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Beyond the Minimal Supersymmetric Standard Model

by

Csaba Csáki

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Author ....
Department of Physics
April 9, 1997

Certified by ....
Lisa Randall
Associate Professor of Physics
Thesis Supervisor

Accepted by ....
George Koster
Chairman, Physics Graduate Committee

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Abstract

Several topics from the subject of supersymmetric field theories are reviewed. We first present the structure of the minimal supersymmetric standard model. Next we discuss the issue of the doublet-triplet splitting problem in supersymmetric grand unified theories. We show some necessary conditions which a viable model has to satisfy and present three models which successfully solve the doublet-triplet splitting problem. We extend the simplest model to include fermion masses as well. Phenomenological consequences of this and other models are reviewed as well.

The next topic to be reviewed is confinement in $N = 1$ supersymmetric gauge theories. We first summarize the low energy behavior of supersymmetric gauge theories and discuss the special case of SUSY QCD for different number of flavors. Then we present two necessary conditions for a theory to be "s-confining", which allows us to give a complete list of such theories based on a single gauge group.

Finally we review the subject of dynamical supersymmetry breaking. We present the criteria for a model to break supersymmetry and discuss the method of Dine, Nelson, Nir and Shirman for finding new models of dynamical supersymmetry breaking. Using this method we find a large new class of models of dynamical supersymmetry breaking based on the gauge group $SU(n) \times SU(m) \times U(1)$, and analyze these theories in detail.

Thesis Supervisor: Lisa Randall
Title: Associate Professor of Physics

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Chapter 1

Introduction

Supersymmetry (SUSY) is the most popular of theories beyond the standard $SU(3) \times SU(2) \times U(1)$ model. The major phenomenological motivation of SUSY is that it solves the "hierarchy problem". The problem arises because of the presence of elementary scalars in the standard model. These scalars receive quadratically divergent loop corrections to their masses, and one needs a huge fine tuning between the bare tree-level mass term and the loop corrections (usually cut off at a scale $\Lambda \sim M_{Planck}$) to set $M_{weak} \ll M_{Planck}$.

Supersymmetry automatically solves this problem by the cancelation of all quadratic divergences between fermionic and bosonic loops. There are other hints in favor of supersymmetry as well. As we will discuss in Chapter 3, the particle content of the minimal supersymmetric standard model (MSSM) automatically guarantees the unification of gauge couplings at $M_{GUT} \sim 10^{16}$ GeV.

A theoretical motivation for supersymmetry comes from string theories. It has been shown that in order to have a stable vacuum in string theory (that is to avoid the presence of tachyonic states) one needs to consider supersymmetric string theories (superstrings).

Here we will discuss several topics from the subject of supersymmetric field theories. First we review the structure of the minimal supersymmetric standard model. We discuss the field content and the superpotential, and review the description of supersymmetry breaking. The issue of electroweak symmetry breaking within the MSSM ("radiative symmetry breaking") is taken up next. Finally we close that chapter by reviewing the tree-level mass matrices of the sparticles (the superpartners of the standard model particles).

Chapter 3 deals with the subject of supersymmetric grand unified theories (SUSY GUTs), specifically with the doublet-triplet splitting problem which arises in SUSY GUTs. After a brief introduction to SUSY GUTs, Section 3.1 describes the doublet triplet splitting problem and the known possible solutions to it. Section 3.2 reviews the most economical solution, the "Higgs as pseudo-Goldstone boson" mechanism. Three possible implementations of this mechanism are given in Section 3.3, where one of the models is extended to incorporate fermion masses as well. This model serves as an existence proof for complete SUSY GUTs solving both the doublet triplet splitting problem and the fermion mass hierarchy problem. Phenomenological consequences of the Higgs as pseudo-Goldstone boson mechanism are described in Section 3.4.

Chapter 4 deals with the issues of non-perturbative effects in asymptotically free supersymmetric gauge theories. After a brief introduction we review the recent results by Seiberg on the low-energy behavior of SUSY QCD. These results will also be important in Chapter 5. The remainder of Chapter 4 focuses on the confining supersymmetric gauge theories.

In Section 4.2 we define what we mean by "s-confining" theories and in the following derive two necessary conditions that an s-confining theory has to obey. Using these conditions we are able to find all s-confining theories based on a single gauge group. These theories are listed in Section 4.4.

Finally, the issue of dynamical supersymmetry breaking is reviewed in Chapter 5. We first motivate the importance of dynamical supersymmetry breaking and give the necessary conditions for a model to break supersymmetry. Section 5.3 describes the classic models by Affleck, Dine and Seiberg [49], while the $SU(N) \times SU(M) \times U(1)$ models are reviewed in Section 5.4. We first describe the $SU(4) \times SU(3) \times U(1)$ model, where we derive the exact low-energy superpotential and explicitly supersymmetry breaking. Then we repeat the analysis for the general $SU(n) \times SU(m) \times U(1)$ models, which break supersymmetry as well.

Chapter 2

The Minimal Supersymmetric Standard Model (MSSM)

In this chapter we review the structure of the MSSM [1, 2, 3, 4, 5, 6, 7]. We first motivate the particle content of the theory by examining the quantum numbers of the known standard model particles and by the requirement of anomaly cancelation.

Once the particle content is fixed we can write down the most general renormalizable superpotential. However such a superpotential will contain terms breaking lepton and baryon number which leads us to the concept of R-parity conservation.

The question of supersymmetry breaking is discussed next. We list the possible soft breaking terms. However the Lagrangian involving the most general soft breaking terms is phenomenologically intractable because of the appearance of many new parameters. It also leads to some unacceptable predictions. To reduce the number of parameters we restrict ourself to the case with universal soft breaking terms at the GUT scale. We motivate the need for universal soft breaking terms by the apparent unification of gauge couplings in the MSSM and by the absence of flavor changing neutral currents. Then we discuss radiative electroweak symmetry breaking. Radiative breaking arises because the one loop corrections involving the large top Yukawa coupling change the sign of the soft breaking mass parameter of the up-type Higgs doublet, this way introducing a nontrivial minimum in the Higgs potential.

Finally we give an overview of the possible mixings in the MSSM and enumerate the physical (mass eigenstate) fields together with the mass matrices.

2.1 Particle content and superpotential

The Standard Model (SM) of particle physics enjoys an unprecedented success: up to now no single experiment has been able to produce results contradicting this model. Particle theorists are nevertheless unhappy with this theory. The most important features of the SM that are technically allowed but nevertheless theoretically unsatisfying are the following:

a. There are too many free parameters

b. The $SU(2) \times U(1)$ group is not asymptotically free

c. Electric charge is not quantized

d. The hierarchy problem.

While the first three problems can be taken care by introducing grand unification, the mystery of the hierarchy problem remains unsolved in GUTs as well. The hierarchy problem is associated with the presence of elementary scalars (Higgs) in the SM. The problem is that in a general QFT containing an elementary scalar the mass of this scalar would be naturally at the scale of the cutoff of the theory (if the SM were the full story then the Higgs mass would be naturally of $\mathcal{O}(M_{Pl})$) due to the quadratically divergent loop corrections to the Higgs mass. If one wants to protect the scalar masses from getting these large loop corrections one needs to introduce a new symmetry. The only known such symmetry is supersymmetry (SUSY), which relates fermions and bosons to each other.[1]

In this chapter we will review the minimal extension of the SM that includes SUSY, the Minimal Supersymmetric Standard Model (MSSM) [1, 2, 3, 4, 5, 6, 7]. We will assume that the reader is familiar with both the structure of the SM and with N=1 global SUSY.

The SM is a spontaneously broken $SU(3) \times SU(2) \times U(1)$ gauge theory with the matter fields being

$$\begin{aligned} \text{leptons: } L_i = \begin{pmatrix} \nu \\ e \end{pmatrix}_{L_i} &= (1,2,-\tfrac{1}{2}) \\ e_{R_i} &= (1,1,-1) \\ \text{quarks: } Q_i = \begin{pmatrix} u \\ d \end{pmatrix}_{L_i} &= (3,2,\tfrac{1}{6}) \\ u_{R_i} &= (3,1,\tfrac{2}{3}) \\ d_{R_i} &= (3,1,-\tfrac{1}{3}) \\ \text{Higgs: } H = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix} &= (1,2,\tfrac{1}{2}) \qquad i = 1,2,3, \end{aligned} \tag{2.1}$$

where i is the generation index, L and R refer to left and right handed components of fermions and the numbers in parenthesis are the $SU(3) \times SU(2) \times U(1)$ quantum numbers.

The MSSM is an extension of the $SU(3) \times SU(2) \times U(1)$ gauge theory with N=1 SUSY (which will be broken in a specific way, see Section 3).

The rules of building N=1 SUSY gauge theories are to assign a vector superfield (VSF) to each gauge field and a chiral superfield (χSF) to each matter field. The physical particle content of a VSF is one gauge boson and a Weyl fermion called gaugino, and of the χSF is one Weyl fermion and one complex scalar [9, 10]. The VSF's transform under the adjoint of the gauge group while the χSF's can be in any representation. Since none of the matter fermions of the SM transform under the adjoint of the gauge group we can not identify them with the gauginos. Thus we have to introduce new fermionic SUSY partners to each SM gauge boson.

If we now look at the matter fields of the SM listed above we see that the only possibility to have two SM fields as each others superpartner would be to have $\tilde{H} = i\tau_2 H^*$ as a superpartner of L. However this is phenomenologically unacceptable since L carries lepton number 1, while H lepton number 0, and the superpartners must carry the same gauge and global quantum numbers. Thus we conclude that we have to introduce a new superpartner field to every single field present in the SM: scalar partners to the fermions (called sleptons and squarks), fermionic partners to the Higgs (Higgsino) and gauge bosons (gaugino).

[1] Another way of solving the hierarchy problem is to assume that the Higgs is not an elementary scalar but a bound state of fermions, which idea leads to technicolor theories.

χSF	SU(3)	SU(2)	U(1)	B	L
L_i	1	2	$-\frac{1}{2}$	0	1
$\bar{E}_i$	1	1	1	0	-1
Q_i	3	2	$\frac{1}{6}$	$\frac{1}{3}$	0
$\bar{U}_i$	$\bar{3}$	1	$-\frac{2}{3}$	$-\frac{1}{3}$	0
$\bar{D}_i$	$\bar{3}$	1	$\frac{1}{3}$	$-\frac{1}{3}$	0
H_1	1	2	$-\frac{1}{2}$	0	0
H_2	1	2	$\frac{1}{2}$	0	0

Table 2.1: The χSF's of the MSSM with their gauge and global quantum numbers. $i = 1,2,3$.

However we can see that we have introduced one extra fermionic $SU(2)$ doublet Higgs with $SU(3) \times SU(2) \times U(1)$ quantum numbers (1,2,$\frac{1}{2}$). This is unacceptable because of the Witten anomaly and because of the $U(1)$ anomaly that it causes. Thus we need to introduce one more $SU(2)$ doublet with opposite $U(1)$ charge. The need for this second Higgs doublet can also be seen in a different way: in the SM one needed only one Higgs doublet to give masses to both up and down type quarks, because one was able to use both H and $\tilde{H}$ in the Lagrangian. However in SUSY theories the superpotential (the only source of Yukawa interactions between only matter fields and its partners) must be a holomorphic function of the fields thus both H and $\tilde{H}$ can not appear at the same time in the superpotential [9]. This again calls for the need of two Higgs doublet χSF's, one with the quantum number of H, the other with the quantum numbers of $\tilde{H}$. The final resulting χSF content of the MSSM is given in Table 2.1. (We use the conjugate fields $\bar{U}, \bar{D}, \bar{E}$ because in the superpotential we can not use conjugation anymore.)

Once the particle content is fixed one can try to write down the most general renormalizable Lagrangian for this N=1 SUSY $SU(3) \times SU(2) \times U(1)$ theory. It is known from the structure of N=1 SUSY gauge theories that the Lagrangian is completely fixed by gauge invariance and by supersymmetry, except for the choice of the superpotential, which could contain all possible gauge invariant operators of dimensions not greater than 3. In our case this means that

$$\begin{aligned} W &= (\lambda_u^{ij} Q^i H_2 \bar{U}^j + \lambda_d^{ij} Q^i H_1 \bar{D}^j + \lambda_e^{ij} L^i H_1 \bar{E}^j + \mu H_1 H_2) + \\ &+ (\alpha_1^{ijk} Q^i L^j \bar{D}^k + \alpha_2^{ijk} L^i L^j \bar{E}^k + \alpha_3^i L^i H_2 + \alpha_4^{ijk} \bar{D}^i \bar{D}^j \bar{U}^k). \end{aligned} \tag{2.2}$$

The terms in the first pair of parenthesis correspond to the SUSY extension of the ordinary Yukawa interactions of the SM and an additional term ("μ-term") breaking the Peccei-Quinn symmetry of the two doublet model. However the terms is the second pair of parenthesis break baryon and lepton number conservation. Thus as opposed to the SM where the most general renormalizable gauge invariant Lagrangian automatically conserved baryon and lepton number, here one has to require some additional symmetries to get rid of the B and L violating interactions that are phenomenologically unacceptable. The easiest way to achieve this is to introduce R-parity and require R-parity conservation.[2] Under R-parity

$$\begin{aligned} &H_1, H_2 \rightarrow H_1, H_2, \\ &Q, \bar{U}, \bar{D}, L, \bar{E} \rightarrow -(Q, \bar{U}, \bar{D}, L, \bar{E}) \\ &\theta \rightarrow -\theta, \end{aligned} \tag{2.3}$$

which means that

$$\begin{aligned} \text{(ordinary particle)} &\rightarrow \text{(ordinary particle)} \\ \text{(superpartner)} &\rightarrow -\text{(superpartner)}. \end{aligned} \tag{2.4}$$

Note that this Z_2 group is a subgroup of a $U(1)_R$ symmetry where the R-charges of the χSF's are:

$$\begin{aligned} R &= 1 \ \text{ for } \ H_1, H_2 \\ R &= \frac{1}{2} \ \text{ for } \ L, \bar{E}, Q, \bar{U}, \bar{D}. \end{aligned} \tag{2.5}$$

However the imposition of the full $U(1)_R$ symmetry forbids Majorana masses for the gauginos which are phenomenologically needed. There are two possible solutions to this problem. One could impose only the Z_2 subgroup, R-parity, which forbids the B,L violating terms in the superpotential, but allows for gaugino mass terms.[3] If however one imposes the full $U(1)_R$ symmetry then this symmetry has to be spontaneously broken to its Z_2 subgroup leading to complications with the resulting Goldstone boson. We will not discuss this possibility further here. In both cases however R-parity is an unbroken symmetry of the theory.

As a consequence of R-parity conservation superpartners can be produced only in pairs, implying that the lightest superpartner (LSP) is stable if R-parity is exact. Most of the experimental detection modes of SUSY are based on this fact [11].

2.2 SUSY breaking, radiative breaking of $SU(2) \times U(1)$

In the previous section we have seen the particle content and the superpotential of the MSSM. However we know that this can not be the full story for two reasons:

-SUSY is not yet broken

-$SU(2) \times U(1)$ is not yet broken.

First we discuss SUSY breaking. SUSY was invented to solve the hierarchy problem. However SUSY can not be an exact symmetry of nature since in this case many of the superpartners should have been observed by experiments. One has two possibilities for SUSY breaking, either explicit or spontaneous breaking. While theoretically spontaneous breaking of SUSY is much more appealing, one nevertheless has to rule out this possibility in the context of MSSM. To see the reason behind this we have to examine the scalar quark mass

[2]One could forbid the appearance of the B,L breaking terms by imposing different symmetry requirements. For example a Z_2 subgroup of B×L known as matter parity could achieve this goal as well. The point is that once those terms are absent R-parity will necessarily be a symmetry of the Lagrangian.

[3]R-parity as defined in Eq. 2.3 is actually Z_2 subgroup of the continuous R-symmetry of 2.5 combined with a baryon and lepton number transformation. The value of the R-parity can be given by $R = (-1)^{3B+L+2S}$, where B is the baryon number, L the lepton number and S the spin of a given particle.

matrix in detail [12]. The most general scalar mass matrix in N=1 SUSY gauge theories is given by [10]

$$M^{2a}_{\ \ c} = \left[\begin{array}{cc} \bar{W}^{ab}W_{bc} + \frac{1}{2}D^a_\alpha D_{\alpha c} + \frac{1}{2}D^a_{c\alpha}D_\alpha & \bar{W}^{abc}W_b + \frac{1}{2}D^a_\alpha D^c_\alpha \\ \bar{W}^b_{abc} + \frac{1}{2}D_{\alpha a}D_{\alpha c} & W_{ab}\bar{W}^{bc} + \frac{1}{2}D_{\alpha a}D^c_\alpha + \frac{1}{2}D^c_{\alpha a}D_\alpha \end{array} \right], \tag{2.6}$$

where $W_a = \frac{\partial W}{\partial \phi_a}|_{\phi=\langle\phi\rangle}$, $W_{ab} = \frac{\partial^2 W}{\partial\phi_a\partial\phi_b}|_{\phi=\langle\phi\rangle}$, etc. and $D_\alpha = g_\alpha \phi^{\dagger a} T_{\alpha b}^{\ a}\phi_b$, $D_{\alpha c} = \frac{\partial D_\alpha}{\partial \phi_c}|_{\phi=\langle\phi\rangle}$, etc., W is the superpotential, the ϕ_a's are the complex scalars of the χSF's, the g_α's are the gauge couplings, and the $T_{\alpha b}^{\ a}$'s are the generators of the gauge group in the representations of the χSF's.

Specifying this matrix to the squarks we note that since all the squark VEV's must vanish (so as color and electric charge are unbroken symmetries) $D^a_\alpha = 0$ for the squarks. On the other hand quarks get their masses solely from the superpotential thus $\bar{W}^{ab}W_{bc}$ is nothing but the square of the quark mass matrix m. Since electric charge and color are not broken one needs to have $D_1 = D_2 = 0$ (where 1 and 2 here are $SU(2)$ indices) and $D_i = 0$ ($i = 1, ...8$ of $SU(3)$). Thus the only possible non-vanishing D-terms are D_3 and D_Y. Therefore the squark mass matrices can be written in the form

$$M^2_{2/3} = \left[\begin{array}{cc} m_{2/3}m^\dagger_{2/3} + (\frac{1}{2}gD_3 + \frac{1}{6}g'D_Y)1 & \Delta \\ \Delta^\dagger & m^\dagger_{2/3}m_{2/3} - \frac{2}{3}g'D_Y 1 \end{array} \right] \tag{2.7}$$

for the charge 2/3 squarks and

$$M^2_{1/3} = \left[\begin{array}{cc} m_{1/3}m^\dagger_{1/3} + (-\frac{1}{2}gD_3 + \frac{1}{6}g'D_Y)1 & \Delta' \\ \Delta'^\dagger & m^\dagger_{1/3}m_{1/3} + \frac{1}{3}g'D_Y 1 \end{array} \right] \tag{2.8}$$

for the charge -1/3 squarks. Here $m_{1/3}$ and $m_{2/3}$ are the 3 by 3 quark mass matrices in generation space. $M^2_{2/3}$ and $M^2_{1/3}$ are thus 6 by 6 matrices for the 3 generations of left and right handed squarks. The exact form of Δ and Δ' is not important for us. One may notice that (as a consequence of the tracelessness of the group generators) the sum of the D-terms appearing in the two squark matrices is zero. Therefore at least one of the appearing D-terms is non-positive. Assume for example that $\frac{1}{2}gD_3 + \frac{1}{6}g'D_Y \leq 0$. But if β is the normalized eigenvector of the quark mass matrix $m_{2/3}$ corresponding to the smallest eigenvalue m_0 we get that

$$(\beta^\dagger, 0)M^2_{2/3}\left(\begin{array}{c} \beta \\ 0 \end{array} \right) \leq m_0^2. \tag{2.9}$$

Therefore there must be a charged scalar state with mass less than the mass of either the u or d the quark which is experimentally excluded.

Thus we conclude that we need to introduce explicit SUSY breaking terms in order to circumvent the previous argument. However these terms must be such that the solution of the hierarchy problem is not spoiled. Such terms are called soft SUSY breaking terms, and those are the terms that do not reintroduce quadratic divergences into the theory.

The philosophy behind these soft breaking terms is the following: there is a sector of physics that breaks SUSY spontaneously. This is at much higher energy scales than the weak scale. SUSY breaking is communicated in some way (either through gauge interactions or through gravity) to the MSSM fields and as a result the soft breaking terms appear. One popular implementation of this idea is to break SUSY spontaneously in a "hidden sector", that is in a sector of fields that do not interact with the SM particles ("visible sector") except through supergravity which will mediate the SUSY breaking terms to the visible sector. This mechanism with minimal supergravity generates universal soft breaking terms for the visible sector fields at the Planck scale.

Thus one has to handle the MSSM as an effective theory, valid below a certain scale (of new physics), and the soft breaking terms will parameterize our ignorance of the details of the physics of the SUSY breaking sector.

The most general soft SUSY breaking terms are [13]

i. gaugino mass terms

ii. scalar mass terms

iii. scalar quadratic and trilinear interaction terms.

Thus if one wants to implement this program consistently one has to add a separate mass term for each scalar and gaugino and add each quadratic and trilinear interaction term appearing in the superpotential with different coefficients to the Lagrangian:

$$-\mathcal{L}_{soft} = \sum_{i=Q_i,\bar{U}_i,\ldots} m_i^2|\phi_i|^2 + \left(\sum_{i=1,2,3} M_i\lambda_i\lambda_i - B\mu H_1 H_2 + \right.$$

$$\left. + \sum_{ij} A_u^{ij}\lambda_u^{ij}Q^i H_2 \bar{U}^j + \sum_{ij} A_d^{ij}\lambda_d^{ij}Q^i H_1 \bar{D}^j + \sum_{ij} A_e^{ij}\lambda_e^{ij}L^i H_1 \bar{E}^j + h.c. \right) \quad (2.10)$$

This would mean that we introduce 17 new real and 31 new complex parameters into the theory. There are two major problems with this:

-not every set of (m_i, M_i, B, A_k^{ij}) parameters is allowed by phenomenology

-there are too many new parameters to handle the phenomenology.

Let's first see what the requirements for the soft breaking parameters are. The two most serious restrictions come from the requirements that

1. large flavor changing neutral currents (FCNC) and lepton number violations are absent

2. the theory should not yield too large CP violation.

One can easily understand why a general set of soft breaking parameters introduces large FCNC's. Let's look at the $K^0 - \bar{K}^0$ mixing. In the SM one gets contributions from the diagrams shown in Fig. 2-1. However in the MSSM one has additional contributions from the diagrams of Fig. 2-2, where the intermediate lines are now gauginos and squarks, and the cross denotes the soft breaking squark masses. In Fig. 2-2 the usual CKM factors appear at the vertices. Thus the leading part of this diagram is proportional to $V^\dagger M^2 V$, where V is the CKM matrix. The successful implementation of the GIM mechanism in the SM in $K^0 - \bar{K}^0$ mixing is based on the fact that the diagrams are proportional to $V^\dagger V = 1$. However if M^2 is an arbitrary matrix then $V^\dagger M^2 V \neq 1$. Thus we can see that in order to maintain the successful GIM prediction in the MSSM one has to require that $M^2 \approx m^2 1$, that is squarks must be nearly degenerate.

Very similar arguments hold for the $\mu \to e\gamma$ process which will result in the need of nearly degenerate sleptons.

The second constraint on the soft breaking terms comes from the fact that the SM can account for all the measured CP violation. Thus there is no need for extra sources of CP violation in the MSSM, therefore it is usually assumed that the soft breaking parameters are real.

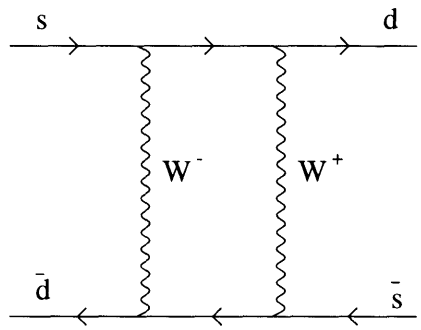


Figure 2-1: Diagrams contributing to $K^0 - \bar{K}^0$ mixing in the SM.

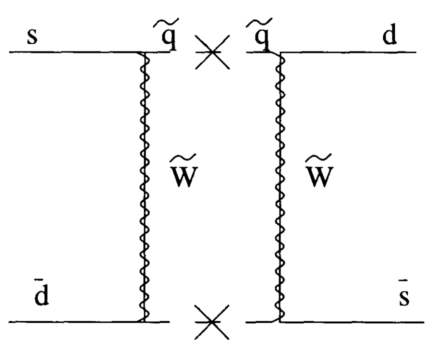


Figure 2-2: Additional diagrams contributing to $K^0 - \bar{K}^0$ mixing in the MSSM.

Thus we have seen what the phenomenological constraints on these soft breaking parameters are. Now we present a set of assumptions that satisfy these constraints and at the same time highly reduce the number of free parameters of the model:

1. Gaugino unification (common mass for the gauginos at the Planck scale)
2. Unification of soft masses (common soft breaking mass terms for the scalars at the Planck scale)
3. Unification of the soft breaking trilinear couplings A_k^{ij} (common trilinear soft breaking term for each trilinear term at the Planck scale)
4. All soft breaking parameters are real.

As one can see these assumptions greatly reduce the number of independent free parameters of the theory. However one has to stress that these are just assumptions, with no solid basis of origin. The strongest argument in favor of these assumptions is that if one takes a supergravity theory in which SUSY is broken in a hidden sector and SUSY breaking is communicated to the visible sector by gravity then one gets flavor independent mass terms, real universal A-terms at the Planck scale and real gaugino masses, provided one assumes that the Kähler potential of the supergravity theory is minimal.

The argument for gaugino unification is the following. It is experimentally indicated that gauge couplings do unify in the MSSM [14]. However the 1-loop RGE for the gaugino masses is given by [5]

$$\frac{d}{dt}\left(\frac{\alpha_i}{M_i}\right) = 0, \;\; i = 1,2,3 \;\; t = \log\left(\frac{\Lambda}{M_{GUT}}\right). \tag{2.11}$$

Here $\alpha_i = g_i^2/4\pi$, g_i are the gauge couplings and M_i the gaugino masses. The ratios of gauge couplings to gaugino masses are scale invariant. Thus if gauge couplings unify so must the gaugino masses.

If one accepts these arguments then the independent soft breaking terms are A_0, m_0, B and $M_{1/2}$ (at the Planck scale), and the soft breaking Lagrangian at the Planck-scale is given by

$$\begin{aligned} -\mathcal{L}_{soft}|_{M_P} = m_0^2 \sum_{i=Q_i,\bar{U}_i,\ldots} |\phi_i|^2 + \Bigg[M_{1/2} \sum_{i=1,2,3} \lambda_i\lambda_i - B\mu H_1 H_2 + \\ + A_0\big(\sum_{ij} \lambda_u^{ij} Q^i H_2 \bar{U}^j + \sum_{ij} \lambda_d^{ij} Q^i H_1 \bar{D}^j + \sum_{ij} \lambda_e^{ij} L^i H_1 \bar{E}^j\big) + h.c. \Bigg], \end{aligned} \tag{2.12}$$

and the Lagrangian at the weak scale can be obtained by running down these universal parameters from the Planck-scale[4] to the weak scale. This procedure will yield sufficiently degenerate squark and slepton masses, and if the soft breaking terms are real at the Planck scale then they will not obtain imaginary parts at the weak scale either. Therefore the above assumptions do satisfy the phenomenological constraints and at the same time they greatly increase the predictive power of the theory. Often these assumptions about the soft breaking terms are assumed to be part of the definition of the MSSM. However one can not overemphasize the fact that these assumptions are ad hoc and need not necessarily be

[4]Usually the Planck-scale and the GUT-scale are not distinguished and it is common to assume that 2.12 is still valid at the GUT-scale $\approx 10^{16}$ GeV.

satisfied.

In the remainder of this section we will discuss the breaking of $SU(2)\times U(1)$. The Higgs potential without soft breaking terms is given by

$$V_{SUSY}(H_1,H_2)=\mu^2(|H_1|^2+|H_2|^2)+\frac{g^2}{2}(H_1^\dagger\vec{\tau}H_1+H_2^\dagger\vec{\tau}H_2)^2+\frac{g'^2}{2}(H_1^\dagger H_1-H_2^\dagger H_2)^2. \quad (2.13)$$

The minimum of this potential is at $\langle H_1\rangle=\langle H_2\rangle=0$, thus we need to incorporate the soft breaking terms to get electroweak breaking. The full Higgs potential at the Planck (GUT) scale is

$$\begin{aligned} V(H_1,H_2)|_{GUT} &= (\mu^2+m_0^2)(|H_1|^2+|H_2|^2)-B\mu(H_1H_2+h.c.)+ \\ &+ \frac{g^2}{2}(H_1^\dagger\vec{\tau}H_1+H_2^\dagger\vec{\tau}H_2)^2+\frac{g'^2}{2}(H_1^\dagger H_1-H_2^\dagger H_2)^2. \end{aligned} \quad (2.14)$$

This potential still does not break $SU(2)\times U(1)$. This can be seen in the following way: in order to have a nontrivial minimum of the Higgs potential

$$m_{H_1}^2|H_1|^2+m_{H_2}^2|H_2|^2-m_{12}^2(H_1H_2+h.c.)+\frac{g^2}{2}(H_1^\dagger\vec{\tau}H_1+H_2^\dagger\vec{\tau}H_2)^2+\frac{g'^2}{2}(H_1^\dagger H_1-H_2^\dagger H_2)^2 \quad (2.15)$$

the quadratic coefficients have to fulfill the following inequalities:

$$\begin{aligned} & m_{H_1}^2+m_{H_2}^2>2|m_{12}^2| \\ & |m_{12}^2|^2>m_{H_1}^2m_{H_2}^2. \end{aligned} \quad (2.16)$$

The first inequality is required so that the potential remains bounded from below for the equal field direction $H_1=H_2$, while the second is required so that the quadratic piece has a negative part enabling a nontrivial minimum. We can see that the potential in Eq. 2.14 can not fulfill both inequalities at the same time thus electroweak symmetry is not broken at the tree level. However radiative corrections can change this situation. To calculate these radiative effects one needs to evaluate the one loop effective potential:

$$V_{1-loop}=V_{tree}(\Lambda)+\Delta V_1(\Lambda), \quad (2.17)$$

where V_{tree} is the tree level superpotential with running parameters evaluated at a scale Λ and ΔV_1 is the contribution of one loop diagrams to the effective potential evaluated by the method of Coleman and Weinberg. The running of the parameters in the tree level potential is generated by the one loop RGE's. $V_{tree}+\Delta V_1$ is Λ independent up to one loop order. If we choose the scale Λ to be close to the scale of the masses of the particles of the theory (in our case $\Lambda\simeq M_{weak}$) ΔV_1 will not contain large logarithms, thus the leading one loop effects will arise due to the running of the parameters of the tree level potential between the Planck and the weak scale. To estimate the running effects on the Higgs parameters we neglect all Yukawa couplings with the exception of the top Yukawa coupling (this is the only large Yukawa coupling so it is reasonable to assume that the largest effects will be caused by it). Then the RGE's for the soft breaking mass terms of the scalars participating in the top Yukawa coupling of the superpotential are [5]:

$$\frac{dm_{H_2}^2}{dt}=\frac{3}{5}g_1^2M_1^2+3g_2^2M_2^2-3\lambda_t^2(m^2+A_t^2)$$

$$\frac{dm_{\tilde{t}}^2}{dt} = \frac{16}{15}g_1^2M_1^2 + \frac{16}{3}g_3^2M_3^2 - 2\lambda_t^2(m^2 + A_t^2)$$
$$\frac{m_{\tilde{Q}_3}^2}{dt} = \frac{1}{15}g_1^2M_1^2 + 3g_2^2M_2^2 + \frac{16}{3}g_3^2M_3^2 - \lambda_t^2(m^2 + A_t^2), \qquad (2.18)$$

where $m^2 = m_{H_2}^2 + m_{\tilde{t}}^2 + m_{\tilde{Q}_3}^2$, $t = \frac{1}{16\pi^2}\log\frac{M_{GUT}^2}{\Lambda^2}$, Λ is the energy scale, g_i are the gauge couplings, A_t the soft breaking trilinear parameter corresponding to the top Yukawa coupling, M_i the gaugino masses and λ_t the top Yukawa coupling.

One can see that the contributions of the gauge and Yukawa loops are independent of each other and the contributions of the gauge loops are independent of the soft breaking masses m_i^2. Thus one can solve Eq. 2.18 by setting the gauge couplings to zero and at the end add the gauge contribution to the resulting solution. Therefore one has to solve the following equation:

$$\frac{d}{dt}\begin{pmatrix} m_{H_2}^2 \\ m_{\tilde{t}}^2 \\ m_{\tilde{Q}_3}^2 \end{pmatrix} = -\lambda_t^2\begin{pmatrix} 3 & 3 & 3 \\ 2 & 2 & 2 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} m_{H_2}^2 \\ m_{\tilde{t}}^2 \\ m_{\tilde{Q}_3}^2 \end{pmatrix} - \lambda_t^2A_t^2\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}. \qquad (2.19)$$

This differential equation can be solved easily if one neglects the running of of λ_t and A_t. The solution corresponding to the universal boundary condition at $t = 0$ ($\Lambda = M_{GUT}$) $m_{H_2}^2 = m_{\tilde{t}}^2 = m_{\tilde{Q}_3}^2 = m_0^2$ in the limit $t \to \infty$ is given by

$$m_{H_2}^2 = -\frac{1}{2}m_0^2$$
$$m_{\tilde{t}}^2 = 0$$
$$m_{\tilde{Q}_3}^2 = \frac{1}{2}m_0^2. \qquad (2.20)$$

Thus we can see that the radiative corrections due to the top Yukawa coupling want to reverse the sign of the soft breaking mass parameter of the up-type Higgs, which is enough to satisfy the conditions for electroweak breaking of Eq. 2.16 at the weak scale. The gauge loops will yield additional positive contributions proportional to $M_{1/2}^2$, and the solution to 2.19 is more complicated if one takes the running of λ_t and A_t into account. However the most important feature of the solution in 2.20 is unchanged: appropriate choices of the input parameters $M_{1/2}, m_0, A_0$ and λ_t will drive the soft breaking mass parameter of the up-type Higgs (and only of the up-type Higgs) negative which will result in the breaking of electroweak symmetry. This mechanism is called radiative electroweak breaking.

Thus as we have seen loop corrections usually modify the Higgs potential such that at the weak scale $SU(2) \times U(1)$ is spontaneously broken. However it is not enough to require that the symmetry is broken, it has to reproduce the correct SM minimum. The Higgs potential at the weak scale can be written as

$$(m_{H_1}^2 + \mu^2)|H_1|^2 + (m_{H_2}^2 + \mu^2)|H_2|^2 - B\mu(H_1H_2 + h.c.) +$$
$$\frac{g^2}{2}(H_1^\dagger\vec{\tau}H_1 + H_2^\dagger\vec{\tau}H_2)^2 + \frac{g'^2}{2}(H_1^\dagger H_1 - H_2^\dagger H_2)^2. \qquad (2.21)$$

The VEV's of the Higgs doublets are

$$\langle H_1 \rangle = \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \langle H_2 \rangle = \begin{pmatrix} v_2 \\ 0 \end{pmatrix}, \tag{2.22}$$

and we define $\tan\beta = v_2/v_1$, $v^2 = v_1^2 + v_2^2$. Minimizing the Higgs potential we find that to fix the W,Z masses at their experimental values it is necessary that [5]

$$\begin{aligned} \mu^2 &= \frac{m_{H_1}^2 - m_{H_2}^2 \tan^2\beta}{\tan^2\beta - 1} - \frac{1}{2} M_Z^2 \\ B &= \frac{(m_{H_1}^2 + m_{H_2}^2 + 2\mu^2)\sin 2\beta}{2\mu}, \end{aligned} \tag{2.23}$$

where all parameters are to be evaluated at the weak scale.

With this we are now able to determine the free parameters of the MSSM. In the soft breaking sector we had $m_0, M_{1/2}, A_0$ and B. In the Higgs sector we have μ and $\tan\beta$, and since the top mass is experimentally not well measured and λ_t tends to run to an IR fixed point at M_Z, $\lambda_t(M_G)$ is basically an unknown parameter of the theory as well. However from Eq. 2.23 μ^2 and B are determined (but not the sign of μ). Therefore the MSSM with R-parity, universal soft breaking parameters and radiative electroweak breaking is determined by 5+1 parameters: $m_0, M_{1/2}, A_0, \tan\beta, \lambda_t$ and the sign of μ.

2.3 Sparticle masses

In this section we present the possible mixings between the superpartner fields and list the tree level mass matrices [1, 5, 15].

2.3.1 Sfermions

In principle one must diagonalize 6 by 6 matrices corresponding to the mixing of the L and R scalars of the 3 generations. To simplify this we neglect intergenerational mixings and take only L-R mixing into account.

Squarks

The mass matrices in the L-R basis are for each generation of up-type scalars is

$$M_{\tilde{u}_{L,R}}^2 = \begin{pmatrix} m_{\tilde{Q}}^2 + m_u^2 + (\frac{1}{2} - \frac{2}{3}\sin^2\theta_W)D & m_u(A_u - \mu\cot\beta) \\ m_u(A_u - \mu\cot\beta) & m_{\tilde{u}}^2 + m_u^2 + \frac{2}{3}\sin^2\theta_W D \end{pmatrix}, \tag{2.24}$$

where the mass parameters with a tilde refer to the soft breaking squark mass parameters while the mass parameters without tilde are the usual quark masses, $D = M_Z^2 \cos 2\beta$.

The down-type mass matrix is

$$M_{\tilde{d}_{L,R}}^2 = \begin{pmatrix} m_{\tilde{Q}}^2 + m_d^2 - (-\frac{1}{2} - \frac{1}{3}\sin^2\theta_W)D & m_d(A_d - \mu\tan\beta) \\ m_d(A_d - \mu\tan\beta) & m_{\tilde{d}}^2 + m_d^2 - \frac{1}{3}\sin^2\theta_W D \end{pmatrix}. \tag{2.25}$$

The only source of intergenerational mixing is the superpotential, thus in the more general case the diagonal elements m_u^2 and m_d^2 must be exchanged to $v_{2,1}^2(\lambda_{u,d}^\dagger \lambda_{u,d})_{ij}$, where

ij are generation indices. However since the CKM mixing is small and the soft breaking mass terms are large compared to the quark masses, these effects are usually negligible.

Sleptons

In the same notation the sneutrino masses are:

$$M_{\tilde{\nu}}^2 = M_{\tilde{L}}^2 + \frac{1}{2}D \tag{2.26}$$

while for $\tilde{e}, \tilde{\mu}, \tilde{\tau}$ the mass matrices are

$$M^2_{\tilde{e}_{L,R}} = \begin{pmatrix} m_{\tilde{L}}^2 + m_e^2 - (\frac{1}{2} - \sin^2\theta_W)D & m_e(A_e - \mu\tan\beta) \\ m_e(A_e - \mu\tan\beta) & m_{\tilde{e}}^2 + m_e^2 - \sin^2\theta_W D \end{pmatrix}. \tag{2.27}$$

2.3.2 The scalar Higgs sector

We use the notation

$$H_1 = \begin{pmatrix} h_1^0 \\ h_1^- \end{pmatrix}, \quad H_2 = \begin{pmatrix} h_2^+ \\ h_2^0 \end{pmatrix}. \tag{2.28}$$

The tree level masses are calculated from the mass matrices

$$\begin{aligned}
\frac{1}{2}\frac{\partial^2 V_{tree}}{\partial(Imh_i^0)\partial(Imh_j^0)} &= \frac{1}{2}M_A^2\sin 2\beta \begin{pmatrix} \tan\beta & 1 \\ 1 & \cot\beta \end{pmatrix} \\
\frac{1}{2}\frac{\partial^2 V_{tree}}{\partial(Reh_i^0)\partial(Reh_j^0)} &= \frac{1}{2}M_A^2\sin 2\beta \begin{pmatrix} \tan\beta & -1 \\ -1 & \cot\beta \end{pmatrix} + \frac{1}{2}M_Z^2\sin 2\beta \begin{pmatrix} \cot\beta & -1 \\ -1 & \tan\beta \end{pmatrix} \\
\frac{\partial^2 V_{tree}}{\partial h_i^- \partial h_j^+} &= \frac{1}{2}M_{H^\pm}^2\sin 2\beta \begin{pmatrix} \tan\beta & 1 \\ 1 & \cot\beta \end{pmatrix},
\end{aligned} \tag{2.29}$$

where $M_A^2 = m_{H_1}^2 + m_{H_2}^2 + 2\mu^2$, $M_{H^\pm} = M_W^2 + M_A^2$, $i,j = 1,2$.

The first mass matrix has eigenvalues 0 (GB eaten by the Z) and M_A^2 (CP odd scalar). The second matrix gives the masses for the light and heavy Higgs bosons:

$$M_{H,h}^2 = \frac{1}{2}\left[(M_A^2 + M_Z^2) \pm \sqrt{(M_A^2 + M_Z^2)^2 - 4M_A^2M_Z^2\cos^2 2\beta}\right]. \tag{2.30}$$

The third matrix has eigenvalues 0 (charged GB's eaten by $W^\pm$) and $M_{H^\pm}^2$ (charged scalars).

It is important to mention that for some of the Higgs masses the 1-loop corrections can be significant. For example from the above formula one would get that $m_h \leq M_Z$, while including 1-loop corrections the corresponding bound will be modified to $m_h \leq 150$ GeV [15].

