# Power System Dynamic Load Modeling

by

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B.S., Swarthmore College (1995)

Submitted to the Department of Electrical Engineering and Computer Science in partial fulfillment of the requirements for the degree of

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## Abstract

Modeling is necessary to monitor and control a modern power system. One of the primary elements of a power system is the load. Accurate load models which capture important behaviors and dynamics are becoming increasingly important due to changes in the way power systems are operated (e.g., deregulation).

A physically-based load modeling framework is used to create aggregate load models. An aggregate load is split into component elements which contain individual devices that share common physical characteristics. In particular, thermostatic loads, induction motors and fluorescent lights were studied. A novel approach to thermostatic load modeling was developed, which involves creating simulated heaters which are always on or off; this approach allows both changes to system voltage and outside temperature to be accommodated without delays. A third order induction motor model was reduced using synchronic modal equivalencing. We concluded that static models were adequate for fluorescent lights in their normal mode of operation. Some work on load modeling was also done to motivate the splitting of aggregate loads into load elements.


Thesis Supervisor: Bernard C. Lesieutre
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# *Dedication*

## To **my parents:**

They helped me to dream bigger dreams than country folk from rural Pennsylvania are generally encouraged to dream.

## To **Jessica Wong:**

She helps make this country bumpkin happy.

# *Acknowledgements*

First of all, I'd like to thank Bernie Lesieutre for being a good advisor and all around nice guy. He's denied me the anecdotes and horror stories of which some of my friends seem to have an abundance; I think it's a fair trade.

I'd also like to acknowledge the members of my oral qualification exam committee, Professor Athans, Professor Lang and Professor Verghese. The whole experience was generally unpleasant (mostly, through no fault of theirs), but, in retrospect, I feel some of their comments have filtered into this thesis. I guess it wasn't totally all pain and no gain.

I'll give a blanket acknowledgement to the students of 10-082, including an honorary nod to Babak Ayazifar because he eats lunch on our side of the lab on occasion. The roaches, rats and dim lighting would be hard to endure without good company. I'd especially like to thank Jesko Hagee for writing his thesis before I did, so he could suffer through getting the thesis templates to work and all of that.

Thanks also go out to Erik Thoen for writing his thesis after I did. I saw this as a form of moral support, even if it was not intended that way.

Vivian Mizuno was not necessarily directly involved in my thesis, but she is generally overworked and underappreciated. LEES would cease to function without her.

In an attempt to not be such a pointy headed engineering type, I'd like to thank my yoga instructor, Barbara. If it wasn't for her, I would have a much more difficult time maintaining a semblance of calm.

In the same vein, Rob Howell is a constant source of help in my struggles to make sense of the information available to us, the general public. To anyone else struggling to keep your head above the muck foisted on us by the sorry excuse for a media in this country, you are not alone.

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*Chapter 1*

# *Background*

One type of element on the power system is a load; a load is a collection of devices whose main interaction with the power system is as a consumer of electricity. This thesis will discuss a particular approach to load modeling, physically-based load modeling, which can be used to model different types of loads. To place the topic of load modeling into proper perspective, a short background discussion of power systems and where load models fit into the general framework of power system modeling is presented in this chapter.

## 1.1 Power systems

Power systems are an integral part of daily life for many people; these people depend on their local utilities to provide them with electricity on demand. Accurate models for power systems are necessary to allow the operators and maintainers of the system to provide this electricity to the consumers in a safe, efficient and environmentally sound manner; these models are used to analyze and manage the transfer of energy from the utility to the consumer. Recent changes in the way utilities operate in the United States, such as the construction of fewer power plants and the deregulation of the power industry, present new challenges to achieving safe and economical distribution of power. These changes have resulted in a renewed interest in good models for power systems.

In an attempt to simplify the problem of modeling a complex power system, the power system is often conceptualized as being composed of three different general types of elements: generators, the power grid and loads. The problem is then reduced to modeling each of these types of elements and reintegrating the elements to create a total power system model. Different methods are used to model the three different types of elements because they perform qualitatively different tasks with respect to the overall power system.

Models of generators are dynamic because the physical processes involved in the operation of a generator are dynamic. A generator converts mechanical energy to electrical energy using a rotating shaft in a magnetic field. The important physical laws for this situation are Newton's laws of motion for rotational systems and Faraday's law, both of which are dynamic. Typically, generator models are more detailed then the models for the other types of elements. One reason for this greater detail is engineers are able to study generators under conditions they can control and measure. Another reason for this greater detail is the relative scarcity of individual generators connected to the power system as compared to individual power grid or load elements. Of course, probably the most important reason for the greater detail is detailed generator models are necessary for accurate system analysis and control.

Often, models of distribution elements are static because the processes which govern distribution element behavior are instantaneous when compared to times scales of interest for power system studies. Occasionally, more detailed dynamic models are necessary for studying certain phenomena.

Postponing discussion of loads temporarily, we will draw some conclusions about power system modeling in general based on the above discussion of generator and distribution element modeling. First, the degree of detail of a model is primarily governed by the degree of detail which power system engineers feel is necessary for study and control. Second, the degree of detail of a model is also governed by the accessibility of detailed information about an element from which to create a detailed model. These effects are also present in the modeling of load elements.

At present, load models are primarily static with the exception of dynamic models used to represent the portion of the load consisting of induction motors. Load models are typically not very detailed for three reasons. First, some engineers believe detailed load models are unnecessary. Second, studying loads under controlled conditions that approximate the conditions of an operating power system is difficult. Third, the number of individual devices which comprise a load element on the power system is large. A more detailed discussion of the generally-used load modeling practice will be presented in the next section.

## 1.2 Present load modeling

Now that the framework in which load models reside has been explained, a more detailed analysis of the present state of load modeling can be pursued. This overview of load modeling practice is necessary as a starting point and a basis for comparison for the load modeling in this thesis.

One crucial thing to note is the elements typically referred to as loads in power system studies are actually composed of many different individual devices. We will refer to collections of individual devices as aggregate loads, which will include conglomerations of different types of load devices and collections of many separate, similar load devices. Some of the individual devices included in these aggregate loads are part of the transmission and distribution networks. For example, online tap changers are sometimes included as part of the aggregate load. These inclusions are often by default; the behavior of some power system devices are not modeled anywhere else, so they become part of the load model [8].

When modeling aggregate loads, the overwhelming majority of electrical utilities use static models according to a recent survey [8]. One of the static models used is a polynomial load model:

$$P = P_0\left[a_1\left(\frac{V}{V_0}\right)^2 + a_2\left(\frac{V}{V_0}\right) + a_3\right], \tag{1.1}$$

$$Q = Q_0\left[a_4\left(\frac{V}{V_0}\right)^2 + a_5\left(\frac{V}{V_0}\right) + a_6\right], \tag{1.2}$$

where $P$ is real power into the load; $Q$ is the reactive power into the load; $V$ is the voltage across the load; $V_0$, $P_0$ and $Q_0$ are the initial voltage, real power and reactive power, respectively, into the load; and $a_1$ through $a_6$ are parameters used to fit the model to the characteristics of the load. This polynomial model is a combination of three different simpler static load models. The first term of each equation is a constant impedance model; the second term of each equation is a constant current magnitude at constant power factor term; the third term of each equation is a constant power term. A generalization of this model is the exponential load model:

$$P = P_0\left(\frac{V}{V_0}\right)^{np}, \tag{1.3}$$

$$Q = Q_0\left(\frac{V}{V_0}\right)^{nq}, \tag{1.4}$$

where the exponents $np$ and $nq$ can now take on values other than one, two or three. The equation for the exponential model can be the sum of more than one of the exponential terms shown. In either the exponential or polynomial models, a frequency dependent term may be multiplied by some of the terms, if appropriate.

Static load models cannot be used to model essentially dynamic phenomena. As a consequence, a dynamic induction motor model is sometimes combined with a static model when dynamic effects are deemed to be important to the phenomena being studied.

Some work has been done on dynamic load models in addition to the induction motor models mentioned earlier. A dynamic load model referred to as the "exponential recovery model" is presented in [2]. This model is an extension of a generic static load model. The differential equation for real power is given below,

$$T_p \frac{dP}{dt} + P = P_s(V) + k_p(V) \frac{dV}{dt}, \tag{1.5}$$

where $P_s$ is the static functional relationship between voltage and power for the load, $T_p$ is a time constant and $k_p$ is a nonlinear function. A similar relationship holds for reactive power. The term "exponential recovery model" derives from the model's behavior after a voltage change. After a voltage disturbance, the power exponentially approaches a new steady state. We will be using this model for comparison purposes when thermostatic loads are studied.

Another possible dynamic load model is presented in [11]. The general form of the equation is shown below,

$$T_L \frac{dG}{dt} = P(V) - V_0^2 G \left[ \frac{V}{V_0} \right]^2, \tag{1.6}$$

where $T_L$ is a time constant and $G$ is the conductance of the load.

For further references on present practice in load modeling, we refer the reader to [8], [9], and [10].

## 1.3 Reasons to develop improved load models

Though the present load models may be adequate representations for some studies, examples of situations where the present load models are inadequate have been studied. In a study of the Swedish blackout of 1983, initial calculations indicated the system should not have failed [17]. A more detailed analysis had to be performed in order to model the actual occurrence. One of the contributing factors to the initial erroneous results were the load models used. The authors concluded that for large voltage and frequency excursions the load models must have dynamics and include nonlinear stationary relationships.

Another issue that arises is the existence of solutions to the network/load equations. Sufficient conditions on static load models have been derived that guarantee the existence of a solution to the network/load equations [7]. In certain situations, some static load models can lead to no solution to the network/load equations, which is not physically realistic.

Fitting parameters for a static load model to data taken from an aggregate load is only guaranteed to represent that aggregate load under similar conditions. If the aggregate load being studied is not truly static, inferring the response of the load to certain conditions from a static load model may not be valid if the load model was derived under conditions different from those being studied. As an example, a model for a load containing a substantial number of fluorescent lights is substantially different depending on whether the voltage is low enough to extinguish the lights.

One method of incorporating dynamic effects into the load model is to include a dynamic induction motor model in the aggregate model. Part of the rational for this is that historically induction motors have consumed up to seventy percent of the electrical energy in the United States [8], and they are, therefore, the largest source of dynamic effects. While this is true, if some other load device had dynamics which affected the response of the aggregate load, the induction motor model would not necessarily be able to capture these dynamics.

As a last note, improving load models can also have less dramatic, but substantial benefits. If the present load models are generating overly pessimistic results, an improved load model will show that a more efficient use of the present system is viable. If the present load models are generating overly optimistic results, an improved load model will show where possible problems could occur. As mentioned in [8], even if safety is the primary concern, no one static load model yield the most conservative results in every situation.

*Chapter 2*

# *Overview*

To overcome some of the shortcomings of the present load models, we are proposing a different approach to developing aggregate load models. In this chapter, the important features of our aggregate load model are outlined. In addition, the particulars of this modeling strategy which were studied for this thesis are presented.

## 2.1 Physically-based load modeling

Instead of modeling aggregate loads using static load models with the possible addition of induction motor models, we propose to use a physically-based approach to load modeling. In a manner analogous to the way a power system is separated into different types of elements (generators, loads and power grid) for modeling, the aggregate load can be separated into smaller elements, each of which contain similar types of loads. For example, an aggregate load for a residential neighborhood could be split into baseboard heaters, air conditioners, hot water heaters, electronics, lighting, and induction motors. Each of these smaller aggregate load elements are modeled based on their physical characteristics and, then, the total aggregate load is created by combining these smaller aggregate elements.

