Developing a New CMBS Hedging Tool: A Property Price Index-Based Synthetic

by

Juthatham Bo Chirathivat
Bachelor of Architecture, 2003
Cornell University

Submitted to the Department of Architecture
In Partial Fulfillment of the Requirements for the Degree of

Master of Science in Real Estate Development

at the

Massachusetts Institute of Technology

September 2007

Signature of Author ____
Department of Urban Studies and Planning
July 27, 2007

Certified by ____
David Geltner
Professor of Real Estate Finance
Department of Urban Studies and Planning
Thesis Supervisor

Accepted by ____
David Geltner
Chairman, Interdepartmental Degree Program in
Real Estate Development

Developing a New CMBS Hedging Tool:
A Property Price Index-Based Synthetic

by

Juthatham Bo Chirathivat

Submitted to the Department of Architecture
On July 27, 2007 in Partial Fulfillment of the Requirements for the Degree of
Master of Science in Real Estate Development

Abstract

By isolating credit as a distinct asset class, credit derivatives provide new vehicles for synthetically trading and transferring credit exposure of commercial real estate without buying or selling the physical assets. Recent developments of CMBS index-based synthetics, namely the CMBX, have allowed systematic market exposure to a basket of CMBS credit default swaps. The creation of these credit derivatives indices has enabled market participants to trade rating-specific risk, hedge against market-wide credit risk, and express a macro view within the CMBS sector.

This thesis identifies the key underlying source of credit default risk as the commercial real estate market itself, and explores the concept of a CMBS default risk synthetic that is based on transaction-based commercial property price index movements. Such indices would allow investors to more precisely target and hedge the particular risk in their CMBS portfolios that is exposed to specific commercial real estate markets tracked by the indices. The thesis proposes a methodology for the new synthetic product to approximately replicate the credit loss behavior of specific rated tranches of a CMBS.

This thesis utilizes Monte Carlo simulation to test the hedging performance of the proposed property price index-based synthetic, considering both cash flow correlation and hedge ratio analyses. The results reveal that the effectiveness of the hedge varies depending on the investor's horizon or degree of temporal precision the investor seeks in the hedge, as well as the target tranche rating. The hedge ratio is very dynamic throughout the life of the synthetic, suggesting that the investor buying the synthetic for hedging purposes would need to rebalance his position accordingly.

The author believe that the possibility of utilizing commercial property price indices to structure equity index-based credit derivatives, as demonstrated by methodologies in this thesis, will enhance investment and risk management strategies for CMBS investors, facilitating access to the breadth and depth of existing real estate equity indices. Further pioneering efforts in the development of credit derivatives will be a catalyst for a tremendous growth in the CMBS market.

Thesis Supervisor: David M. Geltner
Title: Professor of Real Estate Finance

Acknowledgements

I would like to give my sincerest appreciation to Professor David Geltner for his inspiration, insightful guidance, and enthusiastic support for my thesis. Thank you for making this thesis possible, and more importantly, a fun and wonderful learning experience.

I wish to acknowledge Neal Elkins for his innovative ideas and real world advice on this thesis.

I owe very special thanks to my parents and my brother for their unconditional love, and for giving me moral, intellectual, and emotional support throughout my twenty-seven years. I am forever grateful for their encouragement and unwavering faith in me.

I also would like to thank all my friends and professors who have made the past year at MIT an amazing journey of discovery.

Table of Contents

Introduction

Commercial Mortgage-Backed Securities (CMBS) provide an important source of funding for commercial real estate in the U.S. According to the Commercial Mortgage Securities Association, CMBS issuance in the U.S. alone reached a record of $207 billion in 2006. The recent surge of liquidity in real estate capital markets led to unusually high commercial property price appreciation, which in turn suppressed CMBS delinquency rates, and also led to complacency in loan underwriting. Investors in the CMBS market are of course subject to credit risk, and the recent risky loan underwriting[1] combined with credit problems in some sectors of the home mortgage industry have raised alarms about the possible need for more and better tools for hedging and trading commercial CMBS credit risk exposure. Together with recent developments in the bond market credit risk derivatives industry, this suggests that the time may be opportune for the development of innovative credit risk derivative products aimed at the U.S. CMBS and commercial mortgage industry.

Indeed, recent developments in CMBS credit derivatives are already beginning to provide new vehicles for trading and transferring credit risk of commercial real estate synthetically without buying or selling the physical assets. The credit derivative is often referred to as the *synthetic CMBS*. The development and growth of real estate credit derivatives will further enhance CMBS' reputation as a mainstream asset class in the fixed income market, and allow investors to broaden and enhance credit risk management capabilities.

Much of the recent development in CMBS credit derivatives has focused on tools to hedge against systematic market exposure. Existing synthetic trades in the CMBS market are designed as a default swap directly based on a CMBS index, such as the CMBX. This thesis explores the concept of a CMBS default risk synthetic, based on commercial property price index movements. Such indices track the key underlying source of credit risk, the commercial real estate market itself, and might allow investors to more precisely target risk exposures in their particular CMBS portfolios, or to more efficiently trade the risk they want to buy or sell. This thesis proposes and examines a new product idea to approximately

---

[1] Moody's Investors Service, US CMBS and CRE CDO 1Q 2007 Review: Conduit Credit at the Turning Point?

replicate the credit loss behavior of specific tranches of specific CMBS that have exposure to commercial real estate markets tracked by transaction price indices. The proposed property index-based CMBS synthetic would be tradable as a derivative contract to hedge CMBS credit risk. This thesis uses Monte Carlo simulation analysis to test and explore the hedging performance of the product by analyzing the cash flow correlation between the synthetic payoffs and the credit losses experienced by specific tranches of a hypothetical CMBS under plausible stylized assumptions about the future evolution of the relevant real estate market as tracked by the index. Use of the synthetic product is also examined in terms of the implied hedge ratios.

This thesis outline is as follows. Chapter One reviews related literatures on credit risk, embedded options in commercial mortgages, and mortgage valuation techniques. Chapter Two reviews two existing CMBS credit derivatives in the market: the single-name credit default swap and the CMBS index-based derivative. Chapter Three outlines the underlying assumptions and methodology in developing our proposed property index-based synthetic product idea. Chapter Four presents the results from the hedge analysis.

Chapter One: Commercial Mortgage Credit Risk

Mortgage holders face risks from interest rate movement, prepayment, and default. Interest rate risk and prepayment risk can be hedged in interest rate futures and options markets, thus this thesis focuses on the default risk. In an attempt to structure credit derivatives for hedging CMBS credit risk, this thesis first seeks to define the determinants of credit risk in commercial mortgages. This chapter studies the underlying mortgages on commercial properties as the fundamental source of credit risk in CMBS.

Embedded Options in a Commercial Mortgage

A CMBS deal is backed by a pool of commercial mortgages that can be treated as a fixed income instrument with a set of embedded options, namely the option to prepay and the option to default on the mortgage. If the market value of remaining scheduled mortgage payments is considered the underlying bond, then valuing a mortgage is equivalent to valuing this bond together with the options. These options are the prime determinants of the mortgage's credit risk, and are the main focus of this chapter.

CMBS are passive in the sense that prepayments and defaults are not due to the rational decision of the issuer, but to the decision of the borrowers of the collateralizing mortgages, who have no active interest in the mortgage backed securities. Thus, the default and prepayment depend on the borrowers' behavior on the underlying mortgages.

Prepayment as a call option on long-term debt

Literature on mortgage valuation views the right to prepay a mortgage at anytime as a call option to buy the mortgage from the lender. In other words, paying off the loan is equivalent to exercising a call option on the bond, with an exercise price equal to the loan principal amount. This call option value depends on the dynamics of interest rates in the bond market, namely the term structure of interest rates. When refinancing mortgage rate decreases, repayments arise, shortening the duration of bondholders' portfolios and decreasing their yields. Thus, interest rate volatility increases the value of the borrower's option to refinance and the yields.

A mortgage with a call option is referred to as a callable debt. Although callable debt is a standard feature in residential mortgage-backed securities (RMBS), call protection is a standard contract feature on virtually all mortgages contained in a CMBS pool, insulating the prepayment risk. During the call protection period, commercial mortgages are subject to prepayment lockouts, defeasance, yield maintenance, and prepayment penalties to protect bondholders.

Prepayment lockout, the most stringent form of call protection, is the period during which the borrower is contractually prohibited from prepaying the loan. The yield maintenance requires the borrower to pay a penalty to the lender if the loan is prepaid. The penalty is calculated as the difference between the present value of the loan's remaining cash flows at the time of prepayment and principal prepayment. Under the defeasance approach, the borrower invests the amount in U.S. Treasury securities, whose cash flow equals or exceeds the remaining payments of the mortgage loan, instead of passing it through to the investors. The treasuries replace the building as collateral for the loan. The remaining cash flow structure remains intact through to the final maturity date. Prepayment penalties require the borrower to pay a fixed percentage of the unpaid balance of the loan. The penalties usually decline as the loan ages. Different prepayment penalty structures significantly affect optimal prepayment decisions and the value of delaying the option exercise. The rationale offered for these restrictions is that large underwriting costs must be recouped in order for commercial mortgages to be profitable for the originator.

Default as a put option on real estate prices

The right to rationally default on a mortgage is a put option held by the borrower to sell the collateral property to the lender in exchange for abandoning payments. In other words, default is equivalent to exercising the put option imbedded in the mortgage contract. As a result, the lender, not the borrower, bears the ultimate cost of default since a rational borrower may find it financially optimal to simply default from their loan under certain circumstances, thereby forcing ownership on the lender. Under this interpretation, default is the result of a rational profit-maximizing decision by the borrower.

Because of the call protection in commercial mortgages, the put option to default dominates the call option to prepay. The lockout provisions that essentially preclude prepayment in the commercial mortgage market are so prevalent that much of the research (Titman and Torous [1989], Vandell [1992], Riddiough and Thompson [1993]) simply assumed away prepayment risk to focus exclusively on default risk. Titman and Torous [1989] empirically investigate a contingent claim model of commercial mortgage pricing, and find that the observed default premia for a sample of non-prepayable fixed-rate mortgages can be explained by the model. They purely focus on the valuation effect of default risk.

A mortgage, therefore, can be written as a combination of a riskfree bond (with the same term maturity and coupon as the loan) and a default option or put contract sold to the borrower. We can also envision the mortgage as the combination of the property value and a call option sold to the borrower. The fundamental relationship between property value and mortgage default behavior is the backbone of this thesis.

The value of the mortgage liability can be written in the form:

$$M_t = B_t - P_t = V_t - C_t$$

where,

B_t is the value of the riskfree bond

P_t is the value of the default put option to the mortgage holder

V_t is the value of the property

C_t is the value of the call option to the mortgage holder

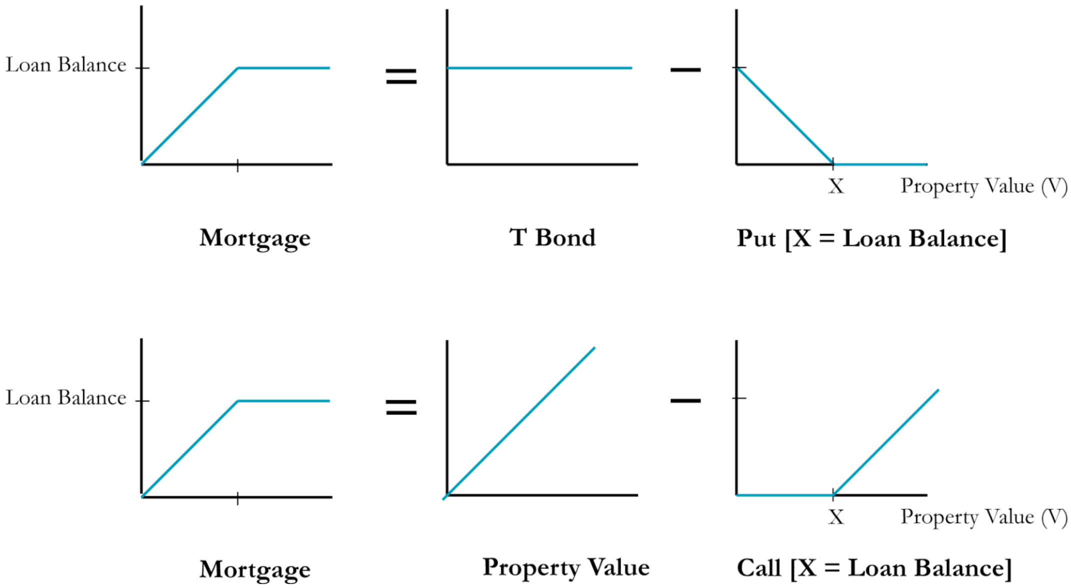


Figure 1: Mortgage Option Theory

The default risk can also be considered as a commercial mortgage interest rate spread over the risk free rate, relating the spread to the default option value.

Mortgage Yield = T Bond Yield + Default Premium

In CMBS, the equivalent benchmark for riskfree yield is the LIBOR swap yield.

CMBS Yield = LIBOR Swap Yield + Default Premium

Therefore, CMBS spread to swaps, which is the difference between CMBS yield and swap yield, reflects the capital market's evaluation of default risk in CMBS.

Figure 1 clearly illustrates that a mortgage can be valued separately as a combination of a fixed income instrument and a put option. A put option is a contingent claim on the underlying asset of the mortgage, the commercial real estate itself. Therefore, this thesis focuses solely on credit risk in CMBS as determined by the put option to default on the commercial real estate. The wide use of non-recourse clause limits the borrower's liability to the real estate asset in the event of default. Since an option value depends on the volatility of the underlying asset, the value of the put option is contingent upon the commercial real estate price movement. In other words, the property price movement can be modeled as the stochastic process that governs commercial mortgage default behavior. Unlike prepayment, which is affected by interest rate movements, default behavior is affected by property price movements, as proxies in this thesis by the real estate price index.

Case and Shiller [1996] explore the idea of using index-based options driven by movements in residential real estate prices as the basis for hedging mortgage default risk. They model the relationship between house prices and default rates using a distributed lag. The results suggest that periods of high default rates strongly follow real estate price declines. They present evidence that the value of mortgage portfolios does depend on risks of price change in real estate residential markets. They conclude that mortgage holders should have strong incentive to hedge in real estate futures and options markets since the values of their portfolios depend on the current price of the collateral real estate. Although their focus is on house price index, this thesis explores the same logic for commercial property price index.

Option Pricing Approach to Mortgage Valuation

The fundamental relationship between prepayment risk, default risk, and mortgage valuation can be theoretically characterized in a framework based on option-pricing theory. Theoretical pricing models of mortgages as derivative assets give rise to a contingent claim model of commercial mortgage pricing, often termed the option-pricing approach to mortgage valuation. In the arbitrage-free perfect capital markets, commercial mortgages are treated as contingent claims, allowing mortgage value to depend on underlying stochastic processes. Mortgage valuation often uses the Monte Carlo method to simulate the price paths of these stochastic processes; this is called the forward pricing techniques. The Monte Carlo method for mortgage valuation allows for the calculation of the expected present value of future cash flow stream, discounted at the appropriate rate. This procedure requires predetermining the criteria of borrower's decision to default on the mortgage.

Many researchers have tried to identify the loan parameters that explain commercial mortgage default. The standard contingent claims approach to mortgage pricing infers that default is a function of loan attributes such as loan-to-value ratio (LTV) and debt service coverage ratio (DCR). The LTV is the ratio between the loan amount and the property value. The DCR is the Net Operating Income (NOI) divided by mortgage payment per period. The loan parameters for commercial mortgage default determine the number of triggers underlying the contingent-claim model for pricing commercial mortgages. The three stochastic processes that are modeled are term structure of interest rate, property price, and property income.

The Single-Trigger Model: Property Value

The single-trigger model identifies two sources of uncertainty: the term structure and the value of the commercial property. The interest rates follow the mean reverting process, as defined by Cox, Ingersoll and Toss [1985].

$$dr = K(\theta - r)dt + \sigma_r \sqrt{r} dz_r$$

where,

r is the current spot interest rate

K is the speed of reversion parameter

θ is the long run value towards which the spot rate is expected to revert

$\theta_r \sqrt{r}$ is the standard deviation of changes in the current spot rate

dz_r is a standard Wiener process[2]

Despite the continued stochastic disturbances, the interest rate reverts toward a trend rate θ, at a rate dictated by K.

[2] A standard Wiener process, often called Brownian motion, is defined as $\varepsilon\sqrt{dt}$.
ε is a random variable in a standard normal distribution; that is, a normal distribution with a mean of zero and standard deviation of 1.

