Real-time Multi-Period Truckload Routing Problems

by

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Dedicated to my beloved country

THAILAND

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Abstract

In this thesis we consider a multi-period truckload pick-up and delivery problem dealing with real-time requests over a finite time horizon. We introduce the notion of postponement of requests, whereby the company can postpone some requests to the next day in order to improve its operational efficiency. The postponed requests must then be served on the next day. The daily costs of operation include costs associated with the trucks' empty travel distances and costs associated with postponement. The revenues are directly proportional to the length of job requests. We evaluate the profits of various re-optimization policies with the possibility of postponement. Another important notion of trucking operation corresponds to repositioning strategies which exploit probabilistic knowledge about future demands. A new repositioning strategy is proposed here to provide better decisions. For both notions, extensive computational results are provided under a general simulation framework.

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Chapter 1

Introduction

The explosion of information technologies and the importance of the logistics industry have led researchers to focus on dynamic vehicle routing and scheduling problems. The advances in telecommunications have enabled companies to use real-time information to enhance the performance of sophisticated decision tools in the area of vehicle routing. To achieve competitive advantages, companies must be able to effectively process real-time information as it arrives. The Dynamic Vehicle Routing Problem is an important class of the logistics industry where intelligent use of real-time information can differentiate one company from another by means of superior service.

In a real-time setting, service requests arise dynamically throughout the course of the system's operation. Any assignment decision is made based upon the information gathered until that point. In addition, known current information as well as historical data can be combined to predict partial future demand's information. Future demands, including customer locations and service times, are subject to random variations over time. With respect to the Dynamic Vehicle Routing Problem, the use of probabilistic knowledge about future requests could help improve the efficiency of fleet management.

In addition to exploiting probabilistic knowledge about demand, we consider a multi-period version of the problem, which, in practice, arises with common service level agreements. Specifically, some level of service in signed contracts requires the assurance of a number of days within which the request has to be met. In this context, companies have flexibility in serving their customers within the commitment period. Instead of either fulfilling or rejecting a service request on the day of that request, service level agreements allow a company to postpone the request and carry it out within a specified period.

The practical importance of the Dynamic Vehicle Routing Problem is evident in a variety of its real-world applications, e.g., courier services, repair services, and ambulance dispatching. One case of the Dynamic Vehicle Routing Problem is the Dynamic Multi-Vehicle Truckload Pick-up and Delivery Problem, where several vehicles must visit and service a number of customers. The problem includes the consideration of single load capacity and of time windows within which to start service at customers; additional restrictions on trucks and services must be accounted for when seeking to maximize revenues.

1.1 Motivations

Freight transportation constitutes a significant fraction of the economy of most nations. For example, the findings of the U.S. Department of Transportation[1] show that, in 1998, over 15 billion tons of freight valued at over $9 trillion were transported across the U.S. transportation system. The truck fleet accounted for 80 percent of the total value of U.S. shipments that year. Today, the truck mode is predicted to have the fastest growth in terms of the value of shipments over the next two decades. Trucks are expected to carry over 75 percent more tons in 2020, representing a large share of total tonnage. The economic importance of the freight transportation is one of our motivations to use operations research tools in order to improve its efficiency.

Operations research tools for vehicle routing and scheduling problems have long been developed and successfully implemented in practice. This development has provided significant improvement. As information technologies have considerably improved, the field of dynamic vehicle routing has emerged and has currently become an

[1] http://www.ops.fhwa.dot.gov/freight/documents/faf_overview.pdf

important dimension in trucking operations. Even small advancement in the development of new operations research tools can potentially contribute to a large reduction in operational costs. The development and implementation of real-time optimization models are the focus of our research.

1.2 Literature Survey

Although the vehicle routing problem as well as its real-time version have been widely studied in the literature, by and large, the only solutions that have been successfully applied are those of the static vehicle routing problem. In other words, there remains much needed additional work for the development of real-time practical applications.

Generally, Vehicle Routing Problems (VRPs) involve job scheduling vehicles in an efficient order such that job requirements are met. Deterministic versions of static VRPs, have been widely addressed in the literature. Bodin et al. (1983 [9]), and Fisher (1995 [2]) provide extensive surveys of the VRPs and present solutions to the problems. Bienstock et al. (1993 [8]) and Bramel et al. (1993 [10], 1997 [11]) provide probabilistic analyses of various heuristic approaches to solve the deterministic version of these static VRPs. One can also refer to Ball et al. (1995 [2]) for more surveys concerning the probabilistic analyses of the VRPs.

The stochastic versions of static VRPs have also been of interest to many researchers. For example, the VRPs with Poisson-distributed loads were examined by Golden and Stewart (1978 [17]). Jaillet (1985 [20], 1988 [21]) introduces the Probabilistic Traveling Salesman Problem which involves the appearance of jobs that are described in terms of probability distributions. Bertsimas (1988 [4]), and Bertsimas and Howell (1993 [3]), further examine theoretical aspects of the problem and present some heuristic approaches to tackle it. Later, Laporte et al. (1994 [22]) pose the problem as an integer program and solve it by using a branch-and-cut method.

The dynamic version of the Traveling Salesman Problem (TSP), which involves the dynamic assignment of resources to tasks, was introduced by Psaraftis (1988 [29]). Bertsimas and van Ryzin (1991 [7], 1993a [5], 1993b [6]) also examine the dynamic version of Traveling Repairman Problem. They investigate a dynamic routing problem in the Euclidean plane with random on-site service times. They evaluate the performance of various dispatching rules according to the average time spent by customers in the system, based on queuing-theory calculations. Gendreau et al. (1999 [14]) have proposed a general heuristic strategy to tackle the dynamic VRP with time windows. The heuristic for inserting a new arrival of request is presented to improve the quality of solutions of a tabu search. The denial of jobs, costs of operations, and waiting time are taken into consideration in their work. Ichoua et al. (2000 [18]) further considered methods that allow vehicle diversions for dynamic VRPs with time windows. They state that the number of unfulfilled requests can be reduced if diversion is allowed. Larsen et al. (2002 [23]) consider this problem in the context of partial dynamic VRPs and examine the effects of the degree of dynamism on the quality of solution. Regan et al. (1995 [30], 1996a [31], 1998 [32]) evaluate truckload pickup and delivery problems in a real-time setting when the vehicle diversion is taken into account. Various local rules for the real-time assignment of vehicles are examined in their work. Later, Lund et al. (1996 [26]) suggest a sequential scheme such that when new demand occurs, the dispatcher performs a re-optimization. Consequently, Yang et al. (1998 [34]) introduce re-optimization real-time policies into truckload pickup and delivery problems, and these policies are tested under a more general setting. Prior to a study from Powell et al. (2002 [28]), most of the research in the dynamic VRPs was considered only in terms of one-day horizon. Powell et al. have conducted research on truckload transportation with a horizon of several days, servicing a geographically dispersed region. For more extensive surveys of the dynamic aspect of the vehicle routing problem, one may refer to Spivey (2004 [33]).

The stochastic version of dynamic VRPs has also attracted increasing attention. For example, Gendreau et al. (1996 [15]) analyze dynamic VRPs under the influence of stochastic customer demands. Generally, the stochastic version of the VRPs has been solved as a stochastic mathematical program by using a tabu-type algorithm. However, this approach requires large computational time in order to obtain a high-quality solution; therefore, it is less useful in a real-time environment. A mathematical formulation considered in Powell et al. (2000a [16]) provides a formulation of the static version of the dynamic VRP in this thesis. We explore relevant works and summarize them in the following three subsections.

1.2.1 Dynamic Multi-Vehicle Truckload Pick-up and Delivery Problems

Yang, Mahmassani, and Jaillet (1998 [34]) present the dynamic vehicle routing on a continuous time, continuous space, and infinite horizon approach. Yang et al. (2004 [35]) introduce near-optimal procedures and evaluate them against various local rules for the dynamic assignment of vehicles in real-time. They explore optimization-based approaches for the Multi-vehicle Truckload Routing Problem. These approaches use a mathematical formulation in order to instantaneously re-optimize the current solution in the real-time environment of truckload pick-up and delivery problems. At the arrival of each request, the real-time version of the Truckload Routing Problem is solved as a static version, using the appropriately corresponding mathematical formulation of the instance of the problem. Thus, we refer to the static version as the off-line problem. The solution to the off-line problem provides a new routing assignment. The work done by Yang et al. also considers the notion of diversion, which is discussed in Ichoua et al. (2000 [18]). A truck can be re-assigned to different jobs if the jobs have not been serviced. The analysis in Yang et al. accounts for the performance of local rules and near-optimal approaches under varying input parameters. The results reveal that a re-optimization policy outperforms the other local rules.

1.2.2 A Priori Dynamic Pick-up and Delivery Problems

Another important issue in the field of dynamic vehicle routing is how to effectively utilize information about future requests. Powell (1996 [27]) shows that it is advantageous to take into consideration the prediction of future demands. Later, the issue of repositioning an idle vehicle or an empty-moving vehicle in anticipation of future requests is examined in Larsen, Madsen, and Solomon (2004 [24]). The problem addressed in this work is the Traveling Salesman Problem with time windows. The distribution region is divided into a number of smaller regions, hereafter called zones. Each region corresponds to a Poisson process with different arrival rates and has an idle point for vehicles to reposition at. The objective was to minimize a weighted sum of total lateness and total travel time for all requests. When a vehicle finished a job, the best idle location would be derived from an algorithm. The idle location would then be assigned to the vehicle if there was enough flexibility before the next service request. Larsen et al. focus on overnight mail service providers in which a fleet of vehicles visit a site of request in order to either pick up or deliver a package. The problem is further investigated in Ichoua et al. (2006 [19]) under a similar framework which related to long-distance express mail services. When the customers called a central office to pick up their mail, the vehicles were then sent to collect the mail and bring it back to the central office. Some heuristics that take into account future customer requests are evaluated in a simulation framework in their work.

1.2.3 Dynamic Multi-Period Pick-up and Delivery Problems

In most dynamic pick-up and delivery problems, a trucking company makes a decision only about either acceptance or rejection of requests within the horizon of one day. Some research studies have divided the requests into known requests (off-line requests) and unknown requests (on-line requests) in the context of one day. Therefore, it is interesting to look into the multi-period dimension of dynamic problems. The multi-period dimension in the context of dynamic routing is analyzed by studying simple algorithms and their competitive ratio in Angelelli et al. (2008 [1]). They introduce the dynamic multi-period routing problem, capturing the characteristics of postponement policies. In particular, they consider the following case: there are customers that require services at the beginning of each period. Some of the customers allow their service to be postponed to a later day. Thus, in each period there are immediate customers as well as postponed customers requesting to be served. The objective is to minimize the total travel distance over the planning horizon of two consecutive periods. In the present thesis, we address the flexibility of postponement by looking into a simple problem and analyze the impacts of postponement under various situations through a simulation framework.

1.3 Research Objectives

The objective of this thesis is to explore key issues of dynamic vehicle routing problems for truckload pickup-and-delivery service. We develop dynamic operational policies to determine how to effectively assign the fleet of trucks to a sequence of jobs in different zones. Revenue management policies through acceptance/rejection/postponement decisions under various demand situations are analyzed. The demands are categorized as postponable and non-postponable requests. Then, we evaluate the performances of dynamic decision policies to assess their benefits in various situations.

We build on the optimization-based approaches of the Multi-vehicle Truckload Routing Problem which is discussed in Yang et al.(2004 [35]). Various policies are investigated through computer simulation to gain insights into the a priori dynamic problems described in Section 1.2.2 and the dynamic multi-period problems described in Section 1.2.3.

One distinctive feature in this thesis is the approach that exploits probabilistic knowledge of future requests. The issue of relocating vehicles in the a priori dynamic problems is investigated in the context of the dynamic truckload problem. We apply repositioning strategies, using some probabilistic rules, to the truckload pick-up and delivery problem. The repositioning strategies are examined under various degrees of dynamism and various degrees of heterogeneity of pre-defined areas of distribution.

With respect to the previously cited multi-period dynamic problems, we analyze the problems in a general simulation framework. The requests can be serviced within two days and are categorized as postponable or non-postponable. If a request is non-postponable, it has to be either fulfilled on the day of request or rejected. Postponable requests may be served on the following day if beneficial and feasible; otherwise, they are either accepted or rejected on the day of request.

With regard to the three kinds of problems introduced above, our contribution is the development and evaluation of empirical experimentation on a general expectation-based heuristic. Moreover, an extensive simulation-based study on multi-period problems is conducted and simple online policies are proposed. In order to assess the performance of the proposed policies, we compare them with a full re-optimization approach that does not involve any sophisticated decisions. The full re-optimization approach method instantaneously re-optimizes the current solution as more information arrives.

The study of the "real-time" vehicle routing problem through a computational analysis is divided into two main domains in this thesis: repositioning decisions and postponement decisions. The key questions that will be addressed in our research are the following:

Repositioning Decisions

- Is the knowledge of future demands always useful for the management of a fleet of vehicles?
- Under what conditions would a repositioning strategy be beneficial?
- Does the degree of heterogeneity of the distribution region impact the results of repositioning strategies?

Postponement Decisions

- Under what conditions can postponement strategies generate more profits? Would more requests be captured?
- Does the requests that end up being postponed belong to some class that is easy to be recognized?

1.4 Outline of the Thesis

This thesis is organized as follows. Chapter 2 offers an overview of the dynamic routing problems dedicated to truckload pick-up and delivery service problems over multiple periods. Chapter 3 discusses the off-line problems in a real-time environment. Chapter 4 describes proposed policies for repositioning decisions and addresses the key characteristics of postponement issues. The general framework of simulation used to evaluate the proposed policies is reported in Chapter 5. In Chapter 6, the empirical results obtained from extensive computational simulation are discussed and conclusions are summarized. Chapter 7 discusses future directions for relevant research topics. Appendix A displays the numerical data obtained from the two domains of simulation. Appendix B details how to obtain expected Euclidean distances between two points in a two-dimensional Euclidean space. Appendix C describes theoretical methodologies of how probabilistic events are generated.

Chapter 2

Conceptual Framework

This chapter provides an overview and formal definition of the problems related to real-time truckload routing problems. In Section 2.1, we first introduce the problems. Later in section 2.2, we provide the detailed notation necessary for mathematical formulations.

2.1 Problem Definition

2.1.1 The Real-Time Truckload Routing Problem

We consider a trucking company routing a fleet of trucks, which serve a dynamic series of customers within several geographical zones. Each job, issued by a customer, is a request of picking up and delivering a load throughout a distribution region. A truck can carry only one job at a time and must deliver it to the destination before servicing another job. All trucks are assumed to move at the same constant speed. The pick-up location, the delivery location, the earliest pick-up time, and the deadline of the job are provided by customers when they request their service. A single depot of trucks is located at the center of the region. All trucks leave the depot in the morning and have to return to the depot at the end of the day.

The issue of relocating an empty truck in anticipation of future demand through exploitation of probabilistic knowledge is addressed in this thesis. The requests unfold over time in a number of geographical zones, according to a Poisson process with different arrival rates. Each zone has an idle point at which trucks can be repositioned. The idle point corresponding to a zone is located at the center of that zone. If the probability of having at least one request at an idle point is high enough, a truck that has finished serving its current client will be asked to go to the idle point. To decrease the inefficiency in repositioning, the truck will be repositioned if its next already accepted service allows the truck enough time to insert the additional job with high probability.

Acceptance or rejection of a job is determined within a response time specified by the customer. The price of each accepted job is proportional to the distance between the job's pick-up and delivery locations. The company incurs operating costs subject to the distance traveled by trucks. The objective of the company is to maximize the net profit by implementing an efficient strategy for handling this sequence of jobs. The strategy needs to address decisions related to job acceptance/rejection, postponement, job-truck assignment, as well as repositioning in a real-time environment.

2.1.2 The Multi-Period Real-time Truckload Routing Problem

We add here the option of postponement, associated with jobs that can be postponed to a later day. Under a certain contract, a customer can indeed decide whether the job can be postponed or not, and how long it can be postponed. In case of the postponement of a job, the company will incur a cost associated with that postponement. In this thesis, the time frame of the operation is assumed to be three consecutive days. No job exists at the start of the first day. The company may postpone some jobs, servicing them on the next day. At the beginning of the second day, some jobs may have already been postponed from the first day, and there may be more jobs arriving during that day. On the last day, we assume that no jobs can be postponed. Finally, we assume that each job can be serviced in a period of one day.

2.2 Problem Statement for the Multi-Period Real-time Truckload Routing Problem

2.2.1 Notations

$\mathcal{IP}$	The set of idle points, where vehicles can be repositioned
$\mathcal{K}$	The set of trucks indexed from 1 to K
τ^{cls}	The deadline at the depot (all trucks must be back at the depot by that time)
v	The constant speed of trucks, which is the same for all trucks
$\mathbf{o}_i$	The pick-up location of job i
$\mathbf{d}_i$	The delivery location of job i
τ_i^{arv}	The arrival time of job i
τ_i^{avl}	The earliest pick-up time of job i
T_i^{pck}	The pick-up time width
τ_i^{pdn}	The deadline for pick-up of job i
τ_i^{ddn}	The deadline for delivery of job i
T_i^{res}	The amount of allowable time it takes to respond to job i
L_i	The length of job i, i.e., the time needed to go from $\mathbf{o}_i$ to $\mathbf{d}_i$
p_i	A boolean variable indicating if job i is postponable or not
$\mathbf{o}_{ip}$	The location of idle point ip, where $ip \in \mathcal{IP}$
$D(\mathbf{p},\mathbf{q})$	The Euclidean distance between two locations, $\mathbf{p}$ and $\mathbf{q}$ (see Appendix B)

2.2.2 Dynamic Model Statement

At the beginning of each day ($t = 0$), all K trucks are idle at the central depot and they must be back at the depot by the end of that day ($t = \tau^{cls}$). The time evolution of the system on each day is indexed by a continuous variable $t \in [0, \tau^{cls}]$.

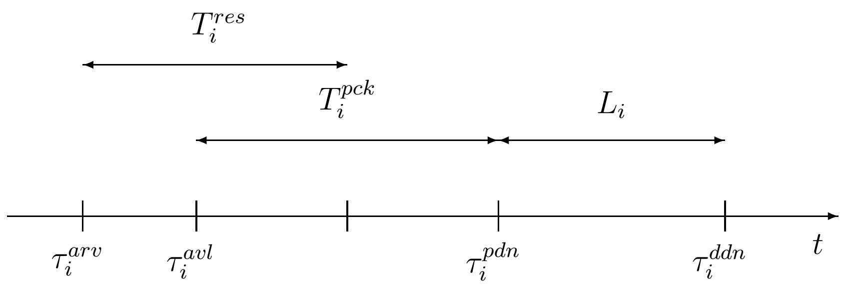


Figure 2-1: Time Line of a Job Request

At any given time $t \geq 0$, the truck positions, job pick-up and delivery locations are assumed to be points in a bounded region of the Euclidean plane. During the day, the dynamics of the system is driven by a sequence of job requests, which are indexed by i according to the order of job arrival. The information about job i is available at its arrival time τ_i^{arv}, and is characterized by a vector $\mathbf{J}_i$ as follows:

$$\mathbf{J}_i = (\mathbf{o}_i, \mathbf{d}_i, T_i^{res}, \tau_i^{avl}, \tau_i^{ddn}, p_i). \tag{2.1}$$

The traveling time between the pick-up location $\mathbf{o}_i$ and the delivery location $\mathbf{d}_i$ of job i is denoted by L_i. T_i^{res} gives the company the flexibility of whether to take job i, i.e. the final decision of job i must be done within an amount of T_i^{res} time. τ_i^{ddn} denotes the latest delivery time of job i. So, a truck must pick up a job no later than $\tau_i^{pdn} = \tau_i^{ddn}$ - L_i.

At the instant of job arrival, the new demand information is included into the system. This information be added to the sequence $(\mathbf{J}_i, \tau_i^{arv})_{i \geq 1}$, which completely describes all requests. The company faces a series of decisions, including job acceptance/rejection decisions, postponement decisions, and job-truck assignment decisions on accepted jobs. Any decision at time t is based on the information up to time t and some probabilistic knowledge about future demand. Temporary decisions that have been made related to jobs and assignment are implemented, when appropriate, until the arrival of the next job, which will trigger a re-optimization. Conversely, those decisions may be changed when the next job arrives. Since new information revealed at a job arrival can impact decisions, we refer to a job arrival epoch as *a decision epoch*. For example, a re-optimization[1] can be used at a decision epoch. After time $\tau_i^{arv} + T_i^{res}$, the decision about job i must have been finalized and cannot then be changed. In case of acceptance, the job may be fulfilled on the day of the request or postponed for the next day.

2.2.3 States

A state $\mathbf{S}_d(t)$ at instant t on the dth day is described by the following state vectors:

$$\mathbf{S}_d(t) = \{\mathbf{s}_d(t), \mathbf{Q}_d(t), \mathbf{l}_d(t), L_d(t), P_d(t)\},$$

where

$$\mathbf{s}_d(t) = (s_d^k(t))_{k\in\mathcal{K}} \in (\{-1, 0, +1\} \cup \mathcal{IP})^{|\mathcal{K}|} \text{ contains the status of each truck,}$$

$$s_d^k(t) = \begin{cases} 0 & \text{truck } k \text{ is idle,} \\ -1 & \text{truck } k \text{ is moving empty toward its next assigned load,} \\ +1 & \text{truck } k \text{ is moving loaded,} \\ ip & \text{truck } k \text{ is moving to the idle point } ip \ (ip \in \mathcal{IP}). \end{cases}$$

$\mathbf{Q}_d(t) = (Q_d^k(t))_{k\in\mathcal{K}}$ contains the ordered list of non-completed jobs assigned to each truck,

$\mathbf{l}_d(t) = (\mathbf{l}_d^k(t))_{k\in\mathcal{K}}$ contains each truck's location at time t,

$L_d(t) =$ the set of temporarily rejected jobs,

$P_d(t) =$ the set of jobs being postponed.

[1]The process of re-optimizing the assignment plan incorporating new demand information. This process solves the off-line problem and returns a near-optimal solution at each decision epoch.

2.2.4 Transitions

$\mathbf{S}_d(t)$ completely describes the dynamics of the system on the dth day under a given policy. Given a sequence of requests $(\mathbf{J}_i, \tau_i^{arv})_{i\geq 1}$, consider the post state denoted as $\mathbf{S}_d(t^+)$, where $t^+ = t + \delta t$ $(\delta t \to 0)$

1. If t is a decision epoch, i.e. $t = \tau_j^{arv}$ for a job j:

   The state parameters are then fully updated according to the result from the policy, discussed in Chapter 4.

2. If t is not a decision epoch:

   Let $t < \tau_j^{arv}$ and job j will be the next job arriving into the system. The state parameters are updated as follows:

   (a) update $L_d(t)$ and $P_d(t)$:

   For all $i \in L_d(t)$ such that $t \leq t^+ = \tau_i^{arv} + T_i^{res} \leq \tau_j^{arv}$, $L_d(t^+) = L_d(t) \backslash \{i\}$; in addition, if p_i is true, $P_d(t^+) = P_d(t) \cup \{i\}$

   (b) update on the idle trucks (any k such that $s_d^k(t) = 0$) :

   If ip is the best idle point that is a result of the repositioning policy, discussed in 4, $s_d^k(t^+) = ip$, $\mathbf{l}_d^k(t^+) = \mathbf{l}_d^k(t)$, and $Q_d^k(t^+) = \emptyset$. If repositioning is not beneficial, $s_d^k(t^+) = 0$, $\mathbf{l}_d^k(t^+) = \mathbf{l}_d^k(t)$, and $Q_d^k(t^+) = \emptyset$.

   (c) update on the trucks moving empty:

   - Truck k is moving toward job i: (any k such that $s_d^k(t) = -1$)

     Truck k will arrive at $\mathbf{o}_i$ at time $t' = t + D(\mathbf{l}_d^k(t), \mathbf{o}_i)/v$. Hence, for $t^+ < \min\{t', \tau_j^{arv}\}$, set $s_d^k(t^+) = -1$, $Q_d^k(t^+) = Q_d^k(t)$, and $\mathbf{l}_d^k(t^+) = \mathbf{l}_d^k(t) + (\mathbf{o}_i - \mathbf{l}_d^k(t))(t^+ - t)/(t' - t)$.

     If $t' \leq \tau_j^{arv}, s_d(t') = +1, \mathbf{l}_d^k(t') = \mathbf{o}_i$, and $Q_d^k(t') = Q_d^k(t)$. Otherwise, t' is the decision epoch.

   - Truck k is moving towards idle point ip: (any k such that $s_d^k(t) = ip$)

     Truck k will arrive at $\mathbf{o}_{ip}$ at time $t' = t + D(\mathbf{l}_d^k(t), \mathbf{o}_{ip})/v$. Hence, for $t^+ < \min\{t', \tau_j^{arv}\}$, set $s_d^k(t^+) = -1$, $Q_d^k(t^+) = Q_d^k(t)$, and $\mathbf{l}_d^k(t^+) = \mathbf{l}_d^k(t) + (\mathbf{o}_{ip} - \mathbf{l}_d^k(t))(t^+ - t)/(t' - t)$.

If $t' \leq \tau_j^{arv}, s_d(t') = 0, \mathbf{l}_d^k(t') = \mathbf{o}_{ip}$, and $Q_d^k(t') = Q_d^k(t)$. Otherwise, t' is the decision epoch.

