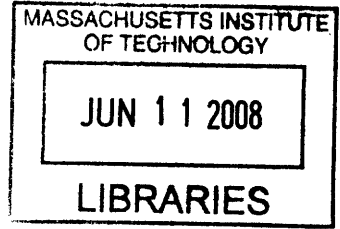


# Essays in Incomplete Markets

by  
Vasia Panousi



Submitted to the Department of Economics  
in partial fulfillment of the requirements for the degree of  
Doctor of Philosophy in Economics

ARCHIVES

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY  
[June 2008]  
May 2008

© Vasia Panousi, MMVIII. All rights reserved.

The author hereby grants to MIT permission to reproduce and distribute publicly paper and electronic copies of this thesis document in whole or in part.

Author .....  
.....  
Department of Economics  
May 15, 2008

Certified by .....  
.....  
George-Marios Angeletos  
Professor of Economics  
Thesis Supervisor

Certified by .. .....  
.....  
Mikhail Golosov  
Dornbusch Career Development Assistant Professor of Economics  
Thesis Supervisor

Accepted by .....  
.....  
Peter Temin  
Elisha Gray II Professor of Economics  
Chairman, Department Committee on Graduate Students



# Essays in Incomplete Markets

by

Vasia Panousi

Submitted to the Department of Economics  
on May 15, 2008, in partial fulfillment of the  
requirements for the degree of  
Doctor of Philosophy in Economics

## Abstract

This thesis studies the macroeconomics of incomplete markets. Chapter 1 studies the effects of capital taxation in a dynamic heterogeneous-agent economy with uninsurable entrepreneurial risk. Unlike either the complete-markets paradigm or Bewley-type models where idiosyncratic risk impacts only labor income, here it is shown that capital taxation may actually stimulate capital accumulation. This possibility emerges because of the general-equilibrium effects of the insurance aspect of capital taxation. Chapter 2, which is joint work with George-Marios Angeletos, revisits the macroeconomic effects of government consumption in the neoclassical growth model augmented with idiosyncratic investment (or entrepreneurial) risk. Under complete markets, a permanent increase in government consumption has no long-run effect on the interest rate, the capital-labor ratio, and labor productivity, while it increases work hours due to the familiar negative wealth effect. Under incomplete markets, the very same negative wealth effect now causes a reduction in risk taking and investment. This in turn leads to a lower risk-free rate and, under certain conditions, also to a lower capital-labor ratio, lower productivity and lower wages. Chapter 3 uses annual Greek data to test the validity of the Permanent Income Hypothesis (PIH) versus the “Keynesian” view that consumption responds to current income. The PIH is rejected by all tests, and so is the simple excess sensitivity hypothesis with a constant marginal propensity to consume. Subsequently, the relevance of a more sophisticated excess sensitivity version, with a time-varying marginal propensity to consume, is determined. It is shown that liquidity constraints, in the form of the spread between private interest rates on loans and deposits, negatively affect the marginal propensity to consume out of current disposable labor income. This result disappears when total disposable income is considered.

Thesis Supervisor: George-Marios Angeletos  
Title: Professor of Economics

Thesis Supervisor: Mikhail Golosov  
Title: Dornbusch Career Development Assistant Professor of Economics



## Acknowledgments

I am deeply indebted to my primary advisor, George-Marios Angeletos, for his constant support and guidance. I am extremely grateful to my advisors Mike Golosov and Ivan Werning for extremely constructive feedback and discussions. I would like to thank Daron Acemoglu, Olivier Blanchard, Sylvain Chassang, Peter Diamond, Simon Gilchrist, Jerry Hausman, Jiro Kondo, Dimitris Papanikolaou, James Poterba, Catarina Reis, Robert Townsend, Harald Uhlig, and seminar participants at MIT for useful comments.



# Contents

<b>I</b>	<b>Capital Taxation with Entrepreneurial Risk</b>	<b>13</b>
1.1	Introduction . . . . .	13
1.2	The Model . . . . .	18
1.2.1	Preferences . . . . .	18
1.2.2	Entrepreneurs . . . . .	18
1.2.3	Laborers . . . . .	19
1.2.4	Government . . . . .	19
1.2.5	Finite lives and annuities . . . . .	20
1.3	Equilibrium . . . . .	20
1.3.1	Individual Behavior . . . . .	20
1.3.2	Equilibrium definition . . . . .	22
1.3.3	General equilibrium . . . . .	22
1.4	Steady State . . . . .	24
1.4.1	Characterization of aggregates . . . . .	24
1.4.2	Characterization of invariant distributions . . . . .	25
1.5	Steady-State Effects of Proportional Capital Taxation . . . . .	25
1.6	Simulations, Parameter Choice, and Benchmark Model . . . . .	27
1.6.1	Simulations . . . . .	28
1.6.2	Parameter choice . . . . .	29
1.6.3	Steady-state aggregates and distributions . . . . .	32
1.7	Effects of Capital-Income Taxation . . . . .	34
1.7.1	Steady state . . . . .	34
1.7.2	Dynamics of eliminating the capital-income tax . . . . .	37
1.8	Extension: Introducing Publicly Traded Sector . . . . .	44
1.9	Conclusions . . . . .	45
<b>2</b>	<b>Revisiting the Supply-Side Effects of Government Spending Under Incomplete Markets</b>	<b>47</b>
2.1	Introduction . . . . .	47
2.2	The basic model . . . . .	50
2.2.1	Households and firms . . . . .	50
2.2.2	Government . . . . .	52
2.2.3	Equilibrium definition . . . . .	52

2.3	Equilibrium . . . . .	52
2.3.1	Individual behavior . . . . .	52
2.3.2	General equilibrium . . . . .	54
2.4	Steady State . . . . .	56
2.4.1	Characterization . . . . .	56
2.4.2	A graphical representation . . . . .	56
2.4.3	The long-run effects of government consumption . . . . .	58
2.4.4	Numerical simulation . . . . .	59
2.5	Endogenous labor . . . . .	61
2.5.1	GHH preferences . . . . .	61
2.5.2	KPR preferences . . . . .	62
2.5.3	Hand-to-mouth workers . . . . .	63
2.5.4	The long-run effects of government consumption with endogenous labor . . . . .	64
2.6	Dynamic responses . . . . .	65
2.7	Conclusion . . . . .	69
<b>3</b>	<b>Consumption Behavior in Greece: Alternative Explanations, Identification and Interpretation</b>	<b>71</b>
3.1	Introduction . . . . .	71
3.2	Consumption and the variance of consumption innovations . . . . .	73
3.3	Bivariate VAR on income and saving . . . . .	74
3.4	Bivariate VAR hypothesis tests . . . . .	75
3.5	Single equation tests and interpretations . . . . .	76
3.5.1	A single equation test of the PIH . . . . .	76
3.5.2	Unemployment as proxy for liquidity constraints . . . . .	76
3.5.3	The weak instruments problem . . . . .	77
3.5.4	Estimation of the single consumption equation (3.3) . . . . .	78
3.5.5	A more sophisticated version of the Keynesian view . . . . .	79
3.6	Conclusions . . . . .	81
<b>A</b>	<b>Appendix to Chapter 1</b>	<b>83</b>
<b>B</b>	<b>Appendix to Chapter 2</b>	<b>93</b>
<b>C</b>	<b>Appendix to Chapter 3</b>	<b>97</b>



# List of Figures

1-1	Lorenz Curves for Wealth and Consumption . . . . .	33
1-2	Wealth Distribution for Entrepreneurs and Laborers . . . . .	34
1-3	Steady State and Capital-Income Taxation . . . . .	35
1-4	Robustness Checks . . . . .	36
1-5	Dynamics of Incomplete Markets: Eliminating the Capital-Income Tax . . .	39
1-6	Dynamics of Complete Markets: Eliminating the Capital-Income Tax . . . .	40
1-7	Short-Run vs Long-Run Welfare Implications in Cross-Section . . . . .	41
1-8	Short Run (SR) vs Long Run (LR): Human Wealth and Saving Returns . . .	43
2-1	The steady state and the effects of higher government spending. . . . .	57
2-2	Dynamic responses to a permanent shock with KPR preferences. . . . .	67
2-3	Dynamic responses to a permanent shock with hand-to-mouth agents. . . . .	68



# List of Tables

1.1	Benchmark Calibration Values . . . . .	29
1.2	Steady-State Aggregates . . . . .	32
1.3	Distribution of Wealth in the US and in the Model. . . . .	33
1.4	Dynamics of Eliminating the Capital-Income Tax. . . . .	38
1.5	Short-Run Welfare Implications of Eliminating the Capital-Income Tax. . . . .	41
2.1	The steady-state effects of the size of government. . . . .	60
2.2	Long-run effects of a permanent 1% increase in government consumption. . . . .	61
2.3	Long-run effects with endogenous labor. . . . .	65
3.1	Likelihood Ratio (LR) Tests. . . . .	75
3.2	The Role of Unemployment. . . . .	78
3.3	The Role of Liquidity. . . . .	79
3.4	Time-Varying Marginal Propensity to Consume. . . . .	80



# Chapter 1

## Capital Taxation with Entrepreneurial Risk

### Abstract

This chapter studies the effects of capital taxation in a dynamic heterogeneous-agent economy with uninsurable entrepreneurial risk. Although it allows for rich general-equilibrium effects and a stationary distribution of wealth, the model is highly tractable. This permits a clear analysis, not only of the steady state, but also of the entire transitional dynamics following any change in tax policies. Unlike either the complete-markets paradigm or Bewley-type models where idiosyncratic risk impacts only labor income, here it is shown that capital taxation may actually stimulate capital accumulation. This possibility emerges because of the general-equilibrium effects of the insurance aspect of capital taxation. In particular, for the preferred calibrated version of the model, when the tax on capital is 25%, aggregate output is 2.5% higher than what it would have been had the tax rate been zero. Turning to the welfare effects of a reform in capital taxation, it is shown how these effects depend on whether one focuses on the steady state or also takes into account transitional dynamics, as well as how they vary in the cross-section of the population (rich versus poor, entrepreneurs versus non-entrepreneurs).

### 1.1 Introduction

This chapter studies the macroeconomic and welfare effects of capital-income taxation in an environment where agents face uninsurable idiosyncratic risk to the return of their investment choices. Such risk is empirically important for entrepreneurs and wealthy agents, who, even

though they represent a small fraction of the population, yet they hold most of an economy's wealth. In this context, capital taxation raises an interesting tradeoff between the distortion of investment incentives and the provision of insurance against idiosyncratic capital-income risk. On the one hand, capital taxation comes at a cost, since it distorts agents' saving decisions. On the other hand, it has benefits, since it provides agents with partial insurance against idiosyncratic investment risk. This suggests that a positive tax on capital income could be welfare-improving, even if it reduced capital accumulation.

Most surprisingly though, it is shown that a positive tax on capital income may actually stimulate capital accumulation. Indeed, the steady-state levels of the capital stock, output and employment may all be maximized at a positive value of the capital-income tax. This possibility emerges because of the general-equilibrium effects of the insurance aspect of capital taxation. This result stands in stark contrast to the effect of capital taxation both under complete-markets models, and under incomplete-markets models with uninsurable labor-income risk alone. In these models, capital-income taxation, irrespectively of whether it is welfare-improving or not, necessarily discourages capital accumulation.

*Model.* This chapter represents a first attempt to study the effects of capital-income taxation in a general-equilibrium incomplete-markets economy, where agents are exposed to uninsurable idiosyncratic investment risk. The framework builds on Angeletos (2007), who develops a variant of the neoclassical growth model that allows for idiosyncratic investment risk, and studies the effects of such risk on macroeconomic aggregates. Here, as in Angeletos's model, agents own privately held businesses that operate under constant returns to scale. Agents are not exposed to labor-income risk, and they can freely borrow and lend in a riskless bond, but they cannot diversify the idiosyncratic risk in their private business investments. Abstracting from labor-income risk, borrowing constraints, and other market frictions, isolates the impact of the idiosyncratic investment risk and preserves the tractability of the general-equilibrium dynamics. The present model extends Angeletos's model in the following ways. First, a government is introduced, imposing proportional taxes on capital and labor income, along with a non-contingent lump-sum tax or transfer. Second, agents have finite lives, which ensures the existence of a stationary wealth distribution. Third, there is stochastic, though exogenous, transition in and out of entrepreneurship, which helps capture the observed heterogeneity between entrepreneurs and non-entrepreneurs without the complexity of endogenizing occupational choice. Fourth, labor supply is endogenous. Clearly the first element is essential for the novel contribution of the chapter. The other three improve the quantitative performance of the model and demonstrate the robustness of the main result.

*Preview of results.* The main result of the chapter is that an increase in capital-income taxation may in fact increase capital accumulation. The intuition behind this result comes from recognizing that the overall effect of the capital-income tax on capital accumulation can be decomposed in two parts. The first part captures the response of capital to the tax in a setting with endogenous saving but exogenously fixed interest rate. This is isomorphic to examining the effects of the capital tax in a "small open economy." In this context, it is shown that an increase in the capital-income tax unambiguously decreases the steady-

state capital stock. The second part, which is the core result of this chapter, captures the importance of the general-equilibrium adjustment of the interest rate for wealth and capital accumulation. Here, an increase in the tax reduces the effective variance of the risk agents are exposed to. This reduces the demand for precautionary saving, and therefore increases the interest rate, which in turn increases steady-state wealth. With decreasing absolute risk aversion, wealthier agents are willing to undertake more risk, and hence they will increase their investment in capital. In other words, the general-equilibrium effect of the interest rate adjustment is a force that tends to increase investment and the steady-state capital stock.

For plausible parameterizations of the closed economy, the general equilibrium effect dominates for low levels of the capital-income tax, so that steady-state capital at first increases with the tax. In particular, for the preferred calibrated version of the model, the steady-state capital stock is maximized when the tax on capital is 40%. When the tax on capital is 25%, aggregate output is 2.5% higher than what it would have been had the tax rate been zero. The result that the steady-state capital stock is inversely U-shaped with respect to the capital-income tax is robust for a wide range of empirically plausible parameter values. Furthermore, the tax that maximizes steady-state capital is increasing in risk aversion or the volatility of the idiosyncratic risk. This finding reinforces the insurance interpretation of the tax system.

Subsequently, the chapter examines the aggregate and welfare effects of eliminating the capital-income tax. This is because an extensive discussion has been conducted within the context of the complete-markets neoclassical growth model about the welfare benefits of setting the capital-income tax to zero. In light of the main result here, revisiting this discussion is worthwhile. In particular, the aggregate and welfare effects are presented from two different perspectives. On the one hand, one might be interested in examining the welfare of the current generation immediately after the policy reform, taking into account the entire transitional dynamics of the economy towards the new steady state with the zero tax. On the other hand, one might be interested in examining the welfare of the generations that will be alive in the distant future, i.e. at the new steady state. For convenience, the first exercise will be referred to as the study of the short-run welfare implications of eliminating the capital-income tax, whereas the second exercise will be referred to as the study of the long-run welfare implications of eliminating the capital-income tax.

First, consider the macroeconomic effects of eliminating the capital-income tax. When markets are complete, investment increases in the short run, and it is also higher at the new long-run steady state with the zero tax, relative to the old steady state with the positive tax. By contrast, in the present model of incomplete markets, investment falls in the short run, as well as in the long run.

Second, consider the welfare effects of eliminating the capital-income tax. These vary across the different types of agents, the different levels of wealth, and the short-run and long-run perspectives. In the short run, poor agents, whether entrepreneurs or non-entrepreneurs, prefer the zero tax. This is because most of their wealth comes from wage income, and, with capital fixed, the present value of wages increases due to a fall in the interest rate. Rich agents, on the other hand, prefer a positive tax, since they benefit more from insurance

provision.

In the long run, all types of agents, and at all levels of wealth, prefer a positive tax on capital income. However, the cost of switching to a zero-tax regime is much higher for poorer than for wealthier agents of all types. This is because, in the long run, the elimination of the tax decreases the steady-state capital stock, thereby decreasing the present value of wages. Therefore poorer agents will suffer the most, since human wealth constitutes a big part of their total wealth.

*Literature review.* This chapter focuses on entrepreneurial risk, because such risk is in fact empirically relevant, even in a financially developed country like the United States. For example, Moskowitz and Vissing-Jørgensen (2002) find that 75% of all private equity is owned by agents for whom such investment constitutes at least half of their total net worth. Furthermore, 85% of private equity is held by owners who are actively involved in the management of their own firm. Given this evidence about the United States, one expects that entrepreneurial risk must be even more prevalent in less developed economies, where a large part of production takes place in small unincorporated businesses and where risk-sharing arrangements are much more limited.

This chapter relates to the strand of the macroeconomic literature discussing optimal taxation and the effects of taxation. However, most of this literature has focused on labor income risk. Chamley (1986) and Judd (1985) first established the result of zero optimal capital taxation in the long run when markets are complete. Atkeson, Chari and Kehoe (1999) generalized this result to most of the short run for an interesting class of preferences, and to the case of finitely lived agents. Aiyagari (1995) extended the complete-markets framework to include uninsurable labor income risk and borrowing constraints. In this context, when only a limited set of policy instruments are available, it becomes optimal to tax capital in the long run: a positive capital tax increases welfare, but it unambiguously lowers the level of the capital stock.

