C.E. Thesis case

THESIS.

AN INVESTIGATION AND DEVELOPMENT OF A CERTAIN SLOPE DEFLECTION METHOD AND ITS APPLI- CATION TO RIGIDLY FRAMED STRUC- TURES .

Prepared by

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B.S: Tongshan Engineering College.

for the

Fulfilment of M.S. Degree in the

Department of Civil Engineering

of the

Massachusettes Institue of Technology, Mass.

June, 1922.

June 1, 1921.

Prof. A. L. Merrill,

Secretary of Faculty,

Massachusetts Institute of Technology.

Dear Sir;

In accordance with the requirement for the degree of Master of Sience in Civil Engineering, I herewith submit my thesis entitled " The investigation and development of a certain slope-deflection method for designing rigidly framed structures."

Respectfully submitted,

Kalgan Shih.

Contents.

Preface.

This work is prepared for the fulfilment of M.S degree in the Institute. In doing it a time of about 200 hours has been spent, of which the most part was devoted in reading and researching. Pains have been taken to present the method in unique and simple manner and to develop all the necessary expressions, so that the easiness of the application of the method can be achieved. Examples are also given at the end of the work, they serve the purposes of illustrating the method of application, checking up the correctness of this method, and, at the same time, giving a brief comparison of this method with those existing.

Owing to the limited time and the large amount of work involved, some points might have been overlooked, although great care has been taken by the writer.

The writer wishes to acknowledge his indebtedness to Professor Charles M. Spofford for the inspiring instruction received from him as a student in his class on advanced structures. He particularly wishes to express his appreciation of the valuable instruction and criticism on this work from Professor Hale Sutherland, under whom the writer has taken the course on advanced structural design.

Kalgan Shih

May, 1922.

INTRODUCTION.

1. A review of previous methods on rigidly framed structure.

It is believed that a wide knowledge of the analysis of rigidly framed structure is generally required in gaining economies and in securing effective designs. Notwithstanding the importance, the close investigation of the stresses as they actually occur in this kind of structure is not usually attempted in practice. The reason is simply due to the fact that the existing methods for investigation requires either considerable time to work out formulas for statically indeterminate quantities to suit a particular case in question, or laborious work to apply the different expressions already determined to get so many unknown quantities. In view of the economy of the time element, several very approximate methods are in general use, although they sometimes give results which are serious in error. The methods that have been so far developed can be classified as follows:

a) The approximate methods. They are originated by various authorities (Spofford, Fleming, Smith, Burt, Thayer, etc). The methods consist of certain combination of the following assumptions:

1. Point of contraflexure of columns at mid-height.

2. Point of contraflexure of beams at mid-length.

3. Direct stresses in columns are propoltional to the distance of the column from neutral axis.
4. The shear on all interior columns is equal and the shear on each exterior column is equal to one half the shear on interior column.
5. Shear is distributed amongst columns in proportion to their moments of inertia.
6. Shear is distributed amongst columns in proportion to area of vertical rectangle of which the column is the axis.

As the methods are common in practice and can be found in papers written by respective authors mentioned above, so they will not be given here in further detail.

b) The more exact methods. These consist of the methods based on slope and deflection originated by Dr. C.A.Melick and Mr. E.F.Jonson. The original work is laborious and is almost inpractical for buildings several stories high.

The further development of the slope and deflection method by W. M. Wilson and several other men (bulletins 80 and 108, Eng. Exp. Sta of Univ. Of Ill.) rendered this method generally applicable. Expressions for various cases have been derived. It is considered simple, although in some cases its application is restricted. (See Conclusion, Part 4).

c) The exact method. This method is a development by method of least work. The well-known papers regarding to this method are by Prof. A. Smith (Journal of Western Society of Engineers, Vol. XX) and by Dr. Mikishi Abe, (Bulletin 107, Eng. Exp. Sta. of Univ. of Ill..). The method is exact and the direct stresses can be taken into account. It is, of course, very long.

2. General outline of the present method.

The method herein developed and investigated belongs to Class b, the more exact method, of the previous discussion, or, in other words, the slope deflection method treated in more unique manner. The preference of this method will be discussed in the conclusion of Part IV of this work.

The work is devided into 4 parts with an introduction at its beginning, in which, besides some general discussion, a section for getting primary sections of a reinforced concrete frame is also given. Several formulas derived from the writer's experience are presented, which, when being applied, needs only a single slide rule operation and are deemed simple and helpful.

The first part of the work, Part I, is the basis of the whole. All the theories and derivations are included in it. They serve in the main to get the characteristics of a given frame. By characteristics of a frame are here meant the following:

Taking each member of the frame separately and assuming one end being fixed, while the other end being subjected to a unit moment and a shear resulted therefrom, the expressions for the points of contraflexures of the members can be obtained. The position of the points of contraflexure does not change for any magnitude of the accually applied moment. (Characteristic I)

Taking each joint of the frame separately and assuming a unit moment being applied to the joint. Now since it can be easily proven that in the strained position all the members meeting at one point are subjected to the same change of slope, the relation of transmission of the moment, or the moment factors, can be derived. The moment factor does not change for any magnitude of the actually applied moment. (Characteristic II)

At the end of Part I a summary of the procedure for computation is also given.

Most of the theories given in this part are taken from Dr. Strassener's work appeared in the paper " Forscherarbeiten auf dem Gebiete des Eisenbetons, Hefte 26". The expressions for moment factors for cases where four members meeting at a joint are the writer's own development from the same principles; these complete the missing link of Dr. Strassener's work.

The second part of this work, Part II, treats the method of application of the development given in Part I to the case of a frame subjected to vertical loads. Section 2, the separation of moments and their signs, shows the unique manner of this method, in which lies its superiority. The two sections on several short-cut methods for finding moments of rigid members and Criteria for maximum combined stresses in beams and columns are collected from various sources and seem to be helpful.

The third part of this work, Part III, treats the method of application of the development given in Part I to the case of a frame subjected to horizontal loads. On account of the fact that the tops of all columns are subjected to a deflection in this case, expressions for moments due to unit deflection are derived. Now since the deflection and the resulted moments are always in direct proportion to each other, advantage can be taken from this fact for determining the moment of each member due to an assumed deflection unity of each story of the frame. From these moments the horizontal shear subjected to each column can be found. The sum of the shears of all the columns in each story should naturally be equal to the external force applied at that story, which causes the assumed unit deflection. by multiplying the moment due to the unit deflection by the ratio

$$\frac{\text{Acctually applied force}}{\text{External force causing unit defl.}}$$

the acctual moment of each member can be obtained.

Dr. Strassener gives some hint for attacking such problem in his paper mentioned above, but the method has not been treated by him in unique and simple manner. The present method is a collection of the developments of various engineers appearing in "Schweizerische Bauzeitung" and put up in simple and collective manner by the writer. It should be noted that this method is more simple than Wilson's method by reducing two equations from each story.

The fourth part of the work, Part IV, gives a complete illustration of the method and conclusions.

3. <u>Foundamental principles and assumptions of the present slope deflection method.</u>

The principles and the assumptions of the method in successive sections given in this work are almost the same as are given in Wilson's paper, Bulletins 80 and 108, Eng'g experimental station of University of Illinois. The treatment and the applications of the principles and assumptions are entirely different from those given in Wilson's work and the preference of the present method will be discussed in the conclusions in Part 4. The principles and assumptions adopted can be summarized as follows:

1) The moment at an end of a member of a frame is a function of the changes in the slopes of the tangents to the elastic curve of the member at its ends and of the deflection of one end of the member relative to the other end.

2) In the strained position, all the columns and girders which intersect at one point have been subjected to the same change in slope.

Upon these two principles the Part 1 of this work is based.

3) The change in the length of a member due to direct stress is equal to zero.

4) The horizontal deflection of the tops of all columns due to vertical load is equal to zero.

Upon these two assumptions the part 11 of this work is based.

5) The horizontal deflections of the tops of all columns of a story due to horizontal load are equal.

Upon assumptions 3 and 5 the Part 111 of this work is based. Other minor assumptions are:

6) The connections between the columns and girders are perfectly rigid.

7) The length of a girder is the distance between the neutral axes of the columns which it connects and the length of a column is the distance between the neutral axes of the girder which it connects.

8) The deflection of a member due to the internal shearing stresses is equal to zero.

9) The wind load is resisted entirely by the rigid frame.

4. Method of getting primary sections of the frame for investigation.

In all the methods discussed in section 1, except the approximate ones, it is required to know all the sections of the members before investigation. This is, however, the hardest task, which the designer has to encounter, if any exact computation is intended to make at all. Owning to this fact, it seems to the writer to be favorable to include this section in this work.

Now since the scope of structural conditions is so wide that the degree of rigidity varies with all factors, like supports, joints, materials, etc., it is impossible to present any method which can be applied to all cases. In the following discussion it is, therefore, confined to reinforced concrete frames, which has absolute rigidity and the degree of accuracy of the new slope deflection method herein presented can be warranted.

The formulas given below are derived from the writer's experience and are based upon 1 : 2 : 4 concrete ($f_c = 650$. $f_s = 16000$ and $n = 15$), which is the mixture used in almost all cases of building frames. Though the derivation of the formulas is so simple and their forms do not differ much from ordinary ones, yet the merit of a single slide rule operation can hereby obtained.

Formula 1. For rectangular beams. (standard notations)

$$d = \sqrt{\frac{M}{108 \cdot b}}$$

This formula is derived from ordinary chart for rectangular beams. For the specified stresses of 1 : 2 : 4 concrete $\frac{M}{bd^2} = 108$ and whence we have the formula.

Formula 2, For T-beams, neutral axis lying in stem.

$$d = \sqrt{\frac{M}{35 \cdot b'}}$$

in which b' is the assumed flange width, and according to the specification of Joint Committee

a) It should not exceed $\frac{1}{4}$ of the span length of the beam.

b) Its overhanging width on either side of the web shall not exceed 6 times the thickness of the slab.

c) It shall not exceed the distance c-c of the beams.

This formula is derived by examining a series of T-beams together with the charts given on page 364-365 of Hool and Johnson's Concrete Eng'rs Handbook. It is found that the design of T-beams is always limited by the stress of steel, while the stress of concrete is generally far below its limit. The latter stress ranges from 300 to 400 for competent design with cost ratio r = 70 approximately. Now let draw a horizontal line on diagram 8, page 364, H. and J.'s book, which passes the curve f_c= 350 and so place the line that half of the curve is above and half below it, it will be found that this horizontal line corresponds $\frac{M}{bd^2} = 35$ and whence we have the formula. It has been tested that this formula satisfies most of the cases, especially for getting primary sections.

Formula 3. For getting reinforcing steel of rectangular and T-beams.

$$A_s = \frac{M}{14000 \cdot d}$$

This formula is obtained by substituting $j = \frac{7}{8}$ into the general one $A_s = \frac{M}{jdf_s}$.

Formula 4. For columns centrally loaded.

$$A_c = \frac{P}{C}$$

Where A_c= the net area of cross section of the column, P= the centrally applied load and C a constant given in the table below, its value is different with different values of "p", the percentage of reinforcement.

1:2:4 (2000-lb. concrete) n=15.

f_c=450 for ordinary lateral ties.*

P	0.010	0.015	0.02	0.025	0.030	0.035	0.040	0.045	0.050
C	513	545	576	608	639	671	702	734	765

The values of C in the above table are computed from the expression $C=f_c[1+(n-1)p]$, the formula is a general one and requires no further discussion.

For beams reinforced with compressive steel and columns subjected to eccentric loading, formulae given in various text books are supposed to be used.

* Specification of Joint Committee.

Part 1.

THEORIES FOR GETTING THE CHARACTERISTICS OF A GIVEN RIGIDLY FRAMED STRUCTURES

The foundamental theories from which this part of the work is based, are originated by Prof. W. Ritter in his book " Anwendung der graphische Statik". During the last ten years research works have continuously been made in Germany to study the effect of rigidity of any kind of framed structure with the intention to obtain a method of solution, which should be practical in actual use and, at the same time, will give a degree of accuracy not far from the exactness. In year 1916 Dr. Strassener published a long article treating this subject in "Forscherarbeiten im Gebiete des Eisenbetons" and since then several articles regarding this work have appeared during the years 1917-1921 in Schweizerische Bauzeitung. In both of Dr. Strassener's work and the articles in Schweizerische Bauzeitung the expressions given for moment factors are confined to joints where only three members meet. In order to solve problems like rigidly framed buildings, cases always occur where four members meet together, like interior bays of the building, consequenty expressions for these, equations 10 to 14, are developed by the writer.

1. <u>The change of slope of a beam.</u>

Let, M_a = the moment applied at the left end of a beam.

M_b= " " " " " right " " " "

θ_a= the slope angle at the left end of a beam due to $M_a = M_b = 1$

θ_b= the slope angle at the right end of a beam due to $M_a = M_b = 1$

β= the slope angle at left end due to M_b= 1, or the same at right end due to M_a= 1

and let x = the distance of any section from left end of the beam in question and x' the same from right end, then the expressions for the slope angles are:

$$\theta_a = \frac{1}{L}\cdot\int\frac{x'dx}{E\cdot I_x}, \quad \theta_b = \frac{1}{L}\cdot\int\frac{x\cdot dx}{E\cdot I_x},$$

$$\text{and} \qquad \beta = \frac{1}{L^2}\cdot\int\frac{x'x\cdot dx}{E\cdot I_x} \qquad \cdots\cdots(1)$$

For beams of constant cross section, $\theta_a = \theta_b = \theta$

$$\theta = \frac{L}{2EI} \quad \text{and} \quad \beta = \frac{L}{6EI}\ ^* \qquad \cdots\cdots(1_a)$$

The derivation of equation (1) can be obtained very easily by following the discussions given in Morley's Strength of Materials, pages 170-179, or by Prof. Swain's theory given in Transactions of A. S. C. E. 1918.

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* The limit of integration in equation (1) is from 0 to L.

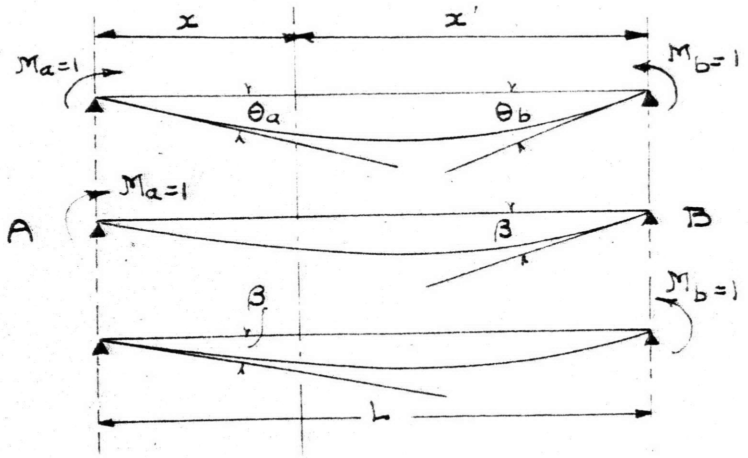


Fig. 1.

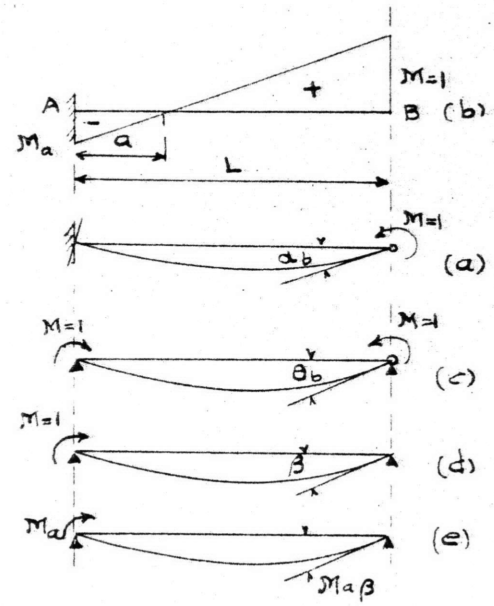


Fig. 6

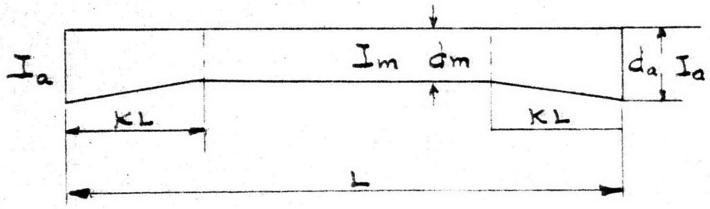


Fig. 2

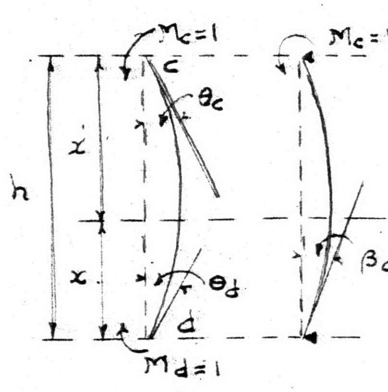


Fig. 3

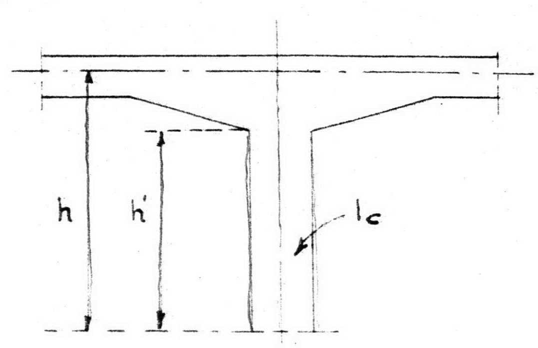


Fig. 4

In case that the cross section of the beam is not constant and its moment of inertia varies, the integral symbols of the above expressions can be rrplaced by the symbol "Σ" and by looking the value dx as a part of the beam having a length of x . The solution of each expression can be done graphically without difficulty.

In view of practical use, two tables are hereby reproduced from "Forscherarbeiten auf dem Gebiete des Eisenbetons, heft xxvi". In order to facilitate the use of the tables, the following equation is given:

$$\theta = \frac{L}{2EI_m} \cdot \varphi_a , \qquad \beta = \frac{L}{6EI_m} \cdot \varphi_b \quad \cdots (1_b)$$

Equation (1_b) is only applicable to symmetrical beams, which type is used in almost all cases in building construction. The factors $\frac{L}{2EI_m}$ and $\frac{L}{6EI_m}$ are the slope angles θ and β respectively for beams of constant moment of inertia I_m, and the factors φ_a and φ_b are constants found either from table 1 or table 11 corresponding to the type of haunches that the beam has. These constants depend also upon the following items of the beam:

KL = length of haunch, and

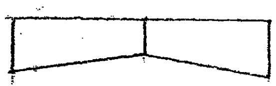
$$C = \sqrt[3]{\frac{I_a}{I_m}} - 1 \text{ , or} \quad \cdots\cdots\cdots\cdots (2)$$

$$C = \frac{d_a}{d_m} - 1 \text{ for beams having constant width}$$

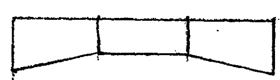
Table I. For beams with straight haunches.

φ_a = upper value, φ_b = lower value

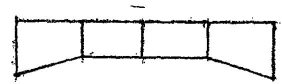
k \ n	0	0.4	0.8	1.0	1.2	1.4	1.6	2.0	3.0
1/2	1.0 1.0	.612 .685	.432 .518	.375 .460	.331 .414	.295 .376	.266 .344	.222 .294	.156 .215
1/3	1.0 1.0	.742 .843	.621 .758	.583 .729	.554 .705	.530 .686	.511 .669	.481 .643	.438 .601
1/4	1.0 1.0	.806 .907	.716 .856	.688 .839	.665 .825	.648 .813	.633 .803	.611 .787	.578 .761
1/5	1.0 1.0	.845 .939	.773 .905	.750 .894	.732 .884	.718 .876	.706 .869	.689 .859	.663 .842



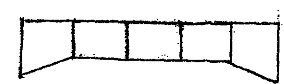
$k = \frac{1}{2}$



$k = \frac{1}{3}$



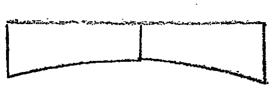
$k = \frac{1}{4}$

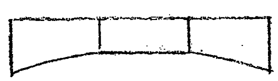


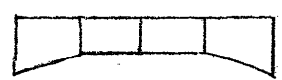
$k = \frac{1}{5}$

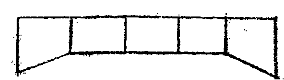
Table II. For beams with parabolic haunches, φ_a= upper value, φ_b= lower value.

K \ C	0	0,4	0.8	1.0	1.2	1.4	1.6	2.0	3.0
1/2	1.0 1.0	.730 .820	.592 .710	.545 .670	.507 .635	.475 .605	.449 .579	.406 .535	.336 .459
1/3	1.0 1.0	.819 .913	.728 .859	.696 .839	.671 .821	.650 .806	.632 .793	.604 .771	.557 .732
1/4	1.0 1.0	.865 .949	.796 .917	.772 .905	.753 .895	.738 .886	.724 .878	.703 .865	.668 .842
1/5	1.0 1.0	.892 .967	.837 .946	.818 .939	.803 .931	.790 .925	.779 .920	.762 .911	.734 .896


K = 1/2


K = 1/3


K = 1/4


K = 1/5.

2. <u>The change of the slope of a column.</u>

Let, M_c = the moment applied at the top of a column,

M_d = " " " " " foot " " "

θ_c = the change of slope at the top of a column due to $M_c = M_d = 1$

θ_d = the change of slope at the foot of a column due to $M_c = M_d = 1$

β_c = the change of slope at top due to $M_d = 1$, or the change of slope at foot due to $M_c = 1$.

The slope angles are expressed as (fig. 3):

$$\theta_c = \frac{1}{h} \cdot \int \frac{x \cdot dx}{EI}, \quad \theta_d = \frac{1}{h} \int \frac{x' \cdot dx}{EI}, \quad *$$

$$\text{and} \quad \beta_c = \frac{1}{h^2} \int \frac{x' \cdot x \, dx}{EI} \quad \cdots\cdots\cdots (3)$$

It is generally that the columns are built as shown in fig. 4 and in that case

$$\theta_d = \frac{h'(2h-h')}{2hEI_c} \quad *$$

$$\theta_c = \frac{h'^2}{2hEI_c} \quad \cdots\cdots\cdots\cdots (3_a)$$

$$\beta_c = \frac{h'^2(3h-2h')}{6h^2EI_c}$$

wher h' is the clear height of the column and h is the distance between the foot of the column and the neutral axis of the adjourning beams.

With constant $I = I_c$ throughout the column, then $h = h'$, and $\theta = \theta_c = \theta_d$, hence

$$\theta = \frac{h}{2EI_c}, \quad \text{and} \quad \beta_c = \frac{h}{6EI_c} \qquad \ldots\ldots\ldots\ldots (3_b)^*$$

The strengthening of the top part of the column due to beam haunches does not exert an influence so serious as the haunches to the beam itself, however, the effect depends largely upon their relative stiffness. With strong column and weak beams, i.e. beams easily subjected to deformation, the effect is very small and h' will be almost equal to h. In the contrary, the whole strengthening effect should be taken into consideration to warrant the correctness of the design.

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* The derivation of equations (3), (3_a) and (3_b) can be made without difficulty from the references given in section 1. The limits of integration of equation (3_a) are from 0 to h'.

3. Ascertaining of moment of inertias.

In reinforced concrete buildings the moment of inertia for beams and girders is generally not the same throughout the member, it is therefore necessary to compute and use the least moment of inertia for a given member. A chart for determining that for T-sections is reproduced here from the same book refered above, which can be used for T-columns and sometimes for beams and girders. The moment of inertia is expressed as :-

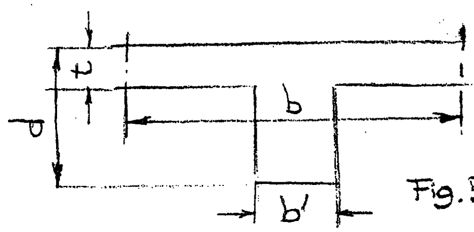


Fig.5

$$I = \frac{b \cdot d^3}{12} \cdot u \quad \cdots\cdots\cdots\cdots (4)$$

Where u is a constant obtained from the chart.