(d) update on the trucks moving loaded: (any k such that $s_d^k(t) = +1$)

Let job i be the first element in $Q_d^k(t)$. Truck k will arrive at $\mathbf{d}_i$ at $t' = t + D(\mathbf{l}_d^k(t), \mathbf{o}_i)/v$. Hence, for $t^+ \leq \min\{t', \tau_j^{arv}\}$, set $s_d^k(t^+) = +1$, $Q_d^k(t^+) = Q_d^k(t)$, and $\mathbf{l}_d^k(t^+) = \mathbf{l}_d^k(t) + (\mathbf{d}_i - \mathbf{l}_d^k(t))(t^+ - t)/(t' - t)$. If $t' \leq \tau_j^{arv}, s_d(t') = 0, \mathbf{l}_d^k(t') = \mathbf{d}_i$, and $Q_d^k(t') = Q_d^k(t) \backslash \{i\}$. Otherwise, t' is the decision epoch.

2.2.5 Objective

The definition of time-dependent set of variables that keep track of the dynamics of the system.

A binary variable indicating whether or not truck k is moving empty at time t is described by

$$E_d^k(t) = \begin{cases} 1 & \text{if } s_d^k(t) = -1 \text{ or } s_d^k(t) = ip, \\ 0 & \text{otherwise.} \end{cases}$$

A set of jobs which have been requested by time t is described by

$$N_d(t) = \{i | \tau_i^{arv} \leq t\}.$$

A set of jobs which has been permanently accepted by time t is described by

$$A_d(t) = \{i | \tau_i^{arv} + T_i^{res} \leq t, i \notin L_d(\tau_i^{arv} + T_i^{res})\}.$$

A set of jobs which has been permanently rejected by time t is described by

$$Z_d(t) = \{i | \tau_i^{arv} + T_i^{res} \leq t, i \in L_d(\tau_i^{arv} + T_i^{res})\}.$$

A set of jobs which are completely serviced by time t is described by

$$Y_d(t) = \{i | \tau_i^{dln} \leq t, i \in A_d(t)\}.$$

The objective function of the system

In order to assess the performance of the system under a certain policy, we define some relevant parameters. Let α be the revenue per unit distance of service movement, and let γ be the postponement penalty (a multiplier of the revenue parameter α). The number of days is denoted as DAY. The cumulative net revenue up until time t on dth day, $R_d(t)$, is explained by the following equation.

$$R_d(t) = \alpha \sum_{i \in A_d(t)} vL_i - \gamma\alpha \sum_{i \in P_d(t)} vL_i - v \int_0^t E_d^k(\tau) d\tau \tag{2.2}$$

$R_d(t)$ expresses the net revenue of fleet's operation on the dth day. The revenues are generated by the serviced jobs, and are explained by $\alpha \sum_{i \in A_d(t)} vL_i$. The operational costs includes: the cost associated with postponement, explained by $\gamma\alpha \sum_{i \in P_d(t)} vL_i$; and the cost of empty movement of the trucks, explained by $v \int_0^t E_d^k(\tau) d\tau$.

The total net revenue of the trucking company operating up to time t on the DAY^{th} day, $T_{DAY}(t)$, is as follows:

$$T_{DAY}(t) = \sum_{d=1}^{DAY-1} R_d(\tau^{cls}) + R_{DAY}(t) \tag{2.3}$$

Chapter 3

Mathematical Model for the Off-line Problems

3.1 The Multiple Vehicle Pick-up and Delivery Problem

3.1.1 Definition of the Re-optimization Problem

The off-line problem considers K trucks as in the Real-Time Truckload Routing Problem, described in the previous chapter. A near-optimal routing assignment is returned by the algorithm. Suppose that t is the time when the re-optimization routine is started. Define τ_k^{int} as the initial available time of truck k and $\mathbf{b}^k$ as the initial location of the truck. Both variables are updated according to the dynamics of the system, previously discussed in Subsection 2.2.4. If truck k is moving loaded with job i, $\mathbf{b}^k$and τ_k^{int} are $\mathbf{d}_i$ and the finishing time of job i, respectively. If truck k is moving empty or idle, τ_k^{int} and $\mathbf{b}^k$ are t and the location of truck k, respectively. The jobs considered in this off-line problem are those that have been requested so far, but have not been serviced (picked up). Let N denote the number of such jobs.

At each decision epoch, the N jobs may consist of permanently accepted jobs, temporarily accepted jobs, and temporarily rejected jobs. The formulation needs to impose constraints to guarantee the service of those permanently accepted jobs. For the jobs that had previously been temporarily decided upon, their status may be changed due to any re-optimization. Note that a new job can always be rejected with the company continuing the previous plan. The off-line problem and the re-optimization will be used interchangeably hereafter.

3.1.2 Re-optimization Model Statement

Notations

$\mathcal{N}$	The set of jobs
$\mathcal{N}_0$	The set of jobs and the depot, indexed by 0
$\mathcal{A}$	The set of accepted jobs that have not yet been serviced
$\mathbf{b}^k$	The initial location of truck $k \in \mathcal{K}$
$\mathbf{o}_i$	The pick-up location of job i
$\mathbf{d}_i$	The delivery location of job i
$\mathbf{o}_0 = \mathbf{d}_0$	The location of the depot
τ_i^{avl}	The earliest pick-up time of job i
T_i^{pck}	The pick-up time width
τ_i^{ddn}	The deadline for delivery of job i
L_i	The length of job i, i.e., the time needed to go from $\mathbf{o}_i$ to $\mathbf{d}_i$

Empty-movement Times

D_{ij}^k	The amount of time to travel in the Euclidean plane between $\mathbf{d}_i$ and $\mathbf{o}_j$, $\forall k \in \mathcal{K}, \forall (i,j) \in \{(i,j) \mid i \neq j, \forall i,j \in \mathcal{N}\}$
$D_{\mathbf{b}^k i}^k$	The amount of time to travel in the Euclidean plane between location $\mathbf{b}^k$ and $\mathbf{o}_i$, $\forall k \in \mathcal{K}, \forall i \in \mathcal{N}_0$

D^k_{i0} The amount of time to travel in the Euclidean plane between $\mathbf{d}_i$ and the depot, $\forall k \in \mathcal{K}, \forall i \in \mathcal{N}$

Empty-movement Costs

C^k_{ij} The costs associated with traveling in the Euclidean plane between $\mathbf{d}_i$ and $\mathbf{o}_j$, $\forall k \in \mathcal{K}, \forall (i,j) \in \{(i,j)|i \neq j, \forall i,j \in \mathcal{N}\}$

$C^k_{\mathbf{b}^k i}$ The costs associated with traveling in the Euclidean plane between $\mathbf{b}^k$ and $\mathbf{o}_i$, $\forall k \in \mathcal{K}, \forall i \in \mathcal{N}_0$

C^k_{i0} The costs associated with traveling in the Euclidean plane between $\mathbf{d}_i$ and the depot, $\forall k \in \mathcal{K}, \forall i \in \mathcal{N}$

Decision Variables

$$x^k_{ij} = \begin{cases} 1 & \text{if truck } k \text{ visits job } j \text{ immediately after job } i, \\ 0 & \text{otherwise.} \end{cases}$$

$$\forall i,j \in \mathcal{N} \cap \{i \neq j\}, \forall k \in \mathcal{K}$$

$$x^k_{ii} = x^k_i = \begin{cases} 1 & \text{if job } i \text{ is rejected by truck } k, \\ 0 & \text{otherwise.} \end{cases} \qquad \forall i \in \mathcal{N}, \forall k \in \mathcal{K}$$

$$z_i = \begin{cases} 1 & \text{if job } i \text{ is rejected,} \\ 0 & \text{otherwise} \end{cases} \qquad \forall i \in \mathcal{N}$$

$$x^k_{\mathbf{b}^k i} = \begin{cases} 1 & \text{if truck } k \text{ visits job } i \\ & \text{immediately after its initial location } \mathbf{b}^k, \\ 0 & \text{otherwise.} \end{cases} \qquad \forall i \in \mathcal{N}, \forall k \in \mathcal{K}$$

$$x^k_{\mathbf{b}^k0} = \begin{cases} 1 & \text{if truck } k \text{ visits the depot} \\ & \text{immediately after its initial location } \mathbf{b}^k, \\ 0 & \text{otherwise.} \end{cases} \qquad \forall k \in \mathcal{K}$$

$$x^k_{i0} = \begin{cases} 1 & \text{if truck } k \text{ ends up idle at the depot} \\ & \text{immediately after the visit at node } i \text{ ,} \\ 0 & \text{otherwise.} \end{cases} \qquad \forall i \in \mathcal{N}, \forall k \in \mathcal{K}$$

$$x^k_{0\mathbf{b}^k} = 1 \quad \text{(for consistency of the problem formulation)} \qquad \forall k \in \mathcal{K}$$

$$x^k_{i\mathbf{b}^k} = 0 \quad \text{(for consistency of the problem formulation)} \qquad \forall i \in \mathcal{N}, \forall k \in \mathcal{K}$$

$$x^k_{\mathbf{b}^k\mathbf{b}^k} = 0 \quad \text{(for consistency of the problem formulation)} \qquad \forall k \in \mathcal{K}$$

$$t^{pck}_i \in \Re^+ \text{ denotes the pick-up time of job } i \qquad \forall i \in \mathcal{N}$$

$$t^k_0 \in \Re^+ \text{ denotes the arrival time of truck } k \text{ at the depot} \qquad \forall k \in \mathcal{K}$$

Objective function of the model

$$\text{Maximize} \quad \alpha \sum_{i \in \mathcal{N}} L_i \times (1 - z_i) - (\sum_{j \neq i \in \mathcal{N}} \sum_{i \in \mathcal{N}} C_{ij} \sum_{k \in \mathcal{K}} x^k_{ij} + \sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{N}_0} C_{\mathbf{b}^k j} \times x^k_{\mathbf{b}^k j} + \sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{N}} C_{i0} \times x^k_{i0}) \tag{3.1}$$

The first term in the objective function is the revenue generated by the accepted jobs. The first term in the parenthesis is the cost associated with the empty movement between two jobs. The second term in the parenthesis is the empty-movement cost associated with traveling from the initial location of trucks to the first job. The last term is the cost of empty movement between the last job to the depot. The objective is to maximize the profit, which equals to the revenue minus the total costs.

Constraints of the model

$$\sum_{j \in \mathcal{N}_0 \cup \{\mathbf{b}^k\}} x_{ij}^k = 1, \qquad \forall i \in \mathcal{N}_0 \cup \{\mathbf{b}^k\}, \forall k \in \mathcal{K} \tag{3.2}$$

$$\sum_{i \in \mathcal{N}_0 \cup \{\mathbf{b}^k\}} x_{ij}^k = 1, \qquad \forall j \in \mathcal{N}_0 \cup \{\mathbf{b}^k\}, \forall k \in \mathcal{K} \tag{3.3}$$

$$x_{\mathbf{b}^k 0}^k \leq x_i^k, \qquad \forall i \in \mathcal{N}, \forall k \in \mathcal{K} \tag{3.4}$$

$$\sum_{k \in \mathcal{K}} x_i^k = z_i + (|\mathcal{K}| - 1), \qquad \forall i \in \mathcal{N} \tag{3.5}$$

$$\sum_{k \in \mathcal{K}} (D_{\mathbf{b}^k i}^k + \tau_k^{int}) \cdot x_{\mathbf{b}^k i}^k \leq t_i^{pck}, \qquad \forall i \in \mathcal{N} \tag{3.6}$$

$$(D_{\mathbf{b}^k 0}^k + \tau_k^{int}) \cdot x_{\mathbf{b}^k 0}^k \leq t_0^k, \qquad \forall k \in \mathcal{K} \tag{3.7}$$

$$t_i^{pck} \geq \tau_i^{avl}, \qquad \forall i \in \mathcal{N} \tag{3.8}$$

$$t_i^{pck} \leq \tau_i^{avl} + T_i^{pck}, \qquad \forall i \in \mathcal{N} \tag{3.9}$$

$$t_i^{pck} + L_i + D_{ij}^k \leq t_j^{pck} + M \cdot (1 - x_{ij}^k), \qquad \forall i, j \in \mathcal{N}, \forall k \in \mathcal{K} \tag{3.10}$$

$$t_i^{pck} + L_i + D_{i0}^k \leq t_0^k + M \cdot (1 - x_{i0}^k), \qquad \forall i \in \mathcal{N}, \forall k \in \mathcal{K} \tag{3.11}$$

$$t_i^{pck} + L_i \leq \tau_i^{ddn}, \qquad \forall i \in \mathcal{N} \tag{3.12}$$

$$t_0^k \leq \tau^{cls}, \qquad \forall k \in \mathcal{K} \tag{3.13}$$

$$z_i = 0, \qquad \forall i \in \mathcal{A} \tag{3.14}$$

$$t_0^k, t_i^{pck} \geq 0, \qquad \forall i \in \mathcal{N}, \forall k \in \mathcal{K}$$

$$z_i \in \{0, 1\}, \qquad \forall i \in \mathcal{N}$$

$$x_{ij}^k \in \{0, 1\}, \qquad \forall i, j \in \mathcal{N}_0 \cup \{\mathbf{b}^k\}, \forall k \in \mathcal{K}$$

Constraints 3.2, 3.3, 3.4, and 3.5 represent an assignment problem. Specifically, constraint 3.4 implies that if truck k goes to the depot directly from its initial location, no job can be assigned to truck k. Constraint 3.5 ensures that each job can be served by at most one truck. In other words, there should be at least $k-1$ trucks rejecting a job. Constraint 3.6 enforces that truck k arrives at the pick-up location of job i after $D_{\mathbf{b}^k i}^k + \tau_k^{int}$ if i is the first job serviced by k. Similarly, Constraint 3.7 enforces that truck k arrives at the depot after $D_{\mathbf{b}^k i}^k + \tau_k^{int}$ if it serves no job. Constraints 3.8

and 3.9 ensure that any job's pick-up time is not earlier than its earliest pick-up time, and it can be picked up no later than after the allowable T_i^{pck} time window. Constraint 3.10 imposes the arrival at the pick-up location of job j within $L_i + D_{ij}^k$ on any truck k if job j is to be fulfilled after job i. Similarly, constraint 3.11 enforces that truck k arrives at the depot at least $L_i + D_{i0}^k$ if job i is the last job of the day. Constraint 3.12 confirms that any jobs must be delivered before the deadline if they are serviced. Constraint 3.13 ensures that truck k must arrive at the depot before it is closed. For any job i that is in Set $\mathcal{A}$, it is permanently accepted and will never be rejected; so, z_i is set to 0 as expressed in Constraint 3.14.

3.2 The Multiple Vehicle Pick-up and Delivery Problem with Postponement

The off-line problem in the Multiple Vehicle Pick-up and Delivery Problem with Postponement is quite similar to the one in Section 3.1. There are additional considerations associated with postponement. First, the off-line problem in the context of postponement consists of a re-optimization with some additional constraints and a slightly different objective function. Second, the postponable requests that are rejected, as a result of the re-optimization problem, may be postponed and fulfilled the next day. Therefore, an algorithm for ensuring the feasibility of postponing requests is needed. Finally, since postponed requests are known before the start of each day, we perform an "over-night" optimization incorporating all known requests to obtain an optimal routing solution. The solution of the over-night optimization provides a basis input for the subsequent re-optimization during the day. In the subsequent sections, let us divide the off-line problem in three subproblems and treat each of them in a different section.

3.2.1 Definition of the Re-optimization Problem

The re-optimization problem in the context of postponement needs to include a guarantee of service for the previously postponed requests. Additional constraints ensuring this commitment must be included in addition to the constraints described in section 3.1.2. The flexibility gained by implementing postponement will, however, come at the expense of a discount associated with that postponement. In mathematical terms, this will be done by adding another term accounting for the cost of postponement (a penalty) to the objective function of the re-optimization problem. This term is shown in equation 3.1. This penalty is seen as a discount on the revenue for each job's postponement. In this thesis, we will allocate the revenue generated from a postponed job on the day that the job is serviced, but we will allocate the cost of postponement on the arrival day of the job.

3.2.2 Definition of the Feasibility of Postponement Problem

Prior to postponing a job request, it is important to ensure that there exists enough capacity for the request to be serviced on the next day. When a trucking company employs a postponement strategy and signs a service agreement contract with many clients, there may be a lot of jobs being postponed. Such a case will definitely cause congestion in the system on the following day if the arrival rate of requests is relatively high. To ensure enough capacity to fulfill a postponed request, a feasibility-check algorithm is needed. The objective is to find a feasible assignment that will be used for the next day, taking into account all postponed jobs known up to the time the algorithm is called. At a decision epoch, the feasibility-check algorithm is called, and immediately returns its decision about postponement before the re-optimization. The status of a postponable request will be changed to non-postponable if postponing the request causes infeasibility on the next day. Such a request will then be re-considered, whether to be serviced or not, only on the day of its request.

3.2.3 Definition of the Over-night Optimization Problem

The over-night optimization is simply an optimization algorithm that runs during the night before the operation of the next day. Its mathematical formulation slightly differs from the formulation of a re-optimization problem. First, input parameters of the over-night optimization is different. The initial available time of all trucks is set to $t = 0$. Their initial location is the depot. Second, all postponed jobs are considered as non-postponable jobs. The over-night optimization is assumed to be "fully optimal" because of unlimited time in seeking an optimal solution. The solution to the over-night optimization problem provides a starting solution for the subsequent "re-optimization" during the day.

3.2.4 Re-optimization Model Statement

Additional Notations

On a given day, let

$\mathcal{P}$	be the set of postponed jobs from previous day
$\mathcal{M}$	be the set of postponable jobs on current day

Objective function of the model

$$\text{Maximize} \quad \alpha \sum_{i \in \mathcal{N}} vL_i \times z_i - \Big(\sum_{j \neq i \in \mathcal{N}} \sum_{i \in \mathcal{N}} C_{ij} \sum_{k \in \mathcal{K}} x_{ij}^k + \sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{N}_0} C_{\mathbf{b}^k j} \times x_{\mathbf{b}^k j}^k + \sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{N}} C_{i0} \times x_{i0}^k \Big) - \gamma \sum_{i \in \mathcal{M}} vL_i \times z_i \quad (3.15)$$

Compared to equation 3.1, the last added term is regarded as the cost associated with a discount on prices for postponable jobs. A postponable job $j \in \mathcal{M}$ that is rejected, is then automatically postponed if feasibility permits. For example, if a solution to the re-optimization problem is to reject job j, the job may be serviced on the following day depending on the availability of trucks. In case of postponement, the client will get a discount price of the service. In our construction of the problem, this cost of discount incurs on the day of the request, and the generating revenue will pay out on the next day.

Additional constraints of the model

$$z_i = 0, \qquad \forall i \in \mathcal{P} \tag{3.16}$$

This constraint is to ensure that the postponed jobs from the previous day will be fulfilled today.

Chapter 4

Dynamic Operation Policies

4.1 The Repositioning Decisions

We introduce a policy that allows idle truck to reposition to a more "attractive" area, where it is highly likely that there will be a new request. We assume that the requests for pick-up are distributed across the region according to a Poisson distribution with parameter λ_p which varies over zones. We assume that delivery locations are uniformly distributed throughout the region. The trucking company may take advantage of the repositioning strategy to effectively manage the fleet of trucks.

A predefined region is shown in Figure 4-1. Each zone p has an idle point located at its center, where a repositioned truck will be waiting. The requests for pick-up in zone p are generated dynamically according to a Poisson distribution with parameter λ_p. The expected number of new customers in the zone, where a truck has repositioned is equal to $\triangle\tau_p \cdot \lambda_p$, where $\triangle\tau_p$ is the amount of time between the time that the truck has arrived in zone p and the time it will have to leave for servicing the next job previously listed in its queue.

To determine whether to reposition a vehicle toward an idle point or have it remain at its current location, we have to compare the trade-offs between the two options. To derive any potential benefits from repositioning to an idle point and ensuring feasibility, there are three criteria we consider in making a repositioning decision.

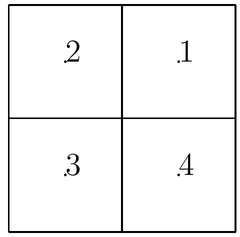


Figure 4-1: Predefined Region

4.1.1 Feasibility Condition for Repositioning

This condition ensures that if a vehicle is repositioned at an idle point, this will not prevent it to service the next accepted job it may have had in its queue. The repositioned vehicle must arrive at an idle point no later than the time it has to leave for servicing the next job within its pick-up deadline. Clearly, there is no benefit in repositioning a vehicle if no new job is assigned to the vehicle while repositioned. The trucking company will only incur extra empty-movement cost. The following conditions substantially reduce the chance of such a situation.

4.1.2 Rate-of-Requests Condition for Repositioning

The probability of receiving at least one new request in zone p during the time interval $\triangle\tau_p$ must be sufficiently high. For this purpose, a threshold $\hbar$ is introduced as a minimum probability of repositioning. A vehicle will not be repositioned at an idle point if the corresponding probability is less than the value of the threshold. Formally, given that the requests arrive according to a Poisson process with intensity λ_p in zone p, the probability of receiving m requests during $\triangle\tau_p$ can be expressed as

$$P\{X = m\} = \frac{1}{m!}(\lambda_p \triangle \tau_p)^m e^{-\lambda_p \triangle \tau_p}, m = 0, 1, 2, \dots \tag{4.1}$$

where X is defined as the number of new requests in zone p over $\triangle\tau_p$.

This means that the probability of receiving at least one request during the time interval $\triangle\tau_p$ is

$$P\{X \geq 1\} = 1 - e^{-\lambda_p \triangle \tau_p} \tag{4.2}$$

Hence, the vehicle will not be repositioned and remains idle at its current position if

$$P\{X \geq 1\} < \hbar. \tag{4.3}$$

Otherwise, the vehicle may be repositioned at one of those idle points satisfying this condition.

4.1.3 Flexibility Condition for Repositioning

Once the feasibility condition and the rate-of-requests condition have been satisfied, the flexibility condition will be investigated. Whether a vehicle is repositioned or not is also determined by the possibility that the insertion of new job l (if it is served) would violate the deadline of the queuing job j. Figure 4-2 demonstrates a "good" repositioning, which new job l can be inserted between job i and job j. However, the new job, namely l, could violate job j as mentioned earlier. Figure 4-3 illustrates the situation where a repositioned vehicle may receive a new request that conflicts with the schedule of the next job, namely j, in the queue. When the vehicle is idle and is asked to reposition, it will arrive at an idle point at time $\tau_p = \tau_i^{idle} + T_{ip}$. If the vehicle then accepts the new job l that has the pick-up location in zone p, the total amount of time to complete this job and arrive at the pick-up location of job j will be at least $T_{pl} + L_l + T_{lj}$. $\triangle w$ is the amount of waiting time at idle point p prior to leaving for job l. We can express

$$\triangle w = \tau_l^{arv} - \tau_p. \tag{4.4}$$

$$\mathbb{E}[\triangle w] = \frac{1}{\lambda_p} \tag{4.5}$$

As depicted in Figure 4-3, if job l is served and finished after the pick-up deadline for job i (τ_i^{ddn}), the acceptance of job l will inevitably conflict with the accepted job j. If job j has been permanently accepted, the acceptance of job l would result in an intractable situation for the fleet management because the repositioned truck could not service job l and the other trucks might be occupied. Thus, the vehicle cannot accept job l even if it would be more profitable to do so. In that case, the repositioning decision is not successful, leading to the unnecessary cost of empty-movement. Therefore, to protect against this situation, a "flexibility" parameter is introduced for each possible idle point considered by the truck. This parameter is a random variable indicating the amount of the remaining time before the pick-up deadline of job j (τ_j^{ddn}) after the repositioned vehicle had waited, serviced job l, and arrived at the pick-up location of job j. The flexibility parameter for the idle point p is given by the following equation:

$$\epsilon_p = \tau_j^{pdn} - (\tau_i^{idle} + T_{ip} + \triangle w + T_{pl} + L_l + T_{lj}) \quad \forall p \in \mathcal{IP} \tag{4.6}$$

By construction, this heuristic probably leads to penalizing idle points that are further away from the vehicle's current location; conversely, it may favor the nearest idle point. A vehicle is considered to reposition at idle point p if the value of the flexibility parameter of idle point p is greater than that of the current location ϵ_0: $\epsilon_p > \epsilon_0$. Since these parameters are random variables, we will use their expected value for comparison. If the expectation of ϵ_p is greater than zero where $\epsilon_p = \max_{z \in \mathcal{IP}}\{\epsilon_z\}$, the vehicle will be repositioned at the idle point p, hereafter called "p-repositioning." The expectation of all flexibility parameters are then compared. Equation 4.7 shows the mathematical expression of the expectation of ϵ_p.

$$\mathbb{E}[\epsilon_p] = \tau_j^{pdn} - (\tau_i^{idle} + T_{ip} + \frac{1}{\lambda_p} + \mathbb{E}[T_{pl}] + \mathbb{E}[L_l] + \mathbb{E}[T_{lj}]) \quad \forall p \in \mathcal{IP} \tag{4.7}$$

Appendix B details how to compute the expected distances (the last three terms in equation 4.7) in Euclidean space. The expected Euclidean distance between two uniformly distributed random points can be presented in a closed-form equation. Appendix B also includes the discussion of the expected distance from a fixed point to a random point in a square region. Therefore, the expectations in equation 4.7 can be computed and yield the value of $\mathbb{E}[\epsilon_p]$.

In conclusion, p-repositioning takes place when p satisfies all three criteria: Feasibility Condition; Rate-of-Requests Condition; and Flexibility Condition. If there exists more than one candidate of p, the p that has the highest value of $\mathbb{E}[\epsilon_p]$ will be chosen as p-repositioning.

