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PLASMA CYCLOTRON ECHOES

THOMAS B. SMITH

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TECHNICAL REPORT 458

JUNE 1, 1967

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
RESEARCH LABORATORY OF ELECTRONICS
CAMBRIDGE, MASSACHUSETTS

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The research reported in this document was made possible in part by support extended the Massachusetts Institute of Technology, Research Laboratory of Electronics, by the JOINT SERVICES ELECTRONICS PROGRAMS (U. S. Army, U. S. Navy, and U. S. Air Force) under Contract No. DA 28-043-AMC-02536(E).

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Thomas B. Smith

This report is based on part of a thesis submitted to the Department of Physics, M. I. T., 1966, in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

(Manuscript received May 1, 1967)

Abstract

We present a single-particle model that is sufficient to explain observed radiation peaks from weakly ionized laboratory plasma immersed in a magnetic field subsequent to one, two or three short driving RF pulses at the gyro frequency. It is shown that, depending upon the regime of operation, these echoes can result from any combination of arbitrary speed dependences in the gyro frequency, collision frequency, or diffusion of the electrons between and after the pulses.

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I. INTRODUCTION

Several observations of the radiation of echoes from laboratory plasmas subsequent to one or more driving RF pulses have recently been reported.[1-5] The plasmas were immersed in a static magnetic field and the pulse frequency was adjusted to the gyro frequency of the electrons. The cleanest experiments are performed with weakly ionized plasmas with pulse strengths low enough that the electrons suffer predominantly elastic collisions with neutral atoms. Occasionally, the electron density was sufficiently high that Coulomb effects appeared.[5] The model described in this report does not apply to this regime, but the qualitative arguments presented here will still bear an element of truth.

These experiments are interesting for two reasons. First, they provide information about the collision processes of electrons and their diffusion along and across the field lines. Second, although the physics is purely classical, certain of these experiments exhibit discrete behavior strikingly analogous to the two- and three-pulse spin echo experiments of Hahn.[6]

In Section II, wherein we describe the experiments more fully, it may be seen that the two- and three-pulse experiments display discrete radiative echoes at times determined solely by the interpulse intervals, whereas the one-pulse echoes are actually reverberations, in the sense that they are peaks in the radiation whose occurrence times depend upon the plasma parameters.

Section III gives the essentials of our model. It will be seen that the echoes come about when the phases of the gyrating electrons cohere partially, because of transients induced in their motion by the pulses. Eventually, all memory of the pulses is destroyed by collisions and only the much smaller incoherent contribution remains.

To calculate the echoes, a single-particle model is assumed. Within the framework of this model several echo-producing mechanisms have been proposed:

(i) Nonlinear driving force during the pulses, because of the finite size of the gyro-radius,

(ii) Speed-dependent collisions,

(iii) Speed-dependent gyro frequency,

(iv) Speed-dependent diffusion.

Mechanism (i) has been discussed elsewhere[9] and is negligible.

In Section IV one-pulse echoes are discussed in the light of mechanisms (ii) and (iii). The general result (Eq. 18) shows that echo reverberations are the rule rather than the exception when either (ii) and/or (iii) operates. This result, however, leaves several integrations undone. To better understand the problem, several special cases are worked out. Notably, when collisions may be ignored one obtains the Hirshfield-Wachtel expression (Eq. 24) which applies when the gyro-speed dependence is due to a weakly relativistic mass shift of the electron.

Section V discusses two-pulse echoes when mechanisms (ii) and (iii) operate. The general result (Eq. 32) shows that echoes occur for any speed dependence of the collision and/or gyro frequencies. If these be proportional to the energy, then the special case (Eq. 35) follows.

Section VI treats the three-pulse echoes when (ii) and (iii) apply. Because the third pulse occurs at long times the velocity of all of the electrons is randomized by scattering between the second and third pulse. Memory of the first interval, τ, is retained through the energy distribution if the collisions are elastic. The general result (Eq. 40) shows echoes at times $n\tau$ after the third pulse. As an example, the gyro-speed dependence is ignored and the collision frequency is taken to be proportional to the energy.

As we shall see, two- and three-pulse echoes require for their existence a slight spread in gyro frequencies. Normally, this is achieved by means of a transverse variation in the magnetic field. A concomitant longitudinal variation may also exist. In Section VII the effects of this longitudinal variation as the electrons' motion carries them along the field are considered. It is found that each member of the two-pulse echo train is damped by its own simple multiplicative parameter, whereas the three-pulse echoes suffer much more complicated and damaging effects. If, however, the field is fairly uniform longitudinally, it is shown (see Sec. VII) how transverse speed-dependent diffusion can itself cause appreciable three-pulse echoes. This effect is not normally important during the lesser times involved with the two-pulse echoes because the diffusion is slow.

II. EXPERIMENTS

Plasma echoes of the kind described in this report were first measured by R. M. Hill and D. E. Kaplan,[1] who are still active in this field.[11] They observed echoes in an afterglow plasma whose temperature was roughly ambient. They used Ar, Ne, Kr, He, and N_2, with electron densities from $10^6\ cm^{-3}$ to $10^{11}\ cm^{-3}$ and gas pressures from 0.5 μ to 400 μ. Typical values of electron and neutral density are $10^9\ cm^{-3}$ and $10^{13}\ cm^{-3}$, respectively. The plasma was a few centimeters in size and was immersed in a slightly inhomogeneous magnetic field of approximately 1 kilogauss. The best echoes occurred when the exciting high-frequency pulses were adjusted to the mean gyro frequency and propagated across with their electric vector perpendicular to the magnetic field. Pulse widths were short enough so that their frequency content was sufficiently broad to (almost equally) excite all of the electrons, and their strength large enough to permit one to ignore the velocity of the electrons before the pulse sequence. Typically, the inhomogeneity was 1 per cent in a kilogauss field and the pulsewidth was approximately 10^{-8} second. The power of the pulses corresponded to a few watts over a few square centimeters.

The Hill-Kaplan experiments,[1] and those of Harp, Bruce, and Crawford[12] are of two types, which are drawn schematically in Fig. 1. For the two-pulse experiment a

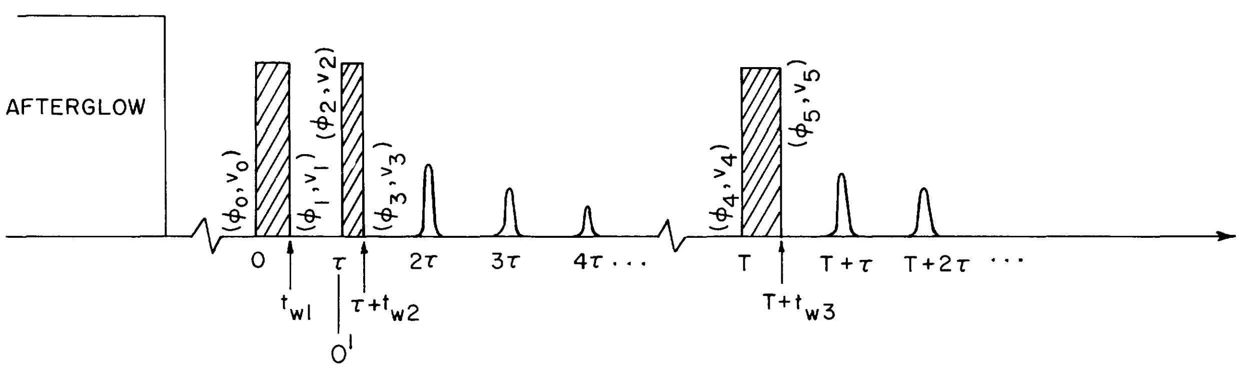

Fig. 1. Schematic representation of two- and three-pulse echoes.

pulse is sent in at time t = 0, followed by a second pulse at time $\tau \cong 10^{-7}$ second. Subsequently, sharp echoes are received at times $t = 2\tau, 3\tau, \ldots$. The second type of experiment retains the two pulses but includes a third at a time T which is much greater than τ. An echo train is then observed at times $n\tau$ after the third pulse. The peak power of these echoes is typically measured in milliwatts.

The envelopes of the peak power of the echoes, measured as the interpulse intervals are varied, are characterized by decay times that depend upon the type of gas used and upon the plasma parameters, and are further complicated and diminished by the diffusion of the electrons between pulses from one point to another in the inhomogeneous magnetic field. The three-pulse echoes are especially interesting, for they are quite persistent, even for times T as long as, but usually less than, 1 msec.