2.3.3 Charginos

Charginos are mixtures of the charged Higgsinos and the charged gauginos ($\tilde{W}_{1,2}$). The mass matrix is given by ($\lambda^\pm = (\tilde{W}_2 \pm i\tilde{W}_1)/\sqrt{2}$):

$$\begin{pmatrix} \lambda^+ & \tilde{h}_2^+ & \lambda^- & \tilde{h}_1^- \end{pmatrix} \begin{pmatrix} 0 & 0 & M_2 & -g_2 v_1 \\ 0 & 0 & g_2 v_2 & -\mu \\ M_2 & g_2 v_2 & 0 & 0 \\ -g_2 v_1 & -\mu & 0 & 0 \end{pmatrix} \begin{pmatrix} \lambda^+ \\ \tilde{h}_2^+ \\ \lambda^- \\ \tilde{h}_1^- \end{pmatrix} . \tag{2.31}$$

The eigenvalues are

$$M_{\tilde{C}_{1,2}} = \frac{1}{2} \left[(M_2^2 + \mu^2 + 2M_W^2) \pm \sqrt{(M_2^2 + \mu^2 + 2M_W^2)^2 - 4(M_2\mu - M_W^2 \sin 2\beta)^2} \right] . \tag{2.32}$$

2.3.4 Neutralinos

Neutralinos are the mixture of neutral Higgsinos and the neutral gauginos $(\tilde{B}, \tilde{W}_3)$. The mass matrix is given by

$$\begin{pmatrix} i\tilde{B} & i\tilde{W}_3 & \tilde{h}_1^0 & \tilde{h}_2^0 \end{pmatrix} \frac{1}{2} \begin{pmatrix} -M_1 & 0 & g'v_1/\sqrt{2} & -g'v_2/\sqrt{2} \\ 0 & -M_2 & -g_2 v_1/\sqrt{2} & g_2 v_2/\sqrt{2} \\ g'v_1/\sqrt{2} & -g_2 v_1/\sqrt{2} & 0 & \mu \\ -g'v_2/\sqrt{2} & g_2 v_2/\sqrt{2} & \mu & 0 \end{pmatrix} \begin{pmatrix} i\tilde{B} \\ i\tilde{W}_3 \\ \tilde{h}_1^0 \\ \tilde{h}_2^0 \end{pmatrix} . \tag{2.33}$$

Chapter 3

Supersymmetric Grand Unified Theories[1]

As mentioned in the Introduction, one of the main motivations for considering supersymmetric theories is the unification of gauge couplings. Calculating the running of the coupling constants in the standard model, it turns out that the gauge couplings do not meet at one point. However changing the particle content to that of the MSSM, the one-loop β-functions change, and the gauge couplings automatically unify at $M_{GUT} \sim 10^{16}$ GeV [14]. This is a strong evidence for the MSSM itself being embedded into a large gauge group, which breaks at M_{GUT} to $SU(3) \times SU(2) \times U(1)$.

The simplest such model is to embed the MSSM into an $SU(5)$ theory, and introduce an $SU(5)$ adjoint ($\mathbf{24}$) chiral superfield Σ, and an $SU(5)$ flavor $H + \bar{H}$ ($= \mathbf{5} + \bar{\mathbf{5}}$).

A superpotential for the field Σ

$$W = \frac{1}{2} M \mathrm{Tr}\Sigma^2 + \frac{1}{3}\lambda \mathrm{Tr}\Sigma^3 \tag{3.1}$$

forces a VEV

$$\langle \Sigma \rangle = \frac{M}{\lambda} \begin{pmatrix} 2 & & & & \\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ & & & & -3 \end{pmatrix}, \tag{3.2}$$

which breaks $SU(5)$ to $SU(3) \times SU(2) \times U(1)$, while the fields $H + \bar{H}$ will provide the Higgs doublets of the MSSM. The matter fields can be incorporated in the usual way using the representations $(\mathbf{10} + \bar{\mathbf{5}})_i$, $i = 1, 2, 3$. The superpotential giving rise to ordinary fermion masses is given by

$$W = A_{ij} H 10_i 10_j + B_{ij} \bar{H} 10_i \bar{5}_j. \tag{3.3}$$

3.1 The doublet-triplet splitting problem

The model above (and all other SUSY GUTs) have a potentially devastating problem. The $SU(5)$ representations $H, \bar{H}$ are to contain the MSSM Higgs doublets H_1 and H_2,

[1]Based on research done in collaboration with Lisa Randall and Zurab Berezhiani reported in Refs. [34, 40].

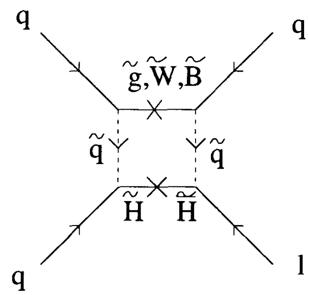


Figure 3-1: The diagram leading to proton decay via the color triplet Higgs fields.

whose masses we know must be in the range of few hundreds of GeV. Moreover, by $SU(5)$ symmetry the $H, \bar{H}$ fields contain now color-triplet components as well. However, the fermionic component of these color triplet fields mediate proton decay via the diagram in Fig. 3-1. To suppress proton decay to an acceptable level, one must require that the mass of the color triplets are $\sim M_{GUT}$. Thus the color triplets of the $SU(5)$ representations $H, \bar{H}$ must have mass $\sim 10^{16}$ GeV, while the doublets from the same $SU(5)$ representations have mass $\sim 10^2$ GeV. The doublet-triplet splitting problem is to naturally explain the splitting of $\mathcal{O}(10^{14})$ of the different components of $H, \bar{H}$, without introducing fine-tuning into the theory. Next we review the possible suggested solutions to the doublet-triplet splitting problem [16].

3.1.1 Fine tuning

One can just tune the parameters of the superpotential in order to achieve splitting of the doublet and the triplet masses. Consider the superpotential

$$W = \frac{1}{2} M_\Sigma \mathrm{Tr}\Sigma^2 + \frac{1}{3}\lambda \mathrm{Tr}\Sigma^3 + M_H H\bar{H} + fH\Sigma\bar{H}. \tag{3.4}$$

The Σ VEV is given by

$$\langle\Sigma\rangle = \frac{M_\Sigma}{\lambda}\begin{pmatrix} 2 & & & & \\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ & & & & -3 \end{pmatrix}, \tag{3.5}$$

which yields a mass matrix

$$\begin{pmatrix} M_H + 2f\frac{M_\Sigma}{\lambda} & & & & \\ & M_H + 2f\frac{M_\Sigma}{\lambda} & & & \\ & & M_H + 2f\frac{M_\Sigma}{\lambda} & & \\ & & & M_H - 3f\frac{M_\Sigma}{\lambda} & \\ & & & & M_H - 3f\frac{M_\Sigma}{\lambda} \end{pmatrix}. \tag{3.6}$$

One can see that if $M_H - 3f\frac{M_\Sigma}{\lambda} = 0$, then the doublet masses are zero, while the triplet masses are of order M_{GUT}. However this requires a tuning of the parameters of order 10^{-14}, which makes this model very unattractive.

3.1.2 The "sliding singlet"

Witten noticed [17], that this fine tuning can be achieved naturally, if one introduces a gauge singlet S and modifies the superpotential to

$$W = \frac{1}{2}M_\Sigma \mathrm{Tr}\Sigma^2 + \frac{1}{3}\lambda \mathrm{Tr}\Sigma^3 + f'SH\bar{H} + fH\Sigma\bar{H}. \tag{3.7}$$

Now the $\bar{H}$ equation of motion sets

$$(f'S + f\Sigma)H = 0, \tag{3.8}$$

and assuming that

$$\langle H \rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ v \end{pmatrix} \tag{3.9}$$

we get that

$$f'\langle S \rangle = 3f\frac{M_\Sigma}{\lambda}, \tag{3.10}$$

which is automatically the right value needed to set the doublet mass terms to zero. However, it is crucial for this solution, that the only place S appears in the Lagrangian is in the superpotential term $f'SH\bar{H}$. However, after soft SUSY breaking terms are introduced, loop corrections will generate additional S-dependent terms in the scalar potential and ruin this elegant solution [18].

3.1.3 The "missing partner" mechanism

In Minimal $SU(5)$, the best proposed solution to the doublet-triplet splitting problem is the Missing Partner Mechanism [19, 20]. The idea here is to give the triplet a mass through a Dirac mass term involving a more complicated representation of $SU(5)$ which has the property that it contains a triplet but not a doublet. The 50 is the smallest representation with this feature. One therefore constructs the mass terms

$$W \supset \lambda 5_H \overline{50}_H \langle 75_H \rangle + \lambda' \bar{5}_H 50_H \langle 75_H \rangle \tag{3.11}$$

so that the triplets, but not the doublets are massive. A model with additional symmetry to forbid a direct mass term for the 5 and $\bar{5}$ incorporates an additional 75 [20].

This is a very nice idea, but seems unlikely to be the resolution of the dilemma. There are several problems with this model. First of all, the large rank of the representations is disturbing. From a theoretical perspective, one has yet to find string theory examples containing these large rank representations. Another problem is that the gauge coupling grows very rapidly, so that the theory is strongly coupled not far above the GUT scale. Although this might be acceptable, it is certainly a problem at the level of nonrenormalizable operators which we discuss shortly.

A further problem is that one cannot leave the states in the remainder of the 50 massless, since they contribute like an extra doublet pair to unification, which we know is too much. One can solve this problem for example by adding a mass term $M 50\,\bar{50}$ (though this is forbidden by the symmetry of Ref. [20]). But then there is nothing in the symmetry structure of the theory which could forbid the term $(5)(\bar{5})(75)(75)/M_p$ which is the product of two allowed terms in the superpotential divided by a third and is therefore allowed by the symmetry, no matter what it is. If one believes Planck suppressed operators consistent with the symmetries are present, the doublet has much too big a mass. This problem is exacerbated in the case the coupling blows up at a low scale, because it is probably the associated strong scale which would suppress such operators.

A more compact implementation of the Missing Partner Mechanism was proposed for Flipped $SU(5)$ [21]. The idea is again to pair up the Higgs with "something else". Here the something else is a 10_H for the 5_H and a $\bar{10}_H$ for the $\bar{5}_H$, the subscript refers to the Higgs sector to distinguish these fields from the ordinary matter fields. These 10_H and $\bar{10}_H$ fields are not dangerous because the nonsinglet nontriplet fields are eaten when $SU(5)$ breaks. Hence one has eliminated the necessity for the additional mass term.

The 10_H contains a $\bar{3}$ but no color singlet weak doublet. The 10_H and $\bar{10}_H$ get VEVs breaking the $SU(5) \times U(1)$ gauge group to the standard model. The triplet Higgs in 5_H pairs with the triplet in 10_H and the remaining fields in the 10_H are eaten by the massive vector bosons.

This model might work. However, flipped $SU(5)$ is not really a unified model since the gauge group is $SU(5) \times U(1)$ which is not a semisimple group. If it is embedded in a larger gauge group, the problem should be solved in the context of the larger gauge group. The other feature we find disturbing is that there are many assumptions about the vacuum structure. At tree level, there is a D-flat, F-flat direction, and the loop corrections have to be such as to generate the desired minimum. In Ref. [21], the correct ratio of gauge and Yukawa couplings was assumed so that the VEVs for 10_H and $\bar{10}_H$ were at the GUT scale while the 5_H and $\bar{5}_H$ VEVs are small and the VEVs of an $SU(5)$ singlet generated the "μ" term. It is certainly easier to evaluate the vacuum when it is determined at tree level, as it will be in our preferred model.

3.1.4 The "missing VEV" mechanism

Other solutions have been proposed for models which incorporate $SU(5)$ as a subgroup, for example $SO(10)$. The "missing VEV" or Dimopoulos-Wilczek mechanism [22] is probably the most popular $SO(10)$ solution. The idea is again to pair the triplet and not the doublet Higgs with something else so that the triplet, but not the doublet is massive. In this model, the way this is done is that the VEV aligns so that the triplet, but not the doublet, gets a mass. (This would not have been possible in minimal $SU(5)$ due to the tracelessness of the adjoint.)

Again, this mechanism seems very nice at first glance, but worrisome at second. For if this were all there was, you would have four light doublets, not two. You need to give the extra doublets a mass, and the problem is how to do this without reintroducing a problem with proton decay. A series of papers by Babu, Barr and Mohapatra [23, 24, 25, 26] showed possible ways to make the DW mechanism into a more complete model.

The first example [23] had two sectors giving VEVs aligning in different orientations, one responsible for the triplet mass, and one responsible for the doublet mass. They thereby achieved strong suppression of proton decay. There was an additional field to complete the breaking of $SO(10)$ to the standard model, and an additional adjoint to couple the two sectors together (eliminating a massless Goldstone) without misaligning the DW mechanism. The total field content in this model is uncomfortably large – $3(16)+3(10)+3(45)+2(54)+\bar{16}+16$, leading to fairly big threshold corrections and the blowing up of the gauge coupling before M_{Pl}. Other problems with this particular model was that some operators which would have been allowed by the symmetries of the model needed to be forbidden, and that nonrenormalizable Planck mass suppressed operators could be dangerous.

This last problem was addressed in their second model, where they sacrifice strong suppression of proton decay but generate a natural model, in the sense that they include all operators permitted by their assumed symmetry structure. The field content of this model was $3(16) + 2(10) + 3(45) + (54) + (\bar{126}) + (126)$. Discrete symmetries were sufficient to forbid any unwanted terms from the potential. However, the field content was still quite large, and high rank representations were required.

The third model incorporated a smaller field content and no high rank representations, so it should be more readily obtainable from string models. In this model, the authors achieved the DW form with higher dimension operators, so no (54) was required. There was a $\bar{16} + 16$ to complete the breaking to the standard model.

However, without three adjoints, there were intermediate scale pseudo-Goldstone bosons. The authors resolved this problem by canceling the fairly large corrections to unification of couplings (due to the light charged fields) by large threshold corrections. Although this might work, it is at the edge of parameter space.

Another nice model based on $SO(10)$ is the model of Babu and Mohapatra [26] which allows for a 10-16 mixing and therefore a Higgs sector which distinguishes the up and down quark masses. However, this model had a few small (but not very small) parameters, a flat direction and therefore vacuum degeneracy at tree level, extra singlets, and a complicated superpotential.

To summarize, there are some interesting models in the literature, primarily based on clever group theory structure. However most models suffer from one of the following problems.

- There is the problem of actually implementing the potential to get the desired minimum and light Higgses. The minimum can sometimes be destabilized with higher order terms. Also some models have flat directions so the vacuum needs to be carefully thought through.

- It is necessary to ensure the light particle spectrum is compatible with gauge coupling unification. Most solutions rely on pairing up the triplet higgsinos (not doublet) with "something else". "Something else" can be a problem (with gauge unification).

- The particle representation is cumbersome. This leads to the questions of whether it is derivable from strings or whether the coupling blows up before the Planck scale. In any case, models with large particle content seem unappealing and unlikely.

The problem is clear. Minimal $SU(5)$ relates doublets and triplets! Almost always, the solution relies on a compromise at the edge of parameter space or tuned parameters or setting some couplings to zero in the potential. This is a good introduction to the Higgses as pseudo-Goldstone bosons model which we will argue is an exception to the discussion above. Rather than relying on pairing the Higgs in complicated ways, the theory relies on a spontaneously broken symmetry under which the Higgses are Goldstone bosons. This distinguishes the doublets from the triplets in a very nontrivial way, so that it is natural to obtain light doublets when the remaining fields are heavy. The originally proposed model [27, 28, 29] involved gauged $SU(5)$ symmetry and a global $SU(6)$ symmetry which was implemented by tuning potential parameters. A better model [30, 31, 32, 33] was later proposed which admits the possibility for justifying the large global symmetry with discrete symmetries. In fact, as we will see, one can construct a simple model to implement this idea [34].

3.2 The Higgs as pseudo-Goldstone boson mechanism

One of the most economical and satisfying explanations for why the Higgs doublets are light could be that they are pseudo-Goldstone bosons (PGB's) of a spontaneously broken accidental global symmetry of the Higgs sector [27]. The Higgs sector of the chiral superfields is defined with the use of matter parity. Under this Z_2 symmetry all matter fields (fermion fields) change sign while the Higgs fields are invariant. When Yukawa couplings are incorporated (couplings of the Higgs sector to matter fields), the accidental global symmetry is explicitly broken; however, because of supersymmetric nonrenormalization theorems the Higgs masses can only be of order of the supersymmetry breaking, or weak scale.

The first attempts to build such a model were made by requiring that the chiral superfields of a given gauge group are put together into a representation of a bigger global symmetry group [27, 28, 29]. For example the $24, 5, \bar{5}$ and 1 of an $SU(5)$ gauge group could form the 35 adjoint of $SU(6)$. While the global $SU(6)$ breaks to $SU(4)\times SU(2)\times U(1)$, the gauged $SU(5)$ breaks to $SU(3)\times SU(2)\times U(1)$, and the uneaten PGB's are in two $SU(2)$ doublets [27, 28]. Other similar models were discussed in ref. [29].

Unfortunately this model requires even more fine tunings of the parameters of the superpotential than the usual fine tuning solution of the doublet-triplet splitting problem. For example in the $SU(5)$ model mentioned above this would mean that for the general superpotential

$$\begin{aligned} W = &\frac{1}{2}M\mathrm{Tr}\Sigma^2 + \frac{1}{3}\lambda\mathrm{Tr}\Sigma^3 + \mu\bar{H}\Sigma H + \alpha\bar{H}H + \rho_1 Y + \frac{\rho_2}{2}Y^2 + \\ &\rho_3 Y^3 + \rho_4\mathrm{Tr}\Sigma^2 Y + \rho_5\bar{H}HY, \end{aligned} \tag{3.12}$$

where the fields $\Sigma, H, \bar{H}, Y$ are the SU(5) fields transforming according to $24, 5, \bar{5}, 1$ the following relations have to hold in order to have the larger global SU(6) invariance:

$$\begin{aligned} &\alpha = M = \rho_2,\ \ \mu = \lambda,\ \ \rho_3 = -\frac{2}{3}\left(\frac{2}{15}\right)^{\frac{1}{2}}\lambda, \\ &\rho_4 = \frac{1}{\sqrt{30}}\lambda,\ \ \rho_5 = -2\left(\frac{2}{15}\right)^{\frac{1}{2}}\lambda. \end{aligned} \tag{3.13}$$

These relations are very unlikely to be a result of a symmetry of a higher energy theory. Thus this version does not tell much more than the original fine tuned $SU(5)$ theory.

A much more appealing scenario is that the accidental symmetry of the superpotential arises because two sectors of the chiral superfields responsible for gauge symmetry breaking do not mix and thus the global symmetry of this sector is $G\times G$ instead of the original gauge group G [30, 31, 32, 33]. This accidental symmetry could be a result of a discrete symmetry that forbids the mixing of the two sectors so this scenario might well be a consequence of a symmetry of a larger theory. During spontaneous symmetry breaking $G \times G \to G_1 \times G_2$ while the diagonal G (which is the original gauge group) breaks to $SU(3) \times SU(2) \times U(1)$.

The D-terms of the group $G \times G$ in this scheme of spontaneous symmetry breaking (SSB) vanish in order to preserve supersymmetry. Because supersymmetry is preserved, the requirement for "total doubling" [27] is fulfilled, so associated to every Goldstone boson there is also a pseudo-Goldstone boson in a chiral multiplet which is massless only by supersymmetry. Therefore, all the scalars in a Goldstone chiral superfield are light, not only one of the scalar components. We will refer to both as PGB's throughout the paper. The genuine Goldstone bosons remain massless even after adding the soft SUSY breaking terms, while the pseudo-Goldstone bosons get masses at the order of the weak scale at this stage. The remaining massless states get masses during the running down from the GUT scale to the weak scale due to the symmetry breaking Yukawa couplings.

These Yukawa couplings have to break the accidental global symmetry of the Higgs sector explicitly. Otherwise the couplings of the Higgs doublets (which are identified with the uneaten PGB's of the broken global symmetry) to the light fermions would vanish. Thus there would be no source for the light fermion masses. The nonvanishing of the couplings of the Higgs fields to the light fermions (especially to the top quark) is also essential for radiative electroweak breaking. Thus it is necessary that in these models the Yukawa couplings explicitly break the accidental symmetry of the Higgs sector.

Explicit symmetry breaking terms in the Higgs sector can yield additional contributions to the μ-term of the Higgs potential (the models presented in section 4 will contain such explicit breaking terms). There can also be additional contributions to the μ-term from nonrenormalizable contributions to the Kähler potential [35].

An example of models of this kind was given in refs. [30, 31, 32, 33]. In this case $G = SU(6)$, and the accidental global symmetry is broken to $SU(4) \times SU(2) \times U(1) \times SU(5)$. There are exactly two light doublets in this model, so the low energy particle content is just that of the MSSM. The Higgses are naturally light (they are PGB's), while the triplets have masses of $\mathcal{O}(M_{GUT})$.

Although such a model is very appealing in principle, it is not clear that it holds up to more detailed scrutiny. The first problem is to construct a potential with the desired symmetries and symmetry breakings. The second problem is to generate a fermion mass spectrum compatible with observation.

The flavor problem can be addressed by enhancing the field content and including nonrenormalizable operators [32, 33]. However, the first problem is very difficult. The models of Refs. [31, 32] do not give the correct minimum without fine tuning and are therefore unacceptable. The model of Ref. [33] has the correct symmetry and gives the desired minimum if one incorporates only the renormalizable terms in the superpotential. However, once nonrenormalizable terms are incorporated it is very difficult to construct acceptable models without fine tuning.

In the following we outline many constraints for model building. We consider only models of the second type; that is, models where the accidental global symmetry arises as a result of two nonmixing sectors of the Higgs fields. In supersymmetric theories it may happen that some operators are unexpectedly missing from the superpotential even though they are allowed by all symmetries of the theory. However we will use the most pessimistic assumption, that is, all terms consistent with all the symmetries are present and they are as big as they can be (suppressed only by the appropriate powers of M_{Pl}). Our philosophy for constructing models is to find discrete symmetries that forbid the dangerous mixings of the two sectors. These could be either R-type or usual discrete symmetries.

(We assume that all other symmetries of the theory above the Planck scale are broken with the exception of some possible discrete symmetries. These should actually be gauge type discrete symmetries so that they are not destroyed by large gravitational corrections. This implies that these discrete symmetries could possibly have anomalies. However these discrete anomalies can always be canceled by adding extra gauge singlets transforming nontrivially under the discrete symmetries.)

We find for such models a general feature that the more one suppresses mixing terms, the more fine tuning is necessary in the superpotential to maintain the correct values of the vacuum expectation values (VEV's) if the VEV's of all fields in the Higgs sector are comparable. To find a way out we either need to introduce small mass parameters or fields with small (or zero) VEV's.

However, the small parameter which can be used to build these models is necessarily present in models with supersymmetry breaking. Based on the analysis of requirements for a successful model, we show how to exploit the supersymmetry breaking scale to create models which have an accidental global $SU(6) \times SU(6)$ symmetry. In these models the Higgs is naturally light, and there are no problems with the triplets. Alternatively we will show how to use the second possibility (the presence of fields with zero VEVs) to build another class of natural models without the use of any small parameter.

In the following subsection, we review the $SU(6)$ model and give a simple example for a model that is acceptable if one incorporates only renormalizable terms into the superpotential. In Section 3.2.2 we first discuss the requirements for building an acceptable potential and show why it is difficult to get a natural model. First we consider the use of alternative $SU(6)$ representations to forbid the dangerous mixing terms. Then we discuss the possibility of using restrictive discrete symmetries for this purpose. We draw the conclusion that if mixing terms are suppressed there must be either small parameters or fields with zero VEV in the theory. In the next section we present three different models that naturally fulfill all the requirements for the superpotential.

The first model uses a small mass parameter, namely the weak scale, to get the correct magnitudes of VEV's of the fields in the Higgs sector. The second model does not use any small mass parameter, but exploits the presence of fields with zero VEV's to obtain an acceptable theory. In the third model, we assume the appearance of the GUT scale by an unspecified dynamical origin. With these three models we demonstrate that the idea of having the Higgses as pseudo-Goldstone particles can be naturally implemented. Finally we show, that the simplest model can be extended to include fermion masses as well.

3.2.1 A review of the $SU(6)$ model

In the $SU(6)$ model of Refs. [30, 31, 32, 33], the gauge group of the high energy GUT theory is $SU(6)$. The accidental symmetry of the Higgs part of the superpotential arises because there are two sectors involving two different fields which do not mix in the potential, so that an accidental global $SU(6) \times SU(6)$ symmetry is preserved. The fields suggested in Refs. [30, 31, 32] to realize this idea were Σ in an adjoint 35 representation and $H, \bar{H}$ in $6, \bar{6}$ representations of $SU(6)$. Their $SU(5)$ decomposition is

$$\begin{aligned} \Sigma &= 35 = 24 + 6 + \bar{6} + 1 \\ H &= 6 = 5 + 1 \\ \bar{H} &= \bar{6} = \bar{5} + 1. \end{aligned} \tag{3.14}$$

Then one of the sectors consists of the fields $H, \bar{H}$ and the other of Σ. The accidental symmetry is realized if mixing terms of the form $\bar{H}\Sigma H$ are not present in the superpotential. If the fields Σ and $H, \bar{H}$ develop VEV's of the form

$$\langle\Sigma\rangle = V \begin{pmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & 1 & & \\ & & & & -2 & \\ & & & & & -2 \end{pmatrix}, \quad \langle H\rangle = \langle\bar{H}\rangle = U \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \tag{3.15}$$

then one of the global $SU(6)$ factors breaks to $SU(4)\times SU(2)\times U(1)$, while the other to $SU(5)$. Together, the VEV's break the gauge group to $SU(3)\times SU(2)\times U(1)$.

The Goldstone bosons (GB's) coming from the breaking $SU(6)\to SU(4)\times SU(2)\times U(1)$ are (according to their $SU(3)\times SU(2)\times U(1)$ transformation properties):

$$(\bar{3},2)_{\frac{5}{6}} + (3,2)_{-\frac{5}{6}} + (1,2)_{\frac{1}{2}} + (1,2)_{-\frac{1}{2}}, \tag{3.16}$$

while from the breaking $SU(6)\to SU(5)$ the GB's are

$$(3,1)_{-\frac{1}{3}} + (\bar{3},1)_{\frac{1}{3}} + (1,2)_{\frac{1}{2}} + (1,2)_{-\frac{1}{2}} + (1,1)_0. \tag{3.17}$$

But the following GB's are eaten by the heavy vector bosons due to the supersymmetric Higgs mechanism (the gauge symmetry is broken from $SU(6)$ to $SU(3)\times SU(2)\times U(1)$):

$$(3,1)_{-\frac{1}{3}} + (\bar{3},1)_{\frac{1}{3}} + (3,2)_{-\frac{5}{6}} + (\bar{3},2)_{\frac{5}{6}} + (1,2)_{\frac{1}{2}} + (1,2)_{-\frac{1}{2}} + (1,1)_0. \tag{3.18}$$

Thus exactly one pair of doublets remains uneaten which can be identified with the Higgs fields of the MSSM. One can show that the uneaten doublets are in the following combinations of the fields $\Sigma, H, \bar{H}$:

$$h_1 = \frac{Uh_\Sigma - 3Vh_H}{\sqrt{9V^2+U^2}}, \tag{3.19}$$

$$h_2 = \frac{U\bar{h}_\Sigma - 3V\bar{h}_{\bar{H}}}{\sqrt{9V^2+U^2}}, \tag{3.20}$$

where h_H and $\bar{h}_{\bar{H}}$ denote the two doublets living in the SU(6) field H and $\bar{H}$, while h_Σ and $\bar{h}_\Sigma$ denote the two doublets living in the $SU(6)$ adjoint Σ .

In order to get the correct order of symmetry breaking we need to have $\langle\Sigma\rangle\sim M_{GUT}$, $\langle H\rangle = \langle\bar{H}\rangle > \langle\Sigma\rangle$. In this case the gauge group is broken as

$$SU(6)\to SU(5)\to SU(3)\times SU(2)\times U(1)$$

In the case of opposite ordering of the magnitudes of the VEV's we would get

$$SU(6)\to SU(4)\times SU(2)\times U(1)\to SU(3)\times SU(2)\times U(1),$$

which would give unreasonably large threshold correction to the RG values of $\sin^2\theta_W$.

The biggest question of this model is how to realize the necessary suppression of mixing terms like $\bar{H}\Sigma H$ in the superpotential and thus achieve the desired vacuum. We want to find discrete symmetries that forbid the mixing of the two sectors. These could be either R-type or usual discrete symmetries.

In Ref. [31] a Z_2 discrete symmetry $\bar{H} \to -\bar{H}$, $S \to -S$, $H \to H$, $\Sigma \to \Sigma$ (S is an $SU(6)$ singlet) was suggested to forbid the mixing term $\bar{H}\Sigma H$. But in the supersymmetric limit the $H, \bar{H}, S$ VEV's were all zero, so these VEV's come from the soft breaking terms, and consequently

$$\langle H \rangle \sim (m M_{GUT})^{\frac{1}{2}} \approx 10^8 \text{GeV}, \tag{3.21}$$

where m is a mass parameter of the order of the weak scale. Consequently unreasonably large fine tuning is needed to obtain $\langle H \rangle > M_{GUT}$.

One can overcome this problem by introducing more fields into the theory [33]. One can take for example two adjoints Σ_1, Σ_2 instead of just one and a discrete Z_3 symmetry under which $\Sigma_1 \to e^{\frac{2\pi i}{3}} \Sigma_1$ and $\Sigma_2 \to e^{-\frac{2\pi i}{3}} \Sigma_2$, while $H, \bar{H}, S$ are invariant. Then the most general renormalizable superpotential is of the form

$$\begin{aligned} W(S, H, \bar{H}, \Sigma_1, \Sigma_2) = aS(\bar{H}H - \mu^2) - \frac{M'}{2}S^2 - \frac{\gamma}{3}S^3 - m\bar{H}H + \\ \alpha S \text{Tr}\Sigma_1\Sigma_2 + M\text{Tr}\Sigma_1\Sigma_2 + \frac{\lambda_1}{3}\text{Tr}\Sigma_1^3 + \frac{\lambda_2}{3}\text{Tr}\Sigma_2^3. \end{aligned} \tag{3.22}$$

which automatically has the global $SU(6) \times SU(6)$ symmetry. The VEV's are:

$$\langle S \rangle = \frac{m}{a} \tag{3.23}$$

$$\langle \Sigma_1 \rangle = \frac{\alpha \frac{m}{a} + M}{(\lambda_1^2 \lambda_2)^{\frac{1}{3}}} \begin{pmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & 1 & & \\ & & & & -2 & \\ & & & & & -2 \end{pmatrix} \tag{3.24}$$

$$\langle \Sigma_2 \rangle = \frac{\alpha \frac{m}{a} + M}{(\lambda_2^2 \lambda_1)^{\frac{1}{3}}} \begin{pmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & 1 & & \\ & & & & -2 & \\ & & & & & -2 \end{pmatrix} \tag{3.25}$$

$$\langle H \rangle = \langle \bar{H} \rangle = \left[\mu^2 + \frac{M'm}{a^2} + \frac{\gamma m^2}{a^3} - \frac{12(\alpha \frac{m}{a} + M)^2 \alpha m}{a^2 \lambda_1 \lambda_2} \right]^{\frac{1}{2}} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}. \tag{3.26}$$

so that this model gives the correct order of VEV's if $m, M, M' \sim M_{GUT}$. There is no renormalizable mixing term allowed by the discrete symmetry that could destroy the accidental $SU(6) \times SU(6)$ symmetry.

But the problem is that it is not sufficient to consider only renormalizable operators. Nonrenormalizable operators scaled by inverse power of M_{Pl} can potentially introduce large breaking of the global $SU(6) \times SU(6)$ symmetry, which in turn yields large contributions to the PGB masses. In particular, in the above example the term $\frac{1}{M_{Pl}} \bar{H} \Sigma_1 \Sigma_2 H$ is allowed and gives an unacceptably big correction to the PGB masses if present. Namely, the Higgs doublets would acquire masses $\sim M_{GUT}^2/M_{Pl} \approx 10^{13}$ GeV.

3.2.2 Requirements and constraints for the superpotential

We have seen in the previous section that even if mixing terms of the renormalizable superpotential are forbidden by some discrete symmetries, the possible nonrenormalizable operators can still break the accidental global symmetry and thus spoil the solution to the doublet triplet splitting problem.

The origin of the nonrenormalizable operators can be of two forms: they either come from integrating out heavy ($\mathcal{O}(M_{Pl})$) particles from tree level diagrams or they can be a consequence of nonperturbative effects.

The dangerous mixing terms coming from integrating out the heavy fields can be easily forbidden by some additional requirements on the Planck scale particles, for example by requiring that all the Planck mass fields are matter (fermion) fields. In this case the nonrenormalizable terms arising from integrating out the heavy fields can only yield Yukawa terms. But we know that Yukawa terms are irrelevant from the point of view of PGB masses (the accidental global symmetry is a symmetry of the Higgs sector only). This assumption on the heavy fields is usually fulfilled by the interaction terms introduced in models for light fermion masses (e.g. [32, 33]). In those models we want to generate exactly additional Yukawa terms suppressed by Planck masses. Thus matter parity can be used to forbid all dangerous nonrenormalizable mixing terms arising from tree diagrams. Loop diagrams are naturally proportional to supersymmetry breaking.

Even if the above assumption for the superpotential involving heavy fields is valid, there is still the possibility of Planck mass suppressed operators in the superpotential which violate the global symmetry. Although the nonrenormalization theorem prevents these operators from being generated perturbatively if they were not present at tree level, we will take the attitude that all operators consistent with the low energy gauge and discrete symmetries are present, both in the Kähler potential and in the superpotential. We ask the question whether it is possible with this assumption to still maintain an approximate global symmetry which can guarantee that the Higgs doublet is sufficiently light.

The first observation is that the Kähler potential will always permit symmetry breaking terms, suppressed only by two powers of M_{Pl}, for example:

$$\frac{1}{M_{Pl}^2} H^\dagger \Sigma^\dagger \Sigma H. \tag{3.27}$$

There is no symmetry which can prevent such a term. However, although such terms do break the accidental global symmetry, they do not lead to generation of a mass term for the PGB's.

However, the PGB mass terms will be generated if the global symmetry is broken in the superpotential. In the remainder of this section, we show that it is extremely difficult to prevent mixing in the superpotential.

We now summarize the requirements for the superpotential of a realistic model.

1. The mass terms for the PGB's (which are identified with the Higgs doublets of the MSSM) resulting from the symmetry breaking mixing terms should be suppressed at least by a factor of 10^{-13} compared to the GUT scale. In this case the masses of the PGB's will

be at the order of 1000 GeV.

2. The VEV's of the fields Σ and $\bar{H}, H$ should be naturally (without tuning) at the order of the GUT scale (10^{16} GeV).

3. The triplets contained in the Σ field should have GUT-scale masses not to cause too large proton decay. One might think that the same requirement holds for the triplets contained in the fields $\bar{H}, H$. However these triplets are eaten by the heavy SU(6) gauge bosons and are not dangerous for proton decay.

There are two approaches one could imagine to prevent mixing through nonrenormalizable operators. One might try to find a representation of $SU(6)$ which breaks $SU(6)$ to $SU(4) \times SU(2) \times U(1)$ but does not allow mixing. Alternatively, one can search for more restrictive discrete symmetries.

We have found no solution with alternative representations. It is also very difficult to realize the second solution if we try to use M_{Pl} as the only mass scale in the theory. We show that one either needs to introduce small mass scales into the theory or to use fields that have zero VEV's to overcome all constraints listed in the following subsections.

In the next two subsections we consider the above two possibilities for model building. We show that the above requirements necessarily lead us to consider the kind of models presented in the next section [34].

Alternative representations

Let us first consider the possibility of achieving the desired symmetry breaking pattern with alternative representations of $SU(6)$. One can consider symmetric, antisymmetric, or mixed representations. We don't want to replace the the $H, \bar{H}$ fields because the $\bar{H}$ field is capable of splitting the light fermions from the heavy ones through the renormalizable operator $15\bar{H}\bar{6}$, see [32], so we only consider replacing the Σ field. If the representation is symmetric, one does not achieve the desired symmetry breaking pattern. An antisymmetric representation (for example a 15 looks promising) can achieve a good symmetry breaking pattern since it can break $SU(6)$ to $SU(4) \times SU(2)$. Thus with an additional $U(1)$ gauge group $SU(6) \times U(1)$ could break to $SU(4) \times SU(2) \times U(1)$ (much like the flipped $SU(5)$ model of [21]). Furthermore, it looks naively as if it can forbid undesired mixing terms such as $15\bar{6}\bar{6}$ because of the antisymmetry of 15_{ij} . However, in order to cancel anomalies, one must introduce additional fields, either $\bar{15}$ or $\bar{6}+\bar{6}'$. But this addition makes mixing already possible through 15 $\bar{15}\bar{H}H$ or in the other case through $15\bar{H}\bar{6}$. Larger representations do not help because we require a representation that is capable to break $SU(6)$ to $SU(4) \times SU(2) \times U(1)$.

Discrete symmetries

The next possibility is to look for more restrictive discrete symmetries. Throughout this subsection we will assume that the only mass scale present in the theory is M_{Pl} and that all fields have VEV's of the order of the GUT scale. It turns out that under these assumptions even with additional fields, it is extremely difficult to find a satisfactory superpotential with no unnaturally small parameter. We first summarize the reasons why it is difficult to find a satisfactory potential without fine tuning. We subsequently elaborate and illustrate each point in more detail. To be explicit, we assume all fields in the Higgs sector have VEV's of order $10^{-3}M_{Pl}$, the lowest possible value, in order to obtain the maximum suppression in higher dimension mixing operators. This ratio might in fact be larger; one would then need to suppress mixing operators still further. For this value, we require that the mixing term is at least of dimension four greater than the terms in the superpotential which respect the symmetry and generate the VEV's for the Σ and $H, \bar{H}$ fields. The PGB masses will then be at most $(M_{GUT}/M_{Pl})^4 M_{GUT} \sim 1000$ GeV.

It is easy to see that with just the fields $\Sigma, H, \bar{H}$ we can not obtain a successful superpotential. The reason for this is that in order to get nonzero VEV's for the fields we need to have at least two terms in both sectors of the accidental global symmetry (one sector contains the adjoint Σ and breaks $SU(6)$ to $SU(4) \times SU(2) \times U(1)$ while the other H and $\bar{H}$ and breaks $SU(6)$ to $SU(5)$). Then the quotient of the two terms can always multiply a term in the other sector, thereby generating unwanted mixing. Explicitly, if there are terms in one sector of the form

$$\frac{1}{M_{Pl}^{a-3}}\mathrm{Tr}\Sigma^a + \frac{1}{M_{Pl}^{b-3}}\mathrm{Tr}\Sigma^b, \tag{3.28}$$

then $\mathrm{Tr}\Sigma^{b-a}$ transforms trivially under an abelian discrete symmetry (even if it is an R-type symmetry). The presence of the terms

$$\frac{1}{M_{Pl}^{2c-3}}(\bar{H}H)^c + \frac{1}{M_{Pl}^{2d-3}}(\bar{H}H)^d \tag{3.29}$$

in the other sector then means that terms such as

$$\frac{1}{M_{Pl}^{2c+b-a-3}}(\bar{H}\Sigma^{b-a}H)(\bar{H}H)^{c-1} \tag{3.30}$$

are allowed. The number $b-a$ cannot be arbitrarily big if the dimensionful fields have VEV's of order M_{GUT}. This is because in order to balance the two terms in equation 3.28, there must be a small coefficient of order ϵ_G^{b-a} where $\epsilon_G = M_{GUT}/M_{Pl} \approx 10^{-3}$. So in order for the mixing term to be suppressed by 10^{-13}, the mixing must be suppressed by at least ϵ_G^4. But then $b-a \geq 4$ and there must be a small parameter in the potential of order ϵ_G^4, which is badly fine tuned.

So we have established that one requires additional fields, that there must be at least two operators in each of the two nonmixing sectors (one involving the H and $\bar{H}$ fields and one involving only the Σ field), and that the quotient of operators in the superpotential from the same sector must involve negative powers of at least one field, so that such symmetry invariants are not holomorphic functions of the fields.

The next point is that in order to prevent fine tuning, the superpotential should contain operators of similar dimension. The argument which we just gave without additional singlets can readily be generalized (if the singlet VEV is of the same order as those of other fields) to show that in order to prevent fine tuning, the dimension of the operators in the potential which are balanced at the minimum should have comparable dimension. Furthermore, a term of dimension d will yield a mass term for the non PGB's of the order $M \approx M_{GUT}\epsilon_G^{d-3}$. A very high dimension operator without a large coefficient will yield masses for the triplet fields much less than M_{GUT}. Thus according to our requirement 3 the terms containing the Σ field should have low dimensions so that the triplets contained in Σ have sufficiently large masses.

Of course, one can consider cases where not all VEV's are the same, but then VEV's are larger than M_{GUT} and mixing terms will be less suppressed.