4. Characteristic point of contraflexure and moment factor.

By characteristic point of contraflexure is meant the zero moment point in a rigid member resulted by applying a unit moment at one end.

By moment factor is here understood as a facter by which the moment applied at a rigid joint is to be multiplied so as to obtain the actual portion of the moment resisted by each elastic member meeting at the joint.

Now let the charateristic point of contraflexure and its distance from either end of a beam or a column be denoted in the way as shown in fig. 7a and let

α_a = the change of slope at the left end of a beam due to $M_a = 1$,

α_b = the change of slope at the right end of a beam due to $M_b = 1$,

α_c = the change of slope at the top of a column due to $M_c = 1$, and

α_d = the change of slope at the bottom of a column due to $M_d = 1$.

then from Fig. 6_b, we have

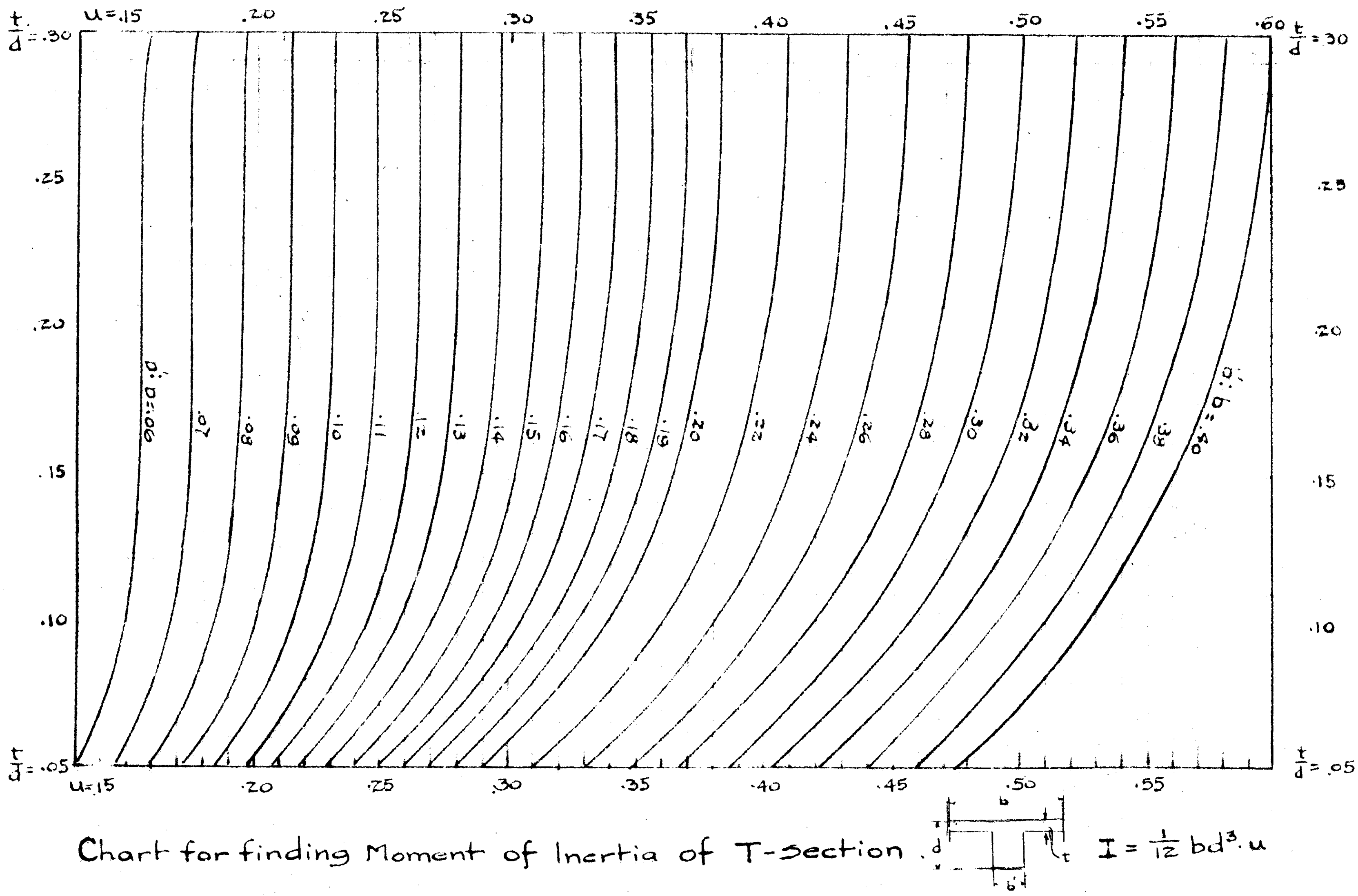


Chart for finding Moment of Inertia of T-section. $I = \frac{1}{12} b d^3 \cdot u$

The moment M=1 applied at B will produce a moment M_a at A. The magnitude may be obtained by simple proportion, whence,

$$M_a = -\frac{a}{L-a}$$

Now by definition θ_b is the slope angle produced by moments M=1 both at ends A and B (fig. 6c), then from figures 7a_c_d_e it is evident that

$$\alpha_b = \theta_b - \beta + M_a\beta$$

$$= \theta_b - \left(1 + \frac{a}{L-a}\right)\beta$$

$$= \theta_b - \frac{L}{L-a}\beta$$

With the same conception and draw similar figures we can derive expressions for α_a, α_c, and α_d. Thus we have

$\alpha_a = \theta_a - \frac{L}{L-b}\cdot\beta$, at left end of a beam

$\alpha_b = \theta_b - \frac{L}{L-a}\cdot\beta$, at right end of a beam . . (5)

$\alpha_c = \theta_c - \frac{h}{h-d}\cdot\beta_c$, at top of a column

and $\alpha_d = \theta_d - \frac{h}{h-c}\cdot\beta_c$, at foot of a column

The values of θ and β are given by previous equations, and mostly, for beams either of constant cross section or of symmetrical in form, $\alpha_a = \alpha_b = \alpha$. The values of a and b will be given later.

To ascertain the moment factors the transmission of the moment should be carefully considered. In order to avoid confusion the following sets of notations are used:

n_a = a fraction of the moment applied at a continuous column, which is taken by the column above.

n_b = a fraction of the moment applied at a continuous column, which is taken by the column below.

n_r = a fraction of the moment applied at a continuous beam, which is taken by the beam at right.

n_l = a fraction of the moment applied at a continuous beam, which is taken by the beam at left.

n_{ul} = a fraction of the moment taken by a column, when the moment is transmitted to it from any member adjourning its upper end.

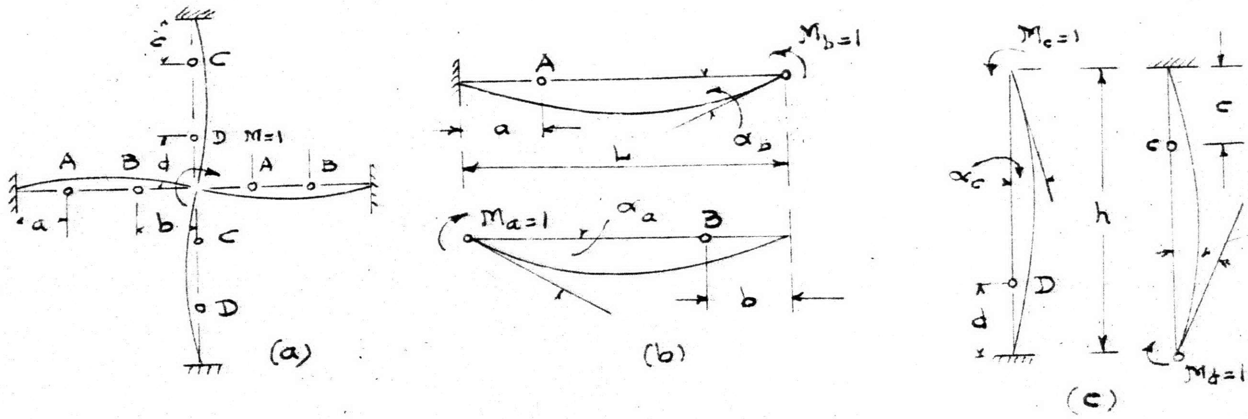
n_{lu} = a fraction of the moment taken by a column, when the moment is transmitted to it from any member adjourning its lower end.

n_{rl} = a fraction of moment taking by a beam, when the moment is transmitted to it from any member adjourning its right end.

n_{lr} = a fraction of moment taking by a beam, when the moment is transmitted to it from any member adjourning its left end.

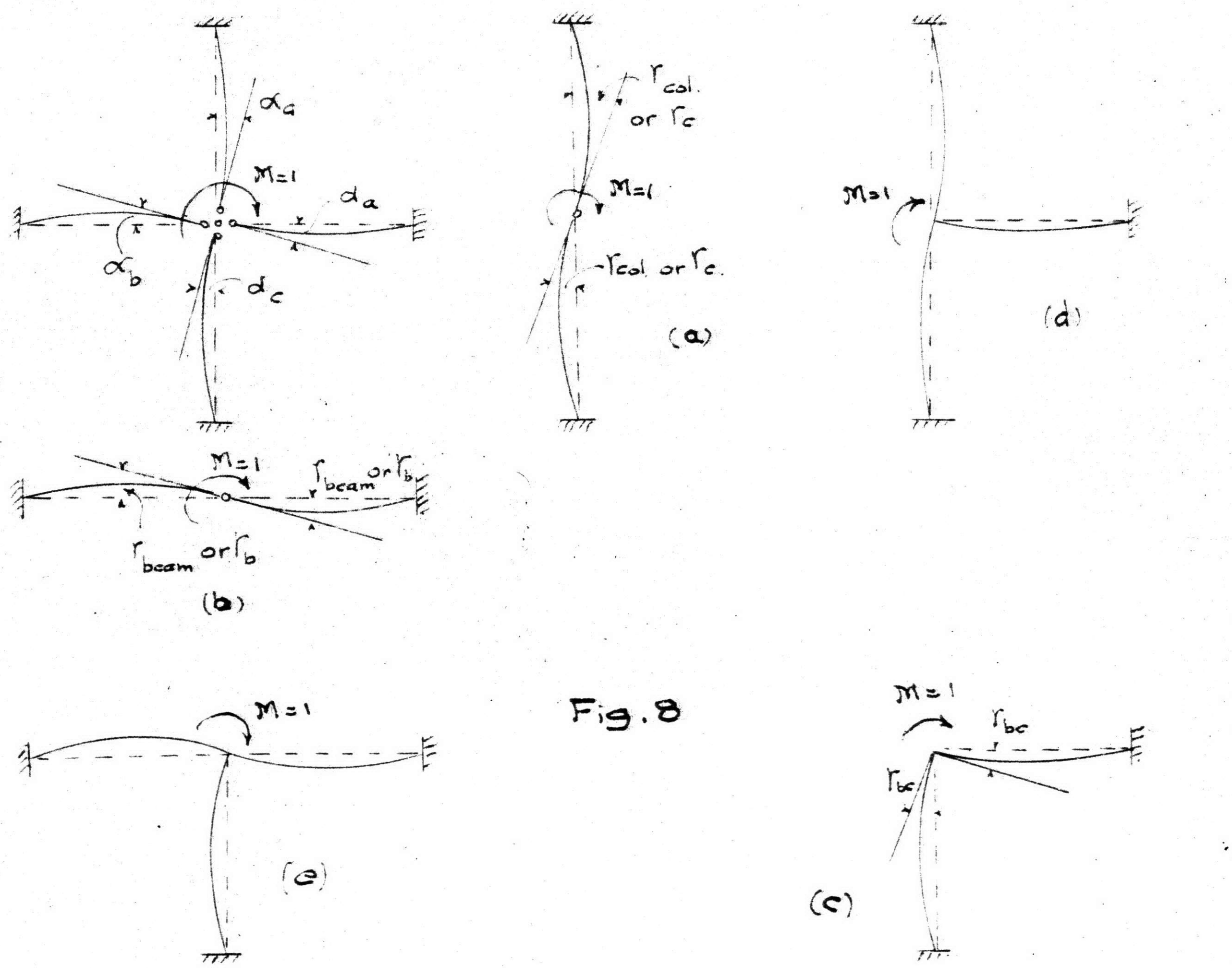
For clear understanding of these notations figure 16 given in the section on separation of moments will be refered.

Let us denote the resulting slope angle common to two rigidly connected bars due to $M = 1$ by "γ" with suffixes



A B C D M=1 A B
a b c d
(a)
A M_b=1 α_b a L
M_a=1 α_a B b
(b)
M_c=1 α_c c C D d h
M_d=1
(c)

Fig. 7



α_a M=1 α_b α_c
r_col. or r_c
M=1
r_col. or r_c.
(a)
M=1
(d)
M=1 r_beam or r_b
r_beam or r_b
(b)
M=1
(e)
M=1 r_bc
r_bc
(c)

Fig. 8

b, c, and bc representing the angles common to beams, columns, beam and column respectively.

Now let the four members given in figures 7 b andc be arranged as shown in figure 8, first consider the two vertical members, or columns, be connected rigidly and are continuous and a moment $M = 1$ is incurred at the joint, fig. 8 a, then the slope at the top of the lower column

$$= n_b \alpha_c = (1-n_a)\alpha_c$$

and of the bottom of the upper column

$$= n_a\alpha_d = (1-n_b)\alpha_d$$

By assumption II these two slopes should be equal to the common slope angle γ_c at the joint, therefore we have,

$$n_a = \frac{\alpha_c}{\alpha_c + \alpha_d}$$

$$n_b = \frac{\alpha_d}{\alpha_c + \alpha_d} \qquad \cdots\cdots\cdots(6)$$

and

$$\gamma_c = \frac{\alpha_c \alpha_d}{\alpha_c + \alpha_d}$$

Similarly from fig. 8b we can derive

$$n_r = \frac{\alpha_b}{\alpha_a + \alpha_b}, \qquad n_l = \frac{\alpha_a}{\alpha_a + \alpha_b}$$

$$\gamma_b = \frac{\alpha_a \alpha_b}{\alpha_a + \alpha_b}. \qquad \ldots\ldots\ldots\ldots (7)$$

Again consider the case shown in fig.8c. we have, for a moment coming from column to beam,

$$\gamma_{bc} = n_{rl}\alpha_b = (1 - n_{rl})\,\alpha_c$$

and for amoment coming from beam to column,

$$\gamma_{bc} = n_{ul}\alpha_c = (1 - n_{ul})\,\alpha_b$$

therefore,

$$n_{rl} = \frac{\alpha_c}{\alpha_c + \alpha_b}, \qquad n_{ul} = \frac{\alpha_b}{\alpha_b + \alpha_c}$$

$$\gamma_{bc} = \frac{\alpha_c \alpha_b}{\alpha_b + \alpha_c} \qquad \ldots\ldots\ldots\ldots (8)$$

In the similar way we can derive expressions for n_{lr}, n_{lu}, etc. By examining the equations (6), (7), and (8), the following rule holds true:

If a moment M=1 is applied at the joint of two bars, x and y, then the portion of the moment which will come to bar x,

$$n_x = \frac{\alpha \text{ at bar } y}{\alpha \text{ at bar } x + \alpha \text{ at bar } y} \qquad \ldots\ldots\ldots\ldots (9)$$

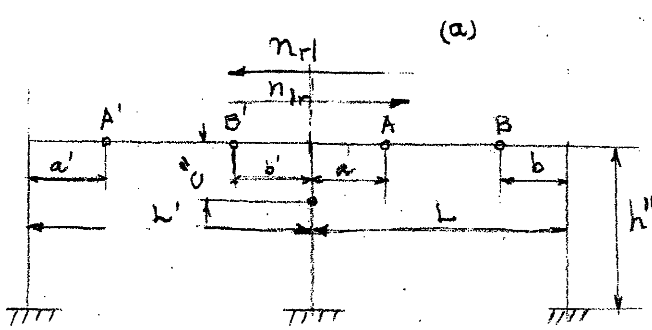
and the portion of the moment which will come to bar y

$$n_y = \frac{\alpha \text{ at bar } x}{\alpha \text{ at bar } x + \alpha \text{ at bar } y} \qquad \ldots\ldots\ldots\ldots (9)$$

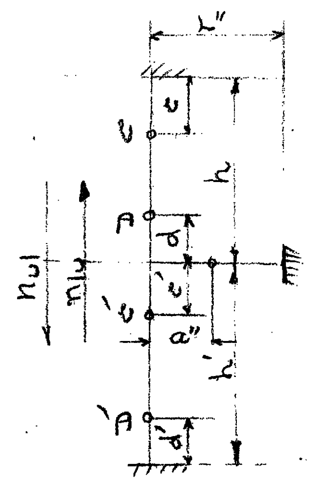
By α it is understood the angle caused by a moment M=1 applied only to the bar in question.

Fig. 9.

(a)



(b)



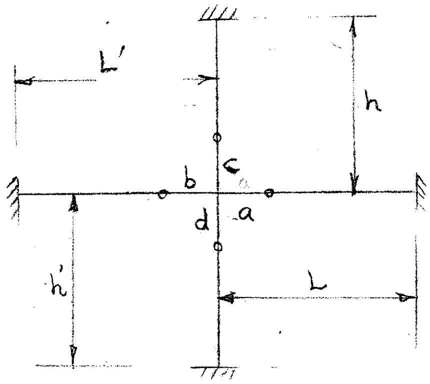


Fig. 10

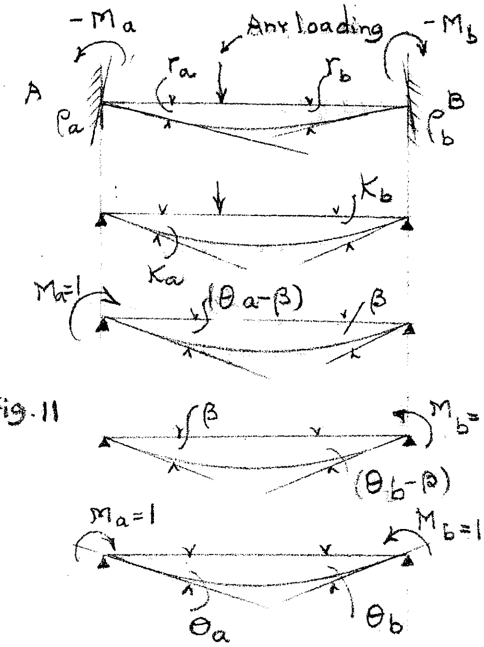


Fig. 11

So far we only have considered a combination of two members, for cases as shown in fig. 8c and 8d, equation (9) can still be applied. By looking the two continuous members as a single member and according to assumption (2) we have,

$$n_{rl} = \frac{\gamma_c}{\gamma_c + \alpha_b}, \qquad n_{lr} = \frac{\alpha_b}{\alpha_b + \gamma_c}$$

$$n_{ul} = \frac{\gamma_b}{\gamma_b + \alpha_c}, \qquad n_{lu} = \frac{\alpha_c}{\alpha_c + \gamma_b} \qquad \cdots\cdots\cdots (10)$$

In the same way we can obtain the value of n for other combination of members.
To find the values of a, b, and c, d, distances of point of contraflexure from ends of beams and columns respectively, as shown in figures 9 and 10 a general expression is here first derived:

Let ρ = change of slope of the support due to M=1 and k = change of slope at ends of the member due to subjected load. Using other notations as used before, then refer to fig.(11) it is evident

$$-\rho_a M_a = k_a + M_a(\theta_a - \beta) + M_b\beta$$
$$-\rho_b M_b = k_b + M_b(\theta_b - \beta) + M_a\beta \qquad (11)_a$$

If the member is only subjected to a moment, then

$$-\rho_a M_a = 0 + M_a(\theta_a - \beta) + M_b\beta$$
$$-\rho_b M_b = 0 + M_b(\theta_b - \beta) + M_a\beta \qquad (11)_b$$

Now since $M_a = \frac{-a}{1-a} + M_b$ (Fig. 6), by substitution we can find

$$a = \frac{l\beta}{\theta_{a} + \beta a} \qquad \text{..........(11)}$$

Similarly for expressions of b, c, and d.

From equation (11) and keeping in mind that ρ is the angle made by the support due to M=1 the following expressions can be derived:

From fig. (9)

$$a = \frac{L\beta}{\theta_{a} + n_{rl}\alpha_{b'}} \qquad b' = \frac{L'\beta'}{\theta_{b'} + n_{lr}\alpha_{a}}$$

$$c' = \frac{h'\beta_{c'}}{\theta_{c'} + n_{lu}\alpha_{d}}, \qquad d = \frac{h\beta_{c}}{\theta_{d} + n_{ul}\alpha_{c'}} \qquad \text{........(12)}$$

Where n_{rl}, n_{ul}, etc. are those given in equation (8)

$$c''' = \frac{h''\beta_{c'}}{\theta'_{c} + \gamma_{b}}, \qquad a'' = \frac{L''\beta}{\theta_{a} + \gamma_{c}} \qquad \text{...........(13)}$$

From Fig. (10)

Same expressions as equation (12) except n_{rl}, n_{ul} etc. are those given in equation (10) .....(14)

For beams and columns of constant moment of inertia, the expressions in equations (5), (12) and (14) can be very much simplified and in that case,

$$\beta = \frac{L}{6EI}, \text{ and } \beta_c = \frac{h}{6EIc}$$

$$\theta = \frac{L}{2EI} = 3\beta, \text{ and } \theta_c = \frac{h}{2EIc} = 3\beta c$$

Equation (5) is simplified to

$$\alpha_a = \beta\left[3 - \frac{L}{L-b}\right], \quad \alpha_b = \beta\left[3 - \frac{L}{L-a}\right].$$

$$\alpha_c = \beta c\left[3 - \frac{h}{h-d}\right], \quad \alpha_d = \beta_c\left[3 - \frac{h}{h-c}\right] \qquad \text{............(15)}$$

and all values of θ's in equations (12), (13) and (14) can be substituted by 3-times their corresponding β's.

5. The procedure of computation

For investigating an existing building or a building, of whose members the sections have already been approximately determined by the method given in the introductory, the distances of the points of contraflexure, a, b, c, and d from each end of the member have first to be found. In order to do this the following steps have to be taken:

(1) Determine the moment of inertia of each member by the aid of the given chart, if necessary.

(2) Determine the values of θ and β of each member by the aid of equation (1) and (3) and by the given tables, if nacessary.

(3) Beginning from the columns of the lowest story, which are generally fixed or rigid. The distance d will be:

Taking care of the strengthening effect of the column head,

$$d=\frac{3h-2h'}{2h-h'}\cdot\frac{h'}{3} \quad \cdots\cdots\cdots\cdots\cdots (16)$$

and when the same is neglected, $d=\frac{h}{3}$ ········ (17)

In the first case the angle at the top of the column,

$$\alpha_c=\frac{h'^3}{4EIc(3h[h-h']+h'^2)} \quad \cdots\cdots (18)$$

and in the second,

$$\alpha_c=\frac{h}{4EIc} \text{ or } \alpha_c=\frac{3}{2}\beta_c \quad \cdots\cdots\cdots\cdots (19)$$

(For derivation of equations 16, 17, 18, and 19 see appendix.)

In order to proceed the computation of angle "γ_c" of each column, it is nacessary first to determine "α_d", (equation 6) in determining the latter, the distance "c" of the next upper column has to be first assumed (equation 5). $c=\frac{1}{4}\cdot h$ is in most of the cases close enough.

With this assumption and the values of γ_c thus found we can determine, from left to right the distance "a", and from right to left the distance "b" by equation (13) for the beam close to the wall column, and by equation (14) for beams in interior spans.

(4) After knowing the location of points of contraflexure so proceed to determine the distance "d" of the columns in the story next above. In doing that use equation (7) to compute γ_b, equation (10) n_{1u}, and finally equation (14) gives the required distance d.