More recently, J. M. Wachtel and J. L. Hirshfield[2] have exhibited and explained the radiation subsequent to a single driving pulse when using a nearly monoenergetic electron beam in a very homogeneous magnetic field with a good vacuum. They found a series of decreasing maxima in the power that was radiated. Their magnetic field was roughly 150 Gauss, with an inhomogeneity of less than .05 per cent, and the transverse energy of their beam could be varied between 1 and 5 keV. The drift energy of the electrons along the field was less than 10 eV.

The experiments of Crawford and his co-workers[12] are similar to those of Hill and Kaplan. By working with a magnetic field of very good longitudinal homogeneity, they have reduced the destructive effects of longitudinal diffusion to a sufficiently low point so as to observe as many as 20 two- and three-pulse echoes. Also, they have demonstrated that speed-dependent diffusion of the electrons perpendicular to the magnetic field can produce large three-pulse echoes of long lifetime.

III. THE MODEL

Because the plasma is assumed to be weakly ionized we consider an ensemble of noninteracting electrons which suffer collisions only with neutrals. The collisions are assumed to be speed-dependent and elastic but catastrophic, i.e., all phase memory, but little energy, is lost. The factor $e^{-\nu t}$ is used to describe the proportion of particles that have not suffered collisions up to time t. Graphs of the collision frequency, ν, as a function of speed are given by S. C. Brown.[13] The plasma is immersed in a magnetic field which we assume to be straight, directed along the z-axis, and slightly inhomogeneous. This produces a spread in gyro frequencies which is needed to produce two- and three-pulse echoes, but tends to damp single-pulse echoes.

The small inhomogeneity is assumed to occur over transverse distances considerably larger than the electron's gyro radius. Actually, as the electrons move along the field lines the field seen by a given electron will vary. We shall, as a first approximation, consider any particular member of our system of electrons to be located in a constant local field. Diffusion of the electrons across the lines and the effect of their motion along them are introduced in Section VII, wherein we shall show that diffusion can, in fact, cause echoes, as well as attenuate them.

In addition to the speed dependence of the collision frequency, we assume that the gyro frequency is speed-dependent. Several such speed dependences are possible. Gould, for example, has pointed out that the perturbative solution of an ensemble of electrons moving in a <u>uniform</u> magnetic field but with a static nonuniform electric field exhibits both a speed dependence of gyro frequency and a static spread in gyro frequencies, in all respects equivalent to the a spread that we have visualized as being due to a transverse gradient. Other possibilities are the gyro speed dependences, owing to the relativistic mass shift and the large size of the electron orbit in the inhomogeneous field.

The general picture is that of an ensemble of noninteracting electrons that suffer speed-dependent collisions and exhibit speed-dependent gyro frequencies. Each electron is specified by a complete set of parameters. Generally, this complete set consists of the value of the magnetic field in which the particle finds itself and the velocity just before the first pulse. If we wish to include the effects of diffusion across the field lines and the motion along them, then we may need additional parameters. Having labeled each member of the ensemble, we then follow its motion through a sequence of pulses. The final velocity distribution of the ensemble gives the radiation which occurs subsequent to the pulse sequence.

In particular, if we assume that the plasma is small, dilute, and but weakly relativistic, then the radiation will be dipole and reabsorption by the plasma may be neglected. The dipole radiation from a system of N electrons is

$$P(t) = \frac{2}{3}\frac{e^2}{c^3}\left(\sum_{k=1}^{N} \vec{A}_k(t)\right)^2 , \tag{1}$$

where $\vec{A}_k$ is the acceleration of the k^{th} ensemble member. The sum may be split up

$$P(t) = \frac{2}{3}\frac{e^2}{c^3}\left(\sum_{k=1}^{N} A_k^2(t) + \sum_{j\neq k}^{N} \vec{A}_j(t)\cdot\vec{A}_k(t)\right) . \tag{2}$$

The first sum is the incoherent contribution, and the second is coherent. If the complete set of parameters is labeled $(a, b, c, \ldots)$, then, for a large number of electrons, we may replace the sum by an integral,

$$\sum_k \vec{A}_k \longrightarrow N \int f(a, b, \ldots)\, \vec{A}_{a,b,\ldots}\, dadb \ldots ,$$

where Nfdadb... tells the number of electrons specified by $(a, b, \ldots)$. We have chosen f to be normalized to unity. We thus have

$$P(t) = \frac{2}{3}\frac{e^2}{c^3}\left\{N \int f_{a,b,\ldots}\, A^2_{a,b,\ldots}\, dadb \ldots + N^2\left(\int f_{a,b,\ldots}\, \vec{A}_{a,b,\ldots}\, dadb \ldots\right)^2\right\}.$$

The first term is incoherent and would dominate but for the transients induced in the ensemble by the pulses. We need to consider here only the coherent part, and if we ignore collisional radiation, we may write the acceleration as being due to the magnetic field,

$$(A_x+iA_y) \cong |\omega_c|(V_x+iV_y),$$

where $|\omega_c|$ is the magnitude of the center gyro frequency and (V_x, V_y) refers to the transverse velocity. The coherent radiation is, therefore,

$$P_{coh.}(t) = \frac{2}{3}\frac{e^2}{c^3} N^2 |\omega_c|^2 \left| \left\langle (V_x+iV_y)\right\rangle_{a,b,\ldots} \right|^2 . \tag{3}$$

The angular brackets indicate an ensemble average, with $f_{ab\ldots}$ used. Maxima in the power are evidenced by maxima in the magnitude of the averaged transverse velocity.

IV. RADIATION AFTER ONE PULSE

Let us denote by ω the local gyro frequency for a given member,

$$\vec{\omega} = -\left(\frac{e\vec{B}_o}{mc}\right) = -(|\omega_c| + \delta)\left(\frac{\vec{B}_o}{B_o}\right), \tag{4}$$

where $\vec{B}_o$ is the local static magnetic field, and ω_c is the center gyro frequency, a constant, and δ gives the field in which an electron moves. The magnitude of the electric charge is e. As we have said, δ will change, for a given electron as it moves, but for the present we neglect diffusion. During a short pulse, say 10^{-8} second, we may ignore collisions. The motion during a pulse is, then,

$$\frac{d\vec{V}}{dt} = \vec{V} \times \vec{\omega} - \frac{e}{m}\vec{E} - \frac{e}{mc}\vec{V} \times \vec{B}, \tag{5}$$

where $\vec{\omega}$, given by Eq. 4, is directed along the static magnetic field, and $(\vec{B}, \vec{E})$ are the pulse fields. We choose $\vec{\omega}$ to lie along z and send in a plane wave propagating along x with its fields $(\vec{E}, \vec{B})$ lying along y and z. Echo radiation will result for other directions and polarizations, but this is simplest. The equation of motion may conveniently be written

$$\frac{d}{dt}V^+ + i\omega V^+ = (-)\, i\frac{e}{m}E_y + i\frac{e}{mc}V^+B_z, \tag{6}$$

where

$$V^+ \equiv V_x + iV_y \tag{7}$$

is the transverse velocity in rotating coordinates. Denoting the pulse width by t_w, we find the velocity just after the pulse to be

$$V^+ \equiv V_1\, e^{-i\phi_1} = V_o\, e^{-i(\omega t_w + \phi_o)} \cdot \left(1 - \Lambda_o g_o\, e^{i\phi_o}\right). \tag{8}$$

To get this result, we used the well-known solution to a first-order differential equation with the right-hand side of (6) as a source. (V_o, ϕ_o) and (V_1, ϕ_1) are the transverse speed and phase just before and immediately after the pulse. Λ_o represents the ratio of the speed increment associated with the pulse to the initial speed,

$$\Lambda_o = \left(\frac{eE_o t_w}{mV_o}\right). \tag{9}$$

g_o is an integral of order one in magnitude,

$$g_o = \frac{i}{t_w}\int_0^{t_w} e^{i\omega\xi}\left(\frac{E_y}{E_o} - \frac{V^+}{c}\frac{B_z}{E_o}\right)d\xi. \tag{10}$$

The pulse fields are to be evaluated along the particle trajectory. E_o represents the pulse strength, and if we choose

$$E_y = B_z = E_o(e^{i(kx-wt)} + c.c.),$$

then

$$g_o = \frac{i}{t_w}\int_0^{t_w} e^{i\omega\xi}\left(1-\frac{V^+}{c}\right)(e^{i(kx-w\xi)}+c.c.)\,d\xi. \tag{11}$$

The trajectory position, x, is a function of time and renders the driving force nonlinear during the pulse. This can cause echoes, but has been shown by Hermann and Whitmer[9] to be of small import for weakly ionized gases. If we locate the gyro center by x_o, then $kx = kx_o$ + (terms of order V/c). Ignoring the nonlinear driving terms is equivalent, therefore, to ignoring terms of order V/c. Doing this and setting the pulse frequency, w, equal to the center gyro frequency, $-|\omega_c|$, gives

$$g_o \cong \frac{i}{t_w} e^{-ikx_o}\int_0^{t_w} e^{-\delta\xi}\,d\xi. \tag{12}$$