Real Estate price movements follow a lognormal process. This means that real estate price is expected to appreciate at a constant rate, but the actual rate of appreciation is being constantly disturbed in a random walk manner.

$$dP = (r - \beta_p)Pdt + \sigma_P P dz_P$$ [3]

Where,

P is property value

α_P is the expected total return on the property

β_P is the continuous property income payout rate

σ_P is the volatility parameter of property returns

dz_P is a standard Wiener process

Thus, α_P-β_P is the instantaneous mean rate of appreciation in property value. This proportionate growth is disturbed by unpredictable events represented by the noise process dz_P. The unanticipated changes in the value of the property are assumed to be correlated with unanticipated changes in the instantaneous riskfree interest rate. With the stochastic processes described, the instantaneous correlation between changes in property prices and interest rates is ρ_{Pi}, and a partial differential equation can then be derived.

Most commercial mortgage pricing studies use this backward single-trigger[4] framework to estimate default risk premium on commercial mortgages (Hilliard, Kau, and Slawson [1998]). In this approach, borrowers are assumed to behave rationally when making default decisions, defaulting only when the mortgage value exceeds the property value; therefore, a key predictor of default incidence is LTV at loan origination.

Fabozzi [2001] proves that the default probability for a loan increases as LTV increases. Simultaneously, the cost to mortgage holders of default also rises; therefore, the correlation between default losses and LTV is non linear, and the mortgage holders would ideally need a

---

[3] See footnote 2.

[4] Single-trigger model is defined as a property price CMBS default model, relying only on LTV. A double-trigger model is defined as property price and property cash flow CMBS default model, relying on both LTV and DCRR.

dynamic hedge. Fabozzi further emphasizes the greater importance of property value and the LTV in assessing credit risk of CMBS, than a temporal phenomenon such as the frequency of default, as determined by DCR. His studies show that the relationship between DCR and default probability is weaker than that between LTV and default probability. One possible explanation is that the borrowers can negotiate the payment rescheduling and the debt restructuring with the lenders.

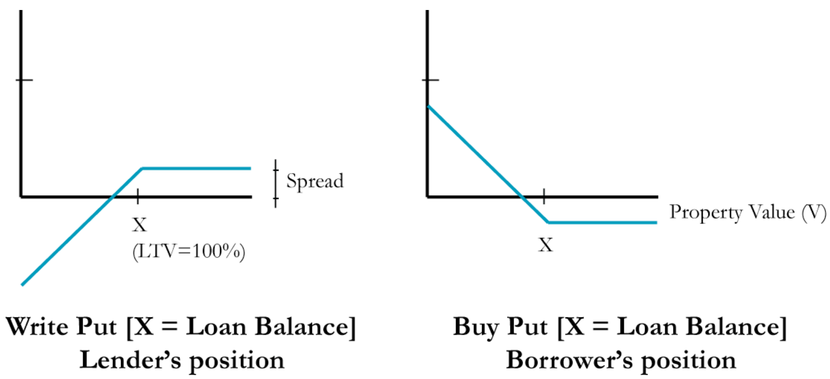


Figure 2: Put as Default Option

If LTV at origination of a particular commercial mortgage is 80%, mortgage investors would be considered writers of 20% out-of-the-money[5] put options on underlying real estate. Once the market value of the property decreases more than 20% of its beginning value, the embedded put option on the loan becomes in-the-money, and provides an incentive for the borrower to default. If the property value does not fall below 20%, writers of the put option realize a gain, which often equals to the put spread. The property value is equal to the loan value when the put is at-the-money. Therefore, the default trigger, also referred to as the strike price, is at LTV equal to 100%.

---

[5] In option theory, the term "out-of-the-money" refers to an option position that would lead to a negative cash flow if exercised immediately. If P is the property price and X is the strike price, a put option is out of the money when P>X, is in-the-money when P<X, and is at-the-money when P=X Clearly, an option will be exercised only if it is in the money.

The Double-Trigger Model: Property Value and Property Income

Recent contributions to the literature recognize the limitation of using a single-trigger, property price-only, commercial mortgage default model. The single-trigger model ignores the borrower's cash flow position (solvency), and assumes that the borrower will default whenever the equity position falls below a critical level, even if the property generates sufficient net operating income to cover expenses and debt service. Recent literature, such as one done by Ciochetti, Deng, Gao, and Yao [2002], has focused on a structural model that incorporates both cash flows and property value for predicting default. In this double-trigger model, the borrower must incur a negative cash flow position in addition to an adverse net equity position to trigger default. They conclude that an asset-value based model alone cannot fully explain default incidence.

Tu and Eppli [2003] support the findings of Ciochetti et al. with a double-trigger model that interactively uses property prices and property cash flows to estimate default risk, assuming that a borrower's default decision is based on both contemporaneous measures of LTV and DCR. They also consider the balloon risk, the risk that the borrower may not be able to refinance the mortgage at maturity. Their Monte Carlo simulation results reveal that mortgage-pricing models based solely on LTV overestimate the historically observed probability and risk premium of default.

Ambrose and Sanders [2003] attribute default to both factors: the frequency of default, which is driven by DCR, and the loss severity, which is driven by the LTV at the time of underwriting. They state that neither measure alone is conclusive. Even if DCR falls below one, the borrower should be able to sell the property for more than the value of the loan without defaulting. On the other hand, if LTV increases to 100%, as long as property cash flow is not less than the debt service amount for an extended period of time, the borrower has little incentive to default, and thus forgo the value of the put option.

The double-trigger model includes property cash flow as another stochastic variable. Period property income is determined by multiplying the property value by the property income payout rate. Since interest rate and are correlated, changes in payout is specified as a function of the contemporaneous interest rate, which is assumed to follow a lognormal process.

$$d\beta_P = \lambda dr + \sigma_\beta dz_\beta$$ [6]

Where,

β_P is the property income payout rate

λ is an estimated parameter

r is the interest rate

σ_β is a volatility parameter of the payout rate

dz_β is a standard Wiener process

Unlike the property value model, which has a time-decreasing threshold (mortgage balance), the cash flow model has a constant default threshold (amortized mortgage payment). Given the parameters of the stochastic processes, spot interest rates, property values, and property income are calculated for each period. These variables are then used to project borrower default behavior.

This chapter introduces and defines the default determinant of a mortgage as an embedded put option that derives its value from the underlying property. The put option on the mortgage is the basic building block of the emerging credit derivatives designed to hedge the default risk of CMBS. This thesis extends the property value-based mortgage fundamental concept to the CMBS market, relying on commercial property price movement as the determining CMBS default risk. This thesis will also assume that commercial borrowers are value maximizers that default only when doing so is optimal from a purely financial perspective. Therefore, the borrowers default only when the property value falls below the mortgage value.

The concept of the proposed synthetic product is to provide an investor a payoff that would hedge against the cash deficiency from defaults, as if the investor owns a portfolio of put

[6] See footnote 2.

options on commercial mortgages underlying a CMBS portfolio. This thesis develops a methodology that utilizes commercial property price indices as the underlying indices on which the proposed CMBS derivative, referred to as property price index-based CMBS synthetic, is based. The next chapter will discuss the two most widely used types of CMBS credit derivatives: the single-name CDS and the CMBS index-based CMBX.

Chapter Two: The Evolution of CMBS Credit Derivatives

A credit derivative, often referred to as a "*synthetic*", is a bilateral financial contract that transfers credit exposure from one counter party to another without a transfer of the underlying asset. Its value is derived from the performance of the asset it references. Credit derivatives isolate credit as a distinct asset class, essentially unbundling the credit risk component from the market risk component of the underlying referenced security. The simplest and most widely used form of credit derivatives is the credit default swap (CDS), which is conceptually similar to credit insurance, an insurance policy associated with a specific loan against credit risk for which the purchaser of the insurance pays a regular premium.

Credit derivatives have many practical applications. They allow investors to enhance credit risk management capabilities, reduce transaction costs, express credit views, and target risk exposures in their portfolios that cannot be created through investing exclusively in traditional cash markets. As a speculative tool, investors can gain leverage by harnessing and exploiting commercial mortgage and real estate expertise. Investors can then focus and amplify their abilities to act based on their analyses of opportunities in the commercial mortgage loan and CMBS markets.[7]

While the initial growth of credit default swaps was in single-name trades, recent growth has expanded to portfolio-level trades in new sectors such as mortgage-backed securities (MBS) and asset-backed securities (ABS). CMBS is the latest asset class to join the family of credit derivatives indices. The latest credit derivatives products allow systematic market exposure to a basket of credit default swaps through index trades in a particular sector, such as corporates (CDX), home equity loans (ABX), and commercial mortgages (CMBX) markets. The creation of these CDS indices has enabled market participants to target exposure to specific rated tranches of their chosen sectors.

---

[7] Manzi, J., D. Berezina., M. Adelson. "Synthetic CMBS Primer." Nomura Fixed Income Research. September 5, 2006

Single-Name CMBS Credit Default Swap Synthetic

Synthetic CMBS deal structures involve the removal of the credit risk associated with a pool of commercial mortgages by means of a credit default swap. A credit default swap is an agreement between two counterparties in which one party wishes to gain exposure to a particular reference asset (i.e., *"the protection seller"),* while the other party wishes to eliminate exposure to the same asset (i.e., *"the protection buyer"*). Selling protection using a CDS differs from actually owning a cash bond in several ways. First, the mortgage borrower is not directly involved in a CDS contract. Instead, the borrower is merely referenced in a private contract between the protection seller and buyer. A CDS can be created even if a certain reference asset is not available in the cash bond market. Also, A CDS contract allows the shorting of credit risk by buying protection in a contract. Furthermore, as a derivative contract, a CDS contract does not require an initial investment. The synthetic form of credit risk offers flexibility for hedging or expressing a view.[8]

The development of CDS on CMBS highly depends on consistent, reliable contract standardization to enhance liquidity thereby promoting buyers and sellers to enter into offsetting trades and improve the efficiency of closing out transaction. The International Swaps & Derivatives Association (ISDA) has been at the forefront in the credit derivative trading technology and template standardization of asset-backed securities. Since the standardized documentation in June 2005, single-name CDS market has grown tremendously. Liquidity improved as trades may now be initiated with one counterparty and closed prior to maturity with another. CMBS contracts are designed differently from the traditional CDS contracts in the corporate world. They have been tailored to accommodate certain nuances specific to CMBS.

[8] Whetten, M., and J. Manzi. "The CMBX: the Future is Here." Nomura Fixed Income Research. March 23, 2006

The Basic Deal Structure

A credit default swap enables the protection buyer to isolate and reduce its credit exposure associated with a pool of commercial mortgage, and the protection seller to synthetically gain exposure to the same entity. The protection buyer pays a periodic fixed spread called a default swap premium to a protection seller over the life of the transaction, in return for which the seller will make a payment on the occurrence of a specified credit event to offset the negative price result. The spread is usually quoted as a basis point multiplier of the benchmarked notional value of the reference entity. As demand to buy protection increases, so does the required spread, indicating there is a greater default probability. The economic payoff to the protection seller is the decline in market value from the par due to the credit event, generally equal to the loss if the cash bonds were held by the protection seller.

In a CMBS, the reference asset is essentially a basket of CMBS securities. A cash flow exchange mechanism of a default swap is illustrated below:

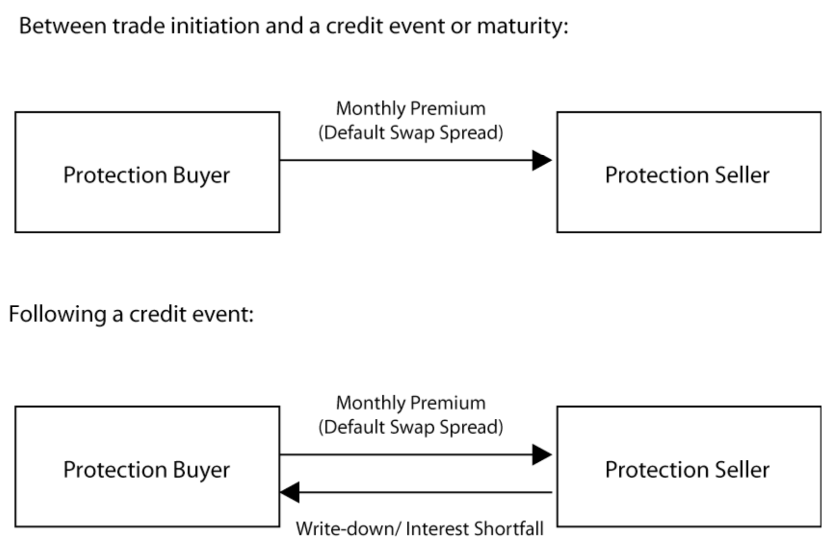


Figure 3: CDS Cash Flow Exchange

The Credit Events or Floating Amount Events

Credit events for CMBS are intended to capture any events that affect the cash flow of the reference asset. They are the underpinning mechanics of a synthetic exposure. Unlike cash security, where the investor owns the security whether it is impaired or in default, the definition of credit events governs contingent credit protection by the synthetic investor. In most CMBS contracts, there are three credit events:

1. *Principal write-down*

   A principal write-down refers to a reduction in the outstanding balance of the reference CMBS pool. In a principal write-down, the protection seller compensates the protection buyer in the amount of the write-down. The contract ends if the reference asset is fully written down. A partial write-down amount proportionately reduces the notional value of the reference asset that the default swap premium is based on. In this case, the protection buyer would continue paying the premium on the proportionately reduced notional amount, and the protection seller pays nothing.

2. *Principal shortfall*

   A principal shortfall occurs when the reference CMBS fails to pay off principal by the legal maturity date.

3. *Interest shortfall*

   An interest shortfall occurs when the interest passed through from the underlying mortgage loan is less than the interest to be paid on the security. CMBS subordinate classes often suffer from interest shortfalls. There are three methods that determine the mechanic of the compensation, which relates to the protection seller's limit of economic exposure, in the event of an interest shortfall.

   - Fixed-cap

     In a Fixed-cap method, the protection buyer is compensated for interest shortfall up to the default swap premium. The protection buyer is not compensated for the interest shortfall amount that exceeds this premium. In the case that the shortfall is equal to or higher than the swap premium, the premium paid by protective buyer offsets the interest shortfall amount, resulting in net zero payment.

- Variable cap

In a Variable cap method, the protection buyer is compensated for interest shortfall up to a floating rate such as LIBOR plus the default swap premium.

- No cap

In this method, the protection buyer is compensated for the entire amount of interest shortfall. While the economics of the no cap method more closely resemble that of owning a cash bond, it may result in the protection seller repaying excess shortfalls out of his own pocket.

The 3 methods of interest shortfall treatment in a CDS is illustrated below[9]:

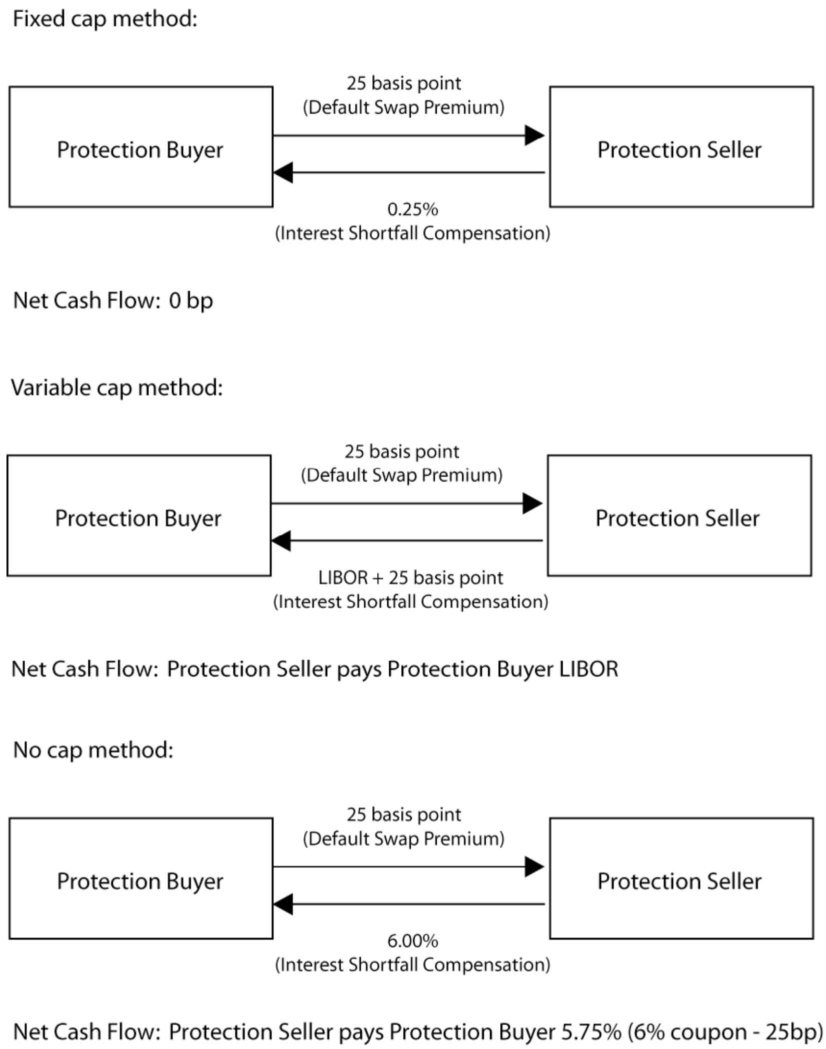


Figure 4: CDS Interest Shortfall Mechanisms

Under each method, any subsequent recovery of interest shortfall and principal write-down from the underlying security triggers a pro-rated reimbursement back from the protection buyer to the protection seller. The amount of this reimbursement is called *"Additional Fixed Payment."*

[9] The assumption is that the underlying security has coupon payment of 6%. The CDS has a premium of 25 basis point. For simplicity, the assumption is that the underlying security is suffering 100% interest shortfall.