τ_i^{idle}	The time that the truck becomes idle at location $\mathbf{d}_i$
τ_p	The time that the truck arrives at the idle point p
τ_l^{pck}	The pick-up time of new job l
τ_j^{pdln}	The deadline of pick-up of job j
$\triangle w$	The amount of time the truck is idle at the idle point p
ϵ	The amount of time that exceeds the pick-up deadline of the next job in queue (flexibility parameter)
T_{ip}	The amount of time from $\mathbf{d}_i$ to reposition at the idle point p
T_{pl}	The amount of time from the idle point p to $\mathbf{o}_l$
L_l	Time length of job l
T_{lj}	The amount of time from $\mathbf{d}_l$ to $\mathbf{o}_j$

Table 4.1: Time Variables in Figure 4-2 and Figure 4-3

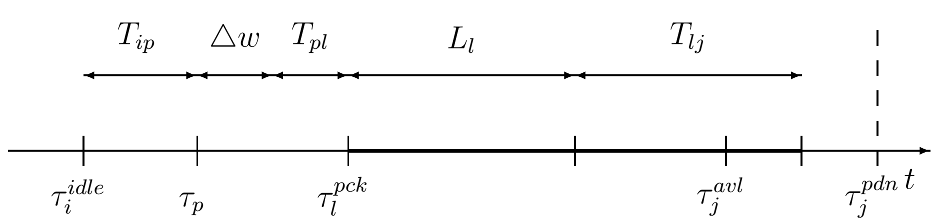


Figure 4-2: Advantageous Situation for Repositioning Policies

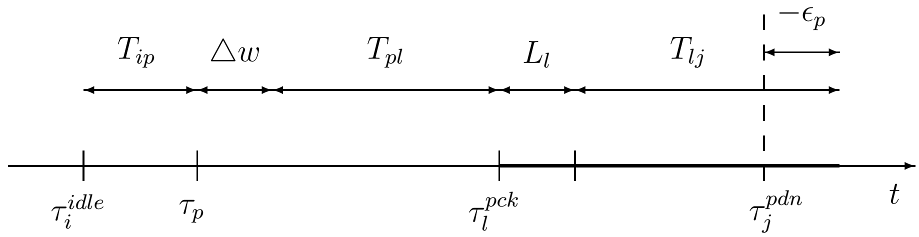


Figure 4-3: Disadvantageous Situation for Repositioning Policies

4.2 The Postponement Decisions

4.2.1 Types of Requests

Two types of requests are considered in this thesis: postponable and non-postponable requests. Each request is assumed to require a service of length not exceeding a period of one day. A non-postponable request is defined as a request that cannot be postponed. On the other hand, a postponable request can be postponed either due to infeasibility or economic reasons. However, there is a cost associated with postponement.

4.2.2 Postponement Objective

The company begins with no job on the first day. At the beginning of each following day, some requests are already known, which are the ones that have been previously postponed. The requests that arrive during the day are called "on-line requests." During the day of operation, the company faces decisions whether or not to fulfill requests. If a job cannot be served on the day of the request, it may be served the next day. Hence, postponement may be beneficial because the company may take advantage of postponing a postponable request in order to accept a non-postponable request on that day.Given the flexibility of postponement, the company may gain benefits from postponing some requests to the next day. Some non-postponable requests may not be fulfilled if all trucks are occupied with either non-postponable or postponable requests. Consequently, having the flexibility to postpone some requests can reduce the congestion of fleet operation on the current day. However, the company contractually incurs a cost of postponement and makes a commitment to serve the postponed job on the next day; this might in turn lead to present tough decisions on the following day due to this commitment.The objective of re-optimization in multi-period setting is to maximize revenue, given the length of the information horizon[1]. The information horizon includes the remaining part of the current day and may partially include the information for the next day. Since the optimization time in real-time settings is generally limited, utilizing excessive information may potentially result in a poor-quality solution. Conversely, the shorter information horizon may not allow the company to fully exploit the availability of the vehicles. Thus, the length of the information horizon plays an important role in effective management of the fleet. If the information horizon consists of the information on the current day only, we may end up postponing many postponable requests to the next day. As a consequence, the postponed requests which are assured to be served on the next day will make it difficult to deal with possible future requests. On the other hand, if the information horizon consists of two consecutive days, we may be able to manage the fleet more effectively on both days depending on the performance of computing machines.

We will be investigating whether a policy that allows postponement is beneficial. In such a policy, three types of the off-line problem, previously discussed in Section 3.2, will be faced throughout the operation.

[1]Information horizon is the horizon which the re-optimization routine considers. It is defined as an amount of time in our case.

4.3 Real-time Policies

4.3.1 Single-Period Policies (Repositioning)

A Benchmark Policy

OPT1 serves as a benchmark policy for evaluating the other policies in the context of repositioning. It is a re-optimization policy without any sophisticated methodology. The basic re-optimization solves an off-line problem at the instance of a job arrival. The information horizon includes the remaining time of the day. This policy takes into consideration all possible re-allocation of trucks and all acceptance/rejection decisions. OPT1 optimizes the acceptance and re-allocation decisions as if no future job would ever be requested.

Advanced Policies

REPO is a re-optimization policy that builds on OPT1 and incorporates the repositioning decisions, excluding the consideration of the flexibility condition. In addition, p-repositioning will take place when equation 4.3 is satisfied and p yields the highest value of equation 4.2. Section 4.1 has described the key characteristics of repositioning strategy. In this way, REPO exploits probabilistic knowledge of future request arrivals.

FREPO is a re-optimization policy that builds on REPO and incorporates the consideration of the flexibility condition as described in Section 4.1.3.

4.3.2 Multi-Period Policies (Postponement)

A Benchmark Policy

OPT3 serves as a benchmark policy in the context of postponement. It is a re-optimization policy that uses OPT1 throughout the time horizon of three consecutive days. The information horizon does not look ahead into the next day;

however, the information horizon includes the remaining time of the current day.

An Advanced Policy

POS is a re-optimization policy that builds on OPT3 and incorporates the postponement decisions. Before the fleet of trucks starts to operate in each day, the over-night problem has been solved in addition to solving the off-line instances during the day. POS considers the postponement decisions as detailed in Section 3.2.

Chapter 5

Simulation Framework

An event-based simulation defined in Larson and Odoni (1981 [25]) is used to evaluate the proposed policies under various probabilistic settings and varying parameters. The policies are tested under several scenarios in such a way that computational time remains manageable.

5.1 Simulation Framework for Repositioning Policies

Notation	Description
α	The revenue per unit of service distance
λ_0	The intensity of the merged Poisson process throughout the region
λ_p	The intensity of the Poisson process corresponding to zone p
ρ	The ratio of high intensity to low intensity used for varying the degree of heterogenous of a predefined region

Table 5.1: Notations for the repositioning policies in addition to Table 2.2.1

5.1.1 Time Horizon

The time horizon is assumed to be over one day, from 8:00AM to 4:00PM, counted as 480 minutes. The next section describes how job requests are generated over the time horizon.

5.1.2 Generation of Job Requests

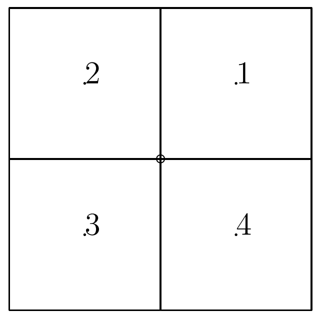
The homogeneous Poisson processes are used to generate new requests in order to obtain τ_i^{arv}. Figure 5-1(a) shows the distribution region, represented by a unit square, which is partitioned into different zones. As can be seen from the figure, the region is divided into 4 different zones 1, 2, 3, and 4. Each zone corresponds to a different intensity rate, i.e., λ_p for zone $p \in \{1, 2, 3, 4\}$. The intensity of the merged Poisson process λ_0 of the whole distribution region is $\sum_{p=1}^{4} \lambda_p$. Any particular request of the merged process has probability $\frac{\lambda_p}{\sum_{p=1}^{4} \lambda_p}$ of originating from zone p. The degree of concentration of the pick-up locations depends on ρ. On the other hand, the destination locations of the jobs are independently identically uniformly distributed throughout in the region. The time window associated with each request is generated as follows:

- T_i^{adv}'s are drawn independently from a uniform distribution with the interval $[T_{min}^{adv}, T_{max}^{adv}]$. So, $\tau_i^{avl} = \tau_i^{arv} + T_i^{adv}$.

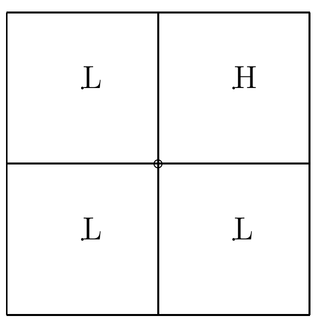
- T_i^{pck}'s are drawn independently from a uniform distribution with the interval $[T_{min}^{pck}, T_{max}^{pck}]$. So, $\tau_i^{pdn} = \tau_i^{avl} + T_i^{pck}$.

- T_i^{res}'s are set constant.

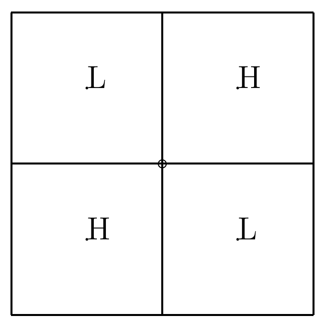
To ensure that no request requires a truck to work beyond the closing time of the depot, such a request will be eliminated and the one generated before will be the last request of the day.



(a) Distribution region divided into four zones. Each zone has an idle point at its center



(b) Distribution region type 1



(c) Distribution region type 2

Figure 5-1: Figure 5-1(a) shows the region divided into the four zones. The idle point p is located at the center of zone p. Figure 5-1(b) and Figure 5-1(c) show the region type 1 and the region type 2, respectively. The letter “H” designates a high-request zone, while the letter “L” designates a low-request zone.

5.1.3 Testing Scenarios

The distribution region is a unit square, representing $50 \times 50\ km^2$ area. The fleet size is set to 15 trucks with all trucks moving at the same constant speed of 40 kms/hour. The choice of $\alpha = 4$ implies that the revenue per unit of service distance is four times higher than the loss revenue per unit of empty distance. Table 5.2 summarizes the numerical values that are assigned to the fixed parameters.

Parameters	K	α	T^{pck}_{min}	T^{pck}_{max}	T^{adv}_{min}	T^{adv}_{max}	T^{res}	τ^{cls}
Value	15	4	10	60	10	90	5	480

Table 5.2: Main parameters used for all scenarios

The expected distance between two random points in a unit square is approximately 0.5214 (see Appendix B). A good limit for the average rate of traffic intensity is the inverse of the average distance times the speed, which is equal here to 0.026. In addition, the expected inter-arrival time for demands in the whole distribution region is chosen to be $1/(K\lambda_0)$. The appropriate value of λ_0 is bounded above by 0.026.

There are two types of zone considered in our simulation. Figure 5-1(b) and 5-1(c) present the two different types of the region. To explore the impact of the degree of heterogeneity on the repositioning policies, we introduce a "Hi-Lo ratio" ρ, which is the ratio of the high intensity to the low intensity of Poisson processes. In addition, we evaluate the repositioning policies by calculating total profits (= operating revenues - costs of movement) and acceptance rates (= accepted requests / total requests). For every set of input parameters, and for every policy under investigation, we simulate 40 independent runs. The analysis is drawn from the sample mean of all independent runs.

Notation	Description
α	The revenue per unit of service distance
λ_0	The intensity of the merged Poisson process throughout the region
μ	The probability of an arriving request being postponable
γ	The discount rate for a unit of distance associated with the postponement of jobs

Table 5.3: Notations for the postponement policies in addition to Table 2.2.1

5.2 Simulation Framework for Postponement Policies

5.2.1 Time Horizon

The time horizon is assumed to be over three days. A daily working service is from 8:00AM to 4:00PM, counted as 480 minutes.

5.2.2 Generation of Job Requests

The pick-up and delivery locations of the jobs are independently identically uniformly distributed. The time window associated with each request is generated the same as in Section 5.1.2. The uniformly distributed requests are generated dynamically according to a Poisson distribution with parameter λ_0. Any particular request has the probability μ of being a postponable request.

5.2.3 Testing Scenarios

The computational analysis has been conducted on several scenarios with various parameters of the requests over the distribution region of 50×50 km^2. The fleet size is 15 vehicles, each of which moving at the same constant speed of 40 kms/hr. The input parameters include γ, μ, and λ_0. For every set of input parameters, and for every policy under investigation, we simulate 40 independent runs. The analysis is drawn from the sample mean of all independent runs. In postponement, the 2^{nd} day represents the expected long-run horizon of a fleet operation. Two aspects of the results are evaluated: profit and acceptance rate. The profits shown hereafter are calculated with operating revenues (deducted by the discount) minus costs of movement. The acceptance rates are the ratio of the sum of postponed and accepted requests to the sum of total arriving requests and postponed requests on the 2^{nd} day.

Chapter 6

Computational Results

Each scenario is simulated a total of 40 independent runs. The variation of the results, shown in Table 6.1 and Table 6.2, suggests that 40 independent runs are sufficient to differentiate the performance of the repositioning policies. However, it is important to note that the sample mean of the measures may not be a good approximation to assess individual policies. In case of postponement, Table 6.3 and Table 6.4 show that the variation of results is statistically insignificant, by means of the variances, when there are 40 independent runs. Therefore, the sample mean of all runs is used as our approximation for the comparison of the policy measures. All computational experiments have been processed on a 1.7GHz Intel Pentium IV machine with 512MB of RAM.

The re-optimization calls the CPLEX solver to solve instances of the off-line problems. To assure timeliness and robustness of the solution, the number of jobs is limited to 40. Moreover, the re-optimization is allocated a maximum time of 3 minutes for each optimization.

6.1 Results for Repositioning Policies

To determine the range of interesting values for the parameters, λ_0, ρ, and $\hbar$, several preliminary computational tests were performed on samples with the fixed parameters given in Table 5.2. The value of λ_0 was then investigated at 0.025 while varying the other two parameters. The value of ρ was chosen from [1,3] and the value of $\hbar$ was investigated over the range $[0.8, 1]$.

Figure 6-1 and Figure 6-2 present the results obtained from both types of region in terms of profit and acceptance rate. In region type 1, it can be seen from Figure 6-1 that the profits generated from REPO are under 3% less than the profits generated from OPT1 for all values of ρ. On the other hand, our proposed FREPO generated more profits than OPT1. These results indicate that REPO does not improve the profit. In region type 2, none of the strategies improves the chance of profits. Nonetheless, FREPO performs 1.5% better than REPO in terms of profits.

The performance of FREPO and REPO is further compared. Figure 6-3 and Figure 6-4 indicate that higher acceptance rates can be gained in both regions by implementing FREPO and REPO. In particular, REPO can capture more customers than FREPO in both regions as evidenced by the greater acceptance rates. Interestingly, the percentage differences shown in Figure 6-5 and Figure 6-6 demonstrate that as the value of ρ increases, the relative acceptances rates of both FREPO and REPO decreases.

The effect of heterogeneity is present here as we can see from the considerable decrease of the percentage differences in profits between REPO and OPT1, obtained from both regions. REPO yields the worst profits among all three strategies in both regions. Although REPO exploits probabilistic knowledge of future demands, the empty-movement cost can be significant, leading to highly excessive costs. Since repositioning must require extra distances, only "good" repositionings could generate more revenues that profitably cover the extra empty-movement cost. When there are dispersed concentrations of job requests (as in region type 2), even FREPO cannot capture the significant empty movement costs. In such dispersed concentrations of requests, the region must be partitioned in a more efficient pattern so as to reduce the distances among idle points. As a result of the flexibility condition, FREPO can successfully eliminate the unnecessary repositionings and generate higher profits than OPT1. This case only occurrs in the region where demand is centralized in one zone. Clearly, the acceptance rates of both FREPO and REPO are higher than OPT1 since both of them allow much more flexibility in optimizing the selection of the requests to fulfill.

6.2 Results for Postponement Policies

To determine the interesting values for the parameters, γ, μ, and λ_0, several computational tests were preliminarily performed in the samples with the fixed parameters given in Table 5.2. The value of λ_0 was varied from 0.010 to 0.025, holding the other two parameters constant at some values. In particular, the value of μ was investigated over the range $[0.0, 1.0]$ and γ was tested at either 0.0 or 0.1.

As a result of the tests, POS yields statistically greater profits when $lambda_0$ is at least 0.010. One reason is that postponement leads to excessively penalized costs ($\gamma = 0.1$); when the system is not congested by requests, there can be inessential postponement and the choice of which jobs are to be postponed is very limited. In consequence, postponed jobs could reduce the flexibility of job scheduling in low-demand scenarios because they require trucks to service at a specified period of time. For instance, when there are fewer jobs to be served, some low-revenue jobs can be postponed. These jobs could obstruct the possibility of accepting higher-revenue jobs on the later day. The results show that POS does not produce profits that are significantly greater than OPT3 does when $\lambda_0 < 0.010$. As can be seen in Table 6.3, when $\lambda_0 = 0.010$ and $\mu \in \{0.25, 0.5\}$, the profits obtained from POS are very close to the profits obtained from OPT3. In case of λ_0 being greater than 0.019 with high values of μ, there are too many jobs that are postponed from the first day to the second day. The computational performance was limited in such a case; in addtion, the acceptance rates are relatively low even when $\lambda_0 = 0.019$. The result for $\lambda_0 > 0.019$ is not simulated. We then investigate various scenarios with $\lambda_0 \in \{0.010, 0.015, 0.019\}$.

The characteristics of the long-run fleet operation with continuous postponement decisions can be represented by the results on the 2^{nd} day with enough number of independent runs. Figure 6-7 displaying the percentage differences in profits between POS and OPT3 indicates that postponement is on average statistically beneficial since all the bars are positive. When μ is at 1.00, more than 2% profit margins can be derived from postponement decisions even if the discount rate is 10%. This is attributed to the fact that the flexibility of postponement is fully utilized. Two different regimes of profits. may be differentiated. There is a significant change in profits when λ is changed from 0.010 to 0.015. Since there are many more jobs to be selected for postponing, postponement can significantly improve the chances of profit. Yet, when profits between $\lambda_0 = 0.015$ and $\lambda_0 = 0.019$ are compared, profits are not considerably increased since the system is already saturated with sufficient requests.

On the other hand, the acceptance rates are substantialy decreased with the increase in λ_0. Figure 6-8 shows the negative bars in all cases when λ_0 is at least 0.015. When λ_0 is low, acceptance rates are not different because only a few postponements have occurred. By and large, postponement decisions can greatly exacerbate the number of accepted jobs in those high-demand scenarios. In conclusion, when the demand in the distribution region is high enough, the postponement decisions can help select highly profitable jobs, trading off with rejecting many low-revenue jobs.

We further investigate the role of the heterogeneity in postponable demand. In our construction, the value of μ signifies the heterogeneity in postponable demand. The result confirms our intuition that the increase in the degree of the heterogeneity in postponable demands can improve the chance of profits. As shown in Figure 6-7, when the value of μ increases, the percentage differences of POS compared to OPT3 is also increased in all values of λ_0. Conversely, the acceptance rates also decrease with the increasing μ. As in Figure 6-8, this behavior becomes significant at higher λ_0. Figure 6-9 and Figure 6-10 reveal that the distribution of percentage differences are quite deviated; however, the trend of increasing profits with the increase in the degree of the heterogeneity clearly exists.

Another issue we have raised is that: does the requests that end up being postponed belong to some class that is easy to be recognized? In an extreme case, if there are two jobs (high-revenue job and low-revenue job) to be postponed, would the POS policy tend to prioritize the high-revenue job or vice versa? The data used to analyze are collected from information about the jobs that were postponed from the 2^{nd} day to the 3^{rd} day. We use boxplots, as shown in Figure 6-11 and Figure 6-12, for describing the characteristics of postponed jobs and compared them with randomly-generated jobs. In all cases, we can see that the median of revenues for all policies is very close. Only those jobs that generate extreme revenues (either very high or very low) may not be postponed.

6.3 Summary of Findings

6.3.1 Repositioning Decisions

- Is the knowledge of future demands always useful for the management of a fleet of vehicles?

  As evidenced by the failure to increase profits in both regions when implementing REPO, the exploitation about the knowledge of future demands is not always beneficial if improperly implemented. If the knowledge of future demands is used in an ineffective management, a trucking company is unlikely to gain benefits from it. Moreover, since the forecast is never 100% correct, attention needs to be paid on the implementation of any use of such knowledge. However, this is not to say that the knowledge of future demands is not informative.

- Under what conditions would a repositioning strategy be beneficial?

  One important condition we have shown is that when the distribution region is heterogenous in demands, implementing FREPO is beneficial in a demand-centralized region.

- Does the degree of heterogeneity of the distribution region impact the results of repositioning strategies?

  Besides using an effective algorithm to reposition vehicles, the distribution region must be partitioned in a proper pattern to reduce the distances among idle points. This has been shown through the failure of FREPO to improve the expected profits in region type 2.

6.3.2 Postponement Decisions

- Under what conditions can postponement strategies generate more profits? Would more requests be captured?

  The use of postponing requests has sufficiently been proven in our simulation to be a good alternative for fleet management. POS yields a significant increase in profits; however, it may be not beneficial when the arrival rate of requests is too low. The derived profits obtained from POS is increased with the increase in the heterogeneity of postponable demands. The flexibility of choosing what jobs to be postponed is even greater if all requests are postponable. The result has revealed that approximately 6% profit margin can be gained in case of $\lambda_0 = 0.019$. An increase in profits can be obtained with increasing number of postponable requests even though the company offers its customer a 10% discount. However, the company is making higher profits at the expense of lower acceptance rates, which decrease with the increasing number of postponable requests.

- Does the requests that end up being postponed belong to some class that is easy to be recognized?

  None of the requests will be given a priority of postponement. Jobs will be treated equally when they are considered for postponement. Note that only those jobs with extreme revenues on either side are less likely to be postponed.