We can generalize the above argument about the superpotential containing only the fields $\Sigma, \bar{H}, H$ to the case when the superpotential also includes an additional $SU(6)$ singlet. To have nonzero VEV's for the fields we need at least two terms that contain $\bar{H}H$ and two that contain Σ in the superpotential, while all these four terms may contain the $SU(6)$ singlet field S. Thus generally the superpotential will have the form (if there is no mixing of the two sectors)

$$(\bar{H}H)^a S^b + (\bar{H}H)^c S^d + \mathrm{Tr}\Sigma^e S^f + \mathrm{Tr}\Sigma^g S^h. \tag{3.31}$$

Without loss of generality we can assume that $d > b$, $f > h$ and $d > f$. If $f > h$ we require that $g > e$; otherwise the operator $\mathrm{Tr}\Sigma^{e-g}S^{f-h}$ (which is just the quotient of the last two terms and thus invariant under all discrete symmetries) would be holomorphic and could multiply either term of the Σ sector to give a non-suppressed mixing term. But because $g > e$, the operator $\Sigma^{g-e}S^{h-f+d}(\bar{H}H)^c$ is holomorphic. This operator is allowed by the discrete symmetries, because it is the product of two terms of the superpotential divided by a third term. Therefore the dimension of the allowed mixing term is equal to the dimension of one of the terms originally present in the superpotential plus the difference of the dimension of two terms present in the superpotential. (It is easy to see that this is also true for the case $e > g$.) Thus the necessary fine tuning is equal to the suppression factor of the mixing term. If we want to suppress mixing by ϵ_G^4 we will need fine tuning of the same order (to balance terms of different dimensions). To illustrate this argument we present a model where although mixing terms are suppressed sufficiently we need unreasonably large fine tuning to get the correct VEV's. In this model the superpotential is given by

$$\begin{aligned} & W(\Sigma, H, \bar{H}, S) = \frac{\alpha}{M_{Pl}^4} S^5 \bar{H}H + \beta S \mathrm{Tr}\Sigma^2 + \\ & \frac{\gamma}{M_{Pl}^3}(\bar{H}H)^3 + \frac{\delta}{M_{Pl}^4}\mathrm{Tr}\Sigma^7, \end{aligned} \tag{3.32}$$

where the discrete charges for the fields $\Sigma, \bar{H}H, S$ are $Q_\Sigma = \frac{25}{61}, Q_{\bar{H}H} = \frac{38}{61}, Q_S = \frac{3}{61}$ and the R-charge of the superpotential is $\frac{53}{61}$. (The transformation of the fields under the discrete symmetry is given by $\Phi \to e^{2\pi i Q}\Phi$.) Then the first allowed mixing term is $\bar{H}\Sigma^5 H S^4$, suppressed by 4 dimensions compared to $\mathrm{Tr}\Sigma^7$ or $S^5\bar{H}H$. The equations of motion for this theory are

$$\begin{aligned} & \frac{5\alpha S^4(\bar{H}H)}{M_{Pl}^4} + \beta \mathrm{Tr}\Sigma^2 = 0 \\ & \frac{\alpha S^5}{M_{Pl}} + 3\gamma(\bar{H}H)^2 = 0 \\ & 2\beta S\Sigma + \frac{7\delta}{M_{Pl}^4}(\Sigma^6 - \frac{1}{6}\mathrm{Tr}\Sigma^6) = 0. \end{aligned} \tag{3.33}$$

If the VEV of Σ has the form

$$\langle \Sigma \rangle = V \begin{pmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & 1 & & \\ & & & & -2 & \\ & & & & & -2 \end{pmatrix} \tag{3.34}$$

then the solution for V is

$$V = M_{Pl}\left[\left(\frac{-3\gamma}{\alpha}\right)\left(\frac{12\beta}{5\alpha}\right)^2\left(\frac{2\beta}{147\delta}\right)^{13}\right]^{\frac{1}{61}}. \tag{3.35}$$

The number multiplying M_{Pl} should be 10^{-3}, so even if we assume that this is the 13/61st power of a combination of the parameters this combination must be $(10^{-3})^{\frac{61}{13}} \sim 10^{-12}$. Thus we can see explicitly in this model that the amount of fine tuning (10^{-12}) is equal to the suppression factor of the dangerous mixing terms.

One might think that we can overcome this problem by introducing even more fields into the theory. If we could find a superpotential where the number of terms contained in the superpotential is equal to the number of fields in the superpotential we could assign arbitrarily different R-charges to the fields in the superpotential and thus forbid mixing terms. However this is not possible. The reason is the following: suppose we have n fields and n polynomial terms in the superpotential. Let's call these terms $A_i, i = 1,\ldots$n, where A_i is a polynomial of the fields $\Phi_k, k = 1,\ldots$n. The superpotential is then

$$W(\Phi_i) = \sum_{k=1}^{n} \alpha_k A_k(\Phi_i). \tag{3.36}$$

The equations of motion are $\frac{\partial W}{\partial \Phi_i} = 0$. If neither of the VEV's is zero then we can also write these equations in the form $\Phi_i \frac{\partial W}{\partial \Phi_i} = 0, i = 1,\ldots$n. Thus we get a system of equations

$$\sum_{k=1}^{n} \beta_{ik}\bar{A}_k = 0, \quad i = 1,\ldots \text{n}, \tag{3.37}$$

where $\bar{A}_k = A_k(\langle\Phi_j\rangle)$. This is a set of n linear homogeneous equations for the terms $\bar{A}_k$. There are two possibilities: the determinant of the coefficients β_{ik} is either zero or nonzero. To have it zero requires fine tuning of the parameters in the superpotential and even then we can not have all VEV's determined by the superpotential because the equations are linearly dependent so there are in fact fewer equations than n. If the determinant is nonzero then the only possibility is to have $\bar{A}_k = 0$ for $k = 1,\ldots$n. This implies that at least one of the VEV's is zero contrary to our assumption.

Thus we need at least n+1 terms in the superpotential to have the VEV's of all fields determined of the correct size without fine tuning. But this means that we can not choose the R-charges of the fields arbitrarily. Generally these connections among the R-charges make it very difficult to find an acceptable superpotential that both determines the VEV's at the right scale without fine tuning and has the mixing terms sufficiently suppressed. In all cases we examined with only low dimensional operators for the Σ field in the superpotential we were either able to find allowed unsuppressed mixing terms or fine tuning was required to set the VEV's to the right scale.

3.3 Three models in which the supersymmetric Higgs particles are naturally pseudo-Goldstone bosons

We have shown in the preceding section that one cannot construct a model based on low energy discrete symmetries without a small parameter if neither of the VEV's of the fields of the Higgs sector is zero. However, low energy supersymmetry *must* contain a small parameter, namely the weak scale, or equivalently, the supersymmetry breaking scale. In the first subsection, we show how one can exploit this small parameter to generate models which naturally respect the accidental global $SU(6) \times SU(6)$ symmetry. Our models differ from the model in Ref. [31] in that we exploit the supersymmetry breaking scale, but we do *not* need to tune the parameters. We naturally balance small terms against each other.

In the second subsection we present a different class of models. These models contain fields with zero VEV's; thus the no-go arguments of the previous section are not valid here. These models include two mass parameters: all mass terms are proportional to the GUT-scale while the nonrenormalizable operators are suppressed by the Planck-scale. In the third model we assume the appearance of a dynamical scale (related to the GUT scale) but do not specify its origin.

These three models serve as existence proofs for models which implement the $SU(6) \times SU(6)$ symmetry. Based on the considerations of the previous section, we expect the simplest successful models will have features of one of the models presented below.

3.3.1 Model 1

In these models we give superpotentials which together with the soft breaking terms give the correct values of VEV's. This is similar to the model of Ref. [31] but there the superpotential contained only renormalizable terms. Consequently the soft breaking terms alone were not enough to set the VEV's to the right scale and additional fine tuning was required.

The essential observation is that the triplets from $H, \bar{H}$ are eaten by the heavy gauge bosons and thus we don't need $\mathcal{O}(M_{GUT})$ mass terms for these fields. The superpotential is given by

$$W(\Sigma, \bar{H}, H) = \frac{1}{2} M \mathrm{Tr}\Sigma^2 + \frac{1}{3}\lambda \mathrm{Tr}\Sigma^3 + \alpha \frac{(\bar{H}H)^n}{M_{Pl}^{2n-3}} \tag{3.38}$$

If one assigns a discrete Z_n symmetry under which $\bar{H}H \to e^{2\pi i/n}\bar{H}H$ and Σ is invariant then these terms are the lowest order allowed ones. In the supersymmetric limit

$$\langle\Sigma\rangle = \frac{M}{\lambda} \begin{pmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & 1 & & \\ & & & & -2 & \\ & & & & & -2 \end{pmatrix}, \quad \langle H\rangle = \langle\bar{H}\rangle = 0. \tag{3.39}$$

The scalar potential (including the soft breaking terms) will have the form:

$$\begin{aligned} V(\Sigma, \bar{H}, H) = {} & \mathrm{Tr}|M\Sigma + \lambda\Sigma^2 - \frac{1}{6}\lambda \mathrm{Tr}\Sigma^2|^2 + \frac{n^2\alpha^2}{M_{Pl}^{4n-6}}(\bar{H}H)^{2n-2}(|H|^2 + |\bar{H}|^2) + \\ & Am\lambda \mathrm{Tr}\Sigma^3 + A' m\alpha \frac{(\bar{H}H)^n}{M_{Pl}^{2n-3}} + BMm\Sigma^2 + m^2(\mathrm{Tr}\Sigma^2 + |H|^2 + |\bar{H}|^2) + \mathrm{D-terms} \end{aligned} \tag{3.40}$$

where m is a mass parameter of the order of the weak scale while A, A', B are dimensionless parameters. The D-terms have to vanish not to have supersymmetry breaking in the visible sector. The soft breaking terms shift the Σ VEV only by a small ($\sim m$) amount. However for the $\bar{H}, H$ terms we have the possibility of a new minimum appearing due to the soft breaking terms. To find this we minimize the $\bar{H}, H$ part of the potential (using $\langle H \rangle = \langle \bar{H} \rangle = U(1,0,0,0,0,0)$ which is a consequence of the vanishing of the D-terms).

$$V(U) = \frac{2n^2|\alpha|^2}{M_{Pl}^{4n-6}} U^{4n-2} + \frac{A' m \alpha}{M_{Pl}^{2n-3}} U^{2n} + 2m^2 U^2. \tag{3.41}$$

Minimizing this potential we will get for one of the minima

$$U = \left[\frac{1}{2n^2 \alpha (4n-2)} (-nA' + \sqrt{n^2 (A')^2 - 16n^2(2n-1)}) M_{Pl}^{2n-3} \mu \right]^{\frac{1}{2n-2}} \tag{3.42}$$

The magnitude of the VEV is determined by the factor

$$\left[M_{Pl}^{2n-3} \mu \right]^{\frac{1}{2n-2}} = \left(\frac{\mu}{M_{Pl}} \right)^{\frac{1}{2n-2}} M_{Pl} \tag{3.43}$$

For $n < 4$ we get a smaller scale than M_{GUT} which is not acceptable. However for $n \geq 4$ the resulting scale always lies between the GUT scale and the Planck scale. (For $n = 4, 5, 6$ we get $U \approx 1.5\ 10^{16}, 7\ 10^{16}, 2\ 10^{17}$ GeV.) Thus all these cases yield naturally the correct values of the $H, \bar{H}$ VEV's. The first mixing term allowed by the Z_n symmetry is $\frac{1}{M_{Pl}^{2n-2}} (\bar{H} H)^{n-1} (\bar{H} \Sigma H)$. However the resulting mass for the PGB's is

$$U \left(\frac{U}{M_{Pl}} \right)^{2n-3} = \mu \left(\frac{\mu}{M_{Pl}} \right)^{2n-2} < \mu. \tag{3.44}$$

This means that all models with $n \geq 4$ yield an acceptable theory with the correct order of VEV's and naturally suppressed mixing terms. The possibility that the $H, \bar{H}$ VEV's are between the GUT and the Planck scale may even be welcome from the point of view of fermion masses (see Ref. [32]), and $\langle H \rangle > M_{GUT}$ is also required for the unification of couplings.

We can not use this method for getting GUT-scale VEV's for the sector containing the field Σ because then the triplets of Σ would get too small masses and would spoil the proton stability. This is however not the case for the $\bar{H}, H$ fields because the triplets from $\bar{H}, H$ are eaten by the $SU(6)$ gauge bosons. Fortunately it suffices to use this method for only one of the sectors because then mixing terms are already sufficiently suppressed. This leads us to the choice of the operator $(\bar{H} H)^n / M_{Pl}^{2n-3}$, while we have no restriction for the other sector. Alternatively we could use for example the superpotential

$$M \mathrm{Tr} \Sigma_1 \Sigma_2 + \frac{1}{3} \lambda_1 \mathrm{Tr} \Sigma_1^3 + \frac{1}{3} \lambda_2 \mathrm{Tr} \Sigma_2^3 + \frac{\alpha}{M_{Pl}^{2n-3}} (\bar{H} H)^n \tag{3.45}$$

with the additional Z_3 discrete charges $Q_{\Sigma_1} = 1/3$, $Q_{\Sigma_2} = -1/3$, $Q_{\bar{H}H} = 0$ (similar to the model of Ref. [33]). In this case the mixing term is even more suppressed by the additional discrete symmetry. The lowest order mixing term in this case is

$$(\bar{H} \Sigma_1 \Sigma_2 H)(\bar{H} H)^{n-1} / M_{Pl}^{2n-1}.$$

This model will be used in Section 3.3.5, when we extend it to incorporate fermion masses.

	Σ_1	Σ_2	A	B	T	S	N	$\bar{H}H$
$Z_3^{(1)}$	$\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{1}{3}$	0	0	0	0
$Z_3^{(2)}$	0	0	0	0	$\frac{1}{3}$	$-\frac{1}{3}$	0	$\frac{1}{6}$
Z_2	0	0	0	0	0	0	0	$\frac{1}{2}$
R	$\frac{1}{9}$	$\frac{2}{9}$	$-\frac{1}{9}$	$\frac{4}{9}$	$\frac{5}{9}$	$\frac{1}{9}$	$\frac{2}{3}$	$\frac{1}{9}$

Table 3.1: The discrete charge assignments of the fields of the Higgs sector of Model 2.

3.3.2 Model 2

In this class of models we will use low dimension operators to get the VEV's of the adjoint sector and then use two singlets with zero VEV's to communicate the required values of the VEV's to the $H, \bar{H}$ fields. The adjoint sector consists of two adjoint fields Σ_1, Σ_2 and two $SU(6)$ singlets A, B, while we introduce additional singlets (N, T, S) to get the desired VEV's for $H, \bar{H}$. We use a $Z_3^{(1)} \times Z_3^{(2)} \times Z_2 \times R$ symmetry, where R is a discrete R-symmetry with the charge of the superpotential being $\frac{1}{3}$. The discrete charge assignments of the fields are given in Table 3.1.

The lowest order superpotential allowed by the discrete and gauge symmetries is

$$\begin{aligned}&M\mathrm{Tr}\Sigma_1\Sigma_2 + a\mathrm{Tr}\Sigma_1^3 + b\mathrm{Tr}\Sigma_2^2 A + M'AB + cB^3 + \\&\frac{\alpha}{M_{Pl}} N(\mathrm{Tr}\Sigma_2^3 + \beta T^3) + \frac{\gamma}{M_{Pl}^2} S(T^4 - \delta(\bar{H}H)^2).\end{aligned} \tag{3.46}$$

The VEV's are

$$\begin{aligned}V_1 &= \left(\frac{M^6 M'^3}{3^8 2^5 a^5 b^3 c}\right)^{\frac{1}{9}} \approx [M^6 M'^3]^{\frac{1}{9}} \\V_2 &= \frac{3a}{M} V_1^2 \approx [M^3 M'^6]^{\frac{1}{9}} \\\langle B \rangle &= -\frac{108ba^2}{M^2 M'} V_1^4 \approx [M^6 M'^3]^{\frac{1}{9}} \\\langle A \rangle &= -\frac{3c}{M'} \langle B \rangle^2 \approx [M^3 M'^6]^{\frac{1}{9}} \\\langle T \rangle &= (\frac{12}{\beta})^{\frac{1}{3}} V_2 \approx [M^3 M'^6]^{\frac{1}{9}} \\\langle H \rangle &= \langle \bar{H} \rangle = \frac{\langle T \rangle}{\delta^{\frac{1}{4}}} \approx [M^3 M'^6]^{\frac{1}{9}} \\\langle S \rangle &= \langle N \rangle = 0\end{aligned} \tag{3.47}$$

where V_1 and V_2 are defined by

$$\langle\Sigma_1\rangle = V_1 \begin{pmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & 1 & & \\ & & & & -2 & \\ & & & & & -2 \end{pmatrix}, \quad \langle\Sigma_2\rangle = V_2 \begin{pmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & 1 & & \\ & & & & -2 & \\ & & & & & -2 \end{pmatrix} \tag{3.48}$$

If $M, M' \approx M_{GUT}$ then all fields (with the exception of N and S) have $\mathcal{O}(M_{GUT})$ VEV's. The lowest possible mixing term in the superpotential is $(\bar{H}\Sigma_1 H)(\bar{H}H)AS$ which yields a supersymmetric mass term (so called μ-term) for PGB Higgs doublets $\mu \sim (10^{16}/2\ 10^{19})^4\ 10^{16} \approx 1000$ GeV. One can see that the dangerous mixing term is quite big (compared to the lowest order mixing term of Model 1). One might need some additional suppression factor but no large fine tuning. The feature of this model that there are symmetry breaking terms that yield extra μ-terms for the Higgs doublets (which may also arise in the models presented in the previous subsection) solve a potential problem of these models. Namely, if there are no explicit symmetry breaking terms in the Higgs sector then the 'genuine GB's' will remain exactly massless at the GUT scale even after adding the soft breaking terms. This results in a potential instability of the Higgs potential (a flat direction for $h_1 = h_2^*$), which has to be removed by radiative corrections (essentially due to the large top Yukawa coupling). Explicit global symmetry breaking terms in these models lift this flat direction and remove the instability. However, these symmetry breaking terms at the same time invalidate the specific prediction of the 'Higgs as PGB' scheme (the μ-term is not related to soft SUSY breaking mass term anymore), and we will be left with the general Higgs potential of the MSSM.

In the above model all fields (except S and N) had the same order of VEV's, thus there is no hierarchy between the $H, \bar{H}$ and Σ VEV's. However such a hierarchy may be an attractive feature for generating fermion masses and is also necessary for the unification of couplings. This can be easily achieved in this model by modifying the discrete charges of $\bar{H}H$. We take the $Z_3^{(1)} \times Z_3^{(2)} \times Z_4 \times R$ charges for the $\bar{H}H$ as 0, $\frac{1}{12}$, $\frac{1}{4}$, $\frac{1}{18}$ instead of the charges listed in Table 3.1 (and all other charges are unchanged). Then the only change will be that instead of $S(\bar{H}H)^2$ we have $S(\bar{H}H)^4$ appearing in the superpotential. This will result in an H VEV that is the geometric mean value of M_{Pl} and M_{GUT}, which is desirable for fermion masses. The mixing terms again yield $\mathcal{O}(1000$ GeV) PGB masses.

3.3.3 Model 3

In the third model we assume that some $SU(6)$ singlet fields have VEV's of the order of the GUT scale through some unspecified dynamics.

One possibility to suppress mixing terms is to have at least two fields whose VEV's are naturally zero in the supersymmetric limit and whose presence is required in all dangerous mixing terms. In this case the mixing terms have the form $ST(\bar{H}H)^a\Sigma^b$ where S, T are the fields with vanishing VEV's. Then these mixing terms do not contribute to the Higgs masses because of $\langle S\rangle = \langle T\rangle = 0$ (if we add the soft breaking terms, $\langle S, T\rangle$ will be of $\mathcal{O}(M_{weak})$, so the contribution to the Higgs masses will be also suppressed by a factor of M_{weak}/M_{GUT} which is exactly what we need). Thus such fields with vanishing VEV's can yield the desired suppression of the mixing terms.

One such an example could be a superpotential of the form

$$aS(\bar{H}H - \alpha M^2) + bT(\mathrm{Tr}\Sigma^3 - \beta N^3). \tag{3.49}$$

(We take a cubic term in Σ because the trace of Σ vanishes and a quadratic term would give $SU(35)$ accidental symmetry. N and M should be singlets with respect to the $SU(6)$ group so that their VEV doesn't break the symmetry further and also to avoid a larger accidental symmetry.)

The equation of motion for the S, T singlets sets

$$\begin{aligned} \langle \bar{H}H \rangle &= \alpha \langle M^2 \rangle \\ \mathrm{Tr}\langle \Sigma^3 \rangle &= \beta \langle N^3 \rangle, \end{aligned} \tag{3.50}$$

while the S and T VEV's vanish because of the other equations of motion.

The problem with this model is that the VEV's of the fields M, N and consequently of $\Sigma, H, \bar{H}$ are not determined. To find an acceptable theory based on the superpotential of Eq. 3.49 we need to reintroduce the GUT-scale into our theory by setting the M and N VEV's to the GUT-scale by hand. The origin of this new scale in the theory could be for example a condensation scale of a strongly interacting gauge group (other than the SU(6)). We assume that for some reason the fields M, N acquire VEV's of $\mathcal{O}(M_{GUT})$. Then these VEV's can be communicated to the fields $\Sigma, H, \bar{H}$ without introducing mixing terms through the Eqs. 3.50. (In other words we could say that an effective tadpole term in the superpotential for the fields S and T is generated by integrating out heavy fields that have VEV's of the order of the GUT scale which spontaneously break the discrete symmetry.) But even if we set the M, N VEV's to the desired value the $H, \bar{H}, \Sigma$ VEV's are still not totally determined. This is done by the D-terms and the soft breaking terms. The D-terms vanish if $\langle H \rangle = \langle \bar{H} \rangle$ and $\langle \Sigma \rangle$ is diagonal. Now adding the soft breaking terms will shift the values of the VEV's by terms of the order of the weak scale and also lifts the very high degeneracy of the Σ vacua. Eq. 3.50 fixes only $\mathrm{Tr}\Sigma^3$. After we add the soft breaking terms the only possible Σ vacua are those which break $SU(6)$ to $SU(n) \times SU(6-n) \times U(1)$ depending on the values of the parameters of the soft breaking terms.

To forbid the direct mixing terms (those without the fields S, T) we should set the discrete charges of $\Sigma, H\bar{H}$ to be small, so that the mixing terms require high powers of these fields. If the discrete symmetry is not an R-type then by choosing the charges of $\Sigma, H\bar{H}$ the charges of the other fields are already determined. For example if we take the charges as $Q_S = \frac{15}{16}$, $Q_{\bar{H}H} = Q_{M^2} = \frac{1}{16}$, $Q_T = \frac{20}{21}$ and $Q_{\Sigma^3} = Q_{N^3} = \frac{1}{21}$, the first mixing term without S, T is $(\bar{H}H)^{16}\Sigma^{63}$, or $\bar{H}H, \Sigma$ exchanged to M^2 or N, while the mixing terms involving S, T are automatically suppressed by a factor $(\frac{M_{weak}}{M_{GUT}})$. (These mixing terms are just the products of the operators in the two sectors.)

We can go further and forbid even the mixing terms that include S, T by promoting the discrete symmetry to an R-symmetry. For example if we assign the R-charge for the fields $Q_S = \frac{1}{55}$, $Q_{\bar{H}H} = \frac{2}{165} = Q_{N^2}$, $Q_T = \frac{1}{44}$, $Q_\Sigma = Q_M = \frac{1}{396}$, and that of the superpotential is $\frac{1}{33}$ then all mixing is forbidden to more than 50 orders.

3.3.4 Summary

In this section we have presented three different type of models that all yield acceptable theories. We have shown how to circumvent the difficulties of section 3 and build natural theories with sufficiently suppressed mixing terms.

One might think that the above arguments for building a superpotential are true only for the $SU(6)$ model we have considered. However, it can be shown that alternative models based on the idea of two noninteracting sectors and either an $SU(n)$, $SO(n)$, or E_6 gauge group which do not have additional light doublet or triplet fields are trivial generalizations of the model we have considered, and therefore yield no more compelling solutions [34].

3.3.5 Fermion masses

Next we show that it is possible to extend the successful picture of fermion masses of Refs. [32, 33] with the discrete charges in accordance with a realistic Higgs sector. For the Higgs sector we will use the superpotential presented in Section 3.3.1, extending the discrete symmetry to the fermion sector. We will see a model that is consistent both in the Higgs sector and in the fermion sector. This serves as an existence proof for realistic models implementing the idea of having the Higgs fields as pseudo-Goldstone bosons.

First we briefly review the model of Refs. [32, 33] for fermion masses in the $SU(6)$ model. The minimal anomaly free fermion content of $SU(6)$ that includes one generation of light fermions is

$$15 + \bar{6} + \bar{6}', \tag{3.51}$$

where 15 is the two index antisymmetric representation and $\bar{6}$ is the conjugate of the defining representation. One can add any self adjoint representation and maintain anomaly cancelation. Generally self adjoint representations have invariant mass terms so it is no use adding them to the fermion content. But there are some special cases when this mass term vanishes. For example if we add just one representation 20 of $SU(6)$ (three index antisymmetric representation) then the mass term for this vanishes by antisymmetry (in general if we have odd number of 20's one of them will have a vanishing mass). We remark that the addition of a 20 to the usual particle content of the theory destroys asymptotic freedom of the $SU(6)$ gauge coupling. But this is not a problem since with only one 20 the increase of the coupling is very slow, its value increases only a few percent between the GUT and the Planck scale.

The idea of [32] is to add the extra 20 to the fermion content which will be then

$$(15 + \bar{6} + \bar{6}')_i + 20, \quad i = 1, 2, 3 \tag{3.52}$$

The SU(5) decomposition of these fields is

$$\begin{aligned} 20 &= 10 + \bar{10}, \\ 15 &= 10 + 5, \\ \bar{6} &= \bar{5} + 1. \end{aligned} \tag{3.53}$$

Then the renormalizable Yukawa couplings have the form

$$\lambda^{(1)} 20 \Sigma 20 + \lambda^{(2)} 20\ H\ 15_i + \lambda^{(3)}_{ij} 15_i \bar{H} \bar{6}'_j, \quad i, j = 1, 2, 3 \tag{3.54}$$

(i, j denote generation indices). If we insert the VEV's of $H, \bar{H}, \Sigma$ and the Higgs doublets into Σ (if $\langle H \rangle \gg \langle \Sigma \rangle$, the Higgs doublets live almost entirely in the Σ field, see Eqs. 3.19, 3.20) we get the following mass terms:

$$\lambda^{(2)} \langle H \rangle 10_i \bar{10} + \lambda^{(3)}_{ij} \langle H \rangle 5_i \bar{5}'_j + \lambda^{(1)} Q\ u^c\ h_2, \quad i, j = 1, 2, 3, \tag{3.55}$$

where the decomposition of 10 of $SU(5)$ is $Q + u^c + e^c$. The fermion fields in (3.52) contain altogether four 10's, six $\bar{5}$'s, three 5's and one $\bar{10}$ of $SU(5)$. From (3.55) we see that out of these fields one combination of 10's, three of $\bar{5}$'s and the three 5's and the $\bar{10}$ will get masses of $\mathcal{O}(M_{GUT})$, so the light fermion spectrum is the desired

$$3 \times (10 + \bar{5}), \tag{3.56}$$

while only one light fermion (namely, the up type quark contained in 20) gets a mass from the renormalizable interaction with the Higgs doublet. The reason is that the couplings of the 20-plet explicitly violate the global $SU(6)_\Sigma \times SU(6)_H$ symmetry, so that the Higgs doublet h_2 has non-vanishing coupling to the up type quark from 20, which can be identified with the top quark. Thus, the top mass is naturally in the 100 GeV range. Other fermions stay massless at this level, unless we invoke the higher order operators explicitly violating the accidental global symmetry.

To go further we need to introduce nonrenormalizable operators to give masses to the other fermions. Generally, these operators explicitly violate the accidental global symmetry, since they include both the Σ and $H, \bar{H}$ fields, so that they can provide nonvanishing Yukawa couplings to the Higgs doublets, though suppressed by M_{Pl}. In Refs. [32, 33] these operators were obtained from heavy fermion exchange [36]. For this purpose a specific set of heavy (Planck-scale) vectorlike fermion superfields in nontrivial SU(6) representations was introduced whose couplings with the light fermions and the Higgs superfields yielded the needed structure of nonrenormalizable operators (together with flavor-blind discrete Z_2 [32] or Z_3 [33] symmetries to forbid some unwanted operators). For example, in Ref. [32] the relevant nonrenormalizable operators coming from the specified heavy fermion superfield exchanges were specified by

$$\begin{aligned}
&\frac{1}{M_{Pl}}(20\Sigma)H\,15_i,\ i = 1,2,3, \\
&\frac{1}{M_{Pl}^2}20(\bar{H}\Sigma\bar{H})\bar{6}_3 \\
&\frac{1}{M_{Pl}^2}15_i(\Sigma\bar{H})(\Sigma\bar{6}_j), i,j = 2,3.
\end{aligned} \tag{3.57}$$

The first operator gives mass to the c quark, the second to the b and τ, and the third to the s and μ. These masses will have a proper hierarchy provided that $\langle H \rangle >> \langle \Sigma \rangle$. In the model of ref.[32] the first generation fermions were left massless, however in the model of Ref. [33] they can also get masses of the needed value.

However in our approach all nonrenormalizable operators that are not forbidden by some symmetry are present in the superpotential. In other words, we would like to obtain all masses in general operator analysis, not relying on heavy fermion exchange mechanism [36] with specified fields. Therefore a 'flavor democratic' approach to fermion masses (which means that there are no 'family symmetries' that would distinguish among the generations) is out of question: it would yield too heavy first generation masses. Thus we will need to use family symmetries in constructing the fermion mass terms. The simplest way is just to extend the discrete symmetries used for the stabile picture in the Higgs sector also to the fermion sector.

For the demonstration, we will use the Model 1 presented in Section 3.3.1, which is based on a $Z_3 \times Z_n^{(1)} \times Z_n^{(2)}$ discrete symmetry. The discrete charge assignments are given

	H	$\bar{H}$	Σ_1	Σ_2	20	15_3	$15_{2,1}$	$\bar{6}_{3,1}$	$\bar{6}_2$	$\bar{6}'_3$	$\bar{6}'_{2,1}$
$Z_4^{(1)}$	$\frac{1}{n}$	0	0	0	0	$-\frac{1}{n}$	$-\frac{1}{n}$	0	$\frac{1}{n}$	$\frac{1}{n}$	$\frac{1}{n}$
$Z_4^{(2)}$	0	$\frac{1}{n}$	0	0	0	0	0	$-\frac{2}{n}$	$-\frac{1}{n}$	$-\frac{1}{n}$	$-\frac{1}{n}$
Z_3	0	0	$\frac{1}{3}$	$-\frac{1}{3}$	$\frac{1}{3}$	$-\frac{1}{3}$	0	0	0	$\frac{1}{3}$	0

Table 3.2: The charge assignments of the chiral superfields under the discrete $Z_n^{(1)} \times Z_n^{(2)} \times Z_3$ symmetry.

in Table 3.2. The Higgs sector is given by the superpotential of Eq. 3.45. Because we have now two adjoint fields in one of the sectors of the accidental symmetry the h_1, h_2 Higgs doublet fields live in a linear combination of them. One can show that the uneaten PGB doublets are given by

$$h_1 = \cos\alpha(\cos\gamma\, h_{\Sigma_1} + \sin\gamma\, h_{\Sigma_2}) - \sin\alpha\, h_H$$
$$h_2 = \cos\alpha(\cos\gamma\, \bar{h}_{\Sigma_1} + \sin\gamma\, \bar{h}_{\Sigma_2}) - \sin\alpha\, \bar{h}_{\bar{H}} \tag{3.58}$$

where $\tan\gamma = V_2/V_1$ and $\tan\alpha = 3V/U$. Here $\langle H\rangle = \langle\bar{H}\rangle = U$, $\langle\Sigma_{1,2}\rangle = V_{1,2}$, and $V = (V_1^2 + V_2^2)^{1/2}$.

If $V_1 \sim V_2 \ll U$, as it occurs e.g. for $n = 6$, then the Higgs doublets are dominantly contained in Σ_1 and Σ_2 while almost not contained in H and $\bar{H}$.

The allowed Yukawa couplings in this model together with their physical role are listed below.

- $15_i\bar{H}\bar{6}'_i$: makes the extra $\bar{5}$'s and 5's heavy.
- $20H15_3$: makes the extra 10 and $\bar{10}$ heavy.
- $20\Sigma_1 20$: yields heavy top quark.
- $\frac{1}{M^2}20\Sigma_2\bar{H}\bar{H}\bar{6}_3$: defines $\bar{6}_3$ state and yields b and τ masses.
- $\frac{1}{M^2}20\Sigma_2 H 15_2$: defines 15_2 state and yields charm mass via c-t mixing.
- $\frac{1}{M^2}15_{2,1}\Sigma_1\Sigma_2\bar{H}\bar{6}_2$: gives s, μ masses and Cabbibo mixing.
- $\frac{1}{M^4}15_{2,1}\Sigma_1\Sigma_2\bar{H}(\bar{H}H)\bar{6}_{3,1}$: gives d,e masses and 1-3 mixing.
- $\frac{1}{M^2}15_{2,1}H\Sigma_1\Sigma_2 H 15_{2,1}$: gives u mass.

M denotes the suppression scale of the nonrenormalizable operators. We denote $\epsilon_H = \frac{\langle H\rangle}{M}$, $\epsilon = \frac{\langle\Sigma\rangle}{\langle H\rangle}$. As we noted before, for $n = 6$ $\langle H\rangle$ is at an intermediate scale scale between $\langle\Sigma\rangle$ and M_{Pl}. Since $\langle\Sigma\rangle = M_{GUT} \simeq 10^{16}$ GeV is fixed (as the $SU(5)$ scale) by the gauge coupling unification, we obtain $\epsilon_H \sim \epsilon \sim 1/30$, which can explain the fermion mass hierarchy. A somewhat better fit can be obtained with the slightly larger value $\epsilon_H \sim 0.1$. This could occur if the above listed operators are generated by heavy fermion exchange (the heavy particles should be below the Planck scale, with masses $M \sim 10^{18}$ GeV). The desired value of the parameter ϵ should remain $\sim 1/30$ which fits perfectly to the light generation fermion masses.

The physical consequences of the above listed operators can be summarized as follows (λ denotes the Yukawa couplings of the MSSM, while θ_{ij} denotes the mixing angle of the i'th and j'th generation):

- $\lambda_t \sim 1$
- $\lambda_{b,\tau} \sim \epsilon_H^2$, $\lambda_b = \lambda_\tau$.
- $\lambda_c \sim \epsilon_H^2$

- $\sin\theta_{23} = \sqrt{\frac{\lambda_c}{\lambda_t}} \sim \epsilon_H$.

- $\lambda_{s,\mu} \sim \epsilon\epsilon_H^2$, but the ratio λ_s/λ_μ is not fixed (because there is more than one operator due to the different possible contractions of indices in the operator $15_{2,1}\Sigma_1\Sigma_2\bar{H}\bar{6}_2$).

- $\lambda_{d,e} \sim \epsilon\epsilon_H^4$, but the ratio of the couplings is not fixed again. The ratio λ_e/λ_μ is of order $\epsilon_H^2 \sim \lambda_c/\lambda_t$. However λ_d is a somewhat small.

- $\lambda_u \sim \epsilon\epsilon_H^3$. The ratio λ_u/λ_c is of order $\epsilon\epsilon_H \sim \lambda_\mu/\lambda_\tau\sqrt{\lambda_c/\lambda_t}$.

- $\sin\theta_{12} \sim \mathcal{O}(1)$ (Cabbibo angle).

- $\sin\theta_{13} \sim \frac{\lambda_d}{\lambda_b}$.

All these consequences (except the down mass which must be enhanced by introducing a large Clebsch coefficient) are in qualitative agreement with the experimental values, provided that $\tan\beta$ is small (close to 1). In fact, here we used the general operator analysis consistent with the gauge $SU(6)$ and discrete symmetries. By addressing the specific heavy fermion exchanges, one could also fix the relative Clebsch factors between the down quark and charged lepton masses [32, 33]. Thus we have shown that a consistent model based on the $SU(6)$ gauge group and discrete symmetries can be constructed. In this model the accidental global $SU(6) \times SU(6)$ symmetry is preserved by nonrenormalizable operators in the Higgs superpotential up to sufficiently high order terms, so that the Higgs doublets are PGB's without any fine tuning. On the other hand, Yukawa terms explicitly violating the $SU(6) \times SU(6)$ symmetry yield the necessary Yukawa couplings for the light fermion masses.

3.4 Phenomenological constraints on the Higgs as pseudo-Goldstone boson mechanism

In this section we investigate the restrictions that the assumption that the μ-term is generated purely by the Higgs as PGB mechanism places on the low energy physics. Because of an additional constraint on the Higgs sector $\tan\beta$ is not an independent parameter. We impose this constraint and check whether a suitable standard model minimum exists. This will result in an equation for $\tan\beta$ which is given in Eq. 3.85.

We solve this equation numerically and give the resulting range of $\tan\beta$. The parameter range where this equation can be satisfied is also displayed.

Thus we will show how our assumptions on the GUT physics together with our present knowledge of weak scale phenomenology constrain the parameters of weak scale supersymmetry. This way we could get information on the GUT scale physics if the MSSM parameters are ultimately measured in future colliders.

3.4.1 The μ-term from the Higgs as PGB scheme

In the Higgs as PGB solution to the doublet-triplet splitting problem one assumes that there is an additional global symmetry which, after spontaneous breaking, ensures the lightness of the Higgs particles [27]. For example, in the originally proposed model the $SU(5)$ gauge group is enlarged to an $SU(6)$ global symmetry containing the gauged $SU(5)$. While the $SU(6)$ breaks to $SU(4) \times SU(2) \times U(1)$, the gauged $SU(5)$ breaks to the SM group $SU(3) \times SU(2) \times U(1)$, leaving one pair of $SU(2)$ doublets as uneaten Goldstone bosons [27, 28].

However only one of the scalars in the chiral superfields is a genuine Goldstone boson; the other scalar is massless only on account of supersymmetry. Thus it is not surprising that the soft breaking terms (which do preserve the extra global symmetry) generate mass terms for these non-Goldstone boson light fields and a supersymmetric μ-term for the Higgs fields.

In this section we will explicitly demonstrate how this mechanism works in the above mentioned $SU(5)$ model with $SU(6)$ global symmetry. Although this model is not the most esthetic in that it requires tuning the $SU(5)$ couplings, the simplicity of this model makes it a good example to illustrate the generic features of the Higgs as PGB mechanism. We will also see in Section 3.4.4 that this model can be an effective theory for some energy range for the more realistic model of Refs. [30, 31, 32, 33, 34]. All the results in this section apply more generally as has been shown in Ref. [27].

The Higgs sector of this model consists of one adjoint of $SU(6)$, denoted by Φ,

$$\Phi = 24 + 5 + \bar{5} + 1 = \Sigma + H + \bar{H} + S \tag{3.59}$$

under $SU(5)$. The explicit realization of this decomposition is given in the following way:

$$\Phi = \begin{pmatrix} -\frac{5}{\sqrt{30}}S & H \\ \bar{H} & \Sigma + \frac{1}{\sqrt{30}}S \end{pmatrix}. \tag{3.60}$$

The superpotential is given by

$$W(\Phi) = \frac{1}{2}M\mathrm{Tr}\Phi^2 + \frac{1}{3}\lambda\mathrm{Tr}\Phi^3. \tag{3.61}$$

The VEV is given by

$$\langle\Phi\rangle = \frac{M}{\lambda}\begin{pmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & 1 & & \\ & & & & -2 & \\ & & & & & -2 \end{pmatrix}, \tag{3.62}$$

which breaks the gauged $SU(5)$ to $SU(3)\times SU(2)\times U(1)$, and leaves one pair of Goldstone bosons uneaten, which can be identified as the $SU(2)$ doublets in H and $\bar{H}$. The scalar potential after the inclusion of the soft breaking terms is given by

$$V(\Phi) = \mathrm{Tr}\left|M\Phi + \lambda(\Phi^2 - \frac{1}{6}\mathrm{Tr}\Phi^2)\right|^2 + \left(A_\Phi\lambda\frac{1}{3}\mathrm{Tr}\Phi^3 + B_\Phi M\frac{1}{2}\mathrm{Tr}\Phi^2 + h.c.\right) + m_0^2\mathrm{Tr}|\Phi|^2. \tag{3.63}$$

Now the Φ VEV is shifted to

$$\langle\Phi\rangle = \frac{1}{\lambda}\left[M + (A_\Phi - B_\Phi) + \frac{1}{M}(3A_\Phi B_\Phi - A_\Phi^2 - 2B_\Phi^2 - m_0^2) + \mathcal{O}\left(\frac{1}{M^2}\right)\right]. \tag{3.64}$$

After substituting $\langle\Phi\rangle$ into the potential one finds a mass term for the doublets in $H, \bar{H}$ (denoted by h_u, h_d)

$$\left|\frac{h_u - h_d^\dagger}{\sqrt{2}}\right|^2 \left[2m_0^2 + 2(A_\Phi - B_\Phi)^2\right], \tag{3.65}$$

while at the same time the shift in the Φ VEV generates a mass term for the higgsinos $\tilde{h}_u$

and $\tilde{h_d}$ of the form

$$(A_\Phi - B_\Phi)\tilde{h_u}^c\tilde{h_d} + h.c.. \tag{3.66}$$

Thus, as it was shown in general in Ref. [27], there is a supersymmetric μ-term generated by this mechanism, which is usually a model dependent function of the soft breaking parameters (in the above described model $\mu = A_\Phi - B_\Phi$). The other important lesson from this example is that the combination $1/\sqrt{2}(h_u + h_d^\dagger)$ is the genuine Goldstone boson with no mass term at the GUT scale even after inclusion of the soft breaking terms [27]. Such a mass term is generated only by the explicit symmetry breaking loop corrections.