The Pay-As-You-Go (PAUG) Format

The CDS contract in CMBS does not terminate on the occurrence of a credit event. Instead, it is structured on a "Pay As You Go" (PAUG) basis, which involves ongoing two-way payments over the life of the contract between the counter parties. The PAUG format is intended for the CDS contract to closely mirror the cash flow of the reference bond. A PAUG may also include a physical settlement option. The physical settlement option allows the protection buyer to terminate the contract before maturity by delivering the reference asset. However, if the protection buyer delivers the reference asset for a portion of the CDS notional, the PAUG contract remains effective for the remaining portion of the notional amount. This ISDA credit derivatives documentation standardization of the PAUG basis, introduced in 2005, is one of the major reasons for the dramatic growth of CMBS CDS.

CMBS Index-Based Synthetic: The CMBX

The liquidity of the CMBS credit derivatives, like other derivatives products, highly depends on the availability of a benchmarked index. One of the most significant events in the CDS market was the launch of global indices for CDS of corporate credit risk in 2004, which remarkably increased trading volumes and provided a strong platform for other index CDS products. Daily trading of transparent portfolios and quick dissemination of pricing information led to greater liquidity. Since then, the total notional amount in the credit derivatives market has more than quadrupled. The recent growth has finally expanded into a new sector: the CMBS market.

CMBS is the last major fixed-income asset class to join the family of CDS index derivatives. This section introduces a newly developed CDS index derivatives called CMBX[10], a portfolio-based use of synthetic CMBS. The introduction of the CMBX indices in March

[10] CMBX was launched by Markit Group Ltd., and CDS IndexCo LLC. Markit Group is a provider of independent mark-to-market pricing and valuations, and CDS IndexCo is a consortium of 16 investment banks licensed as market makers. The market makers in the new CMBX index are Bank of America, Bear Stearns, Citigroup, Credit Suisse, Deutsche Bank, Goldman Sachs, J.P. Morgan, Lehman Brothers, Merrill Lynch, Morgan Stanley, Nomura International, RBS, Greenwich Capital, UBS, and Wachovia. Markit serves as the Administration and Calculation Agent for the indices.

2006 is viewed as a potential catalyst for a tremendous growth of the market.[11] Instead of entering into separate single-name credit default swap transactions for each of the CMBS pool owned, an investor can enter into a contract based on the nationally diversified CMBX, which is fairly representative of the overall CMBS credit sector performance. CMBX provides CMBS investors with a more efficient and standardized tool for trading and structuring an exposure to a diverse pool of assets.

The Construction of CMBX

CMBX is a tradable synthetic family of indices consisting of six sub-indices, each based on 25 CMBS deals issued within the past two years. The reference obligation is a pool of pass-through securities backed by a pool of at least 50 separate fixed-rate commercial mortgages from at least 10 unaffiliated borrowers. The six index tranches are based on the rating of the reference obligations, rated AAA through BBB-. A new series of CMBX is issued every six months. The CMBX reference entities are selected through an algorithm that identifies the deals with a minimum size of $700 million, and which satisfy some additional diversity requirements. For example, no state can represent more than 40% of the properties securing the transaction, and no single asset type can represent more than 60% of the properties. These requirements ensure the diversity of the CMBX.

The Mechanics of CMBX

The CMBX market quotes are based on spread rather than price. Protection buyers pay monthly premiums based on a fixed rate, pre-determined on each index. The buyer and the seller of protection settle the net present value when they enter into a contract. For example, if the quoted spread is below the fixed rate, the protection seller pays to the protection buyer, and vice versa. Over the life of a contract, the protection buyer pays the fixed rate amount to the protection seller, based on the current notional amount of the index.[12]

---

[11] Whetten, M., and J. Manzi. "The CMBX: the Future is Here." Nomura Fixed Income Research. March 23, 2006
[12] Whetten, M., and J. Manzi. "The CMBX: the Future is Here." Nomura Fixed Income Research. March 23, 2006

The CMBX index adopts the standardized PAUG template with the three floating amount events— principal write-down, principal shortfall, and interest shortfall— as published by the ISDA. The index uses the fixed cap method for the interest shortfall event. The CMBX index contract does not include a physical delivery option for protection buyers.

An investment in a tranche of the CMBX index is technically analogous to entering into a separate credit derivatives transaction on each of the 25 underlying reference obligations. In other words, the index represents aggregate diversified performance of a basket of single-name CMBS credit default swaps.

Applications of CMBS Credit Derivatives

Hedging credit risk

CMBS CDS contracts can be used as a hedging tool by investors and dealers, both against general spread widening and the potential negative credit performance of an individual line item. CMBS issuers and conduits buy the contracts to hedge the pipeline of loans they hold awaiting securitization. Conduits historically hedge the risk of fluctuating Treasury rates and fluctuating interest rate in the futures and options market, but they had no means to hedge against the risk of CMBS credit spreads widening before the introduction of CMBS CDS. Financial institutions looking to reduce loan portfolio risk following mergers and acquisitions, and banks looking to reduce credit risk of commercial mortgages are also potential protection buyers.

The CMBX provides a more efficient tool for trading and hedging systematic market-wide commercial mortgage credit risk, particularly for investors and hedgers with a diversified CMBS portfolio. Investors can also separate spread risk and credit risk of the CMBS market by rolling into a new index series every six months.

Expressing credit views

CMBS CDS provide investors with convenient means of expressing credit views on the CMBS market. In the CMBS cash market, where shorting a security down the credit curve is not a practical option, an investor looking at a questionable credit can only refrain from buying. With CMBS CDS, investors have the option to convert their negative credit outlook into a position in CDS. This strategy allows investors and B-piece buyers to be selective about CMBS exposure, leveraging their underwriting capabilities.

While the single-name CMBS CDS allows investors to express their credit views towards specific bonds, the approach is inefficient for investors who wish to express a market-wide view. With the CMBX, investors can take advantage of any expectations of market-wide spread tightening or widening. They can also express market-wide rating-specific credit views.

Investors can also buy protection in one CMBX-rated index while sell protection in another CMBX-rated index, expressing a market-wide relative value opinion. Hedge funds may take a highly leveraged and more aggressive position, such as long-short strategy, to exploit relative value between the general market and specific deals based on rating, vintage, and other attributes. They would likely be short-term players, taking advantage of temporary price dislocation in the CMBS market.

Alpha harvesting

The broad-based exposure of CMBX allows investors who benchmark against a CMBS index to hedge their position without being influenced by bond-specific credit concerns. Investors can spend their time harvesting alpha through careful bond selection and either buying protection or selling under-performing cash bonds or selling protection or buying over-performing cash bonds. This strategy would help investors to outperform the given index.

The broad-based exposure of the CMBX eliminates specific bond-level basis risk, and provides a more operationally efficient and low cost method of sector rotation trading. The CMBX indices can be used for various relative values strategies. One strategy is to exploit relative value between the general market and specific deals by taking a long-short position in single-name CMBS CDS and a CMBX index.

Portfolio ramp-up

CDO managers would use the synthetic for managing their portfolios, either for a quick ramp-up or by engaging in a long-short strategy. The ability to choose line items to sell protection on without having to necessarily rely on the output in the new issue market makes CDS contracts a valuable tool for CDO issuers. They can source CMBS collateral purely synthetically or in conjunction with cash securities through a hybrid cash and synthetic deals, taking advantage of asset availability and market spread levels.

The availability of the CMBX has resulted in increased efficiency in synthetically trading and transferring the credit risk of commercial real estate market. The readily available public information on construction and calculation of the index led to increased investors' confidence and ensure transparency. However, while the tradable CMBX is a great broad-based index of CMBS for hedging against default behavior of a diversified pool of mortgages, it is not well tailored to a particular investor having defined exposure to certain markets.

Chapter Three: The Property Index-Based CMBS Synthetic

A CMBS pool is composed of commercial mortgages collateralized by a unique blend of properties. While the tradable CMBX is a great broad-based index of CMBS for hedging against default behavior of a diversified pool of mortgages, it cannot be custom tailored to a particular investor having defined exposure to certain markets. This thesis proposes that a weighted customized composition of commercial property indices, each representing different property types and geographic locations of underlying properties in the CMBS pool, can more closely match real estate exposure of a particular CMBS portfolio. The key assumption of this thesis hinges on the correlation between CMBS default behavior and commercial property value. This thesis focuses on commercial property price index movements as the major factor in explaining commercial mortgage termination behavior.

The proposed property price index-based CMBS synthetics are a set of tranche-specific indices that could in principle be traded directly as a derivative contract between protection buyer and protection seller. The synthetic is designed to closely mirror the cash flow losses due to defaults[13] of the particular market exposure in a portfolio of cash CMBS bonds. Such a trade may also permit an investor to artificially construct a pool of credit exposures that might not be obtainable in cash form. The method of payment can take the form of either a fixed premium like a credit default swap, or an upfront payment like an option.

For the contract to be successfully executed, there needs to be demand on both the short and the long side of the synthetic. The long side consist of protection buyers who either have a CMBS portfolio they want to hedge, or who want to make a bet on a commercial real estate market decline. The protection buyers can be any CMBS market participants ranging from hedge funds, CMBS trading desks, and banks that have concentrated credit exposure in certain market sectors.

---

[13] In this thesis, we shall refer to the cash flow losses due to defaults of properties underlying mortgages in a CMBS as the "*credit loss*" and such risk as the "*credit risk*." The term "*credit risk*" and "*default risk*" may also be used interchangeably.

The short side consists of protection sellers who have positive views towards a particular commercial market, since the protection sellers would only gain from the contract if the premium offsets the payoffs of the synthetic. Another type of protection sellers are those who want to invest in a particular CMBS market but do not have access to it. The protection sellers can assume the default risk of owning a particular tranche of a particular CMBS pool.

This thesis hypothesizes that the proposed synthetic property price index-based derivatives would provide a more tailored hedging alternative for CMBS investors with market and property-specific default risks. The model in this thesis relies on some stylized assumptions, but it offers a proof of concept for the product at a basic level, and provides a useful methodology for further studies.

The first part of this chapter presents some challenges and concepts that lay the basis for structuring property price index-based credit derivatives. The second part of the chapter describes the stylized assumptions of the analysis. The third part illustrates the methodology used to model the proposed synthetic product.

Important Considerations for Index-Based Derivatives

Basis Risk

Basis risk occurs when the desired object to be hedged is different from the underlying asset of the hedging tool used. It is the risk that the synthetic exposure does not perfectly track the desired exposure it was designed to replicate. Both the CMBX and the proposed property index-based CMBS synthetic have some kind of basis risk. The underlying assets of the CMBX are 25 biggest and most recent CMBS deals, which undoubtedly represent accurate default behavior of a diversified CMBS pool. However, it suffers from the basis risk that an investor's CMBS portfolio exposure might not be diversified, and thus not well represented by the general market average index. On the other hand, the underlying asset of the proposed synthetic product is a set of commercial property indices. Basis risk is also present in this model, as the underlying commercial property prices are not perfectly correlated with actual CMBS default behavior. However, the index exposure to commercial market sectors might act as a better surrogate for the CMBS portfolio exposure. An example of commercial property indices that might reduce the overall basis risk of the property index-based synthetic is explained in the next section.

The Use of Real Capital Analytics Transaction-Based Indices

By unbundling a CMBS pool into property type and market-specific tranches, a tailored composition of commercial property price sub-indices can be a better surrogate for structuring property price index-based synthetics. This strategy has numerous applications for investors whose CMBS allocation has concentrated exposure, not well represented by the diversified CMBX. Imagine, for example, an investor whose CMBS portfolio is heavily weighted on a Southern office market who would like to hedge his portfolio from credit default risk. The CMBX might prove to be too diversified for his exposure. His Southern office properties may be better represented by the transaction-based property price index such as the Real Capital Analytics (RCA) index. The property type and geographic sub-indices prove to be very suitable for customizing his exposure into a weighted fraction of a set of these indices. Another advantage of using the RCA property price index-based is its tremendous wealth of commercial real estate investment market data and analysis inherent in the indices. This should gain investors' confidence and improve trading liquidity, as the underlying indices have already proven to be widely accepted benchmarks in the general real estate market.

There are two methods of constructing property price indices: the transaction-based method and the appraisal-based method. An example of a repeat-sales transaction-based index is based on the RCA database developed by MIT/CRE.[14] An example of the appraisal-based index is the NCREIF property price index. The transaction-based method more accurately represents the true property value, as it avoids the smoothing and lagging of commercial price movements in the index of the appraisal-based method. A repeat-sale index includes property sales into account at the earliest possible time, rather than as supporting data in appraisals, which introduces the possibility of a lag. In effect, the appraisal-based indices are subjective and myopic. Although the transaction-based index is more volatile, it leads the appraisal-based index in timely reporting the property value. Thus, it would more accurately capture the short-run volatility and downturn market, tracking the actual defaults in properties.

A set of 29 RCA transaction-based indices has been developed specifically for derivatives trading. It includes national, regional, and MSA-level for four property usage type sectors, as well as top-10 MSA indices by sector. The capability of an investor to compose a weighted index from these sub-indices presents a new possibility for the property index-based CMBS synthetic. In this thesis, we will refer to the customized exposure-weighted index as the *"CMBS portfolio index"* or simply the "*portfolio index."*

Option on a pool vs. Pool of options

After reading the literature review in Chapter One, one may be tempted to conclude that a CMBS investor can eliminate portfolio default risk simply by purchasing real estate put options to cancel the risk incurred from embedded option investments. However, the

---

[14] Refer to http://web.mit.edy/cre/research/credl/tbi.html

strategy is flawed. Mortgages in a CMBS pool are structured into tranches, each with a unique risk profile according to its payment characteristics. The CMBS investor holds in effect a portfolio of options with different strike prices, corresponding to the different tranches in a CMBS structure. A portfolio of options is not the same as an option on a portfolio.

Recall that in traditional mortgage pricing literature, default risk of a mortgage can be calculated as a put option on the collateral property. The logic then follows that default risk of a CMBS is the average of all individual put options on the corresponding collateral properties in the CMBS. These put options have different strike prices corresponding to different LTV of the individual mortgages. More importantly, they also depend on different commercial property price movements of the underlying properties. Therefore, an adjustment needs to be made to account for the individual property price movements.

Adjustment for the Downside Idiosyncratic Risk of the Collateral: the *"k"*

The proposed property price index-based CMBS synthetic has characteristics similar to a put option because its value is contingent upon an extreme downside outcome; its payoff is asymmetrical. If all of the properties underlying all the loans in a given CMBS pool evolved in an exactly identical manner over time, then a portfolio of simple put and call options on a weighted index tracking the weighted average market the properties were located in would indeed suffice to provide a synthetic hedge to the CMBS securities. (A given CMBS tranche would be hedged by a specific combination of puts and calls on the index, with different strike prices to tailor the exposure to the specific tranche). But in reality the individual properties underlying the loans in the pool will evolve to some extent independently, reflecting the realization and accumulation of *idiosyncratic risk* – property specific components of the property price movements that do not echo the overall index. A given property underlying a mortgage in a CMBS pool may do poorly and default even though the overall market and the market-weighted index increase in value. Therefore, a simple option on the market-weighted index would fail to reflect the actual default risk of some underperforming properties in a CMBS pool.

In our proposed synthetic hedge product, we address this issue by conceiving of the property pool as being segregated into components corresponding to each tranche in the CMBS, with each component tracking below the customized market-weighted index, referred to as *the portfolio index,* by a constant rate of drift, *"k"*. The *k* allows the synthetic hedge product to reflect the downside idiosyncratic drift of the underperforming properties that are relevant (ex post) to a given tranche. To an approximation, the drift differentials below the portfolio index reflect the idiosyncratic evolution of individual properties that can result in defaults in lower tranches even when the overall market, the portfolio index, and mortgage pool, are all doing fine.