A related but different normative exercise is conducted by Davila et al. (2007). They examine constrained efficiency, in the spirit of Geanakoplos-Polemarchakis, within a version of Aiyagari's model. This exercise does not allow for risk-sharing through taxes or any other instruments, and instead considers an efficiency concept where the planner directly dictates to the agents how much to invest and to trade. Angeletos and Werning (2006) examine a similar constrained efficiency problem in a two-period version of a model with idiosyncratic investment risk. Albanesi (2006) considers optimal taxation in a two-period model of entrepreneurial activity, in a constrained efficiency setting, and following the Mirrlees optimal policy tradition. The benefit of her approach is that the source of incomplete risk-sharing is endogenously specified as the result of a private information (moral hazard) problem, and that there are no ad hoc restrictions placed on the tax instruments. However, her model does not allow for dynamics, for long-run considerations, or for general-equilibrium effects like those studied in the present chapter.

The growing literature on the effects of borrowing constraints on entrepreneurial choices has examined policy questions, and especially the implications of replacing a progressive with a proportional income tax schedule, in an Aiyagari-type framework, i.e. with decreasing



returns to scale at the individual level, borrowing constraints, and undiversifiable labor income risk. These policy exercises have been conducted from a long-run perspective, without taking into account transitional dynamics. Some examples in this area include Cagetti and DeNardi (2004), Meh (2005), and Li (2002). Benabou (2002) develops a tractable dynamic general-equilibrium model of human capital accumulation with endogenous effort and missing credit and insurance markets. Within this framework he examines the long-run tradeoffs of progressive taxation and education finance. Finally, Erosa and Koreshkova (2007) examine the long-run effects of switching from progressive to proportional income taxation in a quantitative dynastic model of human capital.

This chapter also relates to the branch of the public finance literature that considers the effects of capital taxation on portfolio allocation and risk-taking. Domar and Musgrave (1944) first proposed the idea that proportional income taxation may increase risk-taking, by having the government bear part of the risk facing the agents. This idea was formalized by Stiglitz (1969), within a two-period single-agent model, where asset returns and the level of saving are exogenously given, but where the agent optimally chooses the allocation of his fixed amount of saving between a risky and a riskless asset. Ahsan (1974) extends Stiglitz by endogenizing the intertemporal consumption-saving decision in a two-period model. He shows that the partial-equilibrium effect of capital-income taxation on risk-taking is in general ambiguous. By contrast, in the “small open economy” version of the present model, which allows for wealth accumulation over time but takes the interest rate as exogenously fixed, it is shown that the steady-state capital stock is decreasing in the capital-income tax. This finding highlights that the results here are driven by general-equilibrium effects, which is novel to the literature.

As already mentioned, the present model builds on Angeletos (2007), who abstracted from policy questions and considered instead the effect of investment risk on macroeconomic aggregates. The contribution of the present chapter is to study the effects of capital-income taxation on aggregates and welfare. Angeletos and Panousi (2007), in a framework like the one in Angeletos (2007), examine the effects of government spending on macroeconomic aggregates, but for the case where government spending is financed exclusively through lump-sum taxation.

The rest of this chapter is organized as follows. Section 2 presents the model. Section 3 describes individual behavior and the aggregate equilibrium dynamics. Section 4 characterizes the steady state in terms of aggregates and distributions. Section 5 presents and discusses the main theoretical result. Section 6 presents the calibration methodology and the parameter choices, along with the implications of the preferred calibrated model for aggregates and distributions. Section 7 quantifies the steady-state effects of capital taxation, as well as the short-run and long-run effects of eliminating the capital-income tax. Section 8 examines the robustness of the results to the availability of a safe asset in positive net supply. Section 9 concludes. All proofs are delegated to the appendix.

## 1.2 The Model

Time is continuous and indexed by  $t \in [0, \infty)$ . There is a continuum of agents distributed uniformly over  $[0, 1]$ . At each point in time an agent can be either an entrepreneur, denoted by  $E$ , or a laborer, denoted by  $L$ . The probabilities of switching between these two types are exogenous. In particular, the probability that an agent will switch from being an entrepreneur to being a laborer is  $p_{EL} dt$ , and the probability that he will switch from being a laborer to being an entrepreneur is  $p_{LE} dt$ . The measure of entrepreneurs in the economy at time  $t$  is denoted by  $\chi_t$ .

In what follows, and for expositional simplicity, labor is taken to be exogenous. All proofs in the appendix consider the general case of endogenous labor, where preferences are homothetic between consumption and leisure, i.e. they are of the King-Plosser-Rebelo (1988) specification.

### 1.2.1 Preferences

All agents are endowed with one unit of time. Preferences are Epstein-Zin over consumption,  $c$ , and they are defined as the limit, for  $\Delta t \rightarrow 0$ , of:

$$U_t = \left\{ (1 - e^{-\beta\Delta t}) c_t^{1-1/\theta} + e^{-\beta\Delta t} (E_t [ U_{t+\Delta t}^{1-\gamma} ])^{\frac{1-1/\theta}{1-\gamma}} \right\}^{\frac{1}{1-1/\theta}}, \quad (1.1)$$

where  $\beta > 0$  is the discount rate,  $\gamma > 0$  is the coefficient of relative risk aversion, and  $\theta > 0$  is the elasticity of intertemporal substitution. For  $\theta = 1/\gamma$ , this reduces to the case of standard expected utility,  $U_t = E_t \int_t^\infty e^{-\beta s} U(c_s) ds$ , where  $U(c_t) = \frac{c_t^{1-1/\theta}}{1-1/\theta}$ .

### 1.2.2 Entrepreneurs

When an agent is an entrepreneur, he owns and runs a firm operating a constant-returns-to-scale neoclassical production function  $F(k, l)$ , where  $k$  is capital input and  $l$  is labor input. An entrepreneur can only invest in his own firm's capital, although he supplies and employs labor in the competitive labor market. Capital investment in his firm is subject to uninsurable risk. The idiosyncratic shocks are i.i.d., hence there is no aggregate uncertainty. An entrepreneur can also save in a riskless bond.

The financial wealth of an entrepreneur  $i$ , denoted by  $x_t^i$ , is the sum of his holdings in private capital,  $k_t^i$ , and the riskless bond,  $b_t^i$ :

$$x_t^i = k_t^i + b_t^i. \quad (1.2)$$

The evolution of  $x_t^i$  is given by:

$$dx_t^i = (1 - \tau_t^K) d\pi_t^i + [ (1 - \tau_t^K) R_t b_t^i + (1 - \tau_t^L) \omega_t + T_t - c_t^i ] dt, \quad (1.3)$$

where  $d\pi_t^i$  are firm profits (capital income),  $R_t$  is the interest rate on the riskless bond,  $\tau_t^K$

is the proportional capital-income tax,  $\omega_t$  is the wage rate in the aggregate economy,  $\tau_t^L$  is the proportional labor-income tax,  $T_t$  are the transfers received from the government, and  $c_t^i$  is consumption. Finally, a no-Ponzi game condition is imposed.

Firm profits are given by:

$$d\pi_t^i = [ F(k_t^i, l_t^i) - \omega_t l_t^i - \delta k_t^i ] dt + \sigma k_t^i dz_t^i, \quad (1.4)$$

where  $F(k, l) = k^\alpha l^{1-\alpha}$  with  $\alpha \in (0, 1)$ , and  $\delta$  is the mean depreciation rate in the aggregate economy. Idiosyncratic risk is introduced through  $dz_t^i$ , a standard Wiener process that is i.i.d. across agents and across time<sup>1</sup>. The scalar  $\sigma$  measures the amount of undiversified idiosyncratic risk, and is an index of market incompleteness, with higher  $\sigma$  corresponding to a lower degree of risk-sharing, and  $\sigma = 0$  corresponding to complete markets.

### 1.2.3 Laborers

When an agent is a laborer, he cannot invest in capital, and he can only hold the riskless bond. He also supplies labor in the competitive labor market. Financial wealth for a laborer  $i$  is therefore:

$$x_t^i = b_t^i, \quad (1.5)$$

and its evolution is given by:

$$dx_t^i = [ (1 - \tau_t^K) R_t b_t^i + (1 - \tau_t^L) \omega_t + T_t - c_t^i ] dt. \quad (1.6)$$

### 1.2.4 Government

At each point in time the government taxes capital and bond income at the rate  $\tau_t^K$ , and labor income at the rate  $\tau_t^L$ . Part of the tax proceeds is used by the government for own consumption at the deterministic rate  $G_t$ . Government spending does not affect the utility from private consumption or the production technology. The remaining tax proceeds are then distributed back to the households in the form of non-contingent lump-sum transfers. The government budget constraint is therefore:

$$0 = [ \tau_t^L F_L(\int_i k_t^i, 1) + \tau_t^K ( F_{K_t}(\int_i k_t^i, 1) - \delta ) \int_i k_t^i - G_t - T_t ] dt, \quad (1.7)$$

where  $F_{K_t}(\int_i k_t^i, 1)$  is the marginal product of capital in the aggregate economy, and where  $\int_i l_t^i = 1$ .

---

<sup>1</sup>Idiosyncratic risk is modeled here as coming from uninsurable i.i.d. depreciation shocks. However these shocks could also be modeled as or interpreted as i.i.d. productivity shocks.

## 1.2.5 Finite lives and annuities

All households face a constant probability of death, with Poisson arrival rate  $v dt$  at every instant in time<sup>2</sup>. There is no intergenerational altruism linking a household to its descendants, and utility is zero after death. The discount rate in preferences can then be reinterpreted as  $\beta = \tilde{\beta} + v$ , where  $\tilde{\beta}$  is the psychological or subjective discount rate and  $v$  is the probability of death.

To simplify the analysis, it is assumed that there exist annuity firms permitting all agents to get insurance against mortality risk, by freely borrowing the entire net present value of their future labor income. As a result, all agents have human wealth, denoted by  $h_t$ , and defined as the present discounted value of their net-of-taxes labor endowment plus government transfers:

$$h_t = \int_t^\infty e^{-\int_t^s ((1-\tau_j^K)R_j + v) dj} ((1-\tau_s^L)\omega_s + T_s) ds. \quad (1.8)$$

Then, total effective wealth for an agent, denoted by  $w_t^i$ , is defined as the sum of his financial and human wealth,  $w_t^i \equiv x_t^i + h_t$ . Hence, effective wealth for an entrepreneur is given by:

$$w_t^i = k_t^i + b_t^i + h_t, \quad (1.9)$$

and effective wealth for a laborer is given by:

$$w_t^l = b_t^l + h_t. \quad (1.10)$$

## 1.3 Equilibrium

### 1.3.1 Individual Behavior

Because entrepreneurs choose employment after their capital stock has been installed and their idiosyncratic shock has been observed, and because their production function,  $F$ , exhibits constant returns to scale, optimal firm employment and optimal profits are linear in own capital:

$$l_t^i = l(\omega_t) k_t^i \quad \text{and} \quad d\pi_t^i = r(\omega_t) k_t^i dt + \sigma k_t^i dz_t^i, \quad (1.11)$$

where  $l(\omega_t) \equiv \arg \max_l [F(1, l) - \omega_t l]$  and  $r(\omega_t) \equiv \max_l [F(1, l) - \omega_t l] - \delta$ . Here,  $r_t \equiv r(\omega_t)$  is an entrepreneur's expectation of the return to his capital prior to the realization of his

---

<sup>2</sup>In general, with finite lives and no altruism, Ricardian equivalence might fail, since some of the tax burden associated with the current issue of a bond is borne by agents who are not alive when the bond is issued. For  $v = 0$ , Ricardian equivalence holds in the model, because all agents can freely borrow in the riskless bond. However, none of the results of this chapter hinge on Ricardian equivalence, hence the government budget constraint will be written as in (1.7) for  $v$  positive but small.

idiosyncratic shock, as well as the mean of the realized returns in the cross-section of firms. The key result here is that entrepreneurs face risky, but linear, returns to their investment.

The evolution of effective wealth for an entrepreneur is described by:

$$dw_t^i = [ (1 - \tau_t^K) r_t k_t^i + (1 - \tau_t^K) R_t (b_t^i + h_t) - c_t^i ] dt + \sigma (1 - \tau_t^K) k_t^i dz_t^i. \quad (1.12)$$

The first term captures the expected rate of growth of effective wealth, showing that wealth grows when the total return to saving for an entrepreneur exceeds consumption expenditures. The second term captures the impact of idiosyncratic risk. The evolution of effective wealth for a laborer is described by:

$$dw_t^i = [ (1 - \tau_t^K) R_t (b_t^i + h_t) - c_t^i ] dt. \quad (1.13)$$

Let the fraction of effective wealth an agent saves in the risky asset be:

$$\phi_t^i \equiv \frac{k_t^i}{w_t^i}. \quad (1.14)$$

Let an agent's marginal propensity to consume out of effective wealth be:

$$m_t^i \equiv \frac{c_t^i}{w_t^i}. \quad (1.15)$$

Let  $\mu_t = (1 - \tau_t^K) r_t - (1 - \tau_t^K) R_t$  be the risk premium,  $\rho_t \equiv \phi_t (1 - \tau_t^K) r_t + (1 - \phi_t) (1 - \tau_t^K) R_t$  the net-of-tax mean return to saving for an entrepreneur, and  $\hat{\rho}_t \equiv \rho_t - 1/2 \gamma \phi_t^2 \sigma^2 (1 - \tau_t^K)^2$  the net-of-tax risk-adjusted return to saving for an entrepreneur. The net-of-tax return to saving for a laborer is simply  $(1 - \tau_t^K) R_t$ .

Because of the linearity of the budget constraints (1.12) and (1.13) in assets, and the homotheticity of the preferences, the optimal individual policy rules will be linear in total effective wealth, for given prices and government policies. I.e., for given prices and policies, an agent's consumption-saving problem reduces to a tractable homothetic problem as in Samuelson's and Merton's classic portfolio analysis. Optimal individual behavior is characterized by the following proposition.

**Proposition 1.** *Let  $\{\omega_t, R_t, r_t\}_{t \in [0, \infty)}$  and  $\{\tau_t^K, \tau_t^L, T_t, G_t\}_{t \in [0, \infty)}$  be equilibrium price and policy sequences. If an agent  $i$  is an entrepreneur, his optimal consumption, investment, portfolio, and bond holding choices respectively are given by*

$$c_t^i = m_t^E w_t^i, \quad k_t^i = \phi_t w_t^i, \quad \phi_t = \frac{(1 - \tau_t^K) r_t - (1 - \tau_t^K) R_t}{\gamma \sigma^2 (1 - \tau_t^K)^2}, \quad b_t^i = (1 - \phi_t) w_t^i - h_t. \quad (1.16)$$

*If an agent  $i$  is a laborer, his optimal consumption, investment, and bond holding choices respectively are given by*

$$c_t^i = m_t^L w_t^i, \quad k_t^i = 0, \quad b_t^i = w_t^i - h_t. \quad (1.17)$$

The marginal propensities to consume satisfy the following system of ordinary differential equations:

$$\frac{\dot{m}_t^E}{m_t^E} = m_t^E - \theta\beta + (\theta - 1)\hat{\rho}_t + \frac{\theta - 1}{1 - \gamma} p_{EL} \left[ \left( \frac{m^L}{m^E} \right)^{\frac{1-\gamma}{1-\theta}} - 1 \right] \quad (1.18)$$

$$\frac{\dot{m}_t^L}{m_t^L} = m_t^L - \theta\beta + (\theta - 1)(1 - \tau_t^K) R_t + \frac{\theta - 1}{1 - \gamma} p_{LE} \left[ \left( \frac{m^E}{m^L} \right)^{\frac{1-\gamma}{1-\theta}} - 1 \right]. \quad (1.19)$$

From (1.16) and (1.17) it is clear that optimal consumption is a linear function of total effective wealth, where the marginal propensities to consume depend only on the type of the agent and not on the level of wealth. In other words, all entrepreneurs share a common marginal propensity to consume,  $m_t^E$ , and all laborers share a common marginal propensity to consume,  $m_t^L$ . The fraction  $\phi_t$  of wealth invested in the risky asset by an agent who happens to be an entrepreneur is increasing in the risk premium, decreasing in risk aversion, and decreasing in the effective variance of risk,  $\sigma(1 - \tau_t^K)$ . Because of homotheticity and linearity,  $\phi_t$  is the same across all entrepreneurs, and independent of the level of wealth. The policy for optimal bond holdings follows from (1.9), (1.10), and (1.14). The system of (1.18) and (1.19) is a system of two Euler equations. It shows that the marginal propensities to consume, conditional on being an entrepreneur or a laborer, depend on the process of the corresponding net-of-tax anticipated (risk-adjusted) returns to saving. The last terms in the Euler equations indicate that the marginal propensity to consume for an agent is affected by the probability that he might switch between being an entrepreneur and being a laborer.

### 1.3.2 Equilibrium definition

The initial position of the economy is given by the distribution of  $(k_0^i, b_0^i)$  across households. An equilibrium is a deterministic sequence of prices  $\{\omega_t, R_t, r_t\}_{t \in [0, \infty)}$ , a deterministic sequence of policies  $\{\tau_t^K, \tau_t^L, T_t, G_t\}_{t \in [0, \infty)}$ , a deterministic macroeconomic path  $\{C_t, K_t, Y_t, L_t, W_t, W_t^E, W_t^L\}_{t \in [0, \infty)}$ , and a collection of individual contingent plans  $\{c_t^i, l_t^i, k_t^i, b_t^i, w_t^i\}_{t \in [0, \infty)}$  for  $i \in [0, 1]$ , such that the following conditions hold: (i) given the sequences of prices and policies, the plans are optimal for the households; (ii) the labor market clears,  $\int_i l_t^i = 1$ , in all  $t$ ; (iii) the bond market clears,  $\int_i b_t^i = 0$ , in all  $t$ ; (iv) the government budget constraint (1.7) is satisfied in all  $t$ ; and (v) the aggregates are consistent with individual behavior,  $C_t = \int_i c_t^i$ ,  $L_t = \int_i l_t^i = 1$ ,  $K_t = \int_i k_t^i$ ,  $Y_t = \int_i F(k_t^i, l_t^i)$ ,  $W_t = \int_i w_t^i$ ,  $W_t^E = \int_{i, E} w_t^i$ , and  $W_t^L = \int_{i, L} w_t^i$ , in all  $t$ .