(5) The location of the points of contraflexure for beams above the second story will then be calculated. The angle α for the column below can be found either by equation (5) or (15). In order to do this it is again nacessary to assume the distance $c = \frac{1}{4} h$ of the column in question. In the same way as before we compute γ_c and the distances a, and b, for the beams.

(6) Repeat the process mentioned in (4) and (5), we can obtain all the distances a, b, c, and d for all stories of the building. Beginning from top we can finally determine the last unknown, i.e. the distance c, which was assumed before for each story of the building.

In case that the final value c obtained in (6) varies a great deal compared with the assumed value, so the process may be repeated by assuming c = its final value obtianed in the first computation. IN general, however, the first computation gives result sufficiently accurate, while in the second computation the result is almost exact. The degree of exactness to be attained depends, of course, upon the character of the building and the opinion of the designer.

Note: For $c = \frac{1}{4}$ h and for constant moment of inertia from equation (5), we have,

$$\alpha_d = \frac{h}{EI\ 3.6} \quad \text{or} \quad \alpha_d = \frac{5}{3} \beta_c$$

Appendix.

The Derivation of equations 16, 17, 18, and 19.

From equation (11)

$$d = \frac{h\beta_c}{\theta_d + \rho_d} \qquad \text{.............(A)}$$

From equation (3a)

$$\theta_d = \frac{h'(2h-h')}{2h\cdot EI_c}, \quad \theta_c = \frac{h'^2}{2h\cdot EI_c}, \quad \beta_c = \frac{h'^2(3h-2h')}{6h^2EI_c}.$$

also from equation (5)

$$\alpha_c = \theta_c - \frac{h}{h-d}\cdot\beta_c \qquad \text{.............(B)}$$

Substituting the values of θ_d and β_c in (A), we have

$$d = \frac{h'^2(3h-2h')}{3h'(2h-h') + 2hEI_c\rho_d}$$

For continuous and rigidly fixed columns $\rho_d = 0$, then

$$d = \frac{h'\cdot(3h-2h')}{3\cdot(2h-h')} \qquad \text{.............(16)}$$

If $h = h'$, then $d = \frac{h}{3}$ ....(17)

Similarly assume $\rho_d = 0$ and substituting the values of θ_c, β_c and d in (B), we obtain

$$\alpha_c = \frac{h'^2}{2h\cdot EI_c} - \frac{3h'^2(3h-2h')(2h-h')}{6h^2EI_c3(2h-h')-6hh'EI_c(3h-2h')}$$

$$= \frac{h'^3}{4EI_c(3h[h-h']+h'^2)} \qquad \text{.......(18)}$$

If $h = h'$, then $\alpha_c = \frac{h}{4EI_c}$

or, since $\beta_c = \frac{h}{6EI_c}$, $\alpha_c = \frac{3}{2}\beta_c$ ....(19)

PART 2

MOMEMTS AND STRESSES DUE TO VERTICAL LOADS.

In computing moments and stresses due to vertical loads a general assumption has here been made that the deflection due to vertical load at the top of the columns is equal to zero. The theories followed in this section are mostly given in Prof. W. Ritter's book " Anwendungen der Graphische Statik." It is endeavored, however, in this section to put these theories into practical adoption for the particular use to solve rigidly framed buildings. The use of moment factors developed in part 1 is a special feature, upon which the separation of the moment at each joint is based. This principle can be adopted for any kind of rigidly framed structures. In view of practical use several short cut methods for finding moments of a rigid member are presented here also.

1. Several short-cut methods for finding moments of rigid members.

From equations $(11)_a$, $(11)_b$, and (11) if k_a and k_b are not =0 we have, from equation (11),

$$\rho_a = -\theta_a - \beta \frac{L}{a}, \quad \rho_b = -\theta_b - \beta \frac{L}{b}$$

Substitute these values in $(11)_a$ and simplifying the following expressions result:

$$M_a \frac{L-a}{L} + M_b \frac{a}{L} = -\frac{a}{L} \cdot \frac{k_a}{\beta}$$

$$M_b \frac{L-b}{L} + M_a \frac{b}{L} = -\frac{b}{L} \cdot \frac{k_b}{\beta}$$

Refer to fig. (12) and by geometrical relation, we can see at once,

$$S_a = -\frac{a}{L} \cdot \frac{k_a}{\beta}$$

$$S_b = -\frac{b}{L} \cdot \frac{k_b}{\beta} \qquad \ldots\ldots\ldots (20)_A$$

For members of constant moment of inertia k_a and $k_b = \frac{1}{EI} \frac{A(L-\bar{x})}{L}$,

(Morley's Strength of Materials, page 176.) and $\beta = \frac{L}{6EI}$, (Part I) substitute these expressions into former equation, we obtain,

$$S_a = -\frac{a}{L}\cdot\frac{6A(L-\bar{x})}{L^2} = -\frac{a}{L}\cdot q$$

$$S_b = -\frac{b}{L}\cdot\frac{6A(L-\bar{x})}{L^2} = -\frac{b}{L}\cdot q' \quad \ldots\ldots\ldots\ldots (20)$$

Where A is the bending moment diagram due to the applied load when the member is simply supported.

$\bar{x}$ is the distance of the centroid of the area A from the origin A, and $(L-\bar{x})$ is its distance from B for S_a, & the reverse for S_b.

q and q' are the second factor of the right hand expression of each equation.

If we substitute c amd d for a and b respectively into equation (20), the expressions for S_c and S_d for a vertical member result. All the short-cut methods given below are based on equation (20).

For beams of varying cross section separate discussions are made in Case A and Case B.

— Case A. Uniform distributed load. Fig. 13.—

In this case the moment diagram for a simply supported beam is a parabola, Calling the moment at center =M, then,

$A = 2/3\,ML$, $\quad L-\bar{x} = \frac{L}{2}$, $\quad$ hence $q = q' = 2M$.

From simple geometrical relation of similar triangles it is evident that the graphical solution of S_a and S_b shown in fig. 13 holds. The intersection of the two crossing lines coinsides with the apex of the parabola.

For beams of varying cross section, we have for equation (20),

$$k_a = k_b = \frac{1}{E}\int_0^L \frac{M}{I_x}\,dx, \quad \text{(Morley's, page 183)}$$

$$\beta = \frac{1}{L^2}\int_0^L \frac{x'x\,dx}{EI_x} \qquad \text{(Part I,)}$$

By putting these two angles in the form of a ratio the factor

$\int_0^1 \frac{dx}{I_x}$ both in the numerator and in the denominator cancels out, therefore the variation of moment of inertia of the member does not effect the values of S_a and S_b in this case.

—Case B. Single concentrated load. Fig. 14—

As shown in the figure, the moment diagram for a simply supported beam under this case is a triangle, whose A=1/2 LM, $\bar{x}$ from A = 1/3 (L+x) and from B = 1/3 (L+x'), hence from equation (20)

$$q=\frac{M(L+x)}{L} \quad \text{and} \quad q'=\frac{M(L+x')}{L}$$

To find the values of q and q' without calculation, lay off the length L either side from the load, join the ends with the apex of the moment diagram, and extend the lines as shown, then q and q' are obtained, for by similar triangles the relation

$$q : M = L+x : L$$

$$\text{and} \quad q' : M = L+x' : L$$

exists. Again draw crossing lines in the way shown, then the values of S_a and S_b can be found. A similar relation holds, i.e.

$$S_a : a = q : L$$

$$\text{and} \quad S_b : b = q' : L$$

Now in case that the cross section of the beam varies, the solution of the problem is somewhat difficult here, for the factor $\int \frac{dx}{I_x}$ (compare Case A.) become

$$\int_0^x \frac{dx}{I_x} + \int_0^{x'} \frac{dx}{I_x} \quad \text{in the expression for angle } k,$$

$$\text{and} \quad \int_0^L \frac{dx}{I_x} \quad \text{in the expression for angle } \beta,$$

and these factors will not cancel out when expressed in the form of a ratio, therefore the variation of the cross section of the beam does have effect on the values of S_a and S_b.

To take care of this effect the work is complicated. In order to facilitate speedy solution, a table is reproduced here from "Strassener's Forscherarbeiten". The values of S_a and S_b can be taken directly from the table. For single con-

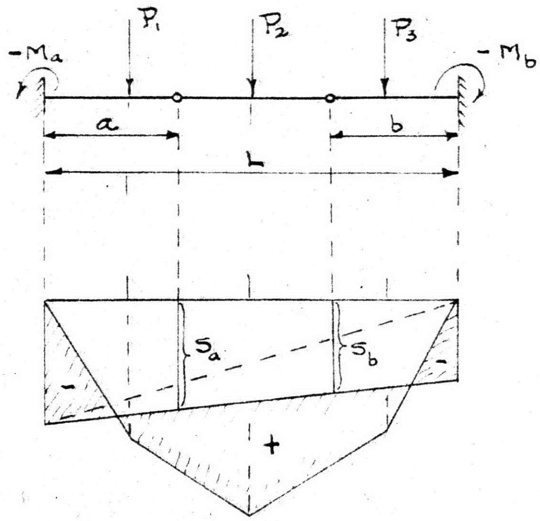


Fig. 12

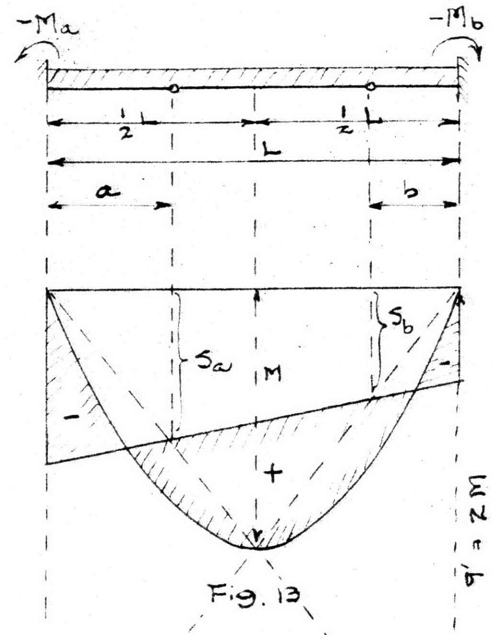


Fig. 13

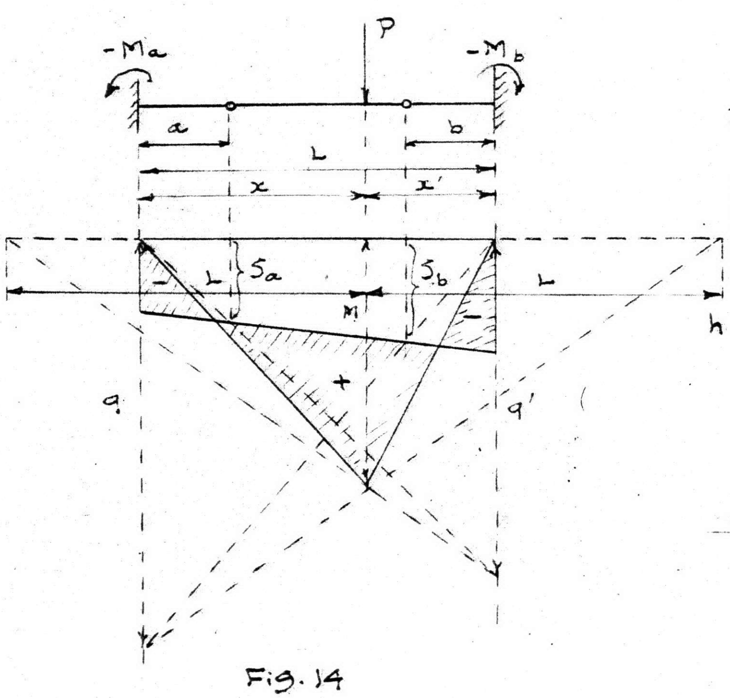


Fig. 14

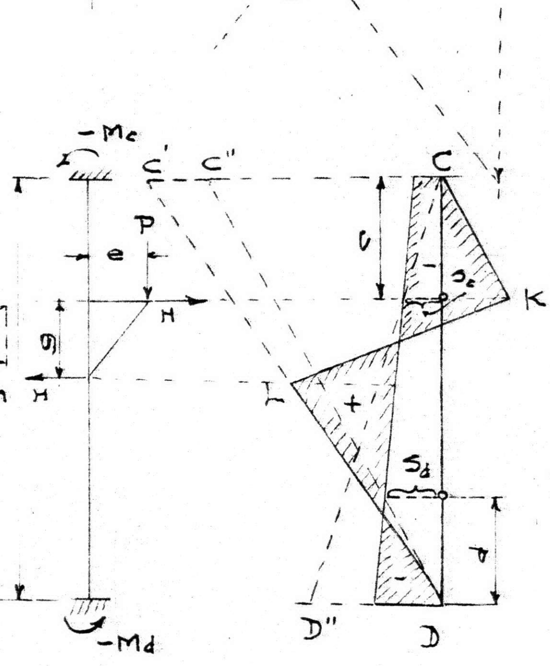


Fig. 15

Table for Values of $-\frac{S_a}{a}$ & $-\frac{S_b}{b}$ for $P=1$, $-\frac{S_a}{a}$, (upper figure), $-\frac{S_b}{b}$, (lower figure).

0 1 2 3 4 5 6 5 4 3 2 1 0

	$\frac{I_m}{I_a}$	For Straight Haunches 1	2	3	4	5	6	For Parabolic Haunches 1	2	3	4	5	6
Const. H	1.0	0.146 0.083	0.255 0.162	0.328 0.234	0.370 0.296	0.385 0.344	0.375 0.375	0.146 0.083	0.255 0.162	0.328 0.234	0.370 0.296	0.385 0.344	0.375 0.375
K = 1/2	0.20	0.117 0.083	0.220 0.165	0.304 0.242	0.365 0.313	0.401 0.369	0.402 0.402	0.123 0.083	0.230 0.164	0.312 0.241	0.368 0.308	0.396 0.361	0.393 0.393
	0.10	0.110 0.083	0.212 0.166	0.298 0.244	0.364 0.318	0.405 0.375	0.408 0.408	0.119 0.083	0.225 0.165	0.309 0.242	0.368 0.311	0.398 0.364	0.396 0.396
	0.05	0.106 0.084	0.206 0.166	0.294 0.246	0.364 0.319	0.407 0.379	0.412 0.412	0.117 0.083	0.223 0.165	0.308 0.243	0.368 0.312	0.399 0.366	0.398 0.398
K = 1/3	0.20	0.117 0.083	0.223 0.165	0.310 0.243	0.370 0.312	0.398 0.364	0.395 0.395	0.125 0.083	0.234 0.164	0.317 0.240	0.371 0.307	0.394 0.358	0.388 0.388
	0.10	0.112 0.083	0.218 0.166	0.307 0.244	0.370 0.315	0.401 0.368	0.398 0.398	0.122 0.083	0.230 0.165	0.315 0.241	0.371 0.309	0.395 0.360	0.389 0.389
	0.05	0.110 0.084	0.215 0.166	0.305 0.245	0.370 0.316	0.402 0.370	0.400 0.400	0.120 0.083	0.229 0.165	0.314 0.242	0.371 0.309	0.396 0.361	0.391 0.391
K = 1/4	0.20	0.122 0.083	0.232 0.165	0.318 0.242	0.372 0.308	0.394 0.358	0.388 0.388	0.129 0.083	0.240 0.164	0.322 0.239	0.371 0.304	0.391 0.353	0.384 0.384
	0.10	0.118 0.083	0.229 0.165	0.317 0.243	0.372 0.309	0.395 0.360	0.390 0.390	0.127 0.083	0.238 0.165	0.321 0.240	0.371 0.305	0.392 0.354	0.385 0.385
	0.05	0.117 0.084	0.227 0.166	0.316 0.243	0.372 0.310	0.398 0.361	0.391 0.391	0.126 0.083	0.237 0.165	0.321 0.240	0.372 0.305	0.392 0.355	0.385 0.385
K = 1/5	0.20	0.126 0.083	0.239 0.165	0.322 0.240	0.372 0.304	0.391 0.354	0.384 0.384	0.132 0.083	0.245 0.164	0.324 0.238	0.371 0.302	0.389 0.350	0.381 0.381
	0.10	0.124 0.083	0.237 0.165	0.321 0.241	0.372 0.306	0.392 0.355	0.385 0.385	0.131 0.083	0.243 0.164	0.324 0.239	0.371 0.302	0.390 0.351	0.382 0.382
	0.05	0.123 0.084	0.236 0.165	0.321 0.241	0.372 0.306	0.393 0.356	0.386 0.386	0.130 0.083	0.243 0.164	0.324 0.239	0.371 0.303	0.390 0.352	0.382 0.382

centrated load multiply the table value by the load and for several concentrated loads or for a combination of concentrated load and uniform load take each load separately and sum up the moments.

Case A and Case B hold also for vertical members.

--Case C: Columns subjected to crane load. Fig. 15.

The applied crane load P causes a moment P·e, which is equavalent to the moment caused by a horizontal couple H as shown. Lay off CC' equal to the moment P·e. Draw line CK parallel to C'D and join LK as shown, then the resulting area bound by the broken lines CK, KL, LD, and CD will be the moment area of the column due to the crane load P with no regard to the continuity of the column. The value of A and $\bar{x}$ can be easily computed. Compute q and q' from equation (20), and lay off the same equal to CC" and DD" in the figure, then join C"D and CD", the values of S_c and S_d are found. The proof is the same as the first two cases.

2. Separation of moments and their signs.

The moments due to vertical load will, in general, be investigated by considering each single span separately as shown in fig. 15, in which it brings out the way of the separation of moments onto the columns and beams surrounding the loaded central span.

If the moments M_a and M_b for the loaded span be determined according to the method given in the previous section, then at the column to the left of the load, the beam will take a moment = $n_{rl} \cdot M_a$ and the total moment which falls to the adjacent columns at left = $\Delta M_a = M_a - n_{rl} M_a$. From this moment the lower column takes an amount = $n_b \Delta M_a$ and the rest falls onto the upper column. Similarily at the column to the right of the load, the beam takes a moment = $n_{lr} \cdot M_b$, the total moment which falls onto the adjacent columns at right = $\Delta M_b = M_b - n_{lr} M_b$, from which the lower column takes a portion = $n_b \Delta M_b$ while the upper takes the rest.

The transition of moment lines for unloaded spans

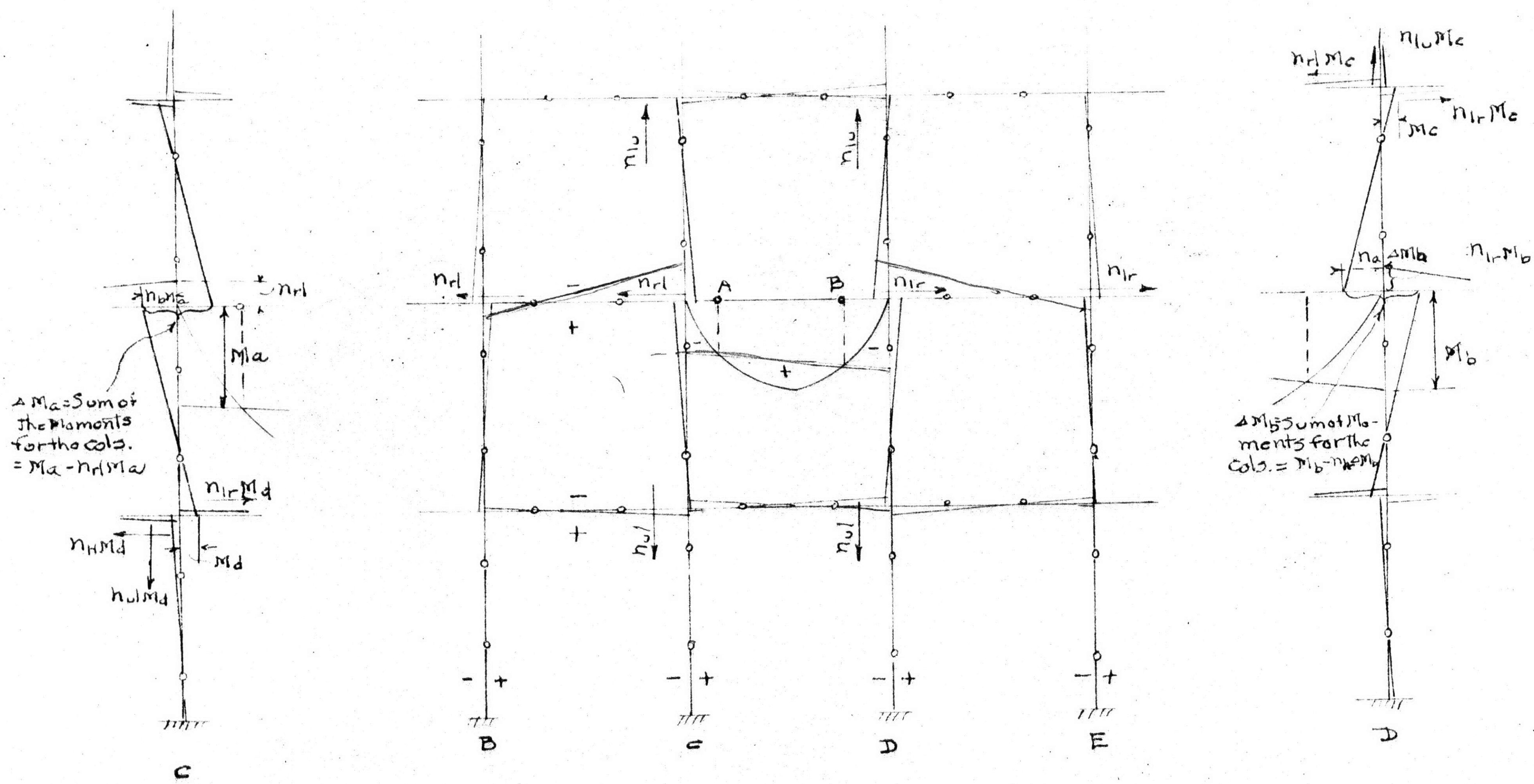

ΔMa=Sum of the Moments for the cols. = Ma - nrl Ma
Ma
nrl
n1r Md
n1l Md
Md
nul Md
n1u
nul
n1r
A
B
ΔMb=Sum of Moments for the cols. = Mb - nrl Mb
Mb
n1u Mc
nrl Mc
n1r Mc
Mc
n1r Mb
B
C
D
E

Fig. 16

is as follows:

For all spans to the right of the loaded the moment line passes through the point B,

for all spans to the left of the loaded the moment line passes through the point A,

for all the columns above the moment line passes through the point C, and

for all the columns below the moment line passes thru. the point D.

The points A, B, C, and D are the points of contraflexure as marked in figure 6, part 1.

For the joints in stories next above or below the moment transmitted over will be sepatated again. Let the moment to be be reseparated be called M_c and M_d, then at the joint next above the reduced column moment = $n_{lu} \cdot M_c$, the total moment to be resisted by the beams = $\Delta M_c = M_c - n_{lu}M_c$, from which the left beam takes a portion = $n_l \Delta M_c$ and the rest falls to the right beam. Similarly at the joint next below the reduced column moment = $n_{ul} \cdot M_d$, the beams have to resist a moment = $\Delta M_d = M_d - n_{ul} \cdot M_d$, from which the beam at left takes the portion $n_l \cdot \Delta M_d$, while that at right takes the rest.

The moment transmitted and to be reseparated becomes smaller and smaller, the further is the span from the loaded one.

At the end columns the moment carried over by the beam should be separated in such a way that it will wholly be taken by the columns, so in that case $\Delta M_a = M_a$ and $\Delta M_b = M_b$.

In the above discussion we have only considered the absolute magnitude of the moments. However, in case that several spans are loaded, the moment due to the load in one span may in most cases cancel that due to the load in the other. In order to get the algebric sum of the moments due to the loadings in various spans and to avoid confusion the following conventional signs will be found helpful=

" The moment is considered positive, if the elastic curvature of the beam axis at the portion considered slopes downward, and nagative, if it slopes upward.