The lowest-rated tranche is most sensitive to credit losses due to defaults; therefore, the effective downside idiosyncratic volatility relevant to it is the highest, implying a greater negative drift differential below the portfolio index (the highest constant *k* rate below the portfolio index). The greater the (negative) drift differential, the lower (more junior) the CMBS tranche hedged by the synthetic. By basing the synthetic hedge on a specific chosen value of the differential drift constant, the synthetic hedge is matched (approximately) to a given tranche of the CMBS that is being targeted. We shall refer to the synthetic with its *k* value thusly defined as a *"synthetic tranche"*.

We can then structure the payoffs to the synthetic product based on the portfolio index reduced by the differential drift so as to mimic to a considerable degree the credit losses incurred by a specific CMBS tranche. The drift differentials in effect adjust the default boundary of the portfolio index. A method a hedger might use to calculate the *k* value he wishes to employ in the synthetic is explained in more detailed in this next section.

The Monte Carlo Method

Although the proposed synthetic product is designed to be based on the RCA-like indices, this thesis uses a simulated commercial property price process as a proxy for the index. The reason is that the RCA index only dates back to 2001, when the overall real estate market has ever since been a bull market. Therefore, the RCA index short history would paint an overly rosy picture of CMBS, and would not be indicative of the possible future defaults. After all, considering that the property and capital markets are late in their cycles, and that the recent underwriting has been more risky, it would not be accurate to extrapolate the past into the future.[15]

Instead, this thesis uses Monte Carlo simulation analysis to simulate independent commercial property price paths representing all possible future paths for the commercial property index, generated from random variables. This method is used to simulate the behavior of the synthetic product and of the actual CMBS pool being hedged. The simulation method would also seek to examine the hedging qualities of the synthetic, including cash flow correlation between the synthetic payoffs and the hypothetical CMBS pool actual losses, and hedge ratio. The result of the analysis would reveal the best *k* value an investor might want to use for the synthetic.

Monte Carlo method of derivative valuation relies on two powerful statistical engines Khinchine's Strong Law of Large Numbers and the Lindeberg-Levy Central Limit Theorem.

KhinChine's Strong Law of Large Numbers states that the if the sample $X_1, X_2 \ldots, X_n, \ldots$ are independent and identically distributed, with $E(X_i) = \mu$ (a finite number) for each i, then with probability one:

$$\overline{X}_n \equiv \frac{1}{n} \sum_{i=1}^{n} X_i \to \mu, \text{ as n} \to \infty$$

[15] Moody's Investors Service, US CMBS and CRE CDO 1Q 2007 Review: Conduit Credit at the Turning Point?

In other words, the sample mean approaches the true mean as the sample size increases.

The Lindeberg-Levy Central Limit Theorem states that the sample mean X_1, X_2, …, X_n, … are independent and identically distributed with $E(X_i) = \mu$, and if $var(X_i) = \sigma^2$ (a finite number) for each i, then:

$$\frac{\sqrt{n}(\bar{X}_n - \mu)}{\sigma} \rightarrow Z \approx N(0,1), \text{ as n} \rightarrow \infty$$

In other words, the sample mean is approximately normally distributed with distribution $N(\mu,\sigma^2/n)$ for large n.

The Stylized CMBS Pool: Simplifying Assumptions for the Analysis

Commercial Mortgages Underlying the Stylized CMBS Pool

The stylized CMBS pool in our analysis represents the actual CMBS pool in an investor's portfolio that is being hedged by our synthetic product. Consider a stylized CMBS pool that consists of 80% average LTV, interest-only commercial mortgages whose only risk is the default risk. Since there is no principal payment, the loan balance is constant throughout the life of the mortgage. The CMBS pool is composed of 100 commercial mortgages, each secured by a single income-producing property. For simplicity, all mortgage contracts and initial property values in the pool are identical. The study assumes that each mortgage is non-recourse and that there is no cross-collateralization among individual mortgages in the pool with other mortgages either within or outside the pool. The mortgage contains a lockout provision that prohibits prepayment during its life, thus the thesis assumes away the interest rate risk. Assume also that there is no transaction costs in the event of default.

The Stylized CMBS Pool Subordination Structure

The subordination structure of our stylized CMBS pool consists of three rated tranches: B, BBB, and AAA, as summarized below.

CMBS Structure	
Total pool par	100
B par	10
Subordination	0
BBB par	30
Subordination	10%
AAA par	60
Subordination	40%

Table 1: CMBS Pool Subordination Levels

The Default Boundary of the CMBS Pool

Our methodology adopts the single-trigger default model that focuses on the real estate default risk. Recall from Chapter One that this default model accounts for property value as the prime determinant for default. Under this assumption, the borrower defaults rationally whenever the property value falls below the loan balance.

The lower-rated tranches provide credit support for the higher-rated tranches. Credit loss due to defaults, which is the difference between mortgage loan balance and property value at the time of default, is deducted from the lowest-rated to the highest-rated tranche. Defaults reduce the face value of the lowest-rated principal-based tranche first. If credit losses cumulatively exceed the face value of the lowest-rated tranche, losses are then allocated to the next highest-ranking tranche until fully allocated.

The stylized pool consists of 60% AAA tranche, 30% BBB tranche, and 10% B tranche. The default order and the subordination structure of each tranche can be converted to a tranche-specific LTV, which represents its default trigger. The lowest-rated tranche would be considered to have the highest LTV, as it is first to default. The higher the LTV, the less property value needs to fall in order to trigger credit losses in the tranche. The B tranche is the first-loss tranche, as it has no credit support; the default risk is the same as the overall pool at 80% LTV. Therefore, its default trigger or ceiling is $100. The BBB tranche has credit support of 10%, therefore the pool would have to suffer 10% of the loan balance before the BBB tranche would be impacted. This means that the underlying property value would have to fall to $90/125 = 72% of its initial value before the BBB tranche loses its par value. In other words, it has default risk similar to a mortgage with a 72% LTV ratio. The AAA tranche has credit support of 40%, therefore the pool would have to suffer 40% of the loan balance before the AAA tranche would be impacted. The AAA tranche has default risk similar to a mortgage with a 48% LTV ratio ($60/125).

The CMBS Synthetic Product

Definition of the Synthetic Product

The synthetic product is designed to hedge against the credit losses of an actual CMBS pool; therefore the payoff structure and boundary of the synthetic product should approximately replicate the default structure and boundary of a referenced CMBS tranche.

To standardize and facilitate the quoting and marking of the synthetic product, a stylized index payoff trigger value is established at $100. For example, if one is trying to hedge a CMBS with an 80% LTV, the synthetic contract would be written with a portfolio index starting value of $125 ($100/0.80), so that the trigger of $100 would correspond to the portfolio index falling to 80% of its starting value.

As another example, if one is trying to hedge a CMBS with a 75% LTV, the synthetic contract would be written with a portfolio index starting value of $133.33 ($100/0.75). The goal is to standardize the synthetic contract by calibrating the synthetic payoff trigger to be $100 notional value in all the contracts.

The Calibration Process

Each synthetic tranche represents the value of the properties that will ultimately cause credit losses to a given corresponding actual CMBS tranche that is the hedge target. From this point forward, we will refer to this value as the *"collateral value."* Recall that the k rates are the drift differentials that model the downside idiosyncratic risk of each synthetic tranche. In effect, the k rates determine the sequential payoff order of the synthetic tranches in the model so as to approximately replicate the default order of the actual CMBS tranches under the same subordination assumptions. The payoff boundary of the synthetic product should therefore approximately replicate the default boundary of a referenced tranche in the CMBS pool.

Recall that for the proposed synthetic to hedge our CMBS pool with an 80% LTV, the synthetic contract would be written with the portfolio index starting value of $125. Therefore, the synthetic tranche starting value (presumed initial collateral value) is also $125. Each synthetic tranche is calibrated to have a payoff trigger of $100. This process, in some sense, is normalizing the actual CMBS tranche's default risk equivalence to an 80% LTV. This standardization will make the synthetic product easier to work with by multiple parties who use the synthetic product for different purposes. The *k* differentials act as a device to standardize the synthetic product for trading. Ideally, an investor should pick a *k* rate such that the synthetic tranche, modeled with the selected *k* rate, would hit the $100 ceiling when the actual target CMBS tranche is expected to experience credit losses.

Consistent with this calibration process and considering that the B tranche encompasses 10% of the par value of the pool, the stylized floor of the B tranche would be $90. Similarly, the BBB tranche encompasses 30% of the par value of the pool, so the stylized floor of the BBB tranche would be $70. The AAA tranche encompasses 60% of the par value of the pool, so the stylized floor of the AAA tranche would be $40. LTV equivalence is calibrated to 80%, and would be terminated once the property value falls below $40. The calibration process is summarized in the Table 2 below.

				No Calibration			**With Calibration**			
		Property Value		**LTV Equivalence**	**Ceiling Trigger**	**Floor Trigger**	**LTV Equivalence**	**Ceiling Trigger**	**Floor Trigger**	
		$								
		125								
		100	=	80% LTV	$100	$90	80% LTV	$100	$90	**B**
B	**10%**	**90**	=	72% LTV	$90	$60	80% LTV	$100	$70	**BBB**
BBB	**30%**									
		60	=	48% LTV	$60	$0	80% LTV	$100	$40	**AAA**
AAA	**60%**									
		0								

Table 2: LTV Default Trigger

The Synthetic Price Process

In this section, we present the price process underlying the portfolio index, and how the synthetic tranches are adjusted from and related to the index. The study utilizes the Monte Carlo method as the engine to simulate future possible price paths of the portfolio index. The index, M_t, is modeled as a lognormal process:

$$M_t = M_{t-1} e^{\mu(dt) + \tilde{Z}_t}$$

where,

μ is the portfolio index continuously-compounded long-run expected return net of property income; i.e., the mean rate of appreciation in property value

Since the paths are simulated in quarterly time steps, dt = 0.25 years

Z_t is the stochastic volatility of the index price path, which is defined as:

$$\tilde{Z}_t = \sigma_m N(0,1)\sqrt{dt}$$

where,

σ_m is the continuously-compounded index volatility per annum

N(0,1) is a random number under the standard normal distribution[16]

The stochastic variable Z_t creates volatility around the long-run drift rate μ of the portfolio index in each period. Together, the portfolio index growth rate is defined as:

$$g_t^m = \mu(dt) + \tilde{Z}_t = \mu(dt) + \sigma_m \text{ N}(0,1)\sqrt{dt}$$

[16] The standard normal distribution is a normal distribution with a mean of 0 and a standard deviation of 1.

Since the starting value of the synthetic tranche B_t is the same as that of the portfolio index M_t,

$B_0=M_0=\$125$

Let t = 1,

$$M_t = M_0 e^{(g_t^m)}$$

Recall that B_t is defined as a constant *k* below M_t growth rate; therefore,

$$B_t = M_0 e^{(g_t^m) - k_B(dt)}$$

In a general term,

$$B_t = B_{t-1} e^{\mu(dt) + \sigma_m N(0,1)\sqrt{dt} - k_B(dt)}$$

Therefore, each tranche is defined as:

$$B_t = B_{t-1} e^{\mu(dt) + \sigma_m N(0,1)\sqrt{dt} - k_B(dt)}$$

$$BBB_t = BBB_{t-1} e^{\mu(dt) + \sigma_m N(0,1)\sqrt{dt} - k_{BBB}(dt)}$$

$$AAA_t = AAA_{t-1} e^{\mu(dt) + \sigma_m N(0,1)\sqrt{dt} - k_{AAA}(dt)}$$

where,

$M_0 = B_0 = BBB_0 = AAA_0 = \$125$, and $k_B > k_{BBB} > k_{AAA}$

The price paths for the portfolio index and each of the tranches are simulated for 20 periods (t = 5 years) with the following CMBS portfolio assumptions:

$\mu = 1\%$

$\sigma_m = 10\%$

These assumptions should be consistent with the forecasts for the particular CMBS portfolio market exposure of the investor. A single simulation of portfolio index price path and the corresponding synthetic tranches is illustrated in Table 3 and Figure 5. This simulation happens to represent a slightly pessimistic market scenario.

Period t	Normal (0,1) _	Random Volatility z(t)	Index Growth g(m,t)	Index M(t)	B B(t)	BBB BBB(t)	AAA AAA(t)
0				**125.00**	125.00	125.00	125.00
1	-1.44	-0.07	-0.07	**117.02**	114.71	115.42	116.33
2	0.38	0.02	0.02	**119.48**	114.81	116.24	118.08
3	0.23	0.01	0.01	**121.12**	114.09	116.23	119.00
4	0.49	0.02	0.03	**124.29**	114.76	117.64	121.40
5	-0.58	-0.03	-0.03	**121.19**	109.68	113.14	117.67
6	-0.49	-0.02	-0.02	**118.69**	105.30	109.29	114.57
7	1.97	0.09	0.10	**130.71**	113.68	118.72	125.43
8	0.97	0.05	0.05	**137.24**	117.00	122.95	130.92
9	-1.13	-0.05	-0.05	**130.39**	108.96	115.21	123.65
10	-0.11	-0.01	0.00	**130.01**	106.50	113.31	122.57
11	-0.05	0.00	0.00	**130.00**	104.39	111.75	121.84
12	-1.00	-0.05	-0.05	**124.27**	97.82	105.37	115.79
13	-0.70	-0.03	-0.03	**120.51**	92.98	100.78	111.62
14	0.19	0.01	0.01	**121.89**	92.20	100.55	112.24
15	-0.80	-0.04	-0.04	**117.64**	87.22	95.72	107.69
16	-0.39	-0.02	-0.02	**115.78**	84.15	92.92	105.37
17	-0.46	-0.02	-0.02	**113.54**	80.89	89.88	102.73
18	1.17	0.06	0.06	**120.38**	84.07	93.99	108.27
19	-0.53	-0.03	-0.02	**117.69**	80.57	90.64	105.23
20	-0.53	-0.03	-0.02	**115.02**	77.19	87.37	102.24

Table 3: Synthetic Price Process Simulation

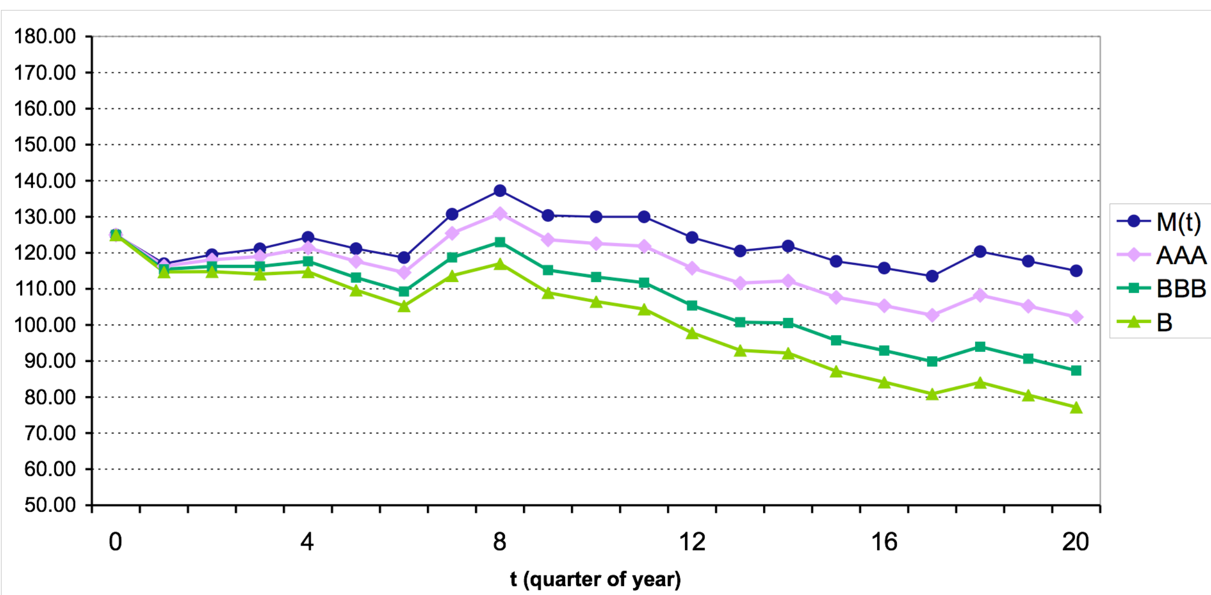


Figure 5: Portfolio Index and Synthetic Tranches Price Paths

The Synthetic Payoff Structure

	B	BBB	AAA
Ceiling	100	100	100
Floor	90	70	40

Table 4: Synthetic Tranche Payoff Boundary

Now that we understand how the synthetic tranche price paths are adjusted from the portfolio index by the *k*, let us examine more closely the payoff structure of each synthetic tranche. Table 4 above summarizes the synthetic tranche payoff boundary, which is derived from the default boundary of the corresponding tranches in the referenced CMBS portfolio.