Thus, the general conclusion is that the Higgs as PGB mechanism generates the following mass term for the MSSM Higgs fields at the GUT scale:

$$V(h_u, h_d)|_{M_{GUT}} = (m_0^2 + \mu^2)|h_u - h_d^\dagger|^2 + \text{ D-terms}, \tag{3.67}$$

where m_0 is the soft breaking mass parameter introduced in Eq. 3.63, while μ is a function of all soft breaking parameters.

The general Higgs potential in the MSSM is given by

$$V(h_u, h_d)|_\Lambda = m_1^2(\Lambda)h_d^\dagger h_d + m_2^2(\Lambda)h_u^\dagger h_u + m_3^2(\Lambda)(h_u h_d + h.c.) + \text{ D-terms} \tag{3.68}$$

where we have explicitly displayed the scale (Λ) dependence of the parameters and where

$$\begin{aligned} m_1^2(\Lambda) &= m_d^2(\Lambda) + \mu^2(\Lambda), \\ m_2^2(\Lambda) &= m_u^2(\Lambda) + \mu^2(\Lambda), \\ m_3^2(\Lambda) &= B(\Lambda)\mu(\Lambda), \end{aligned} \tag{3.69}$$

m_u^2, m_d^2 are the running soft breaking mass terms for the up- and down-type Higgses, $\mu(\Lambda)$ is the running μ-parameter, while $B(\Lambda)$ is the running soft breaking parameter corresponding to the μ-term. From Eq. 3.67 one can see that the specific μ-term generated by the Higgs as PGB mechanism requires that the boundary condition

$$m_d^2(M_G) + \mu^2(M_G) = m_u^2(M_G) + \mu^2(M_G) = -B(M_G)\mu(M_G) \tag{3.70}$$

is satisfied. If one assumes universal soft breaking terms at the GUT-scale, then the first of the equations is automatically satisfied, and we have one additional constraint equation.

As noted already in Refs. [27, 28, 37, 31], Eq. 3.70 means that the number of free parameters in the MSSM is reduced by one. However we have not seen in any of the previous analysis an explicit determination of the results of this additional restriction.

Already the authors of the first papers on this subject noted the importance of Eq. 3.70, and analyzed its consequences. However in these papers [28, 37] a very specific form of the soft breaking terms was assumed and therefore the consequences were not sufficiently general. The authors of Ref. [31] also provide an analysis of the constraint of Eq. 3.70. However their method is not described in sufficient detail for us to compare the results.

3.4.2 Analysis of the constraint arising from the Higgs as PGB mechanism

As we saw in the previous section, the specific form of the μ-term generated by the Higgs as PGB mechanism implies the following constraint on the running mass parameters:

$$m_d^2(M) + \mu^2(M) = m_u^2(M) + \mu^2(M) = -B(M)\mu(M) \tag{3.71}$$

where M is the scale where the additional global symmetry which gives rise to the light Higgses is broken. We make the following assumptions:

A. The μ-term is generated purely by the Higgs as PGB mechanism implying the constraint 3.71.

B. The form of the constraint 3.71 remains valid at the GUT-scale.[2]

Usually the number of free parameters in the MSSM (assuming universal soft breaking terms at the GUT scale and gaugino unification) is 5+1, where the 5+1 are:

1. m_0 - the universal soft breaking mass term,
2. A_0 - the trilinear soft breaking term,
3. $M_{1/2}$ - the gaugino mass,
4. λ_t - the top Yukawa coupling,
5. $\tan\beta$ - the ratio of Higgs VEV's,

and the extra parameter is the sign of the μ parameter.

We can see that this set does not contain either μ or B, since they are determined (at the weak scale) from the requirement of electroweak symmetry breaking (see e.g. [5]):

$$\begin{aligned} \mu^2(M_Z) &= \frac{\bar{m}_d^2 - \bar{m}_u^2 \tan^2\beta}{\tan^2\beta - 1} - \frac{1}{2}M_Z^2, \\ B(M_Z)\mu(M_Z) &= \frac{1}{2}\sin 2\beta(\bar{m}_1^2 + \bar{m}_2^2), \end{aligned} \tag{3.72}$$

where $\bar{m}_d^2, \bar{m}_u^2, \bar{m}_1^2, \bar{m}_2^2$ are the 1-loop corrected values of the above defined soft breaking mass parameters evaluated at the weak scale.

This means that the constraint Eq. 3.71 will determine one additional parameter of the 5, but in a nontrivial way. In our analysis we choose $m_0, A_0, M_{1/2}, \lambda_t$ and the sign of μ to be the independent parameters and $\tan\beta$ will be evaluated in the following way: given the 4 input parameters, one can calculate the soft breaking mass terms at the weak scale through RG running. Thus, we will have an expression for $\mu^2(M_Z)$ and $B(M_Z)$ as a function of $\tan\beta$ for every set of input parameters. Then we scale these expressions back to the GUT-scale and require that Eq. 3.71 is satisfied. This will yield an equation for $\tan\beta$. In our analysis we use the one loop RGE's for the MSSM, retaining only the top Yukawa coupling and the gauge couplings. In this case the RGE's can be solved analytically with the exception of one function, where numerical integration is necessary. The approximate analytical expressions are [38, 39]:

$$m_u^2 = m_0^2 + 0.52M_{1/2}^2 + \Delta m^2$$

[2]Our results do not change significantly when allowing running between M_P and M_G. See the end of Section 3.4.4.

$$m_d^2 = m_0^2 + 0.52M_{1/2}^2$$
$$\mu^2(M_Z) = 2\mu_0^2\left(1-\frac{Y_t}{Y_f}\right)^{1/2}$$
$$B(M_Z) = B_0 - \frac{A_0}{2}\frac{Y_t}{Y_f} + M_{1/2}\left(1.2\frac{Y_t}{Y_f} - 0.6\right), \tag{3.73}$$

where

$$\Delta m^2 = -\frac{3}{2}m_0^2\frac{Y_t}{Y_f} + 2.3A_0M_{1/2}\frac{Y_t}{Y_f}\left(1-\frac{Y_t}{Y_f}\right) - \frac{A_0^2}{2}\frac{Y_t}{Y_f}\left(1-\frac{Y_t}{Y_f}\right) + M_{1/2}^2\left[-7\frac{Y_t}{Y_f} + 3\left(\frac{Y_t}{Y_f}\right)^2\right], \tag{3.74}$$

$$Y_t = \lambda_t^2/4\pi, \tag{3.75}$$

$$Y_t = \frac{2\pi Y_t(M_G)E(t)}{2\pi + 3Y_t(M_G)F(t)}, \tag{3.76}$$

$$E(t) = (1+\beta_3 t)^{\frac{16}{3b_3}}(1+\beta_2 t)^{\frac{3}{b_2}}(1+\beta_1 t)^{\frac{13}{9b_1}}, \tag{3.77}$$

$$\beta_i = \alpha_i(M_G)b_i/4\pi, \tag{3.78}$$

$$t = \log(M_G/\Lambda)^2, \tag{3.79}$$

$$F(t) = \int_0^t E(t')dt', \tag{3.80}$$

$$Y_f = \frac{2\pi E(t)}{3F(t)}, \tag{3.81}$$

for $t = \log(M_G/M_Z)^2$, $M_G = 2 \cdot 10^{16}$ one has $E \simeq 14, F \simeq 293$ in the MSSM. Putting these together with the constraint Eq. 3.71 and with Eq. 3.72 for electroweak breaking one gets the following equation for $\tan\beta$ [40, 41]:

$$m_0^2 + \frac{1}{2(1-\frac{Y_t}{Y_f})^{1/2}}\left(-m_0^2 - 0.52M_{1/2}^2 - \Delta m^2\frac{\tan^2\beta}{\tan^2\beta - 1} - \frac{M_Z^2}{2}\right) =$$
$$-\frac{1}{\sqrt{2}(1-\frac{Y_t}{Y_f})^{1/4}}\left\{-\frac{\tan\beta}{1+\tan^2\beta}\left(\Delta m^2\frac{\tan^2\beta+1}{\tan^2\beta-1} + M_Z^2\right)\right.$$
$$\pm\left[\frac{A_0}{2}\frac{Y_t}{Y_f} - M_{1/2}\left(1.2\frac{Y_t}{Y_f} - 0.6\right)\right]\left(-m_0^2 - 0.52M_{1/2}^2\right.$$
$$\left.\left.-\Delta m^2\frac{\tan^2\beta}{\tan^2\beta-1} - \frac{M_Z^2}{2}\right)^{1/2}\right\}. \tag{3.82}$$

When solving this equation one has to be careful about the sign of $\tan\beta$. In the MSSM one can fix $\tan\beta$ to be positive, because for negative $\tan\beta$ one can redefine the phase of one of the Higgs fields to absorb a minus sign and thus changing the sign of the $B\mu$ term in Eq. 3.70. However in our case the constraint of Eq. 3.70 is not invariant under this phase redefinition. As a result Eq. 3.82 is not invariant under $\tan\beta \to -\tan\beta$, thus as opposed to the MSSM one loses generality by restricting to positive $\tan\beta$. Therefore we solve Eq. 3.82

separately for positive and negative $\tan\beta$.

The $\pm$ in Eq. 3.82 stands for the two possible signs of the μ parameter. Note however, that Eq. 3.82 does not yet include the corrections to m_u^2 and m_d^2 arising from the one loop corrections to the effective potential which are known to be significant and which should be incorporated. The expressions for these corrections are:

$$\Delta m_u^2 = \frac{\partial \Delta V}{\partial v_u^2},$$
$$\Delta m_d^2 = \frac{\partial \Delta V}{\partial v_d^2}, \tag{3.83}$$

and we retain only the top-stop loops for ΔV:

$$\Delta V = \frac{3}{16\pi^2}\left[\frac{1}{2}m_{\tilde{t}_1}^4\left(\log\left(\frac{m_{\tilde{t}_1}^2}{\Lambda^2}\right)-\frac{3}{2}\right)+\frac{1}{2}m_{\tilde{t}_2}^4\left(\log\left(\frac{m_{\tilde{t}_2}^2}{\Lambda^2}\right)-\frac{3}{2}\right)-m_t^4\left(\log\left(\frac{m_t^2}{\Lambda^2}\right)-\frac{3}{2}\right)\right], \tag{3.84}$$

where $m_{\tilde{t}_{1,2}}$ are the stop masses and m_t is the top mass. Eq. 3.82 is then modified to be

$$m_0^2+\frac{1}{2(1-\frac{Y_t}{Y_f})^{1/2}}\left(-m_0^2-0.52M_{1/2}^2-\Delta m^2\frac{\tan^2\beta}{\tan^2\beta-1}-\frac{M_Z^2}{2}+\frac{\Delta m_d^2}{\tan^2\beta-1}\right.$$
$$\left.-\frac{\Delta m_u^2\tan^2\beta}{\tan^2\beta-1}\right)=-\frac{1}{\sqrt{2}(1-\frac{Y_t}{Y_f})^{1/4}}\left\{-\frac{\tan\beta}{1+\tan^2\beta}\left[(\Delta m^2+\Delta m_u^2-\Delta m_d^2)\frac{\tan^2\beta+1}{\tan^2\beta-1}\right.\right.$$
$$\left.+M_Z^2\right]\pm\left[\frac{A_0}{2}\frac{Y_t}{Y_f}-M_{1/2}\left(1.2\frac{Y_t}{Y_f}-0.6\right)\right]\left(-m_0^2-0.52M_{1/2}^2-\Delta m^2\frac{\tan^2\beta}{\tan^2\beta-1}-\frac{M_Z^2}{2}+\right.$$
$$\left.\left.\frac{\Delta m_d^2}{\tan^2\beta-1}-\frac{\Delta m_u^2\tan^2\beta}{\tan^2\beta-1}\right)^{1/2}\right\}. \tag{3.85}$$

3.4.3 Results

We have shown in the previous section that the fact that the μ-term is generated by the Higgs as PGB mechanism implies Eq. 3.82 for $\tan\beta$.

We note that Eq. 3.82 is invariant under the transformation $\mu \to -\mu$, $M_{1/2} \to -M_{1/2}$ and $A_0 \to -A_0$. Therefore one can fix $\mu < 0$, and then the solutions corresponding to positive μ are obtained by taking $M_{1/2} \to -M_{1/2}$ and $A_0 \to -A_0$. Consequently we will show four plots; the first two corresponding to $\tan\beta > 0$ and both signs for $M_{1/2}$ (Figs. 3-2.c and 3-2.d). Subsequently we will give the two plots corresponding to $\tan\beta < 0$, with $M_{1/2} > 0$ in Fig. 3-3 and $M_{1/2} < 0$ in Fig. 3-4. In all these plots we will have $\mu < 0$ fixed. All $\mu > 0$ solutions can be obtained as described above.

The values of m_0 and $|M_{1/2}|$ are bounded from below in order to ensure that the sparticle masses obey the experimental limits [42]. Thus one has to combine the experimental lower bounds on the sparticle masses and the requirement that there is a solution to Eq. 3.85 to get the possible parameter range of the MSSM.

We solved Equation 3.85 numerically for the small $\tan\beta$ regime (we take $1 < \tan\beta < 15$; large $\tan\beta$ would require a fine tuning of order $1/\tan\beta$ in the Higgs sector [43] contrary to the spirit of not tuning parameters). In the following we summarize the main features of the solutions.

If one fixes $\tan\beta > 0$ then the only viable solutions fulfilling the experimental constraints correspond to

-large values of $|A_0|$

-relatively small values of m_0 and $|M_{1/2}|$.

This is illustrated in Fig. 3-2 where we display the allowed parameter space for a fixed value of λ_t and different fixed values of m_0. One gets the same type of plots for other fixed values of λ_t. Figs. 3-2. a and b are presented to ease the reading of the subsequent plots. They both display the $m_0 = 60$ case with detailed explanation of the allowed parameter space. Figs. 3-2. c and d are the same but for four different values of m_0.

However there is an important potential problem with these solutions: the existence of charge and/or color breaking (CCB) minima of the potential of the sleptons and squarks. We use the "traditional" condition for the absence of such minima [44] but we evaluate this condition not at the GUT scale but at the weak scale. This yields the following conditions for the weak scale parameters:

$$\begin{aligned}
&A_e^2 < 3(m_{\tilde{L}}^2 + m_{\tilde{e}}^2 + m_d^2)\\
&A_d^2 < 3(m_{\tilde{Q}}^2 + m_{\tilde{d}}^2 + m_d^2)\\
&A_u^2 < 3(m_{\tilde{Q}}^2 + m_{\tilde{u}}^2 + m_u^2),
\end{aligned} \tag{3.86}$$

where the A_e, A_d and A_u refer to the soft breaking trilinear terms of a given interaction (that is A_0 scaled down to the weak scale) and the m's refer to the soft breaking scalar mass terms also evaluated at the weak scale. We have calculated these parameters in the same one loop approximation (that is including only loops with gauge or top Yukawa couplings), and plotted the allowed region (that is the region where the inequalities 3.86 are satisfied). It is well known that these conditions are neither sufficient nor necessary to avoid the presence of CCB vacua [44]. The full analysis for the absence of these particular CCB vacua has been recently done in [45]. We have checked that these results are sufficiently close to those of the full analysis given in Ref. [45], with the results of Ref. [45] being always even more restrictive than those based on our analysis.

Furthermore even if a CCB vacuum exists that is a global minimum of the scalar potential one has to calculate the tunneling rate from the false MSSM vacuum to the real CCB vacuum for each such solution and only if that is large can one exclude a given point in the parameter space. Thus it is clear that Eq. 3.86 is not the full story. However it can be used as an approximate indicator for the presence of CCB vacua. If one is very far outside the allowed region allowed by 3.86 then that point on the parameter space can be safely excluded. If one is deep inside the allowed region CCB vacua are probably not dangerous.

In Fig. 3-2 we also display the CCB bounds obtained from Eqs. 3.86, along with the allowed parameter space. As one can see from Fig. 3-2, all these solutions to equation 3.85 lay outside the bounds of 3.86 and thus CCB poses a threat to the entire allowed parameter space. Therefore if we take these CCB bounds seriously we have to discard these solutions. This conclusion is not altered if we take different values of λ_t.

In this case we can conclude that one needs to take $\tan\beta < 0$. If we again fix $\mu < 0$ as before. We will get two type of solutions for this case. The solutions to the $M_{1/2} > 0$ case resemble very much the solutions of the previous case; that is one needs to have large $|A_0|$ and small m_0 and $M_{1/2}$. However now larger values of $M_{1/2}$ and m_0 are possible, but the overlap with the CCB allowed region is still very small as illustrated in Fig. 3-3. (There is

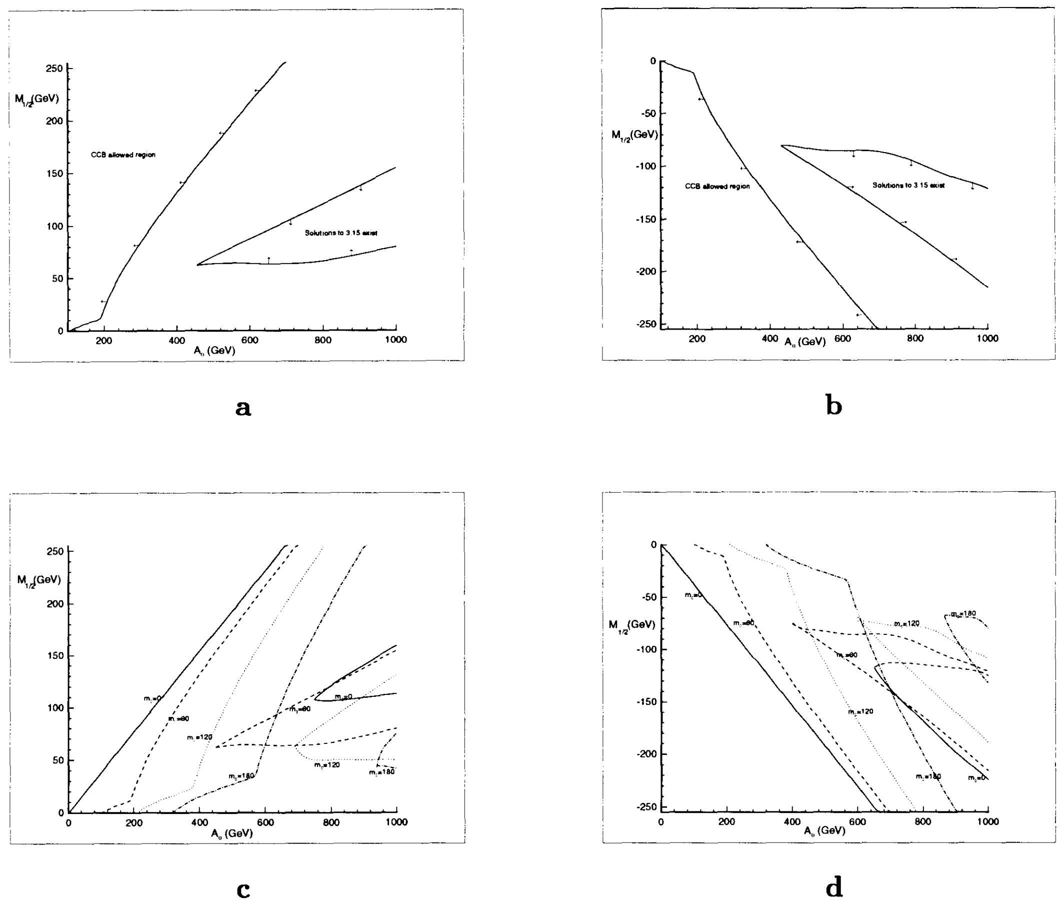


Figure 3-2: The allowed MSSM parameter space for a fixed value of λ_t ($\lambda_t = 1.2$) and for different fixed values of m_0 with $\tan\beta > 0$. **a**: $m_0 = 60$ GeV, $M_{1/2} > 0$. The curve on the right of the plot gives the region where Eq. 3.85 can be satisfied with the above mentioned parameters, while the curve on the left gives the region where CCB vacua are absent. **b**: the same as in a for $M_{1/2} < 0$. In both cases $\mu < 0$. As explained in the text the $\mu > 0$ solutions can be obtained by taking $M_{1/2} \to -M_{1/2}$ and $A_0 \to -A_0$ simultaneously. **c**: As in a for varying m_0. In both c and d the solid line corresponds to $m_0 = 0$ GeV, the dashed to $m_0 = 60$ GeV, the dotted to $m_0 = 120$ GeV and the dash-dotted to $m_0 = 180$ GeV. **d**: As in b for varying m_0.

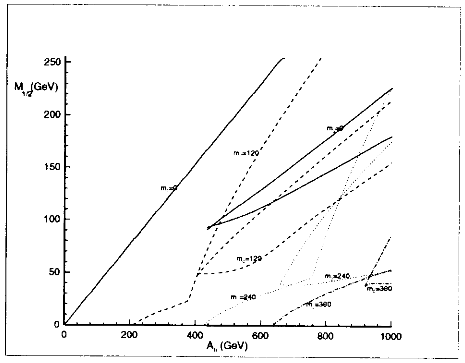


Figure 3-3: The same as Fig. 3-2 but for $\tan\beta < 0$ and positive $M_{1/2}$. The solid line corresponds to $m_0 = 0$ GeV, the dashed to $m_0 = 120$ GeV, dotted to $m_0 = 240$ GeV and the dash-dotted to $m_0 = 360$ GeV. One can see that the overlap of the solutions with the CCB allowed regions is very tiny.

still no overlap for small values of m_0's and tiny overlap for large values.)

The final possibility (negative $\tan\beta$, negative μ and negative $M_{1/2}$) is not so restrictive. In this case one does not get an upper bound on $|M_{1/2}|$ and m_0; instead one gets a lower bound on $|M_{1/2}|$ (which is however more constraining then the usual experimental bounds in the MSSM). One gets a large overlap with the CCB allowed region. This region of overlap is however much more restricted than the region of parameters allowed in the MSSM. This is illustrated in Fig. 3-4 for different values of m_0 and fixed λ_t. Figs. 3-4.a and 3-4.c are again presented to ease the reading of the other two plots, with detailed explanation of the allowed region. Note in Figs. 3-4.b and 3-4.d that for $m_0 = 500, 1000$ GeV one does not get any restriction from the CCB bounds if $|A_0| < 1000$ GeV. Also note that for large values of m_0 the MSSM bound on $M_{1/2}$ is independent of m_0. Different values of λ_t yield similar plots, with the curves somewhat shifted to the right (towards larger values of A_0). This is illustrated in Figs. 3-4.b and 3-4.d.

In summary we find that most of the possible solutions to Eq. 3.85 obeying the CCB bounds correspond to the $\tan\beta, M_{1/2}, \mu < 0$ case with the allowed parameter space displayed in Fig. 3-4.

Since $\tan\beta$ is not a free parameter of the theory it is not surprising that the range of values $\tan\beta$ can take on is much smaller than in the MSSM. There any low value of $\tan\beta$ not too close to 1 can be acceptable for fixed A_0 and m_0 if one varies $M_{1/2}$. In our case however $\tan\beta$ is the solution to Eq. 3.85 and thus will in general not take all values. This is illustrated in Fig. 3-5, where we display the allowed range of $\tan\beta$ for fixed values of λ_t and m_0, while $|M_{1/2}|$ is allowed to vary in the range of 0, 800 GeV. One can see that one finds a smaller region of acceptable vacua than the MSSM together with the experimental constraints would allow.

To conclude this section we summarize the consequences of our analysis. We have seen that the boundary condition 3.70 together with the requirement of an acceptable SM minimum will determine $\tan\beta$ from other input parameters. We have seen that this equation for $\tan\beta$ does not always have solutions which excludes some regions of the MSSM parameter space (which is now reduced to m_0, $M_{1/2}$, A_0, λ_t). These constraints displayed

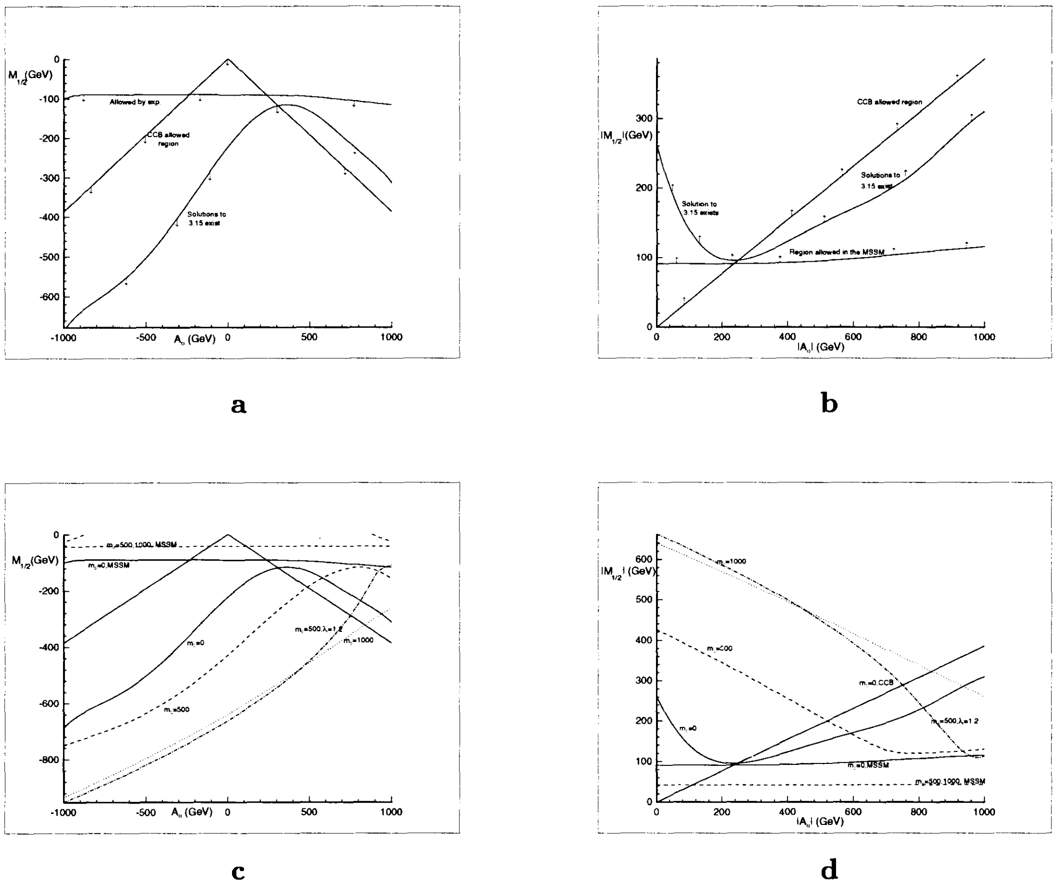
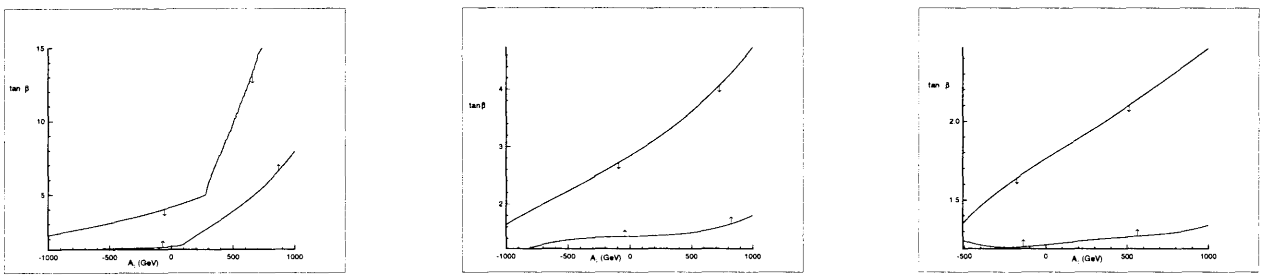


Figure 3-4: The allowed region of parameters for $\tan\beta < 0$ and negative $M_{1/2}$. $\lambda_t = 0.8$ for all four plots (except in b and d where explicitly stated) **a**: $m_0 = 0$ GeV. The upper straight line corresponds to the bound on $M_{1/2}$ in the MSSM, if the values of $\tan\beta$ are varied between 1 and 15. The Λ-shaped curve on the top of the plot gives the region allowed by the absence of CCB vacua, while the lower line corresponds to the bound on $M_{1/2}$ obtained from Eq. 3.85. **b**: The same as in a but for different values of m_0. The solid lines corresponds to $m_0 = 0$ GeV, the dashed lines to $m_0 = 500$ GeV and the dotted line to $m_0 = 1000$ GeV. Note that one does not get any restriction from the CCB bounds for $m_0 = 500, 1000$ GeV when $|A_0| < 1000$ GeV. Also note that for large values of m_0 the MSSM bound on $M_{1/2}$ is independent of m_0. The dash-dotted line corresponds to $m_0 = 500$ GeV, but with $\lambda_t = 1.2$. The increase in λ_t results in the shift of the curves towards larger values of A_0. **c**: The bounds on the absolute value of $M_{1/2}$ as a function of the absolute value of A_0 obtained from a and b. c gives the $m_0 = 0$ case, where the lower straight curve is the MSSM bound, the upper straight curve is the bound from CCB, while the third curve in the middle is the bound obtained by requiring that 3.85 has a viable solution. **d**: The same as in c, where $m_0 = 0$ GeV corresponds to the solid lines, $m_0 = 500$ GeV to the dashed lines and $m_0 = 1000$ GeV to the dotted line. The dash-dotted line corresponds to $m_0 = 500$ GeV but $\lambda_t = 1.2$.



a b c

Figure 3-5: a: The allowed region for $\tan\beta$ if we vary $|M_{1/2}|$ between 0 and 800 GeV. $m_0 = 0$ GeV and $\lambda_t = 0.8$ is fixed. b and c are the same as a with $m_0 = 500,\ 1000$ GeV.

in Figs. 3-4 and 3-5 are the main results of our analysis. If the μ-term is generated by the Higgs as PGB mechanism then the MSSM parameters must be inside the boundaries given in Figs. 3-4 and 3-5 or inside the tiny overlapping regions of Fig. 3-3.

Thus if one ultimately measures these parameters in colliders one can check whether they are indeed in the allowed region or not. If the MSSM parameters are all measured one can also check whether the experimental value of $\tan\beta$ does satisfy Eq. 3.82 or not, thereby testing the assumptions on the GUT-scale Higgs sector.

3.4.4 Implications for realistic models

We finally comment on the validity of our analysis for models implementing the Higgs as PGB scheme. Although the Higgs as pseudo-Goldstone boson idea is perhaps the most natural solution to the doublet-triplet splitting problem in the context of SUSY GUT's, it is difficult to build realistic models that implement this idea in a natural way (see Ref. [34]) without additional light charged particles which disrupt unification.

The only known realistic model is based on the SU(6) gauge group [30, 31, 32, 33, 34] and has an accidental $SU(6)\times SU(6)$ global symmetry of the Higgs sector. This symmetry is achieved by requiring that two sectors of the Higgs fields are are not mixed among each other in the superpotential. The models of Refs. [30, 31, 32, 33, 34] use the $SU(6)$ adjoint Σ and a pair of $SU(6)$ vectors $H, \bar{H}$ for the Higgs sector. Then the superpotential has the form

$$W(\Sigma, H, \bar{H}) = W_1(\Sigma) + W_2(H, \bar{H}) \tag{3.87}$$

up to dimension seven in the superpotential of the Higgs fields. This stringent requirement is necessary so that nonrenormalizable operators breaking the accidental global symmetry do not give too large a mass to the Higgs doublets. In Section 3.3 several suggestions for a superpotential implementing this idea have been presented. From the point of view of the μ-term we can divide them into two categories, according to whether a symmetry breaking term (that is a term that couples the Σ and $H, \bar{H}$ fields) containing seven Higgs sector fields is or is not allowed.[3] A dimension seven operator would give an additional contribution to the μ-term spoiling Eq. 3.85 without destroying the solution to the doublet-triplet splitting problem.

In Model 2 of Section 3.3 such a term is allowed by all symmetries of the Lagrangian and thus may yield a contribution to the μ-term of order 100-1000 GeV. Since the coefficient of this operator is a completely free parameter of the theory the constraint of Eq. 3.85 does not hold and our analysis may not be applied to this theory. Such a theory cannot be tested by the constraints described in this paper.

However, if no dimension seven mixing terms are allowed in the superpotential then there can be no significant extra contribution to the μ-term. This is the case in the simplest model, namely Model 1 of Section 3.3 and also in Model 3.

The superpotential of Model 1 is given by

$$\frac{1}{2}M\mathrm{Tr}\Sigma^2 + \frac{1}{3}\lambda\mathrm{Tr}\Sigma^3 + \frac{\alpha}{M_{Pl}^{2n-3}}(\bar{H}H)^n, \tag{3.88}$$

where $n = 4, 5, 6$. After the inclusion of the soft breaking terms one gets $\langle H\rangle \sim 10^{17}$ GeV $> M_{GUT}$, and at this scale $SU(6)$ is broken to $SU(5)$. If one neglects the small admixture of $H, \bar{H}$ fields in the Higgs doublets then at the $SU(5)$ scale $\langle H\rangle$ we have an $SU(5)$ gauge theory with an "accidental" global $SU(6)$ symmetry of the Higgs sector, since the theory originates from an $SU(6)$ gauge theory. Thus at the $\langle H\rangle$ scale we get as an effective theory exactly the model of Section 3.4.1 since the $SU(5)$ nonsinglet fields of H are eaten by the heavy gauge bosons. This means that the threshold corrections to Eq. 3.70 arising from the fact that the constraint is not generated at the GUT scale but at a somewhat higher scale can be estimated to be of the order

$$\frac{\lambda_t^2}{16\pi^2}\log\left(\frac{\langle H\rangle}{M_{GUT}}\right) \sim 0.01, \tag{3.89}$$

due to the running between the $\langle H\rangle$ and GUT scales. Thus the corrections in this model to Eq. 3.82 are expected to be a few percent and the results of our analysis should not be modified significantly. We have checked that corrections in Eq. 3.70 as large as 10 percent caused only a small shift in the constraint curves. Consequently there was still no overlap between the allowed parameter region and the region allowed by CCB for the $\tan\beta > 0$ case. Therefore the constraints obtained in this analysis should be robust.

[3]The models in Section 3.3 were especially designed such that no symmetry breaking terms containing only six or less Higgs sector fields are allowed, since these would give the doublet Higgses a mass of order $M_{GUT}(\frac{M_{GUT}}{M_{Pl}})^{-3} \sim 10^7$ GeV and thus spoil the solution to the doublet-triplet splitting problem.

Chapter 4

Confinement in $N = 1$ Supersymmetric Gauge Theories[1]

There has been a revolution in the understanding of the low-energy behavior of asymptotically free supersymmetric gauge theories in the past three years. This has been sparked by the work of Seiberg [46] who showed how to describe supersymmetric QCD in the far infrared. In this chapter we will first review Seiberg's results on supersymmetric QCD. Then we focus our attention on a particular case, the $N_f = N_c + 1$ theory. We will describe what the most important features of this theory are, and define a class of theories ("s-confining") which behave similarly. After giving two necessary conditions for "s-confinement", we describe all such theories with a single gauge group.

4.1 The low energy behavior of SUSY QCD

4.1.1 General features of asymptotically free SUSY gauge theories

In this section, we summarize the most important general features of the low-energy behavior of supersymmetric gauge theories. A general $N = 1$ SUSY gauge theory is given by the Lagrangian [9]

$$\mathcal{L} = \int d^4\theta \Phi^\dagger e^{gV} \Phi + \left[\int d^2\theta W(\Phi) + \frac{8\pi}{g^2} \int d^2\theta W_\alpha W^\alpha + h.c. \right],$$

where θ is the Grassmanian superspace coordinate, the Φ's are chiral superfields, the V is the vector superfield in the adjoint representation of the gauge group, g is the gauge coupling and $W_\alpha = -\frac{1}{4}\bar{D}\bar{D}e^{-gV}D_\alpha e^{gV}$ is the field strength chiral superfield. W is the superpotential which is a holomorphic function of the chiral superfields.

The first question about a theory is to ask what the possible vacua of the theory are. This can be inferred by looking for the minimum of the scalar potential of the theory, which in case of the above Lagrangian turns out to be

$$V = g^2 \sum_\alpha \left| \sum_i (\Phi_i)^\dagger_a (T^\alpha)^a_b \Phi^b_i \right|^2 + \sum_i \left| \frac{\partial W}{\partial \Phi_i} \right|^2,$$

[1]Based on research done in collaboration with Martin Schmaltz and Witold Skiba reported in Ref. [61].

where the first term is called the D-term while the second is called the F-term. Let us assume for a moment, that there is no tree-level superpotential in the theory, and ask what the possible vacua are. These are given by the solutions of the equations

$$\sum_i (\Phi_i)^\dagger_a (T^\alpha)^a_b \Phi^b_i = 0. \tag{4.1}$$

The directions in field space satisfying 4.1 are called D-flat directions, or classical moduli space. An important mathematical theorem relates the classical moduli space to gauge invariant operators [47]. More exactly, it states that the classical moduli space (up to gauge transformations) can be parameterized by the independent holomorphic gauge invariant operators X_r.

This has very important consequences for the low-energy behavior of the theory. According to this theorem the classical low-energy theory can be described in terms of the independent gauge-invariant operators X_r, since the potential vanishes along these directions,

$$V(X_r) = 0.$$

If there is in addition a tree-level superpotential, one can calculate from the equations of motion which of the D-flat directions are lifted.

However, this is only the classical description of the theory. What we would be more interested in is whether this simple picture is valid in the quantum theory as well. Indeed, in perturbation theory, the non-renormalization theorem [48] forbids any contributions to the superpotential, therefore the D-flat directions can not be lifted in perturbation theory. However, there might be non-perturbative effects violating the non-renormalization theorem, that give contributions to the superpotential. Such non-perturbative effects are the main interest of the next sections. In the following, we review the nature of these non-perturbative effects in SUSY QCD, following the discussion of Seiberg.

4.1.2 SUSY QCD for $N_f < N_c$: runaway superpotential

Supersymmetric QCD is an $SU(N_c)$ gauge theory with N_f flavors, i.e. N_f chiral superfields in the representation $N_f(\Box + \bar{\Box}) = Q^\alpha_i + \overline{Q}^j_\alpha$, where $i, j = 1, \ldots, N_f$ are flavor indices and $\alpha = 1, \ldots, N_c$ are gauge indices. The theory is asymptotically free only if $N_f < 3N_c$, thus this is the region of N_f we are interested in.

It turns out that the low-energy behavior of the theory is very different depending on the number of flavors N_f. First we look at the case $N_f < N_c$. We assume first that there is no tree-level superpotential in the theory. Then the theory has a large anomaly-free global symmetry given in the table below:

	$SU(N_c)$	$SU(N_f)$	$SU(N_f)$	$U(1)_B$	$U(1)_R$
Q	$\Box$	$\Box$	1	1	$\frac{N_f-N_c}{N_f}$
$\bar{Q}$	$\bar{\Box}$	1	$\Box$	-1	$\frac{N_f-N_c}{N_f}$

,

where $SU(N_c)$ is the gauge group, $SU(N_f) \times SU(N_f)$ are the non-abelian flavor symmetries, $U(1)_B$ is baryon number and $U(1)_R$ is the non-anomalous R-symmetry.

According to the previous section, we first have to form the gauge invariant operators, which parameterize the classical moduli space. For $N_f < N_c$, the independent gauge

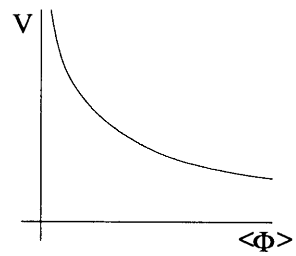


Figure 4-1: The runaway scalar potential resulting from the dynamically generated superpotential term for $N_f < N_c$.

invariant operators are the meson fields

$$M_i^j = Q_{\alpha i}\bar{Q}^{\alpha j}.$$

Their transformation properties under the global symmetries are

	$SU(N_f)$	$SU(N_f)$	$U(1)_B$	$U(1)_R$
M	$\square$	$\square$	0	$2\frac{N_f-N_c}{N_f}$

The superpotential generated by the non-perturbative effects must be invariant under the global symmetries of the theory. To obey the $SU(N_f) \times SU(N_f)$ global symmetry, a dynamical superpotential can only be a function of $\det M$. However, $\det M$ has R-charge $2(N_f - N_c)$. Therefore, global symmetries and holomorphy of the superpotential fix the only possible dynamically generated superpotential term to be

$$W_{dyn} = \left(\frac{\Lambda^{3N_c-N_f}}{\det M}\right)^{\frac{1}{N_c-N_f}}, \tag{4.2}$$

where Λ is the dynamical scale of the theory.

It has been shown by Affleck, Dine and Seiberg in 1984, that for $N_f = N_c - 1$ this superpotential term is indeed generated by instanton effects [49], and the coefficient of the superpotential term is one. If one knows that this term is there for $N_f = N_c - 1$, one can establish its existence for $N_f < N_c - 1$ as well, since one can just add a mass term for one flavor and integrate that flavor from the theory. One finds that the above term is there for any $N_f < N_c$. However the dynamical effect which produces this term for $N_f < N_c - 1$ is gaugino condensation in the unbroken gauge group instead of instantons [49].