If the pulse is short, its spread in frequency covers the spread in gyro frequencies,

$$t_w\delta_{rms} < 1,$$

where δ_{rms} is a measure of this spread, and

$$g_o \cong i\,e^{-ikx_o}. \tag{13}$$

Subsequent to the pulse the electrons gyrate with a frequency that is a function of their net speed, u. Denoting the speed dependence by $\delta(u)$ the net gyro frequency is,

$$\omega(u) = \omega + \delta(u), \tag{14}$$

with ω given by (4). In addition, the phases of those electrons that collide will be randomized by elastic collisions. After a time t, a proportion $e^{-\nu(u)t}$ will have survived collision. The total velocity $\vec{u}$ is the sum of the transverse and longitudinal velocities $\vec{u} = \vec{V} + \vec{u}_p$, where, $\vec{u}_p$ is the velocity along B_o which is unaffected by the pulse. We may now follow the motion of an ensemble member. Just after the first pulse the transverse velocity is given by (8). For later times the coherent velocity is

$$V^+ = V_1\, e^{-i\phi_1}\, e^{-\nu_1 t}\, e^{-\omega_1(t-t_w)}, \tag{15}$$

where the velocity has been weighted to include only those electrons escaping collisions, and the use of subscripts on ν and ω is an abbreviated way of remembering their dependence upon the speed u_1. Equations 8 and 15 combine to give

$$V^+ = V_o\left(e^{-i\phi_o} - \Lambda_o g_o\right) e^{-i\omega t} e^{-(\nu_1+i\delta_1)t}, \tag{16}$$

where ω is given by (4). The speed u_1 is given by

$$\begin{aligned} u_1^2 &= V_1^2 + u_p^2 \\ &= V_o^2\left|1 - \Lambda_o g_o\, e^{i\phi_o}\right|^2 + u_p^2 \\ &= V_o^2\left\{\left(1+\Lambda_o^2\right) + 2\Lambda_o \sin(\phi_o - kx_o)\right\} + u_p^2. \end{aligned} \tag{17}$$

Here we use the simplified form (13) for g_o. The complete set includes V_o, ϕ_o, u_p and the local gyro frequency δ. If the initial phase is random, then the average over it may be performed:

$$\left\langle V^+\right\rangle_{\phi_o} = \frac{V_o}{2\pi} e^{-i\omega t}\int_0^{2\pi} d\phi_o\left(e^{-i\phi_o} - \Lambda_o g_o\right) e^{-(\nu_1+i\delta_1)t}.$$

The arbitrary phase kx_o may be removed from further consideration by transforming to

$$x = \phi_o - kx_o.$$

$$\left.\begin{aligned} &\left\langle V^+\right\rangle_{\phi_o} = \frac{V_o}{2\pi} e^{-i(\omega t + kx_o)}\int_0^{2\pi} dx\left(e^{-ix} - i\Lambda_o\right) e^{-(\nu_1+i\delta_1)t} \\ &\text{where } \nu_1 \text{ and } \delta_1 \text{ are functions of } u_1, \\ &\quad u_1^2 = u_p^2 + V_o^2\left[\left(1+\Lambda_o^2\right) + 2\Lambda_o \sin x\right] \end{aligned}\right\} \tag{18}$$

This is the major result here. The final averaged velocity is obtained by averaging this over the spread, δ, and the initial speeds u_p and V_o. δ appears only in the multiplicative exponent $e^{-i\omega t}$ and produces only a decay resulting from phase mixing. Thus, for interesting one-pulse reverberations, we desire that the inhomogeneity be small. If there were no speed dependence, then ν_1 and δ_1 are constants and may be taken out of the integral, resulting in a simple monotonic decay, i. e., no echoes. When δ_1 and/or ν_1 depend upon speed, echoes are the rule. This should become clear as we now consider several special cases. First, we consider a weak pulse with speed dependence of both δ and ν; second, we look at this result when the collisional dependence predominates; and third, we find the averaged velocity for any pulse strength when collisions may be ignored and the gyro shift is proportional to the energy (Wachtel-Hirshfield).

CASE 1: Weak Pulse

For a weak pulse, $\Lambda_o \ll 1$, we may expand the speed u_1 in powers of Λ_o,

$$u_1 = u_o + \frac{\Lambda_o V_o^2}{u_o} \sin x + \dots , \tag{19}$$

where $u_o^2 = V_o^2 + u_p^2$ is the initial total speed. Similarily, ν_1 may be expanded,

$$\nu_1 = \nu_o + \nu_o' \frac{\Lambda_o V_o^2}{u_o} \sin x + \dots .$$

If the departure of the gyro frequency is proportional to the energy, then $\delta_1 = Bu_1^2$ and,

$$\left\langle V^+ \right\rangle_{\phi_o} \cong V_o \, e^{-i\omega t} \, e^{-\left(\nu_o + iBu_o^2\right)t}$$

$$\cdot \int_0^{2\pi} \frac{dx}{2\pi} \left(e^{-ix} - i\Lambda_o\right) e^{-\left(\frac{\nu_o'}{u_o} + 2iB\right)\Lambda_o V_o^2 t \sin x} .$$

Recognizing Bessel functions, we find

$$\left\langle V^+ \right\rangle_{\phi_o} \cong (-)\, V_o \, e^{-i\omega t} \, e^{-\left(\nu_o + iBu_o^2\right)t}$$

$$\cdot \left\{ J_1(Kt) + i\Lambda_o J_o(Kt) \right\} , \tag{20}$$

where $K = \left(2B - i \frac{\nu_o'}{u_o}\right) \Lambda_o V_o^2$.

If the field is homogeneous and if there is no spread in initial speeds, then the averages over δ, V_o, and u_p need not be performed, and this is the final form. The radiation is given by using (20) in (3). If there is a spread in gyro frequencies, then phase mixing will cause a damping of the velocity which will compete with collisional damping. The characteristic decay time associated with this process will be $1/\delta_{rms}$, with δ_{rms} representing the spread in frequency. If there is an initial spread in speeds, then the averages over V_o, u_p must be performed, but this cannot be done unless the speed dependence of the collision frequency is known.

CASE 2: Collisions Only

When $\delta_1 \ll \nu_1$ we may ignore the speed dependence of the gyro frequency altogether. If this dependence results from the relativistic effect, then we require

$$\frac{|\omega_c|}{2}\left(\frac{u_1}{c}\right)^2 \ll \nu_1. \tag{21}$$

When this applies, the velocity is given by (18), with $\delta_1 = 0$. If, moreover, the pulse is weak, then $\Lambda_o \ll 1$ and the result is given by (20), with the coefficient B equal to zero.

$$\left\langle v^+ \right\rangle_{\phi_o} \cong (i)\, V_o\, e^{-i\omega t}\, e^{-\nu_o t}\left\{ I_1\left(\frac{\nu_o'}{u_o}\Lambda_o V_o^2 t\right) - \Lambda_o I_o(\text{same})\right\},$$

where I_o, I_1 are modified Bessel functions.

This may be further simplified, for the validity of the power-series expansion of ν_1 is under the assumption that the argument of the Bessel functions is small with respect to the argument, $\nu_o t$, of the exponential. We expand the Bessel functions and find

$$\left\langle v^+ \right\rangle_{\phi_o} \cong (-)\,\frac{i}{2}\Lambda_o V_o\, e^{-i\omega t}\, e^{-\nu_o t}\left(2 - \frac{V_o^2}{u_o}\nu_o' t\right). \tag{22}$$

This applies to a monoenergetic beam, and indicates a single maximum at time t_{max},

$$t_{max} = \frac{1}{\nu_o}\left(1 + 2\frac{u_o}{V_o^2}\frac{\nu_o}{\nu_o'}\right). \tag{23}$$

It appears that, by working in a region such that the slope ν_o' is sufficiently negative, one could destroy this echo entirely.

CASE 3: Negligible Collisions

This is the regime applicable to the Wachtel-Hirshfield experiment. Suppose that the converse of condition (21) applies. Then we may ignore ν_1 in the expression (18). If the temperature should be rather warm, then the shift in frequency may be attributed to the relativistic mass shift, with $B = \frac{|\omega_c|}{2}\left(\frac{1}{c}\right)^2$, and we find

$$\left\langle v^+ \right\rangle_{\phi_o} = (-)\, V_o\, e^{-i\omega t}\, e^{-iB\left(u_o^2 + \Lambda_o^2 V_o^2\right)t}$$

$$\cdot\left\{ J_1\left(2B\Lambda_o V_o^2 t\right) + i\Lambda_o J_o(\text{same})\right\}. \tag{24}$$

The power goes as the square of the absolute value of this and indicates a series of decreasing maxima. The special cases (22) and (24) have been derived by Hirshfield. The Hirshfield-Wachtel experiment, described in Section II, shows good qualitative agreement with the result (24).