Recall the following default behavior of the properties underlying a CMBS pool. When current property value exceeds the ceiling boundary, the borrower continues repaying the loan according to contract terms and there is no default. But whenever property value drops below its original loan value, the borrower defaults. The synthetic tranche adopts its payoff structure from the default behavior of the properties underlying the referenced CMBS pool. The payoff structure of the synthetic tranche, therefore, reflects endogenous default decision-making by individual borrowers of the underlying commercial mortgages in the CMBS pool. The synthetic tranche is essentially an index derivative that approximately payoffs within the referenced CMBS tranche default boundary, to hedge against its credit risk.

When the simulated synthetic tranche falls in its payoff boundary, the actual referenced CMBS tranche is assumed to have experienced some cash flow losses due to property defaults. Therefore, the synthetic tranche payoffs the impaired cash flow amount, and its notional par value is written down by the same amount. The payoff is the difference between the synthetic tranche's ceiling and current par value. At each period thereafter, the model tracks the synthetic tranche's minimum to date. Every time its minimum to date drops below its previous minimum, the synthetic tranche pays the difference between the previous minimum and the current minimum to date. This mechanical payoff process continues until the synthetic tranche hits the floor, and exhausts its par value. The payoff mechanism caps each synthetic tranche total payoff at its par value. In our example, the B, BBB, and AAA synthetic tranches' maximum payoffs are their par values of 10, 30, and 60 respectively. Each synthetic tranche's payoff mechanism is the same. The *k* should be calibrated such that a synthetic tranche does not start its payoff period until its more junior synthetic tranche has exhausted its par value.

The following series of tables and graphs present the individual synthetic tranches, for the same simulation scenario as the previous sections. Each graph depicts the synthetic tranche as an adjusted index from the portfolio index by a constant rate of *k*. The shaded area represents the synthetic tranche payoff boundary, as defined by the floor and the ceiling. From this graph, the payoff period can also be estimated. Each graph is accompanied by a payoff schedule. The 2nd column in the payoff schedule is the synthetic tranche price path across the 20 quarters. The 3rd column tracks the synthetic tranche's minimum value to date, which is then used to calculate the payoff in each period as shown in the 4th column. The 5th column keeps track of the remaining par value. The last column calculates the ratio of synthetic payoff in each period to the synthetic tranche notional par value. In this thesis, this ratio will be referred to as *"the synthetic payoff ratio."* The synthetic payoff ratio will be compared to the corresponding ratio for the credit losses of the actual CMBS tranches. In our study, the actual CMBS tranches are modeled by using a hypothetical CMBS portfolio, which is presented in the next section.

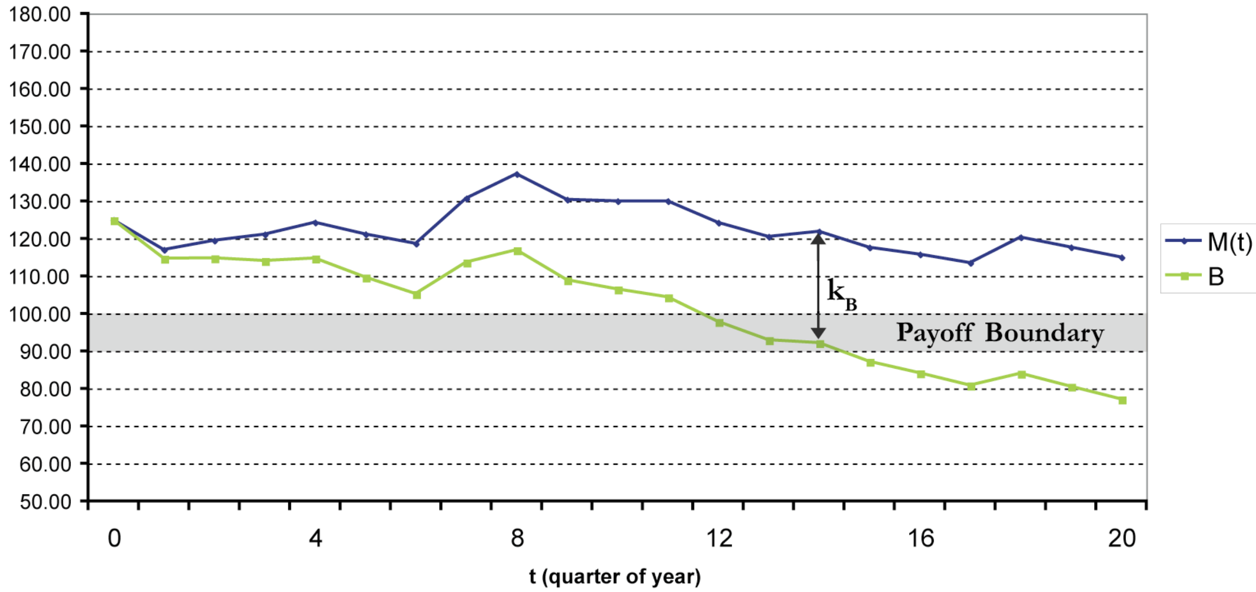


Figure 6: B Synthetic Tranche Price Process

	B Synthetic				
Period t	B_t	track min until floor	Payoff	Par Value	Payoff / $ notional par
0	125	0	0	10	
1	115	0	0	10	0.00
2	115	0	0	10	0.00
3	114	0	0	10	0.00
4	115	0	0	10	0.00
5	110	0	0	10	0.00
6	105	0	0	10	0.00
7	114	0	0	10	0.00
8	117	0	0	10	0.00
9	109	0	0	10	0.00
10	107	0	0	10	0.00
11	104	0	0	10	0.00
12	98	98	2	8	0.20
13	93	93	5	3	0.50
14	92	92	1	2	0.10
15	87	90	2	0	0.20
16	84	90	0	0	0.00
17	81	90	0	0	0.00
18	84	90	0	0	0.00
19	81	90	0	0	0.00
20	77	90	0	0	0.00
Total			10		1.00

Table 5: B Synthetic Tranche Payoff Schedule

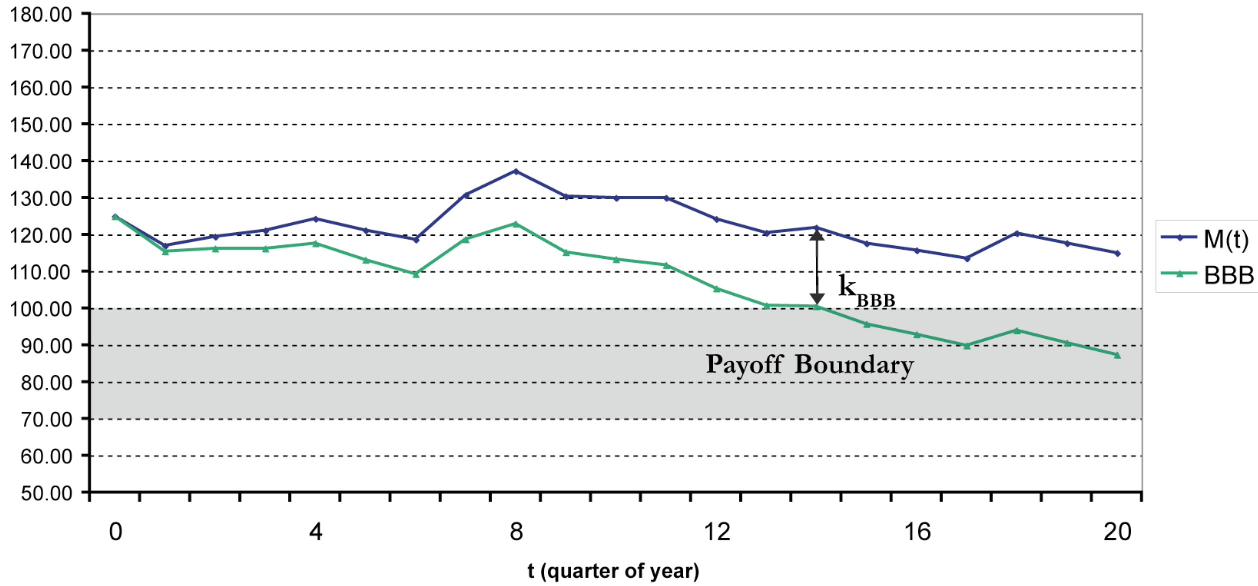


Figure 7: BBB Synthetic Tranche Price Process

	BBB Synthetic				
Period t	BBB_t	track min until floor	Payoff	Par Value	Payoff / $ notional par
0	125	0	0	30	
1	115	0	0	30	0.00
2	116	0	0	30	0.00
3	116	0	0	30	0.00
4	118	0	0	30	0.00
5	113	0	0	30	0.00
6	109	0	0	30	0.00
7	119	0	0	30	0.00
8	123	0	0	30	0.00
9	115	0	0	30	0.00
10	113	0	0	30	0.00
11	112	0	0	30	0.00
12	105	0	0	30	0.00
13	101	0	0	30	0.00
14	101	0	0	30	0.00
15	96	96	4	26	0.13
16	93	93	3	23	0.10
17	90	90	3	20	0.10
18	94	90	0	20	0.00
19	91	90	0	20	0.00
20	87	87	3	17	0.10
Total			13		0.43

Table 6: BBB Synthetic Tranche Payoff Schedule

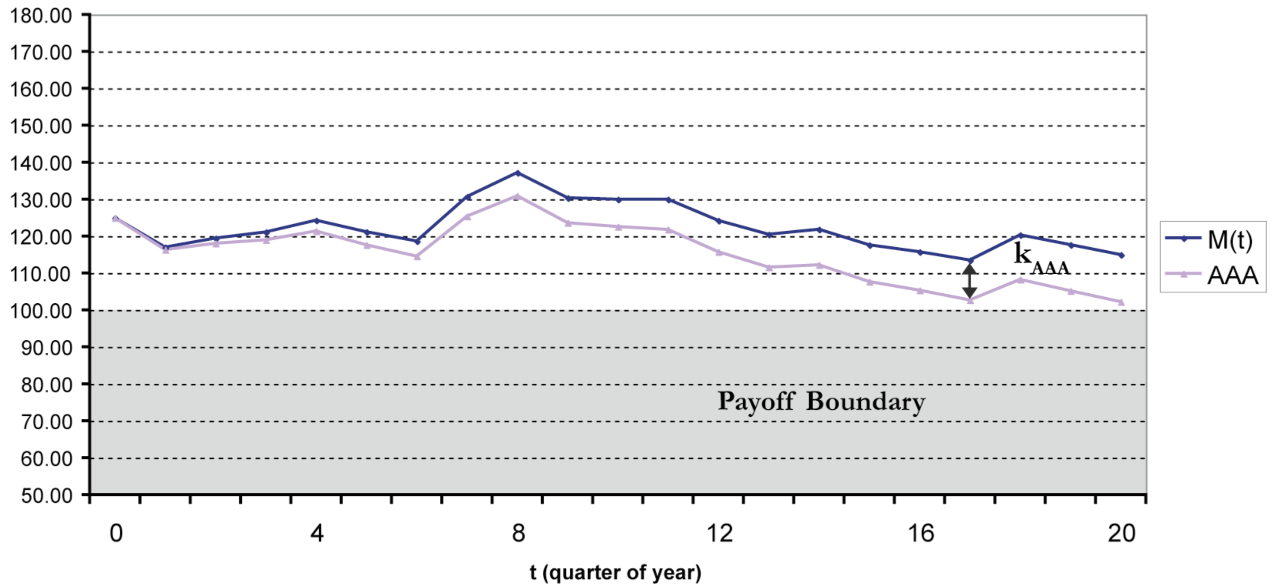


Figure 8: AAA Synthetic Tranche Price Process

Period t	AAA Synthetic				
	AAA_t	track min	Payoff	Par Value	Payoff / $ notional par
0	125	0	0	60	
1	116	0	0	60	0.00
2	118	0	0	60	0.00
3	119	0	0	60	0.00
4	121	0	0	60	0.00
5	118	0	0	60	0.00
6	115	0	0	60	0.00
7	125	0	0	60	0.00
8	131	0	0	60	0.00
9	124	0	0	60	0.00
10	123	0	0	60	0.00
11	122	0	0	60	0.00
12	116	0	0	60	0.00
13	112	0	0	60	0.00
14	112	0	0	60	0.00
15	108	0	0	60	0.00
16	105	0	0	60	0.00
17	103	0	0	60	0.00
18	108	0	0	60	0.00
19	105	0	0	60	0.00
20	102	0	0	60	0.00
Total			0		0.00

Table 7: AAA Synthetic Tranche Payoff Schedule

The Hypothetical CMBS Portfolio

Recall that the primary use of the synthetic is to facilitate the hedging of exposure to real CMBS tranches. Therefore, it is likely that in the real world, potential users of the synthetic would analyze the product by simulating the behavior of the synthetic product and of a real CMBS pool that they would model. In this section, we present such an analysis based on a hypothetical CMBS portfolio.

Such an analysis would seek to suggest the hedging qualities of the synthetic, including cash flow correlation between the synthetic payoffs and the hypothetical CMBS tranche actual losses, and hedge ratio analysis. The analysis would also be used to help the user pick the *k* value they want to use in the synthetic, by fine-tuning the *k* that best suit the particular market exposure and investor CMBS portfolio composition.

The key difference between the synthetic product and the actual CMBS pool is that the latter is composed of actual individual properties that have *idiosyncratic risk* that triggers individual property default behavior. The properties that we model in this section are surrogates for the actual behavior of properties underlying a securitized mortgage in a CMBS pool that the investor is trying to hedge. In our analysis, we model the underlying properties in a hypothetical CMBS pool such that they approximately mimic the timing of actual defaults of the underlying properties in an investor's actual CMBS pool. In this thesis, we will refer to the actual cash flow loss due to defaults experienced by specific tranches of a hypothetical CMBS portfolio as the *"actual credit loss"*, or simply *"credit loss."*

The hypothetical CMBS portfolio models the dispersion of individual property values—the idiosyncratic risk— by an additionally imposed random volatility σ_I around the portfolio index. Similar to the role of *k* in the modeling of synthetic tranches, the idiosyncratic volatility in this model is meant to reflect the downside idiosyncratic risk of individual properties. Some properties would default even though the portfolio average, as represented by the index, does not suggest any property defaults.

Consistent with the portfolio index model, each of the 100 properties in this hypothetical pool is valued at $125 for a total value of $12,500. At 80% LTV, the total loan par value is $10,000. This loan principal value is allocated to each CMBS tranche according to its predefined subordination levels, as shown in Table 8 below.

	Pool	Individual Properties
Property Value	**12500**	125
Loan Par Value	**10000**	100
B Par	**1000**	10
BBB Par	**3000**	30
AAA Par	**6000**	60

Table 8: Hypothetical CMBS Portfolio Par Value and Tranche Allocation

The individual properties, P_t, are modeled as a lognormal process. Since the individual properties represent the underlying collateral in the CMBS pool, they too start at $125. They are modeled to drift with the portfolio index growth rate g_t^m, with an *idiosyncratic volatility* of σ_i in each period.

$$P_t = P_{t-1} e^{(g_t^m) + \sigma_i N(0,1)\sqrt{dt}}$$

where,

$P_0 = M_0 = 125$

σ_i is the individual property idiosyncratic risk that represents the degree of property dispersion around the portfolio index. It also determines how well the index represents the actual market exposure of investor's CMBS portfolio, which should vary accordingly. In this scenario, σ_i is assumed to be 5%.

Recall that $g_t^m = \mu(dt) + \tilde{Z}_t = \mu(dt) + \sigma_m \, N(0,1) \sqrt{dt}$ = portfolio index growth rate

g_t^m is consistent across the portfolio index, synthetic tranches, and the individual properties in each period.

By the use of Monte Carlo simulation method, these properties are simulated simultaneously with respect to the portfolio index and synthetic tranche movements. The same simulation trial as for the index and synthetic tranche price paths, illustrated in Figure 5, is presented below for the properties underlying the hypothetical CMBS portfolio.