Developing a physically-based aggregate load model requires three steps, then. First, suitable elements into which to split the total aggregate load must be determined. Because this method is physically-based, aggregate load elements should be composed of individual loads which share common physical characteristics. For example, an aggregate load element composed of baseboard heaters might be desirable because all heaters are governed by similar processes. Of course, individual heaters will have different particular models, but all of the heaters will have the same basic dynamic structure and share common physical processes.

Second, the load elements must be modeled. Because each load element is selected to incorporate individual loads with common physical characteristics, these models will ideally be based on the physical processes which govern the elements. If the model of a load element is truly representative of the behavior of that load element, the model should work under a variety of conditions.

Third, the individual load element models must be combined to form a total aggregate load model; we must determine how the individual load elements interact and how they can be connected to the rest of our system model. As an adjunct to this step, the parameters of the overall load model must be determined as well. The overall load modeling could be done entirely with *a priori* knowledge of the composition of the aggregate load, but this composition information is unlikely to be completely accurate. In addition, the models for each of the load elements will be a simplification of the true behavior of the load element. Therefore, in a real situation, the overall form of the load model would be determined using this outlined technique, but the specific parameters of the model would be determined using data collected from the actual aggregate load.

A physically-based load model may be more difficult to derive then a static load model because of the necessity to identify the composition of the load and study individual load elements in detail, but there are at least two distinct advantages to this approach. First, a physically-based load model is developed using the actual physical processes present in the load; the form of the model is justifiable physically. Different static load models could seem to model an aggregate load based on measurements of the load; in these cases, a physically-based analysis of the aggregate load may actually indicate which of the possible models is a better representation for the aggregate load. Second, a physically-based load model will have relevant dynamics of the aggregate load that would be omitted by a simpler static load model.

## 2.2 Work completed for this thesis

The bulk of this thesis is concerned with the second step outlined above, modeling load elements. Three common load types were studied: thermostatic heaters, induction motors, and fluorescent lights. These three load types were selected for a variety of reasons. First, they are common loads that are likely to have substantial effects on a typical aggregate load. Second, they are each controlled by very different physical processes, and different techniques are used to model each.

Some work was also completed on the first step outlined above, identifying the load elements for an aggregate load. The identification process is not yet practical for all types of individual loads, but some load components can be distinguished in an aggregate load measurement.

The third step is left as future work. We theorize that a good way to approach the reintegration of the load elements is by putting each load element on a bus. The model of each load element would need to be formulated using input and output variables which were common to all of the load models, and these load models would then be combined in parallel.

## 2.3 Outline

The next three chapters each describe the modeling of one of the load elements: thermostatic heaters, induction motors, and fluorescent lights. The last chapter of the body of the thesis describes the load monitoring work, which is then followed by a summary of the entire thesis.

*Chapter 3*

# *Thermostatic Loads*

One load element of interest is composed of thermostatic heaters. The model developed for the group of thermostatic heaters is referred to in this chapter as the aggregate model, but this aggregate model should not be confused with the total aggregate model composed of many different types of load devices described earlier; the total aggregate load model is not the subject of this chapter.

The load model developed in this chapter is based on the thermal processes which control the thermostat on a thermostatic load. The final approach approach uses a simulation of an individual representative heater to determine the total aggregate load. This approach is novel in that the dynamics of this heater have been separated into two simulations, one for the heater when it is on and one for the heater when it off; these separate estimates are recombined to calculate the total aggregate model.

## 3.1 Brief description of thermostatic loads

Thermostatic loads are devices which are controlled by a temperature gauge. Common examples of thermostatic loads are electric heaters, air conditioners and hot water heaters. Throughout this chapter, we will be primarily concerned with electric heating, but the results are applicable to generic thermostatic loads. For clarity, resistive heating will only be referred to specifically when necessary. Also, the substance being heated is not necessarily the air in a house, but, to avoid cumbersome language, the space being heated will be referred to as a house. As background, this section covers the basics of electric heaters and thermostats.

An electric heater, on the simplest level, is a resistor. An electric current is passed through the resistor which causes energy to be dissipated in the form of heat; the dissipated energy is used to heat the surrounding air. Based on this, an aggregate model for a group of heaters would be simple to develop; the model would consist of resistors in parallel. However, electric heaters are normally connected to some form of thermostat, which complicates the model.

A thermostat is a temperature regulating device. A thermostat attempts to maintain the ambient temperature near some desired temperature. Typically, a thermostat will not have the ability to control the voltage across the electric heater; the thermostat can only control whether the electric heater is on or off. Ideally, the thermostat would turn on the heater as soon as the ambient temperature was colder than the desired temperature and turn off the heater as soon as the ambient temperature was hotter than the desired temperature, but this scheme results in excessively switching the heater on and off. In practice, a "low temperature" and "high temperature" are selected. If the ambient temperature is colder than the "low temperature", the heater will be turned on; if the ambient temperature is warmer than the "high temperature", the heater will be turned off.

More complicated thermostats exist that attempt to compensate for the thermal time constant of the resistor in the electric heater [4]. These types of thermostats could be modeled using similar techniques to the ones outlined in this chapter, but we will develop an aggregate model for electric heaters which includes the effects of the simpler type of thermostat.

## 3.2 Previous models

One of the commonly used aggregate models for electric heaters consists of two models, one for short time scales and one for longer time scales [8]. The basic idea is to simplify the problem by assuming we are only interested in the behavior of the device over a certain time scale.

On short time scales (less than five minutes), the aggregate heater load is modeled as a constant impedance load. The reasoning for this is based on the aggregate load being composed of many individual heaters. Any change in voltage will only affect heaters which are on when the change occurs; the rate at which the ambient temperature drops and the time until it reaches the "low temperature" are not affected by changes in voltage since no current is flowing. In addition, the effect on an individual heater which is on is to lengthen (shorten) the amount of time it is on (off) if the voltage decreases (increases), but the aggregate load experiences a change due to an individual heater only as the heater approaches the time when it will turn off. Therefore, if a voltage change occurs, the number of heaters which are on will remain approximately constant for a short period of time. So, the impedance of the group of heaters, when viewed as resistors in parallel, remains constant.

On longer time scales (more than twenty minutes), the aggregate heater load is modeled as a constant power load. An argument based on energy concepts is used to justify this model. First, we will need to assume that, if the voltage changes, the average ambient temperature of each house being heated remains constant. As long as each heater is still able to increase the temperature of its house to the "high temperature" setting, a constant average temperature is a reasonable approximation. If the temperature for each heater is assumed to increase and decrease linearly as a function of time, the average ambient temperature of each house will actually remain constant. Next, we note that, if the average temperature in the house being heated remains the same, the average power into the house must be constant as well. Therefore, if we ignore any transient behavior, each heater is a constant power load, and, hence, the aggregate heater load is a constant power load on average over a long period of time.

Another possible model which has been proposed is an exponential recovery model [2]. This model responds to a voltage step by immediately reducing the real power consumed. The power consumed then rises exponentially to approach a new steady-state power consumption.

## 3.3 A single heater

In order to model the aggregate heater load, we will first look at a single heater in detail. Because the heater is controlled by a thermostat, we need a model of the thermal process to which the thermostat responds. In the next section, we present a simple model of the relevant thermal system. Following that, we make some observations about how a single heater will respond to a change in voltage.

### 3.3.1 A simple model of the thermal system

An easy way to arrive at a simple model for a house being heated with a heater is to look at the analogous electrical circuit. This model will only capture the first order thermal effects in the house being heated, but these effects should be the dominant ones. A more complicated thermal model could be incorporated if necessary.

As shown in Figure 3.1, the walls of the house are a thermal resistance; the air in the house is a thermal capacitor; the heater is a current source. To complete the circuit analogy, we note that temperature ($\theta$) is analogous to voltage; the outside temperature is a voltage source. We have now reduced our thermal model to a simple first order circuit, shown in Figure 3.2.

The circuit shown is not the complete model. The behavior of the switch must be specified as well.

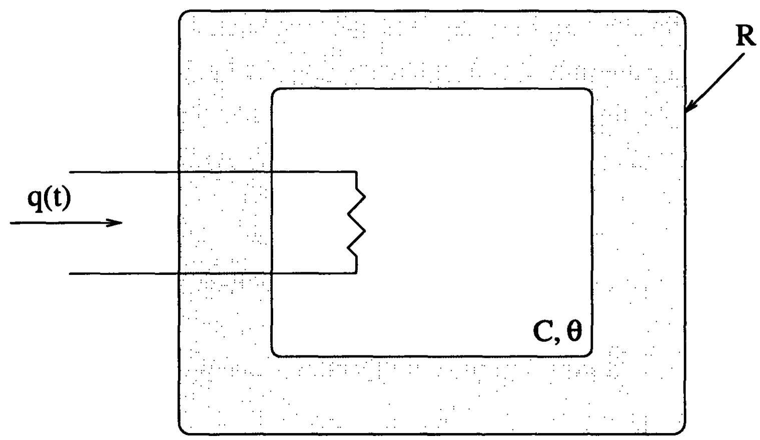


Figure 3.1: A house can be modeled using "circuit" variables, as shown in the schematic above.

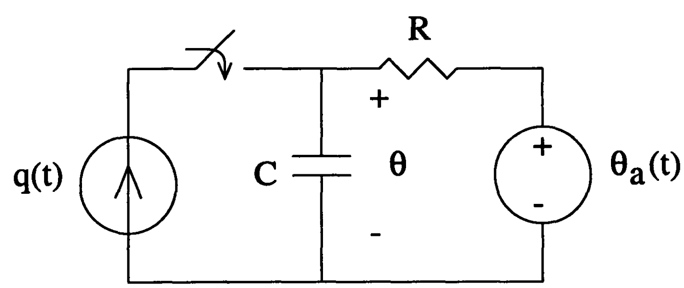


Figure 3.2: Because of its familiarity, a simple first order circuit is a suitable representation for the model shown schematically in Figure 3.1. The switch represents the action of the thermostat, and the voltage source labeled $\theta_a(t)$ models the effect of the temperature outside of the house.

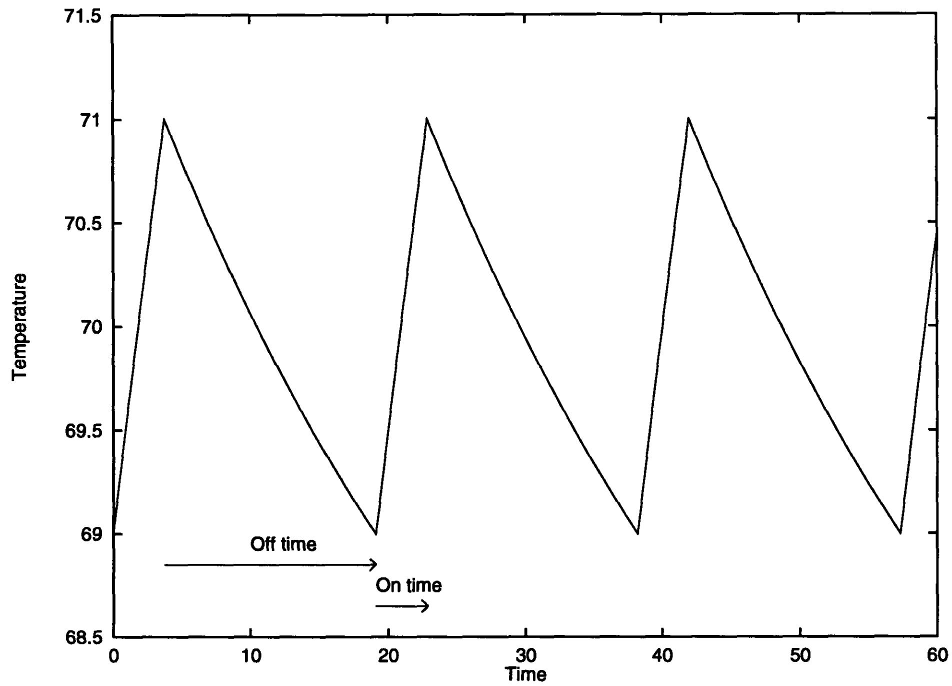


Figure 3.3: A sample response of the first order circuit model to a constant input heat flow is presented here; the "low temperature" was sixty-nine; the "high temperature" was seventy-one; the time constant of the system was thirty. Examples of "off time" and "on time" are also marked.