The maxima described by (24) are easily understood physically. The electrons will have received slightly different energies by the end of the pulse, the result depending upon their phases at the initiation of the pulse. This produces a spread in gyro frequencies whose cumulative effect over subsequent times causes the electrons to coalesce in phase occasionally, and echoes result.

If collisions are rare and a Maxwellian velocity distribution obtains, then the averages over (V_o, u_o) of (24) may be performed. For a weak pulse, $\Lambda_o \ll 1$, this average, when squared, gives a power

$$P(t) \cong \frac{2}{3} \frac{N^2 e^2}{c} \omega_c^2 \left(\frac{kT}{mc^2}\right)^2 \frac{\left(\frac{\lambda_o}{c} \omega_c t\right)^2}{\left\{1 + \left(\frac{kT}{mc^2} \omega_c t\right)^2\right\}^{5/2}}. \tag{25}$$

Here, λ_o is the speed increment associated with the pulse,

$$\lambda_o = \Lambda_o V_o.$$

We have assumed the magnetic field to be homogeneous. A single maximum is indicated at time

$$t_{max} = \sqrt{\frac{2}{3}} \left(\frac{mc^2}{kT}\right) \frac{1}{|\omega_c|}. \tag{26}$$

The reason for a single maximum here is the fact that our initial state is disordered. As we have seen, many maxima result for an initially monoenergetic beam.

V. RADIATION AFTER TWO PULSES

In contrast to the Wachtel-Hirshfield experiment the characteristic feature of the two- and three-pulse experiments is that they require a small inhomogeneity in the magnetic field. Furthermore, the echoes always occur at times determined by the interpulse spacing, although their magnitude varied with the plasma parameters and with the type of gas that was used. Finally, we note that a first pulse sufficiently strong to destroy the past history of the roughly thermal afterglow electrons is used.

We follow the motion of a given electron through the two-pulse process, finding V^+ for times subsequent to the second pulse. The average of this relates to the power through Eq. 3. We allow both the pulse duration and electric field strength to be different for the two pulses. We assume that a given electron stays in the same local magnetic field for the duration of the process. Corrections to this will be examined later. Defining the times as drawn in Fig. 1, with pulse widths t_{w1} and t_{w2} for the first and second pulses, the analysis proceeds as for the one-pulse case. The result is

$$V^+ = e^{-\nu_1\tau} e^{-(\nu_3+i\omega_3)t} \left\{ \left(V_o e^{-i\phi_o} - \lambda_o g_o \right) e^{-i\omega_1\tau} - \lambda_2 g_2 \right\}. \tag{27}$$

The collisional exponentials express the fact that only those electrons which suffer no collisions retain phase memory. We have shifted the time origin to O' in Fig. 1. In general, the integrals g_o and g_2 refer to trajectories during their respective pulses, but when kR is small they may be used in the form (13). λ_o and λ_2 are the speed increments associated with the two pulses. λ_o is given, for example, by (9). If the first pulse is strong enough, then λ_o is much larger than V_o, which may then be neglected in (27). The collision and gyro frequencies ν and ω occur in (27) as functions of the speeds u_1 and u_3 after the first and second pulses. Under the assumption of a strong first pulse, the speed u_1 is given by (16) as

$$u_1^2 = u_p^2 + \lambda_o^2. \tag{28}$$

If the initial parallel velocity, u_p, is of order of magnitude V_o, then we may drop u_p from further consideration. The speed u_3 will be given by the square of (27) with all ν's equated to zero. Under the assumptions of strong first pulse and small gyro radius, V_o is small and g is given by (13). The speed is then

$$u_3^2 \cong V_3^2 = \lambda_o^2 + \lambda_2^2 + 2\lambda_o\lambda_2 \cos \omega_1\tau. \tag{29}$$

With these assumptions the velocity becomes

$$V^+ = (-)\, i\, e^{-\nu_1\tau} e^{-i(\omega t+kx_o)} G_3 \left\{ \lambda_o e^{-i\omega_1\tau} + \lambda_2 \right\}, \tag{30}$$

where

$$G_3 = e^{-(\nu_3 + i\delta_3)t}. \tag{31}$$

G_3 is a function only of $\cos \omega_1 \tau$ and so may be expanded in a Fourier series,

$$G_3(\cos x) = \sum_{-\infty}^{\infty} A_n\, e^{inx}$$

$$A_n = \frac{1}{2\pi} \int_0^{2\pi} G_3\, e^{-inx}\, dx.$$

Therefore we have

$$V^+ = (-i)\, e^{-ikx_o}\, e^{-\nu_1 \tau} \left(\lambda_o\, e^{-i\omega_1 \tau} + \lambda_2 \right) \sum_{-\infty}^{\infty} A_n\, e^{in\omega_1 \tau}\, e^{-i\omega t}.$$

The presence of discrete echoes now appears, for collecting terms gives the general result

$$\left. \begin{aligned} & V^+ = (-i)\, e^{-ikx_o}\, e^{-\nu_1 \tau} \sum_{-\infty}^{\infty} b_n\, e^{-i\omega(t - n\tau)}, \\ & \text{where} \\ & b_n = e^{in\delta_1 \tau} (\lambda_o A_{n+1} + \lambda_2 A_n) \\ & A_n = \frac{1}{2\pi} \int_0^{2\pi} e^{-(\nu_3 + i\delta_3)t}\, e^{-inx}\, dx. \end{aligned} \right\} \tag{32}$$

The generality of this result should be emphasized. We have shown that echoes occur at times $n\tau$ after the second pulse for any speed dependence of the collision and/or gyro frequencies. An echo is due to a partial coalescence in phases of the ensemble members as each follows the motion dictated by the two pulses and by its local field. Mathematically we perform the average over the spread, δ, in frequencies. The dependence upon δ occurs only in the exponents $e^{-i\omega(t-n\tau)}$. When the average upon δ is taken, Eq. 32 becomes a sum of terms distinct in time if $\frac{1}{\delta_{rms}} < \tau$, where τ is the pulse separation, and δ_{rms} gives the spread in gyro frequencies, with their maxima at times $n\tau$ if the distribution of δ is smooth. The power for times near the n^{th} echo is given by (3) and (32):

$$P_n(t) = \frac{2}{3} \frac{e^2}{c^3} N^2 \omega_c^2\, e^{-2\beta_1 \tau}\, |b_n(t)|^2 \left| \left\langle e^{i\delta(t - n\tau)} \right\rangle_\delta \right|^2.$$

If, for example, the spread in δ is due to a spatial variation along y, then

$$\left\langle e^{i\delta(t-n\tau)}\right\rangle = \frac{1}{N}\int_a^b N(y)\, e^{i\delta(y)(t-n\tau)}\, dy,$$

where $N(y)\,dy$ is the number of electrons at y, and N is the total number of electrons. The limits a, b cover the extent of the plasma.

Generally, the peaks of the echoes occur at $n\tau$. In particular, the velocity will have a value at the first echo given by

$$\left\langle V^{+}\right\rangle_{\delta}^{t=\tau} = (-i)\, e^{-ikx_o}\, e^{-\nu_1\tau}\, e^{i\delta_1\tau}$$

$$\cdot \int_0^{2\pi} \frac{dx}{2\pi}\left\{\lambda_o e^{-ix} + \lambda_2\right\} e^{-ix}\, e^{-(\nu_3+i\delta_3)\tau}\, dx. \tag{33}$$

This has been given by Hermann, Hill, and Kaplan.[11] They found in their experiments that the collisional effect completely dominated the speed dependence, δ_3, of the gyro frequency. The reason is that they were working with energies in the electron-volt range, so that the relativistic effect was small. Competition between these effects occurs when

$$\delta_3 \sim \nu_3.$$

If the relativistic effect is the cause of the shift, then we require

$$\frac{|\omega_c|}{2}\left(\frac{u_3}{c}\right)^2 \sim \nu.$$

For a kilogauss field, energies of 10-100 eV are indicated when $\nu \sim 10^6\ \mathrm{sec}^{-1}$.

If, as is applicable to neon, we assume that the collision frequency is proportional to the energy, as is also the gyro frequency, then

$$\nu + i\delta = (a+ib)u^2, \tag{34}$$

and we easily find (see Gould[10])

$$b_n = (i)^n\, e^{inb\lambda_o^2\tau}\, e^{-(a+ib)\left(\lambda_o^2+\lambda_2^2\right)t}$$

$$\cdot \left\{i\lambda_o J_{n+1}[Lt] + \lambda_2 J_n(Lt)\right\}, \tag{35}$$

where $L = 2(a-ib)\,\lambda_o\lambda_2$.