### 1.3.3 General equilibrium

Because individual consumption and investment are linear in individual wealth, aggregates at any point in time do not depend on the extent of wealth inequality at that time. Therefore here, in contrast to other incomplete-markets models, it is not the case that the entire wealth distribution is a relevant state variable for aggregate dynamics. In fact, for the determination of aggregate dynamics, it suffices to keep track of the mean of aggregate wealth, and of the

allocation of total wealth between the two groups of agents. To that end, the fraction of total effective wealth held by entrepreneurs in the economy is defined as:

$$\lambda_t \equiv \frac{W_t^E}{W_t}. \quad (1.20)$$

The aggregate equilibrium dynamics can then be described by the following recursive system.

**Proposition 2.** *In equilibrium, the aggregate dynamics satisfy*

$$\dot{W}_t/W_t = \lambda_t(\rho_t - m_t^E) + (1 - \lambda_t)((1 - \tau_t^K)R_t - m_t^L) \quad (1.21)$$

$$\dot{\lambda}_t/\lambda_t = (1 - \lambda_t)\phi_t\mu_t + (1 - \lambda_t)(m_t^L - m_t^E) + p_{LE}\left(\frac{1}{\lambda_t} - 1\right) - p_{EL} \quad (1.22)$$

$$\dot{H}_t = ((1 - \tau_t^K)R_t + v)H_t - (1 - \tau_t^L)\omega_t - (\tau_t^L\omega_t + \tau_t^K(F_{K_t} - \delta)K_t - G_t) \quad (1.23)$$

$$K_t = \frac{\phi_t \lambda_t}{1 - \phi_t \lambda_t} H_t, \quad (1.24)$$

along with (1.18) and (1.19).

Equation (1.21) shows that the evolution of total effective wealth is a weighted average of two terms. The first term is positive when the mean net-of-tax return to saving for entrepreneurs exceeds their marginal propensity to consume, and is weighted by the fraction of total wealth the entrepreneurs hold in the economy. The second term is positive when the net-of-tax return to saving for laborers exceeds their marginal propensity to consume, and is weighted by the fraction of total wealth the laborers hold in the economy. Equation (1.22) shows the endogenous evolution of the relative distribution of wealth between the two groups of agents. The evolution of  $\lambda$  depends first, on the differential excess return,  $\phi_t\mu_t$ , the entrepreneurs face on their saving, second, on the difference in the level of saving between entrepreneurs and laborers,  $m_t^L - m_t^E$ , and third, on the adjustment made for the transition probabilities. Note here that the evolution of consumption can be recovered by aggregating across individual optimal policies, so that  $C_t^E = m_t^E W_t^E$  and  $C_t^L = m_t^L W_t^L$ . Equation (1.23) shows the evolution of total human wealth, using the government budget constraint  $T_t = \tau_t^L\omega_t + \tau_t^K(F_{K_t} - \delta)K_t - G_t$ , where  $F_{K_t}$  is the marginal product of capital in the aggregate production function  $F(K, 1)$ . Equation (1.24) is the bond market clearing condition. It comes from aggregating across individual capital and bond choices as given in (1.16) and (1.17), adding up, using  $B_t^E + B_t^L = 0$ , and using (1.20). From (1.24) it follows that, for given prices and human wealth, a decrease in  $\lambda$  decreases  $K$ . A fall in  $\lambda$  indicates that the entrepreneurs on average now borrow more from the laborers, hence their wealth will on average be lower. With decreasing absolute risk aversion, this will negatively affect their willingness to take risk, and hence investment and the capital stock will fall.

## 1.4 Steady State

A steady state is a competitive equilibrium as defined in section 3.2, where prices, policies, and aggregates are time-invariant. In what follows, the steady state will be characterized, first in terms of aggregates, and then in terms of the invariant wealth distributions.

### 1.4.1 Characterization of aggregates

In this section, and for expositional purposes, the steady state is characterized for the case of  $\theta = 1$ . The more general cases are treated in the appendix. When  $\theta = 1$ ,  $m_t^E = m_t^L = \beta$  for all  $t$ , and hence aggregate consumption is given by  $C_t = \beta W_t$ , where  $W_t = W_t^E + W_t^L$ . The steady state is the fixed point of the dynamic system in Proposition 2. Let government spending,  $G$ , be parameterized as a fraction  $g$  of tax revenue. The following proposition characterizes the steady state.

**Proposition 3.** (i) *The steady state always exists.* (ii) *In steady state, the capital stock,  $K$ , and the interest rate,  $R$ , are the unique solution to*

$$\lambda = \frac{\beta - (1 - \tau^K) R + p_{LE}}{\beta - (1 - \tau^K) R + p_{LE} + p_{EL}} \quad (1.25)$$

$$F_K(K) - \delta = R + \sqrt{\frac{1}{\lambda(R)} \gamma \sigma^2 (\beta - (1 - \tau^K) R)} \quad (1.26)$$

$$K = \frac{\phi(K, R) \lambda(R)}{1 - \phi(K, R) \lambda(R)} \frac{(1 - \tau^L) \omega(K) + (1 - g) (\tau^L \omega(K) + \tau^K (F_K(K) - \delta) K)}{(1 - \tau^K) R + v} \quad (1.27)$$

Here  $F_K(K)$  and  $\omega(K)$  are, respectively, the marginal product of capital and the wage rate in the aggregate economy. The proof of proposition 3 is left for the appendix<sup>3</sup>. Equation (1.25) captures the relative wealth inequality between the two groups of agents, as a function of the interest rate,  $R$ , and model parameters. Equation (1.26) can be interpreted as describing the behavior of the capital stock in an open economy, where the interest rate is exogenously given. On the one hand, an increase in the interest rate increases the opportunity cost of capital, and thus tends to lower the capital stock. This would be the only effect present under complete markets. On the other hand, an increase in the interest rate might also increase the steady-state wealth of entrepreneurs. With decreasing absolute risk aversion, this increases entrepreneurs' willingness to take risk, and hence it is a force that tends to increase the steady-state capital stock. Overall, one can show that  $K$  increases with  $R$  if and only if  $\theta > \phi/(1 - \phi)$ . Also note from (1.26) that, for given prices,  $K$  will be lower the lower is  $\lambda$ , regardless of the level of  $\tau^K$ . Finally, equation (1.27) would be irrelevant in the

<sup>3</sup>In simulations the steady state is unique for all parameter values. The same is true for the general case of endogenous labor and  $\theta \neq 1$ , although neither existence nor uniqueness have been formally shown for that case.



open economy. In the closed economy, it captures bond market clearing and it determines the equilibrium interest rate.

## 1.4.2 Characterization of invariant distributions

At each point in time, agents die and are replaced by newborn agents, who are endowed with the wealth of the exiting agents. This force generates mean reversion and guarantees the existence of an invariant wealth distribution. Let  $\xi_t^i \equiv w_t^i/W_t$  be the distance between individual and aggregate effective wealth. Let  $\Phi_L$  and  $\Phi_E$  be the conditional invariant distributions for laborers and entrepreneurs respectively. The following proposition characterizes the invariant distributions.

**Proposition 4.** *The conditional invariant distributions  $\Phi_L$  and  $\Phi_E$  are characterized by the following second order linear differential system of two equations*

$$0 = \kappa_1 \xi \frac{\partial \Phi_L}{\partial \xi} + \kappa_2 \Phi_L + p_{EL} \Phi_E,$$

$$0 = \kappa_3 \xi^2 \frac{\partial^2 \Phi_E}{\partial \xi^2} + \kappa_4 \xi \frac{\partial \Phi_E}{\partial \xi} + \kappa_5 \Phi_E + p_{LE} \Phi_L,$$

where  $\kappa_1, \kappa_2, \kappa_3, \kappa_4, \kappa_5$  are determined by steady-state aggregates.

The point to note here is that the tractability of the model allows for a very detailed characterization of the invariant distributions. This is particularly useful for the case of entrepreneurs, since it is reasonable to expect that the distribution of wealth over entrepreneurs will be to a large extent determined by the realization of entrepreneurial returns<sup>4</sup>.

## 1.5 Steady-State Effects of Proportional Capital Taxation

This section proceeds to develop the core of the contribution of this chapter, which is the study of the steady-state effects of capital-income taxation. In particular, the main result here is that an increase in the capital-income tax may actually increase investment and the steady-state capital stock. This possibility arises because of the general-equilibrium effects of the insurance aspect of capital taxation. These effects operate mainly through the endogenous adjustment of the interest rate. For simplicity, and to demonstrate that the qualitative effects do not depend on the finite horizon or the existence of two types of agents, the analysis that follows will assume that  $\lambda = 1$  and  $v = 0$ .

To facilitate the analysis, it is useful to decompose the impact of the capital-income tax on capital in two parts. The first part describes how the steady-state capital stock changes

---

<sup>4</sup>Whereas the tractability of the aggregates follows from Angeletos (2007), the result about the tractability of the invariant distributions is novel to the present chapter.

with the tax,  $\tau^K$ , when the interest rate,  $R$ , is kept constant. This effect corresponds to the case of the “small open economy” version of the model. To this end, let  $K^o(\tau^K, R)$  be the steady-state level of capital in the open economy, where both  $\tau^K$  and  $R$  are parameters. The second part describes the general-equilibrium adjustment of the interest rate in the closed economy, and the subsequent effects of this adjustment on capital accumulation. To this end, let  $R^c(\tau^K)$  be the steady-state interest rate in the closed economy, and let  $K^c(\tau^K) \equiv K^o(\tau^K, R^c(\tau^K))$  be the steady-state capital stock in the closed economy. Then, the total effect of the capital-income tax on steady-state capital can be decomposed as follows:

$$\frac{dK^c}{d\tau^K} = \frac{\partial K^o}{\partial \tau^K} + \frac{\partial K^o}{\partial R} \frac{dR^c}{d\tau^K}, \quad (1.28)$$

where the first term is the open-economy effect, and the second term is the closed-economy or general-equilibrium effect. These effects will next be examined in turn.

The first term on the right-hand-side of (1.28) is the open-economy effect. The following proposition shows that in the open-economy version of the model, an increase in capital-income taxation induces the usual negative incentive effect on capital accumulation.

**Proposition 5.** *In the open-economy version of the model, an increase in the capital-income tax unambiguously decreases the steady-state capital stock.*

Therefore, in the open economy capital falls with the tax, despite the direct insurance aspect of the tax system that is still present. This aspect of the tax system is that the government, through taxation, reduces the variance of net-of-tax returns. Here the government has effectively become a shareholder in private businesses, thereby improving the allocation of risk bearing in the economy and allowing for more risk taking. However, in the open economy this channel is not strong enough to outweigh the distortionary effect of capital taxation on investment. This result stands in contrast to the findings of Ahsan (1974). Ahsan considers the simultaneous determination of the size and the composition of the optimal portfolio, in a two-period model with exogenous returns. He shows that the partial equilibrium effect of an increase in capital-income taxation on risk-taking is in general ambiguous. Ahsan’s result is, in turn, a generalization of Stiglitz (1969), who examines the effects of proportional capital-income taxation in a two-period single-agent model, taking not only returns, but also the level of saving as exogenously given. Hence, once Ahsan’s setting is extended to incorporate endogenous capital return and infinite horizon, the result that the government can increase risk taking and investment in the risky asset by becoming a shareholder in private businesses no longer holds. It is then clear that, on top of the direct insurance role of the government, the endogenous adjustment of the interest rate is also required for the effect of capital taxation on capital to become ambiguous.

The second term on the right-hand-side of (1.28) is the general-equilibrium effect capturing the fact that in the closed economy the interest rate endogenously adjusts to clear the bond market. This term is further decomposed in two forces.

First, an increase in the capital-income tax reduces the effective volatility of risk for entrepreneurs,  $\sigma(1 - \tau^K)$ ; this is the direct insurance effect. Hence the interest rate increases,

essentially because of a reduction in the demand for precautionary saving<sup>5, 6</sup>.

Second, this increase in the interest rate will generate opposing effects on saving and wealth accumulation<sup>7</sup>. On the one hand, the increase in the interest rate increases the opportunity cost of capital, and hence it tends to decrease the steady-state capital stock. This would be the only effect present if markets were complete. On the other hand, in the present model of incomplete markets, the increase in the interest rate increases steady-state wealth, if the substitution effect of the increase in the saving return it induces is strong enough. With decreasing absolute risk aversion, this increases entrepreneurs' willingness to undertake risk, and it therefore tends to increase investment and the steady-state capital stock. The overall effect of  $R$  on  $K$  is summarized in the following proposition.

**Proposition 6.** *In the open-economy version of the model, the steady-state capital stock will be increasing in the interest rate, if and only if  $\theta > \phi/(1 - \phi)$ .*

Hence the product of the two terms on the right-hand-side of (1.28) might be positive. This means that the general-equilibrium effect of insurance provision on the adjustment of the interest rate, and the subsequent effect of this adjustment on wealth accumulation, is crucial for overthrowing the negative open-economy effect of the capital-income tax on capital. The next sections of the chapter will demonstrate how, for empirically plausible parameter values, this general-equilibrium effect will produce the counter-intuitive result that increases in the capital-income tax will at first increase steady-state capital, even with the open-economy effect working in the opposite direction.

## 1.6 Simulations, Parameter Choice, and Benchmark Model

For the quantitative part of this chapter, the benchmark model analyzed so far is extended to include endogenous labor. Preferences are assumed homothetic between consumption,  $c$  and leisure,  $n$ , according to the King-Plosser-Rebelo (1988) specification, and they are defined as the limit, for  $\Delta t \rightarrow 0$ , of:

$$U_t = \left\{ (1 - e^{-\beta\Delta t}) (c_t^{1-\psi} n_t^\psi)^{1-1/\theta} + e^{-\beta\Delta t} (E_t[U_{t+\Delta t}^{1-\gamma}])^{\frac{1-1/\theta}{1-\gamma}} \right\}^{\frac{1}{1-1/\theta}}. \quad (1.29)$$

The appendix presents all proofs for the general case of endogenous labor.

---

<sup>5</sup>Note here that in steady-state the interest rate has to be lower than the discount rate in preferences, otherwise saving and wealth would explode. The reasoning is similar to the reasoning in Aiyagari (1994). By contrast, under complete markets, the steady-state interest rate equals the discount rate.

<sup>6</sup>This intuitive result has not been proven in the context of the infinite horizon model, although a proof is available for the two period version of the closed economy, for small  $\tau^K$ . However, simulations show that in the infinite horizon model, the net interest rate is always increasing in the tax.

<sup>7</sup>This has already been mentioned in section 4.1.

### 1.6.1 Simulations

The dynamic system described in Proposition 2, and generalized to the case of endogenous labor, is highly tractable compared to other incomplete-markets models, where the entire wealth distribution is a relevant state variable for aggregate equilibrium dynamics. The steady state of the system is found by setting the dynamics of all equations to zero. The algorithm first solves for the steady-state aggregates, which are deterministic and characterized by Proposition 3. Subsequently, for any historically given  $(K_0, \chi_0, X_0^E)$ , where  $\chi_0$  is the initial measure of entrepreneurs in the economy, and  $X_0^E$  is the historically given financial wealth of the entrepreneur group, and using as boundary conditions the steady state values of  $(H, m^E, m^L)$ , it integrates backward until the path of  $(K_t, \lambda_t, H_t, m_t^E, m_t^L)$  is close enough to its steady-state value.

The method of finite differences is used on the general version of the system in Proposition 4. The first and second derivatives of the invariant distributions are replaced by their discrete time approximations. The only conditions imposed are that the probability density functions integrate to one, and that they do not explode to the right. The emerging functions  $\Phi_L$  and  $\Phi_E$  are well-behaved and stable.

Subsequently, Monte-Carlo simulations are performed. The processes of dying, of type-switching, and of the idiosyncratic capital-income shocks, are simulated using random number generators for series of 200,000 households and 100,000 years. The wealth distributions generated converge to those produced by the finite differences method, and their variances are stable as time increases. Finally, using these distributions, welfare calculations are performed.

## 1.6.2 Parameter choice

The economy is parameterized by  $(\alpha, \beta, \gamma, \delta, \theta, \sigma, \psi, v, p_{EL}, p_{LE}, \tau^K, \tau^L, G)$ . Table 1.1 presents the parameter choices for the preferred benchmark model calibration.