Let us discuss what the physical meaning of this superpotential term is. If one calculates the resulting scalar potential, it is of the form given in Fig. 4-1. The scalar potential slopes to zero if the fields go to infinity, therefore the theory has no stable vacuum state, the vacuum exhibits runaway behavior. The dynamics of the theory wants to push the fields to larger and larger expectation values. Since the fields are forced to have large VEV's, the gauge group is broken before it could get strong, thus the theory is in the Higgs phase. We remark, that usually the runaway behavior can be stabilized at finite VEV by adding tree-level superpotential terms to the theory.

4.1.3 $N_f = N_c$: confinement with chiral symmetry breaking

The gauge invariants of this theory are the meson fields

$$M_i^j = Q_i^\alpha \bar{Q}_\alpha^j$$

and the baryon and antibaryon fields

$$B = \epsilon_{\alpha_1 \ldots \alpha_{N_c}} Q_{i_1}^{\alpha_1} \ldots Q_{i_{N_c}}^{\alpha_{N_c}} \epsilon^{i_1 \ldots i_{N_c}}$$

$$\bar{B} = \epsilon^{\alpha_1 \ldots \alpha_{N_c}} \bar{Q}_{\alpha_1}^{i_1} \ldots \bar{Q}_{\alpha_{N_c}}^{i_{N_c}} \epsilon_{i_1 \ldots i_{N_c}}.$$

However these fields are not independent, classically there is a constraint $\det M = B\bar{B}$ among these fields.

Following the same kind of reasoning as in the previous section one can show that there can be no superpotential term generated for this theory.

Seiberg argued convincingly [46], that the correct low-energy description is in terms of a confined theory. The confining degrees of freedom are the fields M, B and $\bar{B}$. However, they are not independent, but the classical constraint is modified by quantum effects to $\det M - B\bar{B} = \Lambda^{2N_c}$, where Λ is again the dynamical scale of the theory.

There are several independent checks on this simple picture advocated by Seiberg. First, a confining theory has to satisfy the 't Hooft anomaly matching conditions [50] for the unbroken global symmetries. One can show, that along the quantum modified constraint these conditions are indeed satisfied. Another check is that one can add a mass term for one flavor and integrate this flavor from the theory. The superpotential in the low-energy becomes

$$W = \lambda(\det M - B\bar{B} - \Lambda^{2N_c}) + m M_{N_f, N_f},$$

where λ is a Lagrange multiplier enforcing the quantum modified constraint. After taking the equations of motion with respect to the massive fields one finds that the superpotential of Eq. 4.2 is indeed reproduced.

Note, that the quantum modified constraint implies that some of the fields M, B and $\bar{B}$ necessarily have VEVs. Thus some of the chiral symmetries are broken in the ground state. Therefore the phase of the $N_f = N_c$ theory is confinement with chiral symmetry breaking.

4.1.4 $N_f = N_c + 1$: s-confinement

In this case, the gauge invariant operators are given by

$$M_j^i = Q_j^\alpha \bar{Q}_\alpha^i,$$

$$B^i = \epsilon_{\alpha_1 \ldots \alpha_{N_c}} Q_{i_1}^{\alpha_1} \ldots Q_{i_{N_c}}^{\alpha_{N_c}} \epsilon^{i, i_1 \ldots i_{N_c}},$$

$$\bar{B}_i = \epsilon^{\alpha_1 \ldots \alpha_{N_c}} \bar{Q}_{\alpha_1}^{i_1} \ldots \bar{Q}_{\alpha_{N_c}}^{i_{N_c}} \epsilon_{i, i_1 \ldots i_{N_c}}.$$

These fields obey the classical constraints

$$M_j^i B^j = M_j^i \bar{B}_i = 0, \ \det M (M^{-1})_j^i = B^i \bar{B}_j.$$

Seiberg argued [46], that the correct low-energy description of the theory is in terms of the confining variables M, B and $\bar{B}$. The superpotential terms allowed by the global symmetries and by the holomorphy of the superpotential are $\det M$ and $BM\bar{B}$. The relative coefficient of these terms in the superpotential are fixed by requiring that the theory reproduces the classical limit for large VEVs. This requires that there be a non-vanishing confining superpotential for the fields M, B and $\bar{B}$

$$W = \frac{1}{\Lambda^{2N_c-1}}(BM\bar{B} - \det M). \tag{4.3}$$

When taking the equations of motion with respect to the fields M, B and $\bar{B}$ from the superpotential of Eq. 4.3 the classical constraints are exactly reproduced.

Another check on Seiberg's picture is that the 't Hooft anomaly matching conditions are satisfied everywhere, including the origin of the moduli space. Finally, integrating out one flavor will reproduce the quantum modified constraint of the $N_f = N_c$ theory.

Thus the $N_f = N_c + 1$ theory is confining with a confining superpotential 4.3 and without chiral symmetry breaking. This will be used as a definition of s-confinement in the upcoming sections.

4.1.5 $N_f > N_c + 1$: duality (conformal and free magnetic phases)

Seiberg noted [46], that the $\frac{3}{2}N_c < N_f < 3N_c$ theories have a non-trivial infrared fixed point in their β-functions [51]. Thus he conjectured, that these theories are in the non-abelian Coulomb phase (conformal phase). He also found an equivalent description of the same low-energy physics in terms of a dual gauge group. Seiberg argued, that the theories

	$SU(N_c)$	$SU(N_f)$	$SU(N_f)$	$U(1)_B$	$U(1)_R$
Q	$\square$	$\square$	1	1	$\frac{N_f-N_c}{N_f}$
$\bar{Q}$	$\bar{\square}$	1	$\square$	-1	$\frac{N_f-N_c}{N_f}$

, $W = 0$

	$SU(N_f - N_c)$	$SU(N_f)$	$SU(N_f)$	$U(1)_B$	$U(1)_R$
q	$\square$	$\bar{\square}$	1	$\frac{N_c}{N_f-N_c}$	$\frac{N_c}{N_f}$
$\bar{q}$	$\bar{\square}$	1	$\bar{\square}$	$\frac{N_c}{N_f-N_c}$	$\frac{N_c}{N_f}$
M	1	$\square$	$\square$	0	$2\frac{N_f-N_c}{N_f}$

, $W = Mq\bar{q}$

have the same low-energy limit, and called this phenomenon $N = 1$ duality. This is a weaker version of the electric-magnetic duality conjectured by Olive and Montonen [52].

The consistency checks in favor of the duality are:

- The flat directions of the two theories are in one-to-one correspondence.
- The 't Hooft anomaly matching conditions are satisfied
- Integrating out flavors and higgsing the gauge group is consistent with duality.

Seiberg further noted, that the dual magnetic gauge group $SU(N_f - N_c)$ exists for $N_f < \frac{3}{2}N_c$ as well and is infrared free. He conjectured, that the duality is not only valid in the conformal phase but also in the regime $N_c + 1 < N_f < \frac{3}{2}N_c$, where the dual description is infrared free. He called this phase the free magnetic phase.

Shortly after Seiberg's original work several other $N = 1$ dualities have been discovered [53, 54, 55, 56, 58], and some other confining theories have been discovered as well [57, 59, 60, 61]. In the next sections, we will concentrate on the confining theories analogous to SUSY QCD with $N_f = N_c + 1$.

4.2 S-confinement

Following in Seiberg's footsteps, others have obtained results on a whole zoo of theories [62, 63, 64, 53, 54, 55, 57, 56, 58, 59, 60, 61]. Most of the discovered phenomena follow similar patterns to SUSY QCD, and one is tempted to ask if there is maybe a more general approach than the model-specific trial and error procedure that has been customary thus far.

Whereas a completely general approach that allows one to understand all the obtained results seems impossibly difficult to find, we can make much progress by focusing on the particular phenomenon of confinement. In fact, a frequently occuring and relatively easily identified infrared behavior is "s-confinement". We define an s-confining theory as a theory for which all the degrees of freedom in the infrared are gauge invariant composites of the fundamental fields. Furthermore, we demand that the infrared physics is described by a smooth effective theory in terms of these gauge invariants. This description should be valid everywhere on the moduli space of vacua, including the origin of field space. Finally, we also demand that an s-confining theory generates a dynamical superpotential. At the origin of moduli space all global symmetries of the theory are unbroken and the global anomalies of the microscopic theory are matched by the macroscopic gauge invariants of the effective theory.

The best-known example of a theory which has been conjectured to be s-confining is supersymmetric QCD (SQCD) with N colors and $F = N + 1$ flavors of fundamental and antifundamental matter, Q and $\bar{Q}$ [46, 65]. The gauge invariant confined degrees of freedom are mesons $M = Q\bar{Q}$ and baryons $B = Q^N$, $\bar{B} = \bar{Q}^N$. At the origin of moduli space, all components of the mesons and baryons are massless, and they interact via the confining superpotential

$$W = \frac{1}{\Lambda^{2N-1}}(\det M - BM\bar{B}). \tag{4.4}$$

This description is also valid far from the origin of the moduli space where the large expectation values of the fields completely break the gauge group. In such a vacuum the theory is in the Higgs phase. A smooth gauge invariant description of both the Higgs and confining vacua of the theory can only exist if there is no phase transition between the two regions in moduli space. In particular, there should be no gauge invariant order parameter that distinguishes the two phases.

To understand this in the example of SQCD, note that the quarks transform in a faithful representation of the gauge group $SU(N)$. This implies that arbitrary test charges can be screened by the dynamical quarks because the vacuum can disgorge quark-antiquark pairs to screen charges transforming in any representation of the gauge group. Thus a Wilson loop will always obey a perimeter law because any charges we might want to use to define the Wilson loop can be screened. Our definition of s-confinement above necessitates that an s-confining theory is in such a "screening-confining" phase.

This situation should be contrasted with $SU(N)$ with only adjoint matter or $SO(N)$ with vector matter. In both these cases the matter does not transform in a faithful representation of the gauge group. Now there are charges that cannot be screened by the dynamical quarks, and a Wilson loop can serve as gauge invariant order parameter to distinguish the Higgs and the confining phases. As a result, such theories cannot have a single smooth description of both the Higgs and confining phases of the theory, thus they are not s-confining.

In the next section, we discuss two necessary criteria for a theory being s-confining. In Section 4.4 we apply our conditions to identify all theories with a single gauge group and no tree-level superpotential which s-confine. We give a complete list of the confined spectra and superpotentials for all s-confining theories with an arbitrary SU, SO, Sp, or exceptional gauge group. Finally we comment on possible applications of our results on s-confining theories.

4.3 Necessary criteria for s-confinement

In this section we develop two necessary criteria which allow us to identify all s-confining theories with a simple gauge group and no tree-level superpotential [61]. The first criterion follows from holomorphy of the dynamically generated superpotential, which can be determined using the global symmetries of the theory. This criterion allows us to reduce the number of theories that are candidates for s-confinement to a manageable set. Our second criterion follows from explorations of regions in moduli space which are easier to understand than the origin. As will be demonstrated in Section 4.4, these two conditions combined are sufficient to identify all s-confining theories with a single gauge group and no tree-level superpotential.

4.3.1 The index constraint

In this subsection, we derive a simple constraint on the matter content of s-confining theories which follows from the requirement of holomorphy of the confining superpotential. In theories with a simple gauge group G and no tree-level superpotential, the symmetries are sufficient to determine the form of any dynamically generated superpotential completely [49]. A simple way to prove this makes use of non-anomalous R-symmetries. Define a $U(1)_R$ symmetry as follows: all chiral superfields, except for one arbitrarily chosen field ϕ_i, are assigned zero R-charge. The charge q of the remaining field is determined by requiring anomaly cancelation of the mixed $G^2U(1)_R$ anomaly

$$(q-1)\mu_i - \sum_{j\neq i}\mu_j + \mu_G = q\mu_i - \sum_{\text{all } j}\mu_j + \mu_G = 0, \tag{4.5}$$

where μ_i is the Dynkin index[2] of the gauge representation of the field ϕ_i, and $(q-1)$ is the R-charge of its fermion component. These three terms arise from the contributions of the fermion components of ϕ_i, of all other matter superfields ϕ_j with $j \neq i$, and of the gauge superfields, respectively. The μ_j are the indices of the remaining matter representations, they are multiplied by the R-charges -1 of the fermion components of ϕ_j, and finally μ_G is the index of the adjoint representation of G multiplied by the R-charge $+1$ of the gauginos. R-invariance of the supersymmetric Lagrangian requires the dynamically generated superpotential to have R-charge two. This uniquely fixes the dependence of the superpotential on the field ϕ_i

$$W \propto \left({\phi_i}^{\mu_i}\right)^{2/(\sum_j \mu_j - \mu_G)}. \tag{4.6}$$

To determine the functional dependence on the other superfields, we note that the global symmetries contain a corresponding $U(1)_R$ symmetry for each of the matter superfields, and the superpotential has to have R-charge two under each such R-symmetry. Finally, the dependence on the dynamical scale Λ can be determined by dimensional analysis or using

[2] We normalize the index of the fundamental representations of SU and Sp to 1 and of the vector of SO to 2. This definition ensures invariance of the index when decomposing representations of $SO(2N)$ under the $SU(N)$ subgroup. This is relevant to the flows discussed in Section 4.3.2

an anomalous R-symmetry [62]. The result is

$$W \propto \Lambda^3 \left(\prod_i \left(\frac{\phi_i}{\Lambda} \right)^{\mu_i} \right)^{2/(\sum_j \mu_j - \mu_G)} . \tag{4.7}$$

There may be several (or no) possible contractions of gauge indices, thus the superpotential can be a sum of several terms. We require the coefficient of this superpotential to be non-vanishing, then holomorphy at the origin implies that the exponents of all fields ϕ_i are positive integers. Strictly speaking, we should require holomorphy in the confined degrees of freedom which would imply that the exponents of composites must be positive integers. Since we do not want to have to determine all gauge invariants for this argument, we settle for the weaker constraint on exponents of the fundamental fields. Therefore,[3] $\sum_j \mu_j - \mu_G = 1$ or 2. However, in our normalization of the index, anomaly cancelation further constrains this quantity to be even, thus

$$\sum_j \mu_j - \mu_G = 2. \tag{4.8}$$

This formula summarizes our first necessary condition for s-confinement, which enables us to rule out most theories immediately. For example, for SQCD we find that the only candidate is the theory with $F = N + 1$. Unfortunately, Eq. 4.8 is not a sufficient condition. An example for a theory which satisfies Eq. 4.8 but does not s-confine is $SU(N)$ with an adjoint superfield and one flavor. This theory is easily seen to be in an Abelian Coulomb phase for generic VEVs of the adjoint scalars and vanishing VEVs for the fundamentals. In the following section, we derive another necessary criterion which allows us to rule out theories that satisfy the "index-constraint" but do not s-confine.

4.3.2 Flows and s-confinement

The second condition is obtained from studying different regions on the moduli space of the theory under consideration. A generic supersymmetric theory with vanishing tree-level superpotential has a large moduli space of vacua. By definition, an s-confining theory has a smooth description in terms of gauge invariants everywhere on this moduli space. There should be no singularities in the superpotential or the Kähler potential and there should be no massless gauge bosons anywhere.

Thus, we can test a given theory for s-confinement by expanding around points that are far out in moduli space where the theory simplifies. In the microscopic theory the gauge group gets broken to a subgroup when we go out in moduli space by giving large ($\langle\phi\rangle \gg \Lambda$) expectation values to some fields. In this vacuum, the gauge superfields corresponding to broken symmetry generators get masses through the super-Higgs mechanism and the remaining matter fields decompose under the unbroken subgroup. This "reduced" theory has a smaller gauge group and may be easier to understand. If the original theory was s-confining then its confined description should be valid at this point in moduli space as well. Therefore, the reduced theory is s-confining if the original theory was. This statement

[3]Other solutions exist if all μ_i have a common divisor d, then for $\sum_j \mu_j - \mu_G = d$ or $2d$ the superpotential Eq. 4.8 may be regular. We will argue at the end of Section 4.4.3 that these solutions generically do not yield s-confining theories. Another possibility is that the coefficient of the superpotential above vanishes. There are examples of confining theories with vanishing superpotentials in the literature [82].

can be applied in two directions.
Necessary condition: If the reduced theory does not have a smooth description with only gauge invariant degrees of freedom, then the original theory cannot be s-confining. Sufficient condition: If the original theory is known to be s-confining, then all possible reduced theories (with a remaining unbroken gauge group) which the original theory flows to are s-confining also. The confined spectrum and the confining superpotential of the reduced theories can be obtained by identifying the corresponding points in moduli space in the confined description of the original theory and integrating out all massive fields. In practice, this means identifying the correct gauge invariant fields which have vacuum expectation values and integrating out fields which now have mass terms in the superpotential using their equations of motion.

The reduced theories will always contain some gauge invariant fields in the high-energy description which originally transformed under the now broken gauge generators. These fields do not have any interactions and are irrelevant to the dynamics of the model. They can be removed from the theory. In the confined description the fields corresponding to these gauge singlets are only coupled through superpotential terms which scale to zero when the VEVs are taken to infinity, or which are irrelevant in the infrared.

A non-trivial application of the sufficient condition is given by the flow from $SU(4)$ with an antisymmetric tensor and 4 "flavors" of fundamentals and antifundamentals to $Sp(4)$ with 8 fundamentals. The $SU(4)$ theory is known to s-confine [56]. By giving an expectation value to the antisymmetric tensor the gauge group is broken to $Sp(4)$. All components of the antisymmetric tensor field except for one singlet are "eaten" by the super-Higgs mechanism, and the 4 flavors of fundamentals and antifundamentals become 8 fundamentals of $Sp(4)$. Applying our sufficient criterion, we conclude that the Sp theory is s-confining as well. Its confined spectrum and superpotential can be obtained from the spectrum and superpotential of the $SU(4)$ theory.

A non-trivial example of a theory which can be shown not to s-confine is $SU(4)$ with three antisymmetric tensors and two flavors. This theory satisfies our index condition, Eq. 4.8, and is therefore also a candidate for s-confinement. By giving a VEV to an antisymmetric tensor we can flow from this theory to $Sp(4)$ with two antisymmetric tensors and four fundamentals. VEVs for the other antisymmetric tensors let us flow further to $SU(2)$ with eight fundamentals which is known to be at an interacting fixed point in the infrared. We conclude that the $SU(4)$ with three tensors and $Sp(4)$ with two tensors and all theories that flow to them cannot be s-confining either. This allows us to rule out the following chain of theories, all of which are gauge anomaly free and satisfy Eq. 4.8:

$$\begin{array}{ccccccccc} SU(7) & \rightarrow & SU(6) & \rightarrow & SU(5) & \rightarrow & SU(4) & \rightarrow & Sp(4) \\ \begin{array}{c}\Box\\[-4pt]\Box\\[-4pt]\Box\end{array}\, 2\,\Box\; 4\,\overline{\Box} & & \begin{array}{c}\Box\\[-4pt]\Box\\[-4pt]\Box\end{array}\begin{array}{c}\Box\\[-4pt]\Box\end{array}\,\Box\; 3\,\overline{\Box} & & 2\begin{array}{c}\Box\\[-4pt]\Box\end{array}\overline{\begin{array}{c}\Box\\[-4pt]\Box\end{array}}\,\Box\; 2\,\overline{\Box} & & 3\begin{array}{c}\Box\\[-4pt]\Box\end{array} 2\,\Box\; 2\,\overline{\Box} & & 2\begin{array}{c}\Box\\[-4pt]\Box\end{array} 4\,\Box \end{array} \tag{4.9}$$

Note that a VEV for one of the quark flavors of the $SU(4)$ theory lets us flow to an $SU(3)$ theory with four flavors which is s-confining. We must therefore be careful: when we find a flow to an s-confining theory, it does not follow that the original theory is s-confining as well. The flow is only a necessary condition. However, in all our examples we find that a theory with a single gauge group and no tree-level superpotential is s-confining if it is found to flow to s-confining theories in all directions of its moduli space.

4.4 All s-confining theories

In this section, we present our results which we obtained using the two conditions derived in the previous section. We first created a list of all theories with a single gauge group and matter content satisfying the index constraint. Then we studied all possible flat directions of the individual theories and checked if they only flow to confining theories. We summarize these results in the first table of each subsection. In the first column we list all theories satisfying the index constraint. In the second column we indicate the result of the flows: theories which can be shown to have a branch with an unbroken Abelian gauge group we denote with "Coulomb branch", for theories which can be shown to flow to a reduced theory with a non-Abelian gauge group which is not s-confining we indicate the gauge group of the reduced theory and its matter content, all other theories are s-confining.

After identifying all s-confining theories in this way, we explicitly construct the confined spectra for each s-confining theory. The group theory used to obtain these results can be found in Refs. [66, 67, 68]. We present our results in tables where we indicate the matter content of the ultraviolet theory in the upper part of the table, and the gauge invariant infrared spectrum in the lower part. The gauge group and the Young tableaux of the representations of the matter fields are indicated in the first column. The other groups correspond to the global symmetries of the theory. In addition to the listed global symmetries, there is also a global $U(1)$ with a $G^2U(1)$ anomaly which is broken by instantons.

Finally, we also give the confining superpotentials when they are not too long. We denote gauge invariant composites by their constituents in parenthesis. The relative coefficients of the different terms can be determined by demanding that the equations of motion following from this superpotential reproduce the classical constraints of the ultraviolet theory. This also constitutes an important consistency check: in the limit of large generic expectation values for fields, $\langle\phi\rangle \gg \Lambda$, the ultraviolet theory behaves classically and all its classical constraints need to be reproduced by the infrared description. Checking that all these constraints are reproduced and determining the coefficients is a very tedious exercise which we only performed for some theories. Since we have not determined the coefficients of the superpotential terms for several of the s-confining theories, it may turn out that some of the terms listed in the confining superpotentials have vanishing coefficients.

A more straightforward and also very powerful consistency check is provided by the 't Hooft anomaly matching conditions. We explicitly checked that all global anomalies match between the microscopic and macroscopic degrees of freedom in every theory. Other consistency checks which we performed for a subset of the theories include explorations of the moduli spaces and adding masses for some matter fields and checking consistency of the results. More details on these techniques are described in Ref. [61].

4.4.1 The s-confining $SU(N)$ theories

In this section, we present all s-confining theories based on $SU(N)$ gauge groups. We normalize the Dynkin index and the anomaly coefficient of the fundamental representation to be one. With these conventions, the dimension, index and anomaly coefficient of the smallest $SU(N)$ representations are listed below.

Irrep	Dim	μ	A
$\Box$	N	1	1
Adj	N^2-1	$2N$	0
$\yng(1,1)$	$\frac{N(N-1)}{2}$	$N-2$	$N-4$
$\yng(2)$	$\frac{N(N+1)}{2}$	$N+2$	$N+4$
$\yng(1,1,1)$	$\frac{N(N-1)(N-2)}{6}$	$\frac{(N-3)(N-2)}{2}$	$\frac{(N-3)(N-6)}{2}$
$\yng(3)$	$\frac{N(N+1)(N+2)}{6}$	$\frac{(N+2)(N+3)}{2}$	$\frac{(N+3)(N+6)}{2}$
$\yng(2,1)$	$\frac{N(N-1)(N+1)}{3}$	N^2-3	N^2-9
$\yng(2,2)$	$\frac{N^2(N+1)(N-1)}{12}$	$\frac{N(N-2)(N+2)}{3}$	$\frac{N(N-4)(N+4)}{3}$
$\yng(4)$	$\frac{N(N+1)(N+2)(N+3)}{24}$	$\frac{(N+2)(N+3)(N+4)}{6}$	$\frac{(N+3)(N+4)(N+8)}{6}$
$\yng(2,1,1)$	$\frac{N(N+1)(N-1)(N-2)}{8}$	$\frac{(N-2)(N^2-N-4)}{2}$	$\frac{(N-4)(N^2-N-8)}{2}$

Because the index of a representation of $SU(N)$ grows like N^{k-1} where k is the number of gauge indices, there are very few anomaly free representations which satisfy Eq. 4.8. These representations are listed in Table 4.1. In the first column, we indicate the gauge group and the field content of the theory. In the second column we give the flows which allowed us to rule out s-confinement for a given theory. For those theories which do s-confine we then list the spectra and the confining superpotential in the following tables. For completeness, we also list those s-confining theories which are already known in the literature.

$SU(N)$ with $(N+1)(\Box+\overline{\Box})$ (SUSY QCD) [46]

	$SU(N)$	$SU(N+1)$	$SU(N+1)$	$U(1)$	$U(1)_R$
Q	$\Box$	$\Box$	1	1	$\frac{1}{N+1}$
$\bar{Q}$	$\overline{\Box}$	1	$\Box$	-1	$\frac{1}{N+1}$
$Q\bar{Q}$		$\Box$	$\Box$	0	$\frac{2}{N+1}$
Q^N		$\overline{\Box}$	1	N	$\frac{N}{N+1}$
$\bar{Q}^N$		1	$\overline{\Box}$	$-N$	$\frac{N}{N+1}$

$$W_{dyn} = \frac{1}{\Lambda^{2N-1}}\left[(Q\bar{Q})^{N+1} - (Q^N)(Q\bar{Q})(\bar{Q}^N)\right]$$

$SU(N)$	$(N+1)(\Box+\overline{\Box})$	s-confining
$SU(N)$	$\begin{smallmatrix}\Box\\\Box\end{smallmatrix}+N\overline{\Box}+4\Box$	s-confining
$SU(N)$	$\begin{smallmatrix}\Box\\\Box\end{smallmatrix}+\overline{\begin{smallmatrix}\Box\\\Box\end{smallmatrix}}+3(\Box+\overline{\Box})$	s-confining
$SU(N)$	Adj $+\Box+\overline{\Box}$	Coulomb branch
$SU(4)$	Adj $+\begin{smallmatrix}\Box\\\Box\end{smallmatrix}$	Coulomb branch
$SU(4)$	$3\begin{smallmatrix}\Box\\\Box\end{smallmatrix}+2(\Box+\overline{\Box})$	$SU(2)$: $8\Box$
$SU(4)$	$4\begin{smallmatrix}\Box\\\Box\end{smallmatrix}+\Box+\overline{\Box}$	$SU(2)$: $\Box\!\Box+4\Box$
$SU(4)$	$5\begin{smallmatrix}\Box\\\Box\end{smallmatrix}$	Coulomb branch
$SU(5)$	$3(\begin{smallmatrix}\Box\\\Box\end{smallmatrix}+\overline{\Box})$	s-confining
$SU(5)$	$2\begin{smallmatrix}\Box\\\Box\end{smallmatrix}+2\Box+4\overline{\Box}$	s-confining
$SU(5)$	$2(\begin{smallmatrix}\Box\\\Box\end{smallmatrix}+\overline{\begin{smallmatrix}\Box\\\Box\end{smallmatrix}})$	$Sp(4)$: $3\begin{smallmatrix}\Box\\\Box\end{smallmatrix}+2\Box$
$SU(5)$	$2\begin{smallmatrix}\Box\\\Box\end{smallmatrix}+\overline{\begin{smallmatrix}\Box\\\Box\end{smallmatrix}}+2\overline{\Box}+\Box$	$SU(4)$: $3\begin{smallmatrix}\Box\\\Box\end{smallmatrix}+2(\Box+\overline{\Box})$
$SU(6)$	$2\begin{smallmatrix}\Box\\\Box\end{smallmatrix}+5\overline{\Box}+\Box$	s-confining
$SU(6)$	$2\begin{smallmatrix}\Box\\\Box\end{smallmatrix}+\overline{\begin{smallmatrix}\Box\\\Box\end{smallmatrix}}+2\overline{\Box}$	$SU(4)$: $3\begin{smallmatrix}\Box\\\Box\end{smallmatrix}+2(\Box+\overline{\Box})$
$SU(6)$	$\begin{smallmatrix}\Box\\\Box\\\Box\end{smallmatrix}+4(\Box+\overline{\Box})$	s-confining
$SU(6)$	$\begin{smallmatrix}\Box\\\Box\\\Box\end{smallmatrix}+\begin{smallmatrix}\Box\\\Box\end{smallmatrix}+3\overline{\Box}+\Box$	$SU(5)$: $2\begin{smallmatrix}\Box\\\Box\end{smallmatrix}+\overline{\begin{smallmatrix}\Box\\\Box\end{smallmatrix}}+2\overline{\Box}+\Box$
$SU(6)$	$\begin{smallmatrix}\Box\\\Box\\\Box\end{smallmatrix}+\begin{smallmatrix}\Box\\\Box\end{smallmatrix}+\overline{\begin{smallmatrix}\Box\\\Box\end{smallmatrix}}$	$Sp(6)$: $\begin{smallmatrix}\Box\\\Box\\\Box\end{smallmatrix}+\begin{smallmatrix}\Box\\\Box\end{smallmatrix}+\Box$
$SU(6)$	$2\begin{smallmatrix}\Box\\\Box\\\Box\end{smallmatrix}+\Box+\overline{\Box}$	$SU(5)$: $2(\begin{smallmatrix}\Box\\\Box\end{smallmatrix}+\overline{\begin{smallmatrix}\Box\\\Box\end{smallmatrix}})$
$SU(7)$	$2(\begin{smallmatrix}\Box\\\Box\end{smallmatrix}+3\overline{\Box})$	s-confining
$SU(7)$	$\begin{smallmatrix}\Box\\\Box\\\Box\end{smallmatrix}+4\overline{\Box}+2\Box$	$SU(6)$: $\begin{smallmatrix}\Box\\\Box\\\Box\end{smallmatrix}+\begin{smallmatrix}\Box\\\Box\end{smallmatrix}+3\overline{\Box}+\Box$
$SU(7)$	$\begin{smallmatrix}\Box\\\Box\\\Box\end{smallmatrix}+\overline{\begin{smallmatrix}\Box\\\Box\end{smallmatrix}}+\Box$	$Sp(6)$: $\begin{smallmatrix}\Box\\\Box\\\Box\end{smallmatrix}+\begin{smallmatrix}\Box\\\Box\end{smallmatrix}+\Box$

Table 4.1: All SU theories satisfying $\sum_j \mu_j - \mu_G = 2$. This list is finite because the indices of higher index tensor representations grow very rapidly with the size of the gauge group. We list the gauge group and the field content of the theories in the first column. In the second column, we indicate which theories are s-confining. For the theories which do not s-confine we give the flows to non s-confining theories or indicate that there is a Coulomb branch on the moduli space.

$SU(2N)$ **with** $\begin{smallmatrix}\Box\\ \Box\end{smallmatrix} + 2N\ \overline{\Box} + 4\ \Box$ **[56]**

	$SU(2N)$	$SU(2N)$	$SU(4)$	$U(1)_1$	$U(1)_2$	$U(1)_R$
A	$\begin{smallmatrix}\Box\\ \Box\end{smallmatrix}$	1	1	0	$2N+4$	0
$\overline{Q}$	$\overline{\Box}$	$\Box$	1	4	$-2N+2$	0
Q	$\Box$	1	$\Box$	$-2N$	$-2N+2$	$\frac{1}{2}$
$Q\overline{Q}$		$\Box$	$\Box$	$4-2N$	$-4N+4$	$\frac{1}{2}$
$A\overline{Q}^2$		$\begin{smallmatrix}\Box\\ \Box\end{smallmatrix}$	1	8	$-2N+8$	0
A^N		1	1	0	$2N^2+4N$	0
$A^{N-1}Q^2$		1	$\begin{smallmatrix}\Box\\ \Box\end{smallmatrix}$	$-4N$	$2N^2-2N$	1
$A^{N-2}Q^4$		1	1	$-8N$	$2N^2-8N$	2
$\overline{Q}^{2N}$		1	1	$8N$	$-4N^2+4N$	0

$$W_{dyn} = \frac{1}{\Lambda^{4N-1}}\Big[(A^N)(Q\overline{Q})^4(A\overline{Q}^2)^{N-2} + (A^{N-1}Q^2)(Q\overline{Q})^2(A\overline{Q}^2)^{N-1} + (A^{N-2}Q^4)(A\overline{Q}^2)^N + (\overline{Q}^{2N})(A^N)(A^{N-2}Q^4) + (\overline{Q}^{2N})(A^{N-1}Q^2)^2\Big]$$

$SU(2N+1)$ **with** $\begin{smallmatrix}\Box\\ \Box\end{smallmatrix} + (2N+1)\ \overline{\Box} + 4\ \Box$ **[56]**

	$SU(2N+1)$	$SU(2N+1)$	$SU(4)$	$U(1)_1$	$U(1)_2$	$U(1)_R$
A	$\begin{smallmatrix}\Box\\ \Box\end{smallmatrix}$	1	1	0	$2N+5$	0
$\overline{Q}$	$\overline{\Box}$	$\Box$	1	4	$-2N+1$	0
Q	$\Box$	1	$\Box$	$-2N-1$	$-2N+1$	$\frac{1}{2}$
$Q\overline{Q}$		$\Box$	$\Box$	$3-2N$	$-4N+2$	$\frac{1}{2}$
$A\overline{Q}^2$		$\begin{smallmatrix}\Box\\ \Box\end{smallmatrix}$	1	8	$-2N+7$	0
A^NQ		1	$\Box$	$-2N-1$	$2N^2+3N+1$	$\frac{1}{2}$
$A^{N-1}Q^3$		1	$\overline{\Box}$	$-6N-3$	$2N^2-3N-2$	$\frac{3}{2}$
$\overline{Q}^{2N+1}$		1	1	$4(2N+1)$	$-4N^2+1$	0

$$W_{dyn} = \frac{1}{\Lambda^{2N}}\Big[(A^NQ)(Q\overline{Q})^3(A\overline{Q}^2)^{N-1} + (A^{N-1}Q^3)(Q\overline{Q})(A\overline{Q}^2)^N + (\overline{Q}^{2N+1})(A^NQ)(A^{N-1}Q^3)\Big]$$

$SU(2N+1)$ **with** $\begin{smallmatrix}\square\\ \square\end{smallmatrix} + \overline{\begin{smallmatrix}\square\\ \square\end{smallmatrix}} + 3(\square + \bar{\square})$

	$SU(2N+1)$	$SU(3)$	$SU(3)$	$U(1)_1$	$U(1)_2$	$U(1)_3$	$U(1)_R$
A	$\begin{smallmatrix}\square\\ \square\end{smallmatrix}$	1	1	1	0	-3	0
$\bar{A}$	$\overline{\begin{smallmatrix}\square\\ \square\end{smallmatrix}}$	1	1	-1	0	-3	0
Q	$\square$	$\square$	1	0	1	$2N-1$	$\frac{1}{3}$
$\bar{Q}$	$\bar{\square}$	1	$\square$	0	-1	$2N-1$	$\frac{1}{3}$
$M_k = Q(A\bar{A})^k\bar{Q}$		$\square$	$\square$	0	0	$4N-2-6k$	$\frac{2}{3}$
$H_k = \bar{A}(A\bar{A})^kQ^2$		$\begin{smallmatrix}\square\\ \square\end{smallmatrix}$	1	-1	2	$4N-5-6k$	$\frac{2}{3}$
$\bar{H}_k = A(A\bar{A})^k\bar{Q}^2$		1	$\begin{smallmatrix}\square\\ \square\end{smallmatrix}$	1	-2	$4N-5-6k$	$\frac{2}{3}$
$B_1 = A^NQ$		$\square$	1	N	1	$-N-1$	$\frac{1}{3}$
$\bar{B}_1 = \bar{A}^N\bar{Q}$		1	$\square$	$-N$	-1	$-N-1$	$\frac{1}{3}$
$B_3 = A^{N-1}Q^3$		1	1	$N-1$	3	$3N$	1
$\bar{B}_3 = \bar{A}^{N-1}\bar{Q}^3$		1	1	$-N+1$	-3	$3N$	1
$T_m = (A\bar{A})^m$		1	1	0	0	$-6m$	0

where $k = 0, \ldots, N-1$ and $m = 1, \ldots, N$. The number of terms in the confining superpotential grows quickly with the size of the gauge group. Therefore we only present the superpotential for the $SU(5)$ theory.

$$\begin{aligned} W_{dyn} \;=\; & \frac{1}{\Lambda^9}\Big(M_0^3T_1T_2 + M_1^3 + T_2B_3\bar{B}_3 + T_2H_0\bar{H}_0M_0 + T_2M_1M_0^2 + T_1^3M_0^3 + \\ & T_1^2B_3\bar{B}_3 + T_1^2H_0\bar{H}_0M_0 + T_1^2M_1M_0^2 + T_1B_1\bar{B}_1M_0^2 + T_1H_0\bar{H}_0M_1 + \\ & B_1\bar{B}_1H_0\bar{H}_0 + B_1\bar{B}M_1M_0 + H_1\bar{H}_1M_0 + H_1\bar{H}_0M_0T_1 + \bar{H}_1H_0M_0T_1 + \\ & \bar{H}_1\bar{B}_1B_3 + H_1B_1\bar{B}_3 + H_0B_1\bar{B}_3T_1 + \bar{H}_0\bar{B}_1B_3T_1 + H_1\bar{H}_0M_1 + \bar{H}_1H_0M_1\Big) \end{aligned}$$

Note that the term $T_1M_1^2M_0$ is allowed by all symmetries, however its coefficient is zero, which can be verified by requiring that the equations of motion reproduce the classical constraints.

$SU(2N)$ **with** $\begin{smallmatrix}\square\\ \square\end{smallmatrix} + \overline{\begin{smallmatrix}\square\\ \square\end{smallmatrix}} + 3(\square + \bar{\square})$

	$SU(2N)$	$SU(3)$	$SU(3)$	$U(1)_1$	$U(1)_2$	$U(1)_3$	$U(1)_R$
A	$\begin{smallmatrix}\square\\ \square\end{smallmatrix}$	1	1	1	0	-3	0
$\bar{A}$	$\overline{\begin{smallmatrix}\square\\ \square\end{smallmatrix}}$	1	1	-1	0	-3	0
Q	$\square$	$\square$	1	0	1	$2N-2$	$\frac{1}{3}$
$\bar{Q}$	$\bar{\square}$	1	$\square$	0	-1	$2N-2$	$\frac{1}{3}$
$M_k = Q(A\bar{A})^k\bar{Q}$		$\square$	$\square$	0	0	$4N-4-6k$	$\frac{2}{3}$
$H_m = \bar{A}(A\bar{A})^kQ^2$		$\begin{smallmatrix}\square\\ \square\end{smallmatrix}$	1	-1	2	$4N-7-6m$	$\frac{2}{3}$
$\bar{H}_m = A(A\bar{A})^k\bar{Q}^2$		1	$\begin{smallmatrix}\square\\ \square\end{smallmatrix}$	1	-2	$4N-7-6m$	$\frac{2}{3}$
$B_0 = A^N$		1	1	N	0	$-3N$	0
$\bar{B}_0 = \bar{A}^N$		1	1	$-N$	0	$-3N$	0
$B_2 = A^{N-1}Q^2$		$\begin{smallmatrix}\square\\ \square\end{smallmatrix}$	1	$N-1$	2	$N-1$	$\frac{2}{3}$
$\bar{B}_2 = \bar{A}^{N-1}\bar{Q}^2$		1	$\begin{smallmatrix}\square\\ \square\end{smallmatrix}$	$-N+1$	-2	$N-1$	$\frac{2}{3}$
$T_n = (A\bar{A})^n$		1	1	0	0	$-6n$	0

where $k = 0, \ldots, N-1$, $m = 0, \ldots, N-2$ and $n = 1, \ldots, N-1$. The case of $SU(4)$ is different, because in $SU(4)$ the two-index antisymmetric tensor is self-conjugate. Therefore there is an additional $SU(2)$ global symmetry. The corresponding table is

	$SU(4)$	$SU(2)$	$SU(3)$	$SU(3)$	$U(1)_1$	$U(1)_2$	$U(1)_R$
A	$\begin{smallmatrix}\square\\ \square\end{smallmatrix}$	$\square$	1	1	0	-3	0
Q	$\square$	1	$\square$	1	1	2	$\frac{1}{3}$
$\bar{Q}$	$\overline{\square}$	1	1	$\square$	-1	2	$\frac{1}{3}$
$M_0 = Q\bar{Q}$		1	$\square$	$\square$	0	4	$\frac{2}{3}$
$M_2 = QA^2\bar{Q}$		1	$\square$	$\square$	0	-2	$\frac{2}{3}$
$H = AQ^2$		$\square$	$\overline{\square}$	1	2	1	$\frac{2}{3}$
$\bar{H} = A\bar{Q}^2$		$\square$	1	$\overline{\square}$	-2	1	$\frac{2}{3}$
$T = A^2$		$\square\square$	1	1	0	-6	0

The superpotential for the $SU(4)$ theory is

$$W_{dyn} = \frac{1}{\Lambda^7}\Big(T^2 M_0^3 - 12 T H \bar{H} M_0 - 24 M_0 M_2^2 - 24 H \bar{H} M_2\Big),$$

where the relative coefficients are fixed by requiring that the equations of motion reproduce the classical constraints.

