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Resilient Provision of a Public Good

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Abstract. We present two resilient mechanisms for the provision of a public good. Both mechanisms adopt a knowledge-based benchmark.

Introduction Our first mechanism is appropriate to the case when the players —i.e., the potential beneficiaries of the good— are few in number and/or know each each other quite well. (In this case, for concreteness, we envision the provisioning to occur in a *laboratory*, and refer to the good as —a new piece of— *equipment*, to the players as members of the lab, and to the potential provisioner as the *lab director*.)

Our second mechanism is more appropriate when the players are quite numerous and/or may have only *local knowledge*, that is, each player only knows a few of the other players. (In this case, for concreteness, we envision the provisioning to occur in a *city*, and refer to the good as a *public park*, to the players as citizens, and to the potential provisioner as the *mayor*.)

Notation. Let $N = \{1, \ldots, n\}$ be a set of players, and $\gamma \in \mathbb{R}^+$ the cost (to the "potential provider") of provisioning the good. A player i's valuation of the good is a non-negative real. The profile of all possible valuations of the players is denoted by $\mathbb{V}$. The profile of the players' true valuations is denoted by TV. An outcome is a pair (x, P), where x is a bit indicating whether the good will be provided ($x = 1$) or not ($x = 0$), and P is a profile of prices (real numbers). A payer's utility is $TV_i \cdot x - P_i$. A player i's *general external* knowledge, denoted by GK_i, is i's information about TV_{-i}. A player i's *relevant external knowledge*, denoted by RK^i, is a subprofile in $\mathbb{V}_{-i}$ such that, for each $j \neq i$, RK^i_j is the maximum value with GK_i and less than TV_j. All knowledge of a player is private to him.

In the two mechanisms below, "numbered steps are performed by players, and bullet ones by the mechanism."

1 Our First Mechanism

Mechanism $\mathcal{M}_1$

- Set $x = 0$ and $P_i = 0$ for each player i.

1. Each player i simultaneously and publicly announces a valuation subprofile V^i for players in $-i$.

- Set: $\gamma_i = \sum_{j \in -i} V^i_j$ for each player i, and $\star = \arg\max_i \gamma_i$.
  (We shall refer to player $\star$ as the "star player".)
- If $\gamma_\star < \gamma$, HALT.

2. (If $\gamma_\star \geq \gamma$) Each player i such that $V^\star_i > 0$ publicly and simultaneously announce YES or NO.

- If some player announces NO, reset $P_\star = P_\star + V^\star_i$ for each player i who announces NO, and HALT.
- (If all players announce YES) Reset: (1) $x = 1$; (2) $P_\star = \gamma - \gamma_\star$; and (3) $P_i = V^\star_i$ for each player $i \neq \star$.

Variant. In the last mechanism step replace instruction 2 with the following instruction (2′) $P_\star = \alpha \cdot (\gamma - \gamma_\star)$, where the coefficient α is a constant between 0 and 1 (so as to generate a "surplus" for the lab).

2 Our Second Mechanism

Mechanism $\mathcal{M}_2$

- Set $x = 0$ and $P_i = 0$ for each player i.

1. Each player i simultaneously and publicly announces (A) a subset of players $S_i \subset -i$ and (B) a valuation subprofile V^i for the players in S_i.

- $\forall j$: If $j \notin S_i$ for all $i \neq j$, then set $EV_j = 0$; else, $n_j = \arg\max_{i \neq j} V^i_j$, and set $EV_j = V^{n_i}_j$. Set $K = \sum_j EV_j$.

- If $K < \gamma$, HALT.

2. $(K \geq \gamma)$ $\forall j$ such that $EV_j > 0$ publicly and simultaneously announces YES or NO.

- If some player announces NO: $\forall k$ such that player k announces NO, reset $P_{n_k} = EV_k$. HALT.

- (All players announce YES) (1) reset $x = 1$ and $P_j = EV_j$ for all j; (2) $\forall j$, reset $P_{n_j} = P_{n_j} - (K - \gamma)\frac{EV_j}{K}$.

Variant. In the last mechanism step replace instruction 2 with the following instruction

$$(2') \; P_{n_j} = P_{n_j} - \alpha \cdot (K - \gamma)\frac{EV_j}{K}$$

where the coefficient α is a constant between 0 and 1 (so as to generate a "surplus" for the city).