Parameters	Values
<b>Preferences</b>	
$\beta$	0.022
$\gamma$	10
$\theta$	1
$\psi$	0.75
<b>Technology</b>	
$\alpha$	0.4
$\delta$	0.07
<b>Probabilities</b>	
$v$	0.0067
$p_{EL}$	0.18
$p_{LE}$	0.025
<b>Government</b>	
$\tau^K$	0.25
$\tau^L$	0.35
$G/GDP$	0.2
<b>Risk</b>	
$\sigma$	0.15

**Table 1.1:** *Benchmark Calibration Values*

The parameter values chosen refer to annual data from the United States. The discount rate is  $\beta = 0.022$ . The preference parameter is  $\psi = 0.75$ , which is standard in the macro literature<sup>8</sup>. The income share of capital is  $\alpha = 0.4$ . The depreciation rate is  $\delta = 0.07$ . The probability of death is chosen to be  $v = 1/150$ , a compromise between having an empirically relevant probability of death and allowing for some altruism across generations. The probability of exiting entrepreneurship is  $p_{EL} = 0.18$ . The probability of entering entrepreneurship is  $p_{LE} = 0.025$ . These two values were estimated from the PSID and SCF data, and subsequently used for calibrations, by Quadrini (2000). In Quadrini's model, as well as here, they imply a fraction of entrepreneurs in the total population of 12%<sup>9</sup>, which is in line with the data, as Quadrini and Cagetti and DeNardi (2006) show.

The elasticity of intertemporal substitution is chosen to be  $\theta = 1$ . The empirical estimates of the elasticity of intertemporal substitution vary a lot. Using aggregate British data and correcting for aggregation bias, Attanasio and Weber (1993) estimate  $\theta$  to be about 0.7.

<sup>8</sup>King, Plosser, and Rebelo (1988), and Christiano and Eichenbaum (1992).

<sup>9</sup>The proof can be found in the appendix.

Although the exact estimates from micro data vary across studies and specifications, in most cases they are around 1, especially for agents at the top layers of wealth and asset holdings. For example, using data from the Consumer Expenditure Survey (CEX) and an Epstein-Zin specification, as in the present chapter, Vissing-Jørgensen and Attanasio (2003) report baseline estimates between 1 and 1.4 for stockholders.

The proportional tax on capital income is  $\tau^K = 0.25$ . The Congressional Budget Office Background Paper (December 2006) reports that the average marginal rate at which corporate profits are taxed is 35%, whereas the average marginal rate at which non-corporate business income is taxed is around 26%–27%. The CRS Report for Congress (October 2003) details the capital income tax revisions and effective tax rates due to provisions granted through bonus depreciations of 30% or 50%. If these provisions are taken into account, the average marginal capital income tax is between 20%–25% for non-corporate businesses and between 25%–30% for corporate businesses. The value of  $\tau^K = 0.25$  is chosen to be in the middle of these estimates<sup>10</sup>. The proportional tax on labor income is  $\tau^L = 0.35$ . The Congressional Budget Office Background Paper (December 2006) reports that the median effective marginal tax rate on labor income is 32%, inclusive of federal, state and payroll taxes<sup>11</sup>. Incorporating the distortionary effect of social security taxes would further increase this number, hence the choice made here. The level of government spending,  $G$ , is chosen so that the steady-state government-spending-to-GDP ratio is 20%.

The coefficient of relative risk aversion is chosen to be  $\gamma = 10$ . The empirical estimation of  $\gamma$  is a complicated task, because, as Vissing-Jørgensen and Attanasio (2003) detail, it requires making additional assumptions about the covariance of consumption growth with stock returns, the share of stocks in the financial wealth portfolio, the properties of the expected returns to human capital, and the share of human capital in overall wealth. Using the Consumer Expenditure Survey (CEX), Vissing-Jørgensen and Attanasio find estimates of risk aversion for stockholders in the range of 5–10, but with a broader sample and under different assumptions these estimates go up to 20–30. They also compare their results to Campbell (1996), who estimates  $\gamma$  in the range of 17–25, using data on monthly and annual returns, and assuming that the entire financial portfolio is held in stocks. Dohmen et al. (2005) present evidence on the distribution of risk attitudes in the population, using survey questions and a representative sample of 22,000 individuals living in Germany. The behavioral relevance of their survey is tested by conducting a complementary field experiment, based on a representative sample of 450 subjects. The conclusion is that the survey measure is a good predictor of actual risk-taking behavior. They find that the bulk of the mass in the  $\gamma$ -distribution is located between 1–10. There is, however, a non-negligible mass of estimates in the range of higher values, up to 20. Barsky et al. (1997) measure risk aversion based on survey responses by participants in the Health and Retirement Study to hypothetical situations. These situations were constructed using an economic theorist’s concept of the underlying parameter. They find that most individuals fall in the category that

---

<sup>10</sup>Altig et al. (2001) report a proportional capital income tax of 20% at the federal level, but they also subject capital income to a 3.7% state tax.

<sup>11</sup>This number is also reported by Jokisch and Kotlikoff (2006).

has mean relative risk aversion of 15.8. Cohen and Einav (2005) use a data set of 100,000 individuals' deductible choices in auto insurance contracts, to estimate the distribution of risk preferences. They find that the 82nd percentile in the distribution of the coefficient of relative risk aversion is about 13 – 15.

The standard deviation of the idiosyncratic entrepreneurial returns is chosen to be  $\sigma = 0.15$ . The empirical estimation of the level of idiosyncratic risk facing an entrepreneur is a very difficult task, and has not as yet received much attention in the literature. So far, the most thorough, if not the only, attempt to measure idiosyncratic risk is by Moskowitz and Vissing-Jørgensen (2002). They document poor diversification and extreme concentration of entrepreneurial investment, significant heterogeneity in individual investment choices, and high risk at the individual level due to high bankruptcy rates. However, because of the problems arising when imputing labor income, and because of the lack of sufficient time dimension in the Survey of Consumer Finances (SCF) data, they cannot provide an accurate estimate of the volatility of entrepreneurial returns for unincorporated businesses. In the end they conjecture that the volatility of returns for private firms cannot be lower than the corresponding volatility of publicly held firms, which they find to be about 0.5 per annum. Davis et al. (2006) use the Longitudinal Business Database (LBD), which contains annual observations on employment and payroll for all establishments and firms in the private sector, to estimate the volatility of employment growth rates. They find that in 2001 the ratio of private to public volatility was in the range 1.43 – 1.75. Given that the average annual standard deviation for public firms in the period 1990-1997 was 0.11<sup>12</sup>, and that there is, at least in the context of the present model, a close relationship between volatility of profits and volatility of labor demand, the choice of  $\sigma = 0.15$  could also be justified from this perspective. Finally, this choice generates an annual variance for steady-state consumption growth in the range indicated by the micro data, once consumer heterogeneity is taken into account<sup>13</sup>.

Parameters  $\gamma$  and  $\sigma$  are especially important for the calibrated model, for two reasons. First, they directly influence  $\lambda$ , the fraction of wealth held by entrepreneurs in the economy. In light of the discussions in sections 3.3 and 4.1 about the dynamic and steady-state effects of agent heterogeneity on capital accumulation, the calibrated model's implications about  $\lambda$  are a good criterion of model performance. As will be shown in the next section, the choices  $\gamma = 10$  and  $\sigma = 0.15$ , which seem empirically relevant given the discussion above, produce, without an attempt to match it, a value for  $\lambda$  that is not far from the values documented in the data. Second, parameters  $\gamma$  and  $\sigma$  relate to the interpretation of the capital-income tax as providing insurance. For this reason, comparative statics will also be performed to show how the tax that maximizes the steady-state capital stock varies with risk aversion and the volatility of risk. The main result, that steady-state capital is inversely U-shaped with respect to the capital-income tax, is preserved qualitatively for  $\sigma \in (0, 1)$  and for  $\gamma \in [1, 20]$ .

---

<sup>12</sup>Campbell et al. (2001).

<sup>13</sup>Aït-Sahalia et al. (2001), and Malloy et al. (2006).

### 1.6.3 Steady-state aggregates and distributions

This section examines the quantitative performance of the model in terms of aggregates and wealth distributions. This exercise is interesting for three reasons. First, it indicates how wealth inequality is influenced by the random-walk component introduced in wealth by the idiosyncratic investment risk. Second, it shows how wealth inequality depends on the excess returns to entrepreneurship, which is an interesting question in its own right, but also in view of the impact of agent heterogeneity on capital accumulation and the steady-state capital stock. Third, if the performance of the model can match relevant aspects of the data, this should give some additional confidence in the main quantitative results presented in the next section, about the effects of capital-income taxation on capital accumulation and welfare.

Table 1.2 presents the implications of the model for steady-state aggregates, and compares them to the data from the United States economy. The model's capital-output ratio is 2.6. Investment is 18% of GDP. The safe rate is 1.9%. The steady-state fraction of entrepreneurs,  $\chi^{ss}$ , matches the data by choice of the transition probabilities, as explained in the previous section. Entrepreneurs hold 32% of total wealth in the economy, where the equivalent of  $\lambda$  in the data is the ratio  $X^E/X$ . This is because in the data wealth is defined as total net worth, i.e. it is financial wealth,  $X$ , as defined in the present model, plus housing. The share of total wealth held by entrepreneurs in the data ranges between 35% – 55%. The fraction of entrepreneurs in the top 10% of the population is 22% in the model, whereas in the data this number ranges between 32% – 54%<sup>14</sup>.

	$K/Y$	$I/Y$	$G/Y$	$R$	$\chi^{ss}$	$X^E/X$	$\chi^{ss}$ in top 10%
US Data	2.65	17%	20%	2%	10 – 19%	35 – 55%	32 – 54%
Model	2.6	18%	20%	2.5%	12%	32%	22%

Table 1.2: *Steady-State Aggregates*

As mentioned earlier, the choices of  $\gamma$  and  $\sigma$  were made without any attempt to match  $\lambda$  to the data. This is a good indication of the model's performance, given the significance of agent heterogeneity for capital accumulation.

Next, the wealth distribution generated by the model is examined. Compared to the data, the model generates a much larger fraction of agents at negative levels of wealth, most likely because of the absence of borrowing constraints. The model's conditional wealth distribution, however, does a better job at matching the observed data. The first two rows of Table 1.3 present the percentiles for wealth computed by Quadrini (2000), using the PSID and SCF samples for 1994 and 1992 respectively. The last row is the conditional wealth distribution generated by the benchmark calibrated model.

<sup>14</sup>The data on entrepreneurs and wealth concentration are as reported in Cagetti and De Nardi (2006).

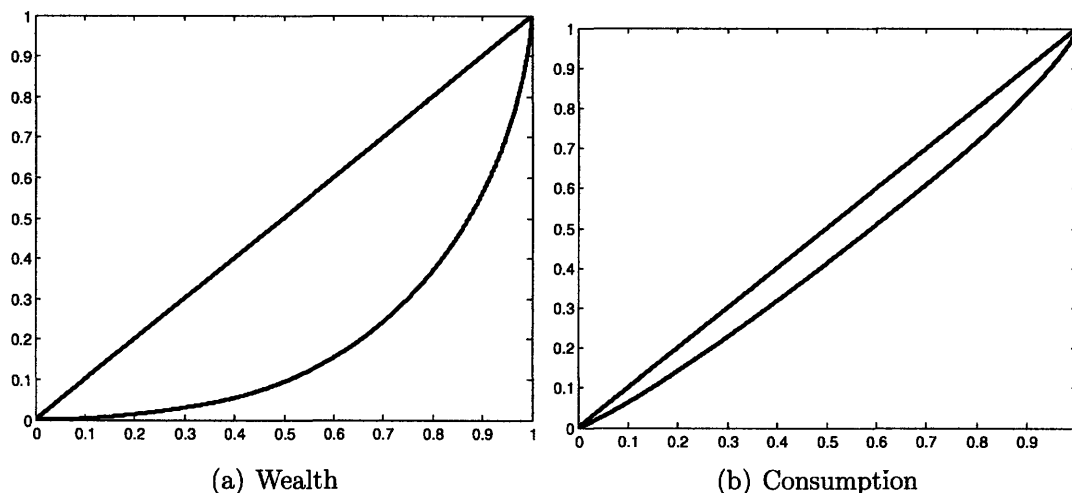


	Top Percentiles				
	30%	20%	10%	5%	1%
SCF	87.6	79.5	66.1	53.5	29.5
PSID	85.9	75.9	59.1	44.8	22.6
Model	75.85	63.13	43.77	29.16	10.12

**Table 1.3:** *Distribution of Wealth in the US and in the Model.*

Aiyagari’s (1994) benchmark calibration predictions for the wealth holdings of the top 5% and the top 1% of the population are 13.1% and 3.2% respectively. Hence the present model highlights how the random-walk component introduced in wealth by entrepreneurial risk helps generate a fatter right tail in the wealth distribution<sup>15</sup>.

Next, Figure 1.1 plots the Lorenz curves for the model’s wealth and consumption distributions. The model produces results in the right direction, in that the distribution of wealth over the population is much more unequal than the distribution of consumption. The model’s Gini coefficient for wealth, conditional on wealth being positive, is 0.61. The model’s Gini coefficient for consumption is 0.14<sup>16</sup>. In the data, the Gini coefficient for total net worth is 0.8, and the Gini coefficient for consumption is 0.32.



**Figure 1-1:** *Lorenz Curves for Wealth and Consumption*

<sup>15</sup>A tractable extension that could improve the model’s prediction about wealth concentration at the top would be to introduce a third state, in which an agent gets to be an entrepreneur operating a very high return or very low risk production function. Then the transition probabilities between the three states can be freely chosen to match desired moments of the wealth distribution. In particular, making the good entrepreneurial state the least persistent and the most likely to transition to the state of being a laborer would increase the precautionary saving, and therefore the wealth concentration, of the very rich agents.

<sup>16</sup>The differences in the Gini coefficients are due to the presence of human wealth: since poorer agents have higher human to financial wealth ratios, they can sustain relatively high consumption. This would not be the case in the presence of borrowing constraints.

Finally, the implications of the model for the wealth distributions over entrepreneurs and laborers are presented. Figure 1.2 plots the conditional distributions of wealth for the two groups. On the horizontal axis is wealth normalized by mean annual income in the economy. On the vertical axis are frequencies. The solid line represents entrepreneurs, and the dashed line laborers. Consistent with the data, the distribution of wealth for the population of entrepreneurs displays a fatter tail than the one for laborers. This is due to the random-walk component that the uninsurable investment risk introduces into entrepreneurial wealth. Furthermore, the entrepreneurial wealth distribution is shifted to the right, and it has lower frequencies at lower levels of wealth. This is due to the higher mean return of the total entrepreneurial portfolio. Finally, the distributions of wealth for both groups have significant mass of people with wealth higher than fifty times mean income. In the model, the laborers at the right tail of the wealth distribution are former successful entrepreneurs.