$SU(6)$ with $\begin{smallmatrix}\square\\ \square\\ \square\end{smallmatrix} + 4(\square + \overline{\square})$

	$SU(6)$	$SU(4)$	$SU(4)$	$U(1)_1$	$U(1)_2$	$U(1)_R$
A	$\begin{smallmatrix}\square\\ \square\\ \square\end{smallmatrix}$	1	1	0	-4	-1
Q	$\square$	$\square$	1	1	3	1
$\bar{Q}$	$\overline{\square}$	1	$\square$	-1	3	1
$M_0 = Q\bar{Q}$		$\square$	$\square$	0	6	2
$M_2 = QA^2\bar{Q}$		$\square$	$\square$	0	-2	0
$B_1 = AQ^3$		$\overline{\square}$	1	3	5	2
$\bar{B}_1 = A\bar{Q}^3$		1	$\overline{\square}$	-3	5	2
$B_3 = A^3Q^3$		$\overline{\square}$	1	3	-3	0
$\bar{B}_3 = A^3\bar{Q}^3$		1	$\overline{\square}$	-3	-3	0
$T = A^4$		1	1	0	-16	4

$$\begin{aligned} W_{dyn} &= \frac{1}{\Lambda^{11}}\Big(M_0 B_1 \bar{B}_1 T + B_3 \bar{B}_3 M_0 + M_2^3 M_0 + T M_2 M_0^3 + \\ &\quad \bar{B}_1 B_3 M_2 + B_1 \bar{B}_3 M_2\Big), \end{aligned}$$

$SU(5)$ **with** $3(\begin{smallmatrix}\square\\\square\end{smallmatrix} + \overline{\square})$

	$SU(5)$	$SU(3)$	$SU(3)$	$U(1)$	$U(1)_R$
A	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\square$	1	1	0
$\bar{Q}$	$\overline{\square}$	1	$\square$	-3	$\frac{2}{3}$
$A\bar{Q}^2$		$\square$	$\overline{\square}$	-5	$\frac{4}{3}$
$A^3\bar{Q}$		$\begin{smallmatrix}\square&\square\\\square&\end{smallmatrix}$	$\square$	0	$\frac{2}{3}$
A^5		$\square\square$	1	5	0

$$W_{dyn} = \frac{1}{\Lambda^9}\left[(A^5)(A^3\overline{Q})(A\overline{Q}^2) + (A^3\overline{Q})^3\right]$$

$SU(5)$ **with** $2\begin{smallmatrix}\square\\\square\end{smallmatrix} + 4\,\overline{\square} + 2\square$

	$SU(5)$	$SU(2)$	$SU(4)$	$SU(2)$	$U(1)_1$	$U(1)_2$	$U(1)_R$
A	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\square$	1	1	0	-1	0
$\bar{Q}$	$\overline{\square}$	1	$\square$	1	1	1	$\frac{1}{3}$
Q	$\square$	1	1	$\square$	-2	1	$\frac{1}{3}$
$Q\bar{Q}$		1	$\square$	$\square$	-1	2	$\frac{2}{3}$
$A\bar{Q}^2$		$\square$	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	1	2	1	$\frac{2}{3}$
A^2Q		$\square\square$	1	$\square$	-2	-1	$\frac{1}{3}$
$A^3\bar{Q}$		$\square$	$\square$	1	1	-2	$\frac{1}{3}$
$A^2Q^2\bar{Q}$		1	$\square$	1	-3	1	1

$$\begin{aligned} W_{dyn} &= \frac{1}{\Lambda^9}\left[(A^3\overline{Q})^2(Q\overline{Q})^2 + (A^3\overline{Q})(A^2Q^2\overline{Q})(A\overline{Q}^2)\right. \\ &+ \left.(A^3\overline{Q})(A^2Q)(A\overline{Q}^2)(Q\overline{Q}) + (A^2Q)^2(A\overline{Q}^2)^2\right] \end{aligned}$$

$SU(6)$ **with** $2\begin{smallmatrix}\square\\\square\end{smallmatrix} + 5\overline{\square} + \square$

	$SU(6)$	$SU(2)$	$SU(5)$	$U(1)_1$	$U(1)_2$	$U(1)_R$
A	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\square$	1	0	3	$\frac{1}{4}$
$\bar{Q}$	$\overline{\square}$	1	$\square$	1	-4	0
Q	$\square$	1	1	-5	-4	0
$Q\bar{Q}$		1	$\square$	-4	-8	0
$A\bar{Q}^2$		$\square$	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	2	-5	$\frac{1}{4}$
A^3		$\square\square\square$	1	0	9	$\frac{3}{4}$
$A^3Q\bar{Q}$		$\square$	$\square$	-4	1	$\frac{3}{4}$
$A^4\bar{Q}^2$		1	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	2	4	1

$$\begin{aligned} W_{dyn} &= \frac{1}{\Lambda^{11}}\left[(A^4\overline{Q}^2)^2(Q\overline{Q}) + (A^4\overline{Q}^2)(A^3Q\overline{Q})(A\overline{Q}^2) +\right. \\ &\left.+(A^3)(A^3Q\overline{Q})(A\overline{Q}^2)^2 + (A^3)^2(A\overline{Q}^2)^2(Q\overline{Q})\right] \end{aligned}$$

Note, that the term $(A^4\overline{Q}^2)(A^3)(A\overline{Q}^2)(Q\overline{Q})$ is allowed by the $U(1)$ symmetries but not by the non-abelian global symmetries.

$SU(7)$ **with** $2\begin{smallmatrix}\square\\\square\end{smallmatrix} + 6\overline{\square}$

	$SU(7)$	$SU(2)$	$SU(6)$	$U(1)$	$U(1)_R$
A	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\square$	1	3	0
$\bar{Q}$	$\overline{\square}$	1	$\square$	-5	$\frac{1}{3}$
$H = A\bar{Q}^2$		$\square$	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	-7	$\frac{2}{3}$
$N = A^4\bar{Q}$		$\square\square$	$\square$	7	$\frac{1}{3}$

$$W_{dyn} = \frac{1}{\Lambda^{13}} N^2 H^2$$

4.4.2 The s-confining $Sp(2N)$ theories

We now discuss the s-confining $Sp(2N)$ theories. First, we again summarize the group theoretical properties of the simplest $Sp(2N)$ representations. Contrary to $SU(N)$ groups there is no chiral anomaly for $Sp(2N)$ groups. The only requirement on the field content is that there is no Witten anomaly, this is satisfied if the sum of the Dynkin indices of the matter fields is even. $Sp(2N)$ is the subgroup of $SU(2N)$ which leaves the tensor $J^{\alpha\beta} = (\mathbf{1}_{N\times N} \otimes i\sigma_2)^{\alpha\beta}$ invariant. Irreducible tensors of $Sp(2N)$ must be traceless with respect to $J^{\alpha\beta}$. One can obtain these irreducible representations by subtracting traces from the $SU(2N)$ tensors. The properties of these representations are summarized in the table below. We use a normalization where the index of the fundamental is one. This normalization is consistent with the $Sp(2N) \subset SU(2N)$ embedding, under which $2N \to 2N$. Thus with these conventions the index of the matter fields does not change under $SU \to Sp$ decompositions. The adjoint of $Sp(2N)$ is the two-index symmetric tensor.

Irrep	Dim	μ
$\square$	$2N$	1
$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$N(2N-1)-1$	$2N-2$
$\square\square$	$N(2N+1)$	$2N+2$
$\begin{smallmatrix}\square\\\square\\\square\end{smallmatrix}$	$\frac{N(2N-1)(2N-2)}{3} - 2N$	$\frac{(2N-3)(2N-2)}{2} - 1$
$\square\square\square$	$\frac{N(2N+1)(2N+2)}{3}$	$\frac{(2N+2)(2N+3)}{2}$
$\begin{smallmatrix}\square\square\\\square\end{smallmatrix}$	$\frac{2N(2N-1)(2N+1)}{3} - 2N$	$(2N)^2-4$

With this knowledge one can again write down all anomaly-free theories for which the matter content satisfies Eq. 4.8. These theories are summarized in Table 4.2. In the first column, we indicate the gauge group and the field content of the theory. The second column gives a possible flow to a non-s-confining theory or if the theory is s-confining, we state that in the second column. The only s-confining theories based on $Sp(2N)$ groups are the two sequences that are already known in the literature. We give the spectra and dynamically generated superpotentials of these theories in the tables below.

$Sp(2N)$	$(2N+4)\,\Box$	s-confining
$Sp(2N)$	$\overset{\Box}{\Box}+6\,\Box$	s-confining
$Sp(2N)$	$\Box\Box+2\,\Box$	Coulomb branch
$Sp(4)$	$2\,\overset{\Box}{\Box}+4\,\Box$	$SU(2)$: $8\,\Box$
$Sp(4)$	$3\,\overset{\Box}{\Box}+2\,\Box$	$SU(2)$: $\Box\Box+4\,\Box$
$Sp(4)$	$4\,\overset{\Box}{\Box}$	$SU(2)$: $2\,\Box\Box$
$Sp(6)$	$2\,\overset{\Box}{\Box}+2\,\Box$	$Sp(4)$: $2\,\overset{\Box}{\Box}+4\,\Box$
$Sp(6)$	$\overset{\Box}{\overset{\Box}{\Box}}+5\,\Box$	$Sp(4)$: $2\,\overset{\Box}{\Box}+4\,\Box$
$Sp(6)$	$\overset{\Box}{\overset{\Box}{\Box}}+\overset{\Box}{\Box}+\Box$	$SU(2)$: $\Box\Box+4\,\Box$
$Sp(6)$	$2\,\overset{\Box}{\overset{\Box}{\Box}}$	$SU(3)$: $\Box\Box+\overline{\Box\Box}$
$Sp(8)$	$2\,\overset{\Box}{\Box}$	$Sp(4)$: $5\,\overset{\Box}{\Box}$

Table 4.2: All Sp theories satisfying $\sum_j \mu_j - \mu_G = 2$. This list is finite because the indices of higher index tensor representations grow very rapidly with the size of the gauge group. We list the gauge group and the field content of the theories in the first column. In the second column, we indicate which theories are s-confining. For the remaining ones we give the flows to non-confining theories or indicate that there is a Coulomb branch on the moduli space.

$Sp(2N)$ **with** $(2N+4)\,\Box$ **[54]**

	$Sp(2N)$	$SU(2N+4)$	$U(1)_R$
Q	$\Box$	$\Box$	$\frac{1}{N+2}$
Q^2		$\overset{\Box}{\Box}$	$\frac{2}{N+2}$

$$W_{dyn} = \frac{1}{\Lambda^{2N+1}}(Q^2)^{N+2}$$

$Sp(2N)$ **with** $\overset{\Box}{\Box}+6\,\Box$ **[59, 60]**

	$Sp(2N)$	$SU(6)$	$U(1)$	$U(1)_R$
A	$\overset{\Box}{\Box}$	1	-3	0
Q	$\Box$	$\Box$	$N-1$	$\frac{1}{3}$
A^k		1	$-3k$	0
QA^mQ		$\overset{\Box}{\Box}$	$2(N-1)-3k$	$\frac{2}{3}$

Here $k = 2, 3, \ldots, N$ and $m = 0, 1, \ldots, N-1$. The number of terms in the superpotential grows quickly with N. For $Sp(4)$ the superpotential is

$$W_{dyn} = \frac{1}{\Lambda^5}\Big[(A^2)(Q^2)^3 + (Q^2)(QAQ)^2\Big].$$

4.4.3 The s-confining $SO(N)$ theories

$SO(N)$ theories[4] are distinct from the SU and Sp theories because contrary to those groups $SO(N)$ has representations which cannot be obtained from products of the vector representations. These are the spinorial representations. A theory can be s-confining only if all possible test charges can be screened by the matter fields. Spinors cannot be screened by matter in the vector representation of SO. Thus, theories without spinorial matter cannot be s-confining. This restricts the number of possible s-confining $SO(N)$ theories, because the Dynkin index of the spinor representation grows exponentially with the size of the gauge group. The biggest group for which Eq. 4.8 can be satisfied with matter including spinor representations is $SO(14)$.

$SO(N)$ theories (for $N > 6$) do not have either chiral or Witten anomalies. We do not consider the $N \leq 6$ theories because they can be obtained from our previous results by using the following isomorphisms: $SO(6) \sim SU(4)$, $SO(5) \sim Sp(4)$, $SO(4) \sim SU(2) \times SU(2)$, $SO(3) \sim SU(2)$, $SO(2) \sim U(1)$.

The spinor representations of $SO(N)$ have different properties depending on whether N is even or odd. For odd N, there is just one spinor representation, while for even N there are two inequivalent spinors. For $N = 4k$ the two spinors are self-conjugate while for $N = 4k + 2$ the two spinors are complex conjugate to each other.

We use a normalization where the index of the vector of $SO(N)$ is 2. The reason is that under the embedding $SO(2N) \supset SU(N)$ the vector of $SO(2N)$ decomposes as $2N \to N + \overline{N}$. If we do not want the index of the matter fields to change under this decomposition we need to normalize the index of the vector to two. The fundamental properties of the smallest $SO(N)$ representations are summarized in the tables below. The adjoint of $SO(N)$ is the two-index antisymmetric tensor.

$SO(2N+1)$		
Irrep	Dim	μ
$\Box$	$2N+1$	2
S	2^N	2^{N-2}
$\begin{smallmatrix}\Box\\\Box\end{smallmatrix}$	$N(2N+1)$	$4N-2$
$\Box\Box$	$(N+1)(2N+1)-1$	$4N+6$

$SO(2N)$		
Irrep	Dim	μ
$\Box$	$2N$	2
S	2^{N-1}	2^{N-3}
$\bar{S},(S')$	2^{N-1}	2^{N-3}
$\begin{smallmatrix}\Box\\\Box\end{smallmatrix}$	$N(2N-1)$	$4N-4$
$\Box\Box$	$N(2N+1)-1$	$4N+4$

Since the vector and the spinors are the only representations that potentially have smaller index than the adjoint, it is clear that candidates for s-confining theories contain only vectors and spinors. For odd N we denote the field content by (s, v), where s is the number of spinors and v is the number of vectors. For even N we use the notation (s, s', v), where s and s' are the numbers of matter fields in the two inequivalent spinor

[4] We do not distinguish between $SO(N)$ and its covering group $Spin(N)$.

representations and v is the number of vectors.

The $SO(8)$ group requires special attention. The reason is that there is a group automorphism which permutes the two spinor and the vector representations. Therefore only relative labelings of the representations are meaningful. For example $(4,3,0)$ and $(0,3,4)$ in $SO(8)$ are equivalent.

With this knowledge of group theory we can write down all theories which satisfy Eq. 4.8. These theories are listed in Table 4.3. Almost all of these theories are s-confining. The only spectrum that has been given in the literature [56] is for $SO(7)$ with $(5,1)$. Below we list the spectra and the confining superpotentials for the s-confining $SO(N)$ theories. Most of the confining superpotentials are very complicated. We only list those where the number of terms in the superpotential is reasonably small.

$SO(14)$ with (1,0,5)

	$SO(14)$	$SU(5)$	$U(1)$	$U(1)_R$
S	64	1	5	$\frac{1}{8}$
Q	$\Box$	$\Box$	-8	0
Q^2		$\Box\Box$	-16	0
S^2Q^3		$\overline{\begin{array}{c}\Box\\\Box\end{array}}$	-14	$\frac{1}{4}$
S^4Q^2		$\Box\Box$	4	$\frac{1}{2}$
S^4Q^4		$\overline{\Box}$	-12	$\frac{1}{2}$
S^6Q^3		$\overline{\begin{array}{c}\Box\\\Box\end{array}}$	6	$\frac{3}{4}$
S^8		1	40	1
S^8Q^4		$\overline{\Box}$	8	1

$$
\begin{aligned}
W_{dyn} = & \frac{1}{\Lambda^{23}}\Big[(S^8Q^4)^2(Q^2) + (S^8Q^4)(S^6Q^3)(S^2Q^3) + (S^8Q^4)(S^4Q^4)(S^4Q^2) \\
& +(S^8)^2(Q^2)^5 + (S^8)(S^6Q^3)(S^2Q^3)(Q^2)^2 + (S^4Q^2)^4(Q^2) + (S^6Q^3)^2(S^4Q^2)(Q^2) \\
& +(S^8)(S^4Q^4)^2(Q^2) + (S^8)(S^4Q^2)^2(Q^2)^3 \\
& +(S^6Q^3)(S^2Q^3)(S^4Q^2)^2 + (S^6Q^3)^2(S^4Q^4)\Big]
\end{aligned}
$$

Note that several terms allowed by $U(1)$ symmetries are not allowed by the full set of global symmetries. For example, the $SU(5)$ contraction in the term $(S^8Q^4)(S^8)(Q^2)^3$ vanishes, since it is not possible to make an $SU(5)$ invariant from the third power of a symmetric tensor and one field in the antifundamental representation. There are more examples of such terms prohibited by non-abelian global symmetries in other theories in this section.

$SO(14)$	$(1,0,5)$	s-confining
$SO(13)$	$(1,4)$	s-confining
$SO(12)$	$(1,0,7)$	s-confining
$SO(12)$	$(2,0,3)$	s-confining
$SO(12)$	$(1,1,3)$	s-confining
$SO(11)$	$(1,6)$	s-confining
$SO(11)$	$(2,2)$	s-confining
$SO(10)$	$(4,0,1)$	s-confining
$SO(10)$	$(3,0,3)$	s-confining
$SO(10)$	$(2,0,5)$	s-confining
$SO(10)$	$(3,1,1)$	s-confining
$SO(10)$	$(2,1,3)$	s-confining
$SO(10)$	$(1,1,5)$	s-confining
$SO(10)$	$(2,2,1)$	s-confining
$SO(10)$	$(1,0,7)$	$SU(4)$ with $3\,\begin{smallmatrix}\Box\\\Box\end{smallmatrix} + 2\,(\Box + \overline{\Box})$
$SO(9)$	$(4,0)$	s-confining
$SO(9)$	$(3,2)$	s-confining
$SO(9)$	$(2,4)$	s-confining
$SO(9)$	$(1,6)$	$SU(4)$ with $3\,\begin{smallmatrix}\Box\\\Box\end{smallmatrix} + 2\,(\Box + \overline{\Box})$
$SO(8)$	$(7,0,0)$	Coulomb branch
$SO(8)$	$(6,1,0)$	Coulomb branch
$SO(8)$	$(5,2,0)$	$SU(4)$ with $3\,\begin{smallmatrix}\Box\\\Box\end{smallmatrix} + 2\,(\Box + \overline{\Box})$
$SO(8)$	$(5,1,1)$	$SU(4)$ with $3\,\begin{smallmatrix}\Box\\\Box\end{smallmatrix} + 2\,(\Box + \overline{\Box})$
$SO(8)$	$(4,3,0)$	s-confining
$SO(8)$	$(4,2,1)$	s-confining
$SO(8)$	$(3,3,1)$	s-confining
$SO(8)$	$(3,2,2)$	s-confining
$SO(7)$	$(6,0)$	s-confining
$SO(7)$	$(5,1)$	s-confining
$SO(7)$	$(4,2)$	s-confining
$SO(7)$	$(3,3)$	s-confining
$SO(7)$	$(2,4)$	$SU(4)$ with $3\,\begin{smallmatrix}\Box\\\Box\end{smallmatrix} + 2\,(\Box + \overline{\Box})$
$SO(7)$	$(1,5)$	Coulomb branch

Table 4.3: All $SO(N)$ theories which contain at least one spinor and satisfy $\sum_j \mu_j - \mu_G = 2$. This list is finite because the index of the spinor representations grows exponentially with N. We list the gauge group of the theory in the first column and the matter content in the second column. As explained in the text, for odd N (s,v) denotes the number of spinors and the number of vectors, while for even N (s,s',v) denotes the numbers of the two inequivalent spinors and vectors. In the third column, we indicate which theories are s-confining. For the remaining ones we give the flows to non-confining theories or indicate that there is a Coulomb branch on the moduli space.

$SO(13)$ **with (1,4)**

	$SO(13)$	$SU(4)$	$U(1)$	$U(1)_R$
S	64	1	1	$\frac{1}{8}$
Q	$\square$	$\square$	-2	0
Q^2		$\square\square$	-4	0
S^2Q^3		$\overline{\square}$	-4	$\frac{1}{4}$
S^2Q^2		$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	-2	$\frac{1}{4}$
S^4Q^4		1	-4	$\frac{1}{2}$
S^4Q^3		$\overline{\square}$	-2	$\frac{1}{2}$
S^4Q^2		$\square\square$	0	$\frac{1}{2}$
S^4Q		$\square$	2	$\frac{1}{2}$
S^4		1	4	$\frac{1}{2}$
S^6Q^3		$\overline{\square}$	0	$\frac{3}{4}$
S^6Q^2		$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	2	$\frac{3}{4}$
S^8Q^3		$\overline{\square}$	2	1
S^8		1	8	1

Note, that one could add the operator S^8Q^4 to the above list without affecting anomaly matching. However, there is a mass term allowed for this operator, and by flowing to this theory from $SO(14)$ with $(1, 0, 5)$ one finds that this mass term is generated. Thus S^8Q^4 is not in the IR spectrum. Similar operators appear in many other s-confining $SO(N)$ theories. Since a mass term is always generated for such operators, we do not include them in any of the forthcoming s-confining spectra.

$SO(12)$ **with (1,0,7)**

	$SO(12)$	$SU(7)$	$U(1)$	$U(1)_R$
S	32	1	7	$\frac{1}{4}$
Q	$\square$	$\square$	-4	0
Q^2		$\square\square$	-8	0
S^2Q^2		$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	6	$\frac{1}{2}$
S^2Q^6		$\overline{\square}$	-10	$\frac{1}{2}$
S^4		1	28	1
S^4Q^6		$\overline{\square}$	4	1

$$W_{dyn} = \frac{1}{\Lambda^{19}}\Big[(S^4Q^6)^2(Q^2) + (S^4Q^6)(S^2Q^6)(S^2Q^2) + (S^4)(S^2Q^2)^2(Q^2)^5 + (S^4)(S^2Q^6)^2(Q^2) + (S^2Q^2)^4(Q^2)^3 + (Q^2)^7(S^4)^2\Big]$$

$SO(12)$ **with (2,0,3)**

	$SO(12)$	$SU(2)$	$SU(3)$	$U(1)$	$U(1)_R$
S	32	$\square$	1	3	$\frac{1}{8}$
Q	$\square$	1	$\square$	-8	0
Q^2		1	$\square\square$	-16	0
S^2		1	1	6	$\frac{1}{4}$
S^2Q^2		$\square\square$	$\bar{\square}$	-10	$\frac{1}{4}$
S^4		$\square\square\square\square$	1	12	$\frac{1}{2}$
S^4Q^2		1	$\square\square$	-4	$\frac{1}{2}$
$S^4Q^{2\prime}$		$\square\square$	$\bar{\square}$	-4	$\frac{1}{2}$
S^6		1	1	18	$\frac{3}{4}$
S^6Q^2		$\square\square$	$\bar{\square}$	2	$\frac{3}{4}$
S^8Q^2		1	$\square\square$	8	1

$SO(12)$ **with (1,1,3)**

	$SO(12)$	$SU(3)$	$U(1)_1$	$U(1)_2$	$U(1)_R$
S	32	1	1	3	$\frac{1}{8}$
S'	$32'$	1	-1	3	$\frac{1}{8}$
Q	$\square$	$\square$	0	-8	0
Q^2		$\square\square$	0	-16	0
$SS'Q^3$		1	0	-18	$\frac{1}{4}$
S^2Q^2		$\bar{\square}$	2	-10	$\frac{1}{4}$
S'^2Q^2		$\bar{\square}$	-2	-10	$\frac{1}{4}$
$SS'Q$		$\square$	0	-2	$\frac{1}{4}$
S^4		1	4	12	$\frac{1}{2}$
S'^4		1	-4	12	$\frac{1}{2}$
$S^2S'^2$		1	0	12	$\frac{1}{2}$
$S^3S'Q^3$		1	2	-12	$\frac{1}{2}$
S'^3SQ^3		1	-2	-12	$\frac{1}{2}$
$S^2S'^2Q^2$		$\square\square$	0	-4	$\frac{1}{2}$
$S^2S'^2Q^{2\prime}$		$\bar{\square}$	0	-4	$\frac{1}{2}$
$S^3S'Q$		$\square$	2	4	$\frac{1}{2}$
S'^3SQ		$\square$	-2	4	$\frac{1}{2}$
$S^3S'^3Q^3$		1	0	-6	$\frac{3}{4}$
$S^3S'^3Q$		$\square$	0	10	$\frac{3}{4}$
$S^4S'^2Q^2$		$\bar{\square}$	2	2	$\frac{3}{4}$
$S'^4S^2Q^2$		$\bar{\square}$	-2	2	$\frac{3}{4}$
$S^4S'^4$		1	0	24	1
$S^4S'^4Q^2$		$\bar{\square}$	0	8	1

$SO(11)$ **with (1,6)**

	$SO(11)$	$SU(6)$	$U(1)$	$U(1)_R$
S	32	1	3	$\frac{1}{4}$
Q	$\Box$	$\Box$	-2	0
Q^2		$\Box\Box$	-4	0
S^2Q^2		$\begin{smallmatrix}\Box\\\Box\end{smallmatrix}$	2	$\frac{1}{2}$
S^2Q^5		$\overline{\Box}$	-4	$\frac{1}{2}$
S^4		1	12	1
S^4Q^5		$\overline{\Box}$	2	1
S^2Q		$\Box$	4	$\frac{1}{2}$
S^2Q^6		1	-6	$\frac{1}{2}$

$SO(11)$ **with (2,2)**

	$SO(11)$	$SU(2)$	$SU(2)$	$U(1)$	$U(1)_R$
S	32	$\Box$	1	1	0
Q	$\Box$	1	$\Box$	-4	$\frac{1}{2}$
Q^2		1	$\Box\Box$	-8	1
S^2Q^2		$\Box\Box$	1	-6	1
S^2Q		$\Box\Box$	$\Box$	-2	$\frac{1}{2}$
S^2		1	1	2	0
S^4		$\Box\Box\Box\Box$	1	4	0
$S^{4\prime}$		1	1	4	0
S^4Q^2		1	$\Box\Box$	-4	1
$S^4Q^{2\prime}$		$\Box\Box$	1	-4	1
S^4Q		$\Box\Box$	$\Box$	0	$\frac{1}{2}$
S^6Q^2		$\Box\Box$	1	-2	1
S^6Q		$\Box\Box$	$\Box$	2	$\frac{1}{2}$
S^8		1	1	8	0
S^8Q		1	$\Box$	4	$\frac{1}{2}$
S^4Q		1	$\Box$	0	$\frac{1}{2}$
S^6		1	1	6	0

$SO(10)$ **with (4,0,1)**

	$SO(10)$	$SU(4)$	$U(1)$	$U(1)_R$
S	16	$\Box$	1	0
Q	$\Box$	1	-8	1
Q^2		1	-16	2
S^2Q		$\Box\Box$	-6	1
S^4		$\begin{smallmatrix}\Box\Box\\\Box\Box\end{smallmatrix}$	4	0
S^6Q		$\Box\Box$	-2	1

$$W_{dyn} = \frac{1}{\Lambda^{15}}\Big[(S^6Q)^2(S^4) + (S^6Q)(S^2Q)(S^4)^2 + (S^2Q)^2(S^4)^3 + (S^4)^4(Q^2)\Big]$$

$SO(10)$ **with (3,0,3)**

	$SO(10)$	$SU(3)$	$SU(3)$	$U(1)$	$U(1)_R$
S	16	$\square$	1	1	0
Q	$\square$	1	$\square$	-2	$\frac{1}{3}$
Q^2		1	$\square\square$	-4	$\frac{2}{3}$
S^2Q		$\square\square$	$\square$	0	$\frac{1}{3}$
S^2Q^3		$\overline{\square}$	1	-4	1
S^4		$\overline{\square\square}$	1	4	0
S^4Q^2		$\square$	$\overline{\square}$	0	$\frac{2}{3}$

$$\begin{aligned} W_{dyn} \ = \ & \frac{1}{\Lambda^{15}}\Big[(S^4Q^2)^3 + (S^4Q^2)^2(S^2Q)^2 + (S^4Q^2)^2(S^4)(Q^2) + (S^2Q^3)^2(S^4)^2 \\ & +(S^2Q)^2(Q^2)^2(S^4)^2 + (S^2Q)^4(Q^2)(S^4) + (Q^2)^3(S^4)^3 + (S^2Q)^6 \\ & +(S^4)(S^2Q^3)(S^4Q^2)(S^2Q) + (S^4Q^2)(S^4)(S^2Q)^2(Q^2) \\ & +(S^4Q^2)(S^4Q)^4 + (S^2Q^3)(S^2Q)^3(S^4)\Big] \end{aligned}$$

$SO(10)$ **with (2,0,5)**

	$SO(10)$	$SU(2)$	$SU(5)$	$U(1)$	$U(1)_R$
S	16	$\square$	1	5	$\frac{1}{4}$
Q	$\square$	1	$\square$	-4	0
Q^2		1	$\square\square$	-8	0
S^2Q		$\square\square$	$\square$	6	$\frac{1}{2}$
S^2Q^3		1	$\overline{\begin{smallmatrix}\square\\ \square\end{smallmatrix}}$	-2	$\frac{1}{2}$
S^2Q^5		$\square\square$	1	-10	$\frac{1}{2}$
S^4		1	1	20	1
S^4Q^4		1	$\overline{\square}$	4	1

$SO(10)$ **with (3,1,1)**

	$SO(10)$	$SU(3)$	$U(1)_1$	$U(1)_2$	$U(1)_R$
S	16	$\square$	1	0	0
$\bar{S}$	$\overline{16}$	1	-3	1	0
Q	$\square$	1	0	-2	1
Q^2		1	0	-4	2
S^2Q		$\square\square$	2	-2	1
$S\bar{S}$		$\square$	-2	1	0
$S^3\bar{S}Q$		$\begin{smallmatrix}\square&\square\\\square&\end{smallmatrix}$	0	-1	1
$S^2\bar{S}^2$		$\square\square$	-4	2	0
S^4		$\overline{\square\square}$	4	0	0
$S^5\bar{S}$		$\square\square$	2	1	0
$S^4\bar{S}^2Q$		$\square$	-2	0	1
$\bar{S}^2Q$		1	-6	0	1
$S^3\bar{S}^3Q^2$		1	-6	-1	2

$SO(10)$ **with (2,1,3)**

	$SO(10)$	$SU(2)$	$SU(3)$	$U(1)_1$	$U(1)_2$	$U(1)_R$
S	16	$\square$	1	1	1	0
$\bar{S}$	$\overline{16}$	1	1	-2	1	$\frac{1}{2}$
Q	$\square$	1	$\square$	0	-2	0
Q^2		1	$\square\square$	0	-4	0
S^2Q		$\square\square$	$\square$	2	0	0
$\bar{S}^2Q$		1	$\square$	-4	0	1
$S\bar{S}$		$\square$	1	-1	2	$\frac{1}{2}$
$S^2\bar{S}^2$		$\square\square$	1	-2	4	1
S^2Q^3		1	1	2	-4	0
$S^3\bar{S}Q$		$\square$	$\square$	1	2	$\frac{1}{2}$
S^4		1	1	4	4	0
$S\bar{S}Q^2$		$\square$	$\overline{\square}$	-1	-2	$\frac{1}{2}$
$S^2\bar{S}^2Q^2$		1	$\overline{\square}$	-2	0	1
$S^3\bar{S}Q^3$		$\square$	1	1	-2	$\frac{1}{2}$

$SO(10)$ **with (1,1,5)**

	$SO(10)$	$SU(5)$	$U(1)_1$	$U(1)_2$	$U(1)_R$
S	16	1	1	5	$\frac{1}{4}$
$\overline{S}$	$\overline{16}$	1	-1	5	$\frac{1}{4}$
Q	$\square$	$\square$	0	-4	0
Q^2		$\square\square$	0	-8	0
S^2Q		$\square$	2	6	$\frac{1}{2}$
$\overline{S}^2Q$		$\square$	-2	6	$\frac{1}{2}$
$S\overline{S}$		1	0	10	$\frac{1}{2}$
S^2Q^5		1	2	-10	$\frac{1}{2}$
$\overline{S}^2Q^5$		1	-2	-10	$\frac{1}{2}$
$S\overline{S}Q^2$		$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	0	2	$\frac{1}{2}$
$S\overline{S}Q^4$		$\overline{\square}$	0	-6	$\frac{1}{2}$
$S^2\overline{S}^2$		1	0	20	1
$S^2\overline{S}^2Q^4$		$\overline{\square}$	0	4	1

$SO(10)$ **with (2,2,1)**

	$SO(10)$	$SU(2)$	$SU(2)$	$U(1)_1$	$U(1)_2$	$U(1)_R$
S	16	$\square$	1	1	1	0
$\overline{S}$	$\overline{16}$	1	$\square$	-1	1	0
Q	$\square$	1	1	0	-8	1
Q^2		1	1	0	-16	2
S^2Q		$\square\square$	1	2	-6	1
$\overline{S}^2Q$		1	$\square\square$	-2	-6	1
$S\overline{S}$		$\square$	$\square$	0	2	0
S^4		1	1	4	4	0
$\overline{S}^4$		1	1	-4	4	0
$S^2\overline{S}^2$		$\square\square$	$\square\square$	0	4	0
$S^3\overline{S}Q$		$\square$	$\square$	2	-4	1
$\overline{S}^3SQ$		$\square$	$\square$	-2	-4	1
$S^2\overline{S}^2Q^2$		1	1	0	-12	2
$S^4\overline{S}^2Q$		$\square\square$	1	2	-2	1
$\overline{S}^4S^2Q$		1	$\square\square$	-2	-2	1
$S^3\overline{S}^3$		$\square$	$\square$	0	6	0
$S^6\overline{S}^2$		1	1	4	8	0
$\overline{S}^6S^2$		1	1	-4	8	0

$SO(9)$ **with (4,0)**

	$SO(9)$	$SU(4)$	$U(1)_R$
S	16	$\square$	$\frac{1}{8}$
S^2		$\square\square$	$\frac{1}{4}$
S^4		$\begin{smallmatrix}\square\square\\\square\square\end{smallmatrix}$	$\frac{1}{2}$
S^6		$\square\square$	$\frac{3}{4}$

$$W_{dyn} = \frac{1}{\Lambda^{13}}\left[(S^6)^2(S^4) + (S^6)(S^4)^2(S^2) + (S^4)^4 + (S^4)^3(S^2)^2\right]$$

$SO(9)$ **with (3,2)**

	$SO(9)$	$SU(3)$	$SU(2)$	$U(1)$	$U(1)_R$
S	16	$\square$	1	1	0
Q	$\square$	1	$\square$	-3	$\frac{1}{2}$
Q^2		1	$\square\square$	-6	1
S^2Q		$\square\square$	$\square$	-1	$\frac{1}{2}$
S^2		$\square\square$	1	2	0
S^4		$\overline{\square\square}$	1	4	0
S^2Q^2		$\overline{\square}$	1	-4	1
S^4Q^2		$\square$	1	-2	1
S^4Q		$\square$	$\square$	1	$\frac{1}{2}$

$SO(9)$ **with (2,4)**

	$SO(9)$	$SU(2)$	$SU(4)$	$U(1)$	$U(1)_R$
S	16	$\square$	1	1	$\frac{1}{4}$
Q	$\square$	1	$\square$	-1	0
Q^2		1	$\square\square$	-2	0
S^2Q		$\square\square$	$\square$	1	$\frac{1}{2}$
S^2		$\square\square$	1	2	$\frac{1}{2}$
S^2Q^3		1	$\overline{\square}$	-1	$\frac{1}{2}$
S^2Q^2		1	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	0	$\frac{1}{2}$
S^4Q^3		1	$\overline{\square}$	1	1
S^2Q^4		$\square\square$	1	-2	$\frac{1}{2}$
S^4		1	1	4	1

$SO(8)$ **with (3,0,4)**

	$SO(8)$	$SU(4)$	$SU(3)$	$U(1)$	$U(1)_R$
Q	8_v	$\Box$	1	3	$\frac{1}{4}$
S	8_s	1	$\Box$	-4	0
Q^2		$\Box\Box$	1	6	$\frac{1}{2}$
S^2		1	$\Box\Box$	-8	0
S^2Q^2		$\begin{smallmatrix}\Box\\ \Box\end{smallmatrix}$	$\bar{\Box}$	-2	$\frac{1}{2}$
S^2Q^4		1	$\Box\Box$	4	1

$$W_{dyn} = \frac{1}{\Lambda^{11}}\Big[(S^2Q^4)^2(S^2) + (S^2Q^4)(S^2Q^2)^2 + (S^2Q^2)^3(Q^2) + (S^2)^3(Q^2)^4 + (S^2Q^2)^2(S^2)(Q^2)^2\Big]$$

$SO(8)$ **with (2,1,4)**

	$SO(8)$	$SU(4)$	$SU(2)$	$U(1)$	$U(1)$	$U(1)_R$
Q	8_v	$\Box$	1	1	0	$\frac{1}{4}$
S	8_s	1	$\Box$	-2	1	0
S'	8_c	1	1	0	-2	0
Q^2		$\Box\Box$	1	2	0	$\frac{1}{2}$
S^2		1	$\Box\Box$	-4	2	0
S'^2		1	1	0	-4	0
S^2Q^2		$\begin{smallmatrix}\Box\\ \Box\end{smallmatrix}$	1	-2	2	$\frac{1}{2}$
S^2Q^4		1	$\Box\Box$	0	2	1
S'^2Q^4		1	1	4	-4	1
$SS'Q$		$\Box$	$\Box$	-1	-1	$\frac{1}{4}$
$SS'Q^3$		$\bar{\Box}$	$\Box$	1	-1	$\frac{3}{4}$

$SO(8)$ **with (3,3,1)**

	$SO(8)$	$SU(3)$	$SU(3)$	$U(1)_1$	$U(1)_2$	$U(1)_R$
Q	8_v	1	1	0	6	1
S	8_s	$\Box$	1	1	-1	0
S'	8_c	1	$\Box$	-1	-1	0
Q^2		1	1	0	12	2
S^2		$\Box\Box$	1	2	-2	0
S'^2		1	$\Box\Box$	-2	-2	0
$SS'Q$		$\Box$	$\Box$	0	4	1
$S^3S'Q$		1	$\Box$	2	2	1
S'^3SQ		$\Box$	1	-2	2	1
$S^2S'^2$		$\bar{\Box}$	$\bar{\Box}$	0	-4	0

$SO(8)$ **with (2,2,3)**

	$SO(8)$	$SU(3)$	$SU(2)$	$SU(2)$	$U(1)_1$	$U(1)_2$	$U(1)_R$
Q	8_v	$\square$	1	1	0	4	0
S	8_s	1	$\square$	1	1	-3	$\frac{1}{4}$
S'	8_c	1	1	$\square$	-1	-3	$\frac{1}{4}$
Q^2		$\square\square$	1	1	0	8	0
S^2		1	$\square\square$	1	2	-6	$\frac{1}{2}$
S'^2		1	1	$\square\square$	-2	-6	$\frac{1}{2}$
$SS'Q$		$\square$	$\square$	$\square$	0	-2	$\frac{1}{2}$
S^2Q^2		$\overline{\square}$	1	1	2	2	$\frac{1}{2}$
S'^2Q^2		$\overline{\square}$	1	1	-2	2	$\frac{1}{2}$
$SS'Q^3$		1	$\square$	$\square$	0	6	$\frac{1}{2}$
$S^2S'^2$		1	1	1	0	-12	1
$S^2S'^2Q^2$		$\overline{\square}$	1	1	0	-4	1

$SO(7)$ **with (6,0)**

	$SO(7)$	$SU(6)$	$U(1)_R$
S	8	$\square$	$\frac{1}{6}$
S^2		$\square\square$	$\frac{1}{3}$
S^4		$\overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}}$	$\frac{2}{3}$

$$W_{dyn} = \frac{1}{\Lambda^9}\left[(S^4)^3 + (S^4)^2(S^2)^2 + (S^2)^6\right]$$

$SO(7)$ **with (5,1) [56]**

	$SO(7)$	$SU(5)$	$U(1)$	$U(1)_R$
S	8	$\square$	1	0
Q	$\square$	1	-5	1
Q^2		1	-10	2
S^2		$\square\square$	2	0
S^4		$\overline{\square}$	4	0
S^2Q		$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	-3	1
S^4Q		$\overline{\square}$	-1	1

$$W_{dyn} = \frac{1}{\Lambda^9}\Big[(S^4Q)^2(S^2) + (S^4Q)(S^2Q)(S^4) + (S^2Q)^2(S^4)(S^2)$$
$$+(Q^2)(S^2)(S^4)^2 + (S^2)^5(Q^2)\Big]$$

$SO(7)$ with (4,2)

	$SO(7)$	$SU(4)$	$SU(2)$	$U(1)$	$U(1)_R$
S	8	$\square$	1	1	0
Q	$\square$	1	$\square$	-2	$\frac{1}{2}$
Q^2		1	$\square\square$	-4	1
S^2		$\square\square$	1	2	0
S^2Q		$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\square$	0	$\frac{1}{2}$
S^2Q^2		$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	1	-2	1
S^4		1	1	4	0
S^4Q		1	$\square$	2	$\frac{1}{2}$

$$W_{dyn} = \frac{1}{\Lambda^9}\Big[(S^4Q)^2(Q^2) + (S^4Q)(S^2Q)(S^2Q^2) + (S^2Q)^2(S^2Q^2)(S^2) + (S^4)(S^2Q^2)^2 + (S^2Q)^2(S^2)^2(Q^2) + (S^2Q^2)^2(S^2)^2 + (S^2)^4(Q^2)^2\Big]$$

$SO(7)$ with (3,3)

	$SO(7)$	$SU(3)$	$SU(3)$	$U(1)$	$U(1)_R$
S	8	$\square$	1	1	0
Q	$\square$	1	$\square$	-1	$\frac{1}{3}$
Q^2		1	$\square\square$	-2	$\frac{2}{3}$
S^2		$\square\square$	1	2	0
S^2Q		$\overline{\square}$	$\square$	1	$\frac{1}{3}$
S^2Q^2		$\overline{\square}$	$\overline{\square}$	0	$\frac{2}{3}$
S^2Q^3		$\square\square$	1	-1	1

$$W_{dyn} = \frac{1}{\Lambda^9}\Big[(S^2Q^3)^2(S^2) + (S^2Q^3)(S^2Q^2)(S^2Q) + (S^2Q^2)^3 + (S^2)^3(Q^2)^3 + (S^2Q^2)^2(S^2)(Q^2) + (S^2Q)^2(S^2)(Q^2)^2 + (S^2Q)^2(S^2Q^2)(Q^2)\Big]$$

The $SO(N)$ theories with $\sum \mu_i - \mu_G = 4$

Our normalization for the indices of SO groups is somewhat non-standard. It follows from demanding that the index is invariant under flows from $SO(2N)$ groups to their $SU(N)$ subgroups. In the normalization where the index of the vector is one rather than two, it is obvious that one can obtain a superpotential that is regular at the origin for $\sum \mu_i - \mu_G = 1$ or 2. In our normalization, this corresponds to $\sum \mu_i - \mu_G = 2$ or 4. We have explicitly checked that none of the $\sum \mu_i - \mu_G = 4$ theories are s-confining by identifying flows to non-s-confining theories.