If collisions are few, there can still be echoes of appreciable magnitude because of the relativistic shift. Unless the energies are fairly high, however, one must wait a long time, during which diffusion mechanisms may damp the effect strongly.

VI. RADIATION AFTER THREE PULSES

The experiment of Hill and Kaplan involves third-pulse times, T, which are so long that virtually all electrons will have suffered phase-destroying collisions between the second and third pulses. Because these elastic collisions are with the neutral atoms, the energy of an electron will slowly be degraded, eventually to near thermal values. For times T that are comparable to the energy relaxation time, however, memory of the first interpulse time, τ, is retained by those electrons that do <u>not</u> suffer phase-destroying collisions between the first two pulses in the shape of their subsequent energy distribution. As before, we shall not discuss the diffusion of the electrons from one field point to another just yet. We must, however, consider the fact that the large transverse velocity acquired after the second pulse is randomized to three dimensions during the second interpulse interval.

If we measure time from the arrival of the third pulse, the velocity, appropriately reduced to exclude those electrons suffering collisions before the second pulse or after the third, is

$$v^+ = e^{-\nu_1\tau}\, e^{-\nu_5 t}\, e^{-i\omega_5 t}\left\{v_4\, e^{-i\phi_4} - \lambda_4 g_4\right\}. \tag{36}$$

To account for the randomization of the velocity into three dimensions, we must use for v_4 only the transverse component of the velocity $\vec{u}_4$. The result must be averaged over all directions of $\vec{u}_4$. Figure 2 indicates the geometry. The angles θ, ϕ locate the direction of $\vec{u}_4$. We find

$$\left.\begin{aligned} u_5^2 &= v_5^2 + u_{p,5}^2 = v_4^2\left|1 - \Lambda_4 g_4\, e^{i\phi_4}\right|^2 + u_4^2 \cos^2\theta \\ v_4\, e^{-i\phi_4} &= u_4 \sin\theta\, e^{i\phi}. \end{aligned}\right\} \tag{37}$$

Finally, to account for energy loss in the interval (τ, T) we must find u_4 in terms of the initial value, v_3. For this, we consider that in elastic collisions the electrons lose a small fraction of their energy to the recoil of the atom or rotation of a molecule, and therefore the energy is degraded at the rate

$$\frac{d(u^2)}{dt} = -\gamma_{(u)} u^2. \tag{38}$$

For atoms with no rotation or vibration,

$$\gamma = 2\,\frac{m}{M}\,\nu. \tag{39}$$

The transverse speed, v_3, is given by (29). We must then regard u_4 as a function of $x = \omega_1\tau$. Under the usual assumption of negligible gyro radius g_4 is given by Eq. 13.

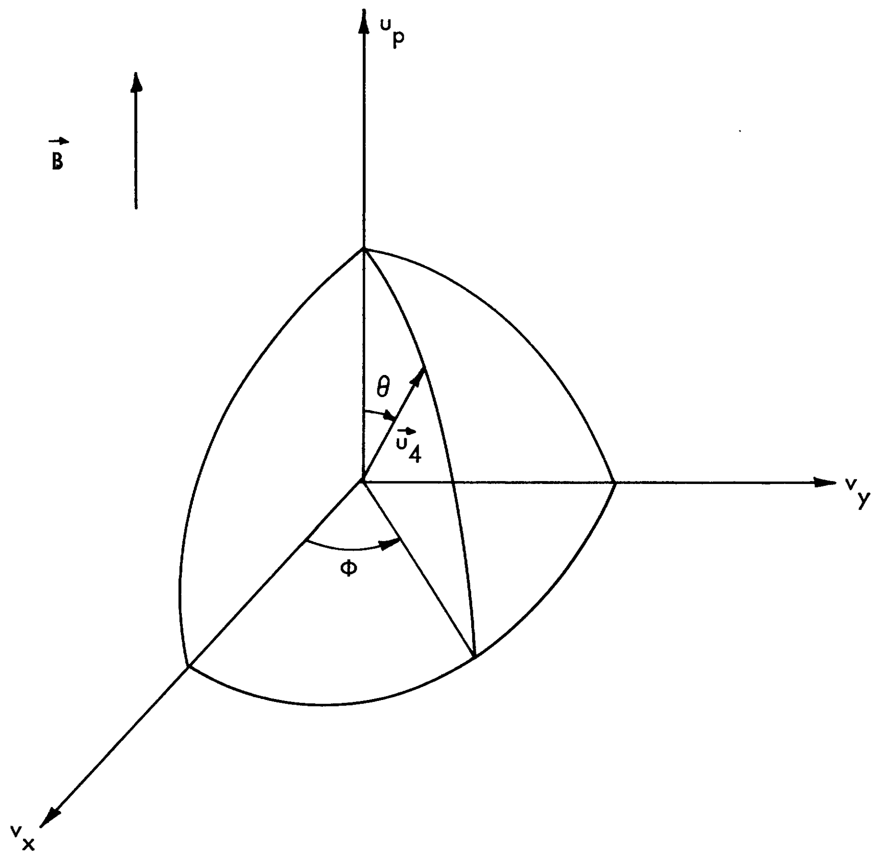

Fig. 2. The randomized velocity $\vec{u}_4$ just before the third pulse.

Echoes at times $n\tau$, $n = 1, 2, \ldots$ are demonstrated in a manner similar to that used in the two-pulse case. Defining

$$G(\cos x) = \left\{u_4(x) \sin\theta\, e^{i\phi} - \lambda_4 g\right\} e^{-(\nu_5 + i\delta_5)t},$$

we make a Fourier expansion in x. The result, averaged uniformly over all directions (θ, ϕ), is

$$\left\langle v^+ \right\rangle = \sum_{n=-\infty}^{\infty} e^{-i\omega(t-n\tau)}\, e^{in\delta_1\tau}\, e^{-\nu_1\tau} \left\langle A_n \right\rangle, \tag{40}$$

where

$$\left\langle A_n \right\rangle = \int_0^{2\pi} \frac{dx}{2\pi}\, e^{-inx} \int_0^{2\pi} d\phi \int_0^{\pi} \frac{d\theta \sin\theta}{4\pi}\, G, \tag{41}$$

where $u_4(x)$ is the solution of (38), with $v_3(x)$ as the initial condition. This is the general result of this section. It shows, by using the arguments of Section V, that speed-dependence of the collision and/or gyro frequencies causes echoes at times $n\tau$ after the third pulse.

Hill and his co-workers, working in a regime where collisional effects dominate, have measured the envelope of the peak power in the first echo, at time τ after the third pulse as T is varied. Henceforth, in keeping with their regime, we ignore the factor δ_5 entirely.

If, as is appropriate to Neon, we assume that the collision frequency is proportional to the energy,

$$\nu = au^2, \tag{42}$$

then the integrations in (41) on ϕ and θ can be performed. Since g is a phase factor, nothing is lost if it be set equal to unity. Thus

$$u_5^2 = u_4^2 + \lambda_4^2 - 2\lambda_4 u_4 \sin\theta\cos\phi,$$

and v_3^2 is given by Eq. 29. The result is

$$\frac{\left\langle v_n^+\right\rangle^{t=n\tau}}{\lambda_4} = e^{-\nu_1\tau}\,\frac{\left\langle A_n\right\rangle}{\lambda_4}$$

$$= \frac{1}{\pi}\, e^{-a\tau\left(\lambda_o^2 + n\lambda_4^2\right)} \int_0^\pi dx \cos nx\, e^{-an\tau u_4^2}$$

$$\cdot\left\{\frac{\cosh\left(2a\lambda_4 u_4 n\tau\right)}{\left(2a\lambda_4^2 n\tau\right)} - \left(1 + \frac{1}{2a\lambda_4^2 n\tau}\right)\frac{\sinh\left(2a\lambda_4 u_4 n\tau\right)}{\left(2a\lambda_4 u_4 n\tau\right)}\right\}. \tag{43}$$

This complicated expression is a function of the independent parameters λ_o, λ_2, λ_4, τ, T and the energy coefficients a and γ that characterize the collision process. It has been evaluated numerically when all three pulses are of equal strength, λ, for two energy relaxation models. When γ is constant, u_4 is given by

$$u_4^2 = v_3^2\, e^{-\gamma T}.$$

The relevant parameters now are γT and $\nu\tau = a\lambda^2\tau$, the collisional damping factor operative between the first two pulses. Graphs of $\frac{\left\langle V^+\right\rangle}{\lambda}$ as a function of these two parameters (dotted lines) are given in Figs. 3 and 4. Shown also (solid lines) are the results when γ is proportional to the energy, $\gamma = \alpha u^2$, for which

$$u_4^2 = \frac{V_3^2}{1 + \alpha\lambda^2 T \left(\frac{\left\langle V_3\right\rangle}{\lambda}\right)^2}.$$