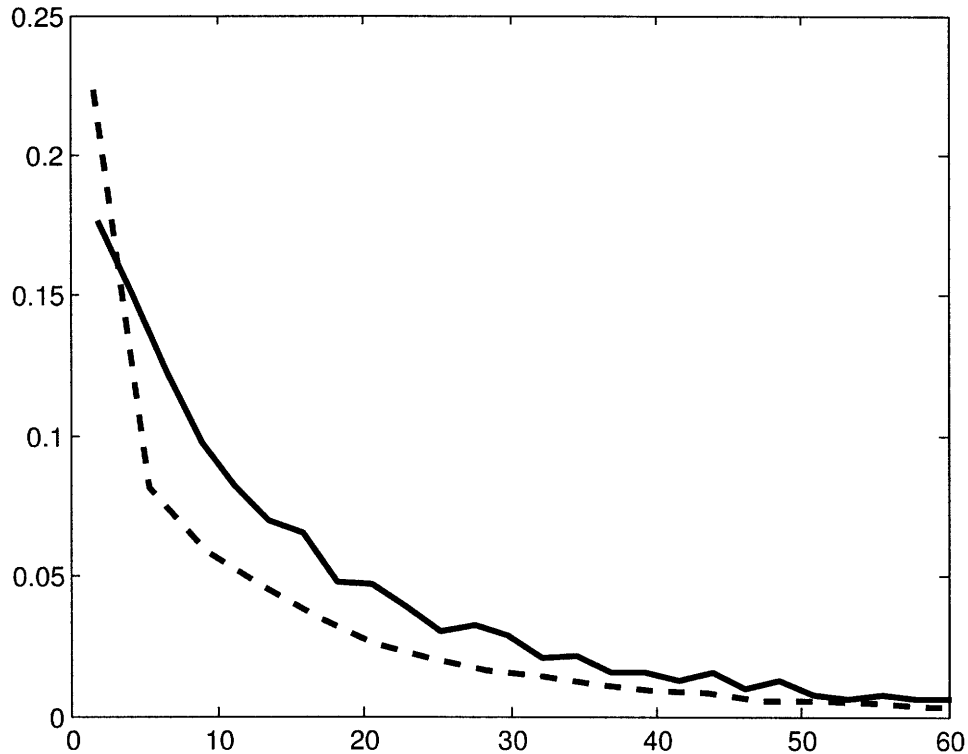


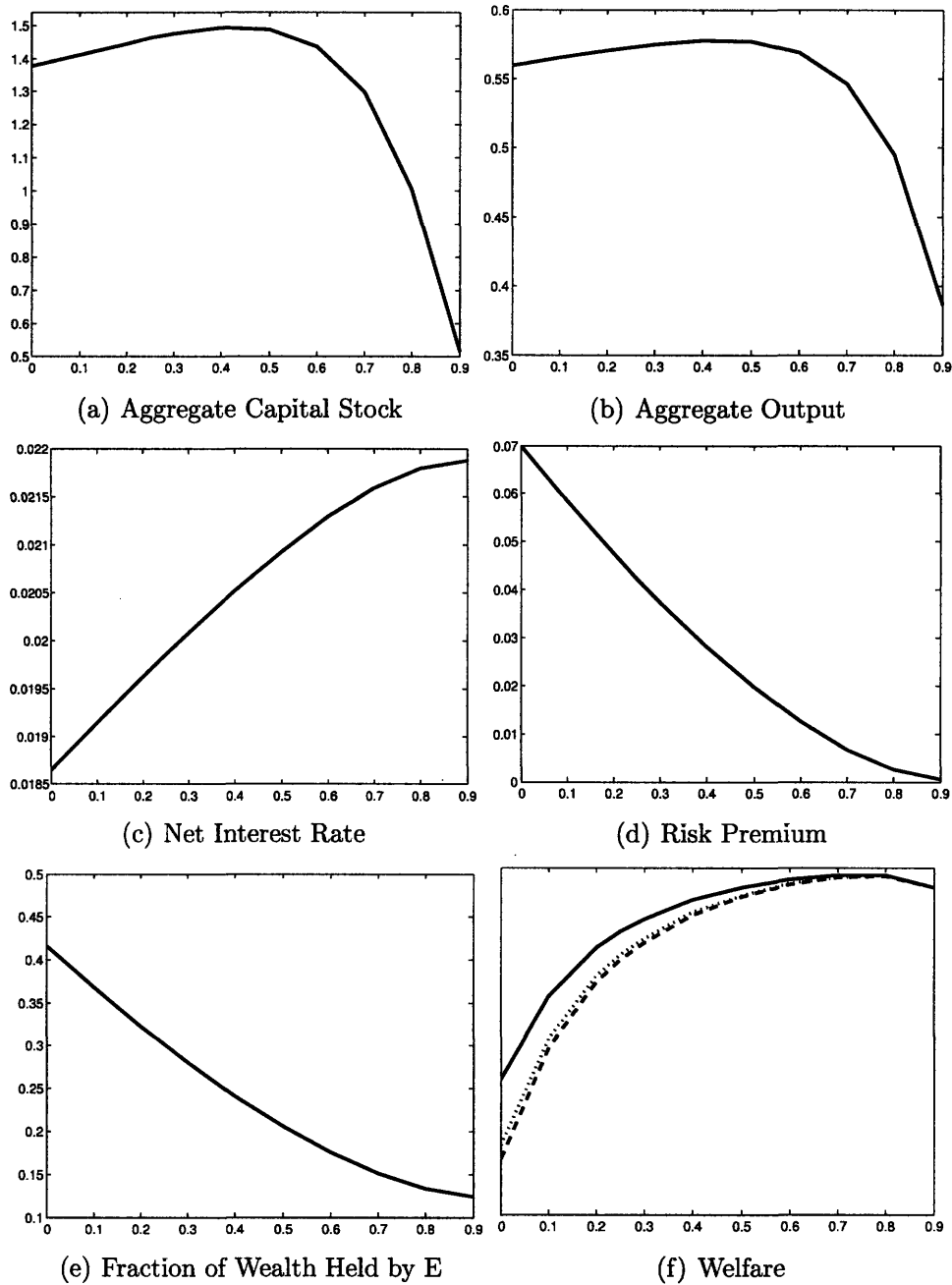
Figure 1-2: *Wealth Distribution for Entrepreneurs and Laborers*

## 1.7 Effects of Capital-Income Taxation

### 1.7.1 Steady state

Having gained some confidence about the overall quantitative performance of the model, this section quantifies the main theoretical result of the chapter, which is that an increase in the capital-income tax increases the steady-state capital stock, when the tax is low enough. This

result is due to the general-equilibrium effect of the insurance aspect of the capital-income tax, and it operates mainly through the endogenous adjustment of the interest rate.



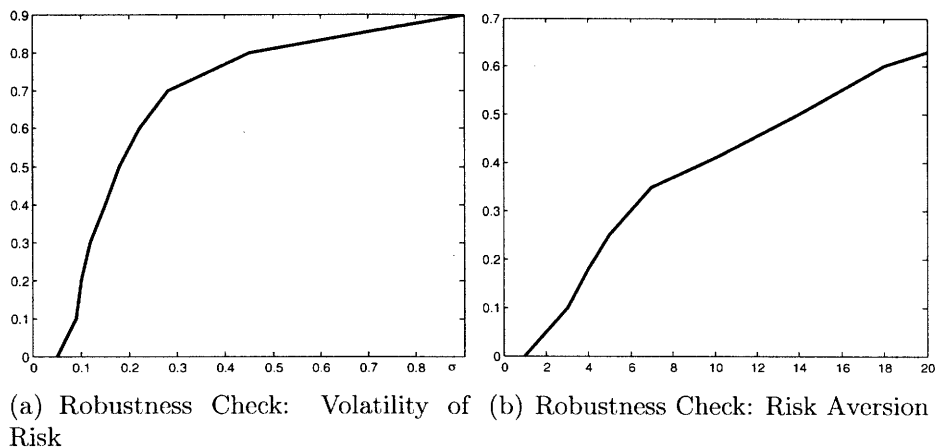
**Figure 1-3: Steady State and Capital-Income Taxation**

Figure 1.3 shows the behavior of the steady-state aggregates, and of welfare, with respect to the capital-income tax, for the benchmark calibrated version of the model. Output (panel (a)), capital (panel (b)), employment, the capital-labor (capital per work-hour) ratio, and output per work-hour are all inversely-U shaped with respect to the capital-income tax, and

they reach a maximum when  $\tau^K = 0.4$ . At this point, steady-state output is 3.3% higher than when  $\tau^K = 0$ , capital per work-hour is 8.3% higher, and output per work-hour is 3.2% higher. At  $\tau^K = 0.4$ , the capital-labor ratio and output per work-hour under complete markets are 21% and 9% lower than when  $\tau^K = 0$ . So for output per work-hour there is a 12% difference between complete and incomplete markets. As shown in Figure 1.3(f), aggregate welfare is maximized at  $\tau^K = 0.7$ , whether for entrepreneurs (solid line), laborers (dashed line), or the economy as a whole (dotted line). Naturally, entrepreneur welfare is higher than laborer welfare for all tax levels<sup>17</sup>.

When the capital-income tax increases, the effective volatility of risk facing an entrepreneur,  $\sigma(1 - \tau^K)$ , decreases. This reduces the demand for precautionary saving, and it therefore increases the interest rate. Figure 1.3(c) shows that the (net) interest rate tends to the discount rate,  $\beta = 0.022$ , as  $\tau^K \rightarrow 1$ . Figure 1.3(d) reinforces this interpretation of the capital-income tax as providing insurance: when the tax increases, the precautionary saving motive becomes weaker, and therefore entrepreneurs are satisfied with a lower risk premium. Figure 1.3(e) shows that the fraction of wealth held by entrepreneurs in the economy is decreasing in the capital-income tax. This result comes from the combination of the weaker precautionary saving motive, the fall in the risk premium, and the increase in the cost of borrowing due to the increase in the interest rate.

Figure 1.4 presents robustness checks with respect to volatility,  $\sigma$ , and risk aversion,  $\gamma$ . On the vertical axis is the tax that maximizes the steady-state capital stock. When either the volatility of risk increases or risk aversion increases, the tax that maximizes the steady-state capital stock increases. These comparative statics reinforce the insurance interpretation of the tax system. They also indicate that the main result of the chapter is robust to the wide range of empirically plausible values of  $\sigma \in (0, 1)$  and of  $\gamma \in [1, 20]$ . In particular, for the low value of  $\sigma = 0.15$ , the capital-income tax that maximizes the steady-state capital stock is positive for all  $\gamma > 1$ .



**Figure 1-4:** *Robustness Checks*

<sup>17</sup>Entrepreneur welfare is also higher than laborer welfare for all levels of wealth, since entrepreneurs are unconstrained in their investment choices.

## 1.7.2 Dynamics of eliminating the capital-income tax

The chapter now proceeds to examine the aggregate and welfare implications of eliminating the capital-income tax. In the standard complete-markets neoclassical growth model, the optimal capital-income tax is zero in the long run, as well as in most of the short run for an interesting class of preferences. Steady-state welfare is also decreasing in the level of the capital-income tax. These findings have initiated an extensive debate as to the possible benefits of eliminating the tax on capital income. By contrast, the main result of the present chapter is that an increase in the capital-income tax may actually increase the steady-state capital stock. In light of this result, it is worthwhile to revisit the discussion on the implications of setting the capital-income tax to zero.

The effects on aggregates and welfare when the capital-income tax is eliminated will be examined from two perspectives. On the one hand, one might be interested in examining the welfare of the current generation immediately after the policy reform, taking into account the entire transitional dynamics of the economy towards the new steady state with the zero tax. On the other hand, one might be interested in examining the welfare of the generations that will be alive in the distant future, i.e. at the new steady state. For convenience, the first exercise will be referred to as the study of the short-run implications of eliminating the capital-income tax, whereas the second exercise will be referred to as the study of the long-run implications of eliminating the capital-income tax.

The present model can in fact examine the short-run implications of policy reforms, because it is very tractable, compared to other incomplete-markets models, where the entire wealth distribution is a relevant state variable. Here only the mean of the wealth distribution is relevant for aggregate dynamics, which constitutes a significant gain in tractability, and allows for the entire dynamic response of the economy, after a policy change, to be considered. This is important, because it has long been recognized that the short-run effects of policy may well be very different from the long-run effects.

Here the economy starts from the steady state described by the benchmark calibration parameters, where the capital-income tax is  $\tau^K = 0.25$ . Subsequently, the tax is set to zero, *ceteris paribus*.

## Aggregate effects

This section presents the short-run and long-run responses of the aggregate variables to the policy reform that eliminates the capital-income tax. The results are compared to the case of complete markets<sup>18</sup>. Table 1.4 shows the response of the aggregates on the impact of the policy reform, as well as at the new steady state, under both complete markets and the present model of incomplete markets. The effects on the interest rate,  $R$ , the risk premium,  $\mu$ , and the investment-output ratio,  $I/Y$ , are in percentage units. The rest of the numbers denote percentage changes.

	Short Run		Long Run	
	Incomplete	Complete	Incomplete	Complete
$L$	-3.17	8.35	-0.1	1.27
$Y$	-1.95	5.001	-2.36	6.96
$C$	2.74	-5.12	-2.13	5.27
$I/Y$	-3.39	5.96	-0.63	2.34
$R_{net}$	-1.33	1.18	-0.12	0
$\mu$	3.11	0	2.75	0
$CE$	2.54	NA	1.56	NA
$X^E/X$	0	NA	11.49	NA

**Table 1.4:** *Dynamics of Eliminating the Capital-Income Tax.*

Under complete markets, a permanent (unanticipated) tax cut leads to an immediate negative jump in consumption and an immediate positive jump in investment. Capital slowly increases and converges to a higher steady-state value, whereas consumption is initially lower and increases over time. In other words, the long-run increase in investment requires an initial period of lower consumption, which in turn allows for a short-run increase in investment as well. By contrast, under incomplete markets, the exact opposite is the case. In light of the main mechanism of the chapter, investment decreases in the long run. This allows for a short-run increase in consumption, and therefore necessitates a short-run fall in investment. In particular, the investment-output ratio falls by more than 3 percentage units. Figures 1.5 and 1.6 plot the impulse responses of the variables when the capital-income tax is eliminated, under incomplete and complete markets respectively.

<sup>18</sup>The complete-markets calibration uses the relevant parameter values from the benchmark Table 1.1.

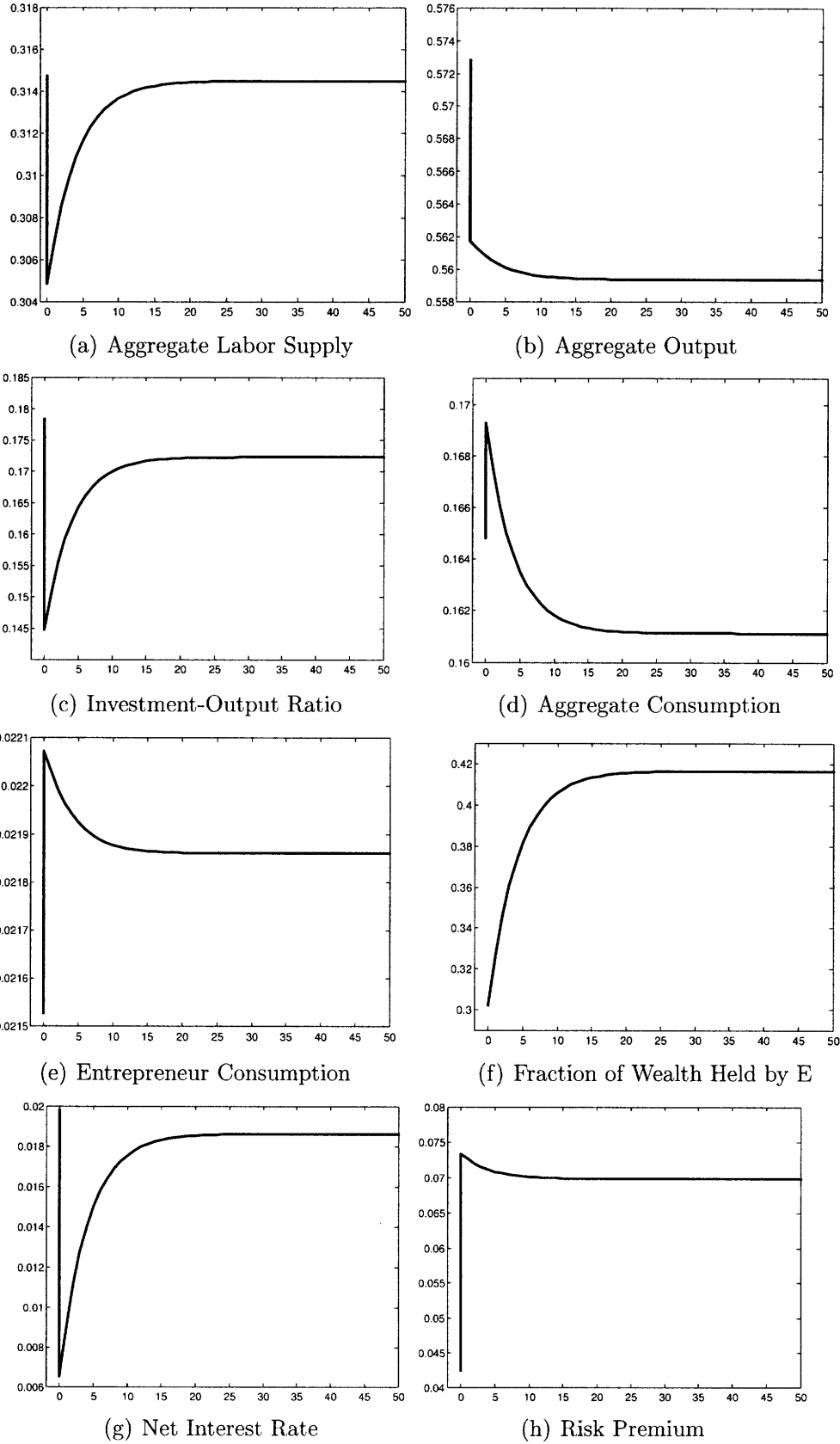
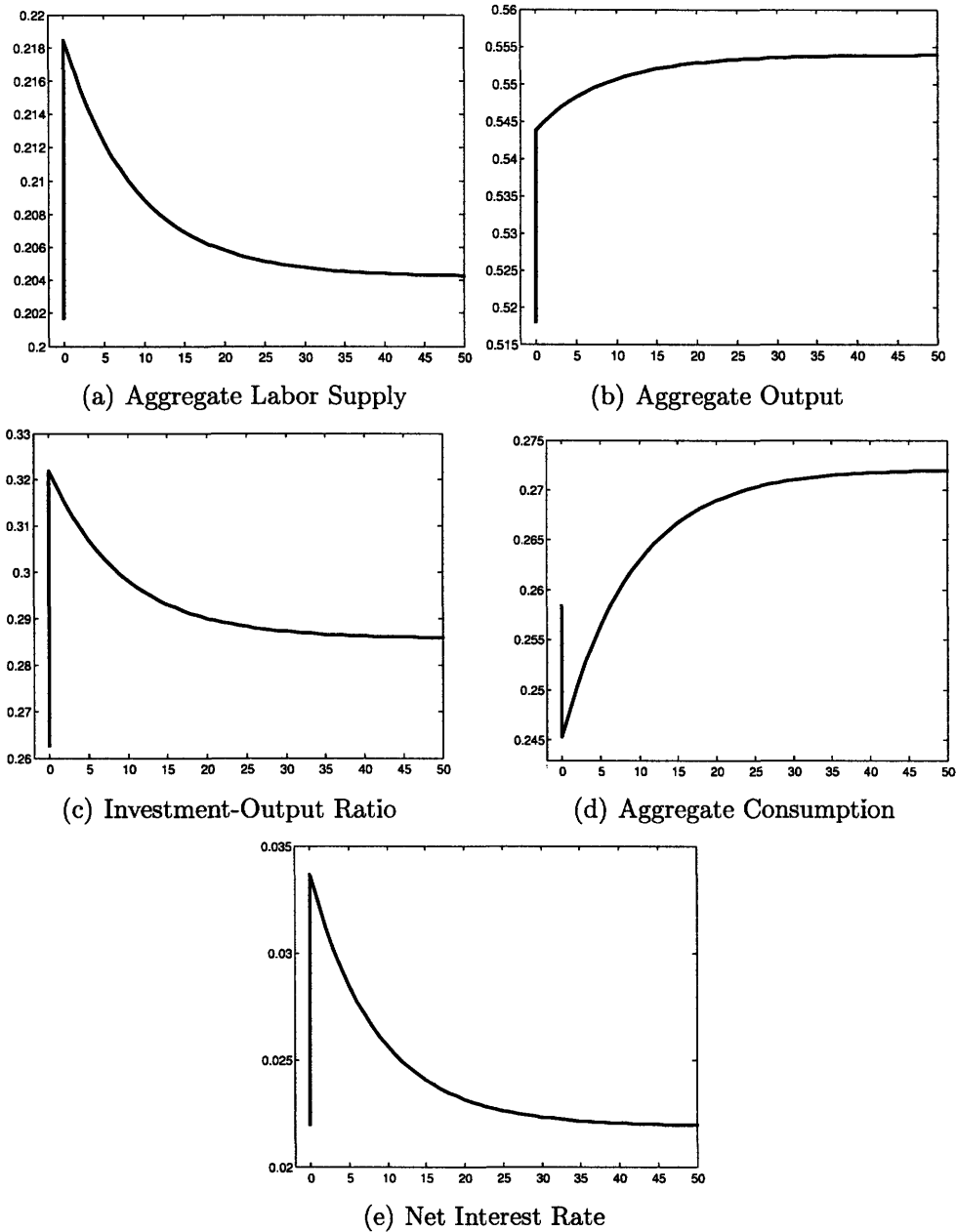