The $\sum \mu_i - \mu_G = 4$ $SO(N)$ theories are examples of the special case where the confining superpotential can be holomorphic at the origin without Eq. 4.8 being satisfied. This can only happen when μ_G and all μ_i have a common divisor. Just like the previously mentioned $\sum \mu_i - \mu_G = 4$ $SO(N)$ theories, such theories are unlikely to s-confine. The reason is that while Eq. 4.8 is preserved under most flows along flat directions, the property that μ_G and all μ_i have a common divisor is not. Thus for most such theories one should be able to find a flow to a non-s-confining theory. We expect that none of these "common divisor" theories s-confine.

4.4.4 Exceptional groups

The analysis for exceptional groups G_2, F_4, E_6, E_7, and E_8 is surprisingly simple. The s-confined spectrum of a G_2 gauge theory with 5 fundamentals has already been worked out in Ref. [57, 58]. The representations of G_2 are real, thus the invariant tensors include the two index symmetric tensor. Furthermore, there are two totally antisymmetric tensors with three and four indices, respectively. Therefore, the confined spectrum is

G_2 with 5 $\Box$ [57]

	G_2	$SU(5)$	$U(1)_R$
Q	7	$\Box$	$\frac{1}{5}$
$M = Q^2$		$\Box\Box$	$\frac{2}{5}$
$A = Q^3$		$\overline{\begin{smallmatrix}\Box\\\Box\end{smallmatrix}}$	$\frac{3}{5}$
$B = Q^4$		$\overline{\Box}$	$\frac{4}{5}$

$$W_{dyn} = \frac{1}{\Lambda^7}\left[M^5 + M^2A^2 + MB^2 + A^2B\right]$$

The F_4, E_6, E_7 and E_8 theories

Theories based on any of the other exceptional gauge groups can be shown to flow to theories which are not s-confining. This is derived most easily by starting with the real group F_4. The lowest dimensional representations of F_4 are the 26 dimensional fundamental representation and the 52 dimensional adjoint. Since any theory with adjoint matter has a Coulomb branch on its moduli space, we can restrict our attention to theories with only fundamentals. By giving an expectation value to a fundamental one can break F_4 to its maximal subgroup $SO(9)$. Under $SO(9)$ the representations decompose as follows: $26 \to 1 + 9 + 16$ and $52 \to 16 + 36$. The 9, 16, 36 are the fundamental, spinor, and adjoint of $SO(9)$. When giving an expectation value to a fundamental of F_4, the spinor component of its $SO(9)$ decomposition is eaten. Thus an F_4 theory with N_f fundamentals flows to an $SO(9)$ theory with N_f fundamentals and $N_f - 1$ spinors. For no N_f is this $SO(9)$ theory s-confining, therefore no F_4 theory s-confines.

Using this result, it is easy to show that none of the groups E_6, E_7, and E_8 s-confine. The lowest dimensional representations of E_6 are the (complex) fundamental and the adjoint. By giving an expectation value to a fundamental, one can flow to F_4, whereas expectation values for an adjoint lead to a Coulomb branch. Thus, E_6 theories cannot be s-confining either.

By giving an expectation value to a field in the 56 dimensional fundamental representation of E_7 one can flow to E_6, while an expectation value for the adjoint again yields a Coulomb branch. For E_8 the lowest dimensional representation is the adjoint, again leading to a Coulomb branch. Thus none of the $E_{6,7,8}$ groups with arbitrary matter are s-confining.

4.5 Summary

In this chapter, we presented all s-confining $N = 1$ SUSY gauge theories based on a single gauge group. These theories are important for the following reasons. First, by integrating out vector-like matter one can find exact results for theories with smaller matter content.

The second possible application of our results on s-confinement is to composite model building. Recently, several examples of models with quark-lepton compositeness have been given [60, 69, 70]. All these models rely on the recent exact results for the infrared spectra of s-confining theories. In these models the dynamically generated superpotentials can be used to give a natural explanation of the hierarchy between the top and bottom quark mass [69]. A toy model based on $Sp(6)$ with an antisymmetric tensor [60] has the interesting feature that it generates three generations of quarks with a hierarchical structure for the Yukawa couplings dynamically. We hope that the wealth of new s-confining theories listed in this paper can be applied to build further interesting and realistic models of compositeness.

Finally, our results can also be applied to dynamical supersymmetry breaking. Ref. [61] has several new examples of supersymmetry breaking models which illustrate different dynamical mechanisms. These models use either s-confining theories, or theories obtained from them by integrating out flavors. Many other new models can be built using our exact results.

Chapter 5

Dynamical Supersymmetry Breaking[1]

In this chapter we will discuss several models of dynamical supersymmetry breaking. We first review the physical motivation to consider dynamical supersymmetry breaking (DSB). Next we discuss the general principles for building models of DSB. We apply these tools in the following sections, where we analyze the theories based on the products groups $SU(n) \times SU(m) \times U(1)$ obtained by decomposing the field content of an $SU(n+m)$ theory with an antisymmetric tensor and $n+m-4$ antifundamentals.

5.1 Why dynamical supersymmetry breaking?

As discussed in the Introduction, supersymmetry solves the hierarchy problem, i.e. it eliminates all dangerous quadratic divergences that could destabilize the ratio $\frac{M_{weak}}{M_{Planck}}$. However, supersymmetry itself does not explain the origin of the hierarchy between these scales. On the other hand, supersymmetry must be broken, and it is natural to expect that it is broken spontaneously. However, we have seen in Chapter 2 that within the MSSM it is not possible to break supersymmetry spontaneously and maintain a viable model. Therefore, the usual assumption is that there are two sectors of physics. One sector breaks supersymmetry spontaneously, but it is not directly coupled to the fields of the MSSM (the visible sector). The MSSM learns of supersymmetry breaking in the other sector only through gravitational or gauge interactions. The weak scale is determined depending on the method of communication of SUSY breaking by $M_{weak} = \frac{M_S^2}{M_{Planck}}$ for gravity mediation and by $M_{weak} = \alpha^n M_S$ $(n = 2, 3)$ for gauge mediation, where M_S is the scale of supersymmetry breaking, while $\alpha = \frac{g^2}{4\pi}$, with g being the gauge coupling. One can see that the above formulae for M_{weak} translate the question of why the weak scale is so small to the question of why the scale of supersymmetry breaking is so small compared to the Planck scale.

This latter question however can be explained by assuming that supersymmetry is dynamically broken. Dynamical breaking means that supersymmetry is not broken at tree-level (or in perturbation theory), but by non-perturbative effects due to strong dynamics. In this case $M_S \propto \Lambda$, where Λ is the dynamical scale of the asymptotically free gauge group.

[1]Based on research done in collaboration with Lisa Randall, Witold Skiba and Robert Leigh reported in Refs. [74, 75].

Since

$$\Lambda = e^{-\frac{8\pi^2}{g^2}} M_{Planck},$$

$M_S \ll M_{Planck}$ can be naturally explained due to the logarithmic running of the gauge coupling.

Thus, to summarize, the motivation for dynamical supersymmetry breaking is that it could naturally explain the smallness of the weak scale compared to the Planck scale.

5.2 The basics of dynamical supersymmetry breaking

In order to complete the program discussed in the previous section, one needs to actually find models which do break supersymmetry dynamically. This turns out to be quite a difficult task. In the following, we review some general arguments which will lead us in looking for models of dynamical supersymmetry breaking.

5.2.1 Witten's argument

Witten suggested to consider the index $\mathrm{Tr}(-1)^F = n_B - n_F$, where n_B is the number of bosonic zero energy states while n_F is the number of fermionic zero energy states [71]. Since supersymmetry is broken if and only if the ground state energy is non-zero, a non-vanishing value of the Witten index indicates unbroken supersymmetry. However if supersymmetry is broken, $n_B = n_F = 0$, and so the Witten index must be vanishing. Since the Witten index can take only integer values, it does not change with continuous changes of the parameters of the theory. Therefore, if one has a non-chiral theory, one can add a mass term to every field and take the masses to infinity. This does not change the value of the Witten index and we are left with a pure Yang-Mills theory. However, Witten showed that pure Yang-Mills theories always have a non-vanishing Witten index and so non-chiral theories can not break supersymmetry. Therefore when looking for models of dynamical supersymmetry breaking one should look for chiral theories.

5.2.2 The ADS conditions for dynamical supersymmetry breaking

Affleck, Dine and Seiberg (ADS) showed a sufficient condition for dynamical supersymmetry breaking [72]. They argued, that supersymmetry is dynamically broken, if the following two conditions are satisfied

- there are no classical flat directions in the theory
- there is a spontaneously broken global symmetry.

Their argument was based on the observation that if there is a broken global symmetry, then there must be a massless scalar in the theory. However, the scalar partner of this field can not be massless since that would imply that there is a flat direction in the theory, which contradicts the first assumption. Therefore they argued that supersymmetry must be spontaneously broken as well.

5.2.3 The method of DNNS

Dine, Nelson, Nir and Shirman noted, that a convenient way to find new models of dynamical supersymmetry breaking is by looking at an existing model and decompose the field content under a subgroup of the original group [73]. This method is guaranteed to yield an anomaly free field content and often turns out to produce new models of dynamical supersymmetry breaking. However, no argument exists based on which one could decide when this method will work and when not.

5.3 The $SU(N)$ model of ADS

The first model known to break supersymmetry dynamically was the $SU(N)$ theory with an antisymmetric tensor A and $N-4$ antifundamental fields $\bar{F}_i$ (with N being odd).

First consider the case of $N=5$. Here there are no classical flat directions, since all gauge invariant operators made out of A and a single $\bar{F}$ vanish. The theory has a non-anomalous $U(1) \times U(1)_R$ symmetry, and ADS argued based on anomaly matching, that one of the $U(1)$ is spontaneously broken in the ground state [72]. Therefore supersymmetry is broken as well.

Supersymmetry is broken for larger N's as well. In this case we need to add a tree-level superpotential

$$W_{tree} = \lambda_{ij} A \bar{F}_i \bar{F}_j$$

in order to lift the classical flat directions, where λ_{ij} is a matrix of maximal $(N-5)$ rank. If we consider the theory before adding the tree-level superpotential, then one has flat directions. Along these flat directions, the $SU(N)$ group is broken to $SU(5)$ with the field content $\mathbf{10} + \bar{\mathbf{5}}$, which is exactly the $SU(5)$ theory considered previously. This $SU(5)$ theory breaks supersymmetry, and the vacuum energy is proportional to the scale of the $SU(5)$ group, $E_{vac} \propto \Lambda_5$. The scale Λ_5 is determined by matching to be $\Lambda_5 = \frac{\Lambda_N^{\frac{2N-3}{13}}}{\Phi^{\frac{2N-10}{13}}}$, where Φ is a generic VEV along the flat directions. Thus the vacuum energy is $E_{vac} \propto \frac{1}{\Phi^{\frac{2N-10}{13}}}$. When the flat directions are lifted by the superpotential, supersymmetry is broken without a runaway vacuum.

Now we can apply the method of DNNS to construct other models of dynamical supersymmetry breaking starting from the presented $SU(N)$ theories. First let us decompose the $SU(5)$ model to $SU(3) \times SU(2)$. The resulting field content is

	$SU(3)$	$SU(2)$
Q	$\square$	$\square$
$\bar{U}$	$\bar{\square}$	1
$\bar{D}$	$\bar{\square}$	1
L	1	$\square$

Adding a tree-level superpotential to this theory

$$W_{tree} = Q\bar{U}L$$

lifts all flat directions. Considering the limit $\Lambda_3 \gg \Lambda_2$, we see that we have $SU(3)$ supersymmetric QCD with two flavors, and thus a superpotential

$$W_{dyn} = \frac{1}{(Q\bar{U})(Q\bar{D})}$$

is generated by the $SU(3)$ dynamics. This superpotential together with the tree-level superpotential break supersymmetry dynamically. This is the famous 3-2 model of Affleck, Dine and Seiberg [49].

One can decompose the $SU(N)$ models similarly to $SU(N-1)\times U(1)$ or to $SU(N-2)\times SU(2)\times U(1)$. Dine, Nelson, Nir and Shirman showed [73], that one can find a suitable tree-level superpotential in both cases to lift the flat directions, and the $SU(N-1)$ group in one case or the $SU(N-2)$ group in the other case has sufficiently few flavors to generate a dynamical superpotential term. Therefore both of these theories break supersymmetry.

However, if one considers other decompositions of the $SU(N)$ model, one does not get a dynamical superpotential in any group factors. We will show in the following sections, that these models nevertheless break supersymmetry [74, 75].

5.4 The 4-3-1 model

In the next two sections we analyze the theories based on the gauge group $SU(n)\times SU(3)\times U(1)$ [74]. Because the gauge dynamics are very different for $n=4$ and $n>4$, we first consider the gauge group $SU(4)\times SU(3)\times U(1)$. The particular models we explore are based on an idea discussed in Ref. [73], where it was suggested to search for models which dynamically break supersymmetry by taking a known model and removing generators to reduce the gauge group. This method is guaranteed to generate an anomaly free chiral theory which has the potential to break supersymmetry. There are several known examples of theories with a suitable superpotential respecting the less restrictive gauge symmetries of the resultant theory, in which supersymmetry is broken without runaway directions. However, there is as yet no proof that this method will necessarily be successful.

Unlike previous models in the literature, neither of the nonabelian gauge groups generates a dynamical superpotential in the absence of the perturbations added at tree level. Because neither factor generates a dynamical superpotential, there is no limit in which the theory can be analyzed perturbatively. Therefore, we derive the exact superpotential for the $n=4$ case which we use to show supersymmetry is broken in the strongly interacting theory.

The $SU(4)\times SU(3)\times U(1)$ model is interesting for several reasons. First, the demonstration of supersymmetry breaking involves a subtle interplay between the confining dynamics and the tree-level superpotential of the theory. Second, this model implements the mechanism of [76, 77] without introducing additional singlets or potential runaway directions. Third, we can lift all the flat directions by a renormalizable superpotential. Fourth, none of the gauge groups generates a dynamical superpotential; the fields are kept from the origin solely by a quantum modified constraint.

In addition, the exact superpotential exhibits several novel features. First, fields with quantum numbers corresponding to classically vanishing gauge invariant operators emerge, and play the role of Lagrange multipliers for known constraints. Second, we find that classical constraints can be modified not only by a constant, but by field dependent terms which vanish in the classical limit. Third, fields which are independent in the classical theory satisfy linear constraints in the quantum theory. By explicitly substituting the solution to the equation of motion for these fields, we show that quantum analogs of the classical constraints are still satisfied.

The $SU(n)\times SU(3)\times U(1)$ theories (to be discussed in the next section) for $n>4$ are less tractable but nonetheless very interesting. We show that it is possible to introduce Yukawa couplings which lift all classical flat directions. We then consider the low-energy limit of this theory. The $SU(3)$ gauge group without the perturbative superpotential is not confining. However, the $SU(n)$ confined theory in the presence of Yukawa couplings induces masses for sufficiently many flavors that there is a dynamical superpotential associated with both the $SU(3)$ and $SU(n)$ dynamics. This low-energy superpotential depends non-trivially on both the strong dynamical scales of the low-energy theory and the Yukawa couplings of the microscopic theory. We consider this model with and without Yukawa couplings which lift the baryon flat directions. In the first case, the theory is too complicated to solve. The form of the low-energy superpotential permitted by the symmetries is nonetheless quite interesting in that it mixes the perturbative and strong dynamics. In the second case, we can explicitly derive that supersymmetry is broken. In either case, there is a spontaneously broken global $U(1)$ symmetry, so we conclude this theory probably breaks supersymmetry and has no dangerous runaway directions when all required Yukawa couplings are nonvanishing.

We first describe the $SU(4) \times SU(3) \times U(1)$ model classically. In particular, we show that the model has no classical flat directions. Then we analyze the quantum mechanical theory in the strongly interacting regime and show that the model breaks supersymmetry. In the next Section, we discuss generalizations to $SU(n) \times SU(3) \times U(1)$.

5.4.1 The classical $SU(4) \times SU(3) \times U(1)$ theory

The field content of the model we study is obtained by decomposing the chiral multiplets of an $SU(7)$ theory with the field content consisting of an antisymmetric tensor and three anti-fundamentals into its $SU(4) \times SU(3) \times U(1)$ subgroup. The fields are:

$$A^{\alpha\beta}(6,1)_6,\ \bar{Q}_a(1,\bar{3})_{-8},\ T^{\alpha a}(4,3)_{-1},\ \bar{F}_{\alpha I}(\bar{4},1)_{-3},\ \bar{Q}_{ai}(1,\bar{3})_4,$$

where $i, I = 1,2,3$ are flavor indices, while Greek letters denote $SU(4)$ indices and Latin ones correspond to $SU(3)$. In this notation $(n,m)_q$ denotes a field that transforms as an n under $SU(4)$, m under $SU(3)$ and has $U(1)$ charge q.

We take the classical superpotential to be

$$\begin{aligned} W_{cl} &= A^{\alpha\beta}\bar{F}_{\alpha 1}\bar{F}_{\beta 2} + T^{\alpha a}\bar{Q}_{a1}\bar{F}_{\alpha 1} + T^{\alpha a}\bar{Q}_{a2}\bar{F}_{\alpha 2} + T^{\alpha a}\bar{Q}_{a3}\bar{F}_{\alpha 3} + \\ &\quad \bar{Q}_a\bar{Q}_{b2}\bar{Q}_{c1}\epsilon^{abc}. \end{aligned} \tag{5.1}$$

We will show shortly that this superpotential lifts all D-flat directions.

From the fundamental fields we can construct operators which are invariant under the gauge symmetries of the theory. We first list those which are invariant under $SU(4)\times SU(3)$ and subsequently construct operators which are also $U(1)$ invariant. Later on it will be important to distinguish operators invariant under the confining gauge groups but which carry $U(1)$ charge.

$$\begin{array}{rcl} M_{iI} = & T^{\alpha a}\bar{Q}_{ai}\bar{F}_{\alpha I} & 0 \\ M_{4I} = & T^{\alpha a}\bar{Q}_a\bar{F}_{\alpha I} & -12 \\ X_{IJ} = & A^{\alpha\beta}\bar{F}_{\alpha I}\bar{F}_{\beta J} & 0 \\ X_{I4} = & \frac{1}{6}A^{\beta\alpha}\bar{F}_{\beta I}\epsilon_{\alpha\gamma\delta\zeta}T^{\gamma a}T^{\delta b}T^{\zeta c}\epsilon_{abc} & 0 \\ \mathrm{Pf}A = & \epsilon_{\alpha\beta\gamma\delta}A^{\alpha\beta}A^{\gamma\delta} & 12 \\ Y_{ij} = & \epsilon_{\alpha\beta\gamma\delta}A^{\alpha\beta}T^{\gamma a}\bar{Q}_{ai}T^{\delta b}\bar{Q}_{bj} & 12 \\ Y_{i4} = & \epsilon_{\alpha\beta\gamma\delta}A^{\alpha\beta}T^{\gamma a}\bar{Q}_{ai}T^{\delta b}\bar{Q}_b & 0 \\ \bar{B} = & \frac{1}{6}\bar{F}_{\alpha I}\bar{F}_{\beta J}\bar{F}_{\gamma K}\epsilon^{IJK}T^{\alpha a}T^{\beta b}T^{\gamma c}\epsilon_{abc} & -12 \\ \bar{b}^i = & -\frac{1}{2}\bar{Q}_a\bar{Q}_{bj}\bar{Q}_{ck}\epsilon^{ijk}\epsilon^{abc} & 0 \end{array} \tag{5.2}$$

$$\bar{b}^4 = \qquad \tfrac{1}{6}\bar{Q}_{ai}\bar{Q}_{bj}\bar{Q}_{ck}\epsilon^{ijk}\epsilon^{abc} \qquad 12$$

The right hand side column indicates the charges of the operators under the $U(1)$ gauge group. All other $SU(4) \times SU(3)$ invariants can be obtained as products of these operators. The classical constraints obeyed by these fields are:

$$\begin{aligned}
&4\, X_{I4}X_{JK}\epsilon^{IJK} - \bar{B}\,\mathrm{Pf}A = 0\\
&\epsilon^{ijk}\epsilon^{IJK}\left(\mathrm{Pf}A\, M_{iI}M_{jJ}M_{kK} - 6Y_{ij}M_{kI}X_{JK}\right) = 0\\
&\epsilon^{ijk}\epsilon^{IJK}\left(\mathrm{Pf}A\, M_{4I}M_{jJ}M_{kK} - 2Y_{jk}M_{4I}X_{JK} + 4Y_{j4}M_{kI}X_{JK}\right) = 0\\
&Y_{i4}\bar{b}^i = 0\\
&\bar{B}\bar{b}^4 - \frac{1}{6}\epsilon^{ijk}\epsilon^{IJK}M_{iI}M_{jJ}M_{kK} = 0\\
&\bar{B}\epsilon^{kij}Y_{ij} - 2\,\epsilon^{kij}\epsilon^{IJK}M_{iI}M_{jJ}X_{K4} = 0\\
&M_{4I}\bar{b}^4 + M_{iI}\bar{b}^i = 0\\
&\epsilon^{ijk}Y_{jk}M_{4I} + 2\,\epsilon^{ijk}M_{jI}Y_{k4} + 4\,X_{I4}\bar{b}^i = 0\\
&\epsilon^{IJK}\epsilon^{ijk}M_{iI}M_{jJ}M_{4K}Y_{k4} = 0.
\end{aligned} \tag{5.3}$$

The completely gauge invariant fields can be formed by taking products of the above $U(1)$ charged fields. However, most of these combinations turn out to be products of other completely gauge invariant operators. As an operator basis we can use the neutral fields from Eq. 5.2 and $E_I = M_{4I}\mathrm{Pf}A$. These operators are subject to the following classical constraints:

$$\begin{aligned}
&\epsilon^{IJK}E_J M_{iK}\bar{b}^i = 0\\
&Y_{i4}\bar{b}^i = 0\\
&\epsilon^{IJK}\epsilon^{ijk}M_{iI}M_{jJ}E_K Y_{k4} = 0\\
&\epsilon^{IJK}\epsilon^{ijk}M_{iI}M_{jJ}Y_{k4}M_{lK}\bar{b}^l = 0
\end{aligned} \tag{5.4}$$

These constraints follow from Eq. 5.3. We have omitted the linear constraints following from Eq. 5.3 which define additional unnecessary fields. These operators obeying the above constraints parameterize the D-flat directions of the theory.

In terms of the invariants defined above we can express the superpotential as

$$W_{cl} = X_{12} + M_{11} + M_{22} + M_{33} + \bar{b}^3. \tag{5.5}$$

We now show that this superpotential suffices to lift all D-flat directions. It is easiest to show this (using the results of Ref. [47]) by demonstrating that the holomorphic invariants which parameterize the flat directions are all determined by the equations of motion (as opposed to parameterizing the flat directions in terms of the fundamental fields). If all holomorphic invariants are determined, we can conclude that all potential flat directions are lifted.

We consider the equations of motion corresponding to the classical superpotential of Eq. 5.1. The equation $\frac{\partial W}{\partial A}$ sets X_{12} to zero if we multiply by A. Forming all gauge invariant combinations from $\frac{\partial W}{\partial \bar{Q}_{ai}}$ we obtain the following. Multiplying $\frac{\partial W}{\partial \bar{Q}_{a3}}$ by $\bar{Q}_{aj}$ gives

$$M_{j3} = 0,$$

similarly for $\frac{\partial W}{\partial \bar{Q}_{a1,2}}$ we obtain

$$M_{12} = 0 \quad M_{22} + \bar{b}^3 = 0 \quad M_{32} - \bar{b}^2 = 0$$
$$M_{21} = 0 \quad M_{11} + \bar{b}^3 = 0 \quad M_{31} - \bar{b}^1 = 0.$$

Next, we multiply the same equations by $\epsilon_{abc} T^{\beta b} T^{\gamma c} A^{\delta\rho} \epsilon_{\beta\gamma\delta\rho}$ to obtain

$$X_{34} = 0 \;\; Y_{24} + 2X_{14} = 0 \;\; Y_{14} - 2X_{24} = 0.$$

Also, by multiplying $\frac{\partial W}{\partial \bar{Q}_{ai}}$ by $\bar{Q}_a \mathrm{Pf} A$ we get

$$E_I = 0.$$

Next, from $\frac{\partial W}{\partial \bar{Q}_a} \bar{Q}_a$ we obtain that

$$\bar{b}^3 = 0.$$

We obtain the remaining equations from $\frac{\partial W}{\partial \bar{F}_{aI}}$. They are:

$$M_{13} - X_{23} = 0 \quad M_{23} + X_{13} = 0 \quad M_{3I} = 0$$
$$E_2 + 4Y_{14} = 0 \quad E_1 - 4Y_{24} = 0 \quad Y_{34} = 0$$

The only solution to these equations sets all operators to be zero. Therefore, our theory does not have flat directions.

In Ref. [72] it was argued that theories which have no flat directions, but preserve an anomaly free R symmetry break supersymmetry spontaneously if the $U(1)_R$ symmetry is spontaneously broken in the vacuum. This follows because there would be a massless pseudoscalar, which is unlikely to have a massless scalar partner. The superpotential of Eq. 5.1 preserves an R symmetry under which the R charges are $R(A) = R(\bar{F}_3) = 0$, $R(\bar{F}_1) = R(\bar{F}_2) = 1$, $R(\bar{Q}_1) = R(\bar{Q}_2) = \frac{5}{3}$, $R(\bar{Q}_3) = \frac{8}{3}$, $R(\bar{Q}) = -\frac{4}{3}$ and $R(T) = -\frac{2}{3}$. Although this symmetry is anomalous with respect to the $U(1)$ gauge group, if it is spontaneously broken, the associated Goldstone boson is nonetheless massless so the argument of Ref. [72] should still apply.

Notice that the classical equations of motion in our theory have a solution only where all fields vanish. In the next section we show that the quantum theory does not permit such a supersymmetric solution, so that supersymmetry is broken.

5.4.2 The quantum $SU(4) \times SU(3) \times U(1)$ theory

In this section we will derive the exact superpotential of the $SU(4) \times SU(3) \times U(1)$ theory. The fact that it is possible to determine the exact superpotential of the theory will enable us to prove that supersymmetry is dynamically broken.

Before proceeding, we list the global symmetries of the microscopic fields, which are useful when constraining the form of the exact superpotential. The global symmetries are:

	$U(1)_A$	$U(1)_{\bar{Q}}$	$U(1)_T$	$U(1)_{\bar{F}}$	$SU(3)_{\bar{F}_I}$	$U(1)_{\bar{Q}_i}$	$SU(3)_{\bar{Q}_i}$	$U(1)_R$
A	1	0	0	0	1	0	1	0
$\bar{Q}$	0	1	0	0	1	0	1	0
T	0	0	1	0	1	0	1	0
$\bar{F}_I$	0	0	0	1	3	0	1	0
$\bar{Q}_i$	0	0	0	0	1	1	3	0
Λ_3^5	0	1	4	0	1	3	1	−2
Λ_4^8	2	0	3	3	1	0	1	0

The only invariants under all global symmetries including $U(1)_R$ are $\mathcal{A} = X_{IJ}X_{K4}\epsilon^{IJK}/\Lambda_4^8$ and $\mathcal{B} = \bar{B}\text{Pf}A/\Lambda_4^8$.

We now identify the proper degrees of freedom. To do so, it is convenient to first take the limit $\Lambda_3 \gg \Lambda_4$ and construct $SU(3)$ invariant operators which are mesons and baryons formed from the $SU(3)$ charged fields, and then to construct the $SU(4)$ bound states of these fields. This gives us the spectrum which matches anomalies of the original microscopic theory, independent of the ratio Λ_3/Λ_4.

Below the $SU(3)$ scale, the theory can be described by an $SU(4)$ theory with an antisymmetric tensor and four flavors. These four flavors are

$$\begin{aligned}
\bar{F}_{\alpha 4} &= \frac{1}{6}\epsilon_{\beta\gamma\delta\alpha}T^{\beta a}T^{\gamma b}T^{\delta c}\epsilon_{abc},\\
F_i^\alpha &= T^{\alpha a}\bar{Q}_{ai},\ i = 1,2,3\\
F_4^\alpha &= T^{\alpha a}\bar{Q}_a,
\end{aligned} \tag{5.6}$$

The three remaining antifundamentals are $\bar{F}_{\alpha I}$, $I = 1,2,3$, the original fields. The $SU(3)$ antibaryons are the $\bar{b}^i$'s of Eq. 5.2, which are singlets under $SU(4)$.

The four-flavor theory with an antisymmetric tensor has been described in Ref. [56]. The confined states of the $SU(4)$ theory are

$$\begin{aligned}
\text{Pf}A &= \epsilon_{\alpha\beta\gamma\delta}A^{\alpha\beta}A^{\gamma\delta}\\
M_{iI} &= F_i^\alpha \bar{F}_{\alpha I}\\
X_{IJ} &= A^{\alpha\beta}\bar{F}_{\alpha I}\bar{F}_{\beta J}\\
Y_{ij} &= A^{\alpha\beta}F_i^\gamma F_j^\delta \epsilon_{\alpha\beta\gamma\delta}\\
B &= \frac{1}{24}F_i^\alpha F_j^\beta F_k^\gamma F_l^\delta \epsilon_{\alpha\beta\gamma\delta}\epsilon^{ijkl}\\
\bar{B} &= \frac{1}{24}\bar{F}_{\alpha I}\bar{F}_{\beta J}\bar{F}_{\gamma K}\bar{F}_{\delta L}\epsilon^{\alpha\beta\gamma\delta}\epsilon^{IJKL}.
\end{aligned} \tag{5.7}$$

Here the indices i and I range from 1 to 4. Note that B, M_{44} and M_{i4} are fields which vanish classically. However, anomaly matching of the microscopic theory to the low-energy theory requires the presence of these fields. Fields other than B, M_{44} and M_{i4} correspond to operators introduced in Eq. 5.2. The low-energy theory consists of the fields listed in Eq. 5.2 and the new fields B, M_{44}, and M_{i4}.

In order to construct the superpotential it is again convenient to consider the limit $\Lambda_3 \gg \Lambda_4$. Below the Λ_3 scale, there is an $SU(4)$ theory with four flavors and an antisymmetric tensor together with the confining $SU(3)$ superpotential of Ref. [46]. The superpotential for the four-flavor $SU(4)$ theory with an antisymmetric tensor has been described in Ref. [56]. We determined the coefficients in the superpotential of Ref. [56] by requiring that the equations of motion reproduce the classical constraints.

In this limit, the superpotential has to be the sum of the contributions from $SU(3)$ and $SU(4)$ dynamics. The exact superpotential is therefore of the form:

$$\begin{aligned} W &= \bar{b}^3 + X_{12} + M_{11} + M_{22} + M_{33} + \frac{1}{\Lambda_3^5}\left(M_{i4}\bar{b}^i - B\right) + \\ & f(\mathcal{A},\mathcal{B}) \cdot \frac{1}{24\,\Lambda_3^5\Lambda_4^8}\Big(24\,BX_{IJ}X_{KL}\epsilon^{IJKL} + 6\,\bar{B}Y_{ij}Y_{kl}\epsilon^{ijkl} - 24\,B\bar{B}\mathrm{Pf}A + \\ & \mathrm{Pf}A\epsilon^{ijkl}\epsilon^{IJKL}M_{iI}M_{jJ}M_{kK}M_{lL} - 12\,\epsilon^{ijkl}Y_{ij}M_{kI}M_{lJ}X_{KL}\epsilon^{IJKL}\Big), \end{aligned} \tag{5.8}$$

where f is an as yet undetermined function of the symmetry invariants $\mathcal{A}$ and $\mathcal{B}$, and $i, I = 1, \ldots, 4$. Therefore, the symmetries together with the limit $\Lambda_3 \gg \Lambda_4$ restrict the superpotential up to a function of $\mathcal{A}$ and $\mathcal{B}$. However, a negative power series in $\mathcal{A}$ or $\mathcal{B}$ would imply unphysical singularities, since there is no limit in which the number of flavors in the $SU(4)$ theory is less than the number of colors. On the other hand, a positive power series in $\mathcal{A}$ or $\mathcal{B}$ would not correctly reproduce the limit where $\Lambda_4 \gg \Lambda_3$. In this limit one has an $SU(4)$ theory with an antisymmetric tensor and three flavors, which yields a quantum modified constraint [64]. Observe the amazing fact that the B equation of motion which involves the superpotential from both the $SU(3)$ and $SU(4)$ terms exactly reproduces this $SU(4)$ quantum modified constraint. This is only true with no further modification of the second term. In fact, this is what permits us to fix the relative coefficient of the two terms in parentheses. Thus we conclude that $f(\mathcal{A},\mathcal{B}) \equiv 1$.

We stress again that each of the fields B, M_{i4}, and M_{44} vanish classically. In the quantum theory, the B field acts as a Lagrange multiplier for the three flavor $SU(4)$ quantum modified constraint. The M_{i4} and M_{44} equations of motion are

$$\begin{aligned} &\epsilon^{ijk}\epsilon^{IJK}\left(\mathrm{Pf}AM_{iI}M_{jJ}M_{kK} - 6Y_{ij}M_{kI}X_{JK}\right) = 6\Lambda_4^8\bar{b}^4 \\ &\epsilon^{ijk}\epsilon^{IJK}\left(\mathrm{Pf}AM_{4I}M_{jJ}M_{kK} - 2Y_{jk}M_{4I}X_{JK} + 4Y_{j4}M_{kI}X_{JK}\right) = 2\Lambda_4^8\bar{b}^i \end{aligned} \tag{5.9}$$

The linear equations for $\bar{b}^i$ and $\bar{b}^4$ can be understood by the fact that they appear as mass terms for M_{44} and M_{i4}. The equations of motion in Eq. 5.9 can be interpreted as quantum modified constraints of a three flavor $SU(4)$ theory with the scales related through the $\bar{b}$-dependent masses.

It is a nontrivial check on the superpotential of Eq. 5.8 that all classical constraints have a quantum analog and vice versa. The quantum modified constraints involving $\bar{b}^i$ and $\bar{b}^4$ are derived by substituting in the solution to their equation of motion. The quantum modified constraints are:

$$4\,X_{I4}X_{JK}\epsilon^{IJK} - \bar{B}\mathrm{Pf}A = \Lambda_4^8 \tag{5.10}$$

$$\epsilon^{ijk}\epsilon^{IJK}\left(\mathrm{Pf}AM_{iI}M_{jJ}M_{kK} - 6Y_{ij}M_{kI}X_{JK}\right) = 6\Lambda_4^8\bar{b}^4 \tag{5.11}$$

$$\epsilon^{ijk}\epsilon^{IJK}\left(\mathrm{Pf}AM_{4I}M_{jJ}M_{kK} - 2Y_{jk}M_{4I}X_{JK} + 4Y_{j4}M_{kI}X_{JK}\right) = 2\Lambda_4^8\bar{b}^i \tag{5.12}$$

$$\epsilon^{IJK}\epsilon^{ijk}M_{iI}M_{jJ}M_{4K}Y_{k4} = 2\,BM_{4I}X_{JK}\epsilon^{IJK} \tag{5.13}$$

$$\bar{B}\epsilon^{kij}Y_{ij} - 2\,\epsilon^{kij}\epsilon^{IJK}M_{iI}M_{jJ}X_{K4} = -2\,M_{i4}M_{jI}\epsilon^{kij}X_{JK}\epsilon^{IJK} \tag{5.14}$$

while the remaining constraints are not modified. The interesting thing to observe in the above equations is that the quantum modifications do not simply involve addition of a constant to the classical field equations. The quantum modification can be field dependent. The classical limit is recovered in Eqs. 5.13, 5.14 because B and M_{i4} are fields which vanish classically. Without a tree-level superpotential M_{i4} is set to zero by the $\bar{b}^i$ equations of motion. However, M_{i4} can be non-vanishing in the presence of a tree-level superpotential. The quantum modifications in Eqs. 5.11, 5.12 do not contain classically vanishing fields, but are proportional to Λ_4, which ensures the correct classical limit. This field dependent modification of constraints is a new feature which is not present when analyzing simple nonabelian gauge groups.

Note that five of our constraints (Eqs. 5.10, 5.11 and 5.12) can be interpreted as the quantum modified constraints on the moduli space of an $SU(4)$ gauge theory with an antisymmetric tensor and three flavors. Such a theory is obtained in several limits. If $\Lambda_4 \gg \Lambda_3$ one trivially has a three flavor $SU(4)$ theory with an antisymmetric tensor. On the other hand, if $\Lambda_3 \gg \Lambda_4$ and any single $\bar{b}$ is non-vanishing one also has a three flavor $SU(4)$ theory with its corresponding quantum modified constraint.

When deriving the constraints in Eqs. 5.10-5.14 from the exact superpotential we frequently encounter expressions containing inverse powers of Λ_4. Such terms are singular in the limit when Λ_3 is held fixed and $\Lambda_4 \to 0$. This is true even for expressions containing the fields B, M_{i4} and M_{44}, since they vanish only in the limit when $\Lambda_3 \to 0$. Therefore all such terms must and do cancel.

5.4.3 Dynamical supersymmetry breaking in the 4-3-1 model

In the low-energy description of our model the $SU(4)$ and $SU(3)$ gauge groups are confined and the only remaining gauge group is the $U(1)$. This $U(1)$ does not play any role in supersymmetry breaking; its purpose is to lift some classical flat directions. Unlike previous examples of dynamical supersymmetry breaking, the superpotential can be completely analyzed in a regime where there are no singularities, either due to a dynamically generated superpotential present in the initial theory, integrating out fields, or particular limits. If the theory breaks supersymmetry, it is simply of O'Raifeartaigh type [78]. In this section, we show that this is the case; there is no consistent solution of the F-flatness equations for the exact superpotential of Eq. 5.8.

We first assume that $\bar{B} \neq 0$. Then the $\frac{\partial W}{\partial Y_{ij}}$ equation of motion implies

$$Y_{ij} = \frac{1}{\bar{B}} X_{KL} M_{iI} M_{jJ} \epsilon^{IJKL}. \tag{5.15}$$

Plugging this expression into the $\frac{\partial W}{\partial X_{IJ}}$ equation of motion, we obtain

$$(\delta^3_S \delta^4_T - \delta^3_T \delta^4_S) + \frac{8}{\Lambda_3^5 \Lambda_4^8} B X_{ST} - \frac{2}{\Lambda_3^5 \Lambda_4^8} \frac{1}{\bar{B}} \epsilon^{ijkl} M_{iM} M_{jN} M_{kS} M_{lT} X_{KL} \epsilon^{MNKL} = 0.$$

However, by using the $\frac{\partial W}{\partial \mathrm{Pf} A} = 0$ equation in the above expression we arrive at a contradiction.

Next we assume that $\bar{B} = 0$, but $B \neq 0$. We can now solve for X using the equation $\frac{\partial W}{\partial X_{IJ}} = 0$:

$$X_{MN} = \frac{\Lambda_3^5 \Lambda_4^8}{8B} \left[(\delta^3_M \delta^4_N - \delta^3_N \delta^4_M) + 48\, \epsilon^{ijkl} Y_{ij} M_{kM} M_{lN} \right]. \tag{5.16}$$

Then we multiply this equation by $\epsilon^{ijkl}\epsilon^{IJMN}M_{kI}M_{lJ}$. The Y_{ij} equation of motion sets the left hand side to zero, while the PfA equation of motion sets the second term on the right hand side to zero. Therefore,

$$\epsilon^{ijkl}M_{iI}M_{jJ}\epsilon^{IJ34} = 0.$$

Using this fact, the PfA equation of motion, and the expression for X_{MN} in Eq. 5.16 we get that $\frac{\partial W}{\partial B} = -\frac{1}{\Lambda_3^5}$, which again means that the equations of motion are contradictory.

Finally we assume that $B = \bar{B} = 0$. Then the $\frac{\partial W}{\partial X_{IJ}}$ equation of motion implies

$$\epsilon^{ijkl}Y_{ij}M_{kI}M_{lJ} = 0$$

for all I, J except $I = 3, J = 4$. Multiplying the $\frac{\partial W}{\partial X_{IJ}}$ equation of motion by $M_{iI}M_{jJ}$ and using the $\frac{\partial W}{\partial \mathrm{Pf}A}$ equation of motion we get that

$$\epsilon^{ijkl}M_{i1}M_{j2} = 0.$$

Using these results the $\frac{\partial W}{\partial M_{i3}}$ equation of motion yields

$$\delta^{i3} - \frac{1}{\Lambda_3^5\Lambda_4^8}\epsilon^{ijkl}Y_{jk}M_{lJ}X_{KL}\epsilon^{3JKL} = 0.$$

Multiplying this equation by M_{i4} implies $M_{34} = 0$, which is in contradiction with the $\frac{\partial W}{\partial \bar{b}^3}$ equation of motion. Thus we have shown that this $SU(4) \times SU(3) \times U(1)$ model breaks supersymmetry dynamically. Since there are no classical flat directions, there should not be runaway directions in this model.