The switch represents the action of the thermostat. In the electrical circuit model, the switch is controlled by the voltage across the capacitor. If the voltage increases above a specified limit, the switch will open. If the voltage decreases below a specified limit, the switch will close.

Using this circuit model, we can derive a first order differential equation for the heating and cooling of the house [3]:

$$\frac{d\theta}{dt} = -\frac{1}{\tau}\left[\theta - \theta_a(t) - w\theta_g(t)\right] \tag{3.1}$$

where $\tau$ is $\frac{1}{RC}$, $\theta_a$ is the effective temperature outside of the house, and $\theta_g$ is the "temperature gain" of the heater, $Rq(t)$. To represent the switch, the variable $w$ is zero when the switch is open (the heater is off) and one when the switch is closed (the heater is on). To make this equation more tangible, a sample simulation using the model is presented in Figure 3.3. When the temperature is increasing, the variable $w$ is one; when the temperature is decreasing, the variable $w$ is zero.

This thermal circuit model is related to the electrical power system through the current source $q(t)$

of the thermal circuit model. The current source in the model is a representation for a heat source. The heat flow into the house is the power dissipated by the heater, which is proportional to the voltage across the heater squared (heat flow corresponds to current in our circuit representation).

### 3.3.2 Voltage changes for a single heater

To facilitate understanding of the response of the aggregate load to changes in voltage, we will first explore what happens to a single heater when the voltage across it is changed. Temporarily, we will keep the discussion general by referring to changes to input heat flow, instead of voltage, because the qualitative analysis which follows does not depend on the thermal source being a resistive heater.

One important thing to note is that, if the input heat flow and the outside temperature are constant, the temperature in the house being heated will oscillate between the high and low temperature settings with a fixed period, as shown in Figure 3.3. For clarity, we will refer to the length of time in a single cycle during which the heater is dissipating heat as the "on time" of the heater. Similarly, we refer to the length of time in a single cycle during which the heater is not dissipating heat as the "off time" of the heater. Examples of these concepts are shown in Figure 3.3. To elaborate on the original note, the "off time" of the heater only depends on the outside temperature and does not depend on the input heat flow. Therefore, if the heat flow changes, the new oscillations will have the same "off time" per cycle if the outside temperature remains fixed. Consequently, we only need to study the response of the "on time" to changes in heat flow if the outside temperature is approximately constant. Temporarily, we will assume the outside temperature is approximately constant, but the final model will not be dependent on this condition.

With steady-state conditions, the "on time" of the heater is simple to calculate. The ambient temperature of the space being heated during its "on time" is the solution of a first order differential equation, specifically Equation (3.1) with $w$ set to one. The initial temperature is the low temperature setting on the thermostat. The ambient temperature rises exponentially from there with a time constant determined by thermal factors of the house being heated (for example, the size of the house). The ambient temperature the system would approach, if the thermostat did not turn off the heater, is the sum of the outside temperature and the "temperature gain" of the heater, $\theta_a + \theta_g$. The "on time" is finished when the ambient temperature reaches the high temperature setting of the thermostat. A change in the heat flow into the house only affects the final temperature the temperature is asymptotically approaching while the heater is on. An increase in heat flow into the house will increase the final temperature the system temperature asymptotically approaches

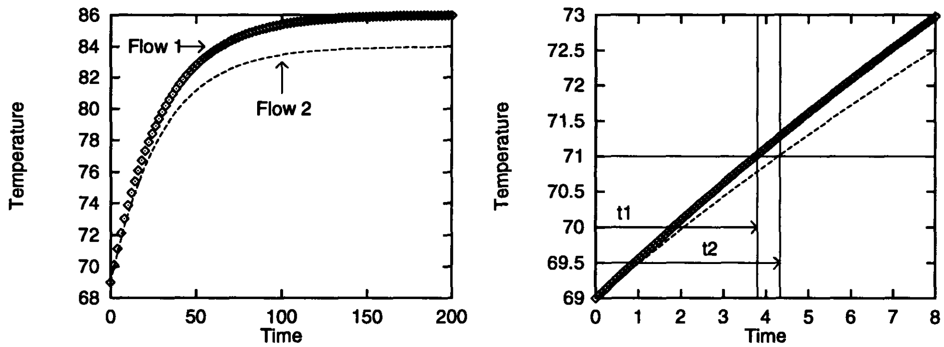


Figure 3.4: The figure on the left is of the temperature in a house (using our model) with two different heat flows into it and without a thermostat to turn off the heater. The heat flow corresponding to the dotted line, Flow 1, is greater than the heat flow corresponding to the solid line, Flow 2; therefore, the temperature corresponding to Flow 1 asymptotically approaches a higher final temperature than the temperature corresponding to Flow 2. The figure on the right is an enlargement of the figure on the left. The "time on" corresponding to Flow 1, t1, is shorter than the "time on" corresponding to Flow 2, t2, as explained in the text.

and decrease the time it takes the system temperature to reach the high temperature setting of the thermostat. In other words, the "on time" of a heater decreases as the heat flow into the house increases. An example simulation is presented in Figure 3.4.

## 3.4 Determining the aggregate heater model

As mentioned earlier, the aggregate heater model is a number of resistors in parallel. Determining the aggregate heater model is equivalent to determining the number of heaters which are on.

### 3.4.1 Assumptions about the distribution of heaters

Some assumptions about the distribution in time of the heaters are necessary to produce an aggregate heater model. In steady-state conditions, each of the heaters being aggregated has a constant period. For simplicity, we will assume the heaters can be treated as having identical characteristics. In reality, the heaters will each have different characteristics, as will the houses they are heating, but to illustrate this concept, we will assume the heaters are identical. So, each of the heaters has the same constant period. The assumption needed is about how each of these constant cycles relates to the other cycles. For example, all of the heaters could be in phase. If this were true, all of the heaters would be either on or off at any given time, as shown in Figure 3.5. We will treat

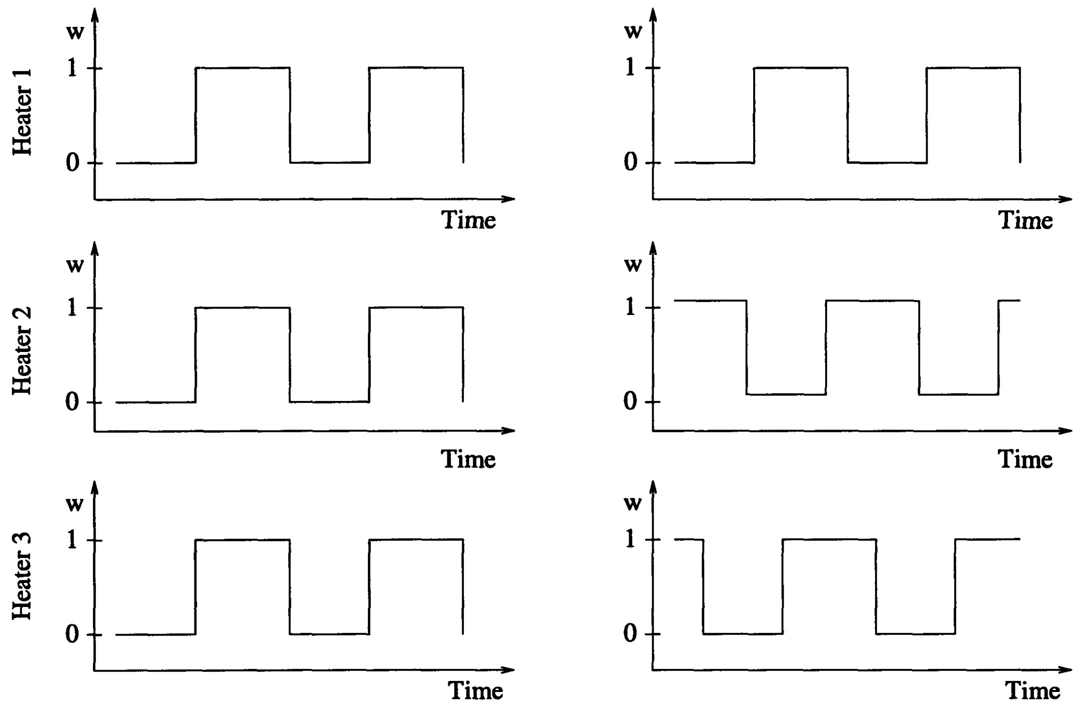


Figure 3.5: The figure above is of two different distributions in time of three different heaters. The three plots of state on the left demonstrate the example in the text of the heaters being in phase. The three plots of state on the right represent a more staggered heater distribution.

the relationship between heaters probabilistically because many factors contribute substantially to it.

The approach taken in this thesis is to view the heater state, $w$ in Equation (3.1), as a random process which takes on the value one at a given instant in time if the heater is on and takes on the value zero if the heater is off. A graph of an individual heater's heater state, $w$, is a sample path of this random process. Each sample path is equally likely, so the expected value of $w$ over all heaters is the percentage of heaters which are on. The overall process is assumed to be ergodic. If any individual heater is observed, the percentage of heaters which are on at any given time is approximately the ratio of the observed heater's "on time" to the length of its period; the expected value of the random process at that time is approximated by the time average of a single sample path.

With this assumption about the distribution of heaters, the number of heaters which are on can be approximated by calculating the length of the period of one representative heater as follows. (As stated earlier, the heaters are not actually identical. Some representative, or average, heater characteristic will need to be determined. If necessary, the process outlined below could also be done for several heaters and then the resulting predictions could be averaged.) The percentage of heaters which are on at any given time can be approximated using the "on time" and the period:

$$\% \text{ of heaters which are on} \approx \frac{t_{on}}{t_{off} + t_{on}}, \tag{3.2}$$

where $t_{on}$ is the "on time" of the observed heater and $t_{off}$ is the "off time" of the observed heater. Assuming a constant ambient temperature on the time scale of interest, $t_{off}$ is a constant, and Equation (3.2) can be equivalently formulated as follows:

$$\% \text{ of heaters which are on} \approx \frac{T - t_{off}}{T}, \tag{3.3}$$

where T is the period of the cycle (i.e., the sum of "on time" and "off time"). Therefore, the percentage of heaters which are on can be approximated using the length of the period of one heater, which can be multiplied by the total number of heaters to calculate the number of heaters which are on.

### 3.4.2 Calculating the instantaneous period

The problem of approximating the number of heaters which are on has been reduced to determining the instantaneous period of the temperature of a single representative heater. The temperature of a heater as a function of time is periodic if the input heat flow and the outside temperature are constant. Hence, the period is well defined in this case. Unfortunately, the case where the input heat flow is constant is not interesting because it implies a static power system voltage. In the case of interest, the input heat flow will be a function of time, so a usable definition of instantaneous period must be found. We decided to calculate the period using averaging techniques because averaging and periods are closely related for periodic functions. (A time-dependent outside temperature can also be incorporated).