Figure 1-5: *Dynamics of Incomplete Markets: Eliminating the Capital-Income Tax*



**Figure 1-6:** *Dynamics of Complete Markets: Eliminating the Capital-Income Tax*

### Welfare effects

This section studies the welfare implications of eliminating the capital-income tax. These implications are represented in terms of a compensating differential for each level of wealth and each type of agent, whether entrepreneur or laborer. I.e., starting from the old regime with  $\tau^K = 0.25$ , the question is what fraction of his financial wealth would an agent be willing to give up in order to avoid the impact of the new regime initiated by the policy change. Depending on the perspective, the new regime is either the short run, when capital

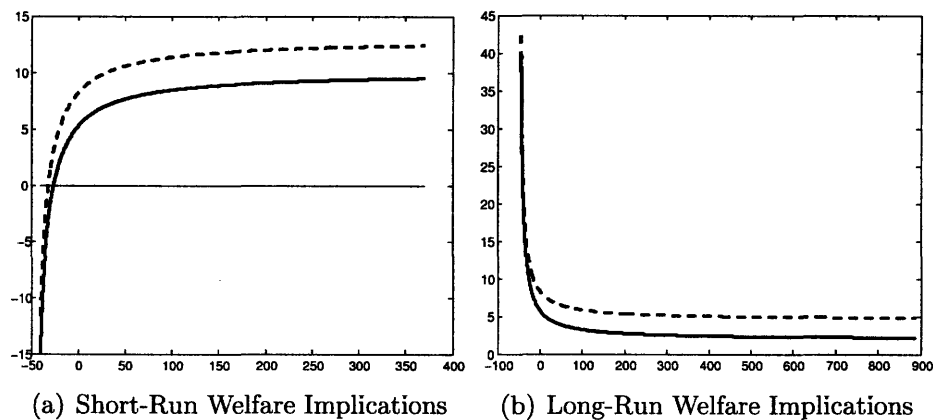


is not allowed to adjust and when the welfare of the current generation is examined, or the long run, when the comparison is between steady states, after capital has adjusted and new generations have entered the economy.

Figure 1.7(a) presents the welfare implications for entrepreneurs (solid line) and laborers (dashed line) in the short run, i.e. at the impact of the policy change. These short-run welfare effects have taken into account the entire transitional dynamics of the economy towards the new steady-state. Financial wealth normalized by annual mean income is on the horizontal axis, and the compensating differentials are on the vertical axis. A negative number on the vertical axis indicates that an agent would have to be paid to be indifferent between the old regime and the regime initiated by the impact of the policy change, and hence he prefers the new regime with the zero capital-income tax. Table 1.5 presents the short-run mean welfare effects over percentiles for entrepreneurs and laborers. The numbers are in percentages.

	Entrepreneurs	Laborers
bottom 1%	-1.4	-0.007
bottom 5%	-2.2	-0.001
bottom 10%	0.004	0.03
1st quintile	8.7	11.93
2nd quintile	20.86	20.62
3rd quintile	22.77	21.98
4th quintile	23.59	22.57
5th quintile	24.06	22.91
top 30%	35.92	34.24
top 10%	12.01	11.48
top 5%	6.04	5.75
top 1%	1.21	1.15

**Table 1.5:** *Short-Run Welfare Implications of Eliminating the Capital-Income Tax.*

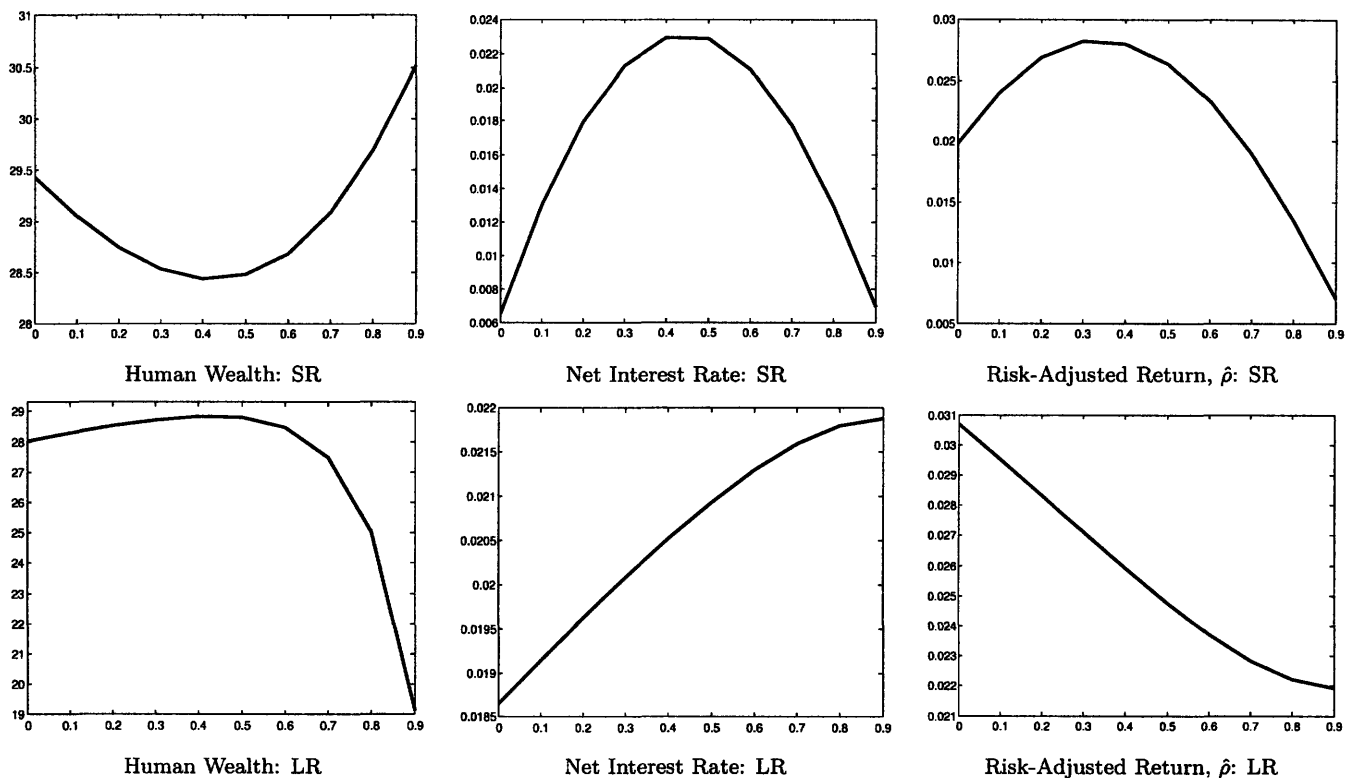


**Figure 1-7:** *Short-Run vs Long-Run Welfare Implications in Cross-Section*

Figure 1.7(a) and Table 1.5 show that in the short run, poor agents, whether entrepreneurs or laborers, prefer the zero capital-income tax regime. As wealth increases, both entrepreneurs and laborers prefer the positive capital-income tax regime. Finally, the mean cost of the tax cut is higher for the middle-class agents than for the very rich. These cross-sectional differences can be explained by referring to the first row of Figure 1.8, which plots the short-run response of human wealth and of the (risk-adjusted) returns to saving for laborers and entrepreneurs respectively, against the tax rate of the policy reform. In the short run, the decrease in the capital-income tax from  $\tau^K = 0.25$  to  $\tau^K = 0$  increases the demand for precautionary saving, and therefore leads to a fall in the interest rate<sup>19</sup>. Since the capital stock is historically given and cannot change, the fall in the interest rate increases human wealth. For poor agents, whether entrepreneurs or laborers, human wealth constitutes a significant part of their total wealth, and hence they benefit a lot from the elimination of the tax. Furthermore, for poor agents the fall in the saving returns does not carry as much weight as the increase in their human wealth. Finally, poor agents do not benefit much from insurance directly, since they invest little or nothing in the risky asset. Therefore, in the short run, poor agents prefer the zero capital-income tax regime, because the elimination of the tax increases their safe income, and safe income is a big part of their total wealth.

---

<sup>19</sup>In the short run, the net interest rate may actually fall if the tax of the reform is very high. This possibility, which does not emerge in the long run, is due to the usual distortionary effect of big tax increases on investment.



**Figure 1-8:** *Short Run (SR) vs Long Run (LR): Human Wealth and Saving Returns*

Figure 1.7(b) presents the welfare implications for entrepreneurs (solid line) and laborers (dashed line) in the long run, i.e. comparing across steady states. Clearly these long-run effects are different from the short-run effects. In the long run, both types of agents and at all wealth levels prefer the steady state with the positive tax, the rich less so than the poor, and the entrepreneurs less so than the laborers. These cross-sectional differences can be explained by referring to the second row of Figure 1.8, which plots the long-run response of human wealth and of the (risk-adjusted) returns to saving for laborers and entrepreneurs respectively, against the tax rate of the policy reform. In the long run, the decrease in the capital-income tax from  $\tau^K = 0.25$  to  $\tau^K = 0$  increases the demand for precautionary saving, and it therefore leads to a fall in the interest rate. This effect is as in the short run. However, in the long run, the general-equilibrium implications of the interest rate adjustment for capital accumulation become relevant. In particular, the fall in the interest rate reduces steady-state wealth and capital accumulation. The fall in the steady-state capital stock dominates the fall in the interest rate, so that steady-state human wealth falls when the capital-income tax is eliminated. This adversely affects poor agents of all types, since human wealth represents a big part of their total wealth. Because the risk-adjusted return for entrepreneurs,  $\hat{\rho}$  increases when the capital-income tax is eliminated, the cost of the policy change is not as high for an entrepreneur as it is for a laborer at any given level of wealth.

In conclusion, the elimination of the capital-income tax has welfare implications that

differ across time and in the cross-section of the population. These differences are due to the general-equilibrium effects of the interest rate adjustment on capital accumulation. In particular, they operate mainly through the different response of human wealth: in the short run, when the capital stock cannot adjust, human wealth increases after the elimination of the capital-income tax, whereas in the long run, when capital accumulation changes endogenously, human wealth falls. Therefore, poor agents prefer a zero capital-income tax in the short run, but a positive capital-income tax in the long run. Rich agents always prefer a positive tax, but less forcefully in the long-run, because in the long run the elimination of the tax increases the mean entrepreneurial portfolio return<sup>20</sup>.

## 1.8 Extension: Introducing Publicly Traded Sector

So far it has been assumed that all investment is subject to uninsurable idiosyncratic risk. This might not be an appropriate assumption for a country like the United States, where private equity actually accounts for about 50% of total financial wealth. To this end a second sector of production is formally introduced. In this sector, all firms are publicly traded, and it is assumed that they can perfectly diversify away all idiosyncratic risks<sup>21</sup>. The mean return to capital in the public sector is lower than in the private sector, otherwise no entrepreneur would invest in the private sector. Both entrepreneurs and laborers can invest in the public sector. Public sector capital is taxed at the rate  $\tau_t^K$ , and public sector labor is taxed at the rate  $\tau_t^L$ . In equilibrium, the marginal product of capital in the public sector is equal to the risk-free rate. The rest of the equilibrium characterization proceeds as in the benchmark model, with bond holdings now replaced by the sum of bond and public equity holdings.

In the public sector, where there is no scope for insurance, an increase in the capital-income tax unambiguously reduces investment, so that public capital is a negative function of the tax. As a result, the overall effect of the tax on the aggregate capital stock is in general ambiguous. In addition though, the increase in the capital tax might now trigger a reallocation of resources away from the low-risk low-productivity public sector towards the higher-risk higher-productivity private sector, thus increasing total factor productivity. As a result, aggregate output may increase with the tax, even if aggregate capital falls.

The model with the public sector is slightly recalibrated, so as to match the US economy aggregates. In steady state, private capital is maximized for  $\tau^K = 0.5$ , whereas public capital falls all the way with the tax. Overall, the aggregate capital stock is falling with the tax, but less so than under complete markets. As a result, when  $\tau^K = 0.4$ , output per work-hour is about 3% lower than it would have been had the tax rate been zero, whereas under complete markets it is about 9% lower. Furthermore, total factor productivity increases by

---

<sup>20</sup>Under complete markets, and starting from the steady state with  $\tau_K = 0.25$ , the average long-run welfare gain (in terms of consumption equivalent) of eliminating the capital-income tax is 1.7%, whereas the average short-run welfare gain is 0.6%.

<sup>21</sup>This is an extreme assumption made here for analytical convenience. In fact, the data indicates that public firms do not have a perfectly diversified shareholder base. Himmelberg et al. (2002), using the World-scope database for panel of public firms across 38 countries, find that the median inside equity ownership share is 40%.

7%. Steady-state welfare is maximized for  $\tau^K = 0.6$ , which is lower than in the benchmark model.

To summarize, in steady state and for  $\tau^K \simeq 40\%$ , output per work-hour is 12% higher than under complete markets when all production takes place in the private sector, and it is 6% higher than under complete markets when the private sector accounts for 50% of financial assets.

## 1.9 Conclusions

This chapter studies the aggregate and welfare effects of capital-income taxation in an environment where agents face uninsurable idiosyncratic entrepreneurial risk. The surprising result emerging is that a positive tax on capital income may actually stimulate steady-state capital accumulation: for empirically plausible calibrated versions of the model, the steady-state levels of the capital stock, output and employment are all maximized for a positive value of the capital-income tax. For the preferred benchmark calibration, when the tax on capital is 25%, the capital stock is 2.5% higher than what it would have been had the tax rate been zero. This result stands in stark contrast to the effect of capital-income taxation in either complete-markets models, or in Bewley-type incomplete-markets models: in these models, capital-income taxation necessarily discourages capital accumulation. The result of the present chapter is due to the endogenous general-equilibrium adjustment of the interest rate and of wealth in the long run.

Although the present chapter provides some useful guidance about the direction of optimal policy, it does not solve for the fully optimal policy. An interesting direction for future research is the formal study of optimal policy, either in the Ramsey tradition (though allowing for lump-sum taxes, as in the present model), or in the Mirrlees tradition of endogenizing the source of market incompleteness and having no ad hoc restrictions placed on the set of available instruments.

The present model focuses on the effects of uninsurable entrepreneurial risk, and abstracts from labor-income risk, borrowing constraints, and decreasing returns to scale at the individual level. Extending the model to include these relevant aspects of the data and revisiting the effects of capital taxation in this richer setting is important, not only to get a better quantitative evaluation of the implications of capital taxation, but also to examine whether the general-equilibrium effects identified here might interact with other sources of market incompleteness in an interesting way. For example, after an increase in the capital-income tax, the increase in steady-state wealth documented here could make borrowing constraints less binding. At the same time, the increase in the steady-state interest rate could also increase the cost of borrowing. Further investigating these rich general-equilibrium interactions will greatly facilitate a better theoretical and quantitative assessment of the implications of fiscal policy in dynamic heterogeneous-agent environments.



# Chapter 2

## Revisiting the Supply-Side Effects of Government Spending Under Incomplete Markets

This chapter represents work with George-Marios Angeletos<sup>1</sup>.

### Abstract

This chapter revisits the macroeconomic effects of government consumption in the neoclassical growth model augmented with idiosyncratic investment (or entrepreneurial) risk. Under complete markets, a permanent increase in government consumption has no long-run effect on the interest rate, the capital-labor ratio, and labor productivity, while it increases work hours due to the familiar negative wealth effect. These results are upset once we allow for incomplete markets. The very same negative wealth effect now causes a reduction in risk taking and investment. This in turn leads to a lower risk-free rate and, under certain conditions, also to a lower capital-labor ratio, lower productivity and lower wages.

### 2.1 Introduction

Studying the impact of government spending on macroeconomic outcomes is one of the most celebrated policy exercises within the neoclassical growth model; it is important for understanding the business-cycle implications of fiscal policy, the macroeconomic effects of wars, and the cross-section of countries. Some classics include Hall (1980), Barro (1981,

---

<sup>1</sup>The authors are grateful to Olivier Blanchard, Chris Carroll, Edouard Challe, Ricardo Caballero, Mike Golosov, Iván Werning and seminar participants at MIT and the 2007 conference on macroeconomic heterogeneity at the Paris School of Economics for useful comments

1989), Aiyagari, Christiano and Eichenbaum (1992), Baxter and King (1993), and McGrattan and Ohanian (1999, 2006).