The parameters now are $\alpha\lambda^2 T$ and $a\lambda^2\tau$. These echoes are produced by speed-dependent

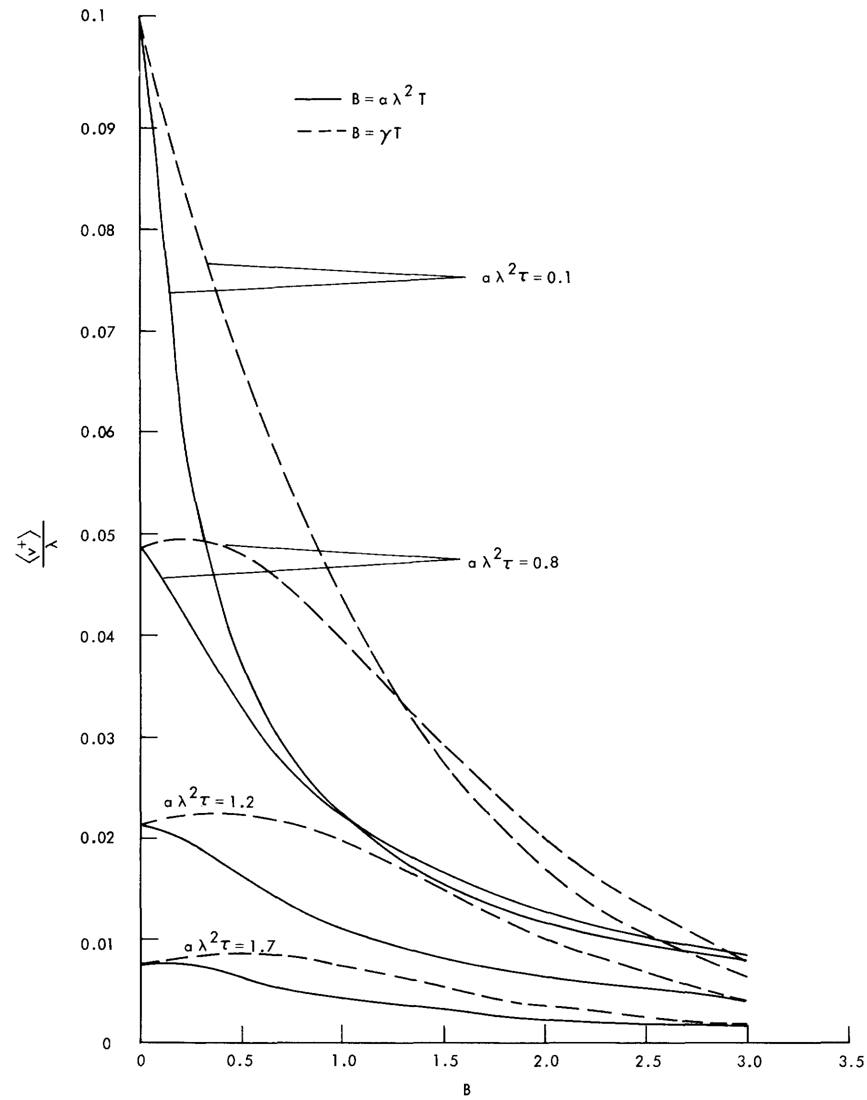

Fig. 3. Variation of the velocity maximum in the first three-pulse collisional echo as the time, T, of arrival of the third pulse is varied.

collisions, and we have used a greatly idealized model, since diffusion effects have been ignored entirely.

The general features of Figs. 3 and 4 can be easily understood. For example, as the first interpulse spacing τ is increased to large values, too many of the electrons will have had their phase coherence destroyed to maintain the echo. As τ is decreased to zero, the echoes must likewise diminish, for then very few electrons will collide and the system is virtually linear. One sees, however, that for this model the echoes continue to increase only until τ reaches rather small times. For this reason, in an actual experimental situation, one should expect the echoes to continue rising as τ is decreased to rather small times. The recent experiments of Crawford and co-workers[12] do seem to indicate that their echoes increase, even for τ very small. They conclude

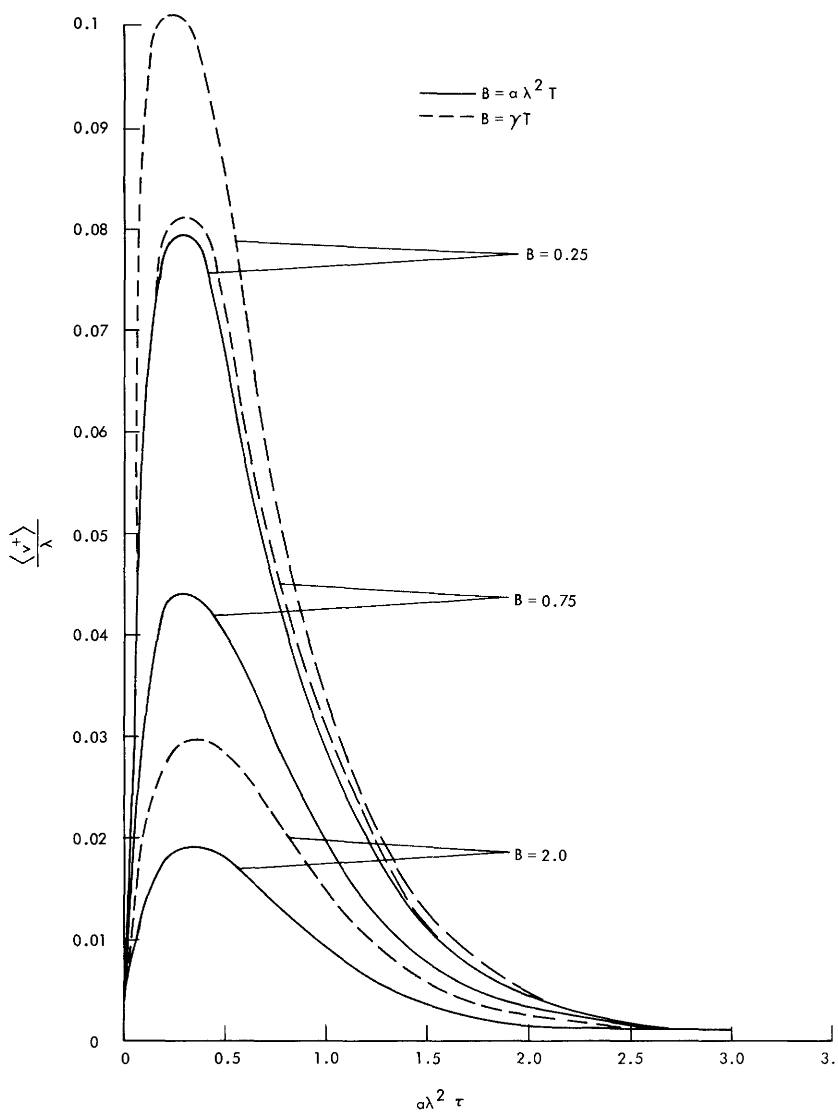

Fig. 4. Variation of the velocity maximum in the first three-pulse collisional echo as the time, τ, between the first two pulses is varied.

that their echoes are produced by the diffusion of electrons across the field lines, a speed-dependent mechanism operating between the second and third pulses, but retaining memory of τ through the speed, u_4. They have described a simple model to show this. We shall see, in general, how echoes can result from diffusion.

If instead one varies T this model shows echoes diminishing, owing to energy degradation. For very short times T, of the order of a few momentum transfer collision times, this model is invalid, for then phase coherence still exists and three-pulse echoes similar to the two-pulse echoes described above can occur. A straightforward extension[14] of the two-pulse model presented above, which is valid for T small, indicates echoes at times $m\tau + nT$, and $p(T-\tau)$ where $m, n, p = 1, 2, \ldots$. There is little intrinsic interest in these echoes, however, for they are in every way similar to their two-pulse cousins. There are scant published data on the variation of three-pulse echoes with T. The data of Hill and his co-workers[11] seem to indicate an initial decrease followed by a rise and gradual fall. The model that we have given here does not explain such behavior. Because T is so very long, it is reasonable to suppose that each experiment has its unique features. The best that we can hope to achieve here is the demonstration of the effects of individual causes.