Table 6.1: Repositioning: Statistics for the percentage differences in profits compared to OPT1 ($N = 40$)

ρ	Statistics	Region Type 1		Region Type 2	
		% REPO	% FREPO	% REPO	% FREPO
1.00	mean	-1.02%	0.15%	-1.02%	0.15%
	$\frac{s.d.}{\sqrt{N}}$	0.77%	0.81%	0.77%	0.81%
2.00	mean	-2.11%	0.51%	-3.40%	-1.52%
	$\frac{s.d.}{\sqrt{N}}$	0.50%	0.55%	0.58%	0.41%
3.00	mean	-2.61%	0.23%	-3.19%	-1.57%
	$\frac{s.d.}{\sqrt{N}}$	0.51%	0.60%	0.77%	0.57%

Table 6.2: Repositioning: Statistics for the percentage differences in profits compared to OPT1 ($N = 40$)

ρ	Statistics	Region Type 1		Region Type 2	
		% REPO	% FREPO	% REPO	% FREPO
1.00	mean	2.68%	1.87%	2.68%	1.87%
	$\frac{s.d.}{\sqrt{N}}$	1.06%	1.08%	1.06%	1.08%
2.00	mean	1.70%	1.59%	1.16%	0.13%
	$\frac{s.d.}{\sqrt{N}}$	0.53%	0.60%	0.45%	0.37%
3.00	mean	1.70%	1.12%	1.09%	0.45%
	$\frac{s.d.}{\sqrt{N}}$	0.49%	0.49%	0.66%	0.41%

Table 6.3: Postponement: Statistics for the percentage differences in profits ($N = 40$)

μ	Statistics	$\lambda_0 = 0.010$		$\lambda_0 = 0.015$		$\lambda_0 = 0.019$	
		$\gamma = 0.0$	$\gamma = 0.1$	$\gamma = 0.0$	$\gamma = 0.1$	$\gamma = 0.0$	$\gamma = 0.1$
0.25	mean	0.30%	0.55%	1.34%	1.78%	1.42%	1.17%
	$\frac{s.d.}{\sqrt{N}}$	0.18%	0.28%	0.40%	0.40%	0.60%	0.63%
0.5	mean	0.97%	0.90%	2.10%	2.24%	2.26%	2.60%
	$\frac{s.d.}{\sqrt{N}}$	0.34%	0.39%	0.53%	0.58%	0.49%	0.53%
0.75	mean	1.61%	1.34%	4.34%	3.77%	4.16%	3.95%
	$\frac{s.d.}{\sqrt{N}}$	0.47%	0.38%	0.96%	0.80%	0.83%	0.83%
1	mean	2.16%	1.89%	5.45%	4.85%	6.15%	4.65%
	$\frac{s.d.}{\sqrt{N}}$	0.53%	0.49%	0.88%	0.77%	1.15%	1.05%

Table 6.4: Postponement: Statistics for the percentage differences in acceptance rates ($N = 40$)

μ	Statistics	$\lambda_0 = 0.010$		$\lambda_0 = 0.015$		$\lambda_0 = 0.019$	
		$\gamma = 0.0$	$\gamma = 0.1$	$\gamma = 0.0$	$\gamma = 0.1$	$\gamma = 0.0$	$\gamma = 0.1$
0.25	mean	-0.04%	0.07%	-0.15%	-0.04%	-1.95%	-2.17%
	$\frac{s.d.}{\sqrt{N}}$	0.04%	0.24%	0.40%	0.40%	0.39%	0.47%
0.5	mean	0.05%	0.18%	-0.71%	-0.51%	-3.33%	-2.94%
	$\frac{s.d.}{\sqrt{N}}$	0.24%	0.35%	0.46%	0.43%	0.45%	0.51%
0.75	mean	-0.13%	-0.22%	-1.14%	-1.14%	-5.35%	-4.49%
	$\frac{s.d.}{\sqrt{N}}$	0.31%	0.28%	0.57%	0.53%	0.74%	0.71%
1	mean	0.27%	0.54%	-0.69%	-1.26%	-6.33%	-6.15%
	$\frac{s.d.}{\sqrt{N}}$	0.21%	0.33%	0.47%	0.42%	1.00%	0.85%

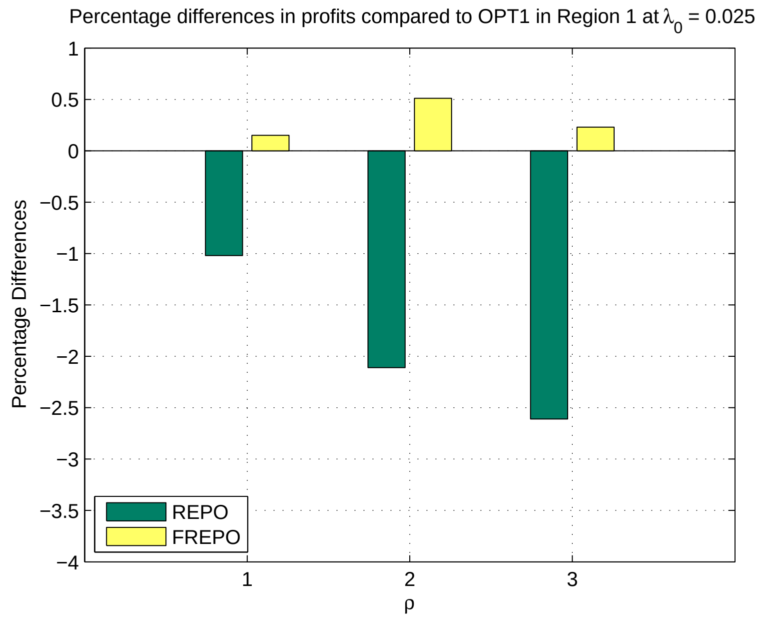


Figure 6-1: The percentage differences of profits between FREPO/REPO and OPT1 performing in region type 1

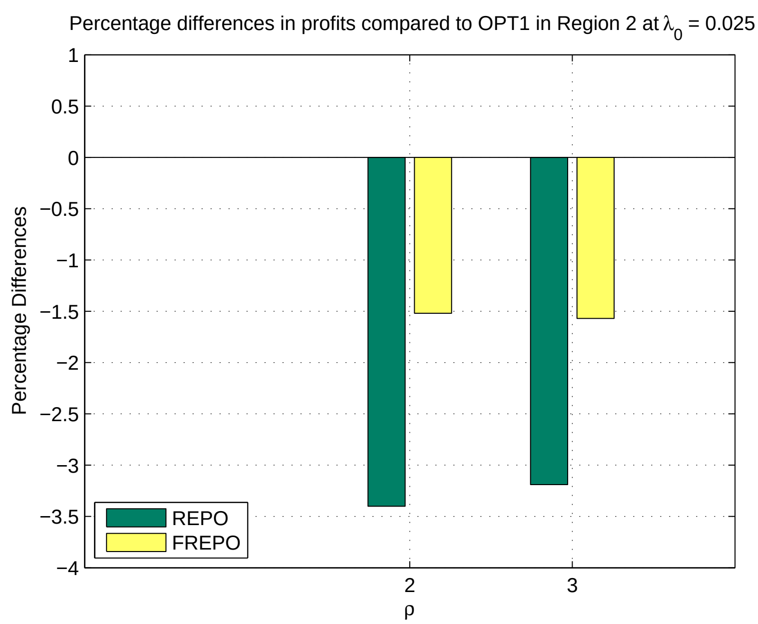


Figure 6-2: The percentage differences of profits between FREPO/REPO and OPT1 performing in region type 2

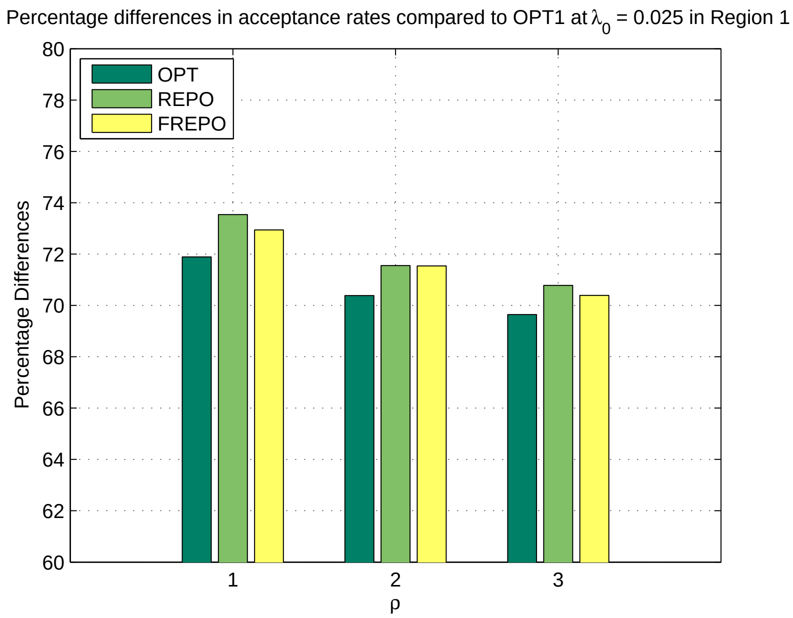


Figure 6-3: The acceptance rates at $\lambda_0 = 0.025$ in region type 1

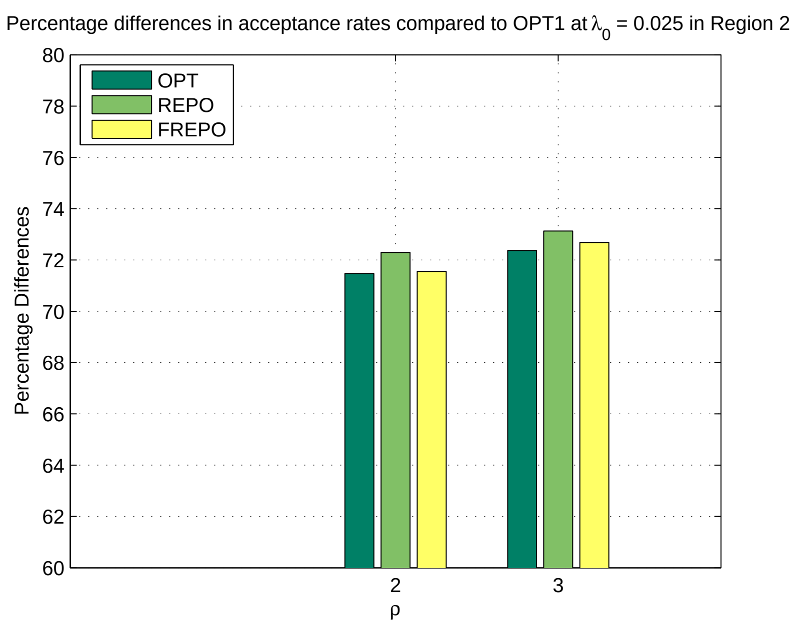


Figure 6-4: The acceptance rates at $\lambda_0 = 0.025$ in region type 2

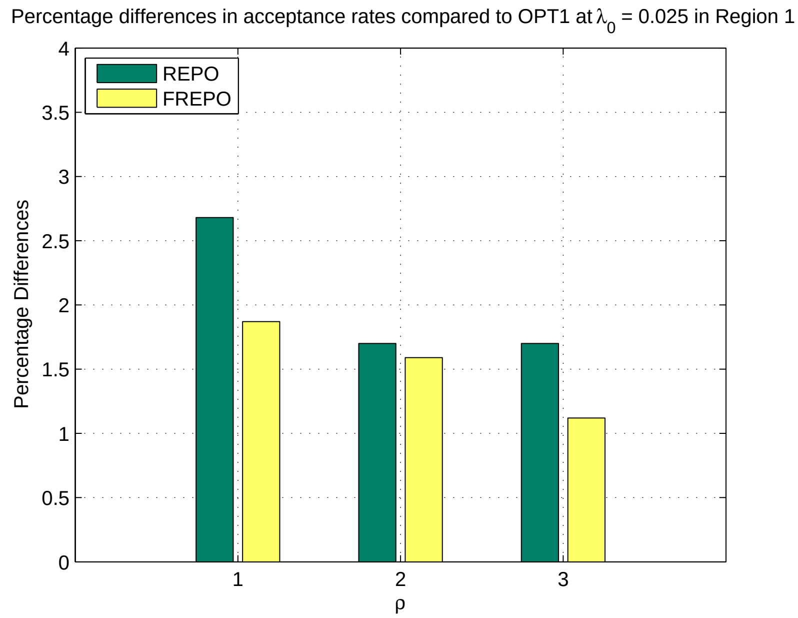


Figure 6-5: The percentage differences of acceptance rates compared to OPT1 at $\lambda_0 = 0.025$ in region type 1

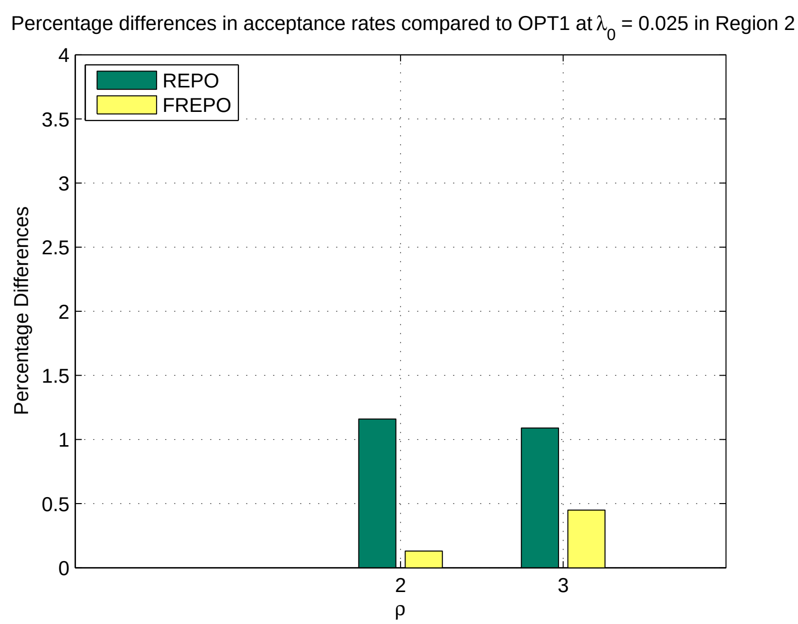


Figure 6-6: The percentage differences of acceptance rates compared to OPT1 at $\lambda_0 = 0.025$ in region type 2

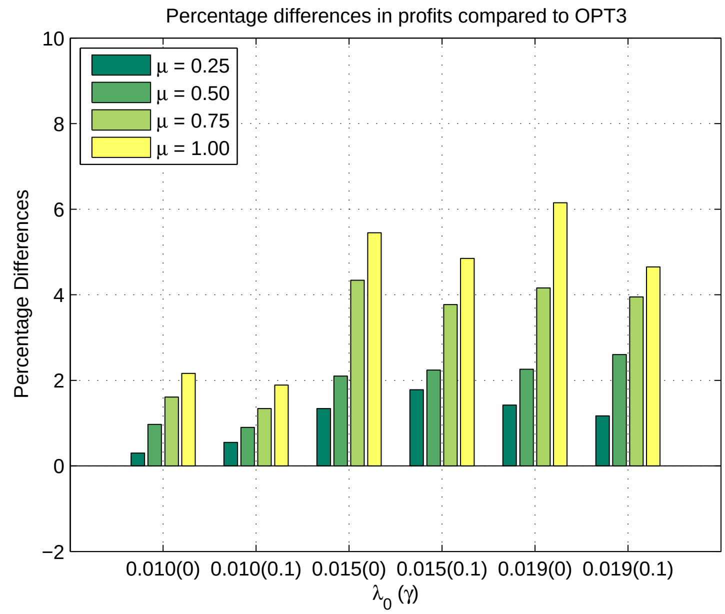


Figure 6-7: The percentage differences of profits between POS and OPT3

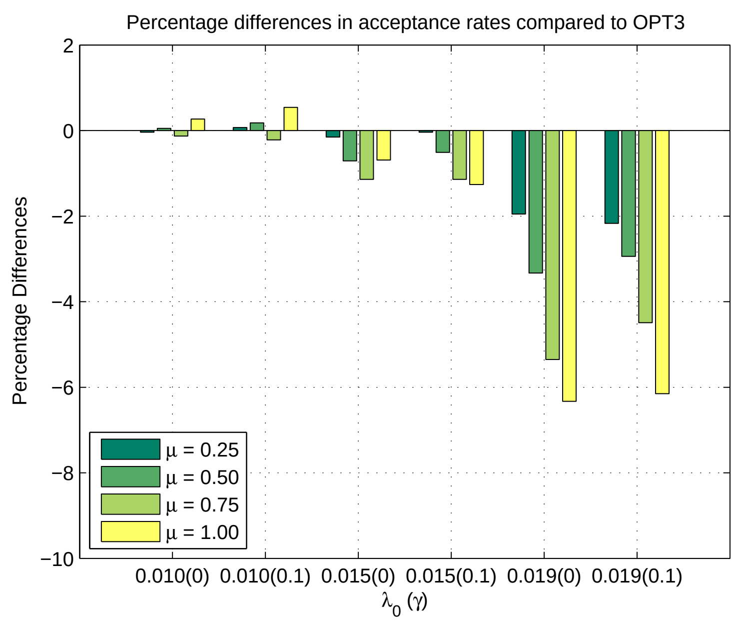


Figure 6-8: The percentage differences of acceptance rates between POS and OPT3

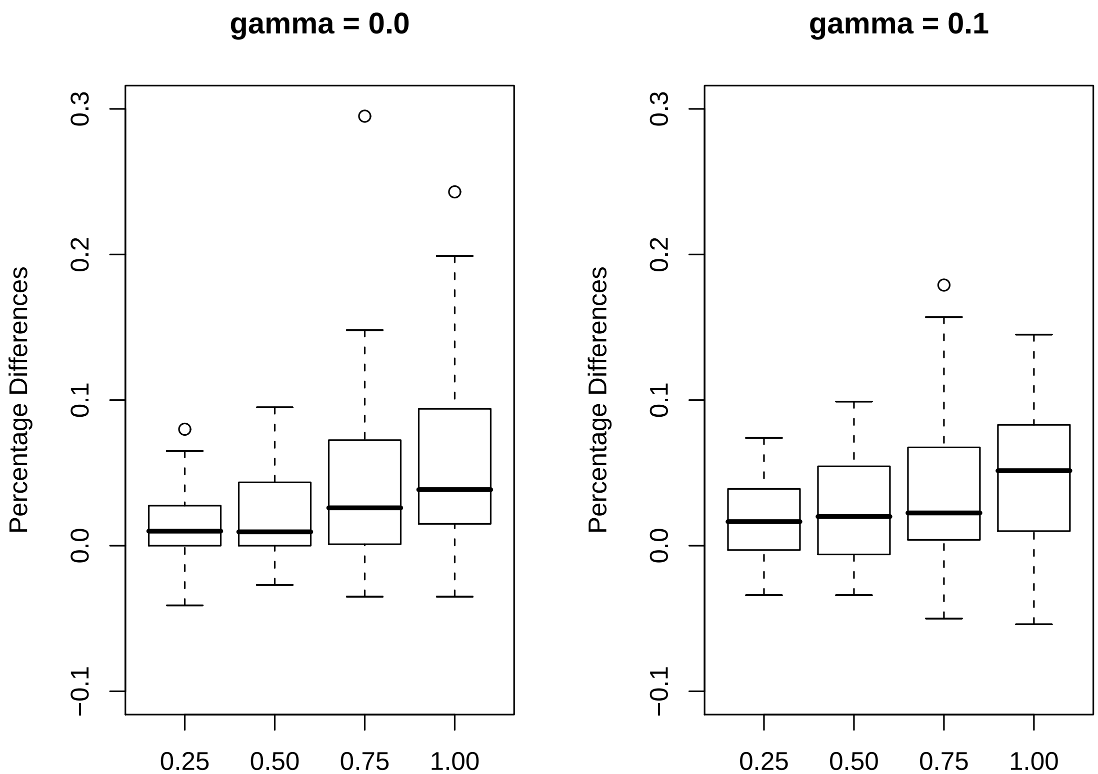


Figure 6-9: The boxplot of the percentage differences of profits between POS and OPT3 at $\lambda_0 = 0.015$

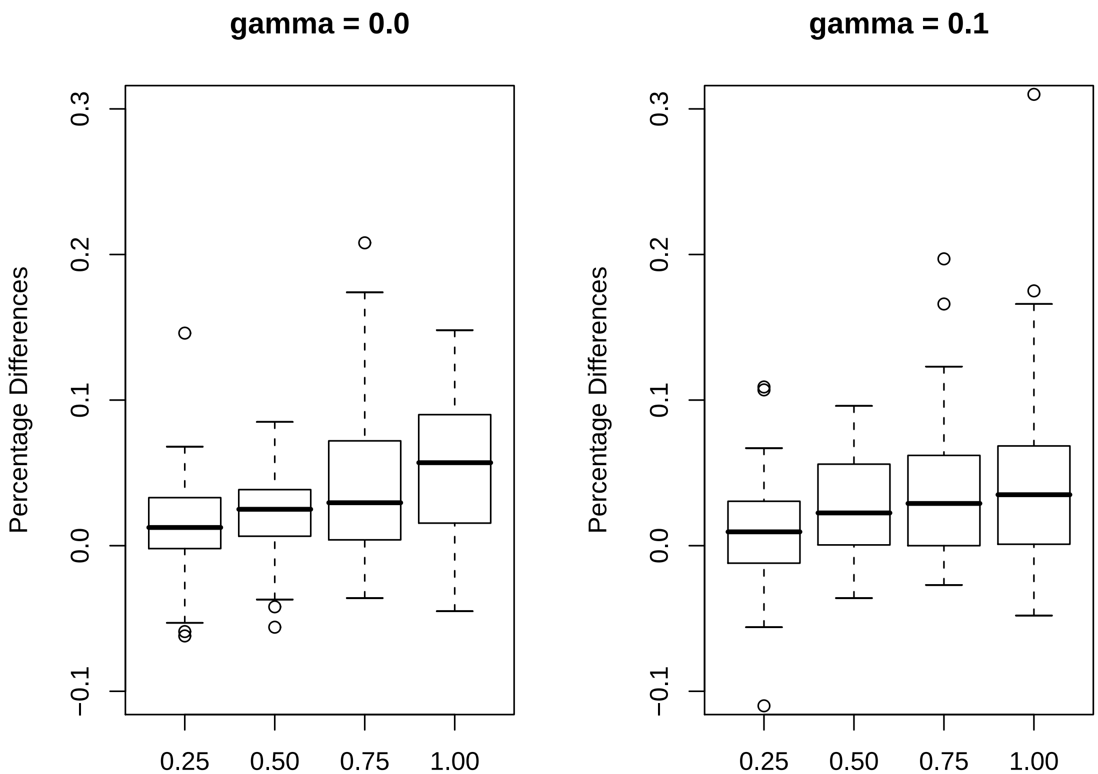


Figure 6-10: The boxplot of the percentage differences of profits between POS and OPT3 at $\lambda_0 = 0.019$

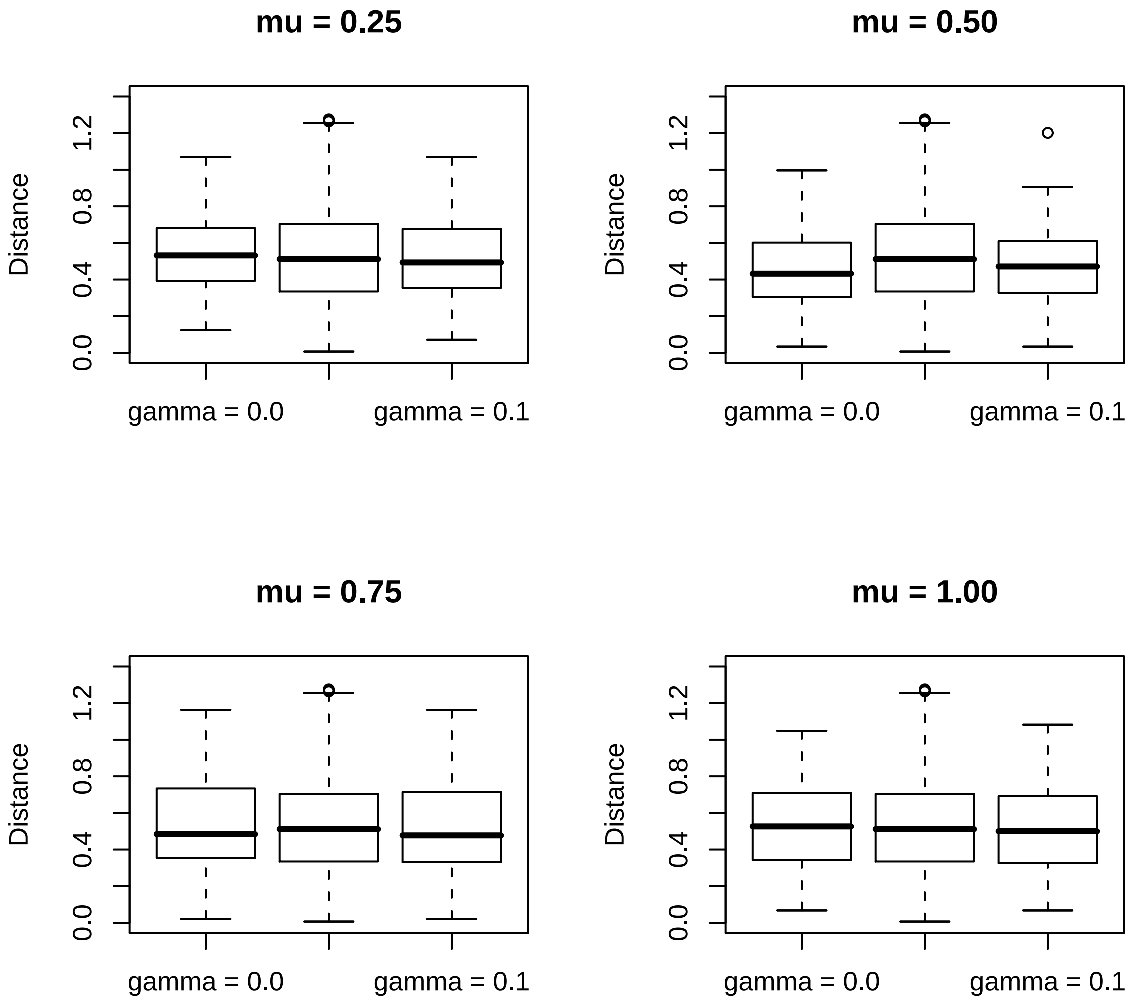


Figure 6-11: The comparison between postponed distances and random distances when $\lambda_0 = 0.015$

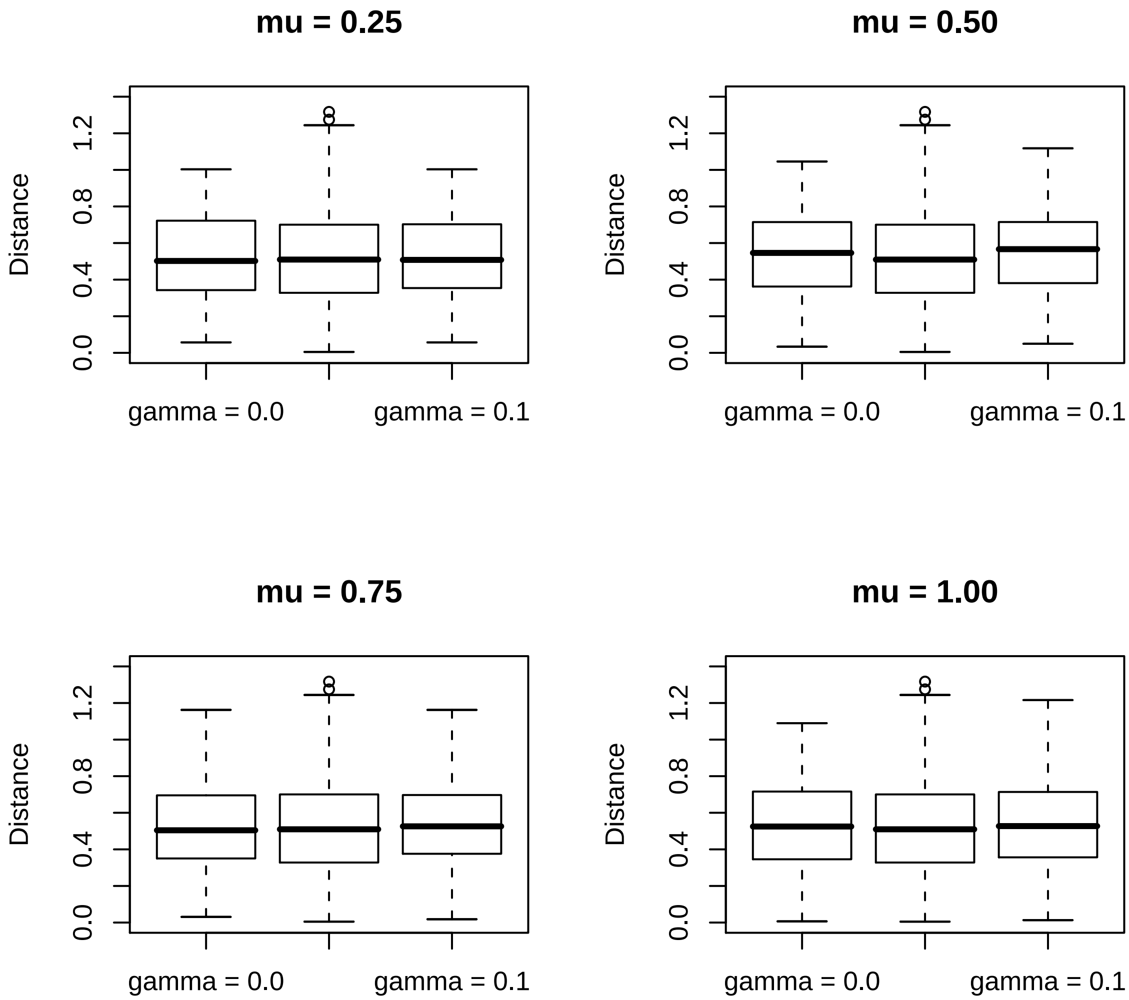


Figure 6-12: The comparison between postponed distances and random distances when $\lambda_0 = 0.019$

Chapter 7

Conclusions

7.1 Summary

In this thesis, we have studied the dynamic vehicle routing problems and developed a real-time solution method to maximize the profits. A new strategy that takes advantage of probabilistic knowledge about future demands to efficiently manage the fleet of trucks is proposed. Several issues related to this strategy were addressed and examined. The results show that the proposed approach provides improvements over the original algorithm in some "critical" situations involving a particular type of heterogeneity of the region with a sufficient arrival rate of request. In particular, when there exists heterogeneity in demands over the distribution region, the proposed strategy is more profitable in a demand-centralized region. Furthermore, the proposed strategy maintains better relationship with the customers as evidenced by the higher acceptance rates. Nonetheless, we have raised a concern over the repositioning strategy when a distribution region has idle points that are not properly arranged throughout the region. That is, the patterns of partitioning the region should also be taken into consideration before any repositioning strategy is implemented.