The moment is considered positive, if the elastic curvature of the column axis at the portion considered slopes to the right, and negative, if it slopes to the left."

In figure 15 the positive moment is drawn below and to the right of the beam and column axes respectively and the negative moment, above and to the left.

Keeping the above signs in mind it is very easy to determine the character of the moments by considering the elastic deflection of the member.

3. Determination of shear and normal forces.

For all members of a building, if their moments at joints are known, we can find the shear and normal forces very easily. For rectangular construction advantage can be taken from the fact that, just as the shear is perpendicular to the normal force, the column axis is perpendicular to the beam axis, therefore we can find the normal force of a column from the shear of a beam and conversely the normal force of a beam from the shear of a column.

Let Q be the normal force or the shear required, Q_o be the same corresponding to a simply supported member, and, as before, M_a and M_b the moment at the left and the right end of a fixed beam, and M_c and M_d the moment at the top and the bottom of a fixed column respectively, then, taking positive shear as that directed to the top at the left section of a beam, and that directed to the left at the lower section of a column, we have :

For the shear of the beam or normal force of the column in question

$$Q = Q_o - \frac{M_a - M_b}{L} \quad \ldots\ldots\ldots (21)$$

For the shear of the column or normal force of the beam in question

$$Q = Q_o + \frac{M_c - M_d}{h} \quad \ldots\ldots\ldots (22)$$

For columns , since they are generally free from external forces, then $Q_o = 0$, and

$$Q = \frac{M_c - M_d}{h} \quad \ldots\ldots\ldots (23)$$

The derivation of the above expressions follows immediately by taking $\Sigma M = 0$ of the member considered.

4. Criteria for maximum combined stresses in beams and columns.

Both the maximum stresses in beams and columns occur from the source of maximum bending moment and direct stress resulting from normal force. For columns, especially those of very slender section, the deflection, which we so far assume to be zero, may cause an additional stress, but the stress due to maximum moment is , in almost all cases , greater than that due to maximum deflection and these two will not occur simultaneously. If we load alternate bays of the structure so as to give maximum bending moment to the column in question, (see discussions given later) the deflection caused by the load on either side is of compensating character and the stress resulted therefrom, if any, is generally small and negligible. From figiure 16 it can be easily seen the following criteria for maximum moments and hence the maximum stresses of the beams and columns:

For beams maximum positive moments are caused by loading all the odd number of bays in both directions from the bay in question, counting the latter zero. The negative moments are caused by loading all the even bays in the direction and all the odd bays away from the direction of the beam end in question, counting in the same manner the loaded bay zero.

For interior columns maximum moments are caused by loading the bay adjacent to the column in question for the full hight of the structure, and, where possible by loading alternate bays in both directions from the bay.

For exterior columns loadings causing maximum moments are similar to interior columns.

For corner columns the moment may be considered as being introduced into the column by the two girders meeting at right angles to each other, and the moments may then be combined to give a diagonal resulting moment.

The maximum stresses in all members will be found by combining the stress caused by the maximum moment with the normal force found by methods given in section 3.

PART 3 .

MOMENTS AND STRESSES DUE TO HORIZONTAL LOADS.

In high building design the stresses caused by the wind load applied in horizontal direction play such an important rôle that a close investigation always seems to be justified. For buildings, of which the width is small compared with its height, the wind stress is always a great factor. In this part of the work the general theories given in Prof. W. Ritter's and Dr. Strassener's books will be followed. It is endeavored, however, that the theories are extended and put in unique manner for the practical solution of high buildings due to horizontal loads.

In dealing with horizontal loads several approximate methods given by various authorities have been known, and they are generally deemed as simple and practical. Nevertheless , none of them seems to be consistant with the actual condition. The exact method given in bulletin 80 of engineering experimental station of the University of Illinois and sevetal other known methods derived from the theory of least work are all too laborious and they are, therefore, restricted in almost all cases in its practical adaptibility.

In the preceeding parts we have made one assumption, that is, the horizontal deflection of a rigid joint due to vertical loads is equal to zero. This assumption, though it does not conform to the theoretical correctness, is reasonable for the solution of the moments and stresses due to vertical loads, since the effect is small and negligible. In dealing with horizontal loads this assumption will no more hold, we have here two kinds of moment to consider:

a. Moments caused by loads considering the rigid joints not being subjected to any deflection.

b. Additional moments caused by resulting deflection.

The present method will be described under various sections given below, from one-story-frame to the general case of many-story frames.

1. Derivation of expressions for moments due to deflections.

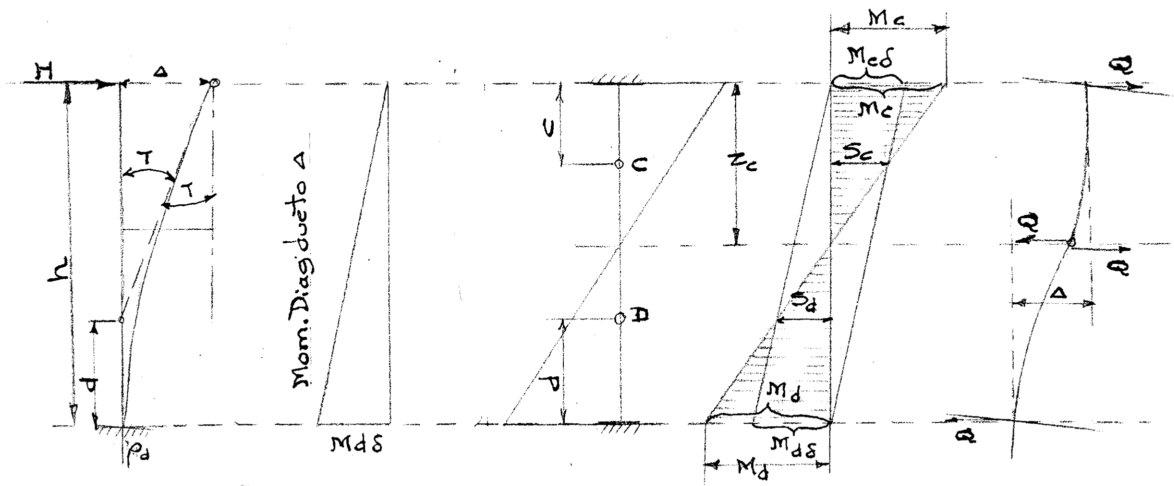


Fig. 17.   Fig. 18.

Considering the case as shown in figure (17) the column has an elastic support at its lower end, the upper end is free to move and subjected to a horizontal load H, then the resulting deflection is

$$\Delta = H\lambda' + Hh^2\rho_d$$

and the angle, through which the top of the column is turned, is

$$\tau = H\alpha' + Hh\rho_d$$

where $\lambda' = h^2(\alpha_d - \beta_c)$ and $\alpha' = h\alpha_d$

For expressions of α_d and β_c refer to Part 1.

By substitution, we have

$$\Delta = Hh^2(\alpha_d + \rho_d - \beta_c)$$

and $$\tau = Hh(\alpha_d + \beta_c) \qquad \cdots\cdots (X)$$

Deviding the former expression by the latter, we have

$$\frac{\Delta}{\tau} = h - \frac{h\beta_c}{\alpha_d + \rho_d}$$

But from equation (11), Part 1

$$\frac{h\beta_c}{\alpha_d + \rho_d} = d$$

therefore $$\frac{\Delta}{\tau} = h - d$$

or, $$\tau = \frac{\Delta}{h-d}$$

The moment at the foot of the column due to the deflection Δ caused by the horizontal load H is then

$$M_{d\delta} = -Hh = -\frac{\tau}{\alpha_d + \rho_d} \qquad \text{(From equation X)}$$

$$= -\frac{\Delta}{(h-d)(\alpha_d + \rho_d)}$$

or, since $\alpha_d + \rho_d = \frac{h\beta_c}{d}$,

$$M_{d\delta} = -\frac{d \cdot \Delta}{(h-d)h\beta_c} \qquad \ldots\ldots\ldots\ldots (24)$$

Equation (24) is the general expression for moment due to deflection, when the deflecting end is free to move.

Again refer to figure 18 the column has elastic supports at both ends and the characteristics of the member, i.e. distances c and d, are already found according to the method given in Part 1. First assume the top end of the column is free to move and is subjected to the shear Q, then we have the moment $$S_d = -M_{d\delta}\frac{h-d}{h} = -\frac{d\Delta}{h^2\beta_c}$$

Similarly by assuming the lower end of the column being free to move and subjected to the horizontal shear Q, we have for

$$S_c = M_{c\delta}\frac{h-c}{h} = \frac{c\Delta}{h^2\beta_c}$$

Now in the actual condition of the column the ends does not move freely. As soon as the deflection Δ occurs at one end, there produces moments M_c and M_d at their respective ends; although these end moments will offset a part of the angular turning of the column ends, they, however, do not have any effect on the value of the moments S_c and S_d, since by hypothesis of Part 1, point C is the zero-moment point of the column due to a moment acting at the bottom end of the column, and point D is the corresponding point of the same due to a moment at the top end.

From above discussion we can have the following expression to find the distance z_c, distance of zero-moment point from the top end:

$$\frac{S_c}{-S_d} = \frac{z_c - c}{h-z_c-d}$$

$$z_c = d + \frac{h-d-c}{1-\frac{S_d}{S_c}}$$

By substituting the values of S_c and S_d in the above expression we have

$$z_c = \frac{ch}{c+d}$$

which gives the location of the zero-moment point.

Now at point C the moment S_c also equal to $Q(z_c - c)$, from which relation we obtain

$$Q = \frac{\Delta}{h^2\beta_c}\cdot\frac{c}{z_c - c}\ .$$

From the figure it is evident that

$$M_c = Q \cdot z_c$$

and

$$M_d = \mp Q(h \mp z_c)$$

By proper substitution the equation for moments is expressed in the following general form:

$$\left.\begin{aligned} M_c &= m \cdot c \\ M_d &= m \cdot d \\ \text{Where } m \text{ is a constant} &= \frac{\Delta}{\beta_c h(h \mp c \mp d)} \end{aligned}\right] \quad \ldots\ldots\ldots\ldots (25)$$

In computing the moments of a building frame the value of Δ in equation (25) can be assumed as unity. After M_c and M_d due to unit deflection are found, we compute the shear Q of each column from the following equation:

$$\left.\begin{aligned} Q &= \frac{(M_d + M_c)}{h} \\ &\text{for moments of opposite signs, and} \\ Q &= \frac{(M_d - M_c)}{h} \end{aligned}\right] \quad \ldots\ldots\ldots\ldots (26)$$

for moments of the same sign.

The sum of the internal shear "P" should be equal to the external load H. The actual moments are found by multiplying the values of M_c and M_d due to unit deflection by the factor $\frac{H}{P}$.

The separation of the moments will be done by the aid of moment factors as described in Part 1 and 11.

In order to make the method clear the application of the method to various cases are given in different sections following.

2. Single-story frame.

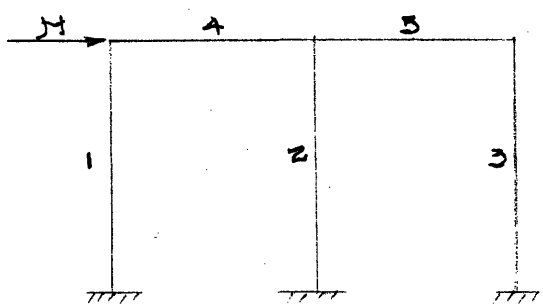


Fig. 19

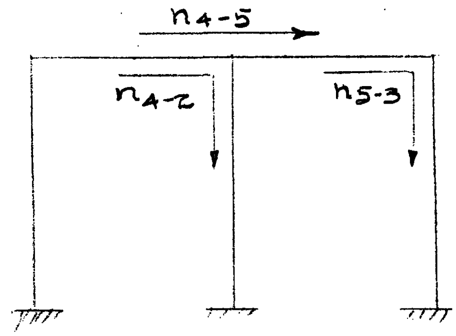
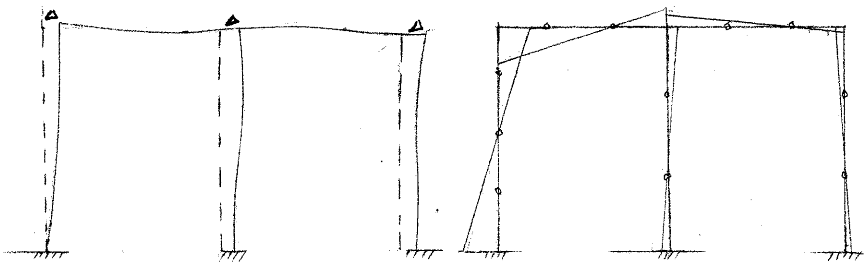


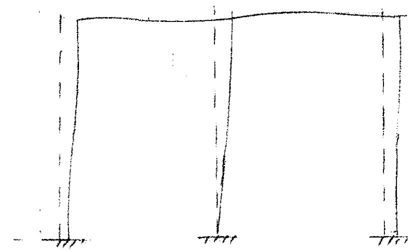
Fig. 20

Let us assume the case given in figure 19, a single story, two span frame acted upon by a horizontal force H at the top girder. The columns may be either hinged, elastic, or fixed.

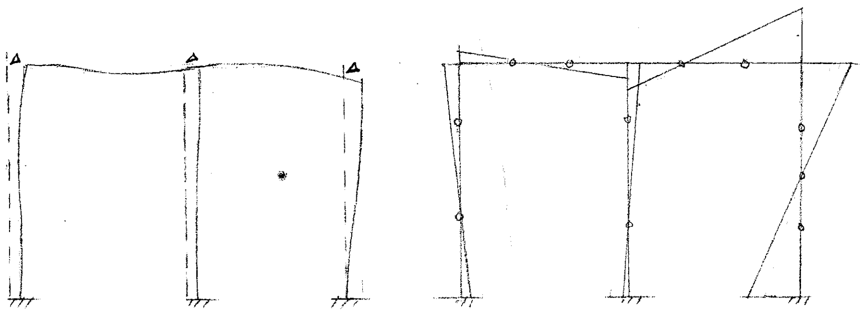
The horizontal force H causes a deflection of the columns at the top end in the direction of the applied horizontal load. The deflections at all columns are the same, because the top ends of the columns are connected by a continuous girder and the effect of normal stress can be neglected. The magnitude of the deflection due to the applied horizontal load is unknown, we can, however, first assume a unit deflection and by the aid of equation (25) the moments due to that deflection can be solved. From the shear calculated therefrom we can determine the force P which produces the assumed deflection. By multiplying the moments by the coefficient $\frac{H}{P}$, the required moments due to the horizontal load H can be found. The procedure of computation is as follows:



(a) Fig. 21 (b)



(a)



(a) Fig. 23 (b)

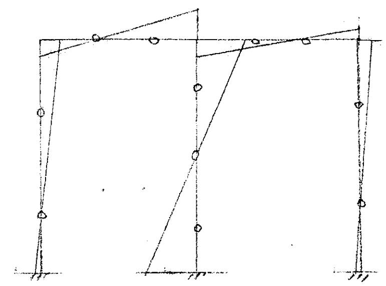
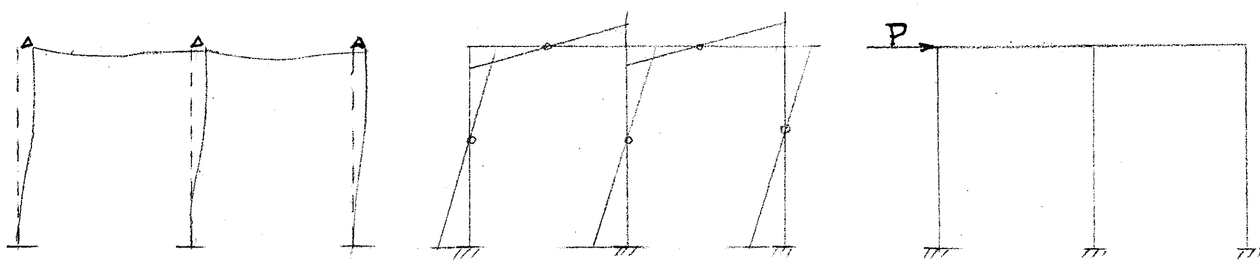


Fig. 22 (b)



(a) (b) (c)

Fig. 24

1. Compute the characteristics of the frame, i.e. distances a, b, c, and d, and the moment factors of the corresponding members by the method given in Part 1.

2 . Treat each column separately and assume the column in question either as fixed or elastic, as the case may be, and the rest as free to move. (Figures 21 a, 22 a, and 23 a.) Compute the moment of each member in each case by equation (25) in which the value of Δ is taken as unity. The reason for taking each column separately is for the convenience of separating the moments by moment factors to each member. The

moment diagrams in each case are shown in figures 21 b, 22 b, and 23 b. In actual computation these diagrams need not be drawn and the work can be done mentally.

3. Taking the algebraic sum of the moments, it gives the combined diagram figure 24 b. The resulting moment in each member is that due to the assumed unit deflection.

4. Apply equation (26) and find the values of Q_1, Q_2, and Q_3, fig. 25, then the sum of the shears must be equal to the load P, the load which causes the unit deflection and produces the moments, or

$$P = Q_1 + Q_2 + Q_3$$

5. Find the coefficient of $\frac{H}{P}$ with which the moments in figure 24 b are to be multiplied to give the required moments of the members of the frame due to the horizontal load H.

Fig. 26 shows a general case for the srparation of moments due to deflection by moment factors.

3. Two-story frames.

For two story frames the moment of each member are due to the combined effect of horizontal loads H_1 and H_2, fig. 27, Through the influence of theseloads the frame changes its shape as shown in fig. 28, i.e. all the joints deflect horizontally in the direction of the load. Now since the magnitude of the horizontal deflection is not known so we have to get at it in some indirect way as before, by the aid of an assumed deflection and the moments resulted therefrom.

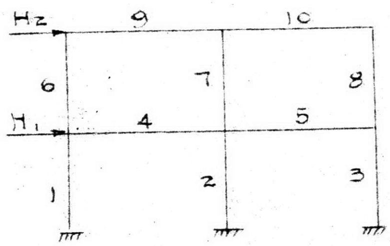


Fig.27

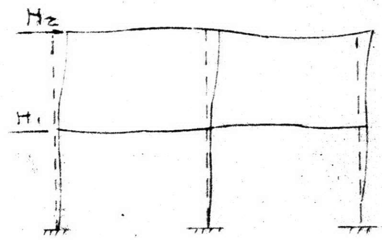
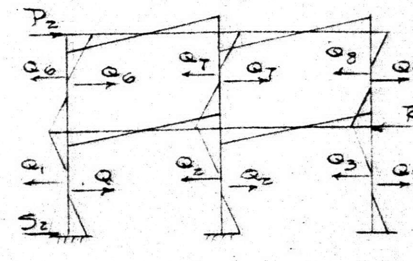
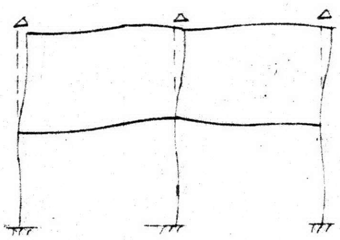


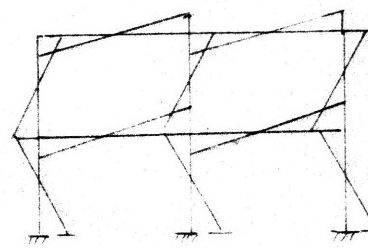
Fig.28



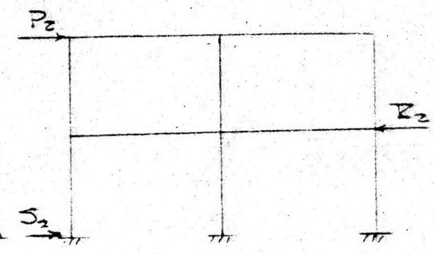
(d)



(a)

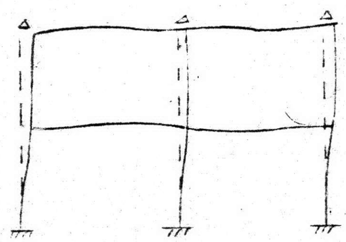


(b)

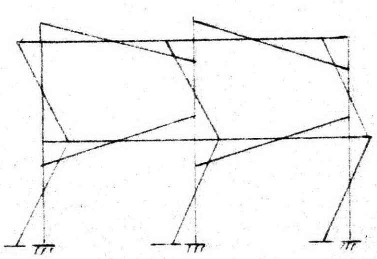


(c)

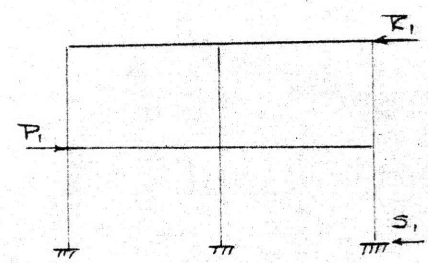
Fig.29



(a)



(b)



(c)

Fig.30

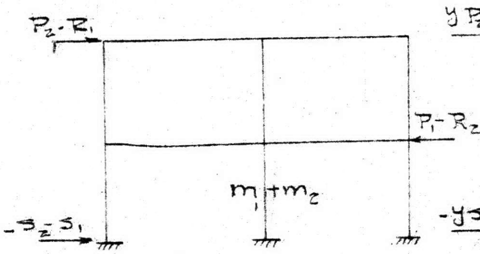


Fig.31

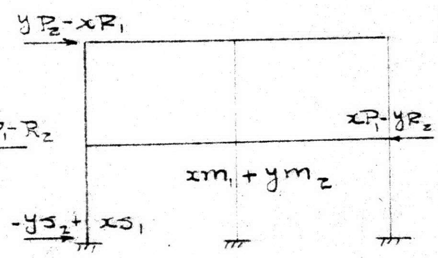


Fig.32

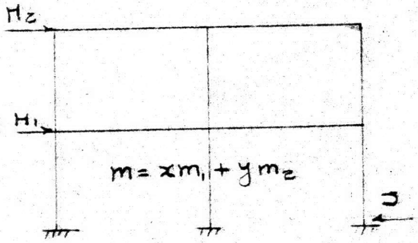


Fig.33

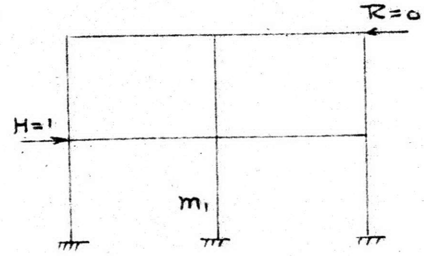


Fig.34

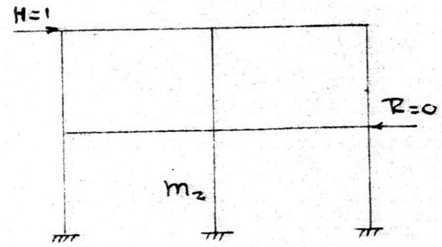


Fig.35

For two story frame we have to consider two conditions of deformation. For the first condition assume the top girder of the frame is subjected to a deflection Δ=unity, while the lower girder remains in its original position (fig. 29 a.). The moment of each member can be computed in the same way as for one story frame, in this we again consider each of the three columns separately and compute moments by equation (25). The moment for each case is separated and the algebraic sums for the three cases are found. The external forces acting on the frame, which produces the deformation of condition 1 are (fig. 29 d)

On upper girder

$$P_1 = Q_6 + Q_7 + Q_8$$

On lower girder

$$R_1 = Q_6 + Q_7 + Q_8 + Q_1 + Q_2 + Q_3$$

and on the footings

$$S_1 = Q_1 + Q_2 + Q_3$$

The suffix given to P, R, and S denote the number of the condition of deflection considered.