One approach would be to use the average temperature of the heater to determine the period. The presumption is that the average temperature over one period will be the same for different steady-state heat flows [8]. Unfortunately, the average temperature of a heater with a constant input heat flow is a function of the heat flow. If the temperature rose and fell linearly, the average temperature would not depend on the input heat flow, but the temperature rises and falls with an exponential characteristic. After an initial investigation, we decided that this was not a fruitful way to proceed.

Another approach would be to use the average of the derivative of temperature. The advantage of this approach is the average of the derivative of temperature for a heater with a constant heat flow input is zero over any integer multiple of the period; a direct application of the fundamental theorem of calculus shows this. (The derivative is not defined when the heater transitions from its off state to its on state or when it transitions from its on state to its off state, but these are only two points of the period, and the temperature is continuous; so, the result still holds because the averaging integral is still well defined.) We can then define the period as the length of time over which the average of the derivative of temperature is zero. This definition of period coincides with the normal definition of period for constant heat flow inputs, but we will need to extend the typical notion of average over a period to account for the function not being periodic in the case of non-constant heat flow inputs.

The average of a function over one period is given by the following equation:

$$\langle x \rangle = \frac{1}{T} \int_{t-T}^{t} x(s)\, ds, \tag{3.4}$$

where $x$ is the function being averaged, $t$ is the current time, and $T$ is the period. If the period is constant, the $T$ term can be moved into or out of the integral without affecting the calculation, but if the period is not constant, moving this term is not permissible. If this term is inside of the integral, the averaging operator is performing a weighted integral, where the function is weighted at each instant in time by the inverse of the period at that time. If the term is outside of the integral, the averaging operator is integrating over the period and then normalizing by the length of the period. Both methods have justifications, so both were explored for this thesis.

### 3.4.3 Averaging as a weighted integral

The first approach was to use an averaging operator with the period term inside of the integral:

$$\langle x \rangle = \int_{t-T(t)}^{t} \frac{x(s)}{T(s)}\, ds. \tag{3.5}$$

This approach can be justified if we view averaging as the index-0 Fourier coefficient [14]. This form of averaging would be particularly appropriate if the higher-order Fourier coefficients were used, as is done in the averaging literature in the field of power electronics [14].

This averaging operator is nonlinear; the importance of this for us is that the average of the derivative is not the derivative of the average. The derivative of the average can be calculated using Leibnitz' rule,

$$\frac{d\langle x\rangle}{dt}=\frac{x(t)}{T(t)}-\frac{x(t-T(t))}{T(t-T(t))}\left[1-\frac{dT}{dt}\right], \tag{3.6}$$

assuming the period is differentiable. The period, in reality, will be slowly varying because the heater itself has a thermal time constant and cannot change the heat flow input instantaneously, so assuming the period is differentiable is reasonable.

We can now apply Equation (3.5) and Equation (3.6) to Equation (3.1).

$$\begin{aligned}
\frac{d\theta}{dt} &= -\frac{1}{\tau}\left[\theta-\theta_a(t)-w\theta_g(t)\right] && (3.7)\\
\langle\frac{d\theta}{dt}\rangle &= \langle-\frac{1}{\tau}\left[\theta-\theta_a(t)-w\theta_g(t)\right]\rangle && (3.8)\\
\int_{t-T(t)}^{t}\frac{1}{T}\frac{d\theta}{ds}\,ds &= -\frac{1}{\tau}\left[\langle\theta-\theta_a(t)-w\theta_g(t)\rangle\right] && (3.9)\\
\int_{t-T(t)}^{t}\frac{d}{ds}\left[\frac{\theta}{T}\right]+\frac{\theta}{T^2}\frac{dT}{ds}\,ds &= -\frac{1}{\tau}\left[\langle\theta-\theta_a(t)-w\theta_g(t)\rangle\right] && (3.10)\\
\frac{\theta(t)}{T(t)}-\frac{\theta(t-T(t))}{T(t-T(t))}+\int_{t-T(t)}^{t}\frac{\theta}{T^2}\frac{dT}{ds}\,ds &= -\frac{1}{\tau}\left[\langle\theta-\theta_a(t)-w\theta_g(t)\rangle\right] && (3.11)\\
\frac{d\langle\theta\rangle}{dt}-\frac{\theta(t-T(t))}{T(t-T(t))}\frac{dT}{dt}+\int_{t-T(t)}^{t}\frac{\theta}{T^2}\frac{dT}{ds}\,ds &= -\frac{1}{\tau}\left[\langle\theta-\theta_a(t)-w\theta_g(t)\rangle\right] && (3.12)
\end{aligned}$$

In the derivation up through Equation (3.12), the variable $T$ has not been restricted in any way; $T$ is not yet what we would call a period. To continue, a restraint must be placed on this equation to ensure that $T$ is the desired quantity we shall call the period. The condition used is the one mentioned earlier; the average of the derivative of temperature must be zero.

$$\begin{aligned}
0 &= \frac{d\langle\theta\rangle}{dt}-\frac{\theta(t-T(t))}{T(t-T(t))}\frac{dT}{dt}+\int_{t-T(t)}^{t}\frac{\theta}{T^2}\frac{dT}{ds}\,ds && (3.13)\\
\frac{dT}{dt} &= \frac{T(t-T(t))}{\theta(t-T(t))}\left[\frac{d\langle\theta\rangle}{dt}+\int_{t-T(t)}^{t}\frac{\theta}{T^2}\frac{dT}{ds}\,ds\right] && (3.14)
\end{aligned}$$

Unfortunately, Equation (3.14) is not an explicit differential equation for the period; the current value of the derivative of the period appears in the integral on the right side of the equation. The other problem with the equation is our constraint on the equation does not uniquely describe the period; during each cycle, the temperature attains each possible value of temperature between the high temperature and the low temperature twice, once while it is increasing and once while it is decreasing. Even if it was possible to easily and reliably use Equation (3.14), the result might not be valid.

### 3.4.4 Averaging as integrating then normalizing

Averaging at each instant in time by integrating over a length of time and then normalizing by the reciprocal of this length of time is the familiar way to perform an average. Unlike the method described above, this method only uses the current period in the calculation, which simplifies the calculation and eliminates the undue influence of past period approximations on the current approximation.

We will again apply the criterion that the average of the derivative over a period must be zero, but, since we are not interested in higher-order Fourier terms, we will use the second definition of average,

$$\langle x \rangle = \frac{1}{T(t)} \int_{t-T(t)}^{t} x(s)\, ds. \tag{3.15}$$

In this case, the relationship of interest is the following:

$$\frac{1}{T} \int_{t-T(t)}^{t} \frac{d\theta}{ds}\, ds = -\frac{1}{\tau} \left[ \frac{1}{T} \int_{t-T(t)}^{t} \theta - \theta_a(t) - w\theta_g(t)\, ds \right] = 0. \tag{3.16}$$

A simplification gives the following result:

$$\theta(t) = \theta(t - T). \tag{3.17}$$

Finding the lengths of time which are candidates for the period is straightforward using this simplification. Of course, this criterion has the same problem as Equation (3.14) does; the period is not uniquely described by it.

A period can still be calculated using this method because we can directly deduce which possible solution that meets our criteria is correct. A sample simulation was performed using this method, and the results are shown in Figure 3.6. As the figure shows, the prediction does not update for considerable lengths of time, despite changes in the actual percentage of heaters which are on. If we recall that the heater is not influenced by the input heat flow while the heater is off, the cause for this discrepancy is clear; some of the heaters will be affected by a voltage change at an instant in time, but, if the representative heater is off at this instant in time, the representative heater will

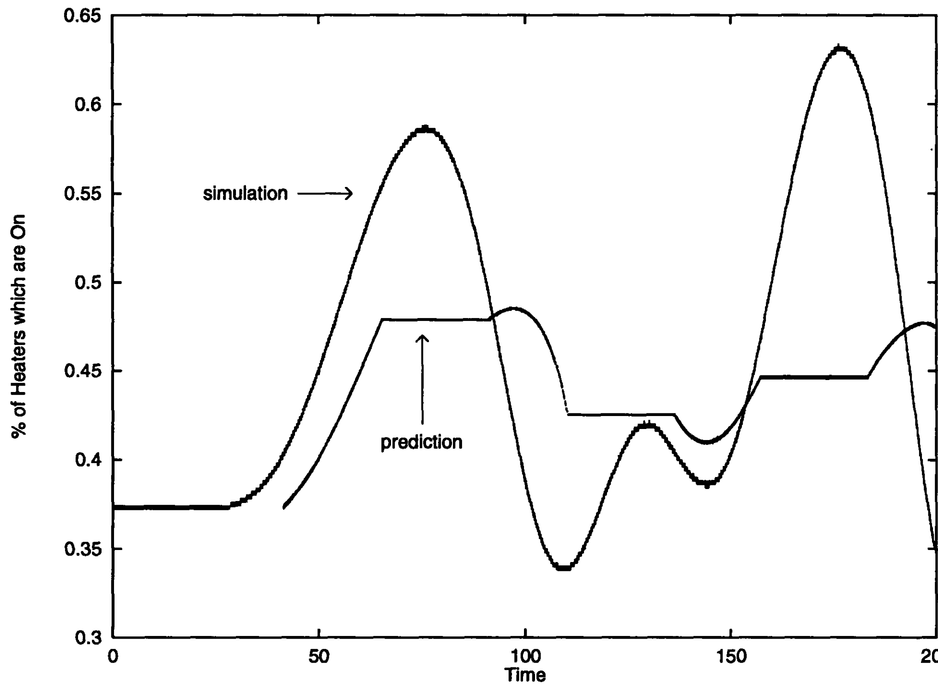


Figure 3.6: The simulation is of 500 identical heaters. The thermal time constants were sixty-four minutes; the low and high thermostat settings were seventy-two and seventy-five degrees; the outside temperature was sixty-six degrees; the "temperature gain" of the heater was initially twenty degrees, but after twenty-two minutes the "temperature gain" oscillated about seventeen degrees. These thermal parameters were adapted from [3].

not be affected.

To resolve this problem, we will again consider how a single heater reacts to a change in voltage. As a side note, a different solution to the one presented below is to take measurements of several heaters and average them because some of them will be on whenever a voltage change occurs.

### 3.4.5 Separating "time on" from "time off"

A separation of the "time on" and "time off" periods of time was used to resolve the problem of an observed heater not being influenced by input heat flow changes while it is off. This separation is justified by noting how a single heater is influenced by a voltage change. If the voltage changes while a heater is off, the heater will continue its cycle normally until it turns on. When the heater turns on, the heater behavior will conform to the expected behavior based on the new input heat flow. In other words, if a heater is off when a voltage change occurs, the behavior of the heater from

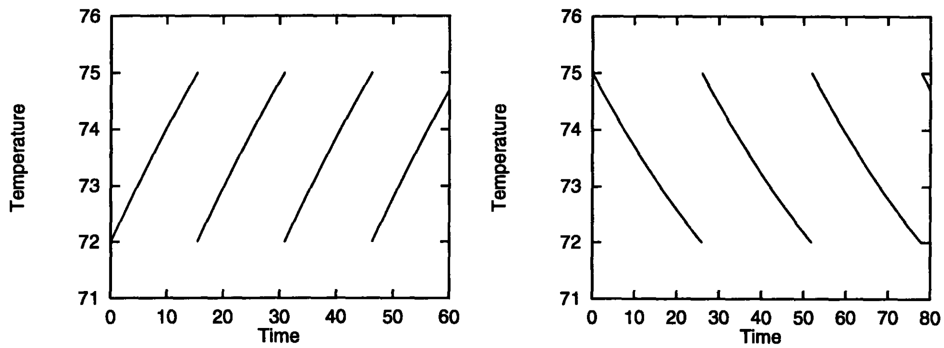


Figure 3.7: The percentage of heaters which are on should be given by $\frac{t_{on}}{t_{off}}$. The figure on the left is used to calculate $t_{on}$, while the figure on the right is used to calculate $t_{off}$. Simulations of the type shown in the figure on the left will be used to model the effects of changes in input heat flow; simulations of the type shown in the figure on the right will be used to model the effects of changes in outside temperature.

that time forward will be indistinguishable from a heater which had the new input heat flow for all time. So, the heaters which are on when the voltage changes will be the only ones to experience a transient due to this voltage change.