VII. EFFECTS OF DIFFUSION

We shall now undertake a simple examination of the electrons' motion as they move from one local field to another. For one-pulse experiments typical times should be chosen so that these effects are small, for we have seen that field inhomogeneities can, through phase mixing, interfere with the processes that we may wish to examine. For the two- and three-pulse experiments, however, we have seen that a spread in gyro frequencies is necessary to produce echoes. The times involved may now be appreciable and, if this spread be due to a field inhomogeneity, then diffusive effects enter perforce.

Our first task will be to explore the damping of the two- and three-pulse echoes considered in Sections V and VI, which is caused by longitudinal inhomogeneities in the magnetic field. Later we shall see that if the field is nearly constant longitudinally, diffusion across them can cause echoes.

We start then by assuming that the magnetic field lies along z but varies in transverse and longitudinal directions and that for the rather strong fields used here each electron stays on a field line, so that we may label them with the parameter δ, which is constant for a given electron. The effects of motion along z will require new parameters, to label the initial longitudinal position and give the longitudinal velocity. The labeling introduced in Eqs. 4 and 14 may be used, but with the single addition of a function $\Delta(z)$ to label the longitudinal dependence of the gyro frequency,

$$\omega(u, z) = -(|\omega_c|+\delta) + \delta(u) + \Delta(z)$$

$$= \omega(u) + \Delta(z). \tag{44}$$

The motion after the pulse is simply

$$V^+ = V_o e^{-i(\omega(u)t+\phi_o)} e^{-i\int_0^t \Delta(z)\, d\xi}. \tag{45}$$

The single addition to the previous analysis is the integrated phase, taken along the particle trajectory. For single-pulse experiments the attenuation will be an average of this exponential over the initial positions and over the spread in longitudinal velocity.

7.1 DAMPING OF TWO-PULSE ECHOES BY LONGITUDINAL DIFFUSION

Consider the two-pulse echoes. Equation 30 describes the velocity as the sum of the increment, λ_o, of the first pulse multiplied by the phase accumulated between pulses plus the increment, λ_2, of the second pulse, all multiplied by the phase accumulated after the second pulse. If we represent the longitudinal position just before the first pulse as z_o, the new expression is

$$V^+_{n.c.} = (-)\, i\, e^{-\nu_1\tau} e^{-i(\omega t+kx_o)} G_3 \left\{\lambda_o e^{-i(\omega_1\tau+\eta)} + \lambda_2\right\} e^{-i\epsilon}, \tag{46}$$

where

$$G_3 = e^{-(\nu_3+i\delta_3)t},$$

$$\eta = \int_0^{\tau} \Delta(z_o+u_p\xi)\, d\xi, \quad \text{and} \quad \epsilon_{(t)} = \int_{\tau}^{t+\tau} \Delta(z_o+u_p\xi)\, d\xi. \tag{47}$$

We assume that the longitudinal velocity is constant in time. As before, G_3 is a function, through V_3^2, of cos x, where now, $x = (\omega_1\tau+\eta)$, for V_3^2 is the square of (46), with all collisional factors equated to zero. The analysis proceeds as before. The result is similar to (32). The only change is that b_n changes form:

$$b_n = e^{in\delta_1\tau}\, e^{i(n\eta-\epsilon)}\, (\lambda_o A_{n+1}+\lambda_2 A_n). \tag{48}$$

All other quantities are unchanged. $V_3^2(x)$, for example, is given by (29).

The average over δ gives discrete echoes. The averages over (z_o, v_z) serve only to depress the magnitude. The damping factor for the n^{th} echo, at $t = n\tau$ is, simply,

$$a_n = \left\langle e^{i(n\eta-\gamma(n\tau))} \right\rangle_{z_o, u_p}. \tag{49}$$

The power in the n^{th} echo is damped by the square, $|a_n|^2$. As an example, for linear field variation, $\Delta = az$ and Maxwellian spread in u_p, the first echo velocity is damped by a factor,

$$a_1 = e^{-\frac{a^2\tau^2 V_T^2}{2}}. \tag{50}$$

Here V_T^2 is the thermal velocity squared, $V_T^2 = \frac{kT}{m}$. The dependence on longitudinal displacement, z_o, vanishes in this special case. Other field variations would give a damping factor which depends upon L, the system dimension. The expression for a_1 given by (50) has been arrived at by Hill and his co-workers by a different route. Typically a_1 can diminish the echo power by a factor of 10.

7.2 EFFECT OF LONGITUDINAL DIFFUSION ON THREE-PULSE ECHOES

Diffusive effects enter during the first interpulse interval through the speed $u_4(x)$ where, from (46), $x = (\omega_1\tau+\eta)$. Between the second and third pulses we may ignore cumulative diffusive effects, for by time T the phases will have been randomized by collisions. After the third pulse, diffusion again takes its toll and we need to multiply the velocity by a phase factor just as we did to arrive at (46). The only added assumption that we make is that the longitudinal position, z_5, just after the third pulse be distributed independently of all other variables. This is reasonable, for an electron has sufficient time before the third pulse to collide many times, changing its longitudinal velocity in a random way. Collisions with the boundary have a similar effect.

The velocity after the third pulse is, then

$$V^+ = e^{-\nu_1\tau}\, e^{-i\omega t}\, G,$$

where

$$G = \left\{u_4(x)\sin\theta\, e^{i\phi} - \lambda_4 g\right\} e^{-(\nu_5+i\delta_5)t}\, e^{-i\int_0^t \Delta(z_5+u_{p,5}\xi)\,d\xi}. \tag{51}$$

Here x is given by $(\omega_1\tau+\eta)$, and z_5 is the longitudinal position just after the third pulse. The parallel velocity at this time, $u_{p,5}$, is given by (37) as $u_4 \cos\theta$. The procedure is to perform a Fourier analysis on x. This has been done several times. Omitting only the average on δ, we obtain

$$\left\langle V^+\right\rangle = \sum_{n=-\infty}^{\infty} e^{-i\omega(t-n\tau)}\, e^{in\delta_1\tau}\, e^{-\beta_1\tau}\left\langle e^{in\eta}\right\rangle_{z_o,u_p}\left\langle A_n\right\rangle, \tag{52}$$

where

$$\left\langle A_n\right\rangle = \frac{1}{2\pi}\int_0^{2\pi} dx\, e^{-inx}\int_0^{2\pi} d\phi\int_0^{\pi} d\theta\,\frac{\sin\theta}{4\pi}\int_{-L}^{L}\frac{dz_5}{2L}\, G. \tag{53}$$

The distribution of the longitudinal positions z_o and z_5 are assumed to be random over the system length, taken here as 2L.

This result indicates that diffusion damping does not produce a simple multiplicative damping constant, a_n, as it did for two pulses. It is our purpose only to estimate the magnitude of diffusion effects. Hill and Kaplan worked with a system of approximate length 2L = 10 cm in a field of roughly quadratic dependence, $\Delta = bz^2$, where $b = 10^7\ cm^{-2}\ sec^{-1}$. The attenuation of the first echo is relevant to their experiments. The diffusion enters first in the factor $\left\langle e^{in\eta}\right\rangle$.

For a random distribution of position z_o between $\pm L$, and with a Maxwellian spread one finds for the first echo,

$$\left|\left\langle e^{i\eta}\right\rangle\right|^2 = \frac{\pi}{4b\tau L^2}\;\frac{1}{\sqrt{1+\frac{b^2V_T^2\tau^6}{36}}}. \tag{54}$$

Here we have assumed that $L\sqrt{b\tau} \gg 1$. With $L \sim 5$ cm, $b \sim 10^7\ cm^{-2}$ sec, $V_T \sim 10^7$ cm sec^{-1}, and $\tau \sim 10^{-7}$ sec one finds

$$\left|\left\langle e^{i\eta}\right\rangle\right|^2 \sim \frac{\pi}{100}.$$

The other diffusive factor occurs in the expression for G and should be integrated as a function of θ. During the time interval after the third pulse, however, the value of the parallel velocity is large and will have some randomness in its value. These uncertainties may significantly alter the phase and hence the expression for the velocity. To get an estimate of the diffusive effect, let us assume $u_p\tau$ to be random, as well as z_5. This diffusive damping then may be taken out of the integral to give, for a square-law dependence of Δ,

$$\left\langle e^{-ib\left\{\tau z_5^2 + u_p z_5 \tau^2 + \frac{u_p^2\tau^3}{3}\right\}}\right\rangle_{z_5, u_p\tau} = \int_{-L}^{L} \frac{dz_5}{2L} \int_{-(L+z_5)}^{(L-z_5)} \frac{dy}{2L}\, e^{-ib\tau\left(z_5^2 + z_5 y + \frac{y^2}{3}\right)}.$$

The limit on the integral over $y = u_p\tau$ is prescribed by the condition that the particles remain inside the boundary at $\pm L$. Completing the square on y and changing variables gives

$$\int_{-L}^{L} \frac{dz_5}{2L} \int_{-L+\frac{z_5}{2}}^{L+\frac{z_5}{2}} \frac{d\xi}{2L}\, e^{-i\frac{b\tau}{3}\xi^2}\, e^{-i\frac{b\tau}{4}z_5^2}.$$

The limits on ξ and z_5 can be approximated by infinity, provided $b\tau L^2$ is somewhat larger than one. Squaring, we find

$$\left|\left\langle e^{-i\int_0^\tau \Delta(z_5+u_p\xi)\,d\xi}\right\rangle\right|^2 \cong \frac{3}{4}\left(\frac{\pi}{b\tau L^2}\right)^2. \tag{55}$$

The results in (54) and (55) when multiplied together give a damping of the power in the first echo of approximately

$$\left(\frac{\pi}{b\tau L^2}\right)^3 \sim 10^{-4}.$$

From our arguments this is perhaps overly pessimistic.