For the second condition a deflection of Δ = unity at lower girder will be considered. The upper part of the frame is free to move, so as soon as a deflection is given to the lower girder, the upper girder will have a deflection greater than Δ. The latter will be assumed as being restricted from producing a deflection greater than Δ, so that the relative position of the upper part of the girder to the lower part does not change. (The deflection of the upper girder cannot be assumed as being restricted to zero, as in that case another moment due to deflection in opposite direction will be resulted, which violets the condition of moment separation). After the moments in three cases are combined and shears found, the external forces will be

(fig. 30 c):

On upper girder,

$$R_2 = Q_6 + Q_7 + Q_8$$

on lower girder,

$$P_2 = Q_6 + Q_7 + Q_8 + Q_1 + Q_2 + Q_3$$

and on the footings.

$$S_2 = Q_1 + Q_2 + Q_3$$

Now we have two load diagrams on hand, figures 29 c and 3o c. The first produces a deflection unity at the upper girder and the second produces the same at the lower. In both of these diagrams is also shown the elastic behavior of the frame against the acting loads. From the moments resulted from these assumed deflections we can determine the moments due to any horizontal load by some simple modification.

Let the moments caused by the loads in fig. 29 c be called m_1 and those caused by the loads in fig.30 c be called m_2, then by addition we have a combined diagram with forces

$P_2 - R_1$ acting on upper girder,

$P_1 - R_2$ " " lower " Fig. 31

and $S_1 - S_2$ " on the supports.

The combined moments are then $= m_1 + m_2$.

It should be kept in mind that this combined load diagram gives a unit deflection both at upper and lower girders. Now in order to conform the actual deflection for a specific case let the loads in diagrams 29 c and 30 c be multiplied by the coefficients x and y respectively, the combined loads, fig. (32), will now be :

$yP_2 - xR_1$ on the upper girder,

$xP_1 - yR_2$ on the lower girder.

and $-yS_2 + x S_1$ on the supports.

Similarly the moments $= xm_1 + ym_2$

It is evident that for the applied loads H_1 and H_2

$$yP_2 - xR_1 = H_2$$

and $$xP_1 - yR_2 = H_1$$

From these two expressions the coefficients x and y can be found and the moments computed from

$$M = xm_1 + ym_2$$

In practice in order to find the maximum moments it is generally required to treat the horizontal loads as moving load and in that case the values of influence moments have to be computed, i. e. the moments due to a horizontal load unity at each panel point separately, thus the conditional equations are as follows:

For a load unity acting at panel point 1, (fig. 34) at lower girder.

$$y_1P_2 - x_1 R_1 = 0$$

$$x_1P_1 - y_1R_2 = 1$$

and $$M_1 = x_1m_1 + y_1m_2$$

For a load unity acting at panel point 2, (fig.35) at upper girder,

$$y_2P_2 - x_2R_1 = 1$$

$$x_2 P_1 - y_2R_2 = 0$$

and $$M_2 = x_2m_1 + y_2 m_2$$

For the load condition shown in fig. 33, $M = M_1H_1 + M_2H_2$

4. Three-and more-story frames.

The method of computation given above can be extended to the solution of three and more story frames. There are as many conditions of deflection as are number of stories. From the load diagrams the elastic behavior of the frame against the influence of horizontal loads will be given. For any specific case each condition of deflection gives a load diagram and a corresponding sets of coefficients. There are always as many conditional equations as are unknown coefficients.

To make the method more evident it is given below for a three story frame the deflection conditions with their corresponding moment- and load-diagrams (figures 32 a-c) and the conditional equations for the determination of coefficients for each case of loading.

For the specific case shown in figure 40 the conditional equations are:

$$xP_1 - yR_2''' + zR_3''' = H_3$$

$$yP_2 - xR_1'' - zR_3'' = H_2$$

$$zP_3 + xR_1' - yR_2' = H_1$$

and the moments

$$M = xm_1 + ym_2 + zm_3$$

If the horizontal loads are taken as moving load, then the conditional equations and expressions for influence moments for a unit load applied at each panel point separately are as follows:

For a load unity acting at first floor girder, Fig. 37, the conditional equations are:

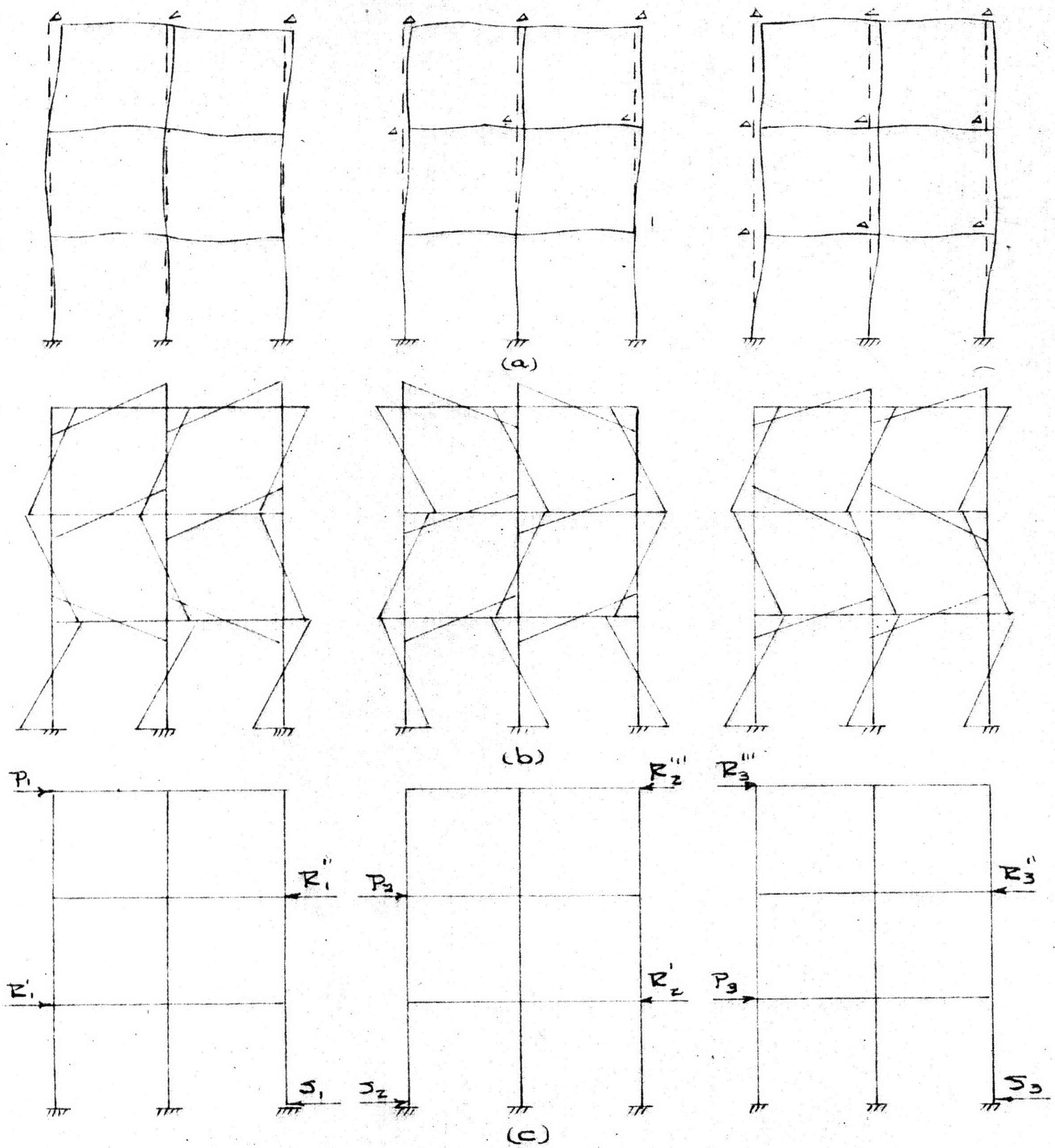


Fig. 36

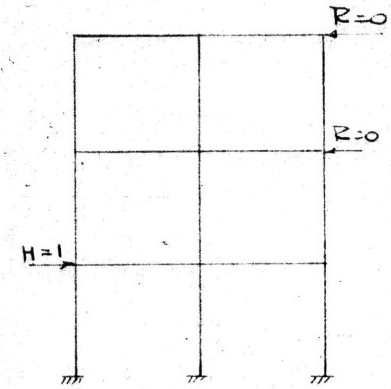


Fig. 37

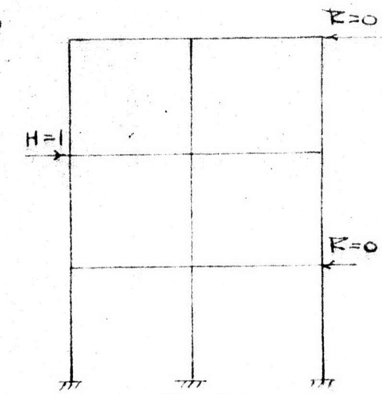


Fig. 38

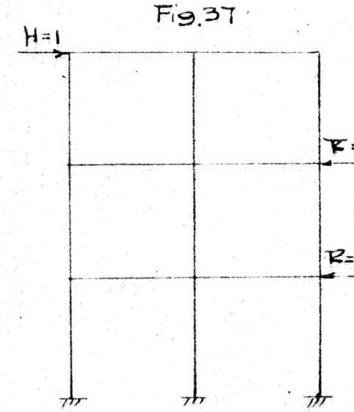


Fig. 39

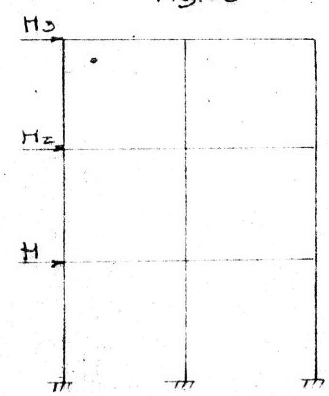


Fig. 40

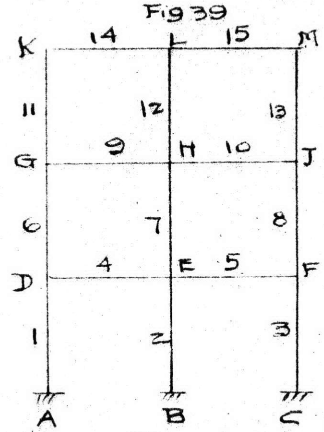


Fig. 41

3-Story-Frame Load & Moment Diagram.

$$x_1P_1 - y_1R_2''' + z_1R_3''' = 0$$

$$y_1P_2 - x_1R_1'' - z_1R_3'' = 0$$

$$z_1P_3 + x_1R_1' - y_1R_2' = 1$$

From which the coefficients x_1, y_1 and z_1 can be solved, The moment expression is

$$M_1 = x_1m_1 + y_1m_2 + z_1m_3$$

For a load unity acting at second floor girder, fig. 38, the conditional equations are:

$$x_2P_1 - y_2R_2''' + z_2R_3''' = 0$$

$$y_2P_2 - x_2R_1'' - z_2R_3'' = 1$$

$$z_2P_3 + x_2R_1' - y_2R_2' = 0$$

and the moment expression is

$$M_2 = x_2m_1 + y_2m_2 + z_2m_3$$

For a load unity acting at the top girder, fig. 39, the conditional equations are:

$$x_3P_1 - y_3R_2''' + z_3R''' = 1$$

$$y_3P_2 - x_3R_1'' - z_3R_3'' = 0$$

$$z_3P_3 + x_3R_1' - y_3R_2' = 0$$

and also the moment expression is

$$M_3 = x_3m_1 + y_3m_2 + z_3m_3$$

For the specific case, fig. 40, the expression for total moment due to the combined effect H_1, H_2 and H_3 is given as

$$M = H_1M_1 + H_2M_2 + H_3M_3$$

In all these cases the signs of the moments should be taken into due consideration.

In computing the moments it is recommended to use the following tabular form:

Typical Tabular Form for Computing Moments of Three-Story-Frame.

Moment of Member at end point		A_1	B_2	C_3	D_1	D_4	D_6			L_{12}	M_{15}	M_{13}
Moment for H=1 at 1st floor Girder	m_1											
" " " " 2nd " "	m_2											
" " " " Top "	m_3											
	$X_1 m_1$											
$M_1 = X_1 m_1 + Y_1 m_2 + Z_1 m_3$	$Y_1 m_2$											
	$Z_1 m_3$											
Moment for 1st loading Condition	M_1											
	$X_2 m_1$											
$M_2 = X_2 m_1 + Y_2 m_2 + Z_2 m_3$	$Y_2 m_2$											
	$Z_2 m_3$											
Moment for 2nd loading Condion	M_2											
	$X_3 m_1$											
$M_3 = X_3 m_1 + Y_3 m_2 + Z_3 m_3$	$Y_3 m_2$											
	$Z_3 m_3$											
Moment for 3rd loading Condition	M_3											
	$H_1 M_1$											
$M = H_1 M_1 + H_2 M_2 + H_3 M_3$	$H_2 M_2$											
	$H_3 M_3$											
Moment due to Specific Load H_1, H_2 & H_3.	M											

5. Special cases.

The usefulness of the methods is not only limited to cases discussed above, but it can be adopted in almost all kinds of framed structure with some modification. By means of this method it is especially simple and exact for the solution of truss-formed frames and truss-bents.

Let take the case shown in fig. 41, a truss-formed frame. It is evident that this truss can be considered as a frame of the same type formerly considered except it is layed in a horizontal position and, instead of one side, both sides are fixed, i.e. fixed at both supports. The deflection condition can be so considered that, while one vertical member is subjected to a deflection Δ due to any vertical load, the ends of other members joining this vertical are all subjected to the same deflection, while the rest verticals are kept in their original position by some imaginary external forces. Here again the moments of all members due to the deflection of each indevidual member for each deflection condition in question are to be determined and these moments combined to give a moment diagram for each particular deflection condition. Figure 43 gives the three cases of deflection conditions with their respective moments and load diagrams.

For the case the truss being subjected to the loads W_1, W_2, and W_3 (fig. 42) the conditional equations for solving the coefficients x, y, and z are as follows:

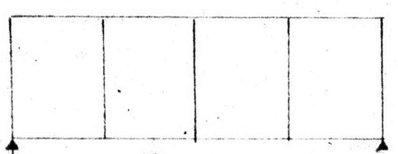
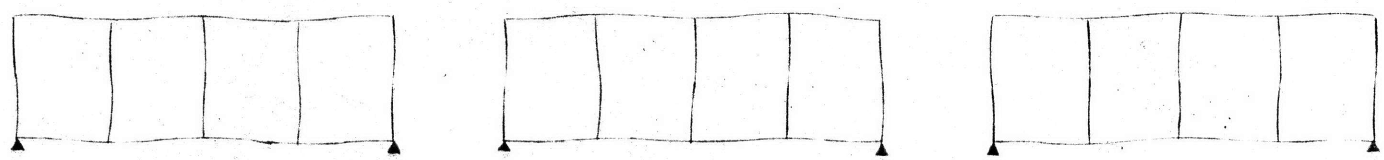


Fig. 41



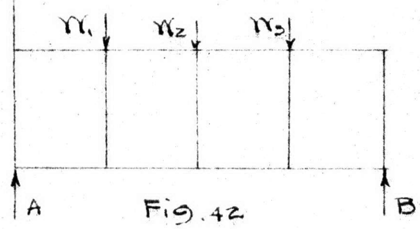
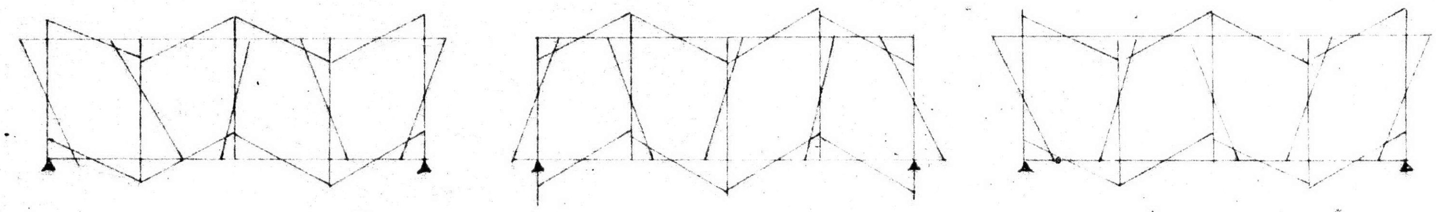


Fig. 42



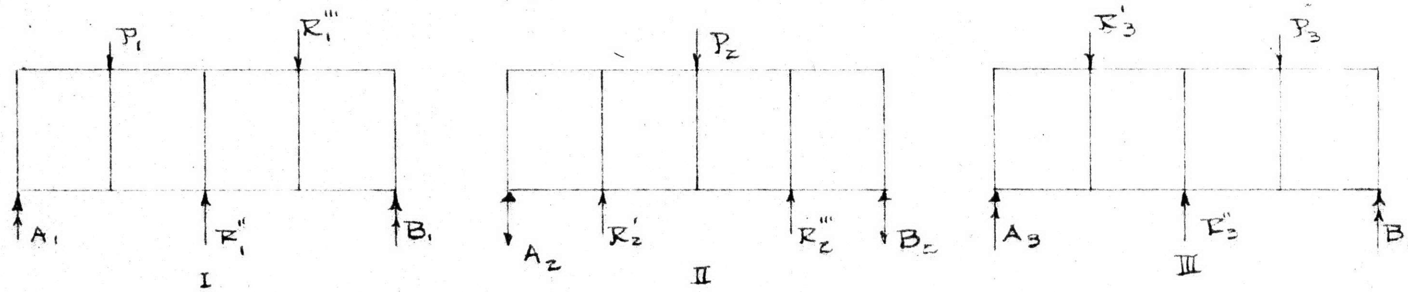


Fig. 43

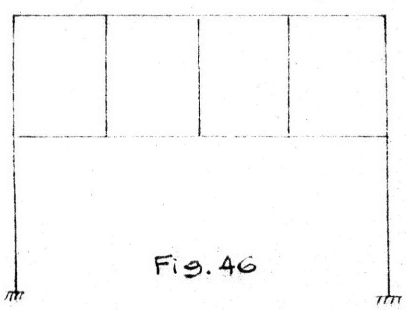


Fig. 46

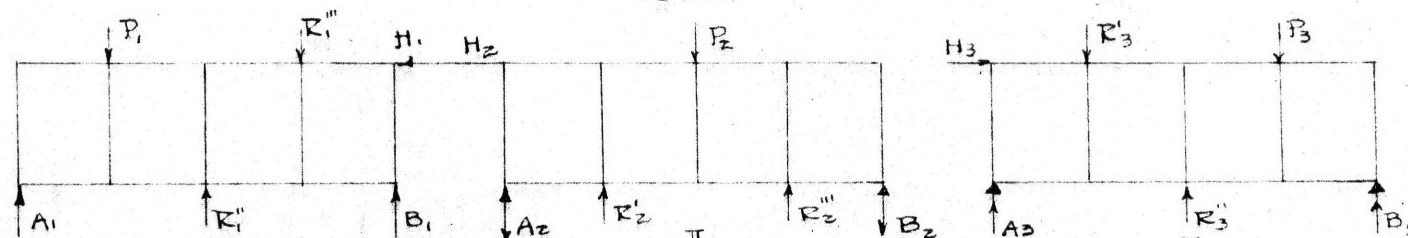


Fig. 44.

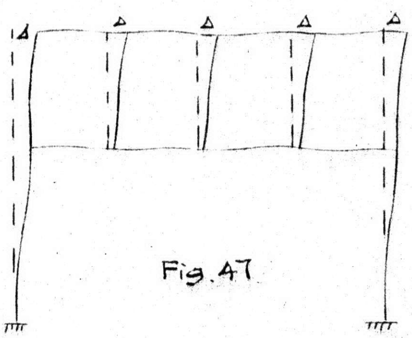


Fig. 47

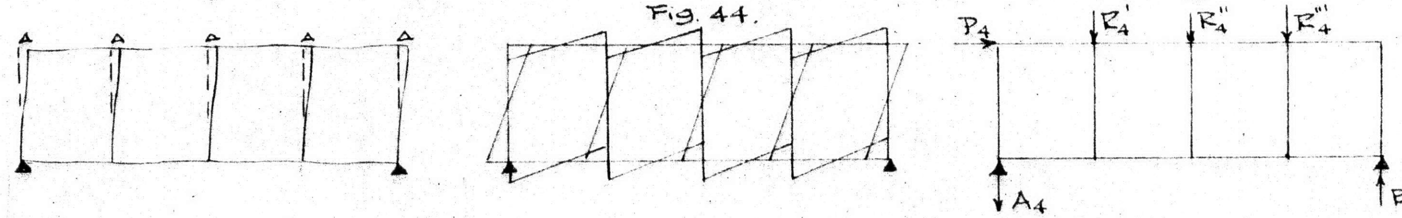


Fig. 45

$$xP_1 - yR_2' + zR_3'' = W_1$$

$$-xR_1'' + yP_2 - zR_3'' = W_2$$

$$xR_1''' - yR_2''' + zP_3 = W_3$$

and the moment expression is

$$M = xm_1 + ym_2 + zm_3$$

If the loads are to be treated as moving load, a unit load can be placed as moving from one panel point to the other and the three sets of coefficients can be solved from similar equations given above.

To find the reactions A and B at the supports the following equations hold:

$$xA_1 - yA_2 + zA_3 = A$$

$$xB_1 - yB_2 + zB_3 = B$$

and $$A + B = W_1 + W_2 + W_3$$

If the truss and the loads are quite unsymmetrical, then there will be an imaginary horizontal force H acting at the upper chords for each load diagram, fig. 44. This force is equal to the algebraic sum of the shears in the verticals, or the unbalanced force in each case. We have now an additional deflection condition to consider, fig. 45. In fig. 45 the value P_4 is evidently equal to the algebraic sum of H's in fig. 44, and the values of R_4', R_4'' and R_4''' are equal to the direct stress of the corresponding verticals, or the end shear of the corresponding upper chords. The four coefficients can be solved from the following four conditional equations:

$$xP_1 - yR_2' + zR_3' + vR_4' = W_1$$

$$-xR_1'' + yP_2 - zR_3'' + vR_4'' = W_2$$

$$xR_1''' - yR_2''' + zP_3 + vR_4''' = W_3$$

$$-xH_1 + yH_2 + zH_3 + vP_4 = 0$$

and the moment expression is

$$M = xm_1 + ym_2 + zm_3 + vm_4$$

In case that there is accually a horizontally applied load H acting on the upper chord, then in solvingthe coefficients the left hand expression should be put equal to H, the applied load, instead of 0.

If the truss is connected with two posts in the form of a bent as shown in fig. 46, then another deflection condition, fig. 47 , has to be taken care of and the conditional equations will be raised to five in number.

6. Abstract on theory of determinants.

From the above discussion it is seen that for each story of the frame there is an unknown coefficient. For buildings more than four or five stories high, the solution of these coefficients may be felt handicapped. It seems to the writer that the use of the method of determinants is preferable. The following abstract on this method is taken from "Hawkes Advanced Algebra." and will be found helpful in most cases.

a) Definations:

Let us solve the equations

$$a_1x + b_1y = c_1 \quad \text{and} \quad a_2x + b_2y = c_2$$

Multiply the first equation by b_2 and the second by b_1, we obtain

$$a_1b_2x + b_1b_2y = b_2c_1$$
$$a_2b_1x + b_1b_2y = b_1c_2$$

Subtracting, we obtain $(a_1b_2 - a_2b_1)x = b_2c_1 - b_1c_2$.

or if $a_1b_2 - a_2b_1 \neq 0$,

$$x = \frac{b_2c_1 - b_1c_2}{a_1b_2 - a_2b_1} \quad \text{and} \quad y = \frac{a_1c_2 - a_2c_1}{a_1b_2 - a_2b_1}$$

We note that the denominators of the expressions for x and y are the same. This denominator we will denote symbolically by the following notation:

$$a_1b_2 - a_2b_1 = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$$

The symbol in the right hand member is called a "Determinant". Since there are two rows and two columns, this determinant is said to be of the "second order". The left hand member of the equation is called the " Development" of the determinant. The symbols a_1, b_1, a_2, b_2 are called " Elements "of the determinant, while the elements a_1 and b_2 are said to comprise its "Principal Diagonal".