Another notion of the dynamic vehicle routing problems we have introduced is postponement. A new policy that allows trucking companies to postpone their requests to the following day is proposed. This policy improves the chances of profits for the company even with a discount of 10% in order to incentivize customers for postponement. This improvement has shown to be more substantial with increasing postponable demands. In addition, there is no priority given to any request when considered for the postponement. When deciding whether to implement a postponement policy, the company need to address the compensation of customers who are willing to postpone their requests. In our case, the discount of 10% is given to those who are willing to postpone; however, the company is trading the profits with the acceptance rates. The relationship with customers could be worse off. Thus, the selection of long-established customers to avoid their rejection should be of great interest to the company.

7.2 Future Work

With respect to future development of repositioning strategies, it should be of interest to develop heuristics that reduce unnecessary repositioning movement in order to effectively exploit the knowledge of future demands. Moreover, different patterns of partitioning the distribution regions are necessary in order to adequately define the importance of repositioning strategies.

More sophisticated scenarios can be extended to analyze the impacts of discounts on postponement. The ratio of postponable requests to non-postponable requests may also be a variable to investigate further, so as to understand how many customers and what kind of customers a company should contract out for postponement if all customers cannot be postponed. It is also important that an algorithm that efficiently tackle large size problems be developed in order to extensively analyze the case with higher arrival rate of requests.

Appendix A

Tables of Numerical Data

Table A.1: Repositioning: Profits of Region Type 1

Index	$\rho = 1$					$\rho = 2$					$\rho = 3$				
	OPT	REPO	FREPO	% (REPO)	% (FREPO)	OPT	REPO	FREPO	% (REPO)	% (FREPO)	OPT	REPO	FREPO	% (REPO)	% (FREPO)
1	157.63	156.19	159.46	-0.92%	1.16%	167.70	164.18	168.20	-2.10%	0.30%	129.60	127.70	133.71	-1.47%	3.17%
2	151.99	159.44	158.27	4.90%	4.13%	143.01	147.00	143.43	2.79%	0.29%	153.31	160.17	158.37	4.48%	3.30%
3	145.95	141.90	140.99	-2.77%	-3.40%	147.66	142.83	154.67	-3.27%	4.75%	156.69	145.80	149.37	-6.95%	-4.67%
4	161.30	154.92	160.97	-3.96%	-0.21%	157.90	152.92	155.54	-3.16%	-1.50%	145.24	143.20	145.07	-1.40%	-0.12%
5	140.63	138.44	138.52	-1.55%	-1.50%	159.10	163.49	162.87	2.76%	2.37%	137.10	137.32	143.05	0.16%	4.35%
6	149.73	143.94	146.04	-3.87%	-2.47%	152.40	138.05	146.79	-9.42%	-3.68%	137.71	133.88	140.13	-2.78%	1.76%
7	158.24	159.53	163.33	0.81%	3.21%	164.14	160.09	161.79	-2.47%	-1.43%	138.32	130.84	137.32	-5.41%	-0.73%
8	163.78	157.64	165.56	-3.75%	1.09%	142.91	138.59	145.72	-3.03%	1.97%	141.21	144.89	145.87	2.60%	3.30%
9	163.28	165.32	172.89	1.25%	5.89%	143.92	139.78	142.82	-2.88%	-0.76%	144.63	142.08	147.39	-1.76%	1.91%
10	145.81	147.34	152.66	1.05%	4.70%	148.05	150.32	150.53	1.53%	1.67%	141.38	140.75	139.53	-0.45%	-1.31%
11	170.52	169.52	168.68	-0.58%	-1.08%	142.12	140.32	141.53	-1.26%	-0.42%	135.99	131.03	141.99	-3.65%	4.41%
12	146.86	150.72	149.06	2.63%	1.50%	165.25	152.60	161.03	-7.65%	-2.55%	150.59	141.00	149.83	-6.37%	-0.50%
13	148.49	144.74	143.75	-2.52%	-3.19%	146.77	144.95	149.86	-1.24%	2.11%	140.56	133.96	136.56	-4.69%	-2.84%
14	155.42	145.01	146.13	-6.70%	-5.98%	156.58	150.71	155.07	-3.75%	-0.96%	117.22	116.04	119.27	-1.01%	1.75%
15	110.44	133.12	139.32	20.54%	26.15%	147.08	140.72	143.93	-4.32%	-2.14%	147.61	143.13	146.30	-3.03%	-0.89%
16	164.76	163.18	161.26	-0.96%	-2.13%	146.04	139.89	150.54	-4.21%	3.08%	154.20	152.91	148.62	-0.84%	-3.62%
17	157.33	148.26	157.11	-5.77%	-0.14%	135.74	134.27	141.35	-1.08%	4.13%	156.61	147.16	153.65	-6.03%	-1.89%
18	139.96	140.88	140.74	0.65%	0.56%	168.66	173.21	169.46	2.70%	0.48%	163.64	157.15	160.80	-3.97%	-1.73%
19	152.26	141.06	147.58	-7.35%	-3.07%	150.59	140.40	143.66	-6.77%	-4.60%	139.40	128.87	137.76	-7.55%	-1.18%
20	140.79	140.61	140.17	-0.13%	-0.44%	115.21	119.21	130.81	3.47%	13.54%	144.71	141.53	142.22	-2.20%	-1.72%
21	168.18	165.15	162.25	-1.80%	-3.53%	146.50	147.55	152.47	0.71%	4.08%	166.28	166.62	166.66	0.20%	0.23%
22	167.42	156.94	172.97	-6.26%	3.31%	159.49	156.31	160.68	-1.99%	0.75%	142.20	136.90	142.97	-3.73%	0.54%
23	154.09	150.18	145.68	-2.54%	-5.45%	143.12	142.21	150.86	-0.64%	5.41%	150.94	146.91	151.72	-2.67%	0.52%
24	142.43	151.44	142.79	6.32%	0.25%	162.24	156.93	158.75	-3.28%	-2.15%	145.29	142.03	143.64	-2.25%	-1.14%
25	145.01	143.48	142.29	-1.06%	-1.87%	141.96	135.30	145.35	-4.69%	2.39%	127.74	135.96	149.32	6.43%	16.89%
26	143.27	145.55	142.05	1.59%	-0.85%	147.45	140.99	148.37	-4.39%	0.62%	143.09	142.00	140.69	-0.77%	-1.67%
27	139.42	125.88	133.03	-9.71%	-4.58%	160.55	153.80	155.06	-4.21%	-3.42%	141.10	140.07	139.53	-0.73%	-1.11%
28	162.14	163.42	163.11	0.79%	0.60%	153.15	154.93	154.16	1.16%	0.66%	158.78	155.48	150.41	-2.08%	-5.27%
29	141.63	138.21	146.12	-2.42%	3.17%	131.98	128.15	126.81	-2.90%	-3.92%	153.79	155.21	155.13	0.92%	0.87%
30	158.70	160.20	162.01	0.94%	2.08%	148.32	140.86	150.57	-5.03%	1.51%	161.94	161.01	171.74	-0.57%	6.05%
31	151.27	145.90	154.95	-3.55%	2.44%	154.08	157.26	150.55	2.06%	-2.29%	142.33	131.49	136.29	-7.62%	-4.24%
32	160.76	157.15	159.52	-2.25%	-0.77%	145.47	138.70	147.25	-4.65%	1.22%	161.95	161.69	165.07	-0.16%	1.93%
33	156.17	158.14	151.37	1.26%	-3.08%	144.30	146.37	140.99	1.44%	-2.29%	140.29	134.11	142.50	-4.41%	1.57%
34	162.47	157.65	154.70	-2.96%	-4.78%	157.95	152.91	149.33	-3.19%	-5.46%	152.15	140.28	143.10	-7.80%	-5.95%
35	161.71	161.94	162.33	0.14%	0.38%	133.14	127.80	137.45	-4.01%	3.24%	148.02	138.11	146.03	-6.69%	-1.34%
36	156.62	151.39	150.25	-3.34%	-4.07%	145.91	134.70	143.95	-7.68%	-1.34%	133.82	126.37	133.70	-5.57%	-0.09%
37	146.67	141.93	145.91	-3.23%	-0.52%	136.84	136.91	144.59	0.05%	5.66%	125.43	120.40	123.31	-4.01%	-1.69%
38	166.87	171.47	169.71	2.76%	1.70%	155.03	151.96	157.82	-1.98%	1.80%	158.08	157.11	158.89	-0.61%	0.51%
39	163.93	151.32	165.37	-7.69%	0.88%	150.88	154.73	149.59	2.55%	-0.86%	157.69	148.53	155.08	-5.81%	-1.66%
40	161.19	162.99	154.35	1.12%	-4.24%	150.92	150.18	147.91	-0.48%	-1.99%	125.65	120.55	127.49	-4.06%	1.46%

Table A.2: Repositioning: Acceptance Rates of Region Type 1

Index	$\rho = 1$					$\rho = 2$					$\rho = 3$				
	OPT	REPO	FREPO	% (REPO)	% (FREPO)	OPT	REPO	FREPO	% (REPO)	% (FREPO)	OPT	REPO	FREPO	% (REPO)	% (FREPO)
1	0.680	0.713	0.680	-14.42%	3.02%	0.582	0.582	0.568	0.00%	-2.35%	0.701	0.709	0.744	1.22%	6.10%
2	0.765	0.756	0.782	-12.12%	-14.54%	0.672	0.704	0.712	4.76%	5.95%	0.654	0.724	0.709	10.84%	8.43%
3	0.761	0.789	0.761	1.37%	-2.84%	0.772	0.798	0.798	3.41%	3.41%	0.740	0.740	0.740	0.00%	0.00%
4	0.727	0.711	0.758	-1.19%	-10.05%	0.718	0.701	0.709	-2.38%	-1.19%	0.654	0.669	0.654	2.41%	0.00%
5	0.771	0.807	0.798	-12.82%	-24.00%	0.672	0.703	0.711	4.65%	5.81%	0.586	0.607	0.579	3.66%	-1.22%
6	0.808	0.828	0.808	-9.91%	-1.26%	0.728	0.696	0.696	-4.40%	-4.40%	0.798	0.819	0.819	2.67%	2.67%
7	0.597	0.597	0.631	-2.14%	26.69%	0.585	0.599	0.577	2.41%	-1.20%	0.757	0.757	0.757	0.00%	0.00%
8	0.762	0.754	0.738	-11.81%	10.34%	0.672	0.697	0.706	3.75%	5.00%	0.841	0.879	0.897	4.44%	6.67%
9	0.675	0.706	0.722	-9.22%	-2.35%	0.612	0.612	0.589	0.00%	-3.80%	0.659	0.667	0.659	1.20%	0.00%
10	0.641	0.656	0.664	9.84%	1.77%	0.704	0.722	0.730	2.47%	3.70%	0.653	0.678	0.661	3.90%	1.30%
11	0.766	0.766	0.758	0.56%	-15.16%	0.770	0.814	0.805	5.75%	4.60%	0.650	0.632	0.667	-2.63%	2.63%
12	0.750	0.806	0.778	1.59%	-0.29%	0.762	0.722	0.730	-5.21%	-4.17%	0.748	0.748	0.748	0.00%	0.00%
13	0.712	0.748	0.730	-5.98%	-7.83%	0.669	0.692	0.692	3.37%	3.37%	0.656	0.680	0.664	3.66%	1.22%
14	0.624	0.624	0.617	29.12%	0.33%	0.806	0.816	0.825	1.20%	2.41%	0.626	0.673	0.673	7.46%	7.46%
15	0.478	0.669	0.669	38.24%	48.97%	0.661	0.679	0.696	2.70%	5.41%	0.712	0.704	0.688	-1.12%	-3.37%
16	0.651	0.644	0.630	17.69%	-0.77%	0.766	0.766	0.775	0.00%	1.18%	0.646	0.661	0.630	2.44%	-2.44%
17	0.741	0.731	0.741	-7.26%	7.74%	0.687	0.696	0.704	1.27%	2.53%	0.798	0.769	0.769	-3.61%	-3.61%
18	0.734	0.766	0.742	-5.43%	-5.82%	0.694	0.716	0.709	3.23%	2.15%	0.691	0.654	0.699	-5.32%	1.06%
19	0.883	0.862	0.872	-19.66%	-22.73%	0.709	0.692	0.701	-2.41%	-1.20%	0.682	0.682	0.701	0.00%	2.74%
20	0.694	0.712	0.694	6.37%	-0.67%	0.738	0.796	0.835	7.89%	13.16%	0.689	0.706	0.681	2.44%	-1.22%
21	0.707	0.722	0.714	-0.84%	3.61%	0.701	0.726	0.701	3.66%	0.00%	0.732	0.748	0.732	2.15%	0.00%
22	0.656	0.672	0.695	2.76%	-3.79%	0.675	0.714	0.675	5.88%	0.00%	0.632	0.669	0.669	5.95%	5.95%
23	0.734	0.742	0.742	5.54%	-7.62%	0.775	0.794	0.824	2.53%	6.33%	0.678	0.703	0.703	3.75%	3.75%
24	0.782	0.812	0.782	-5.80%	-0.56%	0.737	0.763	0.728	3.57%	-1.19%	0.778	0.778	0.769	0.00%	-1.19%
25	0.795	0.795	0.777	-9.77%	-13.14%	0.717	0.717	0.755	0.00%	5.26%	0.690	0.667	0.675	-3.42%	-2.27%
26	0.713	0.722	0.722	6.44%	-11.96%	0.759	0.795	0.786	4.71%	3.53%	0.628	0.642	0.628	2.33%	0.00%
27	0.776	0.766	0.776	-1.28%	-15.75%	0.766	0.712	0.730	-7.06%	-4.71%	0.654	0.685	0.677	4.82%	3.61%
28	0.771	0.831	0.814	-19.32%	-1.55%	0.622	0.667	0.622	7.14%	0.00%	0.759	0.787	0.750	3.66%	-1.22%
29	0.771	0.790	0.781	0.40%	-17.99%	0.775	0.765	0.745	-1.27%	-3.80%	0.633	0.633	0.626	0.00%	-1.08%
30	0.722	0.741	0.731	4.14%	-7.33%	0.752	0.761	0.761	1.14%	1.14%	0.669	0.693	0.709	3.53%	5.88%
31	0.661	0.661	0.677	-5.24%	8.19%	0.627	0.641	0.620	2.25%	-1.12%	0.716	0.743	0.725	3.85%	1.28%
32	0.835	0.807	0.817	-19.52%	-12.97%	0.672	0.656	0.672	-2.33%	0.00%	0.727	0.711	0.695	-2.15%	-4.30%
33	0.623	0.659	0.623	26.77%	15.00%	0.790	0.830	0.800	5.06%	1.27%	0.717	0.708	0.725	-1.16%	1.16%
34	0.752	0.776	0.744	-9.98%	5.42%	0.677	0.692	0.708	2.27%	4.55%	0.793	0.793	0.793	0.00%	0.00%
35	0.669	0.677	0.669	9.86%	6.54%	0.735	0.765	0.755	4.00%	2.67%	0.713	0.704	0.704	-1.22%	-1.22%
36	0.733	0.700	0.708	-7.39%	-18.60%	0.679	0.649	0.709	-4.40%	4.40%	0.597	0.620	0.605	3.90%	1.30%
37	0.824	0.815	0.815	-5.75%	-5.08%	0.777	0.796	0.825	2.50%	6.25%	0.782	0.812	0.772	3.80%	-1.27%
38	0.627	0.664	0.672	-8.25%	15.21%	0.575	0.588	0.588	2.27%	2.27%	0.722	0.746	0.746	3.30%	3.30%
39	0.664	0.672	0.657	-3.29%	-5.60%	0.642	0.664	0.620	3.41%	-3.41%	0.627	0.635	0.643	1.27%	2.53%
40	0.721	0.746	0.689	0.00%	-6.69%	0.721	0.721	0.721	0.00%	0.00%	0.673	0.673	0.673	0.00%	0.00%

Table A.3: Repositioning: Profits of Region Type 2

Index	$\rho = 1$					$\rho = 2$					$\rho = 3$				
	OPT	REPO	FREPO	% (REPO)	% (FREPO)	OPT	REPO	FREPO	% (REPO)	% (FREPO)	OPT	REPO	FREPO	% (REPO)	% (FREPO)
1	157.63	156.19	159.46	-0.92%	1.16%	155.43	140.67	149.00	-9.50%	-4.14%	146.44	140.18	141.75	-4.27%	-3.20%
2	151.99	159.44	158.27	4.90%	4.13%	164.28	150.24	155.83	-8.55%	-5.14%	132.46	130.87	134.46	-1.19%	1.52%
3	145.95	141.90	140.99	-2.77%	-3.40%	155.37	154.65	159.46	-0.46%	2.63%	140.83	137.43	145.26	-2.41%	3.15%
4	161.30	154.92	160.97	-3.96%	-0.21%	160.74	160.82	161.28	0.05%	0.33%	151.42	151.75	156.28	0.22%	3.21%
5	140.63	138.44	138.52	-1.55%	-1.50%	146.79	142.04	154.44	-3.24%	5.21%	146.36	145.56	140.52	-0.55%	-3.99%
6	149.73	143.94	146.04	-3.87%	-2.47%	150.82	144.77	145.32	-4.01%	-3.65%	169.60	168.92	166.42	-0.40%	-1.88%
7	158.24	159.53	163.33	0.81%	3.21%	152.10	141.94	148.48	-6.68%	-2.38%	146.76	153.61	151.54	4.67%	3.26%
8	163.78	157.64	165.56	-3.75%	1.09%	135.21	133.43	133.47	-1.32%	-1.29%	160.86	159.22	161.32	-1.02%	0.29%
9	163.28	165.32	172.89	1.25%	5.89%	153.64	153.99	153.79	0.23%	0.10%	149.70	148.14	149.74	-1.05%	0.03%
10	145.81	147.34	152.66	1.05%	4.70%	143.08	147.72	145.23	3.24%	1.50%	155.80	146.17	151.78	-6.18%	-2.58%
11	170.52	169.52	168.68	-0.58%	-1.08%	156.80	149.43	146.54	-4.70%	-6.54%	135.93	133.36	130.51	-1.89%	-3.99%
12	146.86	150.72	149.06	2.63%	1.50%	163.58	157.78	158.95	-3.54%	-2.83%	151.93	145.65	149.67	-4.13%	-1.49%
13	148.49	144.74	143.75	-2.52%	-3.19%	158.17	147.73	151.83	-6.60%	-4.01%	154.02	153.07	151.25	-0.62%	-1.80%
14	155.42	145.01	146.13	-6.70%	-5.98%	161.71	164.41	154.61	1.67%	-4.39%	145.47	142.76	143.74	-1.87%	-1.19%
15	110.44	133.12	139.32	20.54%	26.15%	134.32	129.55	136.49	-3.55%	1.62%	156.63	155.16	160.08	-0.94%	2.20%
16	164.76	163.18	161.26	-0.96%	-2.13%	154.18	156.50	154.79	1.50%	0.40%	138.71	130.81	136.02	-5.69%	-1.94%
17	157.33	148.26	157.11	-5.77%	-0.14%	151.60	140.87	150.05	-7.08%	-1.02%	140.74	141.62	142.12	0.62%	0.98%
18	139.96	140.88	140.74	0.65%	0.56%	150.16	137.87	146.34	-8.18%	-2.54%	142.94	135.77	145.35	-5.02%	1.69%
19	152.26	141.06	147.58	-7.35%	-3.07%	165.54	154.17	168.26	-6.87%	1.64%	150.98	143.91	149.56	-4.68%	-0.94%
20	140.79	140.61	140.17	-0.13%	-0.44%	163.05	162.25	159.73	-0.49%	-2.04%	162.99	165.06	162.76	1.27%	-0.15%
21	168.18	165.15	162.25	-1.80%	-3.53%	149.27	143.26	144.38	-4.03%	-3.28%	142.20	123.09	129.66	-13.44%	-8.82%
22	167.42	156.94	172.97	-6.26%	3.31%	137.12	124.80	132.43	-8.98%	-3.42%	146.59	127.63	129.84	-12.93%	-11.43%
23	154.09	150.18	145.68	-2.54%	-5.45%	160.25	147.56	155.57	-7.92%	-2.92%	150.94	144.62	147.06	-4.19%	-2.57%
24	142.43	151.44	142.79	6.32%	0.25%	170.46	164.68	165.08	-3.39%	-3.16%	171.20	162.24	162.53	-5.23%	-5.06%
25	145.01	143.48	142.29	-1.06%	-1.87%	162.36	157.97	162.39	-2.70%	0.02%	163.72	159.36	164.18	-2.66%	0.28%
26	143.27	145.55	142.05	1.59%	-0.85%	130.00	126.10	129.90	-3.00%	-0.08%	127.35	116.26	127.91	-8.71%	0.44%
27	139.42	125.88	133.03	-9.71%	-4.58%	146.22	149.62	146.22	2.33%	0.00%	137.53	140.83	143.55	2.40%	4.37%
28	162.14	163.42	163.11	0.79%	0.60%	162.52	150.26	156.64	-7.54%	-3.62%	155.50	155.31	158.28	-0.12%	1.79%
29	141.63	138.21	146.12	-2.42%	3.17%	147.82	148.51	140.66	0.47%	-4.84%	143.93	151.53	146.74	5.28%	1.95%
30	158.70	160.20	162.01	0.94%	2.08%	137.59	124.58	133.60	-9.45%	-2.90%	152.79	144.52	149.09	-5.41%	-2.42%
31	151.27	145.90	154.95	-3.55%	2.44%	162.02	158.05	159.82	-2.45%	-1.36%	155.76	158.50	159.01	1.76%	2.09%
32	160.76	157.15	159.52	-2.25%	-0.77%	129.78	127.83	128.05	-1.50%	-1.33%	163.40	160.93	154.87	-1.51%	-5.23%
33	156.17	158.14	151.37	1.26%	-3.08%	138.30	128.53	138.38	-7.06%	0.06%	161.70	156.48	148.38	-3.23%	-8.24%
34	162.47	157.65	154.70	-2.96%	-4.78%	145.73	146.64	136.06	0.62%	-6.64%	140.48	133.58	132.69	-4.91%	-5.54%
35	161.71	161.94	162.33	0.14%	0.38%	152.46	155.17	155.38	1.77%	1.91%	156.36	124.37	150.72	-20.46%	-3.60%
36	156.62	151.39	150.25	-3.34%	-4.07%	157.45	155.34	155.08	-1.34%	-1.51%	141.48	136.11	140.97	-3.79%	-0.36%
37	146.67	141.93	145.91	-3.23%	-0.52%	157.95	149.57	155.74	-5.31%	-1.40%	158.60	152.72	159.86	-3.71%	0.80%
38	166.87	171.47	169.71	2.76%	1.70%	161.78	159.12	160.36	-1.64%	-0.88%	159.01	147.42	151.79	-7.29%	-4.54%
39	163.93	151.32	165.37	-7.69%	0.88%	165.79	164.99	170.56	-0.48%	2.88%	148.10	136.99	138.17	-7.50%	-6.70%
40	161.19	162.99	154.35	1.12%	-4.24%	144.72	135.66	142.25	-6.25%	-1.71%	147.84	152.64	143.11	3.24%	-3.20%