These studies have all maintained the convenient assumption of complete markets, abstracting from the possibility that agents' saving and investment decisions—and hence their reaction to changes in fiscal policy—may crucially depend on the extent of risk sharing within the economy. This chapter contributes towards filling this gap. It revisits the macroeconomic effects of government consumption within an incomplete-markets variant of the neoclassical growth model.

Apart from introducing undiversified idiosyncratic risk in production and investment, all other ingredients of our model are the same as in the canonical neoclassical growth model: firms operate neoclassical constant-returns-to-scale technologies, households have standard CRRA/CEIS preferences, and markets are competitive. The focus on idiosyncratic production/investment risk is motivated by two considerations. First, this friction is empirically relevant. This is obvious for less developed economies. But even in the United States, privately-owned firms account for about one half of aggregate production and employment. Furthermore, the typical investor—the median rich household—holds a very undiversified portfolio, more than one half of which is allocated to private equity.<sup>2</sup> And second, as we explain next, this friction upsets some key predictions of the standard neoclassical paradigm.

In the standard neoclassical paradigm, the steady-state values of the capital-labor ratio, productivity (output per work hour), the wage rate, and the interest rate, are all pinned down by the equality of the marginal product of capital with the discount rate in preferences. As a result, any change in the level of government consumption, even if it is permanent, has no effect on the long-run values of these variables.<sup>3</sup> On the other hand, because higher spending for the government means lower wealth for the households, a permanent increase in government consumption raises labor supply. It follows that employment (work hours) and output increase both in the short run and in the long run, so as to keep the long-run levels of capital intensity and productivity unchanged.

The picture is quite different once we allow for incomplete markets. The same wealth effect that, in response to an increase in government consumption, stimulates labor supply in the standard paradigm, now also discourages investment. This is simply because risk taking, and hence investment, is sensitive to wealth. We thus find very different long-run effects. First, a permanent increase in government consumption necessarily reduces the risk-free interest rate. And second, unless the elasticity of intertemporal substitution is low enough, it also reduces the capital-labor ratio, productivity, and wages.

The effect on the risk-free rate is an implication of the precautionary motive: a higher level of consumption for the government implies a lower aggregate level of wealth for the households, which is possible in steady state only with a lower interest rate. A lower interest

---

<sup>2</sup>See Quadrini (2000), Gentry and Hubbard (2000), Carroll (2000), and Moskowitz and Vissing-Jørgensen (2002). Also note that idiosyncratic investment risks need not be limited to private entrepreneurs; they may also affect educational and occupational choices, or the production decisions that CEO's make on behalf of public corporations.

<sup>3</sup>This, of course, presumes that the change in government consumption is financed with lump-sum taxes. The efficiency or redistributive considerations behind optimal taxation is beyond the scope of this chapter.



rate, however, does not necessarily imply a higher capital-labor ratio. This is because market incompleteness introduces a wedge between the risk-free rate and the marginal product of capital. Furthermore, because of diminishing absolute risk aversion, the lower the level of wealth, the higher will be the risk premium on investment, and hence this wedge. It follows that, in response to an increase in government consumption, the capital-labor ratio can fall even if the interest rate also falls. Indeed, a sufficient condition for this to be the case is that the elasticity of intertemporal substitution is sufficiently high relative to the income share of capital—a condition easily satisfied for plausible calibrations of the model.

Turning to employment and output, there are two opposing effects. On the one hand, as with complete markets, the negative wealth effect on labor supply contributes towards higher employment and output. On the other hand, unlike complete markets, the reduction in capital intensity, productivity, and wages contributes towards lower employment and output. Depending on the income and wage elasticities of labor supply, either of the two effects can dominate.

The deviation from the standard paradigm is significant, not only qualitatively, but also quantitatively. For our preferred parametrizations of the model, the following hold. First, the elasticity of intertemporal substitution is comfortably above the critical value that suffices for an increase in government consumption to *reduce* the long-run levels of the capital-labor ratio, productivity, and wages. Second, the negative effects on these variables are quantitatively significant: a 1% increase in government spending under incomplete markets has the same impact on capital intensity and labor productivity as a 0.5% – 0.6% increase in capital-income taxation under complete markets. Third, these effects mitigate, but do not fully offset, the wealth effect on labor supply. Finally, the welfare consequences are non-trivial: the welfare cost of a permanent 1% increase in government consumption is three times larger under incomplete markets than under complete markets.

The main contribution of this chapter is thus to highlight how wealth effects on investment due to financial frictions can significantly modify the supply-side channel of fiscal policy. In our model, these wealth effects emerge from idiosyncratic risk along with diminishing absolute risk aversion; in other models, they could emerge from borrowing constraints. Also, such wealth effects are relevant for both neoclassical and Keynesian models. In this chapter we follow the neoclassical tradition, not because of any belief on which paradigm best fits the data, but rather because this clarifies the value of our contribution: whereas wealth effects are central to the neoclassical approach with regard to labor supply, they have been mute with regard to investment.

To the best of our knowledge, this chapter is the first to study the macroeconomic effects of government consumption in an incomplete-markets version of the neoclassical growth paradigm that allows for uninsurable investment risk. A related, but different, exercise is conducted in Heathcote (2005) and Challe and Ragot (2007). These papers study deviations from Ricardian equivalence in Bewley-type models like Aiyagari's (1994), where borrowing constraints limit the ability of agents to smooth consumption intertemporally. In our chapter, instead, deviations from Ricardian equivalence are not an issue: our model allows households to freely trade a riskless bond, thus ensuring that the timing of taxes and the level of debt

has no effect on allocations, and instead focuses on wealth effects on investment due to incomplete risk sharing.

The particular framework we employ in this chapter is a continuous-time variant of the one introduced in Angeletos (2007). That paper studied how idiosyncratic capital-income risk affects aggregate saving, and contrasted this with the impact of labor-income risk in Bewley type models (Aiyagari, 1994; Huggett, 1997; Krusell and Smith, 1998). Other papers that introduce idiosyncratic investment or entrepreneurial risk in the neoclassical growth model include Angeletos and Calvet (2006), Buera and Shin (2007), Caggeti and De Nardi (2006), Covas (2006), and Meh and Quadrini (2006).<sup>4</sup> The novelty of our chapter is to study the implications for fiscal policy in such an environment.

The rest of this chapter is organized as follows. Section 2 introduces the basic model, Section 3 characterizes its equilibrium, and Section 4 analyzes its steady state. The basic model fixes labor supply so as to focus on the most novel results of the chapter, namely the steady-state effects of government consumption on the interest rate and the capital-labor ratio. Section 5 then turns to three extensions that endogenize labor supply. Section 6 examines the dynamic response of the economy to a permanent change in government consumption. Section 7 concludes.

## 2.2 The basic model

Time is continuous, indexed by  $t \in [0, \infty)$ . There is a continuum of infinitely-lived households, indexed by  $i$  and distributed uniformly over  $[0, 1]$ . Each household is endowed with one unit of labor, which it supplies inelastically in a competitive labor market. Each household also owns and runs a firm, which employs labor in the competitive labor market but can only use the capital stock invested by the particular household.<sup>5</sup> Households cannot invest in other households' firms and cannot otherwise diversify away from the shocks hitting their firms, but can freely trade a riskless bond. Finally, all uncertainty is purely idiosyncratic, and hence all aggregates are deterministic.

### 2.2.1 Households and firms

The financial wealth of household  $i$ , denoted by  $x_t^i$ , is the sum of its holdings in private capital,  $k_t^i$ , and the riskless bond,  $b_t^i$ :

$$x_t^i = k_t^i + b_t^i. \tag{2.1}$$

---

<sup>4</sup>Related is also Obstfeld (1994), which assumes a continuous-time Epstein-Zin specification as this chapter, but with an *AK* technology.

<sup>5</sup>We can think of a household as a couple, with the wife running the family business and the husband working in the competitive labor market (or vice versa). The key assumption, of course, is only that the value of the labor endowment of each household is pinned down by the competitive wage and is not subject to idiosyncratic risk.

The evolution of  $x_t^i$  is given by the household budget:

$$dx_t^i = d\pi_t^i + [R_t b_t^i + \omega_t - T_t - c_t^i] dt, \quad (2.2)$$

where  $d\pi_t^i$  is the household's capital income (i.e., the profits it enjoys from the private firm it owns),  $R_t$  is the interest rate on the riskless bond,  $\omega_t$  is the wage rate,  $T_t$  is the lump-sum tax, and  $c_t^i$  is the household's consumption. Finally, the familiar no-Ponzi game condition is also imposed.

Whereas the sequences of prices and taxes are deterministic (due to the absence of aggregate risk), firm profits, and hence household capital income, are subject to undiversified idiosyncratic risk. In particular,

$$d\pi_t^i = [F(k_t^i, n_t^i) - \omega_t n_t^i - \delta k_t^i] dt + \sigma k_t^i dz_t^i. \quad (2.3)$$

Here,  $n_t^i$  is the amount of labor the firm hires in the competitive labor market,  $F$  is a constant-returns-to-scale neoclassical production function, and  $\delta$  is the mean depreciation rate. Idiosyncratic risk is introduced through  $dz_t^i$ , a standard Wiener process that is i.i.d. across agents and across time. This can be interpreted either as a stochastic depreciation shock or as a stochastic productivity shock, the key element being that it generates risk in the return to capital. The scalar  $\sigma$  measures the amount of undiversified idiosyncratic risk and can be viewed as an index of market incompleteness, with higher  $\sigma$  corresponding to a lower degree of risk sharing (and  $\sigma = 0$  corresponding to complete markets). Finally, without serious loss of generality, we assume a Cobb-Douglas specification for the technology:  $F(k, n) = k^\alpha n^{1-\alpha}$  with  $\alpha \in (0, 1)$ .<sup>6</sup>

Turning to preferences, we assume an Epstein-Zin specification with constant elasticity of intertemporal substitution (CEIS) and constant relative risk aversion (CRRA). Given a consumption process, the utility process is defined by the solution to the following integral equation:

$$U_t = E_t \int_t^\infty z(c_s, U_s) ds \quad (2.4)$$

where

$$z(c, U) \equiv \frac{\beta}{1 - 1/\theta} \left[ \frac{c^{1-1/\theta}}{((1-\gamma)U)^{\frac{-1/\theta+\gamma}{1-\gamma}}} - (1-\gamma)U \right]. \quad (2.5)$$

Here,  $\beta > 0$  is the discount rate,  $\gamma > 0$  is the coefficient of relative risk aversion, and  $\theta > 0$  is the elasticity of intertemporal substitution.<sup>7</sup>

Standard expected utility is nested with  $\gamma = 1/\theta$ . We find it useful to allow  $\theta \neq 1/\gamma$  in order to clarify that the *qualitative* properties of the steady state depend crucially on the

---

<sup>6</sup>The characterization of equilibrium and the proof of the existence of the steady state extend to any neoclassical production function; it is only the proof of the uniqueness of the steady state that uses the Cobb-Douglas specification.

<sup>7</sup>To make sure that (A.1) indeed defines a preference ordering over consumption lotteries, one must establish existence and uniqueness of the solution to the integral equation (A.1); see Duffie and Epstein (1992).

elasticity of intertemporal substitution rather than the coefficient of relative risk aversion (which in turn also guides our preferred parameterizations of the model). However, none of our results rely on allowing  $\theta \neq 1/\gamma$ . A reader who feels uncomfortable with the Epstein-Zin specification can therefore ignore it, assume instead standard expected utility, and simply replace  $\gamma$  with  $1/\theta$  (or vice versa) in all the formulas that follow.

## 2.2.2 Government

At each point in time the government consumes output at the rate  $G_t$ . Government spending is deterministic, it is financed with lump-sum taxation, and it does not affect either utility from private consumption or production. The government budget constraint is given by

$$dB_t^g = [R_t B_t^g + T_t - G_t]dt, \quad (2.6)$$

where  $B_t^g$  denotes the level of government assets (i.e., minus the level of government debt). Finally, a no-Ponzi game condition is imposed to rule out explosive debt accumulation.

## 2.2.3 Equilibrium definition

The initial position of the economy is given by the distribution of  $(k_0^i, b_0^i)$  across households. Households choose plans  $\{c_t^i, n_t^i, k_t^i, b_t^i\}_{t \in [0, \infty)}$ , contingent on the history of their idiosyncratic shocks, and given the price sequence and the government policy, so as to maximize their lifetime utility. Idiosyncratic risk, however, washes out in the aggregate. We thus define an equilibrium as a deterministic sequence of prices  $\{\omega_t, R_t\}_{t \in [0, \infty)}$ , a deterministic sequence of policies  $\{G_t, T_t\}_{t \in [0, \infty)}$ , a deterministic macroeconomic path  $\{C_t, K_t, Y_t\}_{t \in [0, \infty)}$ , and a collection of individual contingent plans  $(\{c_t^i, n_t^i, k_t^i, b_t^i\}_{t \in [0, \infty)})_{i \in [0, 1]}$ , such that the following conditions hold: (i) given the sequences of prices and policies, the plans are optimal for the households; (ii) the labor market clears,  $\int_t n_t^i = 1$ , in all  $t$ ; (iii) the bond market clears,  $\int_t b_t^i + B_t^g = 0$ , in all  $t$ ; (iv) the government budget is satisfied in all  $t$ ; and (v) the aggregates are consistent with individual behavior,  $C_t = \int_i c_t^i$ ,  $K_t = \int_i k_t^i$ , and  $Y_t = \int_i F(k_t^i, n_t^i)$ , in all  $t$ .<sup>8</sup>

## 2.3 Equilibrium

In this section we characterize the equilibrium of the economy. We first solve for a household's optimal plan for given sequences of prices and policies. We then aggregate across households and derive the general-equilibrium dynamics.

### 2.3.1 Individual behavior

Since employment is chosen after the capital stock has been installed and the idiosyncratic shock has been observed, optimal employment maximizes profits state by state. By constant

---

<sup>8</sup>Throughout,  $\int_i$  denotes expectation in the cross-section of the population.

returns to scale, optimal firm employment and profits are linear in own capital:

$$n_t^i = \bar{n}(\omega_t)k_t^i \quad \text{and} \quad d\pi_t^i = \bar{r}(\omega_t)k_t^i dt + \sigma dz_t^i, \quad (2.7)$$

where

$$\bar{n}(\omega_t) \equiv \arg \max_n [F(1, n) - \omega_t n] \quad \text{and} \quad \bar{r}(\omega_t) \equiv \max_n [F(1, n) - \omega_t n] - \delta.$$

Here,  $\bar{r}_t \equiv \bar{r}(\omega_t)$  is the household's expectation of the return to its capital prior to the realization of the idiosyncratic shock  $z_t^i$ , as well as the mean of the realized returns in the cross-section of firms. Analogous interpretation applies to  $\bar{n}_t \equiv \bar{n}(\omega_t)$ .

The key result here is that households face risky, but linear, returns to their capital. To see how this translates to linearity of wealth in assets, let  $h_t$  denote the present discounted value of future labor income net of taxes, a.k.a. *human wealth*:

$$h_t = \int_t^\infty e^{-\int_t^s R_j dj} (\omega_s - T_s) ds. \quad (2.8)$$

Next, define *effective wealth* as the sum of financial and human wealth:

$$w_t^i \equiv x_t^i + h_t = k_t^i + b_t^i + h_t. \quad (2.9)$$

It follows that the evolution of effective wealth can be described by

$$dw_t^i = [\bar{r}_t k_t^i + R_t(b_t^i + h_t) - c_t^i] dt + \sigma k_t^i dz_t^i. \quad (2.10)$$

The first term on the right-hand side of (2.10) measures the expected rate of growth in the household's effective wealth; the second term captures the impact of idiosyncratic risk.

The linearity of budgets together with the homotheticity of preferences ensures that, for given prices and policies, the household's consumption-saving problem reduces to a tractable homothetic problem as in Samuelson's and Merton's classic portfolio analysis. It follows that the optimal policy rules are linear in wealth, as shown in the next proposition.

**Proposition 7.** *Let  $\{\omega_t, R_t\}_{t \in [0, \infty)}$  and  $\{G_t, T_t\}_{t \in [0, \infty)}$  be equilibrium price and policy sequences. Then, equilibrium consumption, investment and bond holdings for household  $i$  are given by*

$$c_t^i = m_t w_t^i, \quad k_t^i = \phi_t w_t^i, \quad \text{and} \quad b_t^i = (1 - \phi_t) w_t^i - h_t, \quad (2.11)$$

where  $\phi_t$ , the fraction of effective wealth invested in capital, is given by

$$\phi_t = \frac{\bar{r}_t - R_t}{\gamma \sigma^2}, \quad (2.12)$$

while  $m_t$ , the marginal propensity to consume out of effective wealth, satisfies the recursion

$$\frac{\dot{m}_t}{m_t} = m_t + (\theta - 1)\hat{\rho}_t - \theta\beta, \quad (2.13)$$

with  $\hat{\rho}_t \equiv \rho_t - \frac{1}{2}\gamma\phi_t^2\sigma^2$  denoting the risk-adjusted return to saving and  $\rho_t \equiv \phi_t\bar{r}_t + (1 - \phi_t)R_t$  the mean return to saving.