For a determinant of the third order, we may combine the terms as follows:

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1(b_2c_3 - b_3c_2) - a_2(b_1c_3 - b_3c_1) + a_3(b_1c_2 - b_2c_1)$$

$$= a_1\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - a_2\begin{vmatrix} b_1 & c_1 \\ b_3 & c_3 \end{vmatrix} + a_3\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}$$

We observe that the coefficient of a_1 is the determinant that we obtain by erasing the row and column in which a_1 lies. A similar fact holds for the coefficients of a_2 and a_3. The determinant obtained by erasing the row and column in which a given element lies is called the "Minor" of that element.

Thus $\begin{matrix} b_1 & c_1 \\ b_3 & c_3 \end{matrix}$ is the minor of a_2. We notice that in the above development by minors the sign of a given term is + or − according as the sum of the number of the row and the number of the columnof the element in that term is even or old.

Rules:

1. The development of a determinant of the nth order is equal to the algebraic sum of the terms consisting of letters following each other in the same order in which they found in the principal diagonal but in which the subscripts take on all possible permutations. A term has the positive or the negative sign according as there is an even or an odd number of inversions* in the subscripts.

2. For the development of a determinant by minors write in succession the elements of any row or column, each multiplied by its minor. Give each term a + or a − sign according as the sum of the number of the row and the number of the

---

* If in a series of positive integers a greater integer precedes a less, there is said to be an inversion. Thus in the series 1 4 3 2 there are three inversions.

column of the element in that term is even or odd. Develope the determinant in each term by a similar process until the value of the development can be determined directly by multiplication.

3. The value of one of the variables in the solution of n linear equations in n variables consists of a fraction whose denominator is the determinant of the system and whose numerator is the same determinant, except that the column which contains the coefficients of the given variable is replaced by a column consisting of the constant terms.

Principles:

1. If every element of a row or a column is multiplied by a number m, the determinant is multiplied by m.

2. The value of a determinant is not changed, if the columns and rows are interchanged.

3. If two columns or two rows are interchanged, the sign of the determinant is changed.

4. If a determinant has two rows (or two columns) or any row (or column) being m times any other row (or column), its value is zero.

5. If each of the elements of any row or any column consists of the sum of two numbers, the determinant may be written as the sum of two determinants.

6. The value of the determinant is unchanged if the elements of any row (or column) are replaced by the ele-

ments of that row (or column) increased or diminished by a multiple of the elements of another row or column.

A numerical example for illustrating the use of determinants for the solution of the coefficients for high building frames is given at the end of Part IV.

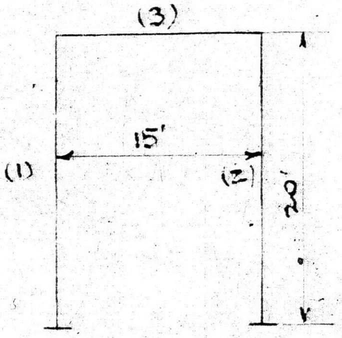
Part 4. Examples.

Example 1.

This example serves the purpose of checking up the method presented in previous pages.
The problem is to investigate the moments & shears of the three members constituting the given rigid bent due to:

(a) A vertical load applied at the center of the girder.

(b) A Horizontal load " " " top " " bent.



Col.s: 2 - 12" [at 25 #, set 8" back to back, webs perpendicular to the paper.
Girder: 12" I at 31.8 #, web vertical.

$A_{1 \& 2} = 14.7$ sq. ins. $I_{1 \& 2} = 331.4$ in^4

$A_3 = 9.26$ sq. ins., $I_3 = 215.8$ in^4.

Computation of characteristics:

$L_{1\&2} = 20'$ or $240''$, $L_3 = 15'$ or $180''$

$E\beta_{1\&2} = \frac{240}{6 \times 331.4} = .1207$, $E\beta_3 = \frac{180}{6 \times 215.8} = .139$. (fomulas 1_a & 3_b)

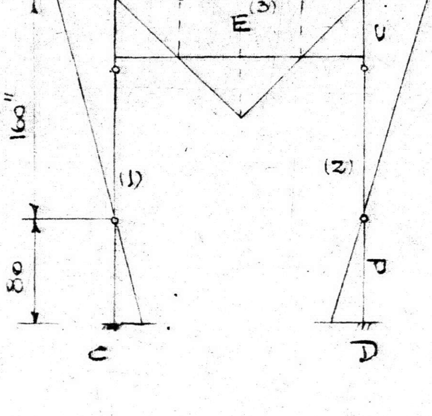
$a_1 = a_2 = \frac{h}{3} = 80''$ (eq. 17)

$E\alpha_{c1} = E\alpha_{c2} = \frac{3}{2} \times .1207 = .1811$ (eq. 19)

$a_3 = b_3 = \frac{180 \times .139}{3 \times .139 + .1811} = 41.83''$ (eq. 11) $L_3 - a_3 = 138.17''$

$E\alpha_{a3} = E\alpha_{b3} = .139\left(3 - \frac{180}{138.17}\right) = .236$ (eq. 15)

$c_1 = c_2 = \frac{240 \times .1207}{3 \times .1207 + .236} = 48.38''$



Investigation of Moments & shears due to vertical load P at center of the girder.

M at center as simply supported = 45P "#

Apply formula 20,

$A = 180 \times \frac{1}{2} \times 45P$, $\bar{x} = 90$, $L - \bar{x} = 90$

$a_3 = b_3 = 41.83$.

$\therefore S_a = S_b = -\frac{41.83}{180} \times \frac{6 \times 90P \times 90 \times 45}{180^2} = 15.68P$

$\therefore M_E = (45 - 15.68)P = 29.32P$

$M_a = M_b = S = -15.68P$. for girder & $+15.68P$ for cols.

& $M_c = M_d = -15.68 \times \frac{80}{160} P = -7.84P$.

All in inch pounds.

Shear for the columns $= \dfrac{-7.84P}{80} = -.098P$

Shear for the girder $= .5P$.

Check by least work method. Formulas obtained from Prof. Spofford's notes.

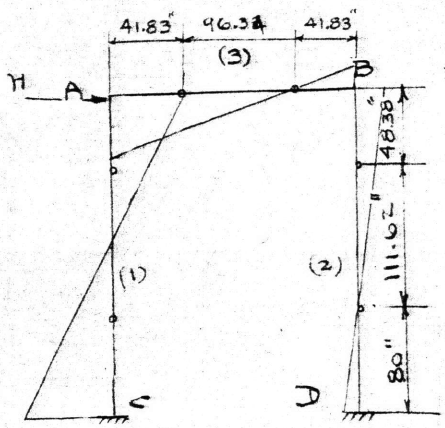
Neglecting direct stress

$$H \text{ or shear in columns} = \dfrac{\dfrac{PL^2}{8}}{\dfrac{1}{3}\dfrac{h^2 l_1}{I} + \dfrac{2hL}{3}}$$

$$= \dfrac{\dfrac{15^2\times 144}{8}P}{\dfrac{20^2\times 144\times 215.8}{3\times 331.4} + \dfrac{2\times 15\times 20\times 144}{3}} = \dfrac{225\times 3P}{8(400\times .651+600)} = .098P^{\#} \text{ check.}$$

$$M_c = M_d = -\dfrac{I}{hI_1+2LI}\cdot\dfrac{PL^2}{8} = \dfrac{-331.4\times 15^2\times 12P}{8(20\cdot 215.8+2\times 15\times 331.4)} = -7.84P^{''\#} \text{ check}$$

Investigation of moments & shears due to horizontal load H applied at top corner of the bent.



Give a deflection $\Delta = 1$ to column (1) and applying equation (25)

$$M_c = \dfrac{1\times 80}{.1207\times 240\times 111.62} = -.02474$$

$$M_a = \dfrac{1\times 48.38}{.1207\times 240\times 111.62} = +.01496$$

for Col. (1)

Equation (25)

M_a at girder $= +.01496$

M_b " " $= -.01496\times\dfrac{41.83}{138.17} = -.00452$

M_b at col. (2) $= +.00452$

$M_d = .00452\times\frac{1}{2} = -.00226$

$$Q = \dfrac{.02474+.01496+.00226+.00452}{240} = \dfrac{.04648}{240} \quad \text{(eq. 26)}$$

Since Col. (2) has to be subjected to a deflection $\Delta = 1$ also, and by looking the moment diagram reversed, we have the same magnitude of Q, thus Total $Q = \dfrac{.04648}{120} = P$ (page 47)

$$\therefore \dfrac{H}{P} = \dfrac{120H}{.04648} = 2580H \text{ (the coefficient.)}$$

Since M_c or M_d due to P which causes $\Delta = 1$ at both Column heads is equal to $-(.02474+.00226) = -0.027$

$\therefore M_c$ or $M_d = -0.027\times 2580H = -69.6$ in.# or -5.8 ft.#

M_a or M_b at cols. $= +(.01496+.00452)\times 2580 = +50.2$ in# or $+4.19$ ft.#

& M_a or M_b at Girder $= +4.19$ ft.# and -4.19 ft.# respectively.

Since M_a & M_b are of the same magnitude, only with the signs different, therefore the point of contraflexure of the girder is at middle.

Let x = the distance of the point of contraflexure of columns from bottom.

Then $\frac{4.19}{5.8} = \frac{20-x}{x}$ $\therefore x = 11.60$ ft.

$\therefore$ Shear in columns $= \frac{5.8H}{11.60} = .5H$ = direct stress in girder

& Shear in girder $\frac{4.19H}{7.5} = .559H$ = " " " Column.

Check by least work method. Formulas obtained from Prof. Spofford's notes.

Neglecting direct stress

$$V = \text{Shear in girder} = \frac{Hh^2}{2hL + \frac{L^2}{3}\frac{I}{I_1}}$$

$$= \frac{H \times 20^2 \times 144}{\left(2 \times 20 \times 15 + \frac{15^2}{3} \times \frac{331.4}{215.8}\right) 144} = \frac{400H}{600 + 115} = .559H \text{ check.}$$

$$nH = \text{Shear in column} = \frac{\frac{I_1}{6I}h^4 + \frac{1}{3}h^3L}{\frac{h^4}{3}\frac{I_1}{I} + \frac{2}{3}h^3L}$$

$$= \frac{12^4\left(\frac{215.8}{331.4 \times 6} \times 20^4 + \frac{1}{3}20^3 \times 15\right)}{12^4 \times 2\left(\frac{215.8}{331.4 \times 6} \times 20^4 + \frac{1}{3} \times 20^3 \times 15\right)} = .5H \text{. check.}$$

$$M_{cord} = \frac{nHh\left(\frac{h}{I} + \frac{L}{I_1}\right) - V\left(\frac{Lh}{I} + \frac{L^2}{2I_1}\right) + H\frac{h^2}{2I}}{\frac{2h}{I} + \frac{L}{I_1}}$$

$$= \frac{.5 \times 20\left(\frac{20}{1.535} + \frac{15}{1}\right) - .559\left(\frac{20 \times 15}{1.535} + \frac{15^2}{2}\right) + \frac{20^2}{2 \times 1.535}}{\frac{2 \times 20}{1.535} + \frac{15}{1}} H$$

$I : I_1 = 1.535 : 1$

$$= \frac{.5 \times 20 \times 28.02 - .559 \times 308 + 130.1}{41.05} = 5.8 \text{ ft.\#} \text{ check.}$$

Example 2

The problem is to investigate the moments of the reinforced concrete building given on next page as an illustration of the application of the methods developed in previous pages. The sections and reinforcements of various members are taken from an existing design made in course 155, reinforced concrete design. The original design is a 3-bay & 4-story building, but in order to make it possible for a single man to handle this problem within very limited time, the center bay & the first floor with the basement of the original design have been omitted; this results a 2-bay & 3-story building for investigation.

Loadings:

Floor	Roof
L.L. 125 #/sq.ft.	L.L. 40#/sq.ft.
D.L. 65#/sq.ft.	D.L. 50#/sq.ft.

Horizotal Wind load 30#/sq.ft of exposed surface.

Computation of moments of inertia:

Columns: - (The outer $1\frac{1}{2}''$ of concrete is assumed for fire protection only.)

(12) $I_c = \frac{\pi \times 11^4}{64} = \pi \times 121 \times 1.89 = 720.0$

$A_s = 1.768$, $d_s = \frac{5.5}{\sqrt{2}} = 3.88$

$I_s = 14 \times 1.768 \times 3.88^2 = 373.0$

Total $= 1093.0 \text{ in}^4$ or $.053 \text{ ft}^4$

(7) $I_c = \frac{\pi \times 16^4}{64} = 256 \times \pi \times 4 = 3220$

$A_s = 2.452$, $d_s = \frac{8}{\sqrt{2}} = 5.66$

$I_s = 14 \times 5.66^2 \times 2.452 = 1104$

Total $= 4324 \text{ in}^4$ or $.209 \text{ ft}^4$

(2) $I_c = \frac{\pi \times 20^4}{64} = 7860$

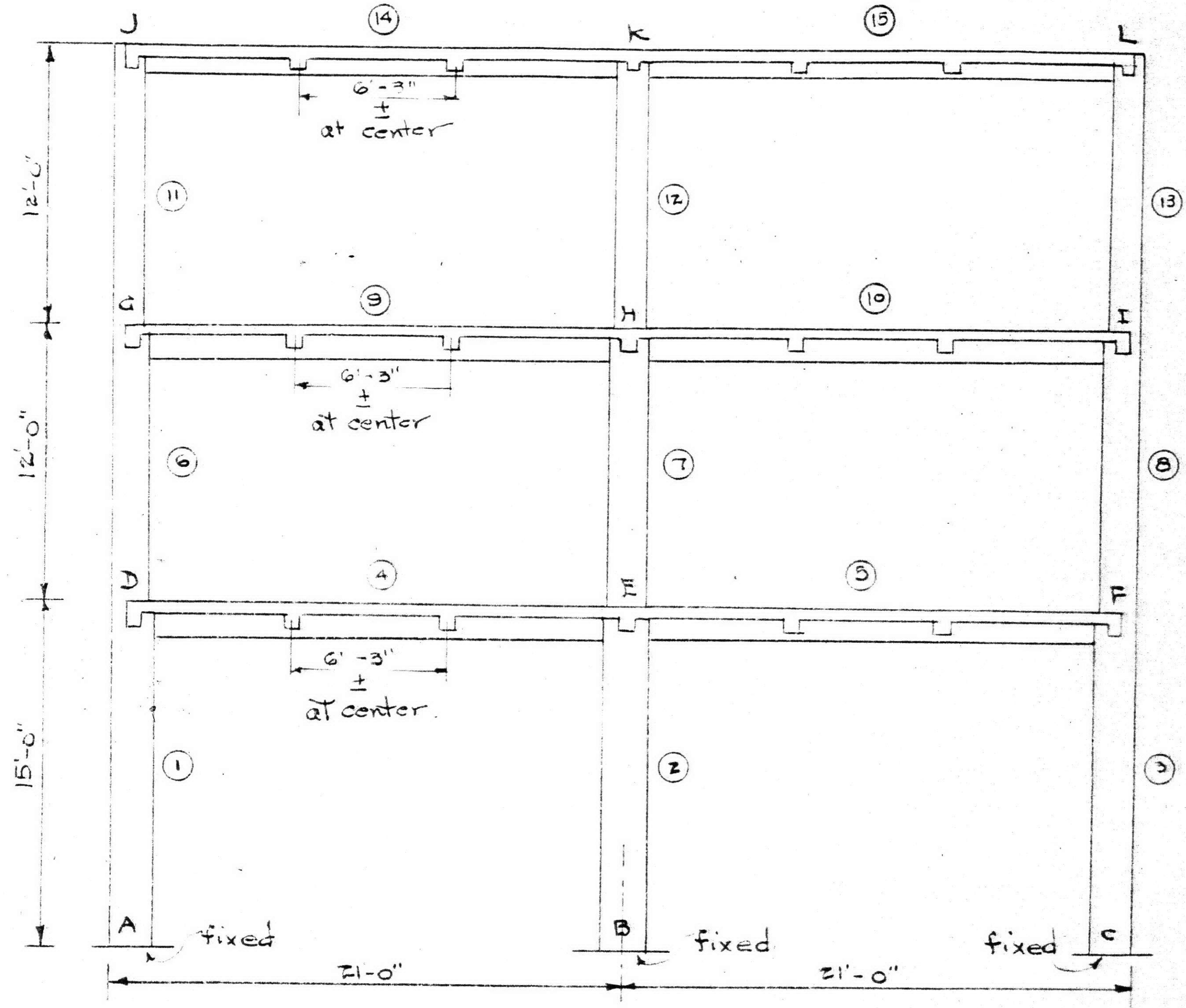
$I_s = 1.226 \times 14 (8^2 + 4^2 + 9.5^2) = 2930$ (Distances of steel bars from N.A. measured graphically)

Total $= 10790 \text{ in}^4$ or $.52 \text{ ft}^4$

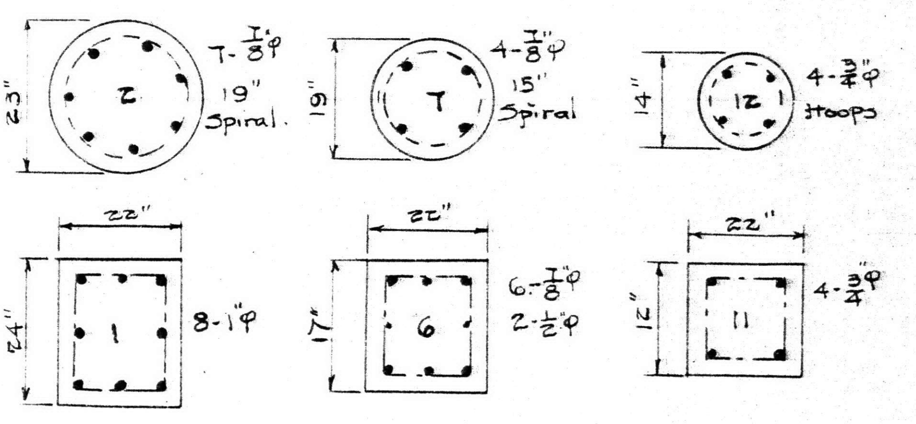
(11) $I_c = \frac{1}{12} \times 19 \times 9^3 = 1152$

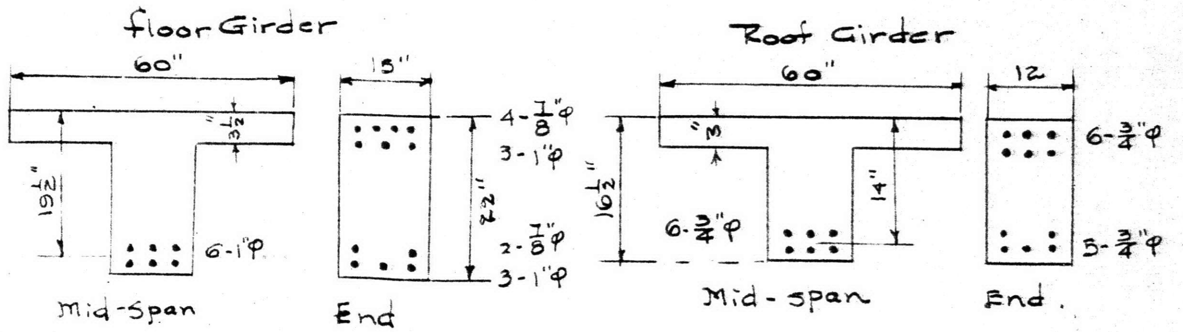
$I_s = 1.768 \times 14 \times 4.5^2 = 501$

1653 in^4 or $.080 \text{ ft}^4$



Note:
Columns spaced 20'-0" c-c laterally in the direction perpendicular to the paper.





(6) $I_c = \frac{1}{12} \times 19 \times 14^3 \quad = 4350$

$I_s = .6013 \times 6 \times 7^2 \quad = 2480$

Total $= 6830$ in^4 or .33 ft^4

(1) $I_c = \frac{1}{12} \times 19 \times 21^3 \quad = 14650$

$I_s = 6 \times .7854 \times 10.5^2 \times 14 = 7280$

21930 in^4 or 1.054 ft^4

Girders: Since the moment of inertia varies in a beam on account of reinforcements, the moments of inertia for both center & end sections are worked out and the smaller value is to be used.

Floor girder center section —

x = c. of g. from top, $A_s = 4.71$

$(x-1.75)\,60 \times 3.5 + (x-3.5) \times 15 \times \frac{(x-3.5)}{2} - 4.71\,(19.5-x) \times 15 = 0$

$x^2 + 30.4x - 221 = 0 \qquad \therefore x = 6.1$ ins.

$I_c = \frac{1}{3} \times 60 \times 6.1^3 - \frac{1}{3} \times 45 \times 2.6^3 \quad = 4276$

$I_s = 4.71 \times 15 \times 13.4^2 \quad = 12700$

Total $= 16980$ in^4 or 0.817 ft^4

Floor girder end section —

$A_s(2\text{-}\frac{7}{8}''\varphi) = 1.2026$, $A_s(3\text{-}1''\varphi) = 2.3562$, $A_s(4\text{-}\frac{7}{8}''\varphi) = 2.4052$

x = c. of g. from bottom

$15x + 14 \times 2.3562(x-1.5) + 14 \times (x-3.5) \times 1.2026 = 15 \times 2.4052(22.0-1.5-x)$
$+ 15 \times 2.3562\,(22.0-3.5-x)$

$137.0\,x = 1500 \qquad \therefore x = 10.94$ ins.

$I_c = \frac{1}{3} \times 15 \times 10.94^3 = \ldots\ldots \quad 6550$

$I_s = \begin{cases} 14\,(2.3562 \times 7.44^2 + 2.4052 \times 9.44^2) = \ldots & 4815 \\ 15\,(2.3562 \times 9.56^2 + 1.2026 \times 7.56^2) = \ldots & 4260 \end{cases}$

Total 15625 in^4

or 0.755 ft^4 (This to be used)

Roof Girder center section —

x = c. of g. from top, $A_s = 2.6508$

$(x-1.5)\,60 \times 3 + (x-3)^2 \times 12 \times \frac{1}{2} - 2.6508 \times 15\,(14.0-x) = 0$

$x^2 + 17.4x - 129 = 0 \qquad \therefore x = 5.6$ ins.

$I_c = \frac{1}{3} \times 60 \times 5.6^3 - \frac{1}{3} \times 48 \times 2.6^3 \quad = 330$

$I_s = 2.6508 \times 15 \times 8.4^2 \quad = 2810$

Total $= 3140$ in^4 or 0.151 ft.^{4} (To be used)

Roof girder end section —

x = c. of g from top, $A_s(3\text{-}\frac{3}{4}''\varphi) = 1.3254$, $A_s(2\text{-}\frac{3}{4}''\varphi) = .8836$

$12(16.5-x) + 14\,[1.3254\,(16.5-1.5-x) + .8836\,(16.5-3.5-x)] = 15\,[1.3254 \times 2 \times (x-2.5)]$

$82.8x = 737 \qquad \therefore x = 8.9''$ (from top) or $7.6''$ from bottom.

$I_c = \frac{1}{3} \times 12 \times 7.6^3 \quad = 1755$

$I_s = \begin{cases} 14\,(1.3254 \times 6.1^2 + .8836 \times 4.1^2) & = 899 \\ 15\,(1.324 \times 2 \times 6.4^2) & = 1630 \end{cases}$

Total 4284 in^4 or .206 ft^4

Computation of characteristics.

Since the frame is symmetrical, the characteristics for half the frame are computed. To compute β, formulas 1a & 3b are used.

Length of Column = distance between c. of g. of girders at each end. As the center lines of columns do not form one continuous line, so, for simplicity the length of girder is taken to be equal to the average distance between column centers of the three stories.

Table for L, I & β

Member	1	2	6	7	11	12	4	9	14
L	14.2	14.2	12	12	12.2	12.2	20.4	20.4	20.4
I	1.054	.520	.330	.209	.080	.053	.755	.755	.151
Eβ	2.24	4.54	6.06	9.56	25.40	38.30	4.50	4.50	22.50

Computation for α, a, b, c & d.

at wall column.

