function } y_i = a_i - b_ip_i + cp_j \\
&= p_i^c \frac{\frac{1}{\beta_i} + b_i}{2b_i} \\
&\leq p_i^c \text{ equality if and only if } \gamma = 0 \text{ i.e. independent market}
\end{aligned}$$

The rest of the proof is easy to do with a graph. First show that as the reaction curves are upward slopping, both Cournot prices are either above or below the Bertrand prices. Then show by contradiction that they cannot be below. Assume that they are below, use the fact that the reaction curves are upward slopping and that for prices below the Bertrand ones the reaction curve of firm 2 is below the one of firm 1.

Exercise 16.10

- competitive equilibrium, $p = mc = 0$ then $Y = 100$

- $y_1 = \frac{100-y_2}{2}$
- $y_1 = y_2 = \frac{100}{3}$
- $Y = \frac{100}{2}$
- $\max_{y_2} y_2(100 - \frac{100-y_2}{2} - y_2)$ then $y_2 = \frac{100}{2}$ $y_1 = \frac{100}{4}$

Exercise 16.12

- $p = mc(y)$ then $y = p$
- $Y^{Comp} = 50p$
- $D^{Comp}(p) = D(p) - Y^{Comp} = 1000 - 100p$
- $\max_q q\frac{1000-q}{100}$ then $q = 500$
- $p = 5$
- $Y^{Comp} = 250$
- $Y^{Total} = 750$