To capture this effect, we propose to simulate the percentage of heaters based on only measurements of a heater which is on. A byproduct of this approach is the ability to incorporate changes in outside temperature, as well, by looking at measurements of a heater which is off. Sample simulations of individual heater characteristics are shown in Figure 3.7.

We have eliminated the need to explicitly calculate the period. We are approximating $t_{on}$ and $t_{off}$ directly. The approximation method is, however, the same as that for the entire period. The derivative of the temperature is no longer a viable criterion because the temperature for each simulation is either always rising or always falling. Instead, the result in Equation (3.17) is applied. As shown in the figures, the application of this result should be unambiguous.

## 3.5 Results

Using heater parameters from [3], the approach outlined in the previous section for calculating the percentage of heaters which are on in an aggregate heater load by simulating "time on" and "time off" separately was tested through simulation. The simulations focused on changes in input heat flow, as opposed to changes in outside temperature, but the same mechanisms should work for both

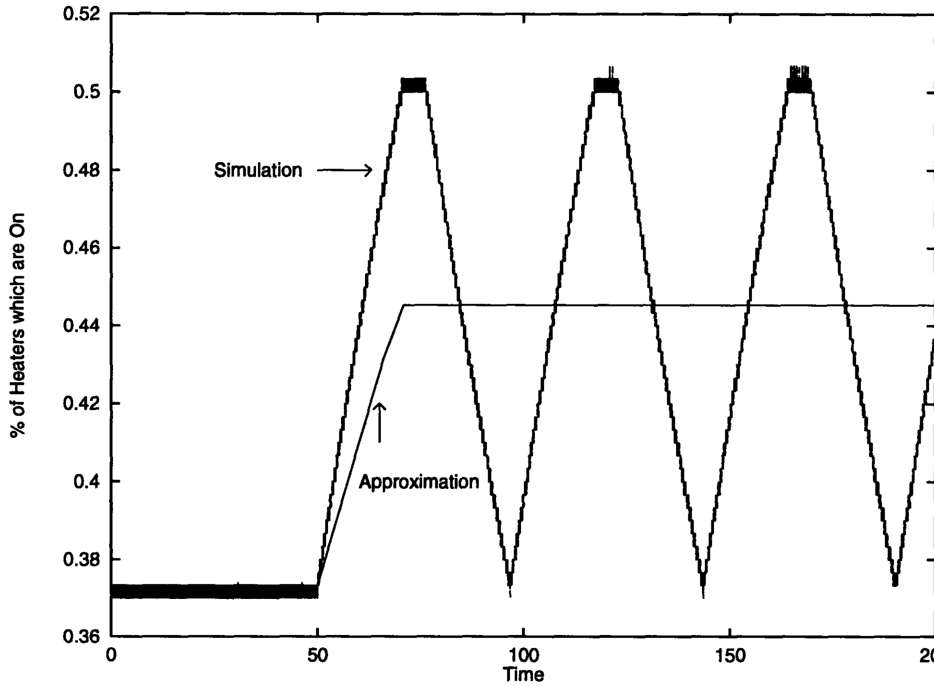


Figure 3.8: The simulation shown is of three hundred heaters. The heaters were distributed uniformly in a deterministic manner. A ten volt step change was simulated after fifty minutes. The oscillations in the simulation are due to the distribution not being uniform for the new input heat flow. In reality, the oscillations would not be as severe because the heaters would not be deterministically distributed and they would not be identical. The approximation was calculated using the separation technique described in this section. (The C code used to generate this approximation is included in Appendix A). The percentage it attains after the step change is the theoretical steady-state percentage for the new heat flow input.

processes. The results shown here are of a simulation of a step change in the input voltage.

## 3.6 Conclusion

The modeling technique discussed in this section is a mixture of simulation and calculation. A single representative heater is simulated and the results of the simulation are used to calculate changes in the percentage of heaters which are on based on changes in input heat flow and outside temperature.

The total heater model is a collection of resistors in parallel. The number of heaters to place in parallel is calculated using measurements based on a single representative heater. These measurements are not of an actual heater but of two conceptual heaters; one of these heaters models only the heater while it is on, and the other models only the heater while it is off. The first simulated heater is used to approximate $t_{on}$, while the second simulated heater is used to approximate $t_{off}$. The value of $t_{on}$ is influenced by the system voltage, while the value of $t_{off}$ is influenced by the outside temperature. The percentage of heaters which are on is given by $t_{on}/(t_{on}+t_{off})$. The total resistance of the aggregate heater load is given by

$$R = \left(1 + \frac{t_{off}}{t_{on}}\right) \frac{1}{gN}, \tag{3.18}$$

where $R$ is the total aggregate resistance, $g$ is the conductance of a single heater, and $N$ is the total number of heaters.

This total aggregate heater model is dynamic. The total resistance depends on $t_{on}$ and $t_{off}$; the calculation of these two lengths of times uses the simulated temperature from a period of time in the past (i.e., the calculation has memory).

Our calculations deviate from previous results in two ways. After a voltage change, the aggregate heater load will attain a new steady-state after all of the heaters which were on during the change turn off. This result is in contrast to the previous expectation that the new steady-state value would only be attained after each of the heaters had completed an entire cycle [8]. Our model also differs from the exponential recovery model. The exponential recovery model exponentially approaches some new steady-state after a voltage disturbance. Our model does not approach the new steady-state exponentially and actually achieves the new steady state in a finite period of time. Achieving the new steady state in a finite period of time is not possible with a standard linear system model, so our model is distinct from this whole class of models, as well as the exponential recovery model in particular.

*Chapter 4*

# *Induction Motors*

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Induction motors are an important class of load models. Induction motors consume over half the electrical energy produced in the United States [8]. Perhaps more importantly, induction motors are dynamic loads, and, consequently, induction motor models are one of the few physically-based load models in common use.

The model of a single representative induction motor is commonly used to model an aggregate induction motor load, and we will also follow this practice [8], [10]. The main goal of this chapter is to explore ways to reduce the order of a third order induction machine model. Reducing the order of this model is advantageous because we would like to aggregate the induction motor with other physically based load models; to make the total aggregate load model manageable, we need to minimize the complexity of each load component model before aggregating.

Several methods to reduce the order of a set of equations might be applicable to this problem, so we will go through an overview of different methods and how they might apply to the problem at hand, during the course of this chapter.

## 4.1 Third order induction motor

The induction motor model we are starting with can be represented as a set of differential algebraic equations [16]:

$$
\begin{aligned}
2H\frac{ds}{dt} &= T_m - E'_D I_D - E'_Q I_Q && (4.1)\\
T_0\frac{dE'_D}{dt} &= -E'_D + s\frac{X_r}{R_r}E'_Q - \frac{X_m^2}{X_r}I_Q && (4.2)\\
T_0\frac{dE'_Q}{dt} &= -E'_Q - s\frac{X_r}{R_r}E'_D - \frac{X_m^2}{X_r}I_D && (4.3)\\
0 &= V_D - E'_D - R_s I_D + X' I_Q && (4.4)\\
0 &= V_Q - E'_Q - R_s I_Q - X' I_D. && (4.5)
\end{aligned}
$$

The state variables in these equations, $s$, $E'_D$ and $E'_Q$, represent the slip of the motor, the real internal voltage and the imaginary internal voltage, respectively. ($E'_D$ and $E'_Q$ are also referred to as "direct" and "quadrature" voltages. The notation used is a result of this terminology.) Also, of interest, $I_D$ and $I_Q$ are the real and imaginary current into the motor; $V_D$ and $V_Q$ are the real and imaginary voltage across the induction motor. Lastly, the remaining variables are parameters related to the induction motor, but they will not concern us directly in our analysis.

We can reduce this system of differential algebraic equations to a set of three differential equations by solving the algebraic constraints for the current and substituting this into the three differential equations. Of course, this substitution will eliminate the current from our set of equations, but, after we have solved the differential equations, we can calculate the current using the algebraic equations. The other undesirable effect of this substitution is the resulting three equations will be nonlinear.

$$
\begin{aligned}
2H\frac{ds}{dt} &= T_M + \frac{E'_D\left(E'_Q X' - V_Q X' - R_s V_D + R_s E'_D\right)}{(X')^2 + R_s^2} - \\
&\quad \frac{E'_Q\left(X'E'_D - X'V'_D + R_s V_Q - R_s E'_Q\right)}{(X')^2 + R_s^2} && (4.6)\\
T_0\frac{dE'_D}{dt} &= -E'_D + s\frac{X_r}{R_r}E'_Q - \frac{X_m^2\left(X'E'_D - X'V_D + R_s V_Q - R_s E'_Q\right)}{X_r\left((X')^2 + R_s^2\right)} && (4.7)\\
T_0\frac{dE'_Q}{dt} &= -E'_Q - s\frac{X_r}{R_r}E'_D - \frac{X_m^2\left(X'E'_Q - X'V_Q - R_s V_D + R_s E'_D\right)}{X_r\left((X')^2 + R_s^2\right)} && (4.8)
\end{aligned}
$$

Now, we can apply reduction methods to our third order nonlinear state space model.

## 4.2 Normal Forms

Our induction motor model is nonlinear; we would prefer it to be linear. Linear systems are relatively well understood and techniques exist for reducing the order of linear models. The normal form representation of a system is the simplest representation for a system; ideally, the normal form representation will be linear. The variables used for the normal form representation are related to the variables of the original model through a nonlinear function. Putting a model into normal form allows us to work with a linear model while retaining the nonlinearities of our original model. (The bulk of this section was adapted from [18]).

The procedure to put a model into normal form is relatively straightforward. The initial steps involve simplifying the linear part of the system. First, we calculate the equilibrium points of our set of equations and change variables by subtracting the equilibrium point corresponding to stable operation from the original variables. This change of variables places the equilibrium point at the origin in terms of the new variables, which simplifies the calculations. Second, we separate the linear part of the system from the rest of the system:

$$\frac{dx}{dt} = f(x) \tag{4.9}$$

$$= \mathbf{A}x + [f(x) - \mathbf{A}x], \tag{4.10}$$

where Equation (4.9) is the state space realization of our nonlinear system and $\frac{dx}{dt} = \mathbf{A}x$ is the linearized version of this system. Third, we change variables again, this time using the change of variables which would put the linearized equation, $\frac{dx}{dt} = \mathbf{A}x$, into Jordan form. At this point, the linear portion of the system has been simplified as much as possible.

The actual normal form procedure takes this system with its linear portion in simplest form and develops a nonlinear map for it. The goal is to create a nonlinear map, which maps this nonlinear system to its linearization. The actual procedure expands the remaining nonlinear portion of the system, which was $f(x) - \mathbf{A}x$ before the Jordan form transformation, using a Taylor's series expansion and removes higher order terms one at a time, i.e., second order terms, then third order terms, etc., by developing nonlinear maps which change variables to eliminate these terms. Fortunately, these maps are simple to compute, and the process of finding them reduces to a linear algebra problem.