7.3 ECHOES PRODUCED BY SPEED-DEPENDENT TRANSVERSE DIFFUSION

As pointed out by Crawford and his co-workers,[12] diffusion transverse to the field lines is speed-dependent and can cause echoes. They avoided destructive longitudinal diffusion by working with fields that varied $\frac{1}{10}$ per cent, or less, along their length. The transverse inhomogeneity was 1 or 2 per cent.

Although the electrons tend to stay tied to their initial field lines, collisions can act by a random-walk process to cause them to move to new fields during the long time T. Because collisions are speed-dependent, echoes result.

After the third pulse an electron will find itself on a new line of force. This is easily included in formula (36) by adding an increment, Δ, to the frequency ω_5,

$$\omega_5' = \omega_5 + \Delta, \tag{56}$$

where Δ is the change in frequency caused by a random walk during the interval (τ, T). Now both the initial field, δ, and the increment, Δ, are random variables, not necessarily independent of one another. If, for example, the number density in space is non-uniform, then they are related, that is, their joint distribution $f(\Delta, T; \delta)$ cannot be factored. This distribution describes the probability of Δ at T when $\Delta = 0$ at τ, and the a priori distribution of δ. Memory of the first interpulse interval is retained because f is a function of the speed $u_4(x)$, where $x = \omega_1\tau$. Performing the average on Δ, we have a new form for G(cos x) which may then be Fourier-analyzed in $x = \omega_1\tau$.

$$G(\cos x) = \{u_4(x) \sin\theta\, e^{i\phi} - \lambda_4 g\}\, e^{-(\nu_5 + i\delta_5)t} \left\langle e^{-it\Delta} \right\rangle_\Delta,$$

where

$$\left\langle e^{-i\Delta t} \right\rangle_\Delta = \int f(\Delta, T; \delta)\, e^{-it\Delta}\, d\Delta.$$

By using this form for G, the result is given by (40) and (41). Furthermore, if the gyro frequency speed dependence may be ignored, the result (43) may be utilized directly with $e^{-i\Delta t}$ inserted in the integrand. Because f depends upon δ the peak may not be exactly at $n\tau$. Explicitly, (43) may be taken over, but with

$$\left\langle e^{-i\delta(t - n\tau)}\, e^{-i\Delta t} \right\rangle_{\Delta, \delta}$$

inserted in the integrand. If, as an idealization, collisions may be ignored other than as a cause of diffusion, then $a \to 0$ in (43), and

$$\frac{\left\langle v^+ \right\rangle^{(n)}}{\lambda_4} = (-)\frac{1}{\pi}\int_0^\pi dx \cos nx \left\langle e^{-i\delta(t - n\tau)}\, e^{-i\Delta t} \right\rangle_{\Delta, \delta}. \tag{56}$$

As an example, consider a linear transverse field variation, $\delta = cy_o$ and $\Delta = c(y - y_o)$, where c is a constant, and y and y_o are the transverse positions at T and 0. We have

$$\frac{\left\langle v^+ \right\rangle^{(n)}}{\lambda_4} = (-)\frac{1}{\pi}\int_0^\pi dx \cos nx \left\langle e^{-icy_o(t - n\tau)}\, e^{-ict(y - y_o)} \right\rangle_{y, y_o}.$$

The simplest random-walk process is described by the joint distribution

$$f(y, y_o) = P(yT \,|\, y_o o)\, f(y_o),$$

where P is the probability of y at T, given y_o at o and

$$P = \frac{1}{\sqrt{4\pi DT}}\, e^{-\frac{(y-y_o)^2}{4DT}},$$

and $f(y_o)$ is the a priori initial distribution of position. D is the diffusion coefficient perpendicular to the magnetic field

$$D = \frac{s^2}{2}\nu,$$

where ν is the collision frequency, and s is the step size caused by one collision. We find easily,

$$\frac{\left\langle V^+ \right\rangle^{(n)}}{\lambda_4} = (-)\frac{1}{\pi}\int_0^\pi dx \cos nx \left\langle e^{-icy_o(t-n\tau)} \right\rangle_{y_o} e^{-DTc^2t^2}.$$

If the initial spatial distribution of electrons, $n(x_o)$, is roughly uniform, then the peak occurs at $t = n\tau$. Then

$$\frac{\left\langle V^+ \right\rangle^{(n)}}{\lambda_4} = (-)\frac{1}{\pi}\int_0^\pi dx \cos nx\, e^{-\frac{s^2}{2} Tc^2 n^2 \tau^2 \nu}.$$

Taking a square-law dependence for the collision frequency, $\nu = au_4^2(x)$, and an exponential decay for the energy, $u_4^2 = V_3^2\, e^{-\epsilon T}$, we find, with $V_3^2 = \lambda_o^2 + \lambda_2^2 + 2\lambda_o\lambda_2 \cos x$,

$$\frac{\left\langle V^+ \right\rangle^{(n)}}{\lambda_4} = (-)\, e^{-\frac{s^2}{2} n^2 c^2 \tau a \left(\lambda_o^2 + \lambda_2^2\right) \tau T\, e^{-\epsilon T}}\, I_n\left(s^2 n^2 c^2 \tau a \lambda_o \lambda_2 \tau T\, e^{-\epsilon T}\right). \tag{57}$$

For equal pulses, $\lambda_o = \lambda_2 = \lambda$ and $a\lambda^2\tau = 1$, the first echo exhibits the dependence

$$\frac{\left\langle V^+ \right\rangle^{(1)}}{\lambda_4} = (-)\, e^{-s^2 c^2 \tau T\, e^{-\epsilon T}}\, I_1\left(s^2 c^2 \tau T\, e^{-\epsilon T}\right).$$

For $s = 10^{-3}$ cm, $c = 10^8$ sec^{-1} cm^{-1}, and $\tau = 10^{-7}$ sec,

$$\frac{\left\langle V^+ \right\rangle^{(1)}}{\lambda_4} = (-)\, e^{-10^3 T\, e^{-\epsilon T}}\, I_1(10^3 T\, e^{-\epsilon T}).$$

This shows a slow rise and fall with a peak in the neighborhood of $T \sim 10^{-3}$ sec $\sim \frac{1}{\epsilon}$.

It may be that the trough in the three-pulse echo described by Hill and co-workers[11] is due to a combination of the collision effects described in Section VI and a diffusion process.

Acknowledgment

The author happily thanks Professor William P. Allis for pointing to the interest in the echo phenomenon and for the careful attention that he gave to the ensuing work.

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1. ORIGINATING ACTIVITY *(Corporate author)* Research Laboratory of Electronics Massachusetts Institute of Technology Cambridge, Massachusetts	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP None

3. REPORT TITLE

Plasma Cyclotron Echoes

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*

Technical Report

5. AUTHOR(S) *(First name, middle initial, last name)*

Thomas B. Smith

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
June 1, 1967	44	15

8a. CONTRACT OR GRANT NO. DA 28-043-AMC-02536(E) b. PROJECT NO. 200-14501-B31F c. d.	9a. ORIGINATOR'S REPORT NUMBER(S) Technical Report 458 9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*

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We present a single-particle model that is sufficient to explain observed radiation peaks from weakly ionized laboratory plasma immersed in a magnetic field subsequent to one, two or three short driving RF pulses at the gyro frequency. It is shown that, depending upon the regime of operation, these echoes can result from any combination of arbitrary speed dependences in the gyro frequency, collision frequency, or diffusion of the electrons between and after the pulses.

DD FORM 1 NOV 65 1473 (PAGE 1)
S/N 0101-807-6811

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14. KEY WORDS	LINK A		LINK B		LINK C	
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Echoes, plasma cyclotron						
Plasma Echo Radiation						