Table A.4: Repositioning: Acceptance Rates of Region Type 2

Index	$\rho = 1$					$\rho = 2$					$\rho = 3$				
	OPT	REPO	FREPO	% (REPO)	% (FREPO)	OPT	REPO	FREPO	% (REPO)	% (FREPO)	OPT	REPO	FREPO	% (REPO)	% (FREPO)
1	0.680	0.713	0.680	3.26%	-4.29%	0.702	0.694	0.702	-1.18%	0.00%	0.651	0.667	0.643	2.38%	-1.19%
2	0.765	0.756	0.782	2.75%	-6.08%	0.786	0.750	0.768	-4.55%	-2.27%	0.718	0.727	0.736	1.27%	2.53%
3	0.761	0.789	0.761	-7.24%	-6.20%	0.706	0.722	0.706	2.25%	0.00%	0.714	0.732	0.714	2.50%	0.00%
4	0.727	0.711	0.758	-16.67%	3.79%	0.605	0.626	0.619	3.37%	2.25%	0.754	0.754	0.754	0.00%	0.00%
5	0.771	0.807	0.798	3.56%	-4.39%	0.798	0.817	0.837	2.41%	4.82%	0.737	0.763	0.728	3.57%	-1.19%
6	0.808	0.828	0.808	-21.59%	-11.18%	0.634	0.672	0.634	6.02%	0.00%	0.718	0.742	0.726	3.37%	1.12%
7	0.597	0.597	0.631	36.30%	22.29%	0.814	0.814	0.823	0.00%	1.09%	0.730	0.765	0.739	4.76%	1.19%
8	0.762	0.754	0.738	-7.40%	-15.18%	0.706	0.731	0.697	3.57%	-1.19%	0.647	0.684	0.662	5.81%	2.33%
9	0.675	0.706	0.722	4.49%	4.27%	0.705	0.721	0.713	2.33%	1.16%	0.703	0.746	0.720	6.02%	2.41%
10	0.641	0.656	0.664	35.83%	8.06%	0.871	0.925	0.903	6.17%	3.70%	0.693	0.685	0.709	-1.14%	2.27%
11	0.766	0.766	0.758	-0.83%	13.80%	0.759	0.750	0.731	-1.22%	-3.66%	0.871	0.911	0.842	4.55%	-3.41%
12	0.750	0.806	0.778	-11.45%	2.75%	0.664	0.664	0.649	0.00%	-2.30%	0.771	0.761	0.771	-1.19%	0.00%
13	0.712	0.748	0.730	-3.75%	-7.73%	0.685	0.693	0.693	1.15%	1.15%	0.657	0.694	0.664	5.68%	1.14%
14	0.624	0.624	0.617	19.21%	16.65%	0.744	0.760	0.736	2.15%	-1.08%	0.728	0.720	0.728	-1.10%	0.00%
15	0.478	0.669	0.669	41.28%	61.36%	0.675	0.709	0.667	5.06%	-1.27%	0.771	0.763	0.763	-1.10%	-1.10%
16	0.651	0.644	0.630	1.69%	18.56%	0.662	0.669	0.677	1.14%	2.27%	0.771	0.790	0.771	2.47%	0.00%
17	0.741	0.731	0.741	-8.51%	7.24%	0.678	0.653	0.669	-3.66%	-1.22%	0.794	0.822	0.804	3.53%	1.18%
18	0.734	0.766	0.742	-8.48%	-15.84%	0.672	0.657	0.649	-2.22%	-3.33%	0.618	0.632	0.647	2.38%	4.76%
19	0.883	0.862	0.872	-25.58%	-20.90%	0.657	0.636	0.679	-3.26%	3.26%	0.698	0.667	0.667	-4.55%	-4.55%
20	0.694	0.712	0.694	2.16%	6.65%	0.709	0.724	0.717	2.22%	1.11%	0.740	0.764	0.756	3.30%	2.20%
21	0.707	0.722	0.714	-3.59%	8.43%	0.681	0.717	0.690	5.19%	1.30%	0.766	0.729	0.738	-4.88%	-3.66%
22	0.656	0.672	0.695	16.74%	-0.50%	0.766	0.748	0.748	-2.44%	-2.44%	0.653	0.613	0.621	-6.17%	-4.94%
23	0.734	0.742	0.742	-3.95%	-1.24%	0.705	0.697	0.721	-1.16%	2.33%	0.725	0.725	0.743	0.00%	2.53%
24	0.782	0.812	0.782	-4.62%	-22.62%	0.746	0.762	0.746	2.13%	0.00%	0.605	0.605	0.605	0.00%	0.00%
25	0.795	0.795	0.777	-14.54%	-8.94%	0.679	0.687	0.694	1.10%	2.20%	0.724	0.732	0.764	1.12%	5.62%
26	0.713	0.722	0.722	6.99%	1.61%	0.763	0.753	0.753	-1.35%	-1.35%	0.724	0.755	0.755	4.23%	4.23%
27	0.776	0.766	0.776	-6.34%	3.38%	0.726	0.761	0.701	4.71%	-3.53%	0.802	0.830	0.840	3.53%	4.71%
28	0.771	0.831	0.814	-12.25%	2.13%	0.677	0.684	0.692	1.11%	2.22%	0.788	0.779	0.788	-1.12%	0.00%
29	0.771	0.790	0.781	-6.20%	-4.65%	0.724	0.732	0.691	1.12%	-4.49%	0.736	0.760	0.760	3.37%	3.37%
30	0.722	0.741	0.731	-4.04%	1.31%	0.693	0.683	0.703	-1.43%	1.43%	0.732	0.724	0.740	-1.11%	1.11%
31	0.661	0.661	0.677	8.35%	14.01%	0.717	0.708	0.717	-1.16%	0.00%	0.754	0.787	0.779	4.35%	3.26%
32	0.835	0.807	0.817	-11.93%	-16.87%	0.735	0.745	0.725	1.33%	-1.33%	0.694	0.701	0.679	1.08%	-2.15%
33	0.623	0.659	0.623	13.43%	-2.01%	0.707	0.716	0.716	1.22%	1.22%	0.611	0.641	0.611	5.00%	0.00%
34	0.752	0.776	0.744	-5.86%	1.62%	0.708	0.743	0.681	5.00%	-3.75%	0.764	0.755	0.764	-1.23%	0.00%
35	0.669	0.677	0.669	4.72%	19.27%	0.701	0.744	0.735	6.10%	4.88%	0.798	0.675	0.763	-15.38%	-4.40%
36	0.733	0.700	0.708	18.52%	3.58%	0.869	0.888	0.869	2.15%	0.00%	0.760	0.779	0.769	2.53%	1.27%
37	0.824	0.815	0.815	-10.48%	-15.06%	0.738	0.746	0.738	1.11%	0.00%	0.700	0.746	0.708	6.59%	1.10%
38	0.627	0.664	0.672	5.96%	11.41%	0.664	0.650	0.657	-2.20%	-1.10%	0.698	0.667	0.698	-4.55%	0.00%
39	0.664	0.672	0.657	1.09%	2.95%	0.671	0.693	0.693	3.19%	3.19%	0.684	0.684	0.658	0.00%	-3.75%
40	0.721	0.746	0.689	-5.27%	3.35%	0.683	0.675	0.683	-1.22%	0.00%	0.745	0.773	0.745	3.66%	0.00%

Table A.5: Postponement: Profits at $\lambda_0 = 0.010$

Index	$\mu = 0.25$					$\mu = 0.50$					$\mu = 0.75$					$\mu = 1.00$				
	OPT3	POS (0)	POS (0.1)	% (0)	% (0.1)	OPT3	POS (0)	POS (0.1)	% (0)	% (0.1)	OPT3	POS (0)	POS (0.1)	% (0)	% (0.1)	OPT3	POS (0)	POS (0.1)	% (0)	% (0.1)
1	98.31	98.31	96.48	0.00%	-1.87%	85.45	85.45	84.81	0.00%	-0.74%	80.78	80.78	79.49	0.00%	-1.59%	82.90	82.90	80.53	0.00%	-2.87%
2	90.44	90.44	90.66	0.00%	0.24%	71.48	76.79	75.43	7.42%	5.52%	70.80	70.80	70.61	0.00%	-0.27%	66.85	69.48	69.90	3.93%	4.56%
3	69.01	69.01	68.96	0.00%	-0.08%	81.88	84.04	85.01	2.64%	3.82%	88.91	88.91	89.84	0.00%	1.04%	91.18	91.18	91.16	0.00%	-0.02%
4	61.89	62.33	60.74	0.71%	-1.87%	92.23	92.02	92.33	-0.22%	0.11%	93.21	96.97	95.04	4.04%	1.97%	74.53	74.53	77.83	0.00%	4.43%
5	96.90	96.90	97.58	0.00%	0.70%	126.31	126.31	118.89	0.00%	-5.87%	87.59	86.49	88.83	-1.25%	1.42%	59.75	66.37	64.40	11.08%	7.79%
6	85.61	85.33	85.99	-0.32%	0.45%	73.45	73.45	72.91	0.00%	-0.74%	105.64	105.64	106.44	0.00%	0.75%	71.47	73.34	71.80	2.63%	0.47%
7	74.13	73.94	72.13	-0.26%	-2.71%	92.95	92.71	93.07	-0.26%	0.12%	62.29	68.72	64.64	10.33%	3.78%	83.27	84.56	82.34	1.55%	-1.12%
8	78.95	78.95	79.83	0.00%	1.13%	64.73	64.73	64.88	0.00%	0.23%	106.74	108.14	106.98	1.32%	0.22%	76.86	79.26	78.09	3.11%	1.59%
9	120.65	120.65	120.61	0.00%	-0.04%	93.13	94.04	94.26	0.97%	1.21%	65.68	66.95	67.68	1.93%	3.04%	88.73	88.47	91.36	-0.29%	2.96%
10	75.65	75.65	76.29	0.00%	0.84%	73.94	80.37	80.32	8.69%	8.63%	76.55	76.55	76.56	0.00%	0.01%	81.82	81.82	81.85	0.00%	0.04%
11	96.70	96.70	99.93	0.00%	3.34%	112.84	112.84	113.46	0.00%	0.55%	69.69	73.04	70.74	4.81%	1.51%	82.72	82.72	84.20	0.00%	1.79%
12	87.82	87.82	87.13	0.00%	-0.78%	65.98	67.29	68.56	1.99%	3.92%	97.57	101.26	100.85	3.79%	3.37%	96.66	97.95	97.31	1.34%	0.68%
13	56.85	60.10	60.06	5.71%	5.65%	69.51	71.63	72.18	3.05%	3.83%	83.87	84.67	85.64	0.95%	2.11%	81.49	81.49	81.34	0.00%	-0.18%
14	70.06	70.06	69.96	0.00%	-0.14%	79.13	81.33	79.01	2.78%	-0.15%	121.76	127.63	127.99	4.82%	5.12%	78.11	78.11	78.25	0.00%	0.17%
15	98.80	98.80	98.93	0.00%	0.13%	84.01	84.01	84.45	0.00%	0.51%	77.03	77.03	76.17	0.00%	-1.11%	87.24	87.71	86.34	0.54%	-1.04%
16	78.53	78.53	78.16	0.00%	-0.47%	73.47	73.47	74.21	0.00%	1.00%	91.06	91.06	90.82	0.00%	-0.26%	115.47	120.54	114.19	4.38%	-1.11%
17	105.54	105.54	107.51	0.00%	1.87%	59.63	59.63	59.58	0.00%	-0.08%	52.87	52.82	52.97	-0.09%	0.19%	89.49	89.49	89.53	0.00%	0.04%
18	58.15	58.15	58.31	0.00%	0.26%	68.85	67.82	68.80	-1.49%	-0.07%	77.83	81.91	79.85	5.25%	2.60%	62.25	62.25	62.49	0.00%	0.39%
19	96.68	95.80	96.89	-0.91%	0.22%	72.26	72.87	74.01	0.84%	2.43%	71.40	70.27	71.12	-1.59%	-0.40%	75.69	79.73	76.16	5.33%	0.61%
20	55.65	55.65	58.56	0.00%	5.22%	66.10	66.10	68.82	0.00%	4.12%	33.17	34.72	34.72	4.67%	4.67%	66.58	73.00	72.00	9.66%	8.15%
21	79.24	79.24	79.26	0.00%	0.02%	97.12	100.98	101.32	3.98%	4.33%	82.69	82.69	80.78	0.00%	-2.31%	114.60	114.42	115.67	-0.15%	0.94%
22	86.20	86.20	86.27	0.00%	0.08%	109.32	112.24	111.53	2.67%	2.02%	84.75	86.57	82.96	2.16%	-2.10%	83.71	83.71	83.60	0.00%	-0.13%
23	108.00	108.00	108.72	0.00%	0.67%	88.20	88.13	87.62	-0.08%	-0.66%	68.82	68.82	69.32	0.00%	0.71%	72.39	72.39	74.60	0.00%	3.06%
24	88.17	88.17	87.96	0.00%	-0.24%	69.21	69.21	69.62	0.00%	0.60%	72.69	72.69	72.85	0.00%	0.22%	105.88	106.12	107.28	0.22%	1.32%
25	87.98	87.98	87.79	0.00%	-0.21%	83.54	83.54	84.08	0.00%	0.65%	98.66	98.66	98.50	0.00%	-0.15%	93.59	93.59	91.26	0.00%	-2.49%
26	84.84	87.34	87.80	2.94%	3.49%	80.88	80.88	80.88	0.00%	0.00%	81.29	81.29	81.78	0.00%	0.59%	74.09	75.82	75.16	2.34%	1.45%
27	67.08	67.08	67.07	0.00%	-0.02%	75.35	75.35	75.25	0.00%	-0.14%	64.55	64.55	64.08	0.00%	-0.72%	52.31	52.31	52.37	0.00%	0.12%
28	86.07	86.81	85.69	0.86%	-0.44%	75.19	75.19	76.38	0.00%	1.58%	64.82	64.82	66.15	0.00%	2.04%	61.40	65.49	65.43	6.67%	6.56%
29	86.36	86.36	86.22	0.00%	-0.16%	81.43	81.43	82.70	0.00%	1.56%	66.56	69.62	70.35	4.60%	5.71%	81.36	81.36	80.91	0.00%	-0.55%
30	98.21	98.21	100.81	0.00%	2.65%	91.55	88.72	90.16	-3.09%	-1.52%	80.33	82.71	84.23	2.96%	4.86%	76.68	81.59	79.44	6.41%	3.60%
31	68.41	70.08	70.28	2.45%	2.74%	70.10	71.29	66.87	1.70%	-4.61%	59.64	59.64	60.12	0.00%	0.80%	103.92	103.92	102.03	0.00%	-1.82%
32	97.71	97.71	97.99	0.00%	0.29%	107.64	107.64	107.16	0.00%	-0.45%	84.82	78.67	85.75	-7.25%	1.10%	87.13	87.13	86.96	0.00%	-0.18%
33	87.99	87.99	88.57	0.00%	0.66%	84.29	85.60	83.84	1.55%	-0.54%	75.85	76.09	72.95	0.32%	-3.83%	103.87	103.87	108.33	0.00%	4.30%
34	87.45	87.45	87.26	0.00%	-0.22%	87.14	87.14	87.06	0.00%	-0.09%	78.03	78.76	78.94	0.94%	1.17%	76.62	76.62	77.69	0.00%	1.40%
35	79.54	79.54	80.27	0.00%	0.92%	86.93	86.93	87.20	0.00%	0.32%	87.02	88.88	86.86	2.14%	-0.18%	85.12	89.14	90.99	4.72%	6.89%
36	96.94	96.94	96.98	0.00%	0.03%	68.97	68.97	69.19	0.00%	0.31%	74.73	78.20	78.41	4.65%	4.93%	102.99	103.97	105.36	0.95%	2.30%
37	106.52	108.11	108.25	1.50%	1.63%	77.30	79.80	79.67	3.22%	3.06%	54.68	54.68	55.96	0.00%	2.33%	88.94	95.09	95.45	6.92%	7.32%
38	96.65	96.65	96.65	0.00%	0.00%	95.23	95.23	95.38	0.00%	0.16%	62.94	64.62	62.25	2.68%	-1.08%	83.00	93.02	91.63	12.08%	10.40%
39	93.71	93.71	94.45	0.00%	0.79%	63.15	64.78	63.90	2.58%	1.20%	81.62	87.29	86.45	6.94%	5.92%	68.35	68.35	68.29	0.00%	-0.08%
40	90.15	89.60	87.72	-0.61%	-2.69%	79.68	79.46	79.64	-0.28%	-0.06%	80.08	84.33	84.31	5.31%	5.29%	93.82	96.75	97.34	3.12%	3.76%

Table A.6: Postponement: Profits at $\lambda_0 = 0.015$

Index	$\mu = 0.25$					$\mu = 0.50$					$\mu = 0.75$					$\mu = 1.00$				
	OPT3	POS (0)	POS (0.1)	% (0)	% (0.1)	OPT3	POS (0)	POS (0.1)	% (0)	% (0.1)	OPT3	POS (0)	POS (0.1)	% (0)	% (0.1)	OPT3	POS (0)	POS (0.1)	% (0)	% (0.1)
1	123.80	126.26	123.45	1.98%	-0.28%	113.50	118.75	122.48	4.62%	7.91%	107.66	110.27	110.20	2.43%	2.36%	132.74	145.98	143.09	9.97%	7.80%
2	108.92	112.30	117.01	3.10%	7.43%	115.08	115.08	111.14	0.00%	-3.42%	143.33	141.60	143.90	-1.21%	0.39%	103.13	123.63	118.04	19.88%	14.46%
3	108.70	109.41	112.44	0.65%	3.44%	139.16	135.36	139.97	-2.72%	0.58%	112.25	125.30	123.31	11.63%	9.85%	108.69	115.39	116.93	6.17%	7.58%
4	118.98	126.16	122.59	6.03%	3.03%	144.92	141.07	151.98	-2.66%	4.87%	121.75	125.68	122.25	3.23%	0.41%	105.34	115.81	116.02	9.94%	10.14%
5	118.43	119.56	118.42	0.96%	0.00%	111.67	115.07	112.42	3.05%	0.67%	125.96	137.22	136.72	8.93%	8.54%	123.19	125.86	132.18	2.17%	7.29%
6	132.96	132.96	131.75	0.00%	-0.91%	132.96	134.67	132.62	1.29%	-0.26%	127.96	130.83	129.53	2.24%	1.22%	109.34	116.62	115.78	6.66%	5.89%
7	133.12	134.33	135.51	0.91%	1.80%	148.55	146.05	152.06	-1.68%	2.37%	154.40	153.26	157.62	-0.74%	2.08%	124.93	131.47	127.13	5.23%	1.75%
8	136.51	137.38	133.01	0.63%	-2.57%	115.34	117.22	115.30	1.63%	-0.04%	120.15	123.00	122.50	2.37%	1.96%	119.39	123.48	126.06	3.42%	5.58%
9	162.64	164.23	161.40	0.97%	-0.76%	132.50	133.42	135.50	0.70%	2.27%	118.11	129.53	121.54	9.68%	2.91%	97.16	111.74	109.44	15.00%	12.64%
10	143.56	137.65	141.64	-4.12%	-1.34%	127.33	133.04	137.98	4.48%	8.36%	90.22	97.98	98.82	8.60%	9.54%	115.27	120.57	116.08	4.60%	0.71%
11	130.19	128.34	128.46	-1.42%	-1.33%	129.93	140.45	139.07	8.10%	7.03%	98.27	101.43	102.69	3.21%	4.49%	131.51	135.12	137.75	2.75%	4.75%
12	143.14	154.66	149.22	8.05%	4.25%	122.19	122.19	119.43	0.00%	-2.25%	132.57	132.35	130.57	-0.16%	-1.51%	129.86	134.34	140.82	3.45%	8.44%
13	135.40	135.40	134.97	0.00%	-0.31%	125.82	127.20	127.18	1.10%	1.08%	116.37	129.09	120.65	10.94%	3.68%	108.64	109.12	111.66	0.44%	2.77%
14	130.23	131.64	137.80	1.08%	5.82%	117.84	121.01	125.27	2.69%	6.30%	154.30	153.59	152.47	-0.46%	-1.19%	129.66	129.66	123.40	0.00%	-4.83%
15	104.11	102.23	105.66	-1.81%	1.48%	110.69	119.55	117.24	8.01%	5.92%	114.47	148.25	134.92	29.51%	17.86%	124.01	125.39	131.26	1.11%	5.85%
16	147.92	148.24	147.13	0.21%	-0.53%	127.55	132.32	130.47	3.74%	2.29%	116.15	117.08	122.82	0.80%	5.74%	122.57	125.40	119.98	2.31%	-2.12%
17	89.32	90.26	90.68	1.05%	1.52%	132.29	133.36	135.53	0.81%	2.45%	105.75	118.93	115.59	12.46%	9.31%	125.50	127.85	128.40	1.87%	2.31%
18	90.14	90.14	93.89	0.00%	4.15%	87.05	87.05	85.90	0.00%	-1.32%	114.66	118.39	117.43	3.25%	2.42%	86.99	108.09	95.96	24.26%	10.32%
19	113.54	117.61	119.25	3.59%	5.03%	104.44	103.64	105.02	-0.77%	0.56%	119.08	119.08	113.09	0.00%	-5.03%	124.92	126.05	127.12	0.90%	1.76%
20	122.09	122.09	123.59	0.00%	1.22%	129.76	128.52	126.81	-0.95%	-2.28%	128.97	128.00	128.12	-0.75%	-0.66%	116.65	121.03	121.76	3.75%	4.38%
21	116.46	116.46	117.88	0.00%	1.22%	115.27	121.77	115.18	5.64%	-0.07%	114.71	119.07	116.59	3.79%	1.63%	121.03	116.76	114.49	-3.52%	-5.41%
22	108.36	111.90	112.82	3.27%	4.12%	93.35	98.77	99.01	5.81%	6.07%	129.41	135.07	134.72	4.37%	4.11%	101.20	103.76	105.04	2.53%	3.80%
23	121.70	124.60	125.09	2.38%	2.78%	118.29	127.28	124.70	7.60%	5.41%	115.60	118.78	120.57	2.76%	4.30%	119.31	124.80	127.77	4.60%	7.09%
24	135.83	131.37	137.92	-3.28%	1.53%	99.35	99.35	109.22	0.00%	9.94%	91.31	103.36	105.64	13.19%	15.69%	131.05	137.15	136.97	4.65%	4.51%
25	123.14	125.47	125.88	1.89%	2.23%	113.00	116.62	115.18	3.21%	1.93%	134.35	129.58	132.61	-3.55%	-1.29%	134.90	135.16	134.55	0.19%	-0.26%
26	101.92	108.56	106.85	6.51%	4.83%	124.89	121.97	122.12	-2.34%	-2.22%	119.55	118.19	119.55	-1.14%	0.00%	117.01	130.17	132.48	11.25%	13.22%
27	128.76	133.32	126.70	3.54%	-1.60%	137.24	134.93	134.29	-1.68%	-2.15%	137.06	137.34	135.34	0.21%	-1.25%	125.99	128.57	127.85	2.04%	1.48%
28	95.75	98.42	100.66	2.79%	5.13%	137.21	137.21	133.84	0.00%	-2.45%	133.95	133.95	133.99	0.00%	0.03%	116.68	127.40	127.49	9.18%	9.26%
29	133.94	135.83	141.25	1.41%	5.46%	109.49	113.82	111.66	3.95%	1.99%	123.23	133.73	135.60	8.52%	10.03%	108.71	120.10	119.40	10.48%	9.84%
30	137.33	139.17	139.82	1.34%	1.81%	113.50	113.50	113.54	0.00%	0.04%	133.20	137.45	135.48	3.19%	1.71%	133.01	134.29	134.77	0.97%	1.32%
31	140.54	146.28	146.03	4.08%	3.91%	144.15	143.00	142.71	-0.79%	-0.99%	122.83	130.15	134.38	5.96%	9.40%	101.51	104.99	100.12	3.42%	-1.37%
32	119.51	118.16	121.97	-1.13%	2.05%	114.67	123.40	123.63	7.61%	7.81%	108.88	114.99	115.74	5.61%	6.29%	121.23	133.83	128.88	10.40%	6.31%
33	121.61	124.89	126.31	2.69%	3.86%	127.72	127.72	126.63	0.00%	-0.86%	124.62	125.42	126.99	0.64%	1.90%	103.25	107.29	103.47	3.91%	0.21%
34	141.77	140.64	145.64	-0.80%	2.73%	109.08	108.56	111.39	-0.48%	2.12%	118.78	120.09	120.32	1.10%	1.30%	105.69	115.87	117.88	9.63%	11.53%
35	118.57	118.57	118.62	0.00%	0.04%	116.75	127.85	127.46	9.50%	9.17%	119.47	119.47	121.44	0.00%	1.65%	141.29	141.63	141.10	0.24%	-0.13%
36	127.14	129.33	130.84	1.73%	2.91%	112.57	117.25	115.99	4.16%	3.04%	104.80	120.35	118.45	14.84%	13.02%	83.27	84.22	83.77	1.14%	0.60%
37	149.56	147.56	148.30	-1.34%	-0.84%	121.17	128.81	127.85	6.31%	5.52%	129.41	132.39	134.20	2.31%	3.70%	151.50	163.16	163.96	7.69%	8.22%
38	121.92	121.92	117.76	0.00%	-3.41%	123.66	123.66	119.81	0.00%	-3.11%	119.03	123.33	127.56	3.62%	7.17%	130.44	131.21	131.01	0.59%	0.44%
39	103.38	109.28	103.59	5.70%	0.20%	118.31	118.38	120.72	0.06%	2.04%	127.40	129.86	123.04	1.93%	-3.42%	102.49	113.03	113.12	10.28%	10.37%
40	98.23	99.24	99.37	1.03%	1.17%	140.93	146.85	145.60	4.21%	3.31%	137.01	137.41	137.50	0.29%	0.35%	120.34	125.55	127.16	4.34%	5.67%