The goal of using this normal form approach is to find a way to map our third order nonlinear differential equation to its linearization. We would then reduce the order of the linearized system and use the inverse of our nonlinear map to transform the reduced order linear system back into the original variables. The final system would be a nonlinear system, but the reduction of the number of variables would have been performed on a linear system, which is an easier task then reducing a nonlinear system.

However, the normal forms process has a number of drawbacks. First, depending on the system to be reduced, the normal forms process may not eliminate certain nonlinear terms. These terms are called resonance terms, and the nonlinear map will transform the original system into its linearized version with these terms added. Using linear algebra techniques, we can identify these terms using the linearized system. Second, the normal form procedure has no convergence properties. At each step when terms of a certain order are eliminated, the remaining higher order terms are affected as well; even if the original systems only has second order nonlinear terms, eliminating these second order terms may generate third order terms. So, eliminating any number of terms is not guaranteed to bring the system closer to being linear though we hope that this is the case. Third, the normal form procedure is designed for systems of the form of Equation (4.9). In some sense, the third order induction motor model is not of this form because the system is not autonomous, i.e., the induction motor model has inputs. At first glance, the input term does not appear to create any problems because we could simply leave it as a parameter. Unfortunately, the mathematics of the computation quickly becomes intractable if the input is left as a variable.

The main stumbling block to developing a normal form for the third order induction motor model was the third problem listed above, the input term. The normal form could not be calculated with the voltage terms left as variables. To circumvent this problem, we attempted to try to treat the input as another state variable. Assuming that we were modeling the response of the motor to a step change in voltage, we created another state equation which stated that the derivative of the input voltage was zero, which is true other then at the instant when the voltage changes. Unfortunately, the normal formed model was very inaccurate at times after the step change. Until normal form type procedures which incorporate an input term are developed, the normal form procedure is not a practical approach to the accurate reduction of our model.

## 4.3 Participation factors

Selective modal analysis is a method for reducing the order of a linear system using the concept of participation factors [12]. Participation factors are a way of measuring how strongly individual modes of a system are coupled with individual state variables of the system. Variables with a large participation factor with respect to a certain mode of a system influence this mode more strongly than variables with a small participation factor with respect to that mode. If the system to be reduced has a subset of modes of interest, the system model could be reduced by creating a model that captures the modes of interest by eliminating variables which have small participation factors in these modes (or, similarly, by keeping only those variables which have large participation factors with the modes of interest).

After the less relevant state variables are identified, the dynamics associated with these state variables must be eliminated in such a way that the desired modes are retained. We do this by ensuring that the input-output characteristic of the simplified version of these less relevant dynamics is accurate for the modes of interest. On a practical level, we accomplish this by taking the Laplace transform of the state equations of the less relevant dynamics. Then, we replace the "$s$" terms by the eigenvalue of the mode we which to keep. This procedure reduces these dynamic state equations to algebraic equations, but they accurately reproduce the dynamic equations for the mode corresponding to the eigenvalue which was substituted. This reduced system will yield identical results to the original system if only the selected mode is excited. Incorporating multiple modes is straightforward with some limitations (see [12]).

Of course, the other modes of the system will not be fully retained, but, if the dynamics of these modes are not of interest, this should not concern us.

A schematic of the process of how the less relevant modes are handled is shown in Figure 4.1.

### 4.3.1 Our third order model and participation factors

In order to determine the modes for our model of the induction motor and whether they can be separated, we need representative numbers for the parameters in our equations. Table 4.1 shows the parameter values we used for our study. (The details of how to perform the calculations in this section are presented in [12].)

By substituting these values into Equations (4.6), (4.7) and (4.8), we get a numerical model for the

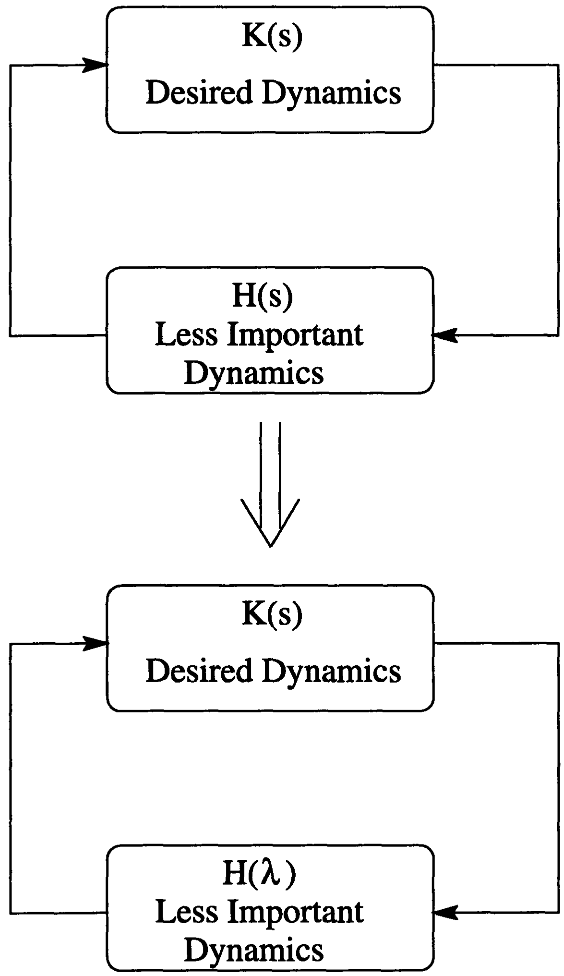


Figure 4.1: A schematic of how the less relevant modes are handled is shown above. The top figure is of the original system with the dynamics we are interested in separated from the rest of the system. The bottom figure shows how the less relevant dynamics are eliminated by converting the dynamic relationship into a static relationship with the use of the eigenvalue of the dynamics of interest.

| $H$ | $T_m$ | $T_0$ | $X_r$ | $R_r$ | $X_m$ | $R_s$ | $X'$ |
|---|---|---|---|---|---|---|---|
| .4238 | .5 | .597 | 3.1487 | .014 | 3.0623 | .024 | .1704 |

Table 4.1: The values in this table are the ones used for computations in this thesis. All values are in per unit. [16]

induction motor, to which we can then try to apply the method described above:

$$.447\frac{ds}{dt} = .5 + .8102E'_D V_D + .8102\left(E'_D\right)^2 + 5.753E'_Q V_D + .8102\left(E'_Q\right)^2 \quad (4.11)$$

$$.597\frac{dE'_D}{dt} = 17.13531862V_D - 18.135E'_D + 224.9sE'_Q + 2.413E'_Q \quad (4.12)$$

$$.597\frac{dE'_Q}{dt} = 2.413V_D - 2.413E'_D - 18.135E'_Q - 224.9sE'_D, \quad (4.13)$$

where we have assumed that the input voltage has a zero phase angle ($V_Q = 0$) as an angle reference. The eigenvalues of the linearized version of this system are $-30$, $-15.3 + j46.4$ and $-15.3 - j46.4$. These eigenvalues are not drastically separated, but the mode corresponding to the eigenvalue $-30$ is slower than the other modes. We will attempt to reduce the system but retain the slower mode. The participation factor for the variable $E'_D$ is .974, so retaining this variable using selective modal analysis seems promising.

Selective modal analysis has not been completely theoretically extended for use with nonlinear systems. Nevertheless, a partial solution to this problem has been tested successfully [13]. We calculated which variable to retain by linearizing our system and computing the participation factors of the system, then we applied the Laplace transform to our linearized equations and replaced $s$ by $-30$ in the two state equations for the less relevant dynamics. The final system consists of the two algebraic linearized equations and the original nonlinear equation of the variable of interest.

As Figure 4.2 shows, the retained variable, $E'_D$, was accurately approximated by the reduced system, including the steady state value . For the other two variables, the desired system mode was retained, but the steady state values which the system approached were erroneous. The steady state values are determined by a combination of the modes of the system, so we cannot expect the selective modal analysis technique to retain them.

## 4.4 Conclusion

We devised a method to reduce the order of our third order induction model if only the slow modes of the system are needed. The result, while not completely theoretically sound, seems to be a good approximation.

A better solution to this problem could be realized in one of two ways. If the normal forms procedure included input terms or selective modal analysis was extended for nonlinear systems, the reduction could grow out of one of these areas.

An alternate approach to reducing the order of motor models using time-scale separation techniques and integral manifolds can be found in [15].

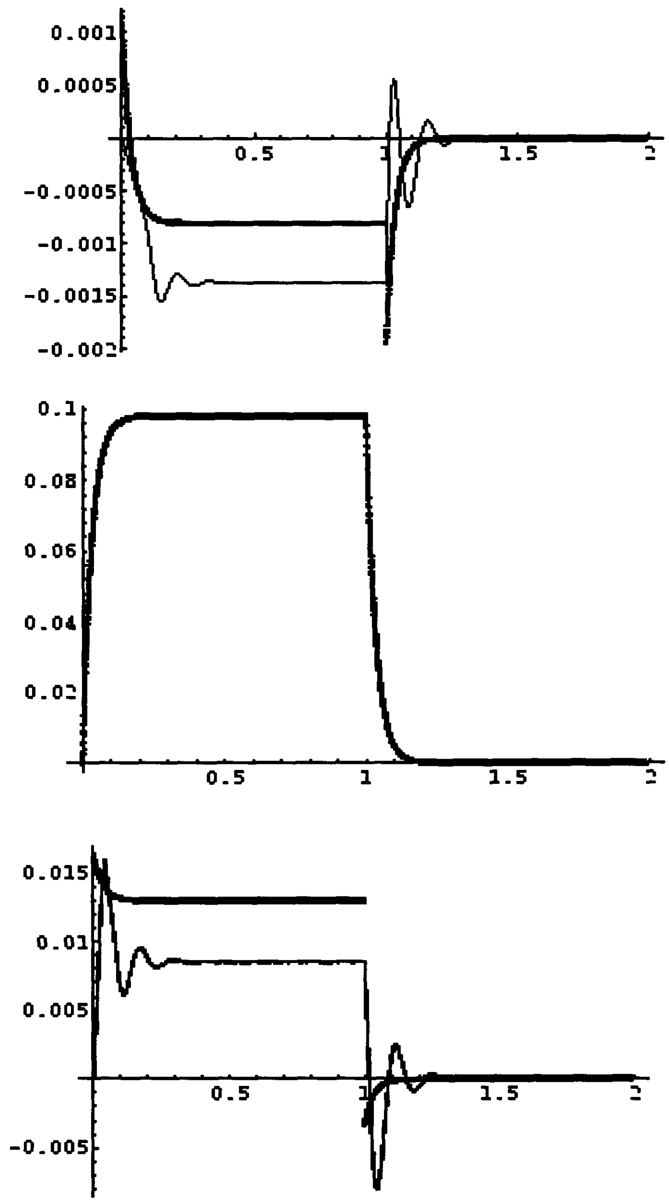


Figure 4.2: The three graphs show deviations from nominal values of the three state variables (in order from top to bottom, $s$, $E'_D$ and $E'_Q$) in response to a pulse down of 0.1 p.u. of the input voltage for one second. The heavier lines correspond to the reduced system using a nonlinear SMA approach. The thinner lines are the response of the full third order system.

*Chapter 5*

# *Fluorescent Lighting*

## 5.1 Introduction

Electronic devices are becoming increasingly important load devices to model. As computers and other electronics become more prevalent, these types of load devices are becoming a larger proportion of the overall load in this country. Electronic load devices seem well suited to a physically-based load modeling approach because the individual devices have well understood circuit models.