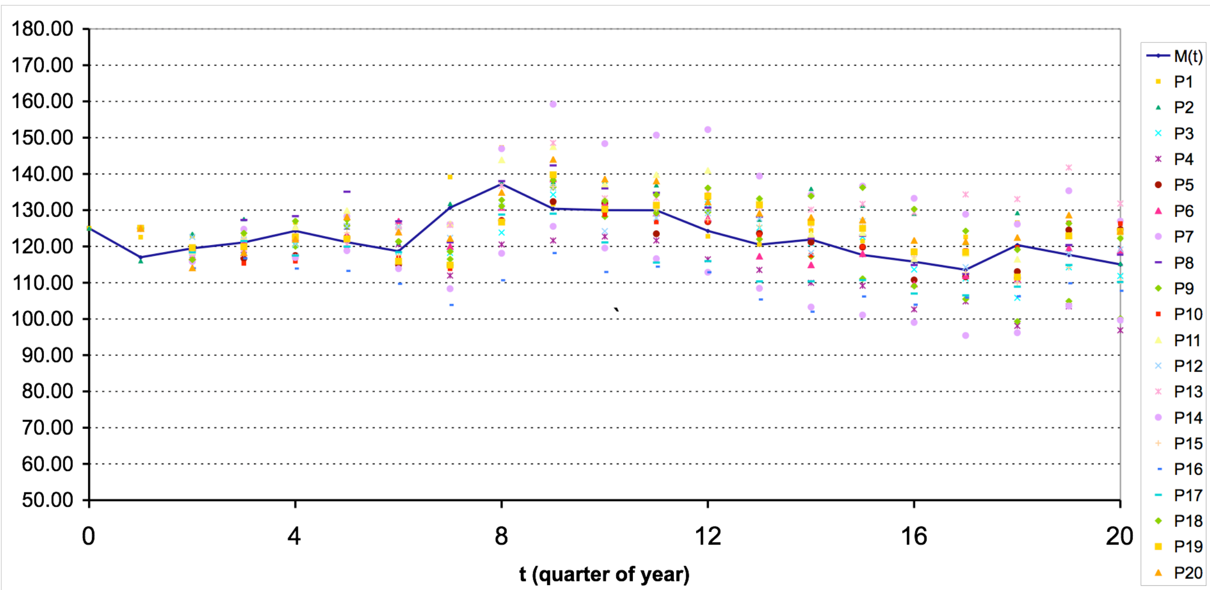


Figure 9: Individual Properties Price Path (20 properties shown)

The Credit Loss Structure of the Hypothetical CMBS Portfolio

In this section, we present how the individual properties' price paths cause defaults in the hypothetical CMBS portfolio, how credit losses are allocated to the par value of each tranche, and how a credit loss ratio is calculated. The hypothetical portfolio is a proxy for the actual CMBS portfolio that an investor is holding.

Each point along the individual properties price path represents the value of a property underlying a securitized mortgage in the CMBS pool. Any default events in the pool disrupt the cash flows to the holders of CMBS based on the subordination structure, reducing the par value of each tranche sequentially from the lowest-rated to the highest-rated. The B, BBB, and AAA synthetic tranches encompass loan par values of $1000, $3000 and $6000 respectively, a total loan par value of $10,000 for the 100 properties.

Recall that a property is considered to default when its value falls below the original loan amount of $100. Each property default is considered as a $100 whole loan loss. The hypothetical CMBS portfolio credit loss model calculates the total CMBS credit losses in each period. The model keeps a running total of the remaining par value of each tranche, having deducted the credit loss allocated to the tranche in each period until it retires from the pool. The ratio of credit loss of the hypothetical CMBS tranche in each period to the tranche original par value is calculated, and will be referred to as *"the credit loss ratio."* For each simulation trial, a pool credit loss schedule is created as shown in Table 9.

		B		BBB		AAA	
Period t	**Default Value**	**Par Value**	**Credit Loss/ $ Par**	**Par Value**	**Credit Loss/ $ Par**	**Par Value**	**Credit Loss/ $ Par**
0		1000		3000		6000	
1	0	1000	0.00	3000	0.00	6000	0.00
2	0	1000	0.00	3000	0.00	6000	0.00
3	0	1000	0.00	3000	0.00	6000	0.00
4	0	1000	0.00	3000	0.00	6000	0.00
5	0	900	0.00	3000	0.00	6000	0.00
6	0	900	0.10	3000	0.00	6000	0.00
7	0	900	0.00	3000	0.00	6000	0.00
8	0	900	0.00	3000	0.00	6000	0.00
9	0	900	0.00	3000	0.00	6000	0.00
10	0	900	0.00	3000	0.00	6000	0.00
11	0	900	0.00	3000	0.00	6000	0.00
12	100	800	0.00	3000	0.00	6000	0.00
13	0	700	0.00	3000	0.00	6000	0.00
14	0	600	0.00	3000	0.00	6000	0.00
15	700	500	0.30	2900	0.00	6000	0.00
16	400	200	0.30	2800	0.00	6000	0.00
17	500	0	0.20	2600	0.03	6000	0.00
18	0	0	0.00	2600	0.00	6000	0.00
19	100	0	0.00	2600	0.07	6000	0.00
20	100	0	0.10	2500	0.13	6000	0.00
Total	1900		1.00		0.23		0.00

Table 9: Credit Loss Schedule of a Hypothetical CMBS Portfolio

Chapter Four: Testing the Hedging Performance of the Synthetic Product

Chapter Four focuses on the mechanics and development of the proposed commercial property price index-based CMBS synthetic. This chapter presents the results from the hedge analysis of the proposed synthetic product. The two important simulation tests to perform are cash flow correlation between the synthetic tranche payoffs and the hypothetical CMBS tranche actual credit losses, and hedge ratio analysis. The cash flow correlation between the synthetic payoff and the hypothetical CMBS tranche actual credit loss would reveal whether or not the proposed synthetic product is an effective hedging tool. The hedge ratio would give us an understanding of the dollar amount of the property index-based synthetics an investor needs to purchase in order to hedge a dollar worth of his CMBS portfolio. In other words, the hedge ratio compares the value of synthetic contract purchases to the value of the CMBS portfolio being hedged.

Through the simulation, the investor can also find the k that results in the highest expected cash flow correlation. In practice, an investor would most likely pick the k from a menu of different basis points. An investor would define the k differential that best matches the risk exposure and CMBS tranche composition in his particular portfolio. A liquid trading in the market may also fine-tune k through supply and demand. In this thesis, the stylized CMBS pool is simple enough to demonstrate the finding of k by the use of Monte Carlo simulation analysis. In our stylized CMBS pool model, k_B is found to be 0.080, k_{BBB} is 0.055, and k_{AAA} is 0.023.

The Monte Carlo simulation facilitates our study as follows. In each simulation, the synthetic price process and payoffs, and the hypothetical CMBS portfolio price process and credit losses are tracked and recorded in a payoff schedule and credit loss schedule, respectively. We will refer to the number of simulation trials as *"n"*. The payoff schedule and the credit loss schedule track the *synthetic payoff ratio* and the *credit loss ratio* in each of the 20 periods. Then we calculate a correlation between the two ratios for the 20 periods in each simulation. Monte Carlo simulation approximates the expectations of the correlation value with a simple arithmetic average of the correlations taken over *n* simulation trials to test and explore the hedging performance of the synthetic product.

The synthetic product can be used as a hedging tool for investors with different horizons in mind. This thesis performs analyses for 3 horizons: a quarter period (*"quarter-by-quarter")*, a 5-quarter period (*"centered rolling 5-quarter average"*), and a presumed five-year lifetime period (*"lifetime"*). These methods of analyses are employed in calculating the synthetic payoff ratios and the credit loss ratios.

The quarter-by-quarter analysis is the most stringent test for hedge quality. It reflects how well each period is hedged. However, many investors may not need the hedge to work precisely in each quarter. They may be able to wait a bit to receive the covering compensation for a credit loss. In this case, a 5-quarter centered rolling average measure, which takes an average of 2 ½ quarters to receive the hedge covering compensation, might be a sufficient method of measure.

The centered-rolling 5-quarter average analysis tests the hedging performance of the synthetic product with rolling averages from the 3rd quarter through the 18th quarter of the synthetic life. For example, for the 3rd quarter, the payoff ratio under this method is the average payoff ratio from the 1st quarter through the 5th quarter inclusive. The payoff ratio for the 4th quarter is the average payoff ratio from the 2nd quarter through the 6th quarter inclusive, and so on. The centered-rolling analysis is a less rigid measure compared to the quarter-by-quarter analysis, as it essentially tests the hedging performance of the synthetic product within a 5 consecutive quarters. It nevertheless is an important analysis because an investor may find it sufficient for the synthetic product to be able to hedge credit losses within a given 5-quarter period.

The lifetime analysis calculates a correlation between the total payoff ratios and the total credit loss ratios over the presumed five-year lifetime of the synthetic for *n* simulations. Then an expected lifetime correlation is calculated by taking an average of the correlations over those *n* simulations. This lifetime analysis is an important analysis for an investor who has a longer horizon hedge objective, to the extent that the user is content for the hedge to occur over the presumed five-year lifetime.

	B Tranche		BBB Tranche		AAA Tranche	
	Mean	Median	Mean	Median	Mean	Median
Period per Period	0.55	0.61	0.70	0.74	0.78	0.86
Centered Rolling 5-qtr Average	0.69	0.84	0.76	0.87	0.85	0.94
5-year Lifetime Period	0.76	-	0.90	-	0.94	-

Table 10: Correlation between Synthetic Payoff and Actual Credit Loss Per Notional Par Value

As the correlation summary table shows, the synthetic provides higher correlation in the higher-rated tranches. As expected, the longer an investor's hedge horizon is, the better the hedge in all the synthetic tranches. Therefore, an investor should be mindful of his hedge objective when testing the performance of the synthetic product.

Quarter-by-Quarter Correlations

This section presents in detail the simulation results of the quarter-by-quarter correlation between the synthetic payoff ratio and the credit loss ratio of the proposed property price index-based synthetic. This thesis uses Crystal Ball™ program as an engine to perform the Monte Carlo simulation. The program accumulates and tabulates the correlation results from each simulation into a histogram, as represented by the top bar graph in Figure 10. The cumulative probability distribution graph is also created after *n* simulations, as well as a statistics table of the correlation results.

Figure 10 shows correlation results for the B synthetic tranche. The mean correlation is 0.55, the median is 0.61, and the standard deviation is 0.35. An important thing to note is that the simulation only counts the trials that involve some credit losses in the hypothetical CMBS tranche. In other words, the correlation evaluates how well each synthetic tranche hedges given some credit losses in that tranche; it excludes any simulation runs for those tranches that do not suffer any credit loss.

As shown in Figures 11 and 12, the mean correlations for the BBB and AAA synthetic tranches are higher at 0.70 and 0.78, and the medians are also higher at 0.74 and 0.86, respectively. The standard deviation for the BBB and AAA synthetic tranches are 0.23 and 0.24, respectively. Note that the number of trials for the AAA synthetic tranche is much lower than that for the B and BBB synthetic tranches. Because the AAA synthetic tranche is the most senior of the three tranches, the number of simulation trials when the AAA tranche does not suffer any credit losses is the highest.

The correlations between the total payoff ratio and the total credit loss ratio throughout the presumed five-year lifetime are 0.75, 0.90, and 0.94 for the B, BBB, and AAA synthetic tranches, respectively. The results show, to the extent that the user is content for the hedge to occur over the presumed five-year lifetime, the synthetic provides substantial higher correlation, particularly in the higher-rated tranches.

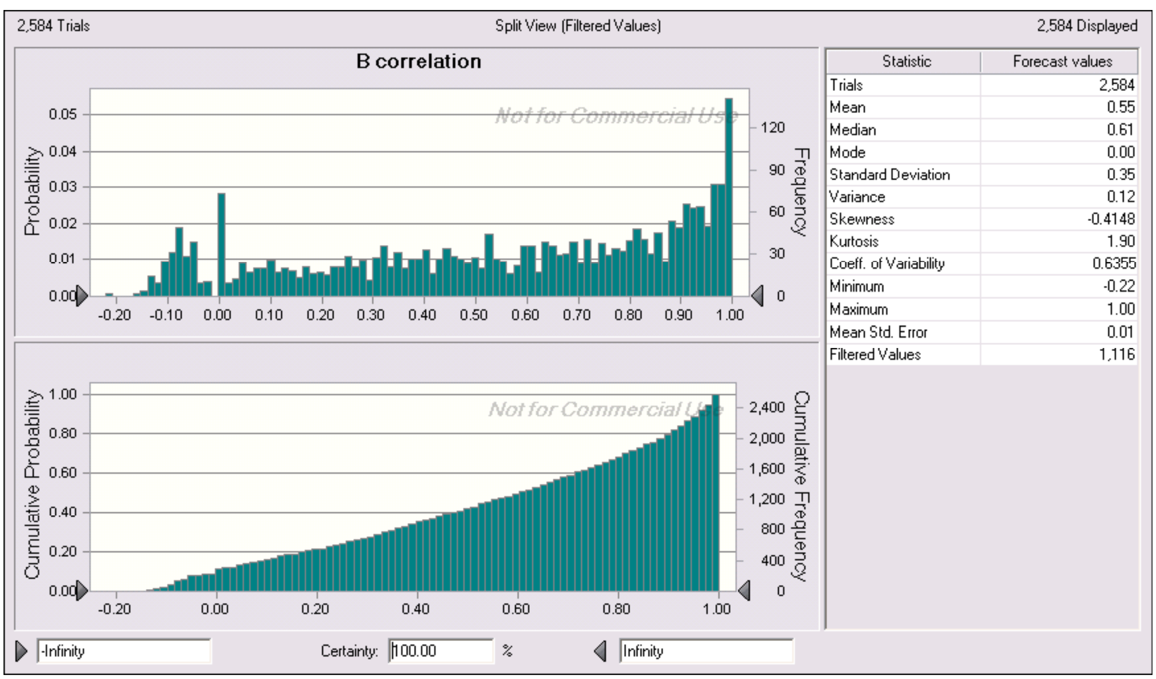


Figure 10: B Tranche Quarter-by-Quarter Correlation Simulation Results

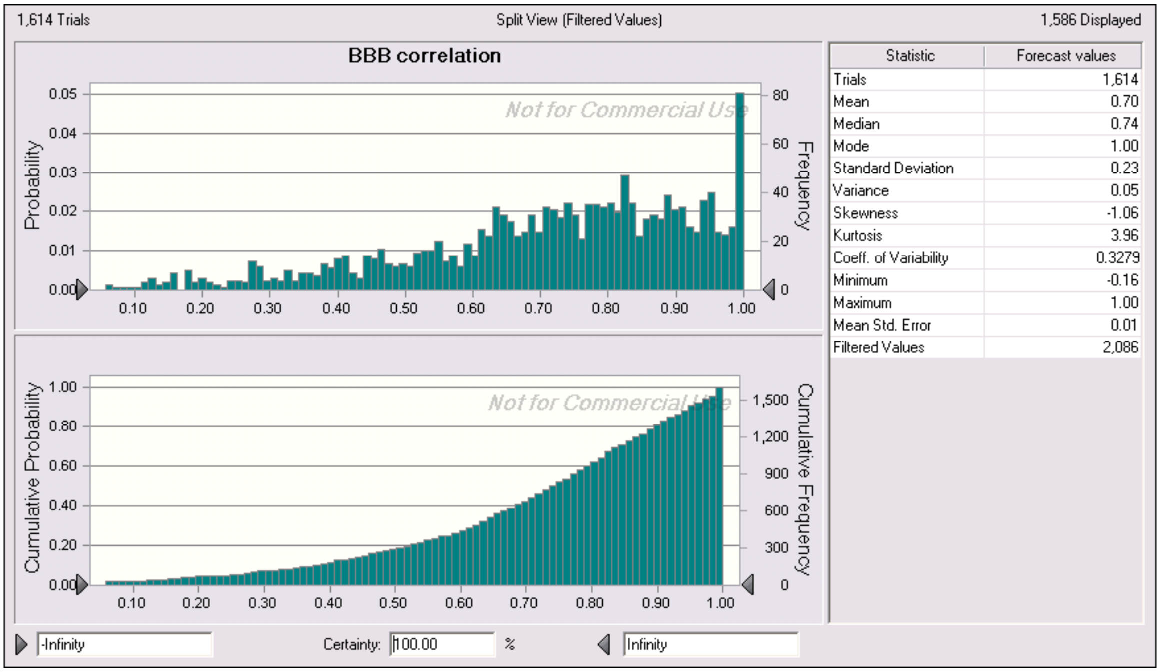


Figure 11: BBB Tranche Quarter-by-Quarter Correlation Simulation Results

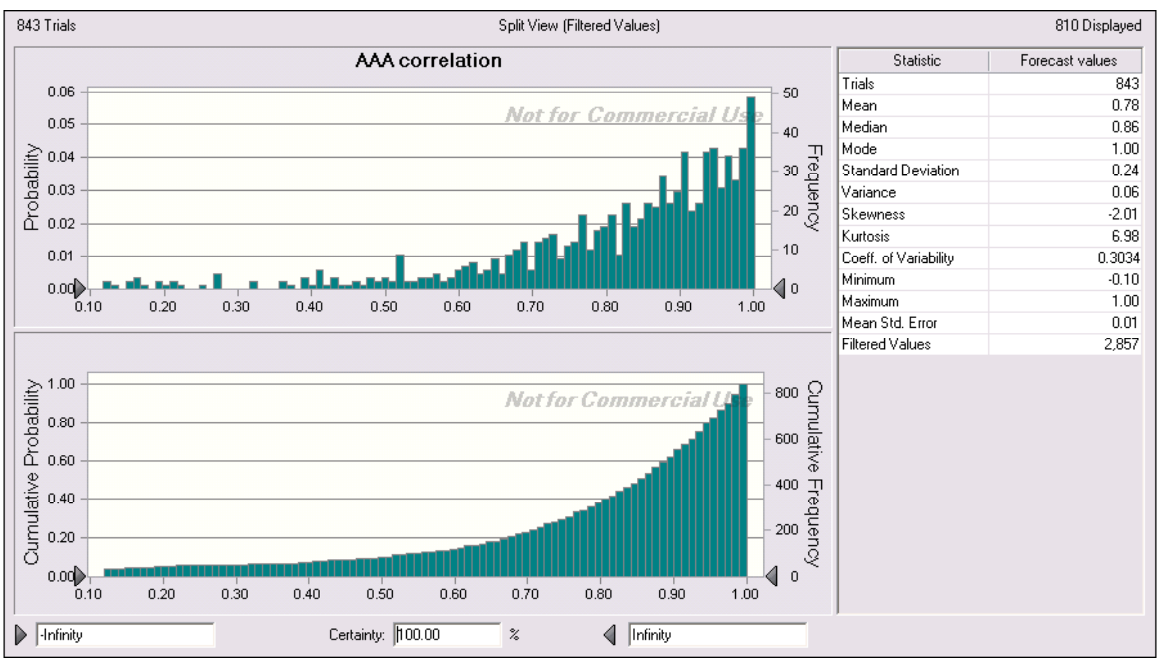


Figure 12: AAA Tranche Quarter-by-Quarter Correlation Simulation Results

Quarter-by-Quarter Hedge Ratio

The hedge ratio defines the amount of synthetic notional value per dollar of actual exposure hedged. In our analysis, we calculate the hedge ratio as the hypothetical CMBS tranche credit loss divided by the synthetic payoff for each of the 20 periods over the five-year simulated life.