Having presented a general proof of supersymmetry breaking, we now give a simpler proof that applies only in a restricted region of parameter space. Assume that Λ_3 is the largest parameter in the theory. The effective superpotential just below the Λ_3 scale is

$$\begin{aligned} W &= \bar{b}^3 + \gamma A^{\alpha\beta}\bar{F}_{\alpha 1}\bar{F}_{\beta 2} + \lambda_1 F_1^\alpha \bar{F}_{\alpha 1} + \lambda_2 F_2^\alpha \bar{F}_{\alpha 2} + \lambda_3 F_3^\alpha \bar{F}_{\alpha 3} + \\ & \quad \frac{1}{\Lambda_3^5}\left(\bar{F}_{\alpha 4}F_i^\alpha \bar{b}^i - \det F_i^\alpha\right), \end{aligned} \tag{5.17}$$

where we use the notation from Eq. 5.6 and we introduced explicitly the Yukawa couplings γ and $\lambda_{1,2,3}$. In terms of the canonically normalized fields, $\lambda_{1,2,3}$ are mass parameters.

Next, we integrate out three of the four flavors to arrive at an $SU(4)$ theory with one flavor and a superpotential

$$W = \bar{b}^3 + \frac{1}{\Lambda_3^5}\bar{F}_{\alpha 4}F_4^\alpha \bar{b}^4. \tag{5.18}$$

To describe the dynamics of the one-flavor $SU(4)$ theory, it is useful to define the effective one-flavor $SU(4)$ scale $\tilde{\Lambda}_4^5$, which is proportional to $\lambda_1\lambda_2\lambda_3\Lambda_3^5\Lambda_4^8$. Below the effective $\tilde{\Lambda}_4$ scale there is a dynamically generated term, so the low-energy superpotential is

$$W = \bar{b}^3 + \frac{1}{\Lambda_3^5}M_{44}\bar{b}^4 + \left(\frac{\tilde{\Lambda}_4^5}{\mathrm{Pf}A\, M_{44}}\right)^{\frac{1}{2}}, \tag{5.19}$$

where $M_{44} = \bar{F}_{\alpha 4}F_4^\alpha$. There are no solutions to the equations of motion. Note that the potential runaway direction is removed by the $U(1)$ D-flatness condition. Therefore supersymmetry is dynamically broken. Observe that supersymmetry breaking in this limit has two sources. First the superpotential generated by the $SU(3)$ and $SU(4)$ gauge groups together does not have a supersymmetric minimum. Second, a Yukawa term in the tree level superpotential is confined into a single field which is also a source of supersymmetry breaking. In fact, the tree-level Yukawa terms have three different important roles in this analysis. They lift the flat directions, they yield mass terms for the $SU(4)$ fields after $SU(3)$ is confining, and they also contribute to supersymmetry breaking by the linear term. The fact that there is a quantum modified constraint in the $\Lambda_4 \gg \Lambda_3$ limit of the theory does not seem to play a major role in the dynamics of supersymmetry breaking.

By symmetries, it can be shown that this simpler proof neglects power corrections proportional to

$$\left(\frac{\gamma^2 \bar{b}^i \mathrm{Pf} A\, M_{44}}{\lambda^4 (\Lambda_3^5)^2}\right)^k .$$

This reflects the fact that here we are studying the effective theory treating Λ_3 as large. The $\bar{b}^4$ equation of motion together with the fact that there are no flat directions imply broken supersymmetry even with these corrections incorporated.

5.5 The $SU(n) \times SU(3) \times U(1)$ theories

In this section we generalize the $SU(4) \times SU(3) \times U(1)$ model to $SU(n) \times SU(3) \times U(1)$, with n even. There are several interesting features of the dynamics of these theories. Without a tree-level superpotential the $SU(3)$ group is not confining. However, the Yukawa couplings of the tree-level superpotential become mass terms when the $SU(n)$ group confines. These mass terms drive the $SU(3)$ group into the confining regime as well. Confinement can change chiral theories into non-chiral ones. In this example Yukawa couplings become mass terms. In fact, the quantum modified constraint associated with the $SU(n)$ group of the initial theory does not appear to play an essential role in the dynamics of supersymmetry breaking. Another interesting phenomena is that even if we remove some of the couplings from the superpotential, so that some flat directions are not lifted, these directions turn out to be lifted in the quantum theory. In particular, once the Yukawa couplings turn into mass terms, the $SU(3)$ antibaryon directions are automatically lifted.

As in Section 5.4, we obtain the field content for these models by decomposing the fields of the $SU(n+3)$ theory with an antisymmetric tensor and $n-1$ anti-fundamentals to $SU(n) \times SU(3) \times U(1)$:

$$\begin{array}{rcl} \begin{smallmatrix}\square\\\square\end{smallmatrix} & \rightarrow & A^{\alpha\beta}(\begin{smallmatrix}\square\\\square\end{smallmatrix},1)_6 + \bar{Q}_a(1,\bar{3})_{-2n} + T^{\alpha a}(\square,3)_{3-n} \\ (n-1)\bar{\square} & \rightarrow & \bar{F}_{\alpha I}(\bar{\square},1)_{-3} + \bar{Q}_{ai}(1,\bar{3})_n, \end{array} \tag{5.20}$$

where $i, I = 1, \ldots, n-1$.

In analogy to the 4-3-1 case, $SU(n) \times SU(3) \times U(1)$ invariants are:

$$\begin{aligned} M_{iI} &= T^{\alpha a}\bar{Q}_{ai}\bar{F}_{\alpha I} \\ X_{IJ} &= A^{\alpha\beta}\bar{F}_{\alpha I}\bar{F}_{\beta J} \\ X_I &= \frac{1}{6}A^{\alpha_n\alpha_{n-1}}\ldots A^{\alpha_4\beta}\bar{F}_{\beta I}\epsilon_{\alpha_n\ldots\alpha_1}T^{\alpha_3 a}T^{\alpha_2 b}T^{\alpha_1 c}\epsilon_{abc} \\ Y_i &= A^{\alpha_n\alpha_{n-1}}\ldots A^{\alpha_4\alpha_3}T^{\alpha_2 a}\bar{Q}_{ai}T^{\alpha_1 b}\bar{Q}_b \end{aligned}$$

$$
\begin{aligned}
\bar{b}_{ij} &= \bar{Q}_a \bar{Q}_{bi} \bar{Q}_{cj} \epsilon^{abc} \\
E_I &= \epsilon_{\alpha_n \ldots \alpha_1} A^{\alpha_n \alpha_{n-1}} \ldots A^{\alpha_2 \alpha_1} T^{\beta a} \bar{Q}_a \bar{F}_{\beta I}
\end{aligned}
\qquad (5.21)
$$

We consider the following superpotential:

$$
\begin{aligned}
W &= X_{12} + X_{34} + \ldots + X_{n-3,n-2} + \bar{b}_{23} + \bar{b}_{45} + \ldots + \bar{b}_{n-2,1} + \\
&\quad M_{11} + M_{22} + \ldots + M_{n-1,n-1}.
\end{aligned}
\qquad (5.22)
$$

Observe the relative shifts in the indices between the X and $\bar{b}$ operators. One can check that not all flat directions are removed without such a shift in the indices.

To demonstrate that all flat directions are lifted, one can use the same method as described in Section 5.4. In this example, we require looking not only at linear equations in the flat direction fields, but also higher order equations, in order to demonstrate that no flat directions remain in the presence of the tree-level superpotential above.

We first use the $\bar{Q}_i$ and $\bar{F}_i$ equations of motion (contracted with $\bar{Q}_k$ and $\bar{F}_j$). One will then find potential flat directions which are labeled by $i = 1, 3, 5, \ldots, 2[n/4] - 1$ with equal values of $X_{2j-1,(2j-1+i)||(n-2)} = \bar{b}_{2j,(2j+i)||(n-2)}$, where $j = 1, 2, 3, \ldots, (n-2)/2$ labels nonvanishing X and $\bar{b}$ fields which are equal along the flat direction. Here, by $[x]$ we denote the greatest integer less than x, while we define $m||n \equiv 1 + (m-1)$ Mod n. There is another set of potential flat directions of the form $X_{2j,(2j+i)||(n-2)} = \bar{b}_{2j-1,(2j-1+i)||(n-2)}$, where again $j = 1, 2, 3, \ldots, (n-2)/2$ and $i = 1, 3, 5, \ldots, 2[n/4] - 1$. In the case when $n = 4k$ and $i = k$, two potential flat directions described above are equal to each other, so they represent just one flat direction. Altogether, there are $(n-2)/2$ potential flat directions. One of these flat directions is lifted trivially by the A equation of motion. To see that the remaining flat directions are lifted requires obtaining quadratic equations in the flat direction of fields by suitably contracting the T equations of motion. These equations can be shown to have only the trivial solution where all fields vanish. We have verified this explicitly in the cases $n = 6, 8, 10$, and 12, but we expect this method to generalize.

One can also verify that the superpotential above preserves two $U(1)$ symmetries, one of which is an R symmetry which is anomalous only with respect to the $U(1)$ gauge group. From the quantum modified constraint it can be shown that at least one of these $U(1)$ symmetries is spontaneously broken. Since the theory has no flat directions and spontaneously breaks a $U(1)$ symmetry, we expect that supersymmetry is broken.

There is a possibility however that in the strongly interacting regime there is a point at which supersymmetry is restored. We now consider the quantum theory and argue that it is likely that supersymmetry is broken.

Without a tree-level superpotential the $SU(3)$ group is not confining for $n > 4$ since $N_f > \frac{3}{2}N_c$. We choose to use fields transforming under $SU(3)$ instead of the $SU(3)$ invariant operators. The D-flatness conditions can then be imposed explicitly. Although in principle one could use holomorphic invariants to parameterize the D-flat directions, the naive application of this method would lead to incorrect results at points of the moduli space where these invariants vanish [80]. Although with careful choice of holomorphic invariants this problem can be circumvented, in practice it is simpler to use the charged fields when the gauge group is not confining.

The $SU(n)$ group has three flavors and an antisymmetric tensor. Therefore $SU(n)$ is confining and gives rise to a quantum modified constraint as described in Ref. [64]. The

$SU(n)$ invariants are:

$$\begin{aligned} X_{IJ} &= A^{\alpha\beta}\bar{F}_{\alpha I}\bar{F}_{\beta J} \\ m_I^a &= T^{\alpha a}\bar{F}_{\alpha I} \\ \text{Pf}A &= \epsilon_{\alpha_n\ldots\alpha_1}A^{\alpha_n\alpha_{n-1}}\ldots A^{\alpha_2\alpha_1} \\ y_a &= A^{\alpha_n\alpha_{n-1}}\ldots A^{\alpha_4\alpha_3}\epsilon_{\alpha_n\ldots\alpha_1}T^{\alpha_2 b}T^{\alpha_1 c}\epsilon_{abc} \end{aligned} \tag{5.23}$$

together with the fields $\bar{Q}_a$ and $\bar{Q}_{ai}$.

The superpotential below the Λ_n scale is

$$\begin{aligned} W &= \alpha^{12}X_{12} + \ldots + \alpha^{n-3,n-2}X_{n-3,n-2} + \beta^{23}\bar{Q}_a\bar{Q}_{b2}\bar{Q}_{c3}\epsilon^{abc} + \ldots + \\ &\quad \beta^{n-2,1}\bar{Q}_a\bar{Q}_{b,n-2}\bar{Q}_{c1}\epsilon^{abc} + \lambda^{11}m_1^a\bar{Q}_{a1} + \ldots + \lambda^{n-1,n-1}m_{n-1}^a\bar{Q}_{a,n-1} + \\ &\quad \eta\Big(\frac{n-2}{3n}\epsilon_{abc}m_{I_1}^a m_{I_2}^b m_{I_3}^c X_{I_4I_5}\ldots X_{I_{n-2}I_{n-1}}\epsilon^{I_1\ldots I_{n-1}}\text{Pf}A - \\ &\quad y_a m_{I_1}^a X_{I_2I_3}\ldots X_{I_{n-2}I_{n-1}}\epsilon^{I_1\ldots I_{n-1}} + \Lambda_n^{2n}\Big), \end{aligned} \tag{5.24}$$

where η is a Lagrange multiplier and we have explicitly included the coupling constants in the tree-level superpotential. In terms of $SU(n)$ invariants, some of the terms in the above superpotential are just mass terms for $(n-1)$ flavors of $SU(3)$, which drive $SU(3)$ into the confining phase. In the presence of these perturbations, non-perturbative $SU(3)$ dynamics will generate a superpotential. Similar results are found in Ref. [79]. We stress again that in the underlying theory these interactions are Yukawa couplings and not mass terms.

To analyze the low-energy theory, we introduce an additional flavor of $SU(n)$ with mass μ. We do this because the $SU(n)$ quantum modified constraint or equivalently anomaly matching shows that $SU(3)$ must be broken below the scale Λ_n in the original theory. With an additional flavor, the origin of moduli space is permitted and $SU(3)$ can remain unbroken. This permits us to derive the confining superpotential with two massless $SU(3)$ flavors. Although the correct theory is only recovered in the limit $\mu \to \infty$, we will analyze the theory in the regime $\mu < \Lambda_n$ and hope one can extrapolate the conclusion that supersymmetry is broken [64].

The superpotential with the additional massive $SU(n)$ flavor is:

$$\begin{aligned} W &= \alpha^{12}X_{12} + \ldots + \alpha^{n-3,n-2}X_{n-3,n-2} + \\ &\quad \beta^{23}\bar{Q}_a\bar{Q}_{b2}\bar{Q}_{c3}\epsilon^{abc} + \ldots + \beta^{n-2,1}\bar{Q}_a\bar{Q}_{b,n-2}\bar{Q}_{c1}\epsilon^{abc} + \\ &\quad \lambda^{11}m_1^a\bar{Q}_{a1} + \ldots + \lambda^{n-1,n-1}m_{n-1}^a\bar{Q}_{a,n-1} + \mu m_n^4 + \\ &\quad \frac{1}{\Lambda_n^{2n-1}}\Big(\text{Pf}A\, m_{I_1}^a m_{I_2}^b m_{I_3}^c m_{I_4}^d X_{I_5I_6}\ldots X_{I_{n-1}I_n}\epsilon_{abcd}\epsilon^{I_1\ldots I_n} + \\ &\quad Y^{ab}m_{I_1}^c m_{I_2}^d X_{I_3I_4}\ldots X_{I_{n-1}I_n}\epsilon_{abcd}\epsilon^{I_1\ldots I_n} + BX_{I_1I_2}\ldots X_{I_{n-1}I_n}\epsilon^{I_1\ldots I_n} + \\ &\quad \bar{B}Y^{ab}Y^{cd}\epsilon_{abcd} + B\bar{B}\text{Pf}A\Big), \end{aligned} \tag{5.25}$$

where the variables are as defined in Eq. 5.23 with an extra $SU(n)$ flavor and

$$\begin{aligned} B &= T^{\alpha_1 a}T^{\alpha_2 b}T^{\alpha_3 c}F^{\alpha_4 4}A^{\alpha_5\alpha_6}\ldots A^{\alpha_{n-1}\alpha_n}\epsilon_{abc}\,\epsilon_{\alpha_1\ldots\alpha_n} \\ \bar{B} &= \bar{F}_{\alpha_1 I_1}\ldots\bar{F}_{\alpha_n I_n}\epsilon^{I_1\ldots I_n}\epsilon^{\alpha_1\ldots\alpha_n} \\ Y^{a4} &= T^{\alpha_1 a}F^{\alpha_2 4}A^{\alpha_3\alpha_4}\ldots A^{\alpha_{n-1}\alpha_n}\epsilon_{\alpha_1\ldots\alpha_n} \\ Y^{ab} &= \epsilon^{abc}y_c. \end{aligned} \tag{5.26}$$

The extra $SU(n)$ flavor is denoted by $F^{\alpha 4}$ and $\bar{F}_{\alpha n}$, and Λ_n is the dynamical scale of the four-flavor $SU(n)$ theory. Here we have not bothered to establish the correct coefficients in the last term in parentheses, since they are irrelevant in the forthcoming analysis.

To arrive at the true low-energy theory, one would integrate out $n-3$ flavors, at which point a superpotential is generated involving Λ_3 for the four flavor theory. Upon integrating out the two remaining heavy flavors, one would generate a complicated superpotential, involving both the Yukawa couplings and the dynamical scales Λ_n and Λ_3. It is however technically difficult to explicitly perform this procedure because of the nonlinear terms induced by the baryon operators in the tree-level superpotential.

If we instead constrain the form of the low-energy superpotential with symmetries and limits, we find that the analysis remains quite complicated, because many terms are permitted by the symmetries and physical limits. We deduce the allowed terms by introducing a parameter $\tilde{\Lambda}_3$ which transforms under anomalous global symmetries associated with the rotation of each field carrying $SU(3)$ gauge charge in the initial microscopic theory. Alternatively, we can define Λ_3 for the two flavor theory, where all heavy flavors have been integrated out. The parameters $\tilde{\Lambda}_3^{9-n}\det(\lambda^{iI})/\Lambda_n^{2n-1}$ and Λ_3^7 have the same charge under all anomalous symmetries so we can describe the low energy dynamics in terms of either one. We also see that if we consider $\tilde{\Lambda}_3$ as a fundamental finite parameter of the initial theory, singularities in the Yukawa couplings λ^{iI} are permitted when we express the result in terms of the low-energy Λ_3, since the appropriate ratio is finite. In essence, the Yukawa couplings become mass terms in the $SU(n)$ confined theory, and appear in the matching of Λ_3 across mass thresholds.

Examples of terms permitted by all symmetries and limits are:

$$\frac{\Lambda_3^7}{\Lambda_n^{2n-1}}\frac{\beta^{ij}}{(\lambda^{iI})^2}(X_{IJ})^{(n-4)/2}\mathrm{Pf}A\, M_I^4\frac{1}{y_a Y^{a4}},$$
$$\frac{\Lambda_3^{14}}{\Lambda_n^{2n-1}}\frac{(\beta^{ij})^2}{(\lambda^{iI})^4}X_{In}(X_{IJ})^{(n-6)/2}\mathrm{Pf}A\frac{1}{(y_a Y^{a4})(y_a M_n^a)},$$

where β^{ij}'s are the coefficients of the baryon operators $\bar{Q}\bar{Q}_i\bar{Q}_j$, and λ^{iI} of the $T\bar{F}_I\bar{Q}_i$ terms in the tree-level superpotential, but the index structure is not specified. These terms mix the effects of the strong dynamics with the tree-level superpotential, which is purely a consequence of integrating out heavy fields. This does not violate the conjecture of Refs. [62, 81], which states that the couplings of the light fields are not mixed into the dynamically generated superpotential.

Because of the complicated superpotential, the analysis of the full theory is difficult. We will therefore consider a simpler version of the theory, in which the baryon couplings, β^{ij}, are zero. This simplified superpotential does not lift all flat directions classically, which might lead to runaway directions in the quantum theory. One can show that these remaining classical flat directions can be parameterized by the baryon operators $\bar{b}_{ij}$. However, in the $SU(n)$ confined theory, these fields are not flat, since the terms proportional to m_{iI}, which are Yukawa couplings in the classical theory, are mass terms in the confined theory. In this case, there is a potential for the baryon fields which drives them towards the origin, and the baryon flat directions are lifted in the quantum theory. This is similar in spirit to what was found in Ref. [76]. In that example however, a quadratic constraint becomes a linear constraint so the flat direction is removed; here we simply see that the $SU(n)$ confined superpotential is such that the baryon fields are not flat. However there is a caveat to this analysis which we discuss shortly.

In this limit it is simple to integrate out the heavy flavors and arrive at the low-energy theory. The resulting superpotential is

$$\begin{aligned} W &= \frac{1}{\Lambda_n^{2n-1}}\Big(y_a m_n^a m_{I_1}^4 X_{I_2 I_3}\dots X_{I_{n-2}I_{n-1}}\epsilon^{I_1\dots I_{n-1}} + BX_{I_1 I_2}\dots X_{I_{n-1}I_n}\epsilon^{I_1\dots I_n} \\ &\quad +\bar{B}Y^{a4}y_a + B\bar{B}\mathrm{Pf}A\Big) + \mu m_n^4 + \lambda^{12}X_{12} + \dots + \lambda^{n-3,n-2}X_{n-3,n-2} \\ &\quad + \frac{\Lambda_3^7}{(Y^{a4}y_a)(m_n^b\bar{Q}_b) - (Y^{a4}\bar{Q}_a)(m_n^b y_b)}. \end{aligned} \tag{5.27}$$

This superpotential clearly breaks supersymmetry since m_n^4 appears only in the term μm_n^4. Since the scales of the $SU(n)$ theory with and without extra flavor are related by $\mu\Lambda_n^{2n-1} = \Lambda_n^{2n}$, this presumably implies that supersymmetry breaking is characterized by Λ_n^{2n-1} in the original theory.

Thus we just showed that if the $SU(n)$ gauge group is confining, supersymmetry is broken. Had supersymmetry not been broken, this would have been a good assumption, since all operators involving fields transforming under the $SU(n)$ are driven to the origin by the classical potential. Because supersymmetry is broken, it is conceivable that the true vacuum is in the Higgs, rather than the confining phase. Nonetheless, we still expect supersymmetry to be broken since there are no classically flat directions in the theory. In this case however, the $\bar{b}$ operators are not lifted by the superpotential. Once the effect of supersymmetry breaking and the Kähler potential are included, the $\bar{b}$ fields presumably have a nontrivial potential. We have not analyzed whether or not this can give rise to runaway directions, should the Higgs phase prove to be the true vacuum.

Having argued that supersymmetry is probably broken for $\beta^{ij} = 0$, we hope that by including the remaining couplings, while lifting the flat directions, does not introduce a supersymmetric minimum. We expect that the arguments presented above indicate that supersymmetry is broken in the full $SU(n)\times SU(3)\times U(1)$ theories.

5.6 The general $SU(N)\times SU(M)\times U(1)$ theories

In the previous two sections we showed that the $SU(n)\times SU(3)\times U(1)$ theories break supersymmetry. There we argued that supersymmetry breaking could be understood as a collusion between separate dynamical effects from the two nonabelian gauge groups. In the first example, the 4-3-1 model based on the gauge group $SU(4)\times SU(3)\times U(1)$, the exact superpotential could be found and the model was an O'Raifeartaigh model with both groups contributing to the final form of the superpotential. In all cases, supersymmetry breaking could be understood by taking a limit in which the gauge coupling of a confining gauge group is the biggest coupling. In this limit, Yukawa couplings which were necessary to lift flat directions turn into mass terms. Many flavors can be integrated out and the gauge dynamics of the second nonabelian gauge factor generated a superpotential which drives fields from the origin leading to the breaking of supersymmetry.

In the models considered in the previous sections, other mechanisms of supersymmetry breaking could appear as well in the limit that one of the gauge couplings dominated. For example, in the particular case of the 4-3-1 model supersymmetry breaking occurs in the strong Λ_3 limit through confinement, analogous to the mechanism of Ref. [82]. On the other hand, if some of the tree level terms are removed, supersymmetry breaking appears due to a quantum modified constraint [76]. Because of these additional descriptions, it was not clear that the quantum modified constraint was not essential to supersymmetry breaking.

In this section, we show that analogous models in which each of the two groups is in one of a confining, free magnetic, or conformal phase (in the limit that we neglect the other coupling) also break supersymmetry, through a conspiracy of dynamical effects from the two gauge groups [75]. Naively, it would appear that such models should allow fields to go to the origin. However, because of the tree-level superpotential and dynamics of one group, the other group can generate a dynamical superpotential in the infrared which forbids the origin and yields supersymmetry breaking.

It is interesting that models in which the theory must be analyzed at low energy in the dual phase can break supersymmetry. It is not essential for the number of flavors to be so small that a dynamical superpotential, a quantum modified constraint, or even confinement occurs in the electric theory. This suggests the possibility of a much larger class of supersymmetry breaking models because of the much less restrictive condition on the size of the initial particle content.

The two models we present in this section are obvious generalizations of the $SU(n) \times SU(3) \times U(1)$ models considered in the previous section. Analogously to the n-3-1 models, supersymmetry breaking can be understood as a result of Yukawa couplings and strong dynamics which make flavors of the second gauge group heavy. In the resulting theory, the origin is forbidden because of a dynamical superpotential from the second gauge group. The mechanism is in some sense independent of the number of flavors in the initial theory. We present two classes of models to illustrate this. In the first class of models, in which one of the gauge groups is confining, supersymmetry breaking occurs through a conspiracy of gauge effects. We then consider a model which must be analyzed in the dual phase. The supersymmetry breaking dynamics for this model is remarkably similar to that of the confining theory, as we will show below.

The fields of the first model can be obtained by decomposing $SU(n+4)$ model with an antisymmetric tensor [49] into its $SU(n) \times SU(4) \times U(1)$ subgroup. The field content is

$$\begin{array}{rcl} \begin{smallmatrix}\Box\\\Box\end{smallmatrix} & \rightarrow & A(\begin{smallmatrix}\Box\\\Box\end{smallmatrix},1)_8 + a(1,\begin{smallmatrix}\Box\\\Box\end{smallmatrix})_{-2n} + T(\Box,\Box)_{4-n} \\ n\cdot\overline{\Box} & \rightarrow & \bar{F}_I(\overline{\Box},1)_{-4} + \bar{Q}_i(1,\overline{\Box})_n, \end{array} \tag{5.28}$$

where $i, I = 1,\ldots,n$. We take the tree-level superpotential to be

$$\begin{array}{c} W_{tree} = A\bar{F}_1\bar{F}_2 + A\bar{F}_3\bar{F}_4 + \ldots + A\bar{F}_{n-2}\bar{F}_{n-1} \\ +a\bar{Q}_2\bar{Q}_3 + a\bar{Q}_4\bar{Q}_5 + \ldots + a\bar{Q}_{n-1}\bar{Q}_1 + T\bar{F}_1\bar{Q}_1 + \ldots + T\bar{F}_n\bar{Q}_n. \end{array} \tag{5.29}$$

A detailed analysis along the lines of Ref. [74] shows that this superpotential lifts all flat directions. The relative shift of the indices in the $A\bar{F}\bar{F}$ and $a\bar{Q}\bar{Q}$ terms is important. Without this shift not all flat directions are lifted. This superpotential preserves an R-symmetry which is anomalous only under the $U(1)$ gauge group.

We analyze this theory in the limit where $\Lambda_n \gg \Lambda_4$. The $SU(n)$ field content is an antisymmetric tensor, four fundamentals and n antifundamentals which give confining gauge dynamics. Below Λ_n, the effective degrees of freedom are the $SU(n)$ invariants [56]

$$\begin{aligned} X_{IJ} &= A^{\alpha\beta}\bar{F}_{\alpha I}\bar{F}_{\beta J} \\ \bar{B} &= \bar{F}_{\alpha_1 1}\ldots\bar{F}_{\alpha_n n}\epsilon^{\alpha_1\ldots\alpha_n} \\ (B_1)^a &= T^{\alpha_1 a}A^{\alpha_2\alpha_3}\ldots A^{\alpha_{n-1}\alpha_n}\epsilon_{\alpha_1\ldots\alpha_n} \end{aligned}$$

$$(B_3)_a = \epsilon_{abcd} T^{\alpha_1 b} T^{\alpha_2 c} T^{\alpha_3 d} A^{\alpha_4\alpha_5} \ldots A^{\alpha_{n-1}\alpha_n} \epsilon_{\alpha_1\ldots\alpha_n}$$
$$M_I^a = T^{\alpha a} \bar{F}_{\alpha I}, \tag{5.30}$$

plus the $SU(n)$ singlets a and $\bar{Q}_i$.

The superpotential is the sum of the tree-level terms from Eq. (5.29) and the confining superpotential [56].

$$\begin{aligned} W &= X_{12} + \ldots + X_{n-2,n-1} + a\bar{Q}_2\bar{Q}_3 + \ldots + a\bar{Q}_{n-1}\bar{Q}_1 + \\ & M_1\bar{Q}_1 + \ldots + M_n\bar{Q}_n + \frac{1}{\Lambda_n^{2n-1}}\Big(B_{3a} M_{I_1}^a X_{I_2 I_3} \ldots X_{I_{n-1}I_n} \epsilon^{I_1\ldots I_n} \\ & + B_1^a M_{I_1}^b M_{I_2}^c M_{I_3}^d X_{I_4 I_5} \ldots X_{I_{n-1}I_n} \epsilon^{I_1\ldots I_n} \epsilon_{abcd} + \bar{B} B_1^a B_{3a}\Big), \end{aligned} \tag{5.31}$$

where small Latin letters denote $SU(4)$ indices.

Note that in the confined theory, some of the Yukawa couplings have become mass terms. To deduce the infrared theory, we integrate out all massive fields. It is technically difficult to integrate out the fields using the full superpotential from Eq. (5.31). For simplicity we set the couplings of all $a\bar{Q}\bar{Q}$ terms to zero. We will argue based on symmetries that the models with the additional baryon operators included still break supersymmetry. It should be noted that the flat directions now present classically are lifted in the quantum theory [76], which is presumably a valid supersymmetry breaking model as well.

Because we have integrated out n massive flavors, the $SU(4)$ theory at low energy has an antisymmetric tensor and only one flavor. This theory dynamically generates a superpotential. The low-energy superpotential is therefore

$$W_{\text{eff}} = X_{12} + \ldots + X_{n-2,n-1} + \frac{1}{\Lambda_n^{2n-1}} \bar{B} m + \left[\frac{\tilde{\Lambda}_4^5}{\text{Pf}a\, m}\right]^{\frac{1}{2}}, \tag{5.32}$$

where $\text{Pf}a = a^{ab}a^{cd}\epsilon_{abcd}$, $m = B_1^a B_{3a}$, and $\tilde{\Lambda}_4$ is the dynamical scale of the effective one flavor $SU(4)$ theory. The equations of motion have set most terms to zero in the Λ_n dependent term. The $\bar{B}$ equation of motion would set $m = 0$. However, this is inconsistent with the $\left[\frac{\tilde{\Lambda}_4^5}{\text{Pf}a\, m}\right]^{\frac{1}{2}}$ term in the superpotential, which drives m from the origin in a theory with no flat directions. Therefore, we conclude that the equations of motion are contradictory, and supersymmetry is dynamically broken.

We have argued that supersymmetry is broken in the theory with $\gamma^{ij} = 0$, where γ^{ij} is the coefficient of the $a\bar{Q}\bar{Q}$ operators in the tree-level superpotential. It is clear that even with nonzero γ^{ij}, supersymmetry is still broken. From symmetries, it can be shown that the neglected terms can correct the superpotential by a power series in

$$\mathcal{A} = \Lambda_n^{-2n+1} (\text{Pf}a)^{\frac{1}{2}} (X_{IJ})^{\frac{n-2}{2}} m^{\frac{1}{2}} (\gamma^{ij})(m^{iI})^{-2}, \tag{5.33}$$

where m^{iI} is the coefficient of the $T\bar{F}_I\bar{Q}_i$ operators. For small γ, these terms could only give a sufficiently large contribution to cancel a nonzero F-term at field values larger than Λ_n. In this case, the theory should have been analyzed in the Higgs phase, which is clearly inconsistent with supersymmetry since there were no flat directions.

As an aside, we note that in the version of the theory without the $a\bar{Q}\bar{Q}$ terms in the superpotential (and hence without the corrections of Eq. (5.33)), there is an additional source of supersymmetry breaking. The terms $X_{12}+\ldots+X_{n-2,n-1}$ in the superpotential lead

	$SU(n-3)$	$Sp(2n-8)$	$SU(n)$	$U(1)$	$SU(n+1)_{\bar{Q}}$	$SU(n+1)_{\bar{F}}$
A	1	1	$\begin{smallmatrix}\Box\\ \Box\end{smallmatrix}$	10	1	1
$\bar{F}$	1	1	$\overline{\Box}$	-5	1	$\Box$
x	$\Box$	$\Box$	1	0	1	1
p	$\Box$	1	1	$5n$	1	1
$\bar{a}$	$\overline{\begin{smallmatrix}\Box\\ \Box\end{smallmatrix}}$	1	1	0	1	1
$\bar{q}$	$\overline{\Box}$	1	$\overline{\Box}$	-5	1	1
l	1	$\Box$	1	0	$\overline{\Box}$	1
M	1	1	$\Box$	5	$\Box$	1
H	1	1	1	0	$\begin{smallmatrix}\Box\\ \Box\end{smallmatrix}$	1
B_1	1	1	$\Box$	$5(1-n)$	1	1

(5.36)

Table 5.1: The field content of the $SU(n) \times SU(5) \times U(1)$ theory after dualizing the $SU(5)$ gauge group.

to supersymmetry breaking due to confinement, as described in Ref. [82]. Here we emphasize the first argument for supersymmetry breaking, which generalizes beyond confining models, as we describe below.

Next, we consider theories based on the gauge group $SU(n) \times SU(5) \times U(1)$ (n even) obtained by reducing the gauge group of the $SU(n+5)$ theory with an antisymmetric tensor and $n+1$ antifundamentals. The mechanism of supersymmetry breaking will turn out to be very similar to the previous models, despite the very different gauge dynamics.

The field content is

$$\begin{array}{rcl} \begin{smallmatrix}\Box\\ \Box\end{smallmatrix} & \rightarrow & A(\begin{smallmatrix}\Box\\ \Box\end{smallmatrix},1)_{10} + a(1,\begin{smallmatrix}\Box\\ \Box\end{smallmatrix})_{-2n} + T(\Box,\Box)_{5-n} \\ (n+1)\cdot\overline{\Box} & \rightarrow & \bar{F}_I(\overline{\Box},1)_{-5} + \bar{Q}_i(1,\overline{\Box})_n, \end{array} \tag{5.34}$$

where $i, I = 1, \ldots, n+1$. The tree-level superpotential is

$$\begin{array}{rcl} W_{tree} & = & A\bar{F}_1\bar{F}_2 + \ldots + A\bar{F}_{n-1}\bar{F}_n + a\bar{Q}_2\bar{Q}_3 + \ldots + a\bar{Q}_n\bar{Q}_1 + \\ & & T\bar{F}_1\bar{Q}_1 + \ldots + T\bar{F}_{n+1}\bar{Q}_{n+1}. \end{array} \tag{5.35}$$

Again a detailed analysis verifies the absence of flat directions.

The $SU(5)$ gauge group has an antisymmetric tensor and n flavors while the $SU(n)$ has an antisymmetric tensor and five flavors. The $SU(5)$ group is in the conformal regime while the $SU(n)$ group is in the free magnetic phase. Although it seems more obvious to dualize the $SU(n)$ which is in the free magnetic phase it is simpler to dualize the gauge group $SU(5)$, as it has an odd number of colors. This duality will increase the number of $SU(n)$ flavors by $n-3$ which takes the theory out of the free magnetic phase.

The dual description of $SU(5)$ with an antisymmetric tensor and n flavors is an $SU(n-3) \times Sp(2n-8)$ gauge theory[56] with the field content given in Table 5.1.

The $SU(n-3)\times Sp(2n-8)$ gauge group in Table 5.1 is the dual of the $SU(5)$ gauge group, while the $SU(n) \times U(1)$ is the remaining original gauge group unchanged by the duality transformation. The $SU(n+1)_{\bar{Q}} \times SU(n+1)_{\bar{F}}$ global symmetries are the non-abelian global symmetries of the original $SU(n) \times SU(5) \times U(1)$ theory.

The superpotential consists of the terms corresponding to the tree-level superpotential of Eq. (5.35) and the terms arising from the duality transformation. It is given by

$$\begin{aligned} W &= A\bar{F}_1\bar{F}_2 + \ldots + A\bar{F}_{n-1}\bar{F}_n + H_{23} + \ldots + H_{n1} + M_1\bar{F}_1 + \ldots + \\ & M_{n+1}\bar{F}_{n+1} + M\bar{q}lx + Hl^2 + B_1p\bar{q} + \bar{a}x^2. \end{aligned} \tag{5.37}$$

As in the $SU(n)\times SU(4)\times U(1)$ models, some of the tree-level Yukawa terms are mapped into mass terms in the dual description. To simplify the theory we again set the coefficients of the $A\bar{F}\bar{F}$ operators to zero, though in this case it is not difficult to leave them in. With this simplification, one can easily integrate out the massive flavors of $SU(n)$ since the $\bar{F}_I$ equations of motion set all M's to zero. There is just one $SU(n)$ flavor remaining and thus there is a dynamically generated term in the superpotential from the $SU(n)$ dynamics. The effective low-energy superpotential is

$$W = H_{23} + H_{45} + \ldots + H_{n1} + Hl^2 + \bar{a}x^2 + \tilde{M}p + \frac{\tilde{\Lambda}_n^{n+1}}{(\tilde{M}\tilde{X}^{(n-4)/2}\mathrm{Pf}\,A)^{1/2}}, \tag{5.38}$$

where $\tilde{M} = B_1\bar{q}$, $\tilde{X} = A\bar{q}\bar{q}$ and $\mathrm{Pf}\,A = A^{n/2}$, while $\tilde{\Lambda}_n$ is the effective $SU(n)$ scale. This superpotential looks very much like the one in Eq. (5.32), with $\tilde{M}$ playing the role of m and p the role of $\bar{B}$. The equations of motion are again contradictory. We again conclude that supersymmetry is broken.

The above analysis neglected the $Sp(2n-8)$ group that appears from dualizing the $SU(5)$ group. This group is however Higgsed by the VEV's of the l fields as a result of the H equations of motion and the terms linear in H in the superpotential. Although instanton terms can be generated in the broken $Sp(2n-8)$ group, these will not involve the fields $\tilde{M}, \tilde{X}, \mathrm{Pf}A$ or p and therefore do not affect the proof of dynamical supersymmetry breaking given above. The $Sp(2n-8)$ dynamics seems to be irrelevant to the analysis of the model.

The dynamics of the general $SU(n)\times SU(m)\times U(1)$ models $(n, m \geq 5)$ obtained in the same way is very similar to that of the $SU(n)\times SU(5)\times U(1)$ model, if one dualizes the $SU(n)$ corresponding to odd n. We expect that a similarly constructed tree-level superpotential lifts all flat directions. One can then show that the resulting low-energy superpotential is in one-to-one correspondence to the superpotential of Eq. (5.38), with the remaining gauge group being $SU(m-3)\times Sp(2m-8)\times SU(m)\times U(1)$ (m is even), which is obtained by dualizing the original $SU(n)$ group. Since the superpotential is exactly of the same form as the one in Eq. (5.38) we conclude that the general $SU(n)\times SU(m)\times U(1)$ models break supersymmetry as well.

The similarities between the $SU(n)\times SU(4)\times U(1)$ and $SU(n)\times SU(5)\times U(1)$ models is intriguing. In both models, the dynamics of the $SU(n)$ group leads to additional flavors of the second gauge group, in one case due to confinement, and in the other case, due to the dual description. In both cases, some of the tree level terms are mapped into mass terms due to dynamical effects in the $SU(n)$ gauge group. After integrating out these massive flavors the other gauge group has only a single flavor remaining besides the antisymmetric tensor and produces a dynamically generated superpotential. This dynamical superpotential together with a piece of the superpotential from the strong dynamics of the first group breaks supersymmetry. Thus supersymmetry breaking in these theories involves a subtle interplay between the gauge dynamics of both groups and the tree-level superpotential.

That these theories (and presumably the general $SU(n)\times SU(m)\times U(1)$ models as well) break supersymmetry suggests the existence of still more models of dynamical supersymmetry breaking. The flavor content of these models can be much larger than one would naively have anticipated by the requirement of a dynamical superpotential, because Yukawa couplings or other interactions in the presence of strong dynamics can change the phase of the theory in the infrared. The low-energy description might then have sufficiently few flavors to break supersymmetry dynamically.

Chapter 6

Conclusions

Supersymmetry is an exciting subject from both the point of view of particle phenomenology and from the more theoretical point of view in field theory. Phenomenologists are interested in supersymmetry because the MSSM and extended versions of it are the most compelling theories of particle physics beyond the standard model. On the other hand, supersymmetric field theories, due to the restrictions of the extra symmetry, are more tractable than non-supersymmetric theories and thus can be used for laboratories of strongly interacting field theories. Therefore supersymmetric theories are interesting even if it turns out that low-energy supersymmetry is not realized in nature.

We have presented several topics from the subject of supersymmetric field theories, touching on both the phenomenological and the theoretical aspects. First we outlined the structure of the minimal supersymmetric standard model. We motivated the choice of matter content and described supersymmetry and electroweak breaking. Next, we reviewed the subject of doublet-triplet splitting in SUSY GUT theories. The doublet-triplet splitting is one of the most severe problems of SUSY GUTs and needs to be addressed by every viable model. We have presented a few possible models that solve this problem, and showed the phenomenological constraints on such theories.

In the second half, we discussed the low-energy behavior of SUSY gauge theories. After reviewing the seminal work of Seiberg, we focused our attention on confining theories. We have shown how one can find all s-confining theories based on a single gauge group and explicitely listed all examples. Next we discussed the issues of dynamical supersymmetry breaking. After a few general arguments we constructed the $SU(n) \times SU(m) \times U(1)$ models obtained by the method of DNNS. We showed that these theories indeed break supersymmetry.

The variety of topics presented here illustrates how important a role supersymmetry is playing in current particle physics. Hopefully, this role will soon be enhanced by the forthcoming generation of collider experiments and by a more systematic understanding of the exact results in supersymmetric gauge theories.

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