This chapter focuses on fluorescent lights with electronic ballasts as an example of an electronic load device. Fluorescent lights can be a significant component of the aggregate load, especially for commercial areas [8]. Most of the new fluorescent lights have electronic ballasts, as opposed to the older magnetic ballasts. These ballasts are used in the newer compact fluorescent lights because they are significantly smaller and lighter then the older ballasts. Many fluorescent lights with more traditional dimensions benefit from electronic ballasts because they reduce the possibility of the light source having a visible modulation by switching the light at a frequency in the kilohertz range. This higher frequency also increases the light output of the fluorescent tube.

A typical circuit for an electronic ballast will be presented and discussed in terms of modeling it as a load.

## 5.2 Electronic ballast circuit for a compact fluorescent light

The schematic shown in Figure 5.1 corresponds to a circuit used to operate a compact fluorescent light [1]. The two diodes and the two capacitors on the left are used to rectify the input voltage and effectively double the peak voltage. The two Mosfets, then, are switched on and off to produce a high frequency current through the fluorescent tube. The inductor, $L$, is necessary to stabilize this high frequency current. The capacitor and PTC are necessary for the ignition phase of the lamp.

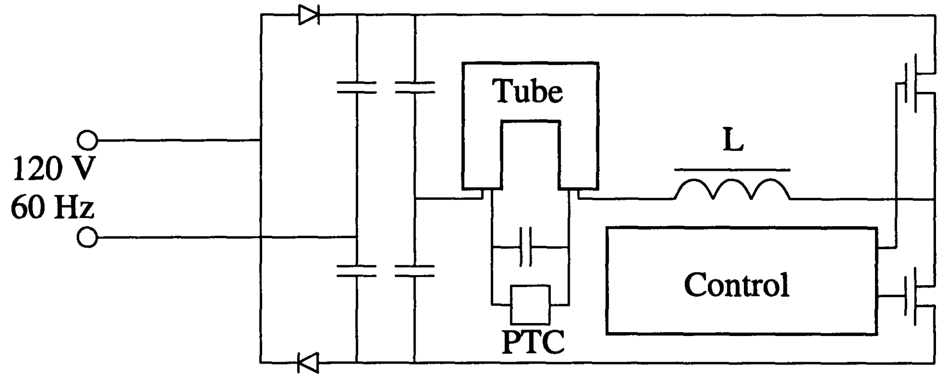


Figure 5.1: A schematic of the circuit used to operate a compact fluorescent light is shown above. The control is used to switch the Mosfets at a frequency of approximately 40 kHz. The PTC is a positive temperature coefficient resistor, which is necessary for the ignition of the lamp. The input is treated as a single phase supply for simplicity.

From the perspective of load modeling, this device is essentially a static device, other than frequency effects. In other words, the transients for a voltage change are negligible for time scales of interest for power systems. The load model stationarity does have two caveats though. First, if the voltage supplied drops below a certain percentage of the anticipated voltage (approximately 80% of nominal), the fluorescent tube will extinguish and the lamp will need to be reignited before normal operation can resume. Second, as mentioned above, the load model may need to incorporate frequency dependent terms. These terms are primarily a result of the initial half-wave rectifier on the circuit. Incorporating a filter as an input stage to the circuit shown would considerably reduce this frequency dependence.

A static relationship, with or without frequency dependent terms, is a common load model for power systems. Fluorescent lighting is one of the loads which is well represented by this practice (typical values can be found in [4]). As part of an aggregate of physically based load models, static load models will probably need to be used to represent at least some of the loads, depending on the actual loads comprising the aggregate load.

## 5.3 Conclusion

At least some electronic loads, including fluorescent lights with electronic ballasts, are well represented by static load models, with the possible addition of frequency dependent terms. More complicated dynamic load models do not seem warranted in these cases.

*Chapter 6*

# *Load Monitoring*

Load monitoring is a necessary practical issue related to physically-based load modeling. In physically-based load modeling, we attempt to model an aggregate load by first breaking it into groups of related loads, such as electric heating loads or induction motor loads. The bulk of this thesis is concerned with modeling these groups after they have been identified, and then combining the models of the different load groups into a single aggregate load. The accuracy of this aggregate load model is limited by our knowledge of the true composition of the load. If there was no accurate way to estimate the composition of the load, a physically-based load model would not be a practical development.

In this chapter, we present one way to estimate the composition of the aggregate load using only measurements of the voltage and current supplied to the entire building [5]. Making load estimates based on aggregate electrical characteristics is advantageous because monitoring individual load devices is not practical; aggregate loads are typically composed of a prohibitively large number of different individual loads.

## 6.1 Processing of measurements

The load monitoring device provides estimates of the steady state spectral envelopes of the aggregate current waveform. The spectral envelopes are the time-varying coefficients, $a_k(t)$ and $b_k(t)$, of the following decomposition of the aggregate current waveform [5]:

$$x(t-T+s) = \sum_{k=0}^{\infty} a_k(t) \cos\left(k\frac{2\pi}{T}(t-T+s)\right) + \sum_{k=0}^{\infty} b_k(t) \sin\left(k\frac{2\pi}{T}(t-T+s)\right), \qquad (6.1)$$

where $T$ is a real period of time and $s \in (0, T]$. The coefficients can be computed using the following equations:

$$a_k(t) = \frac{2}{T}\int_0^T x(t-T+s)\cos\left(k\frac{2\pi}{T}(t-T+s)\right)ds \tag{6.2}$$

$$b_k(t) = \frac{2}{T}\int_0^T x(t-T+s)\sin\left(k\frac{2\pi}{T}(t-T+s)\right)ds. \tag{6.3}$$

The decomposition is closely related to a Fourier series representation if $x(t)$ is a periodic function and $T$ is the period of the function (or any integer multiple of the period). In this case and with the voltage being a cosine with angular frequency $2\pi/T$, $a_1(t)$ is proportional to the real power into the aggregate load and $b_1(t)$ is proportional to the reactive power into the aggregate load.

The prototype load monitoring device measures $a_k(t)$ and $b_k(t)$ for $k = 1, 3, 5$. These measurements are output as voltages, which can be monitored using an oscilloscope, for example.

## 6.2 Analysis of measurements

To determine the composition of the aggregate load, representative measurements must be taken of the individual load types which contribute to the aggregate load; the spectral envelopes of each individual load type need to be estimated. If we treat a set of spectral envelope measurements for an individual load type as a vector (of dimension 6 in the case of the prototype), the set of vectors corresponding to all of the individual load types defines a vector space; if the individual spectral envelopes add linearly, the aggregate spectral envelopes, when viewed as a vector, should be contained in this vector space.

Determining the number of each type of component in the aggregate load flow is, then, a linear algebra problem. Unfortunately, the problem is typically underconstrained because several distinct load types will not have substantial higher order spectral envelopes. Though not every load can be uniquely identified, the spectral envelope estimates still contain useful information, and some of the loads contributing to the aggregate load can be identified. If one of the individual loads had a spectral envelope vector which had a component orthogonal to the spectral envelope vectors of the other individual loads in the aggregate load, this individual load could be properly identified, for example.

Previously, the individual loads which could be identified were estimated using a projection method [5]. The observed aggregate load vector was projected onto the component of the vector corresponding to the individual load to be identified which was orthogonal to the the other individual load vectors. The magnitude of this projection was then normalized by dividing it by the projection of the vector corresponding to the individual load to be identified onto the component of this vector orthogonal to the other individual loads. This process can be described by an equation:

$$\text{number of individual load type} = \frac{\langle \beta_{obs}, \beta_n \rangle}{\langle \beta_l, \beta_n \rangle}, \tag{6.4}$$

where $\beta_{obs}$ is the aggregate load vector, $\beta_l$ is the vector of the load to be identified, $\beta_n$ is a component vector of $\beta_l$ which is orthogonal to the vectors of the other individual load components, and the "number of individual load type" means the quantity of the individual load type which is present in the aggregate load assuming $\beta_l$ is a vector corresponding to a single unit of this load type. In practice, $\beta_n$ is calculated by arranging the vectors of the load types other than the one of interest in a matrix and calculating the nullspace of the transpose of this matrix, and then projecting $\beta_l$ onto this nullspace. With real measurements, the matrix is unlikely to have a nullspace because of uncertainties, but using knowledge of the characteristics of the loads being measured, the matrix of real measurements can be adjusted so that it does have a nullspace (by setting terms which are nearly zero to zero, for example).

We propose to modify this estimation procedure slightly. First, while directly using a vector orthogonal to the vectors of the loads other than the load of interest works well if the nullspace of the transpose of the matrix of vectors of these other loads is one dimensional, this method is ambiguous if this nullspace has dimension greater than one because there is now more than one vector onto which to project. We can generalize this projection procedure easily by noticing that this projection is the least squares solution to the original underconstrained problem. The procedure, then, is to create a matrix of the vectors for all of the loads and find the least squares solution to the following problem:

$$\mathbf{B}x = \beta_{obs}, \tag{6.5}$$

where $\mathbf{B}$ is a matrix formed by using the vectors corresponding to a single unit of each load type as columns and $x$ is the number of units of each load type in the aggregate load arranged as a vector; we are trying to determine $x$. The least squares solution will only be correct for those load types which have a vector component orthogonal to the rest of the load type vectors, so we will only be interested in some of the entries of $x$.

Second, the projection and least squares approach both yield results which are nonphysical. As the least squares solution shows explicitly, the projection allows for the possibility of some of the load components being negative, which is nonsensical because it implies that a negative number of this

| | Instant Starts $\beta_1$ | Motor $\beta_2$ | Rapid Starts $\beta_3$ | Compact Fluorescent $\beta_4$ | Incandescent $\beta_5$ |
|---|---|---|---|---|---|
| $a_1$ | 728 | 60 | 612 | 206 | 960 |
| $b_1$ | -15 | 184 | -108 | -34 | • |
| $a_3$ | -152 | • | • | 154 | • |
| $b_3$ | 144 | • | • | -86 | • |
| $a_5$ | -24 | • | • | -76 | • |
| $b_5$ | 37 | • | • | -94 | • |

Table 6.1: The values in the table are the measurements from the prototype for each of the indicated individual load types. The actual numbers shown are millivolt readings from the prototype device and are proportional to the indicated coefficients. The bulleted entries were too small to be accurately discernible using the oscilloscope; for further calculations, we assumed these entries were zero.

type of load component exists. To correct for this possible problem, a nonnegative least squares method can be used. The nonnegative least squares solution will yield a least squares solution subject to the constraint that a nonnegative number of each load type is present in the aggregate load, which is a physically correct assumption.

## 6.3 Prototype measurements

We tested a prototype device by creating an aggregate load composed of different combinations of the following devices: small induction motor, a bank of instant start fluorescent lamps, a bank of rapid start fluorescent lamps, a compact fluorescent lamp, and an incandescent lamp. The individual vectors corresponding to each of these loads are summarized in Table 6.1.

This prototype device was originally designed for transient event detection [6]. The application was to load balancing and power factor correction at sites where loads turned on and off frequently. The device can detect when particular loads turn on because of their distinct transients. The work presented here is of steady state waveforms as opposed to transients.