Table A.7: Postponement: Profits at $\lambda_0 = 0.019$

Index	$\mu = 0.25$					$\mu = 0.50$					$\mu = 0.75$					$\mu = 1.00$				
	OPT3	POS (0)	POS (0.1)	% (0)	% (0.1)	OPT3	POS (0)	POS (0.1)	% (0)	% (0.1)	OPT3	POS (0)	POS (0.1)	% (0)	% (0.1)	OPT3	POS (0)	POS (0.1)	% (0)	% (0.1)
1	141.43	133.05	136.88	-5.93%	-3.22%	138.07	141.48	144.04	2.47%	4.32%	151.94	152.63	150.16	0.45%	-1.17%	130.57	142.15	130.67	8.87%	0.08%
2	121.25	138.94	134.49	14.60%	10.92%	116.40	122.73	126.97	5.43%	9.09%	148.32	148.49	151.00	0.11%	1.81%	99.61	140.46	130.49	41.01%	31.01%
3	159.91	154.61	163.00	-3.31%	1.93%	132.55	136.82	136.25	3.22%	2.79%	159.63	166.53	159.29	4.32%	-0.21%	158.07	159.35	160.68	0.81%	1.66%
4	164.86	161.60	162.94	-1.98%	-1.16%	136.51	142.97	146.27	4.73%	7.15%	122.75	132.27	132.29	7.75%	7.77%	137.35	150.19	141.12	9.35%	2.75%
5	137.42	137.82	141.29	0.30%	2.82%	120.14	130.32	122.30	8.47%	1.80%	159.15	157.55	161.11	-1.01%	1.23%	138.01	155.67	155.37	12.79%	12.58%
6	141.03	150.18	145.33	6.48%	3.05%	133.58	139.34	141.30	4.31%	5.78%	153.26	154.57	150.51	0.85%	-1.79%	127.20	135.06	127.06	6.18%	-0.10%
7	150.76	151.67	151.76	0.60%	0.66%	137.23	140.37	138.80	2.29%	1.14%	130.30	135.06	137.00	3.65%	5.14%	146.33	145.68	146.52	-0.45%	0.13%
8	143.19	146.50	143.59	2.31%	0.28%	130.92	140.06	139.15	6.98%	6.29%	129.81	138.03	144.95	6.33%	11.66%	117.64	120.98	117.23	2.83%	-0.35%
9	133.73	136.93	138.43	2.39%	3.52%	137.58	142.28	140.97	3.41%	2.46%	133.76	153.07	150.25	14.43%	12.33%	142.32	143.39	142.09	0.75%	-0.16%
10	141.75	146.13	138.26	3.09%	-2.46%	127.40	138.12	134.81	8.41%	5.81%	152.33	155.56	151.21	2.12%	-0.73%	131.29	131.37	135.26	0.06%	3.02%
11	153.52	158.95	158.69	3.54%	3.36%	145.89	146.94	145.95	0.72%	0.04%	137.55	145.66	151.67	5.90%	10.27%	146.57	148.42	145.89	1.26%	-0.47%
12	142.38	133.62	145.06	-6.16%	1.88%	143.25	149.15	144.98	4.11%	1.21%	127.18	128.30	131.07	0.88%	3.06%	145.69	146.49	141.08	0.55%	-3.16%
13	149.65	152.81	158.15	2.11%	5.68%	145.25	152.56	158.15	5.03%	8.88%	137.63	147.66	153.68	7.29%	11.66%	136.14	149.25	147.40	9.63%	8.27%
14	135.44	138.55	134.37	2.30%	-0.79%	138.88	133.68	137.71	-3.74%	-0.84%	148.66	147.41	147.78	-0.84%	-0.59%	164.64	171.57	173.79	4.21%	5.56%
15	159.66	160.41	150.66	0.47%	-5.64%	135.36	132.61	137.17	-2.03%	1.33%	140.21	150.22	143.99	7.14%	2.70%	145.47	153.80	150.68	5.73%	3.58%
16	147.77	145.39	145.20	-1.61%	-1.74%	137.57	138.36	140.17	0.57%	1.89%	140.85	151.73	145.95	7.73%	3.62%	127.70	135.36	127.86	6.00%	0.12%
17	123.88	132.30	137.11	6.79%	10.68%	143.30	144.00	152.92	0.49%	6.71%	136.88	139.22	137.67	1.71%	0.58%	141.23	146.42	148.85	3.67%	5.39%
18	147.46	150.36	144.98	1.97%	-1.68%	140.15	141.76	142.16	1.15%	1.43%	113.60	117.63	120.07	3.55%	5.70%	123.15	124.92	124.98	1.44%	1.49%
19	147.94	147.38	131.68	-0.38%	-10.99%	134.11	134.49	146.92	0.29%	9.56%	146.57	142.72	142.57	-2.63%	-2.73%	151.90	148.18	153.17	-2.45%	0.84%
20	138.72	138.72	135.90	0.00%	-2.04%	134.36	139.19	143.02	3.59%	6.44%	119.04	143.77	142.47	20.78%	19.68%	134.87	143.24	142.17	6.21%	5.41%
21	126.24	128.22	126.29	1.57%	0.04%	146.05	137.90	144.33	-5.58%	-1.18%	140.78	142.43	143.30	1.17%	1.79%	131.84	146.62	153.70	11.21%	16.58%
22	133.61	133.45	135.68	-0.12%	1.55%	151.37	153.86	154.25	1.64%	1.90%	110.30	121.97	113.83	10.57%	3.20%	143.85	150.11	150.97	4.35%	4.95%
23	141.05	141.96	140.54	0.64%	-0.36%	126.66	130.75	127.28	3.23%	0.48%	131.16	135.56	129.98	3.35%	-0.90%	142.60	157.53	149.75	10.47%	5.01%
24	117.48	120.40	123.70	2.49%	5.29%	149.94	151.00	152.88	0.70%	1.96%	123.26	144.75	143.67	17.44%	16.56%	118.38	135.84	139.08	14.75%	17.49%
25	143.05	144.07	147.51	0.72%	3.12%	145.12	146.10	145.26	0.68%	0.10%	138.01	137.11	134.40	-0.65%	-2.61%	127.26	138.90	131.56	9.14%	3.38%
26	143.09	145.62	145.60	1.77%	1.76%	136.96	141.83	144.33	3.56%	5.39%	131.42	133.84	137.46	1.84%	4.59%	121.42	134.96	127.14	11.15%	4.72%
27	123.46	131.10	123.93	6.19%	0.38%	155.61	160.64	160.21	3.23%	2.96%	144.76	139.49	149.39	-3.64%	3.20%	143.34	154.45	150.20	7.75%	4.79%
28	154.05	155.37	150.65	0.86%	-2.20%	131.55	133.94	134.88	1.81%	2.53%	132.25	140.90	145.26	6.54%	9.84%	130.22	138.81	144.59	6.59%	11.04%
29	155.08	149.12	155.40	-3.85%	0.20%	140.79	144.39	144.84	2.56%	2.87%	124.89	131.16	136.81	5.02%	9.55%	149.27	152.38	152.86	2.08%	2.40%
30	133.36	132.66	133.74	-0.52%	0.28%	145.40	148.98	142.78	2.46%	-1.80%	155.01	158.31	154.91	2.12%	-0.07%	145.47	155.67	150.88	7.01%	3.72%
31	150.47	150.06	153.74	-0.27%	2.18%	118.78	119.47	116.42	0.57%	-1.99%	139.79	150.10	139.91	7.38%	0.08%	143.47	151.71	152.87	5.74%	6.55%
32	127.79	132.36	135.99	3.58%	6.42%	148.18	141.89	144.54	-4.24%	-2.45%	130.70	133.33	131.41	2.02%	0.55%	129.38	137.68	138.96	6.42%	7.41%
33	155.74	147.49	151.99	-5.30%	-2.41%	120.93	123.74	124.41	2.32%	2.88%	142.69	155.51	149.45	8.99%	4.74%	135.97	147.76	151.96	8.67%	11.76%
34	124.93	133.20	133.31	6.62%	6.70%	139.16	139.62	147.42	0.33%	5.93%	132.71	132.38	135.39	-0.24%	2.03%	148.32	152.94	151.63	3.12%	2.23%
35	124.07	129.03	125.22	4.00%	0.93%	111.74	118.91	116.12	6.42%	3.92%	145.54	156.31	150.11	7.40%	3.14%	125.96	129.65	125.57	2.94%	-0.30%
36	152.12	157.46	155.67	3.51%	2.33%	137.09	145.47	142.25	6.11%	3.76%	161.13	165.11	164.63	2.47%	2.17%	128.85	123.00	122.63	-4.54%	-4.83%
37	138.09	145.00	143.60	5.01%	3.99%	154.65	150.49	149.05	-2.69%	-3.62%	137.99	134.49	136.38	-2.53%	-1.17%	133.52	152.94	146.66	14.55%	9.85%
38	126.56	128.75	125.00	1.73%	-1.23%	145.95	149.86	142.55	2.68%	-2.33%	146.57	146.97	156.37	0.27%	6.69%	140.07	142.41	136.67	1.67%	-2.43%
39	123.58	124.26	125.74	0.55%	1.75%	154.98	156.43	153.96	0.93%	-0.66%	121.05	120.52	122.73	-0.44%	1.38%	151.47	151.71	146.84	0.16%	-3.06%
40	113.41	113.40	114.50	-0.01%	0.97%	156.19	161.77	156.13	3.58%	-0.04%	149.90	159.83	154.94	6.62%	3.36%	128.22	133.59	137.33	4.19%	7.10%

Table A.8: Postponement: Acceptance Rates at $\lambda_0 = 0.010$

Index	$\mu = 0.25$					$\mu = 0.50$					$\mu = 0.75$					$\mu = 1.00$				
	OPT3	POS (0)	POS (0.1)	% (0)	% (0.1)	OPT3	POS (0)	POS (0.1)	% (0)	% (0.1)	OPT3	POS (0)	POS (0.1)	% (0)	% (0.1)	OPT3	POS (0)	POS (0.1)	% (0)	% (0.1)
1	0.98	0.98	0.97	0.00%	-1.72%	0.96	0.96	0.96	0.00%	0.00%	1.00	1.00	0.98	0.00%	-2.04%	1.00	1.00	0.96	0.00%	-4.17%
2	0.95	0.95	0.95	0.00%	0.00%	0.91	0.93	0.93	2.91%	2.91%	0.98	0.98	0.98	0.00%	0.00%	0.95	1.00	1.00	4.76%	4.76%
3	0.98	0.98	0.98	0.00%	0.00%	0.98	0.98	1.00	0.04%	2.04%	0.98	0.98	1.00	0.00%	2.00%	1.00	1.00	1.00	0.00%	0.00%
4	1.00	1.00	0.98	0.00%	-2.44%	0.96	0.96	0.96	0.00%	0.00%	0.96	0.98	0.96	1.99%	0.07%	1.00	1.00	1.00	0.00%	0.00%
5	0.96	0.96	0.98	0.00%	1.96%	0.96	0.96	0.93	0.00%	-3.08%	0.98	0.98	0.98	0.07%	0.07%	1.00	0.98	1.00	-2.50%	0.00%
6	1.00	1.00	1.00	0.00%	0.00%	0.98	0.98	0.98	0.00%	0.00%	0.96	0.96	0.96	0.00%	0.00%	0.98	0.98	0.98	0.05%	0.00%
7	1.00	1.00	0.98	0.00%	-2.08%	1.00	1.00	1.00	0.00%	0.00%	0.95	0.98	0.95	3.04%	0.14%	0.98	0.98	0.98	0.04%	0.00%
8	0.94	0.94	0.96	0.00%	2.13%	0.90	0.90	0.90	0.00%	0.00%	0.98	0.98	0.97	0.03%	-1.64%	1.00	1.00	1.00	0.00%	0.00%
9	0.98	0.98	0.98	0.00%	0.00%	1.00	0.96	0.96	-3.77%	-3.77%	0.95	0.96	0.96	0.21%	0.21%	0.98	0.96	0.98	-2.00%	0.08%
10	0.98	0.98	0.98	0.00%	0.00%	1.00	1.00	1.00	0.00%	0.00%	0.98	0.98	0.98	0.00%	0.00%	1.00	1.00	1.00	0.00%	0.00%
11	0.93	0.93	0.91	0.00%	-1.64%	0.97	0.97	0.97	0.00%	0.00%	0.98	0.96	0.98	-2.17%	0.00%	0.92	0.92	0.98	0.00%	6.52%
12	0.98	0.98	1.00	0.00%	1.96%	0.93	0.94	0.94	0.15%	0.15%	0.98	1.00	0.98	1.79%	0.06%	0.98	0.98	0.98	0.03%	0.03%
13	1.00	1.00	1.00	0.00%	0.00%	0.95	0.93	0.93	-2.16%	-2.16%	1.00	0.96	1.00	-4.08%	0.00%	0.92	0.92	0.94	0.00%	2.13%
14	1.00	1.00	1.00	0.00%	0.00%	0.95	0.95	0.95	0.11%	0.00%	0.99	0.99	0.97	0.06%	-1.31%	0.98	0.98	0.98	0.00%	0.00%
15	0.91	0.91	0.93	0.00%	1.89%	0.96	0.96	0.98	0.00%	2.17%	1.00	1.00	0.98	0.00%	-2.13%	0.96	0.96	0.96	0.09%	0.00%
16	0.92	0.92	0.92	0.00%	0.00%	0.93	0.93	0.95	0.00%	2.50%	0.91	0.91	0.91	0.00%	0.00%	0.99	0.99	0.99	0.09%	0.00%
17	0.95	0.95	0.96	0.00%	1.85%	0.91	0.91	0.98	0.00%	7.32%	0.95	0.95	0.97	0.00%	2.70%	0.97	0.97	0.98	0.00%	1.79%
18	0.89	0.89	0.89	0.00%	0.00%	0.96	0.96	0.96	0.00%	0.00%	0.90	0.90	0.88	0.24%	-2.04%	0.91	0.91	0.91	0.00%	0.00%
19	1.00	1.00	0.98	0.00%	-1.75%	0.95	0.98	0.97	2.77%	2.70%	0.98	0.98	0.98	0.00%	0.00%	0.97	0.98	0.95	0.14%	-2.56%
20	0.92	0.92	0.97	0.00%	6.06%	1.00	1.00	0.98	0.00%	-2.38%	0.92	0.92	0.92	0.36%	0.36%	1.00	1.00	1.00	0.00%	0.00%
21	0.98	0.98	0.98	0.00%	0.00%	0.98	0.98	1.00	0.03%	1.72%	0.96	0.96	0.94	0.00%	-2.04%	0.95	0.93	0.95	-1.67%	0.00%
22	1.00	1.00	1.00	0.00%	0.00%	0.97	0.98	0.98	1.78%	1.78%	0.92	0.94	0.90	2.30%	-2.17%	0.98	0.98	1.00	0.00%	2.00%
23	0.95	0.95	0.95	0.00%	0.00%	0.96	0.94	0.94	-1.92%	-2.04%	1.00	1.00	0.96	0.00%	-4.26%	0.94	0.94	0.96	0.00%	2.27%
24	0.98	0.98	0.98	0.00%	0.00%	0.96	0.96	0.98	0.00%	2.33%	1.00	1.00	1.00	0.00%	0.00%	0.95	0.95	0.95	0.15%	0.15%
25	0.98	0.98	0.98	0.00%	0.00%	0.98	0.98	0.98	0.00%	0.00%	1.00	1.00	1.00	0.00%	0.00%	0.95	0.95	0.95	0.00%	0.00%
26	1.00	1.00	1.00	0.00%	0.00%	1.00	1.00	1.00	0.00%	0.00%	0.98	0.98	0.98	0.00%	0.00%	0.93	0.93	0.93	0.16%	0.16%
27	0.95	0.95	0.95	0.00%	0.00%	0.96	0.96	0.93	0.00%	-2.27%	0.95	0.95	0.97	0.00%	2.78%	0.97	0.97	0.97	0.00%	0.00%
28	1.00	1.00	1.00	0.00%	0.00%	0.93	0.93	0.98	0.00%	4.65%	0.95	0.95	0.97	0.00%	2.70%	0.98	0.98	0.95	0.12%	-2.27%
29	0.98	0.98	0.98	0.00%	0.00%	0.96	0.96	0.95	0.00%	-1.89%	0.93	0.91	0.93	-2.14%	0.37%	0.97	0.97	0.97	0.00%	0.00%
30	0.96	0.96	0.98	0.00%	1.92%	0.98	0.95	0.95	-3.60%	-3.60%	1.00	0.98	1.00	-2.04%	0.00%	0.98	1.00	0.96	2.33%	-2.22%
31	0.95	0.95	0.95	0.14%	0.14%	0.98	1.00	0.96	2.27%	-2.27%	0.97	0.97	0.97	0.00%	0.00%	0.96	0.96	0.96	0.00%	0.00%
32	0.94	0.94	0.94	0.00%	0.00%	0.95	0.95	0.95	0.00%	0.00%	1.00	0.92	1.00	-8.00%	0.00%	0.96	0.96	0.96	0.00%	0.00%
33	0.98	0.98	0.98	0.00%	0.00%	1.00	0.98	0.98	-1.89%	-1.92%	0.98	0.94	0.94	-4.13%	-4.26%	0.93	0.93	0.98	0.00%	5.56%
34	0.98	0.98	0.96	0.00%	-2.08%	1.00	1.00	1.00	0.00%	0.00%	0.98	0.98	0.98	0.05%	0.05%	1.00	1.00	1.00	0.00%	0.00%
35	1.00	1.00	1.00	0.00%	0.00%	0.98	0.98	0.98	0.00%	0.00%	0.98	0.98	0.98	0.04%	0.00%	0.94	0.98	0.98	4.30%	4.30%
36	0.98	0.98	0.98	0.00%	0.00%	0.95	0.95	0.95	0.00%	0.00%	1.00	1.00	0.98	0.00%	-2.08%	0.91	0.93	0.95	1.89%	3.77%
37	0.97	0.95	0.95	-1.69%	-1.69%	0.93	0.93	0.93	0.13%	0.13%	1.00	1.00	1.00	0.00%	0.00%	0.96	0.96	0.96	0.14%	0.14%
38	0.98	0.98	0.98	0.00%	0.00%	0.92	0.92	0.94	0.00%	2.04%	0.97	1.00	0.95	2.63%	-2.63%	0.92	0.94	0.93	2.66%	0.64%
39	0.96	0.96	0.96	0.00%	0.00%	0.95	1.00	0.95	5.26%	0.13%	0.94	0.96	0.98	2.17%	3.96%	0.97	0.97	0.97	0.00%	0.00%
40	0.95	0.95	0.93	0.00%	-1.89%	0.96	0.96	0.96	0.00%	0.00%	0.96	0.98	0.98	2.27%	2.27%	0.96	0.96	0.95	0.20%	-1.62%

Table A.9: Postponement: Acceptance Rates at $\lambda_0 = 0.015$

Index	$\mu = 0.25$					$\mu = 0.50$					$\mu = 0.75$					$\mu = 1.00$				
	OPT3	POS (0)	POS (0.1)	% (0)	% (0.1)	OPT3	POS (0)	POS (0.1)	% (0)	% (0.1)	OPT3	POS (0)	POS (0.1)	% (0)	% (0.1)	OPT3	POS (0)	POS (0.1)	% (0)	% (0.1)
1	0.94	0.93	0.94	-1.39%	0.00%	0.88	0.92	0.90	3.76%	2.16%	0.92	0.86	0.92	-5.94%	0.00%	0.99	0.99	0.99	0.10%	0.06%
2	1.00	0.97	1.00	-3.33%	0.00%	0.88	0.88	0.86	0.00%	-1.59%	0.88	0.87	0.87	-1.14%	-0.99%	0.95	0.99	0.94	3.69%	-0.75%
3	0.97	0.97	0.98	0.06%	1.75%	0.91	0.84	0.89	-7.20%	-2.25%	0.83	0.84	0.82	1.06%	-0.47%	0.90	0.91	0.93	2.10%	3.80%
4	0.99	0.99	0.96	0.04%	-2.78%	0.85	0.85	0.88	0.00%	2.83%	0.96	0.94	0.94	-1.47%	-1.47%	0.95	0.94	0.96	-1.33%	0.21%
5	0.92	0.92	0.92	0.12%	0.12%	0.93	0.94	0.94	1.77%	1.68%	0.88	0.90	0.90	2.23%	2.23%	0.95	0.94	0.96	-0.92%	0.71%
6	0.99	0.99	0.99	0.00%	0.00%	0.80	0.79	0.81	-1.41%	1.41%	0.97	0.99	0.97	1.43%	0.00%	0.94	0.96	0.96	1.96%	1.96%
7	0.96	0.95	0.95	-1.25%	-1.25%	0.88	0.82	0.83	-6.29%	-5.71%	0.86	0.77	0.79	-10.74%	-7.70%	0.96	0.87	0.91	-9.30%	-5.79%
8	0.85	0.82	0.80	-3.45%	-5.35%	0.97	0.97	0.97	0.05%	0.00%	0.96	0.97	0.97	1.65%	1.61%	0.97	0.97	0.99	0.09%	1.58%
9	0.78	0.78	0.79	-0.39%	0.76%	0.91	0.90	0.91	-0.89%	0.46%	0.97	0.93	0.97	-3.80%	0.14%	0.90	0.91	0.88	1.07%	-1.73%
10	0.85	0.83	0.83	-2.47%	-2.47%	0.93	0.84	0.84	-9.52%	-9.52%	0.98	0.98	1.00	0.16%	1.89%	0.91	0.89	0.88	-2.48%	-4.17%
11	0.96	0.93	0.95	-2.74%	-1.31%	0.92	0.91	0.91	-0.60%	-0.71%	0.95	0.95	0.98	0.25%	3.59%	0.91	0.90	0.91	-1.15%	0.12%
12	0.86	0.89	0.88	2.93%	2.65%	0.91	0.91	0.88	0.00%	-2.90%	0.89	0.88	0.87	-1.10%	-2.17%	0.90	0.91	0.90	0.78%	-0.52%
13	0.94	0.94	0.94	0.00%	0.00%	0.88	0.87	0.86	-1.45%	-2.70%	0.90	0.86	0.83	-3.91%	-7.49%	0.94	0.93	0.93	-1.45%	-1.13%
14	0.90	0.91	0.93	0.25%	2.95%	0.84	0.81	0.84	-4.09%	-0.55%	0.84	0.85	0.84	1.18%	0.00%	0.93	0.93	0.92	0.00%	-1.49%
15	0.97	0.95	0.98	-1.61%	1.64%	0.95	0.99	0.97	3.50%	1.91%	0.90	0.93	0.90	2.66%	0.09%	0.90	0.90	0.85	0.48%	-5.56%
16	0.78	0.81	0.77	3.95%	-1.01%	0.93	0.97	0.95	4.52%	1.65%	0.97	0.96	0.95	-1.18%	-2.41%	0.93	0.92	0.92	-1.30%	-1.41%
17	0.98	0.98	0.98	0.03%	0.03%	0.93	0.90	0.89	-2.44%	-3.47%	0.97	0.97	0.96	0.26%	-1.17%	0.95	0.93	0.95	-2.50%	0.13%
18	0.95	0.95	0.95	0.00%	0.09%	0.98	0.98	0.98	0.00%	0.00%	0.96	0.93	0.95	-2.58%	-1.25%	0.95	1.00	0.92	5.66%	-2.59%
19	0.87	0.88	0.86	0.41%	-0.97%	0.92	0.92	0.94	0.00%	1.80%	0.86	0.86	0.85	0.00%	-1.47%	0.97	0.96	0.96	-1.39%	-1.39%
20	0.90	0.90	0.90	0.00%	0.00%	0.95	0.93	0.93	-1.35%	-1.45%	0.99	0.99	0.99	0.02%	0.02%	0.96	0.96	0.97	0.21%	1.53%
21	0.93	0.93	0.95	0.00%	1.45%	0.84	0.86	0.85	3.03%	1.52%	0.99	0.96	0.93	-2.68%	-5.33%	0.78	0.74	0.74	-4.29%	-5.42%
22	0.96	0.97	0.97	1.43%	1.43%	0.95	0.97	0.95	1.85%	0.18%	0.97	0.94	0.93	-3.58%	-4.73%	0.95	0.95	0.95	0.08%	0.16%
23	0.91	0.95	0.96	4.48%	5.97%	0.96	0.99	0.96	3.12%	0.13%	0.93	0.89	0.88	-3.89%	-5.37%	0.97	0.95	0.95	-2.46%	-2.62%
24	0.90	0.84	0.90	-5.80%	0.00%	0.90	0.90	0.97	0.00%	7.02%	0.91	0.97	0.97	6.20%	6.20%	0.91	0.92	0.92	0.59%	0.59%
25	0.96	0.97	0.96	1.41%	0.11%	0.92	0.91	0.89	-1.41%	-3.08%	0.89	0.88	0.90	-1.30%	0.15%	0.95	0.94	0.93	-1.25%	-2.37%
26	0.92	0.93	0.92	2.09%	0.30%	0.97	0.93	0.94	-4.23%	-2.77%	0.91	0.88	0.88	-3.09%	-2.90%	0.97	0.94	0.96	-2.64%	-1.12%
27	0.91	0.91	0.91	0.25%	0.13%	0.82	0.78	0.81	-4.79%	-1.03%	0.92	0.89	0.87	-4.01%	-5.38%	0.88	0.86	0.85	-3.14%	-4.29%
28	0.95	0.95	0.95	0.08%	0.15%	0.85	0.85	0.86	0.00%	0.19%	0.88	0.88	0.89	0.00%	1.37%	0.93	0.94	0.91	0.48%	-2.44%
29	0.88	0.89	0.92	1.52%	4.22%	0.93	0.92	0.92	-1.04%	-0.92%	0.95	0.98	0.97	2.56%	1.42%	0.91	0.96	0.94	5.11%	3.58%
30	0.92	0.93	0.93	1.27%	1.27%	0.97	0.97	0.97	0.00%	0.00%	0.91	0.87	0.86	-4.40%	-5.71%	0.94	0.91	0.92	-3.30%	-2.28%
31	0.91	0.91	0.89	-0.96%	-2.38%	0.85	0.84	0.85	-1.00%	0.18%	0.94	0.81	0.85	-13.75%	-9.61%	0.97	0.97	0.95	0.05%	-1.59%
32	0.86	0.88	0.90	1.77%	4.43%	0.92	0.90	0.88	-2.10%	-3.50%	0.99	0.99	0.99	0.08%	0.06%	0.96	0.99	0.94	3.19%	-1.21%
33	0.94	0.94	0.96	0.09%	1.58%	0.90	0.90	0.90	0.00%	0.00%	0.96	0.95	0.99	-1.32%	2.78%	0.97	0.95	0.97	-1.44%	0.05%
34	0.91	0.84	0.85	-6.78%	-6.59%	0.94	0.93	0.94	-1.41%	0.17%	0.95	0.94	0.92	-1.59%	-3.26%	0.95	0.96	0.96	0.36%	0.36%
35	0.94	0.94	0.90	0.00%	-4.33%	0.97	0.96	0.94	-1.06%	-3.53%	1.00	1.00	1.00	0.00%	0.00%	0.84	0.83	0.80	-0.64%	-3.92%
36	0.97	0.99	0.97	1.43%	0.08%	0.97	0.99	0.99	1.54%	1.54%	0.91	0.96	0.94	5.51%	3.96%	1.00	1.00	1.00	0.00%	0.00%
37	0.89	0.83	0.84	-5.91%	-4.71%	0.97	0.99	0.99	1.68%	1.63%	1.00	1.00	1.00	0.00%	0.00%	0.90	0.82	0.82	-8.45%	-8.63%
38	0.94	0.94	0.93	0.00%	-1.52%	0.99	0.99	0.96	0.00%	-2.38%	0.94	0.94	0.95	0.42%	1.73%	0.92	0.88	0.88	-4.94%	-5.11%
39	0.88	0.92	0.88	4.62%	0.00%	0.99	0.96	0.97	-2.60%	-1.28%	0.84	0.86	0.82	1.50%	-2.63%	0.94	0.92	0.93	-1.20%	-1.08%
40	0.91	0.93	0.93	2.06%	2.06%	0.82	0.84	0.84	1.47%	2.53%	0.96	0.95	0.95	-1.29%	-1.29%	0.87	0.88	0.90	1.90%	3.40%