By the use of Monte Carlo analysis, the synthetic payoffs and the credit losses are simulated across *n* simulation runs. Then, an expected synthetic payoff and an expected credit loss for each quarter are calculated by averaging the quarterly payoffs and credit losses, separately across the simulation runs. Finally, the hedge ratio is taken between the two averages, and this hedge ratio is reported for each quarter in the five-year life.

By definition, the hedge ratio is higher than 1 in the periods when the credit loss exceeds the payoff, and less than 1 in the periods when the payoff exceeds the loss. A series of bar graphs on the left in Figure 13 show the average magnitude and timing of payoffs and credit losses. It is important to note that on average, the credit loss on the hypothetical CMBS tranche occurs before the synthetic tranche starts to payoff. There is a lag between the credit losses of a hypothetical CMBS portfolio and the synthetic payoff, which needs to be considered when an investor executes his hedging strategy.

This lag causes the hedge ratio to be dynamic through its presumed lifetime, as shown in the series of bar graphs on the right in Figure 13. For the B and BBB synthetic tranches, the general dynamic of the hedge ratios is as follows. The hedge ratio in the first quarter is 1. Then it peaks extremely high in the 2nd quarter, drops dramatically in the next few quarters, until it becomes relatively stable but below 1 through the end of year 5. For the AAA synthetic tranches, the hedge ratio also starts at 1 in the first quarter. It peaks in the 2nd quarter, although much less dramatic than the B and BBB synthetic tranches. Then it drops exponentially through the end of year 5, staying above 1.

The hedge ratio simulation results in this section demonstrate that an investor looking to hedge precisely in each quarter would need to adjust his position drastically in each quarter, especially in the beginning periods of the synthetic life. However, if the investor were content with a lifetime hedge, a single hedge ratio may be sufficient.

	B			BBB			AAA		
Period t	Synthetic Payoff	Credit Loss	Hedge Ratio	Synthetic Payoff	Credit Loss	Hedge Ratio	Synthetic Payoff	Credit Loss	Hedge Ratio
1	0.00	0.00	1.00	0.00	0.00	1.00	0.00	0.00	1.00
2	0.00	0.01	19.46	0.00	0.00	17.06	0.00	0.00	9.10
3	0.01	0.03	5.24	0.00	0.01	7.15	0.00	0.00	7.04
4	0.01	0.04	3.00	0.00	0.01	4.38	0.00	0.00	5.29
5	0.03	0.05	1.82	0.01	0.02	3.24	0.00	0.01	3.76
6	0.04	0.04	1.23	0.01	0.02	2.47	0.00	0.01	3.35
7	0.04	0.04	1.00	0.01	0.02	1.94	0.00	0.01	2.70
8	0.05	0.04	0.83	0.01	0.03	1.68	0.00	0.01	2.64
9	0.05	0.04	0.74	0.02	0.02	1.37	0.00	0.01	2.12
10	0.05	0.03	0.66	0.02	0.02	1.16	0.00	0.01	2.02
11	0.05	0.03	0.60	0.02	0.02	1.07	0.00	0.01	1.77
12	0.05	0.03	0.57	0.02	0.02	0.94	0.00	0.01	1.70
13	0.05	0.03	0.55	0.02	0.02	0.82	0.01	0.01	1.72
14	0.04	0.02	0.55	0.02	0.02	0.72	0.01	0.01	1.60
15	0.04	0.02	0.51	0.02	0.02	0.70	0.01	0.01	1.43
16	0.04	0.02	0.48	0.02	0.02	0.64	0.01	0.01	1.34
17	0.04	0.02	0.55	0.02	0.01	0.62	0.01	0.01	1.45
18	0.03	0.02	0.52	0.02	0.01	0.57	0.01	0.01	1.27
19	0.03	0.02	0.48	0.02	0.01	0.52	0.01	0.01	1.16
20	0.03	0.01	0.48	0.02	0.01	0.50	0.01	0.01	1.04

Table 11: Quarter-by-Quarter Hedge Ratio Analysis

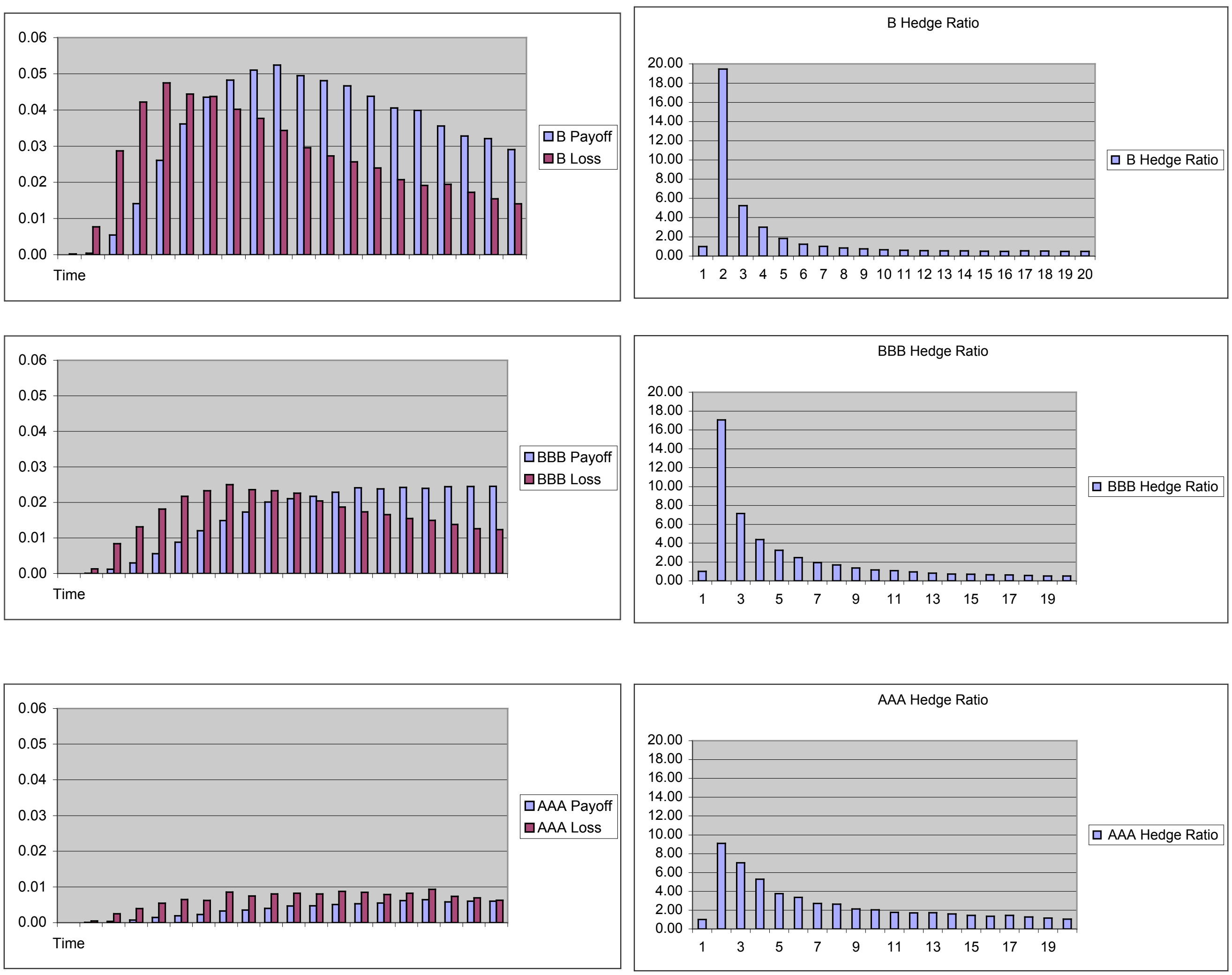
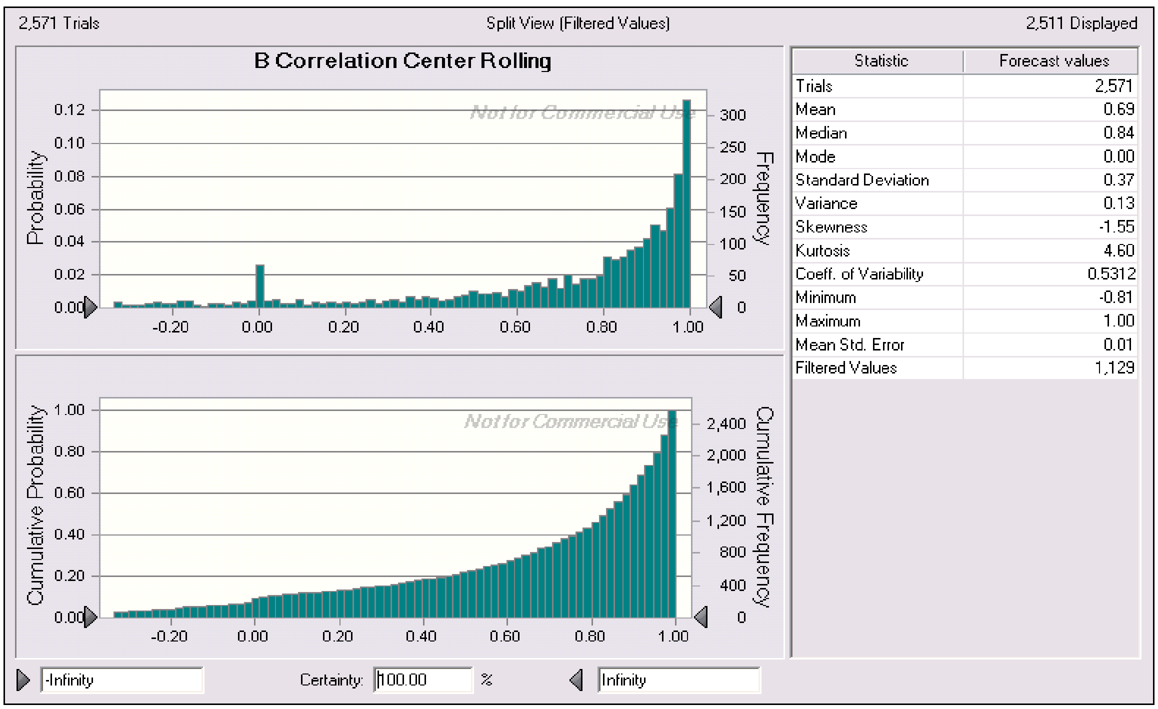


Figure 13: Quarter-by-Quarter Hedge Ratio

Centered-Rolling 5-Quarter Average Correlations

The centered-rolling 5-quarter average correlations are, as expected, higher in all the synthetic tranches compared to the quarter-by-quarter analysis. The correlation means are 0.69, 0.76, and 0.85, the correlation medians are 0.84, 0.87, and 0.94, and the standard deviations are 0.37, 0.29, and 0.22 for the B, BBB, and AAA tranches respectively. This shows that the synthetic product is an effective hedging tool over a 5-quarter time span.



Statistic	Forecast values
Trials	2,571
Mean	0.69
Median	0.84
Mode	0.00
Standard Deviation	0.37
Variance	0.13
Skewness	-1.55
Kurtosis	4.60
Coeff. of Variability	0.5312
Minimum	-0.81
Maximum	1.00
Mean Std. Error	0.01
Filtered Values	1,129

Figure 14: B Tranche Centered-Rolling 5-quarter Average Correlation Simulation Results

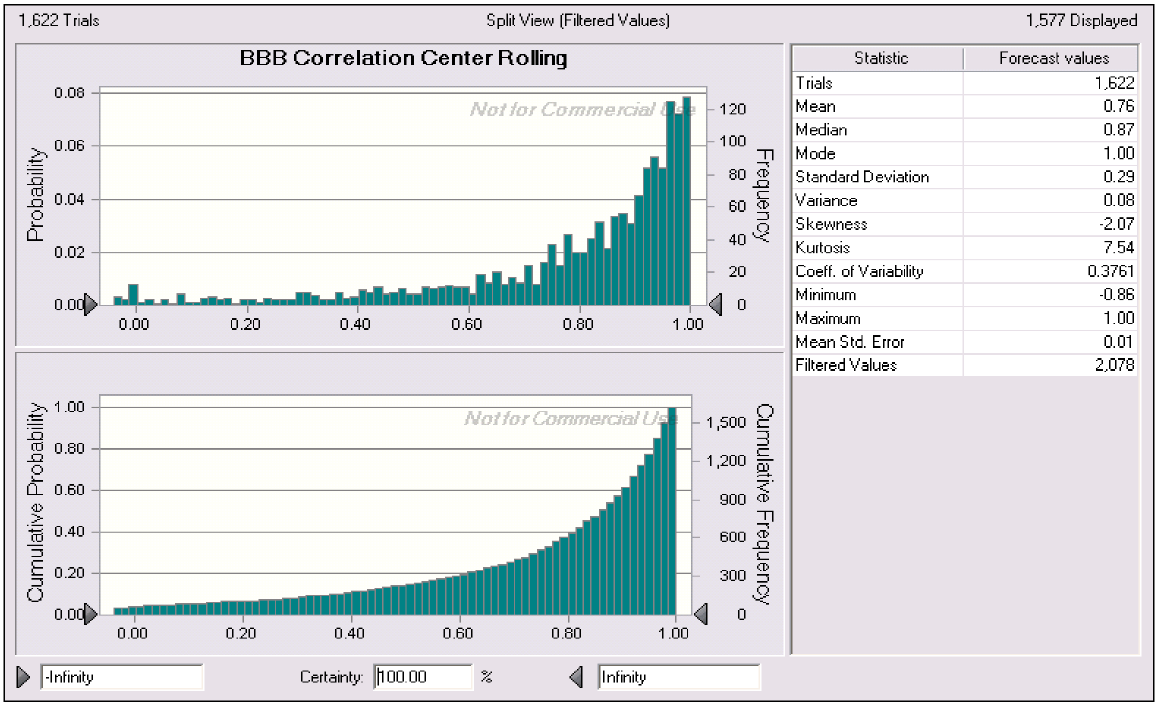


Figure 15: BBB Tranche Centered-Rolling 5-quarter Average Correlation Simulation Results

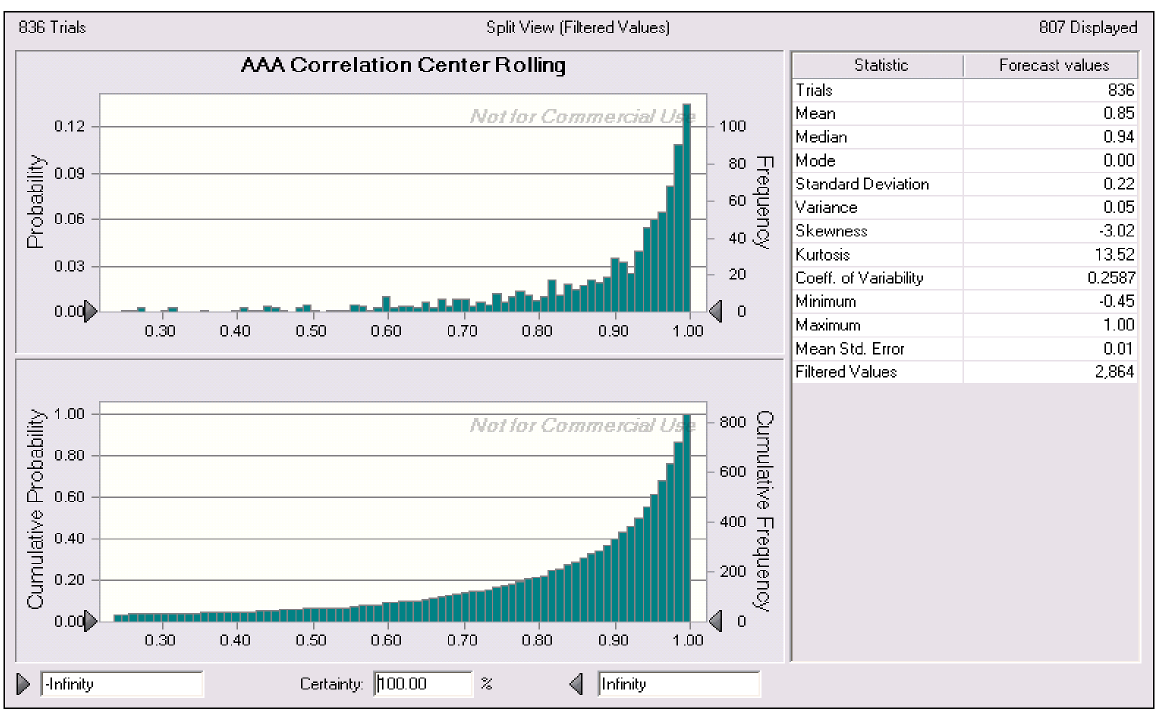


Figure 16: AAA Tranche Centered-Rolling 5-quarter Average Correlation Simulation Results

Centered Rolling 5-Quarter Average Hedge Ratios

The hedge ratio distribution of the centered rolling 5-quarter average analysis are similar to those of the quarter-by-quarter analysis, but much less dramatic and much more uniform: the hedge ratio is at its highest peak in the first period, and decreases exponentially at some rate until the end period for each of the tranches.