$d_1 = \frac{14.2}{3} = 4.73$ (eq. 17)

$E\alpha_{c1} = \frac{3}{2}\beta_1 = \frac{3}{2} \times 2.24 = 3.36$ (eq. 19)

Assume $c_6 = \frac{h_6}{4} = 3$ (page 29)

$E\alpha_{d6} = \frac{5}{3} \times 6.06 = 10.10$ (Footnote, page 30)

$E r_{6-r} = \frac{3.36 \times 10.10}{3.36 + 10.10} = 2.52$ (eq. 6)

$a_4 = \frac{20.4 \times 4.5}{3 \times 4.5 + 2.52} = 5.72$ (eq. 11)

$L_4 - b_4 = 14.79$.

$E\alpha_{a4} = 4.50\left(3 - \frac{20.4}{14.79}\right) = 7.29$

$E r_{4-1} = \frac{7.29 \times 3.36}{7.29 + 3.36} = 2.30$

$d_6 = \frac{12 \times 6.06}{3 \times 6.06 + 2.30} = 3.55$

$h_6 - d_6 = 12 - 3.55 = 8.45$

$E\alpha_{c6} = 6.06\left(3 - \frac{12}{8.45}\right) = 9.58$

Assume $c_{11} = \frac{1}{4}h_{11} = 3.05$

$E\alpha_{d11} = \frac{5}{3} \times 25.40 = 42.40$

$E r_{11-6} = \frac{42.4 \times 9.58}{42.4 + 9.58} = 7.81$

$a_9 = \frac{20.4 \times 4.50}{3 \times 4.50 + 7.81} = 4.30$

at center column.

$d_2 = \frac{14.2}{3} = 4.73$

$E\alpha_{c2} = \frac{3}{2} \times 4.54 = 6.80$

Assume $c_7 = \frac{h_7}{4} = 3$

$E\alpha_{d7} = \frac{5}{3} \times 9.56 = 15.92$

$E r_{2-7} = \frac{6.80 \times 15.92}{6.80 + 15.92} = 4.76$

$L_4 - a_4 = 20.4 - 5.72 = 14.68$

$E\alpha_{b4} = 4.50\left(3 - \frac{20.4}{14.68}\right) = 7.25$ (eq. 15)

$E r_{4-7-2} = \frac{7.25 \times 4.76}{7.25 + 4.76} = 2.87$ (eq. 9)

b_4 or $a_5 = \frac{20.4 \times 4.5}{3 \times 4.5 + 2.87} = 5.61$

$E r_{4-5} = \frac{7.25 \times 7.25}{7.25 + 7.25} = 3.63$

$E r_{4-5-2} = \frac{3.63 \times 6.80}{3.63 + 6.80} = 2.36$

$d_7 = \frac{12 \times 9.56}{3 \times 9.56 + 2.36} = 3.70$

$h_7 - d_7 = 12 - 3.70 = 8.30$

$E\alpha_{c7} = 9.56\left(3 - \frac{12}{8.30}\right) \doteq 14.89$.

Assume $c_{12} = \frac{1}{4}h_{12} = 3.05$

$E\alpha_{d12} = \frac{5}{3} \times 38.30 = 63.80$

$L_9 - a_9 = 20.4 - 4.30 = 16.10$

$E\alpha_{b9} = 4.50\left(3 - \frac{20.4}{16.10}\right) = 7.80$

$E r_{12-7} = \frac{63.80 \times 14.89}{63.80 + 14.89} = 12.07$

$E\alpha_{b9} = 4.50\left(3 - \frac{20.4}{16.10}\right) = 7.80$

$Er_{12-7} = \frac{63.80 \times 14.89}{63.80 + 14.89} = 12.07$

$Er_{12-7-9} = \frac{12.07 \times 7.80}{12.07 + 7.80} = 4.74$

$b_9 = c_{10} = \frac{20.4 \times 4.50}{3 \times 4.50 + 4.74} = 5.03$

$L_9 - b_9 = 20.4 - 5.03 = 15.37$

$E\alpha_{a9} = 4.50\left(3 - \frac{20.4}{15.37}\right) = 7.53$

$Er_{9-6} = \frac{7.53 \times 9.58}{7.53 + 9.58} = 4.21$

$d_{11} = \frac{12.2 \times 25.40}{3 \times 25.40 + 4.21} = 3.86$

$h_{11} - d_{11} = 12.2 - 3.86 = 8.34$

$E\alpha_{c11} = 25.40\left(3 - \frac{12.2}{8.34}\right) = 39.10$

$a_{14} = \frac{22.50 \times 20.40}{3 \times 22.50 + 39.10} = 4.31$

$Er_{9-10} = \frac{7.80 \times 7.80}{7.80 + 7.80} = 3.90$

$Er_{9-10-7} = \frac{3.90 \times 14.89}{3.90 + 14.89} = 3.09$

$d_{12} = \frac{12.2 \times 38.30}{3 \times 38.30 + 3.09} = 3.96$

$h_{12} - d_{12} = 8.24$

$E\alpha_{c12} = 38.3\left(3 - \frac{12.2}{8.24}\right) = 58.25$

$L_{14} - a_{14} = 16.09$

$E\alpha_{b14} = 22.50\left(3 - \frac{20.4}{16.09}\right) = 38.95$

$Er_{14-12} = \frac{38.95 \times 58.25}{38.95 + 58.25} = 23.35$

b_{14} or $a_{15} = \frac{20.4 \times 22.5}{3 \times 22.5 + 23.35} = 5.05$

$L_{14} - b_{14} = 20.4 - 5.05 = 15.35$

$E\alpha_{a14} = 22.50\left(3 - \frac{20.4}{15.35}\right) = 37.65$

$c_{11} = \frac{12.2 \times 22.5}{3 \times 22.5 + 37.65} = 2.61$

(The assumed value of $c_{11} = 3.05$)

$Er_{14-15} = \frac{38.95 \times 38.95}{38.95 + 38.95} = 19.48$

$c_{12} = \frac{12.2 \times 38.3}{3 \times 38.3 + 19.48} = 3.48$

(The assumed value of $c_{12} = 3.05$)

The values inside the brackets below are those formerly obtained.

$h_{11} - c_{11} = 12.2 - 2.61 = 9.59$

$E\alpha_{d11} = 25.4\left(3 - \frac{12.2}{9.59}\right) = 44.0\ (42.4)$

$Er_{11-9} = \frac{44 \times 7.53}{44 + 7.53} = 6.42$

$c_6 = \frac{12.0 \times 6.06}{3 \times 6.06 + 6.42} = 2.96$

(The assumed value of $c_6 = 3.00$)

$h_6 - c_6 = 12 - 2.96 = 9.04$

$E\alpha_{d6} = 6.06\left(3 - \frac{12}{9.04}\right) = 10.12\ (10.10)$

$Er_{6-4} = \frac{10.12 \times 7.29}{10.12 + 7.29} = 4.24$

$c_1 = \frac{14.2 \times 2.24}{3 \times 2.24 + 4.24} = 2.90$

$h_{12} - c_{12} = 12.2 - 3.48 = 8.72$

$E\alpha_{d12} = 38.3\left(3 - \frac{12.2}{8.72}\right) = 61.3\ (63.8)$

$Er_{10-9-12} = \frac{61.3 \times 3.90}{61.3 + 3.90} = 3.67$

$c_7 = \frac{12 \times 9.56}{3 \times 9.56 + 3.67} = 3.53$

(The assumed value of $c_7 = 3.00$)

$h_7 - c_7 = 8.47$

$E\alpha_{d7} = 9.56\left(3 - \frac{12}{8.47}\right) = 16.1\ (15.92)$

$Er_{7-4-5} = \frac{16.1 \times 3.63}{16.1 + 3.63} = 2.96$

$c_2 = \frac{14.2 \times 2.24}{3 \times 2.24 + 2.96} = 3.29.$

The assumed values of c_6, c_7, c_{11} & c_{12} are somewhat off from the actual values, but if we use all the computed values of c_6, c_7, c_{11} & c_{12} and keep the values for α, a, b, c & d the same without revision, the error is very slight according to the statement on page 30, but in order to prove the statement being true, the following revision is made.

Revisions: This revision is not necessary in practical design. The idea to put this in here is to show how slight the error is in the first computation.

The values inside the brackets are those obtained in first computation.

at wall column.

$d_1 = 4.73 \quad (4.73)$

$E\alpha_{c1} = 3.36 \quad (3.36)$

$c_6 = 2.96$ (2.96 computed value)

$E\alpha_{d6} = 6.06\left(3 - \frac{12}{12-2.96}\right) = 10.12 \quad (10.10)$

$Er_{6-7} = \frac{3.36 \times 10.12}{3.36 + 10.12} = 2.54 \quad (2.52)$

$a_4 = \frac{2.04 \times 4.5}{3 \times 4.5 + 2.54} = 5.71 \quad (5.72)$

$L_4 - b_4 = (14.79)$

$E\alpha_{[illegible]} = (7.29)$

$Er_{[illegible]} = (2.30)$

$d_6 = (3.55)$

$h_6 - d_6 = (8.45)$

$E\alpha_{c6} = (9.58)$

$c_{11} = 2.61$ (2.61 computed value)

$E\alpha_{d11} = 25.40\left(3 - \frac{12.2}{12.2-2.61}\right) = 43.9 \quad (42.4)$

$Er_{11-6} = \frac{43.9 \times 9.58}{43.9 + 9.58} = 7.87 \quad (7.81)$

$a_9 = \frac{20.4 \times 4.50}{3 \times 4.50 + 7.87} = 4.30 \quad (4.30)$

$L_9 - b_9 = (15.37)$

$E\alpha_{a9} = (7.53)$

$Er_{9-6} = (4.21)$

$d_{11} = (3.86)$

$h_{11} - d_{11} = (8.34)$

$E\alpha_{c11} = (39.10)$

at center column.

$d_2 = 4.73 \quad (4.73)$

$E\alpha_{c2} = 6.80 \quad (6.80)$

$c_7 = 3.53$ (3.53 computed value)

$E\alpha_{d7} = 9.56\left(3 - \frac{12}{12-3.53}\right) = 15.15 \quad (15.92)$

$Er_{2-7} = \frac{6.80 \times 15.15}{6.80 + 15.15} = 4.70 \quad (4.76)$

$L_4 - a_4 = 20.4 - 5.71 = 14.69 \quad (14.68)$

$E\alpha_{b4} = 4.50\left(3 - \frac{20.4}{14.69}\right) = 7.25 \quad (7.25)$

$Er_{4-7-2} = \frac{7.25 \times 4.70}{7.25 + 4.70} = 2.85 \quad (2.87)$

$b_4 \text{ or } a_5 = \frac{20.4 \times 4.5}{3 \times 4.5 + 2.85} = 5.61 \quad (5.61)$

$Er_{4-5} = (3.63)$

$Er_{4-5-2} = (2.36)$

$d_7 = (3.70)$

$h_7 - d_7 = (8.30)$

$E\alpha_{c7} = (14.89)$

$c_{12} = 3.48$ (3.48 computed value)

$E\alpha_{d12} = 38.30\left(3 - \frac{12.2}{12.2-3.48}\right) = 61.25 \quad (63.8)$

$L_9 - a_9 = (16.10)$

$E\alpha_{b9} = (7.80)$

$Er_{12-7} = \frac{61.25 \times 14.89}{61.25 + 14.89} = 11.97 \quad (12.07)$

$Er_{12-7-9} = \frac{11.97 \times 7.80}{11.97 + 7.80} = 4.72 \quad (4.74)$

$b_9 = c_{10} = \frac{20.4 \times 4.50}{3 \times 4.50 + 4.72} = 5.03 \quad (5.03)$

$Er_{9-10} = (3.90)$

$Er_{9-10-7} = (3.09)$

$d_{12} = (3.96)$

$h_{12} - d_{12} = (8.24)$

$E\alpha_{c12} = ([illegible])$

$a_{11} = (3.86)$

$h_{11} - a_{11} = (8.34)$

$E\alpha_{c11} = (39.10)$

$a_{14} = (4.31)$

$d_{12} = (3.96)$

$h_{12} - d_{12} = (8.24)$

$E\alpha_{c12} = (58.25)$

$L_{14} - a_{14} = (16.09)$

$E\alpha_{b14} = (38.95)$

$E r_{14-12} = (23.35)$

$L_{14} - b_{14} = (15.35)$

$E\alpha_{a14} = (37.65)$

$c_{11} = (2.61)$ check.

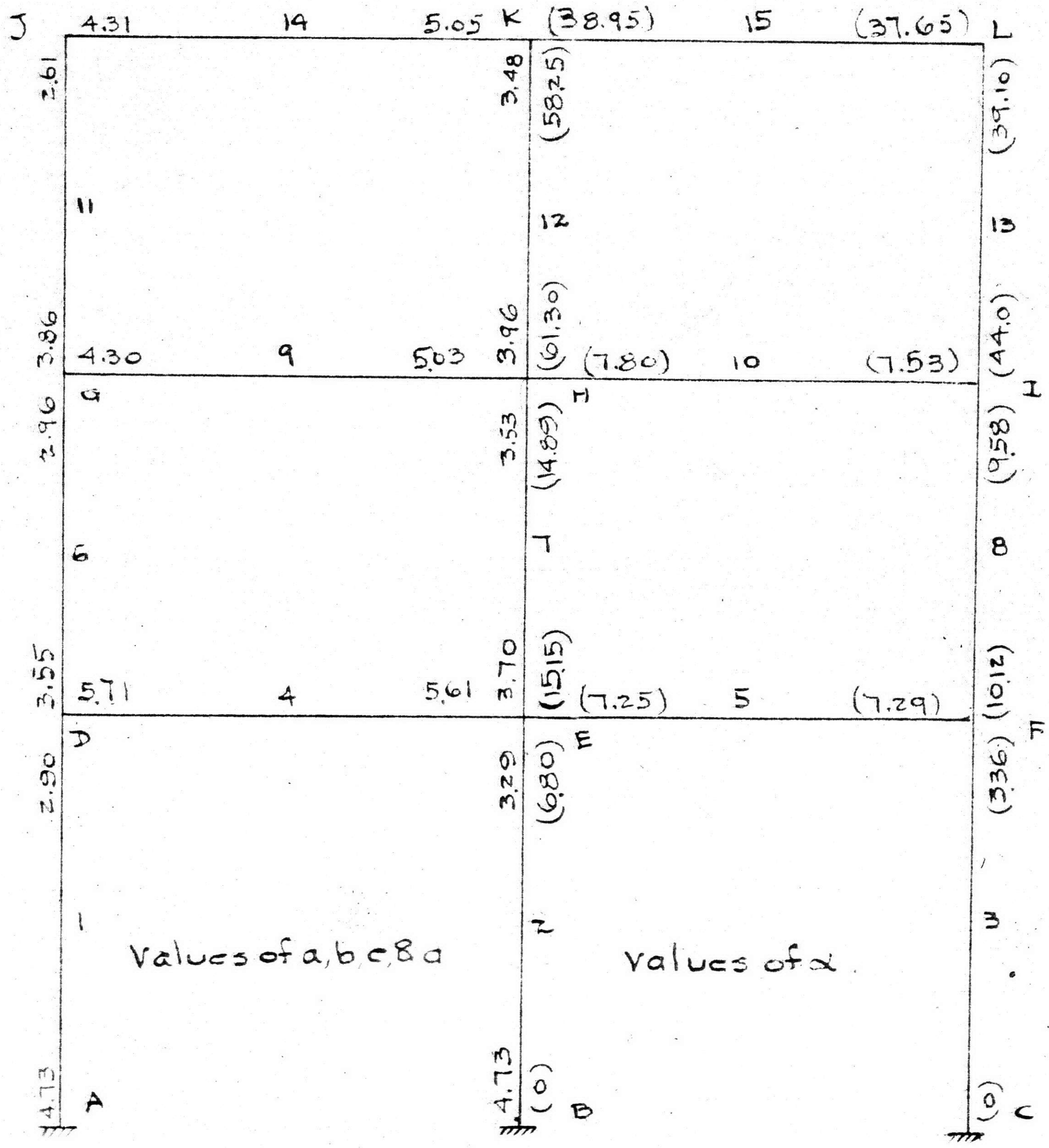
b_{14} or $a_{15} = (5.05)$

$E r_{14-15} = (19.48)$

$c_{12} = (3.48)$ check.

The other values to be followed are identical with the first computation. It is thus seen that the error of some of the values in first computation is very slight and negligible.

Summary of the Values α, a, b, c & d.



Computation of influence Moments.
due to vertical load.

Since each girder carries two concentrated loads transmitted by the two floor beams, 6'-3" apart, symmetrically located, so in order to get the influence moments due to live load a unit load of 1# can be assumed for the concentrated load and, by loading each span successively, the corresponding moments can be obtained. The computation is as follows:-

(Influence moment, by separation, under .20"# is neglected.)

A. Loaded span EF. Girder 5.

Moment at center portion when the girder is simply supported.

$= 1 \times (20.4 - 6.25) \times \frac{1}{2} \times 12 = 84.9"\#$

Apply equation (20), $L - \bar{x} = 10.2'$ (Refer Fig. 14 Placing 2 loads instead of 1)

$A = \frac{84.9}{12} \times (6.25 + 14.15 \times \frac{1}{2}) = 94.27.$

$S_{E_5} = \frac{-5.61 \times 6 \times 94.27 \times 10.2}{20.4 \times 20.4^2} \times 12 = 45.75"\#$, $S_{F_5} = \frac{-5.71 \times 6 \times 94.27 \times 10.2}{20.4 \times 20.4^2} \times 12 = 46.6"\#$

Increment of M per ft. $= \frac{46.6 - 45.75}{20.4 - 5.71 - 5.61} = .0935.$ $M_{center} = 84.9 - 45.75 = 39.15"\#$

$\therefore M_{E5} = 45.75 - 5.61 \times .0935 = -45.23"\#$ $M_{F5} = 46.6 + 5.71 \times .0935 = -47.13.$

Separation of $M_{F5} = -47.13$. refer page 37.

$M_{F3} = +\frac{10.12 \times 47.13}{10.12 + 3.36} = +35.5$, $M_{C3} = -\frac{1}{2} \times 35.5 = -17.75$ $(d = \frac{1}{3}h)$

$M_{F8} = -(47.13 - 35.5) = -11.63$ $M_{I8} = +\frac{11.63 \times 2.96}{12.0 - 2.96} = +3.82$

$M_{I13} = +\frac{3.82 \times 7.53}{44 + 7.53} = +.56$ $M_{L13} = \frac{-.56 \times 2.61}{9.59} = -.15$

$M_{I10} = -(3.82 - .56) = -3.26$ $M_{L15} = +.15$ (x)*

$M_{H10} = \frac{+3.26 \times 5.03}{15.37} = +1.07$

$r_{12-9} = \frac{61.30 \times 7.80}{61.30 + 7.80} = 6.9$

$M_{H7} = +\frac{6.9 \times 1.07}{14.89 + 6.9} = +.34$ $M_{E7} = -\frac{.34 \times 3.70}{8.30} = -.15$ (x)

$M_{H9} = +\frac{.73 \times 61.30}{61.30 + 7.80} = +.65$ $M_{G9} = -\frac{.65 \times 4.30}{16.10} = -.17$ (x)

$M_{H12} = -(1.07 - .34 - .65) = -.08$

Separation of $M_{E5} = -45.23$

$r_{7-2} = \frac{15.15 \times 6.8}{6.8 + 15.15} = 4.7$

$M_{E4} = \frac{-45.23 \times 4.7}{4.7 + 7.23} = -17.8.$ $M_{E7} = +\frac{27.43 \times 6.80}{6.80 + 15.15} = +8.5$

$M_{E2} = -(27.43 - 8.5) = -18.93$ $M_{b2} = \frac{1}{2} \times 18.93 = +9.47$

For signs Refer page 37, + convexity downward & to the right, + (beam, col.) & vice versa.

* (x) means further separation is too small to be considered.

$M_{D4} = +\frac{17.8 \times 5.71}{14.69} = +6.9$

$M_{D6} = -\frac{6.9 \times 3.36}{3.36 + 10.12} = -1.72$ $\qquad M_{96} = \frac{+1.72 \times 2.96}{9.04} = +.56$

$M_{D1} = +(6.9 - 1.72) = +5.18$ $\qquad M_{a1} = -\frac{1}{2} \times 5.18 = -2.59.$

$M_{99} = +\frac{.56 \times 44}{44 + 7.53} = +.48$ $\qquad M_{H9} = -\frac{.48 \times 5.03}{15.37} = -.16\ (x)$

$M_{911} = +(.56 - .48) = +.08\ (x)$

$M_{H7} = \frac{-3.53 \times 8.5}{8.47} = -3.54.$

$r_{9-10} = \frac{7.8^2}{2 \times 7.8} = 3.9$

$M_{H12} = \frac{-3.9 \times 3.54}{3.9 + 61.30} = -.21$ $\qquad M_{H10} = -\frac{1}{2}(3.54 - .21) = -1.67.$

$M_{K12} = +\frac{.21 \times 3.48}{8.72} = +.08\ (x)$ $\qquad M_{I10} = +1.67 \times \frac{4.30}{16.1} = +.44$

$M_{H9} = +1.67$ $\qquad M_{99} = -.44$

$M_{96} = -\frac{.44 \times 44}{9.58 + 44} = -.36$ $\qquad M_{I8} = -.36$

$M_{911} = +.08\ (x)$ $\qquad M_{I13} = +.08\ (x)$

$M_{D6} = +\frac{.36 \times 3.55}{8.45} = +.15\ (x)$ $\qquad M_{F8} = +.15.\ (x)$

B. Loaded span H-I. Girder 10.

M at center portion as simply supported = 84.9"#

A = 94.27, $\quad L - \bar{x} = 10.2'$

$S_{H_{10}} = \frac{-5.03 \times 6 \times 94.27 \times 10.2}{20.4 \times 20.4^2} \times 12 = -41.30''\#$

$S_{I_{10}} = -\frac{4.30 \times 6 \times 94.27 \times 10.2}{20.4 \times 20.4^2} \times 12 = -35.07''\#$

Increment of moment per unit length $= \frac{41.30 - 35.07}{20.4 - 4.30 - 5.03} = .563$

$M_{H10} = -41.30 - 5.03 \times .563 = -44.13''\#$

$M_{I10} = -35.07 + .563 \times 4.3 = -32.65''\#$ $\qquad M_{center} = 84.9 - 32.65 = 52.25''\#$

Separation of $M_{I10} = -32.65$

$M_{I13} = -32.65 \frac{9.58}{44 + 9.58} = -5.85$ $\qquad M_{L13} = +\frac{5.85 \times 2.61}{9.59} = +1.59$

$M_{I8} = +32.65 \times \frac{44}{44 + 9.58} = +26.80$ $\qquad M_{F8} = -\frac{26.8 \times 3.55}{8.45} = -11.26$

$M_{L15} = -1.59, \quad M_{K15} = +\frac{1.59 \times 5.05}{15.35} = +.52$

$M_{K14} = +\frac{.52 \times 58.25}{58.25 + 38.95} = +.31$ $\qquad M_{J14} = -\frac{.31 \times 4.31}{16.09} = -.08\ (x)$

$M_{K12} = +.52 - .31 = +.21$ $\qquad M_{H12} = -\frac{.21 \times 3.96}{8.24} = -.10\ (x)$

$M_{F3} = -\frac{11.26 \times 7.29}{7.29 + 3.36} = -7.71, \quad M_{C3} = +\frac{7.71 \times 4.73}{9.47} = +3.86$

$M_{F5} = -\frac{11.26 \times 3.36}{7.29 + 3.36} = -3.55$ $M_{E5} = +\frac{3.55 \times 5.61}{14.79} = +1.35$

$r_{4-2} = \frac{7.25 \times 6.80}{7.25 + 6.80} = 3.51$

$M_{E7} = -\frac{3.51 \times 1.35}{3.51 + 15.15} = -.25$, $1.35 - .25 = 1.1$, $M_{H7} = +\frac{.25 \times 3.53}{8.47} = +.10$ (x)

$M_{E4} = +\frac{1.1 \times 7.25}{7.25 + 6.8} = +.57$ $M_{D4} = -\frac{.57 \times 5.71}{14.69} = -.22$ (x)

$M_{E2} = +(1.1 - .57) = +.53$ $M_{b2} = -.08$ (x)

Separation of $M_{H10} = -44.13''\#$

$r_{7-9} = \frac{14.89 \times 7.80}{14.89 + 7.80} = 5.11$

$M_{H12} = \frac{5.11\,(+44.13)}{61.30 + 5.11} = +3.4$ $M_{K12} = \frac{-3.4 \times 3.48}{8.72} = -1.36$

$M_{K15} = -\frac{1}{2} \times 1.36 = -.68$ $M_{L15} = +\frac{4.31 \times .68}{16.09} = +.18$ (x)

$M_{K14} = +\frac{1}{2} \times 1.36 = +.68$ $M_{J14} = -$ " $= -.18$ (x)

$M_{L3} = -.18$ $M_{J11} = -.18$

$M_{H10} - M_{H12} = 44.13 - 3.4 = 40.73$

$M_{H9} = \frac{-40.73 \times 14.89}{14.89 + 7.80} = -26.7$ $M_{g9} = +\frac{26.7 \times 4.30}{16.10} = +7.1$

$M_{g11} = -\frac{7.1 \times 9.58}{44 + 9.58} = -1.27$ $M_{j11} = \frac{+1.27 \times 2.61}{9.59} = +.35$

$M_{g6} = +\frac{7.1 \times 44}{44 + 9.58} = +5.83$ $M_{d6} = -\frac{5.83 \times 3.55}{8.45} = -2.45$

$M_{j14} = +.35$ $M_{K14} = -\frac{3.5 \times 5.02}{15.35} = -.12$ (x)

$M_{D1} = -\frac{2.45 \times 10.12}{10.12 + 3.36} = \frac{2.45 \times 10.12}{13.48} = -1.84$, $M_{a1} = +\frac{1}{2} \times 1.84 = +.92$

$M_{D4} = +(2.45 - 1.84) = .61$, $M_{E4} = \frac{+.61 \times 5.61}{14.79} = +.23$ (x)

$M_{H10} - M_{H9} - M_{H12} = 14.03$. $\therefore M_{H7} = -14.03$

$M_{E7} = \frac{+14.03 \times 3.70}{8.30} = +6.26$

$r_{4-5} = \frac{7.25^2}{2 \times 7.25} = 3.63$.