Table A.10: Postponement: Acceptance Rates at $\lambda_0 = 0.019$

Index	$\mu = 0.25$					$\mu = 0.50$					$\mu = 0.75$					$\mu = 1.00$				
	OPT3	POS (0)	POS (0.1)	% (0)	% (0.1)	OPT3	POS (0)	POS (0.1)	% (0)	% (0.1)	OPT3	POS (0)	POS (0.1)	% (0)	% (0.1)	OPT3	POS (0)	POS (0.1)	% (0)	% (0.1)
1	0.90	0.83	0.84	-7.35%	-6.08%	0.85	0.80	0.83	-4.84%	-1.41%	0.84	0.75	0.75	-9.91%	-9.64%	0.78	0.81	0.75	4.21%	-3.57%
2	0.91	0.92	0.88	0.75%	-3.77%	0.90	0.83	0.86	-7.49%	-4.63%	0.88	0.81	0.86	-7.69%	-2.20%	0.72	0.82	0.76	13.92%	5.31%
3	0.75	0.70	0.71	-6.37%	-5.30%	0.78	0.76	0.76	-2.89%	-2.29%	0.86	0.83	0.86	-3.43%	0.26%	0.89	0.77	0.79	-13.65%	-10.83%
4	0.82	0.84	0.82	2.30%	0.20%	0.85	0.87	0.87	2.33%	2.33%	0.94	0.91	0.89	-3.11%	-5.17%	0.85	0.74	0.74	-13.05%	-13.05%
5	0.93	0.89	0.91	-3.44%	-2.05%	0.87	0.85	0.82	-2.09%	-5.00%	0.84	0.77	0.83	-7.88%	-0.57%	0.72	0.74	0.76	3.29%	5.72%
6	0.89	0.89	0.88	0.40%	-0.95%	0.90	0.85	0.87	-5.31%	-2.88%	0.88	0.86	0.82	-2.04%	-6.49%	0.87	0.77	0.77	-10.81%	-11.34%
7	0.79	0.79	0.80	0.00%	1.42%	0.85	0.79	0.78	-7.99%	-8.66%	0.73	0.71	0.71	-2.25%	-1.85%	0.83	0.80	0.80	-4.19%	-3.97%
8	0.88	0.87	0.86	-0.94%	-2.20%	0.89	0.91	0.88	1.73%	-0.88%	0.88	0.85	0.88	-3.07%	-0.52%	0.95	0.95	0.95	0.08%	0.08%
9	0.85	0.86	0.83	1.80%	-2.33%	0.90	0.89	0.90	-0.79%	0.26%	0.93	0.91	0.91	-1.56%	-1.56%	0.91	0.87	0.87	-5.24%	-5.24%
10	0.77	0.79	0.76	3.03%	-1.04%	0.81	0.79	0.76	-2.38%	-6.49%	0.80	0.79	0.75	-1.37%	-5.77%	0.90	0.81	0.82	-10.18%	-8.81%
11	0.80	0.76	0.74	-4.40%	-6.91%	0.84	0.82	0.80	-2.61%	-4.66%	0.82	0.83	0.84	0.67%	2.11%	0.84	0.81	0.81	-4.63%	-4.63%
12	0.85	0.82	0.82	-3.43%	-3.22%	0.85	0.82	0.82	-3.50%	-3.27%	0.94	0.92	0.93	-1.18%	-1.08%	0.84	0.78	0.77	-7.08%	-7.84%
13	0.78	0.77	0.80	-2.37%	2.07%	0.84	0.85	0.84	1.13%	0.35%	0.86	0.77	0.82	-10.78%	-5.13%	0.90	0.78	0.84	-13.46%	-6.85%
14	0.83	0.80	0.77	-2.88%	-7.28%	0.88	0.83	0.86	-5.16%	-2.21%	0.81	0.76	0.78	-5.83%	-3.69%	0.83	0.75	0.76	-9.68%	-8.02%
15	0.80	0.77	0.75	-3.15%	-5.68%	0.93	0.87	0.88	-6.75%	-5.56%	0.82	0.80	0.74	-2.43%	-8.97%	0.78	0.76	0.75	-2.56%	-3.14%
16	0.82	0.82	0.81	0.24%	-1.10%	0.85	0.85	0.86	-0.56%	0.80%	0.90	0.87	0.83	-3.46%	-8.17%	0.90	0.86	0.81	-4.89%	-10.00%
17	0.89	0.83	0.84	-6.92%	-5.50%	0.89	0.85	0.88	-4.83%	-1.51%	0.86	0.84	0.85	-1.75%	-0.90%	0.79	0.74	0.74	-5.98%	-5.72%
18	0.88	0.82	0.83	-6.02%	-5.28%	0.88	0.84	0.86	-4.55%	-2.04%	0.87	0.82	0.86	-5.84%	-1.69%	0.78	0.75	0.73	-3.73%	-5.78%
19	0.90	0.88	0.80	-2.35%	-11.73%	0.94	0.90	0.95	-3.75%	1.60%	0.83	0.77	0.81	-7.32%	-2.47%	0.88	0.82	0.88	-7.64%	-0.34%
20	0.87	0.87	0.88	0.00%	1.22%	0.88	0.84	0.90	-3.53%	2.41%	0.87	0.83	0.84	-3.57%	-2.55%	0.84	0.76	0.80	-9.42%	-4.71%
21	0.82	0.79	0.81	-3.75%	-1.68%	0.84	0.77	0.78	-8.66%	-7.19%	0.85	0.82	0.82	-3.12%	-3.12%	0.79	0.72	0.74	-8.86%	-7.14%
22	0.81	0.80	0.78	-0.75%	-3.17%	0.81	0.77	0.75	-5.43%	-7.83%	0.96	0.91	0.92	-4.84%	-3.93%	0.81	0.73	0.70	-10.33%	-13.53%
23	0.85	0.85	0.82	0.55%	-3.34%	0.87	0.85	0.86	-1.91%	-0.80%	0.93	0.79	0.78	-14.35%	-15.71%	0.88	0.82	0.80	-7.13%	-9.06%
24	0.99	0.94	0.96	-4.20%	-2.84%	0.84	0.79	0.82	-5.66%	-2.14%	0.95	0.89	0.91	-5.81%	-3.57%	0.87	0.90	0.93	3.85%	6.72%
25	0.82	0.80	0.82	-2.00%	0.23%	0.88	0.85	0.87	-3.87%	-1.46%	0.88	0.84	0.80	-4.76%	-9.31%	0.89	0.79	0.79	-10.31%	-10.55%
26	0.85	0.84	0.85	-0.72%	0.60%	0.87	0.84	0.85	-3.89%	-2.73%	0.76	0.73	0.74	-3.88%	-2.27%	0.91	0.86	0.87	-5.40%	-4.28%
27	0.83	0.82	0.80	-0.18%	-3.32%	0.81	0.81	0.82	0.00%	1.15%	0.81	0.65	0.72	-19.00%	-11.29%	0.86	0.86	0.84	0.17%	-2.58%
28	0.77	0.75	0.75	-2.76%	-2.51%	0.80	0.78	0.77	-2.06%	-3.35%	0.85	0.78	0.85	-7.79%	-0.09%	0.88	0.76	0.80	-13.61%	-9.41%
29	0.75	0.72	0.76	-4.33%	0.28%	0.76	0.72	0.71	-6.09%	-7.32%	0.88	0.85	0.88	-3.20%	-0.39%	0.82	0.73	0.71	-10.91%	-13.53%
30	0.87	0.86	0.87	-1.00%	0.53%	0.87	0.88	0.85	1.52%	-2.11%	0.88	0.90	0.89	2.47%	1.18%	0.85	0.70	0.72	-17.18%	-14.49%
31	0.84	0.84	0.81	-0.44%	-3.78%	0.82	0.82	0.79	-0.61%	-3.50%	0.81	0.73	0.73	-10.26%	-10.06%	0.92	0.80	0.81	-13.25%	-12.49%
32	0.87	0.87	0.90	0.52%	3.16%	0.85	0.80	0.82	-5.88%	-3.64%	0.97	0.96	0.97	-1.33%	0.04%	0.85	0.78	0.80	-7.61%	-5.58%
33	0.81	0.77	0.81	-4.96%	-0.49%	0.91	0.89	0.91	-2.50%	0.37%	0.83	0.85	0.78	1.82%	-6.67%	0.74	0.74	0.76	-0.17%	2.18%
34	0.94	0.94	0.95	-0.99%	0.31%	0.82	0.77	0.80	-6.80%	-2.78%	0.83	0.81	0.82	-3.23%	-1.70%	0.78	0.74	0.73	-4.70%	-6.14%
35	0.86	0.83	0.84	-3.19%	-1.67%	0.87	0.84	0.82	-3.36%	-6.11%	0.88	0.75	0.73	-14.18%	-16.31%	0.84	0.83	0.82	-1.33%	-2.84%
36	0.90	0.88	0.89	-1.91%	-0.71%	0.90	0.89	0.86	-0.19%	-4.09%	0.83	0.77	0.78	-7.60%	-5.82%	0.88	0.73	0.75	-17.27%	-15.56%
37	0.93	0.92	0.93	-1.61%	-0.67%	0.87	0.85	0.84	-2.08%	-3.37%	0.84	0.71	0.74	-15.76%	-12.33%	0.78	0.72	0.73	-7.51%	-5.92%
38	0.93	0.92	0.91	-1.41%	-2.82%	0.88	0.88	0.87	-0.99%	-2.10%	0.87	0.81	0.80	-6.52%	-7.56%	0.91	0.81	0.81	-10.63%	-10.63%
39	0.90	0.89	0.92	-1.19%	1.52%	0.78	0.76	0.77	-2.15%	-1.28%	0.92	0.87	0.91	-5.13%	-0.94%	0.79	0.79	0.76	0.24%	-3.21%
40	0.88	0.86	0.88	-2.50%	-1.09%	0.78	0.71	0.67	-8.77%	-13.67%	0.81	0.78	0.78	-3.61%	-3.61%	0.87	0.81	0.82	-6.59%	-5.22%

Appendix B

Expected Euclidean Distances Between Two Points

Definition: The Euclidean distance between two points $\mathbf{p} = (p_1, p_2, \ldots, p_n)$ and $\mathbf{q} = (q_1, q_2, \ldots, q_n)$ in Euclidean n-space, is defined as:

$$D(\mathbf{p}, \mathbf{q}) = D_{\mathbf{pq}} = \sqrt{\sum_{i=1}^{n} (p_i - q_i)^2} \tag{B.1}$$

B.1 The expected distance from an idle point to a random pick-up point

One can refer to Eilon et al. (1971 [13]) and Daley et al. (1976 [12]) for more information about the analysis of the expected distances of random points. Let ϕ_p be the location of the idle point in zone p.

$$\begin{aligned} \mathbb{E}[D(\phi_p, \mathbf{o}_l)] &= \mathbb{E}[D_{\phi_p} \mathbf{o}_l] \\ &= \frac{1}{\sum_p \lambda_p} \sum_p \lambda_p \mathbb{E}[D_{\phi_p} \mathbf{o}_l | \mathbf{o}_l \text{ is in zone } p] \end{aligned} \tag{B.2}$$

We show how to compute $\phi_p = \phi_2$, there are three cases as shown in Figure B-1

Case: Within square, e.g. (b)

$$\begin{aligned}\mathbb{E}[D_{\phi_2}\mathbf{o}_l|\mathbf{o}_l \text{ is in zone 2}] &= \mathbb{E}[\sqrt{(\phi_{x2}-o_{xl})^2+(\phi_{y2}-o_{yl})^2}|\mathbf{o}_l \text{ is in zone 2}] \\ &= 0.5214 \times d \quad \text{for d}\times\text{d square zone}\end{aligned} \tag{B.3}$$

Case: Adjacent squares, e.g. (a), (c)

$$\begin{aligned}\mathbb{E}[D_{\phi_2}\mathbf{o}_l|\mathbf{o}_l \text{ is in zone 1}] &= \mathbb{E}[\sqrt{(\phi_{x2}-o_{lx})^2+(\phi_{y2}-o_{ly})^2}|\mathbf{o}_l \text{ is in zone 1}] \\ &\approx d(1+\frac{d^2}{2d^2}) = 1.5d \quad \text{for d}\times\text{d square zone}\end{aligned} \tag{B.4}$$

Case: Corner point contact squares, e.g. (d)

$$\begin{aligned}\mathbb{E}[D_{\phi_2}\mathbf{o}_l|\mathbf{o}_l \text{ is in zone 4}] &= \mathbb{E}[\sqrt{(\phi_{x2}-o_{lx})^2+(\phi_{y2}-o_{ly})^2}|\mathbf{o}_l \text{ is in zone 4}] \\ &\approx \sqrt{2}d(1+\frac{d^2}{2(\sqrt{2}d)^2}) \approx 1.7678d \quad \text{for d}\times\text{d square zone}\end{aligned} \tag{B.5}$$

From B.2, for $\phi_p = \phi_2$ we will get

$$\mathbb{E}[D(\phi_2, \mathbf{o}_l)] = \frac{1}{\sum_p \lambda_p}\left\{1.5\lambda_1 d + 0.5\lambda_2 d + 1.5\lambda_3 d + 1.7678\lambda_4 d\right\} \tag{B.6}$$

B.2 The expected distance from a random pick-up point to a random delivery point

$$\mathbb{E}[D(\mathbf{o}_l, \mathbf{d}_l)] = \mathbb{E}[D_{\mathbf{o}_l}\mathbf{d}_l]$$

$$= \frac{1}{\sum_p \lambda_p} \sum_p \lambda_p \mathbb{E}[D_{\mathbf{o}_l}\mathbf{d}_l | \mathbf{o}_l \text{ is in zone } p] \tag{B.7}$$

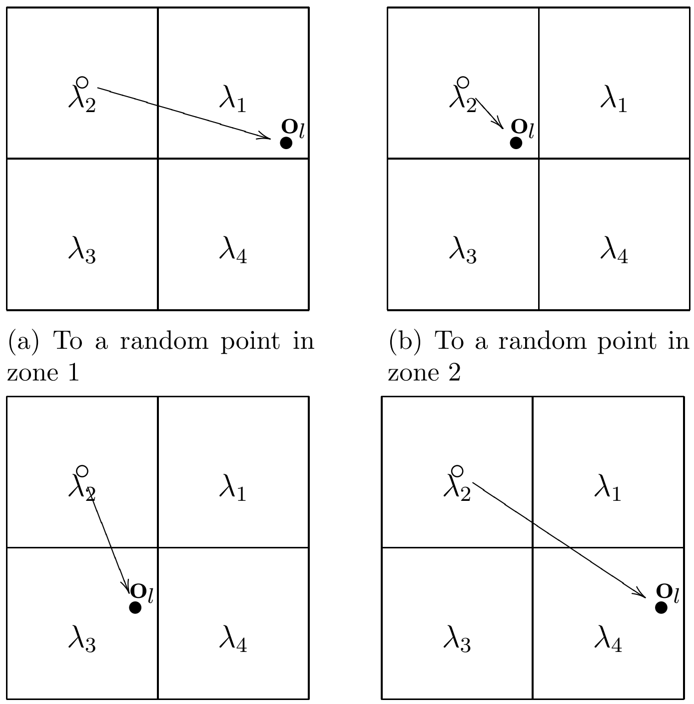
For example, the case of $\mathbf{o}_l$ in zone 1 is shown in Figure B-2

$$\mathbb{E}[D_{\mathbf{o}_l}\mathbf{d}_l | \mathbf{o}_l \text{ is in zone 1}] = \mathbb{E}[\sqrt{(o_{lx} - d_{lx})^2 + (o_{ly} - d_{ly})^2} | \mathbf{o}_l \text{ is in zone 1}]$$
$$\approx 3\sqrt{2}d \approx 4.2426d \quad \text{for d×d square zone} \tag{B.8}$$

B.3 The expected distance from a random delivery point to a fixed pick-up point

$$\mathbb{E}[D(\mathbf{d}_l, \mathbf{o}_j)] = \mathbb{E}[D_{\mathbf{d}_l}\mathbf{o}_j]$$
$$= s[1 + \frac{(2d)^2}{2s^2}] \quad , \tag{B.9}$$

where s = $\sqrt{(o_{xj} - 0.5)^2 + (o_{yj} - 0.5)^2}$



(a) To a random point in zone 1

(b) To a random point in zone 2

(c) To a random point in zone 3

(d) To a random point in zone 4

Figure B-1: The distance from the idle point in zone 2 (ϕ_2) to a random pick-up point in different zone

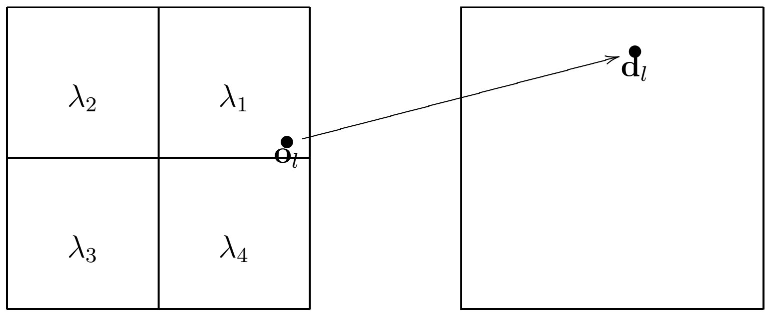


Figure B-2: The distance from a random pick-up point to a random delivery point

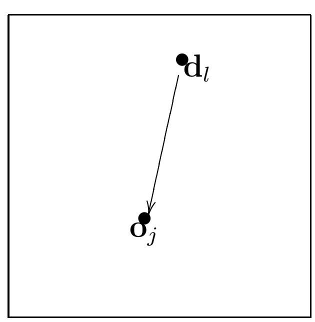


Figure B-3: The distance from a random delivery point to a fixed pick-up point

Appendix C

Simulating Probabilistic Events

The methodological approach of simulating probabilistic events presented in Appendix C is drawn from Larson and Odoni (1981 [25]). We summarize relevant methods used in our simulation in this appendix.

C.1 Discrete Random Variable

The inversion method is used to generate samples from discrete random variables. This is shown in Figure C-1. If y's $\in \{1, 2, 3, 4\}$ are the values that the discrete random variable Y can take and $p_Y(y)$ and $P_Y(y)$ are the pmf and cdf, respectively, of Y, the method consists of (1) drawing a random number r, and (2) finding the smallest value y such that

$$P_X(y_i) = \sum_{\text{all } y \leq y_i} p_X(y) \geq r \qquad \text{(C.1)}$$

In that case we set $y_s = y_i$ (where y_s denotes the sample value of Y). Note that the set $\{1, 2, 3, 4\}$ corresponds to four geographical zones in the region.

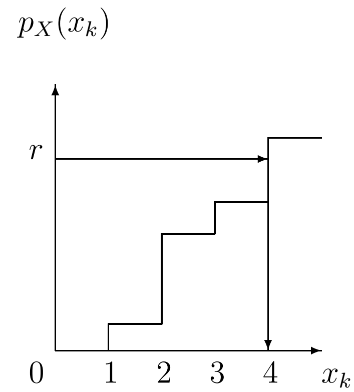


Figure C-1: Use of the inversion method for sampling from discrete distributions.

C.2 Negative Exponential Random Variable

Define T a time interval before a successive event (a new request). It is therefore essential that we be able to generate random sample values, τ_s, of the random variable T with the pdf

$$f_T(t) = \lambda e^{-\lambda t}, \quad \text{for } t \geq 0$$

As we know, the cumulative distribution of T is

$$F_T(t) = \int_0^t \lambda e^{-\lambda \tau} d\tau = 1 - e^{-\lambda t}, \quad \text{for } t \geq 0$$

Let us then set a random number r (uniformly distributed between 0 and 1) equal to $F_T(t)$. We have

$$rF_T(t) = 1 - e^{-\lambda t}$$

or, equivalently,

$$t = \frac{-\ln(1-r)}{\lambda} \tag{C.2}$$

Note that equation C.2 allows us to draw sample observations of T. that is, each time we wish to draw a sample t_s, all we have to do is draw a random number r and use C.2. In fact, since $1 - r$ is also a random number uniformly distributed between 0 and 1, we can also bypass the subtraction. That is, we can use a random number distributed between 0 and 1 directly as follows:

$$\tau_s = \frac{-\ln r}{\lambda} \tag{C.3}$$

C.3 Simulating Poisson Events

Since we consider the sequence of job arrival epochs τ_i^{arv} as the Poisson process arrival times. The Poisson pmf $P\{N(T) = k\}$ is the probability of observing k events in a time interval T when interarrival times are independent with negative exponential distribution. To generate random observations of $N(T)$ for any given T, we follow the procedure shown in Figure C-2, appeared in Larson and Odoni (1981 [25]). We keep generating exponentially distributed time intervals $\tau_{s1}, \tau_{s2}, \tau_{s3}, \ldots$ [by using C.2] until the total length of time represented by the sum of these intervals exceeds T for the first time. That is, we find j such that

$$\sum_{i=1}^{j} \tau_{si} \leq T < \sum_{i=1}^{j+1} \tau_{si}$$

Then our sample observation of $N(T)$ is given by $K_s = j$.

It is important to note that the sum of independent Poisson random variables, X_i's, with the parameter λ_i is a Poisson random variable with the parameter $\sum_i \lambda_i$.

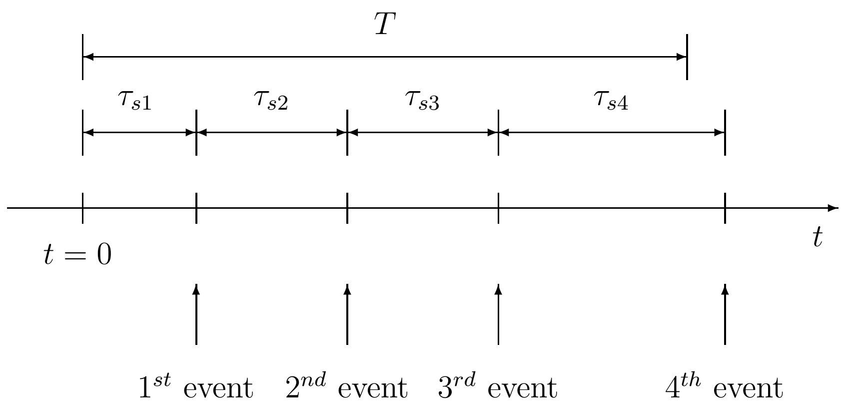


Figure C-2: Generation of random observations from Poisson distribution. In this figure, $k_s = 3$.

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