As shown in Figure 17, there is a trend in the difference in magnitude and timing of the payoffs and credit losses between the tranches. For the B and BBB synthetic tranches, on average, the payoffs are short in hedging the high credit losses in the beginning, but overcompensate for the credit losses towards the end. On the other hand, the AAA synthetic tranche hedge ratio is greater than 1 in all periods, suggesting that the credit loss always exceeds the payoff.

On average, the more senior the synthetic tranche is, the higher the hedge ratio. This means that a higher-rated synthetic tranche investor would need to purchase more notional value of the synthetic per dollar of exposure hedged, compared to a lower-rated synthetic tranche investor.

The B synthetic tranche hedge ratio starts off the lowest among the tranches at about 2.5, and decreases exponentially through the presumed lifetime. Note that in the 7^{th} quarter, the payoff is approximately equal to the credit losses and the hedge ratio is 1. The hedge ratios of the BBB and AAA synthetic tranches are relatively more dynamic, peaking at approximately 4.5 and 5.5 in the first quarter, respectively. The hedge ratios of both synthetic tranches decrease exponentially throughout the presumed lifetime. Note that the BBB synthetic tranche payoff catches up with the credit loss in the 12^{th} quarter, much later than the B synthetic tranche does. In contrast to the hedge ratio of the B and BBB synthetic tranches, the hedge ratio of the AAA synthetic tranche never drops below 1.

Period t	B Synthetic Payoff	B Credit Loss	B Hedge Ratio	BBB Synthetic Payoff	BBB Credit Loss	BBB Hedge Ratio	AAA Synthetic Payoff	AAA Credit Loss	AAA Hedge Ratio
1									
2									
3	0.01	0.03	2.59	0.00	0.01	4.34	0.00	0.00	5.48
4	0.02	0.04	1.95	0.00	0.02	3.16	0.00	0.00	4.84
5	0.03	0.04	1.54	0.01	0.02	2.69	0.00	0.01	4.17
6	0.04	0.05	1.20	0.01	0.02	2.16	0.00	0.01	3.63
7	0.05	0.05	1.00	0.01	0.02	1.84	0.00	0.01	3.31
8	0.05	0.04	0.82	0.01	0.02	1.56	0.00	0.01	2.95
9	0.05	0.04	0.71	0.02	0.02	1.41	0.00	0.01	2.77
10	0.05	0.03	0.65	0.02	0.02	1.26	0.00	0.01	2.51
11	0.05	0.03	0.57	0.02	0.02	1.15	0.00	0.01	2.34
12	0.05	0.03	0.54	0.02	0.02	1.04	0.00	0.01	2.19
13	0.05	0.03	0.59	0.02	0.02	0.94	0.00	0.01	2.03
14	0.05	0.02	0.50	0.02	0.02	0.88	0.00	0.01	1.86
15	0.04	0.02	0.51	0.02	0.02	0.84	0.00	0.01	1.77
16	0.04	0.02	0.53	0.02	0.02	0.79	0.00	0.01	1.69
17	0.04	0.02	0.52	0.02	0.02	0.79	0.01	0.01	1.56
18	0.03	0.02	0.52	0.02	0.02	0.73	0.01	0.01	1.50
19									
20									

Table 12: Centered Rolling 5-Quarter Average Hedge Ratio Analysis

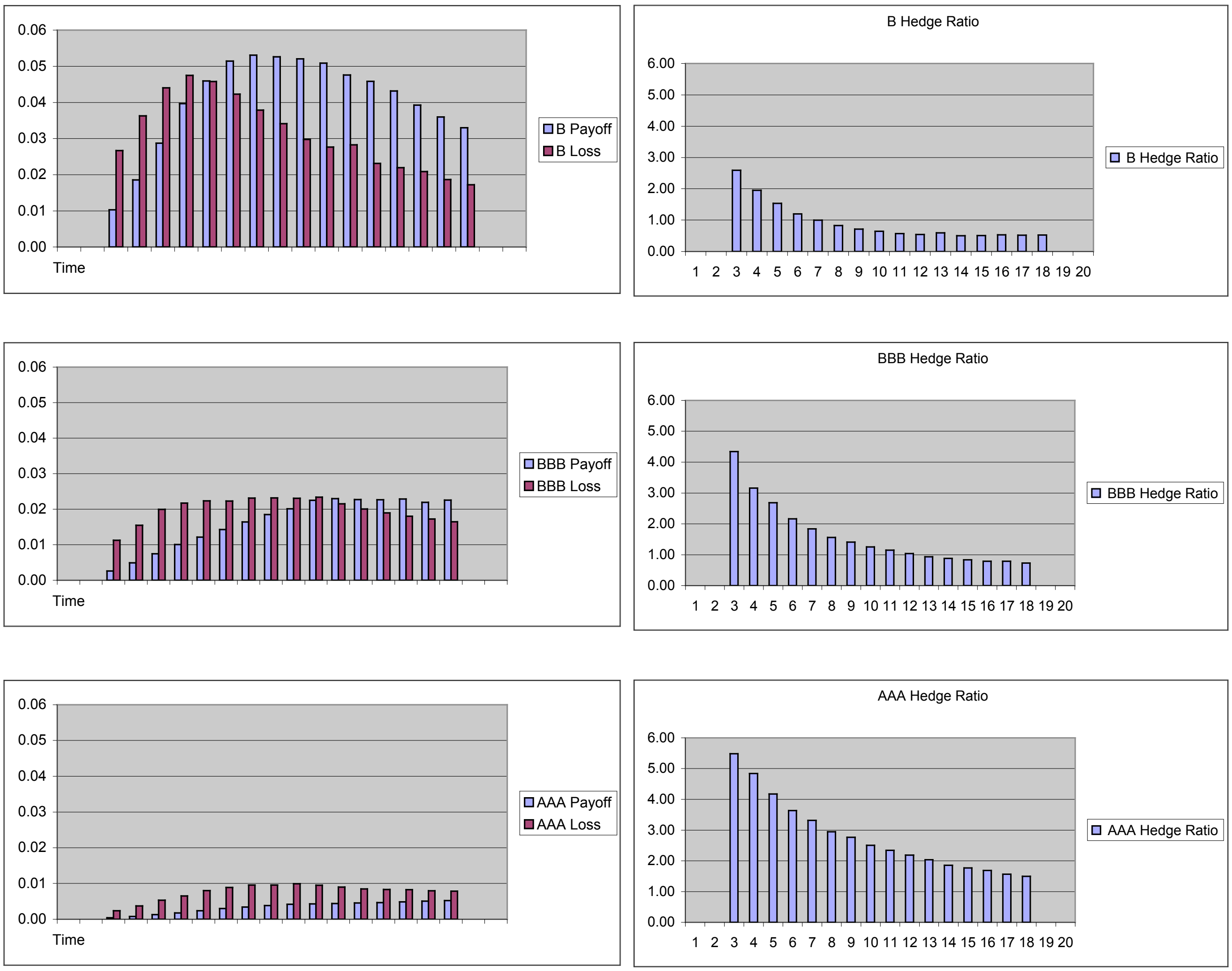


Figure 17: Centered Rolling 5-Quarter Average Hedge Ratio

Conclusion

This thesis has developed a methodology to structure a property price index-based synthetic contract to hedge a particular market exposure of a specific CMBS tranche in an investor's portfolio. The proposed methodology would enable an investor to fine-tune the underlying risk of a CMBS portfolio based on specific geographic and property-type diversification by utilizing a customized set of commercial property price indices. Taking into account the idiosyncratic volatility of the individual properties underlying the CMBS portfolio, this thesis has developed a synthetic product whose payoff behavior approximately replicates the credit loss behavior of a referenced CMBS tranche.

Through Monte Carlo simulation, the thesis explored the hedging performance of the synthetic product, considering both cash flow correlation and hedge ratio analyses. The results reveal that the correlation between the synthetic payoffs and the hypothetical CMBS tranche actual losses, that is, the effectiveness of the hedge, varies depending on the investor's horizon or degree of temporal precision the investor seeks in the hedge. For example, cash flow correlations between the synthetic and the target CMBS tranche are much higher over the five-year presumed lifetime of the risk exposure than over any given 5-quarter time span, and even lower at the completely contemporaneous quarter-by-quarter horizon. The longer the investor is willing to wait to receive the covering compensation for his credit loss, the higher the correlation would be, and the more effectively the synthetic can be used as a hedge. The hedge ratio must also be adjusted accordingly depending on the investor's objective horizon, and the degree of precision of the hedge. For a lifetime hedge, a single hedge ratio may be sufficient. At the other extreme, for the completely contemporaneous quarter-by-quarter hedge, the hedge ratio is very dynamic throughout the life of the proposed synthetic, varying from a very high hedge ratio very early in the life of the risk exposure to a near unity later. The dynamic hedge ratios suggest that the investor buying the synthetics for hedging purposes would need to constantly rebalance his position, or would need a contract that has a varying hedging ratio.

The proposed property price index-based synthetic is particularly suitable for an investor with concentrated market exposure in their portfolio. The proposed synthetic could in principle be traded directly as a type of default swap contract. An investor would purchase notional contract value sufficient to approximately cover his position in the CMBS portfolio, as suggested by the hedge ratio analysis.

To achieve trading liquidity, the methodology developed for the proposed synthetic needs to be highly transparent, and simple for the general market participants to understand. There may be supply/demand imbalances in some transactions because of the distinctly custom tailored exposure of the synthetic trades. This could limit an investor's ability to enter or exit a trade on a timely basis or economically feasible level. Therefore, standardized documentation for the contracts will be exceedingly important to promote operational efficiency. With such need for simplicity, transparency, and standardization in mind, the methodology developed in this thesis is a stylized one. It is designed to demonstrate a proof of concept at a basic level that a CMBS-inspired derivative synthetic structured from a set of commercial property price indices can closely hedge the default risk of a CMBS portfolio that is exposed to specific commercial real estate markets tracked by the indices. Further studies of this concept can incorporate a number of relevant variables such as property recovery rate, a more robust mean-reverting index path process, the effect of stochastic interest rates and property income triggers, as well as the risk-neutral dynamic for synthetic pricing. Some of the stylized assumptions of the CMBS subordination structure and the underlying mortgage pool model construction can also be relaxed to reflect a more complex and realistic actual CMBS issue.

The possibility of utilizing commercial property price indices to structure equity index-based credit derivatives have many implications for the CMBS market. Methodologies such as the one we have demonstrated in this thesis will expand investment and hedging strategies for CMBS investors, facilitating access to the breadth and depth of existing real estate equity indices. As CMBS markets continue to evolve, and commercial property price indices continue to be developed, the credit derivatives market should play an instrumental role in efficiently facilitating an investor's credit risk management and investment opportunities.

References

Ambrose, B., and A. Sanders. 2003. "Commercial Mortgage-Backed Securities: Prepayment and Default." *Journal of Real Estate Finance and Economics*, 26 (2-3): 179-196

Barve, N., and M. Lee. "Credit Default Swaps in CMBS: An Introduction." CMBS World. Spring 2006. 22-28

Case, K.E., and R. Shiller. 1996. "Mortgage Default Risk and Real Estate Prices: The Use of Index-Based Futures and Options in Real Estate." *Journal of Housing Research* 7(2): 243-258

Childs, P.D, S.H. Ott, and T.J. Riddiough. 1996. "The Pricing of Multiclass Commercial Mortgage-Backed Securities." *Journal of Financial and Quantitative Analysis* 31(4): 581-603

Chiochetti, B., Y. Deng, B. Gao, and R. Yao. 2002. "The Termination of Lending Relationships through Prepayment and Default in Commercial Mortgage Markets: A Proportional Hazard Approach with Competing Risks." *Real Estate Economics* 30(4): 595-633

Epperson J.F., J.B. Kau, D.C. Keenan, and W.J. Muller, III. 1985. "Pricing Default Risk in Mortgages." *AREUEA Journal* 13(3): 261-272

Fabozzi, F.J., and D.P. Jacobs. *The Handbook of Commercial Mortgaged-Backed Securities*. 2nd edition. New Hope, PA: Frank J. Fabozzi Associates, 1999.

Geltner, D.M., N.G. Miller, J. Clayton, and P. Eichholtz. *Commercial Real Estate Analysis & Investments*. 2nd edition. Mason, OH: Thompson South-Western, 2007

Hilliard, J.E., J.B. Kau, and V.C. Slawson, Jr. 1998. "Valuing Prepayment and Default in a Fixed Rate Mortgage. *Real Estate Economics* 26(2): 431-468

Kau, J., and D.C. Keenan. 1995. "An Overview of the Option-Theoretic Pricing of Mortgages." *Journal of Housing Research* 6(2): 217-244

Manzi, J., D. Berezina., M. Adelson. "Synthetic CMBS Primer." Nomura Fixed Income Research. September 5, 2006

Philipp, T., N. Levidy, P. Obias, and D. Rubock. "US CMBS and CRE CDO 1Q 2007 Revire: Conduit Credit at the Turning Point?" Moody's Investors Service. Structured Finance Special Report. April 30, 2007

Riddiough, T.J., and H.R. Thompson. 1993. "Commercial Mortgage Default Pricing with Unobservable Borrower Default Costs." *AREUEA Journal* 21 256-291

Schetman, R.M., and M.J. Southwick. "The Evolution of Credit Default Swaps: Single Name to Indices." Cadwalader. The Capital Markets Report. Summer 2006. 1-4

Shimareva, M., T. Fago, J. Story, and J. Lee. "Synthetic Overview for CMBS Investors." Derivative Fitch. Structured Credit Special Report. June 6, 2007

Titman, S., and W. Torous. 1989. "Valuing Commercial Mortgages: an Empirical Investigation of the Contingent-Claims Approach to Pricing Risky Debt." *Journal of Finance* 44(2)

Todd, A., and Y. Iwai. "An Introduction to the CMBX.NA Index and Single-Name CMBS CDS." CMBS World. Spring 2006. 29-35

Tu, C.C., and M.J. Eppli. 2003. "Term Default, Balloon Risk, and Credit Risk in Commercial Mortgages." *The Journal of Fixed Income*: 42-52

Vandell, K.D. 1992. "Predicting Commercial Mortgage Foreclosure Experience." *AREUEA Journal* 20:1 55-88

Whetten, M., and J. Manzi. "The CMBX: the Future is Here." Nomura Fixed Income Research. March 23, 2006

Wong, E. "Investing In and Risk Managing CMBS Synthetically." CMBS World. Spring 2006. 38-40