$M_{E2} = \frac{+3.63 \times 6.26}{3.63 + 6.80} = +2.18$ $M_{b2} = -\frac{1}{2} \times 2.18 = -1.09$

$M_{E5} = -\frac{1}{2}(6.26 - 2.18) = -2.04$ $M_{E4} = +\frac{1}{2}(6.26 - 2.18) = +2.04$.

$M_{F5} = +\frac{2.04 \times 5.71}{14.69} = +.78$ $M_{D4} = -.78$

$M_{F3} = -\frac{.78 \times 10.12}{10.12 + 3.36} = -.58$ $M_{c3} = +.29$

$M_{F8} = +(.78 - .58) = +.20$ (x)

$M_{D1} = -.58$ $M_{a1} = +.29$.

$M_{D6} = +.20$ (x)

C. Loaded span KL. Girder 15.

Ratio of roof load to floor load $= \frac{40}{125} = .32$.

$\therefore$ Moment at center portion as simply supported $= .32 \times 84.9 = 27.2$ "#

$L - \bar{x} = 10.2'$

$$A = \left. \begin{array}{l} 7.075 \times \frac{27.2}{12} \\ + 6.25 \times \frac{27.2}{12} \end{array} \right\} = 2.265 \times 13.33 = 30.22.$$

$$S_{K_{15}} = \frac{-5.05 \times 6 \times 30.22 \times 10.2}{20.4^3} \times 12 = -13.2 \text{ "\#}$$

$$S_{L_{15}} = \frac{-4.31 \times 6 \times 30.22 \times 10.2}{20.4^3} \times 12 = -11.3 \text{ "\#}$$

Increment of moment per ft. $= \frac{13.2 - 11.3}{11.06} = 0.172$

$M_{K_{15}} = -(13.2 + .172 \times 5.03) = -14.07$ "#

$M_{L_{15}} = -(11.3 - 4.31 \times .172) = -10.56$ "#

M center $= 27.2 - 11.3 = 15.9$ "#

Separation of $M_{L15} = -10.56$ "#

$M_{L13} = +10.56$, $M_{I13} = -\frac{10.56 \times 3.86}{8.34} = -4.90$

$M_{I10} = -\frac{4.90 \times 9.58}{7.53 + 9.58} = -2.74$, $M_{H10} = \frac{+5.03 \times 2.74}{15.37} = +.90$

$M_{I8} = -(4.90 - 2.74) = -2.16$, $M_{F8} = \frac{+2.16 \times 3.55}{8.45} = +.91$

$r_{12\text{-}7} = \frac{61.30 \times 14.89}{76.19} = 12.0$ $M_{F5} = +\frac{.91 \times 3.36}{3.36 + 7.29} = +.29$

$M_{H9} = \frac{+.9 \times 12}{12 + 7.80} = +.55$ $M_{E5} = \frac{-.29 \times 5.61}{14.79} = -.11$ (x)

$M_{G9} = -\frac{.55 \times 4.30}{16.10} = -.15$ (x) $M_{F3} = +(.91 - .29) = +.62$

$M_{C3} = -.62 \times \frac{1}{2} = -.31$

$M_{H7} = +\frac{61.3(.9 - .55)}{61.3 + 14.89} = +.28$

$M_{E7} = -\frac{.28 \times 3.7}{8.3} = -.13$ $M_{H12} = -(.35 - .28) = -.07$ (x)

Separation of $M_{K15} = -14.07$.

$M_{K14} = \frac{-58.25 \times 14.07}{58.25 + 38.95} = -8.43$ $M_{K12} = -(14.07 - 8.43) = -5.64$

$M_{J14} = +\frac{8.43 \times 4.31}{16.09} = +2.26$. $M_{J11} = +2.26$

$M_{G11} = -\frac{2.26 \times 3.86}{8.34} = -1.05$

$M_{G9} = +\frac{9.58 \times 1.05}{9.58 + 7.53} = +.59$ $M_{H9} = -\frac{.59 \times 5.03}{15.37} = -.19$ (x)

$M_{G6} = -(1.05 - .59) = -.46$ $M_{D6} = \frac{.46 \times 3.55}{8.45} = +.19$ (x)

$M_{H12} = +\frac{5.64 \times 3.96}{8.24} = +2.71$.

$r_{9-10} = \frac{7.80^2}{2 \times 7.80} = 3.90$, $\quad M_{H7} = + \frac{2.71 \times 3.90}{3.90 + 14.89} = +.56$

$M_{E7} = - \frac{.56 \times 3.70}{8.5} = -.24$

$M_{H9} = +\frac{1}{2}(2.71 - .56) = +1.08 \quad M_{G9} = - \frac{1.08 \times 4.30}{16.1} = -.29$

$M_{H10} = -1.08 \quad M_{I10} = +.29$

$M_{G6} = \frac{-.29 \times 44}{9.58 + 44} = -.24$ (x) $\quad M_{I8} = - \frac{.29 \times 44}{9.58 + 44} = -.24$ (x)

Live Concentrated Load:

The live panel load consists of the load imposed upon the floor area carried by each beam, which is $20' \times (6.25 + 7.08) \times \frac{1}{2}$ = 133.3 sq.ft. Now since in moment separation the unit load for roof span has been reduced by the ratio $\frac{40}{125}$, therefore if we multiply the max. & min. L.L. influence moments given in the table by the floor panel load we have the corresponding L.L. moments.

Floor concentrated load = 133.3 × 125 = 16680 #

Dead Concentrated load:

The floor & roof slabs are of 4" & 3" thickness respectively, the given dead load per ft. is 65 & 50#; in which allowance has been made for the weight of stems of the beams & girders.

As the ratio $\frac{\text{roof load}}{\text{floor load}}$ is different for dead & live load therefore the line headed with "Sum Total" has been computed, each value of which is made up of the following:

$$c + a + e + f + (a+b)\frac{125}{40} \times \frac{50}{65} \quad \text{(See Table)}.$$

If we multiply this by 133.3 × 65 or 8670 # the dead concentrated load, we have the dead load moment.

As the available time is limited the moments due horizontal wind load will not be further investigated.

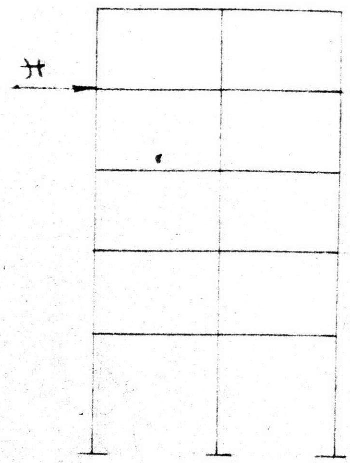
Loaded span	Member	Section	15 M_L	15 C	15 M_K	10 M_i	10 C	10 M_H	5 M_F	5 C	5 M_E	13 M_i	13 M_L	8 M_i	8 M_F	3 M_C	3 M_F	12 M_K	12 M_H	7 M_E	7 M_H	2 M_b	2 M_E
KL		M_{15L}	-10.56	+15.90		-2.74	x	+.90	+.29	x	-.11	-4.90	+10.56	-2.16	+.91	-.31	+.62	x	-.07	-.13	+.28	x	x
		M_{15K}			-14.07	+.29	x	-1.08	x	x	x	x	x	-.24	x	x	x	-5.64	+2.71	-.24	+.56	x	x
	a	Sum	-10.50	+15.90	-14.07	-2.45	x	-.18	+.29	x	-.11	-4.90	+10.56	-2.40	+.91	-.31	+.62	-5.64	+2.64	-.37	+.84	x	x
JK		M_{14J}	x	x	x	+.15	x	-.55	x	x	x	x	x	x	x	x	x	x	+.07	+.13	-.28	x	x
		M_{14K}	-2.26	x	+8.43	-.30	x	-.89	x	x	x	+1.05	-2.26	+.70	-.19	x	x	+5.64	-2.71	+.24	-.56	x	x
	b	Sum	-2.26	x	+8.43	-.15	x	-1.44	x	x	x	+1.05	-2.26	+.70	-.19	x	x	+5.64	-2.64	+.37	-.84	x	x
HI		M_{10I}	-1.59	x	+.52	-32.65	+52.25		-3.55	x	+1.35	-5.85	+1.59	+26.80	-11.26	+3.86	-7.71	+.21	-.10	-.25	+.10	-.08	+.53
		M_{10H}	+.18	x	-.68	x		-44.13	+.78	x	-2.04	x	-.18	x	+.20	+.29	-.58	-1.36	+3.40	+6.26	-14.03	-1.09	+2.18
	c	Sum	-1.41	x	-.16	-32.65	+52.25	-44.13	-2.77	x	-0.69	-5.85	+1.41	+26.80	-11.06	+4.15	-8.29	-1.15	+3.30	+6.01	-13.93	-1.17	+2.71
GH		M_{9G}	+.08	x	-.31	x	x	x	+.22	x	-.57	x	x	x	x	x	x	-.21	+.10	+.25	-.10	+.08	-.53
		M_{9H}	-.17	x	-.56	-7.1	x	+26.7	+.17	x	-2.27	+.27	-.17	-5.83	+2.25	-1.21	+2.42	+1.36	-3.40	-6.26	+14.03	+1.09	-2.18
	d	Sum	-.09	x	-.87	-7.1	x	+26.7	+.39	x	-2.84	+1.27	-.17	-5.33	+2.25	-1.21	+2.42	+1.15	-3.30	-6.01	+13.93	+1.17	-2.71
FE		M_{5F}	+.15	x	x	-3.26	x	+1.07	-47.13	+39.15		+.56	-.15	+3.82	-11.63	-17.75	+35.50	+.08	-.08	-.15	+.34	x	x
		M_{5E}	x	x	x	+.44	x	-1.67			-45.23	+.08	x	-.36	+.15	x	x	x	-.21	+8.5	-3.54	+9.47	-18.93
	e	Sum	+.15	x	x	-2.92	x	-.60	-47.13	+39.15	-45.23	+.64	-.15	+3.46	-11.48	-17.75	+35.50	+.08	-.29	+8.35	-3.20	+9.47	-18.93
DE		M_{4D}	x	x	x	+.17	x	-.65	x	x	x	x	x	x	x	x	x	-.08	+.08	+.15	-.34	x	x
		M_{4E}	x	x	x	-.04	x	-1.51	-.69	x	+17.3	-.16	x	-.20	+1.57	+2.59	-5.18	x	+.21	-8.5	+3.54	-9.47	+18.93
	f	Sum.	x	x	x	+.13	x	-2.16	-.69	x	+17.8	-.16	x	-.20	+1.57	+2.59	-5.18	-.08	+.29	-8.35	+3.20	-9.47	+18.93
Sum Total (see last page)			-32.15	+38.20	-14.56	-48.69	+52.25	-24.08	-49.50	+39.15	-31.22	-13.35	+20.99	+22.53	-16.99	-12.97	+25.93	0	0	0	0	0	0
Max.+ L.L only			+.15	+15.90	+8.43	+.13	+52.25	+26.70	+.68	+39.15	+17.8	+2.76	+11.97	+30.96	+4.73	+6.74	+38.54	+6.87	+6.23	+14.73	+17.97	+10.64	+21.64
Min.- " "			-14.32	x	-15.10	-45.17	x	-48.51	-50.59	x	-48.87	-10.91	-2.58	-8.43	-22.73	-19.27	-13.47	-6.87	-6.23	-14.73	-17.97	-10.64	-21.64

Final moment Table. Moments in 10000 in. lbs.

Member & Section	M_{15L}	M_{15} Cent.	M_{15K}	M_{I10}	M_{10} Cent	M_{H10}	M_{F5}	M_5 Cent.	M_{E5}	M_{I13}	M_{L13}	M_{I8}	M_{F8}	M_{C3}	M_{F3}	M_{K12}	M_{H12}	M_{E7}	M_{H7}	M_{b2}	M_{E2}
D.L.M	-27.06	+33.12	-12.61	-42.20	+45.25	-20.86	-42.90	+33.90	-27.06	-11.58	+18.20	+19.52	-14.71	-11.24	+22.48	0	0	0	0	0	0
+ L.L.M.	+.25	+26.53	+14.07	+.22	+87.10	+44.50	+1.13	+65.30	+29.70	+4.94	+19.96	+51.60	-7.89	+11.25	+64.25	+11.4[illegible]	[illegible]	+24.58	[illegible]	[illegible]	[illegible]
− L.L.M.	-23.90	− x	-25.20	-75.35	− x	-81.00	-84.40	− x	-81.60	-8.20	-4.30	-14.08	-37.90	-32.15	-22.45	-11.4[illegible]	-10.39	[illegible]	[illegible]	[illegible]	-36.[illegible]
Max.	x	+59.65	+1.46	x	+132.35	+23.64	x	+99.20	+2.64	x	+38.16	+71.12	x	+.01	+86.73	+11.46	+10.39	+24.58	+29.97	+17.75	+36.15
Min.	-50.96	x	-37.81	-117.55	x	-101.86	-127.30	x	-108.66	-29.78	x	x	-52.61	-43.39	x	-11.46	-10.39	-24.58	-29.97	-17.75	-36.15

Example 3.

The problem is to solve the values of the coefficients for the frame subjected to the load H by method of determinants. The corresponding equations are given as following:-



$$+3x - 7y + 4z - 1r + 2s = 0$$
$$-2x + 6y - 3z + 4r - 1s = 1$$
$$+5x - 10y + 21z - 6r + 4s = 0$$
$$-7x + 12y - 11z + 4r - 5s = 0$$
$$+3x - 1y + 7z - 9r + 6s = 0$$

From Rule 3, page 66, we can write the coefficients in the folling fractional form:

$$x = \frac{\Pi_x}{D}, \quad y = \frac{\Pi_y}{D}, \quad z = \frac{\Pi_z}{D}, \quad r = \frac{\Pi_r}{D} \;\&\; s = \frac{\Pi_s}{D}.$$

Where Π_x, Π_y etc. are the numerators of the corresponding coefficients, and D their common denominator.

Computation of denominator: Rule 3

$$D = \begin{vmatrix} +3 & -7 & +4 & -1 & +2 \\ -2 & +6 & -3 & +4 & -1 \\ +5 & -10 & +21 & -6 & +4 \\ -7 & +12 & -11 & +4 & -5 \\ +3 & -1 & +7 & -9 & +6 \end{vmatrix}$$

Multiply the first row by $\frac{2}{3}$, $\frac{5}{3}$, $\frac{7}{3}$, and -1 and add the respective respective result to 2nd. 3rd 4th. 5th. row, we have

$$= \begin{vmatrix} +3 & -7 & +4 & -1 & +2 \\ 0 & +\frac{4}{3} & +\frac{2}{3} & +\frac{10}{3} & +\frac{1}{3} \\ 0 & -\frac{5}{3} & +\frac{43}{3} & -\frac{13}{3} & +\frac{2}{3} \\ 0 & -\frac{13}{3} & -\frac{5}{3} & +\frac{5}{3} & -\frac{1}{3} \\ 0 & +7 & +3 & -8 & +4 \end{vmatrix} = 3 \begin{vmatrix} +\frac{4}{3} & +\frac{2}{3} & +\frac{10}{3} & +\frac{1}{3} \\ +5 & +\frac{43}{3} & -\frac{13}{3} & +\frac{2}{3} \\ -\frac{13}{3} & -\frac{5}{3} & +\frac{5}{3} & -\frac{1}{3} \\ +7 & +3 & -8 & +4 \end{vmatrix}$$

Multiply the 1st row by $-\frac{15}{4}$, $+\frac{13}{4}$ & $\frac{21}{4}$ and add the respective result to 2nd, 3rd. & 4th. row, we have.

$$= 3 \begin{vmatrix} +\frac{4}{3} & +\frac{2}{3} & +\frac{10}{3} & +\frac{1}{3} \\ 0 & +\frac{71}{6} & -\frac{189}{12} & -\frac{7}{2} \\ 0 & +\frac{1}{2} & +\frac{25}{5} & +\frac{3}{4} \\ 0 & -\frac{1}{2} & +\frac{51}{2} & +\frac{9}{4} \end{vmatrix} = +3 \times +\frac{4}{3} \begin{vmatrix} +\frac{71}{6} & -\frac{63}{4} & -\frac{7}{12} \\ +\frac{1}{2} & +\frac{25}{2} & +\frac{3}{4} \\ -\frac{1}{2} & +\frac{51}{2} & +\frac{9}{4} \end{vmatrix}$$

Multiply the 1st. row by $-\frac{3}{71}$ & $+\frac{3}{71}$ and add the corresponding result to 2nd & 3rd row, we have

$$= 4 \begin{vmatrix} \frac{71}{6} & -\frac{63}{4} & -\frac{7}{12} \\ 0 & +\frac{3739}{284} & +\frac{55}{71} \\ 0 & +\frac{7053}{284} & +\frac{71}{31} \end{vmatrix} = 4 \times \frac{71}{6} \begin{vmatrix} +\frac{3739}{284} & +\frac{55}{71} \\ +\frac{7053}{284} & +\frac{79}{31} \end{vmatrix} = \frac{142}{3} \left(\frac{3739 \times 79}{284 \times 31} - \frac{7053 \times 55}{284 \times 71} \right)$$

$$= \underline{690.05}.$$

Computation of the numerator of y.

Principal 3

$$N_y = \begin{vmatrix} +3 & 0 & +4 & -1 & +2 \\ -2 & +1 & -3 & +4 & -1 \\ +5 & 0 & +21 & -6 & +4 \\ -7 & 0 & -11 & +4 & -5 \\ +3 & 0 & +7 & -9 & +6 \end{vmatrix} = - \begin{vmatrix} 0 & +3 & +4 & -1 & +2 \\ 1 & -2 & -3 & +4 & -1 \\ 0 & +5 & +21 & -6 & +4 \\ 0 & -7 & -11 & +4 & -5 \\ 0 & +3 & +7 & -9 & +6 \end{vmatrix} = + \begin{vmatrix} +1 & -2 & -3 & +4 & -1 \\ 0 & +3 & +4 & -1 & +2 \\ 0 & +5 & +21 & -6 & +4 \\ 0 & -7 & -11 & +4 & -5 \\ 0 & +3 & +7 & -9 & +6 \end{vmatrix}$$

$$= 1 \begin{vmatrix} +3 & +4 & -1 & +2 \\ +5 & +21 & -6 & +4 \\ -7 & -11 & +4 & -5 \\ +3 & +7 & -9 & +6 \end{vmatrix}$$

Multiply 1st row by $-\frac{5}{3}$, $+\frac{7}{3}$ & -1 and add the respective results to 2nd, 3rd & 4th row, we have.

$$= 1 \begin{vmatrix} +3 & +4 & -1 & +2 \\ 0 & +\frac{43}{3} & -\frac{13}{3} & +\frac{2}{3} \\ 0 & -\frac{5}{3} & +\frac{5}{3} & -\frac{1}{3} \\ 0 & +3 & -8 & +4 \end{vmatrix} = +3 \begin{vmatrix} +\frac{43}{3} & -\frac{13}{3} & +\frac{2}{3} \\ -\frac{5}{3} & +\frac{5}{3} & -\frac{1}{3} \\ +3 & -8 & +4 \end{vmatrix}$$

$$= +3 \begin{vmatrix} +\frac{43}{3} & -\frac{13}{3} & +\frac{2}{3} \\ 0 & +\frac{50}{43} & -\frac{11}{43} \\ 0 & -\frac{305}{43} & +\frac{166}{43} \end{vmatrix} = 3 \times \frac{43}{3} \begin{vmatrix} +\frac{50}{43} & -\frac{11}{43} \\ -\frac{305}{43} & +\frac{166}{43} \end{vmatrix}$$

$$= +43 \left[\frac{50 \times 166}{43 \times 43} - \frac{11}{13} \times \frac{305}{43} \right] = \underline{115}$$

$$\therefore y = \frac{115}{690.05} = .166$$

In the same manner we can compute N_x, N_z, N_r & N_s and from which the coefficients x, z, r & s can be found —

CONCLUSIONS.

As the least work method has the special merit for its exactness, the method herein described possesses the preference for its simplicity. The good features of this method can be summarized as follows;

1. In applying this method it is not necessary to remember any formula, as soon as the general principles involved are understood.

2. The computation can be done with single slide rule operations. There is no long expressions to be computed, this lessens the liability of making numerical errors.

3. As soon as the so-called characteristics have been computed for a given framed structure, the moments can be found for any kind of loading. Influence table can also be prepared with great ease. With the same amount of work it does not seem to be possible to do that in applying both of the least work and the ordinary slope deflection method.

4. The moments of all members and at all points of a member can be computed at one time. This is illustrated in the example of the three story-building.

5. Irregular structures and structures subjected to eccentric load, as crane load, can be investigated without the least difficulty, as the computation of characteristics and the separation of moments can be handled in usual manner.

6. The amount of labor to be put in for investigation can vary according to the degree of accuracy required, as the separation

of moments is quite elastic. It can be carried from a fairly accurate result to one, which is almost exact.

7. The variation of cross section of a member can be taken care of, as it only involves the additional labor to multiply the slope angle by a constant.

The writer wants to add a few words on the necessity of a close investigation for buildings by this method. It is a well-known fact that for high buildings, on account of wind load and the economy of the structure, a close investigation is always necessary. For ordinary buildings up to four to five story high such investigation has never been attempted in ordinary office practice, because the engineer s do not deem it justified to spend the labor and time to do it by means of exact methods known to them. Now as simple as this method appears, it seems to the writer that it does pay the small amount of labor involved in this method, for it eliminates the guesswork which is generally made in designing wall columns or columns subjected to crane load for ordinary office and mill buildings.

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