As expected, only two of the loads had appreciable readings in any of the higher order coefficients. Consequently, only these two loads, the compact fluorescent lamps and the the instant start fluorescent lamps, can be distinguished from an aggregate load measurement. Three different aggregate loads were created; the three combinations and the resulting measurements are summarized in Table 6.2. These measurements, along with the individual measurements summarized in Table 6.1,

| | $\beta_1+\beta_2+$ $\beta_3+\beta_5$ | $\beta_1+\beta_3+$ $\beta_4+\beta_5$ | $\beta_2+\beta_3+$ $\beta_4+\beta_5$ |
|---|---|---|---|
| $a_1$ | 2370 | 2480 | 1835 |
| $b_1$ | 60 | -162 | 42 |
| $a_3$ | -170 | -20 | 130 |
| $b_3$ | 136 | 52 | -82 |
| $a_5$ | -12 | -86 | -64 |
| $b_5$ | 45 | -18 | -76 |

Table 6.2: The values in the table are the measurements from the prototype for each of the indicated aggregate loads, where each aggregate load has one each of the indicated individual loads. The actual numbers shown are millivolt readings from the prototype device and are proportional to the indicated coefficients.

| | $\beta_1+\beta_2+$ $\beta_3+\beta_5$ | $\beta_1+\beta_3+$ $\beta_4+\beta_5$ | $\beta_2+\beta_3+$ $\beta_4+\beta_5$ |
|---|---|---|---|
| $\beta_1$ | 1.03 | .83 | 0 |
| $\beta_2$ | .41 | 0 | .39 |
| $\beta_3$ | 0 | 1.17 | 0 |
| $\beta_4$ | 0 | .69 | .85 |
| $\beta_5$ | 1.66 | 1.06 | 1.70 |

Table 6.3: The values in the table are the number of each individual load type estimated to be in the appropriate aggregate load based on a nonnegative least squares analysis of the aggregate measurements.

were then analyzed to determine whether the individual load types could be identified from the aggregate measurements. The results of this analysis using the nonnegative least squares estimation technique are summarized in Table 6.3.

As expected, the estimates of $\beta_2$, $\beta_3$, and $\beta_5$ are inaccurate. Estimating these three load components is equivalent to solving two equations with three unknowns, which is an underconstrained system. The nonlinear least squares technique seeks to minimize the norm of the vector composed of the number of each load type in the aggregate load, which we would not expect to yield the correct values.

The results were mixed for the two loads we expected to be able to distinguish, $\beta_1$ and $\beta_4$. The analysis of the first aggregate load, $\beta_1+\beta_2+\beta_3+\beta_5$, yielded accurate results; the nonlinear least squares technique estimated that there were no compact fluorescent in the load and there were 1.03 instant start fluorescent in the load. The results of the other two aggregate loads were substantially less accurate though they did have some correspondence to the true load composition. We were able to pinpoint the cause for this inaccuracy.

| | $\beta_1+\beta_2+\beta_3+\beta_5$ | | $\beta_1+\beta_3+\beta_4+\beta_5$ | | $\beta_2+\beta_3+\beta_4+\beta_5$ | |
|---|---|---|---|---|---|---|
| | Sum of Individual | Aggregate Measurement | Sum of Individual | Aggregate Measurement | Sum of Individual | Aggregate Measurement |
| $a_1$ | 2360 | 2370 | 2506 | 2480 | 1838 | 1835 |
| $b_1$ | 61 | 60 | -157 | -162 | 42 | 42 |
| $a_3$ | -152 | -170 | 2 | -20 | 154 | 130 |
| $b_3$ | 144 | 136 | 58 | 52 | -86 | -82 |
| $a_5$ | -24 | -12 | -100 | -86 | -76 | -64 |
| $b_5$ | 37 | 45 | -57 | -18 | -94 | -76 |

Table 6.4: This table compares the measured aggregate load coefficients with what these measurements should be if the coefficients of the individual loads summed linearly. Once again, the values are millivolt readings from the prototype.

The nonlinear least squares analysis of the aggregate load should yield accurate results, as long as the coefficients of the different loads add linearly. Table 6.4 summarizes the relative linearity of the coefficients of each of the aggregate load measurements. Some of the coefficients added linearly, as desired, but others did not. The coefficients $a_3$, $a_5$ and $b_5$ were particularly troublesome. The nonlinearities are caused by the measuring device and are not actually present in the system. The prototype device can be adjusted to eliminate these nonlinearities, but new measurements were not taken in time for the completion of this thesis.

## 6.4 Conclusion

Load monitoring is one possible tool to enable physically-based load modeling to be more practical as a load modeling approach. To be able to apply physically-based load modeling to an aggregate load, the composition of the aggregate load must be known, and the load monitoring device described in this chapter enables us to identify some of the loads in an aggregate load using simple linear algebra techniques.

To test the efficacy of this load monitoring scheme, a prototype load monitoring device was used to take measurements, which were then analyzed. One of the test trials yielded excellent results, but the other two trials were less accurate due to a maladjustment on the prototype load monitoring device. More conclusive results will be available when further adjustments are made to the prototype device.

Further load monitoring techniques will also need to be developed and employed to increase the effectiveness of physically-based load modeling because the load monitoring device is incapable of differentiating between certain classes of load devices [6].

*Chapter 7*

# *Conclusion and Future Work*

## 7.1 Conclusion

This thesis has been an investigation of elements of a physically-based load modeling framework. The basis for this framework is the same principle that created the need for load modeling by defining loads as one element of the power system. The power system is broken up into distinct elements for analysis; similarly, physically-based load modeling takes an aggregate load and breaks it up into distinct elements for analysis. The decomposition of the aggregate load is based on distinct physical characteristics. Individual load devices which share characteristics are grouped together.

The core of this framework is the modeling of groupings of similar loads. To address this core issue, this thesis has been primarily an exploration of applying different modeling techniques to three different load groupings. As pointed out earlier, the three load groupings selected were distinct physically. Fortuitously, the results are also distinct. The thermal loads were determined to be dynamic, and their behavior was explored. The induction motors are dynamic and well modeled, and ways to reduce these models to a minimal form for individual applications were investigated. The fluorescent lights were found to be essentially static devices.

These results show the mixture of findings which can be discovered using physically-based load modeling. Traditional models can be possibly inadequate, as in the case of thermal loads. Traditional models can also be adequate, as in the case of fluorescent loads. Better knowledge of which load models are adequate and inadequate is obviously beneficial.

The decomposition of the aggregate load into suitable elements is crucial to accurate physically-based load modeling. We experimented with one load monitoring device to help with this task. Though no present device can distinguish between all possible interesting load devices, some individual loads have distinct characteristics which enable us to identify their contribution to the composite load.

## 7.2 Future work

The reaggregation of the individual load models was not dealt with in this thesis. This final step in developing a physically-based load model is crucial for validating possible results by comparing with real measurement because actual load measurements are of large aggregates composed of many different load elements. This task would definitely be the next work to be completed on physically-based load models.

After the aggregate load model is developed, real data from a power system should be examined. This data may exhibit important dynamic trends which should be represented in at least one of the load elements. Real data may also by used to verify an aggregate load model and to suggest other possibly important load elements.

Comparing the results from the load models developed with actual load measurements is also important. If some of the assumptions underlying the development of an individual load element are erroneous, this comparison will show possible problem areas.

There is also at least one obvious extension to the work in this thesis. Only three load elements were treated in the course of this work, but there are other possible load elements which could be investigated. For example, as an extension of the thermostatic loads and the induction motors, air conditioning loads could be studied and modeled.

*Appendix A*

# *Thermostatic Load Approximation Program*

---

The code included in this appendix is for approximating the percentage of heaters which are on in an aggregate load using two simulations which represent a single heater. This code was written in C by the author of this thesis.

```
/*************************************************************************/

/* prediction.c - a program to approximate the percentage of heaters which are
   on in an aggregate load.  Only changes to the input voltage are accomadated
   in this version of the program, but the modifications to include changes
   to ambient temperature are minimal.

   written by: james hockenberry
   last modified: 19 January 1997  */

#include <stdio.h>
#include <math.h>

/* Constants are as follows:
   -RF is the heater's resistance.
   -TF is the temperature the house would approach if the heater were off.
   -TAU is the thermal time constant of the house
   -TH is the high temperature setting of the thermostat.
   -TL is the low temperature setting of the thermostat.
   -Tstart is the starting time.
   -Tstop is the stopping time.
   -Tstep is the time step used for the heater simulation
   -Tindex is Tstop/Tstep which is number of points in heater
   simulation.
   -TON is the initial length of the on cycle.
   */

#define RF 720.0
#define TF 66.0
#define TAU 64.0
#define TH 75.0
#define TL 72.0
```

```
#define Tstart 0.0
#define Tstop 200.0
#define Tstep 0.01
#define Tindex 200*100
#define TON 15.43

/**********************************************************************/

main()
{
  /* the variable arrays are as follows:
     -temps is an array to hold the temperatures of heater.
     -temptimes is an array to hold the time index for heater.
     -Tg is an array to hold the temperature gain of the heater during the
     heater simulation.
     -w is an array to hold the state of the heater at each time.
     */

  double temps[Tindex+1];
  double temptimes[Tindex+1];
  double Tg[Tindex+1];
  int w[Tindex+1];

  /* the variables are as follows:
     -i,j,k are used as indices at various points.
     -t is the current time during each simulation.
     -xbar is a counter used to determine the period.
     -period is the current estimate of the period.
     -A, B are constants for heater simulation.
     -V is the current voltage.
     -fout and ftemp are files to hold the period simulation and heater
     simulation, respectively.
     */

  int i,j,k;
  double t,xbar;
  double period;
  double A,B;
  double V;
  FILE *fout, *ftemp;

  /**********************************************************************/

  /* start heater simulation by computing A,B. also, open both files for
     writing */

  A = 1/(1+Tstep/TAU);
  B = (Tstep/TAU)*A;
  fout = fopen("predictionout","w");
  ftemp = fopen("tempout","w");
```

```
/* initialize the temporary variables, the arrays and the state of the
   heater */

temps[0] = TL;
temptimes[0] = 0;
Tg[0] = (pow(120,2)/RF+TF)/TAU;
i=1;
w[0]=1;
V=120;

/* simulate a heater on cycle.  when the simulation reaches the high
   temperature, it automatically resets to the low temperature. */

for (t=Tstart+Tstep;t<=Tstop+.00001;t+=Tstep)
  {
    /* compute the new temperature */
    temps[i] = A*temps[i-1] + B*(TF + V*V/RF);

    /* set the input voltage */
    if (t<50)
      {
        V = 120;
        Tg[i] = (w[i-1]*pow(120,2)/RF+TF)/TAU;
      }
    else
      {
        V = 110;
        Tg[i] = (w[i-1]*pow(V,2)/RF+TF)/TAU;
      }

    w[i] = w[i-1];
    /* if temperature is above high temp, reset it to low temp. */
    if(temps[i]>TH) temps[i] = TL;
    temptimes[i] = t;
    /* dump heater to file */
    fprintf(ftemp,"%lf %lf \n",temptimes[i],temps[i]);
    i++;
  }

/************************************************************************/

/* Find a point in time to start simulation from */
t = 0;
period = TON + 1;
for (j = 0;temptimes[j]<period;j++);
i = 0;

/* compute the period: at each time, integrate the temperature backwards in
   time until 3 degrees have been accumulated, this length of time is the
```

```
     period.  the discontinuities between temp high and temp low are ignored
     to prevent errors.   */
  for (j++;j<Tindex-1;j++)
    {
      xbar = 0;
      for (i=0;xbar<3;i++)
        {
          if (temps[j-i] < temps[j-i-1]);
          else
            xbar += temps[j-i] - temps[j-i-1];
        }
      period = temptimes[j] - temptimes[j-i-1];
      /* Dump the resulting period to the file, along with the percentage of
         heaters which are on */
      fprintf(fout,"%lf %lf %lf\n",temptimes[j],period,
              period/(period + 25.95));
    }
}
```

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