Condition (2.12) simply says that the fraction of wealth invested in the risky asset is increasing in the risk premium  $\mu_t \equiv \bar{r}_t - R_t$ , and decreasing in risk aversion  $\gamma$  and the amount of risk  $\sigma$ .<sup>9</sup> Condition (2.13) is essentially the Euler condition: it describes the growth rate of the marginal propensity to consume as a function of the anticipated path of risk-adjusted returns to saving. Whether higher risk-adjusted returns increase or reduce the marginal propensity to consume depends on the elasticity of intertemporal substitution. To see this more clearly, note that in steady state this condition reduces to  $m = \theta\beta - (\theta - 1)\hat{\rho}$ , so that higher  $\hat{\rho}$  decreases  $m$  (i.e., increases saving out of effective wealth) if and only if  $\theta > 1$ . This is due to the familiar income and substitution effects.

### 2.3.2 General equilibrium

Because individual consumption, saving and investment are linear in individual wealth, aggregates at any point in time do not depend on the extent of wealth inequality at that time. As a result, the aggregate equilibrium dynamics can be described with a low-dimensional recursive system.

Define  $f(K) \equiv F(K, 1)$  as the production in intensive form and let  $\omega(K) \equiv f(K) - f'(K)K$ ,  $\phi(K, R) \equiv \frac{1}{\gamma\sigma^2}(f'(K) - \delta - R)$ , and  $\rho(K, R) \equiv R + \frac{1}{\gamma\sigma^2}(f'(K) - \delta - R)^2$ . Aggregating across the policy rules of the agents and imposing market clearing, we arrive at the following proposition.

**Proposition 8.** *In equilibrium, the aggregate dynamics satisfy*

$$\dot{K}_t = f(K_t) - \delta K_t - C_t - G_t \quad (2.14)$$

$$\frac{\dot{C}_t}{C_t} = \theta(\rho_t - \beta) - (\theta - 1)\frac{1}{2}\gamma\sigma^2\phi_t^2 \quad (2.15)$$

$$\dot{H}_t = R_t H_t - \omega_t + G_t \quad (2.16)$$

$$K_t = \frac{\phi_t}{1 - \phi_t} H_t \quad (2.17)$$

with  $\omega_t = \omega(K_t)$ ,  $\phi_t = \phi(K_t, R_t)$ , and  $\rho_t = \rho(K_t, R_t)$ .

Condition (2.14) is the resource constraint of the economy. The resource constraint does not depend on the degree of market incompleteness. It follows from aggregating budgets across all households and the government, imposing labor- and bond-market clearing, and using the linearity of individual firm employment to individual capital together with constant returns to scale, to get  $Y_t = \int_i F(k_t^i, n_t^i) = F(\int_i k_t^i, \int_i n_t^i) = F(K_t, 1)$ .

Condition (2.15) is the aggregate Euler condition for the economy. It follows from aggregating consumption and wealth across agents together with the optimality condition (2.13)

<sup>9</sup>Clearly, in any equilibrium  $\mu_t$  must be positive, otherwise nobody would invest in capital and an equilibrium would fail to exist.

for the marginal propensity to consume. It also has a simple interpretation. As in the case of complete markets, aggregate consumption growth unambiguously increases with  $\rho_t$ , the mean return to saving. But unlike complete markets, aggregate consumption growth now also depends on  $\frac{1}{2}\gamma\sigma^2\phi_t^2$ , a risk-adjustment term. Whether more risk contributes to a lower or higher marginal propensity to save, and hence whether this new term contributes to lower or higher consumption growth, depends on whether the elasticity of intertemporal substitution,  $\theta$ , is higher or lower than 1. The intuition for this property is the same as the intuition for the impact of the interest rate in a deterministic saving problem, namely the opposing income and substitution effects of a higher rate of return to saving.

Condition (2.16) expresses the evolution of the present value of aggregate net-of-taxes labor income in recursive form. It follows from the definition of human wealth combined with the intertemporal government budget, which imposes that the present value of taxes equals the present value of government consumption.

Finally, condition (2.17) follows from bond-market clearing. More precisely, aggregating bond holdings across agents and imposing bond-market clearing gives  $(1 - \phi_t)W_t - H_t = 0$ , while aggregating investment gives  $K_t = \phi_t W_t$ , and combining the two gives condition (2.17).

These conditions characterize the equilibrium dynamics of the economy with either incomplete or complete markets. In both cases, condition (2.17) ensures that  $\phi_t \in (0, 1)$ . But when markets are complete ( $\sigma = 0$ ), this is possible only if  $f'(K_t) - \delta = R_t$  (meaning arbitrage between bonds and capital). Condition (2.15) then reduces to the more familiar Euler condition  $\dot{C}_t/C_t = \theta [f'(K_t) - \delta - \beta]$ , and one can track the dynamics of the economy merely on the  $(C, K)$  space, using the Euler condition and the resource constraint. When, instead, markets are incomplete,  $\phi_t \in (0, 1)$  is possible only if  $f'(K_t) - \delta > R_t$ , which proves that the marginal product of capital must exceed the risk-free rate. Moreover, the dimensionality of the system now increases by one: along with  $(C, K)$ , we also have to keep track of  $H$ , using condition (2.16).

Still, this is a highly tractable dynamic system, as compared to other incomplete-markets models, where the entire wealth distribution—an infinite dimensional object—is a relevant state variable for aggregate equilibrium dynamics. Indeed, the equilibrium dynamics can be approximated with a simple shooting algorithm: for any historically given  $K_0$ , guess some initial values  $(C_0, H_0)$  and use conditions (2.14)-(2.16) to compute the entire path of  $(C_t, K_t, H_t)$  for  $t \in [0, T]$ , for some large  $T$ ; then iterate on the initial guess till  $(C_T, K_T, H_T)$  is close enough to its steady-state value.<sup>10</sup> In the special case that  $\theta = 1$  (unit EIS),  $m_t = \beta$  and hence  $C_t = \beta(K_t + H_t)$  for all  $t$ . One can then drop the Euler condition from the dynamic system and analyze the equilibrium dynamics with a simple phase diagram in the  $(K, H)$  space.

---

<sup>10</sup>This presumes that a turnpike theorem applies; this is likely to be the case at least for  $\sigma$  small enough, by continuity to the complete-markets case.

## 2.4 Steady State

In this section we study the steady state of the economy (i.e., the fixed point of the dynamic system in Proposition 8) and its comparative statics with respect to the level of government spending.

### 2.4.1 Characterization

Clearly, an equilibrium would fail to exist if the present value of government spending exceeded that of labor income. We thus henceforth parameterize government spending  $G_t$  as a fraction  $g$  of aggregate output  $Y_t$  and impose  $0 \leq g < 1 - \alpha$ .

**Proposition 9.** (i) *The steady state exists and is unique.* (ii) *In steady state, the capital stock  $K$  and the interest rate  $R$  are the unique solution to*

$$f'(K) - \delta = R + \sqrt{\frac{2\theta\gamma\sigma^2(\beta - R)}{\theta + 1}} \quad (2.18)$$

$$K = \frac{\phi(K, R)}{1 - \phi(K, R)} \frac{(1 - \alpha - g)f(K)}{R} \quad (2.19)$$

Output is then given by  $Y = f(K)$ , the wage rate by  $\omega = (1 - \alpha)f(K)$ , and consumption by  $C = (1 - g)f(K) - \delta K$ .

Condition (2.18) follows from the Euler condition (2.15), setting  $\dot{C} = 0$ . Condition (2.19) follows from the bond market clearing condition (2.17), substituting for the steady-state value of  $H$  implied by (2.16), namely  $H = (\omega - G)/R = (1 - \alpha - g)f(K)/R$ .

To better understand the determination of the steady state of our economy, consider for a moment another economy that has the same preferences, technologies and risks but is open to an international market for the riskless bond, thus facing an exogenously fixed interest rate. If  $R \geq 1/\beta$ , then the precautionary saving motive implies that aggregate wealth increases without bound. If, instead,  $R \in (0, 1/\beta)$ , then diminishing absolute aversion ensures the existence of a finite level of aggregate wealth at which the precautionary motive is just offset by the gap between the interest rate and the discount rate.<sup>11</sup> Therefore,  $R \in (0, 1/\beta)$  is both necessary and sufficient for the open economy to admit a steady state. For any such  $R$ , aggregate capital is given by (2.18). There is, however, a unique  $R$  for which the net foreign asset position of the economy is zero, which is precisely what condition (2.19) imposes.

### 2.4.2 A graphical representation

Let  $K_1(R)$  and  $K_2(R; g)$  denote the solutions to, respectively, conditions (2.18) and (2.19) with respect to  $K$ . For any given  $g$ , the intersection of the graphs of these two functions identifies the steady state. To understand how these graphs look like, the next lemma examines the monotonicities of these two functions with respect to  $R$ .

<sup>11</sup>These intuitions are similar to those in Aiyagari (1994).



**Lemma 1.** (i)  $\partial K_1/\partial R > 0$  if and only if  $\theta > \frac{\phi}{1-\phi}$ . (ii)  $\partial K_2/\partial R < 0$  always.

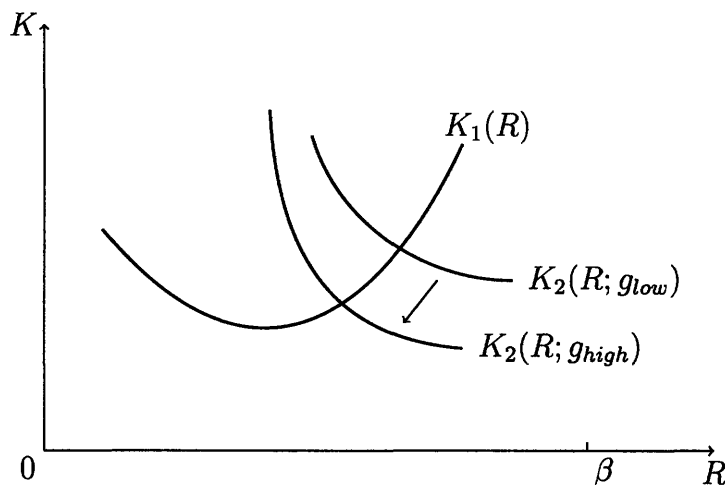
The intuition behind part (ii) is simple. For given  $K$ , and hence given  $\omega$ , an increase in  $R$  reduces both  $H$  and  $\phi(K, R)$ , and thereby necessarily reduces the right hand side of (2.19). But then for (2.19) to hold with the lower  $R$  it must be that  $K$  also falls. It follows that  $K_2(R)$  is a monotonically decreasing function, as illustrated in Figure 1.

The intuition behind part (i) is a bit more convoluted. Recall that condition (2.18) comes from stationarity of aggregate consumption. Clearly, this is equivalent to imposing stationarity of aggregate wealth. Since  $\dot{W}_t = \rho_t W_t - C_t = (\rho_t - m_t) W_t$ , this in turn is the same as imposing  $\rho = m$ . From condition (2.13), on the other hand, we have that the steady-state value of the marginal propensity to consume is given by  $m = \theta\beta - (\theta - 1)\hat{\rho}$ . It follows that aggregate wealth is stationary if and only if

$$\rho + (\theta - 1)\hat{\rho} = \theta\beta, \quad (2.20)$$

where  $\rho$  is the mean return to saving and  $\hat{\rho}$  the risk adjusted return (both evaluated at the steady state  $K$  and for given  $R$ ). Of course, this condition is equivalent to (2.18), but it is more useful for developing intuition.

**Figure 2-1:** *The steady state and the effects of higher government spending.*



First, note that an increase in  $K$  necessarily reduces  $\rho + (\theta - 1)\hat{\rho}$ . To see this, note that an increase in  $K$  reduces  $f'(K)$ . For given  $\phi$ , this reduces  $\rho$  and  $\hat{\rho}$  equally, thus also reducing  $\rho + (\theta - 1)\hat{\rho}$ . Of course, the optimal  $\phi$  must fall, but this only reinforces the negative effect on  $\rho$  (since the portfolio is shifted towards the low-return bond), while it does not affect  $\hat{\rho}$  (because of the envelope theorem and the fact that  $\phi$  maximizes  $\hat{\rho}$ ).

Next, note that an increase in  $R$  has an ambiguous effect on  $\rho + (\theta - 1)\hat{\rho}$ . For given  $\phi$ , both  $\rho$  and  $\hat{\rho}$  increase with  $R$ . But now the fact that  $\phi$  falls works in the opposite direction,

contributing to lower  $\rho$ . Intuitively, though, this effect should be small if  $\phi$  was small to start with. Moreover, the impact of  $\hat{\rho}$  is likely to dominate if  $\theta$  is high. We thus expect  $\rho + (\theta - 1)\hat{\rho}$  to increase with  $R$  if and only if either  $\phi$  is low or  $\theta$  is high. Indeed, we prove that this is the case if and only if  $\theta > \frac{\phi}{1-\phi}$ .

Combining the above observations, we conclude that  $\partial K_1/\partial R > 0$  if and only if  $\theta > \frac{\phi}{1-\phi}$ , which completes the argument behind part (i) of Lemma 1.

In the Appendix, we further show that the steady-state  $\phi$  is a decreasing function of  $R$ . Hence, the condition  $\theta > \frac{\phi}{1-\phi}$  is satisfied if and only if  $R$  is high enough. It follows that  $K_1(R)$  is a U-shaped curve, as illustrated in Figure 1. Intuitively, when  $R$  is close to  $\beta$ , a marginal increase in  $R$  has such a strong positive effect on steady state wealth, that the consequent reduction in the risk premium more than offsets the increase in the opportunity cost of investment, ensuring that  $K$  increases with  $R$ .

As noted earlier, the intersection of the two curves identifies the steady state of the closed economy. The existence and uniqueness of such an intersection is established in the Appendix (see the proof of Proposition 9). What we next seek to understand is how this intersection changes with an increase in government spending.

### 2.4.3 The long-run effects of government consumption

Because  $g$  does not enter condition (2.18), an increase in government consumption does not affect the  $K_1$  curve. On the other hand, because higher  $g$  means lower net-of-taxes labor income, and hence lower  $H$ , an increase in government consumption causes the  $K_2$  curve to shift downwards, as illustrated in Figure 1. This is simply a manifestation of the negative wealth effect of higher lump-sum taxes. Clearly,  $R$  unambiguously falls, whereas the impact on  $K$  depends on whether the two curves intersect in the upward or the downward portion of the  $K_1$  curve. From part (i) of Lemma 1 we know that the intersection occurs in the upward portion if and only if  $\theta > \frac{\phi}{1-\phi}$ . The main result of the chapter is then immediate.

**Proposition 10.** *In steady state, higher government consumption ( $g$ ) necessarily decreases the risk-free rate ( $R$ ), while it also decreases the capital-labor ratio ( $K/N$ ), labor productivity ( $Y/N$ ), the wage rate ( $\omega$ ), and the saving rate ( $s \equiv \delta K/Y$ ) if and only if*

$$\theta > \frac{\phi}{1-\phi}. \quad (2.21)$$

With complete markets, the steady state interest rate is equated to the discount rate ( $R = \beta$ ), and the steady state capital-labor ratio is determined by the equality of the marginal product of capital to the discount rate ( $f'(K/N) - \delta = \beta$ ). It follows that, in the long run, government consumption has no effect on either  $R$  or  $K/N$ ,  $Y/N$ ,  $\omega$ , and  $s$ .<sup>12</sup>

Here, instead, government consumption can have non-trivial long-run effects, even if financed with lump-sum taxation. Because households face consumption risk, they have a

---

<sup>12</sup>This result is true even when labor supply is endogenous (as in Section 5). The only difference is that in the latter case, while  $K/N$  and  $Y/N$  continue to not change,  $N$  changes and hence  $K$  and  $Y$  also change.

precautionary motive to save. Because preferences exhibit diminishing absolute risk aversion, this motive is stronger when the level of wealth is lower. It follows that, by reducing household wealth, higher government spending stimulates precautionary saving. But then the risk-free rate at which aggregate saving can be stationary has to be lower, which proves that  $R$  falls with  $g$ .<sup>13</sup>

The impact of this fall in  $R$  on the capital-labor ratio now depends on two opposing effects. On the one hand, because of diminishing absolute risk aversion, a lower level of wealth implies a lower willingness to take risk, which tends to discourage investment. On the other hand, a lower risk-free rate implies a lower opportunity cost of investment, which tends to stimulate investment. As explained earlier, the wealth effect dominates when  $\theta > \frac{\phi}{1-\phi}$ .<sup>14</sup> Since  $\phi < \alpha$ , this is the case as long as the elasticity of intertemporal substitution is high relative to the income share of capital.

For empirically plausible calibrations of the model, the condition  $\theta > \frac{\phi}{1-\phi}$  is easily satisfied. For example, take the interest rate to be  $R = 4\%$  and labor income to be 65% of GDP. Then  $H$  is about 16 times GDP, or equivalently 4 times  $K$ , if we assume a capital-output ratio of 4. Since in steady state  $\frac{\phi}{1-\phi} = \frac{K}{H}$ , this exercise gives a calibrated value for  $\frac{\phi}{1-\phi}$  about 0.25. This is far lower than most of the recent empirical estimates of the elasticity of intertemporal substitution, which are typically around 1 if not higher.<sup>15</sup> It follows that a negative long-run effect of government consumption on aggregate saving and productivity appears to be the most likely scenario.

## 2.4.4 Numerical simulation

We now numerically simulate the steady state of our economy, to get a first pass at the potential quantitative importance of our results.

The economy is fully parameterized by  $(\alpha, \beta, \gamma, \delta, \theta, \sigma, g)$ , where  $\alpha$  is the income share of capital,  $\beta$  is the discount factor,  $\gamma$  is the coefficient of relative risk aversion,  $\delta$  is the (mean) depreciation rate,  $\theta$  is the elasticity of intertemporal substitution,  $\sigma$  is the standard deviation of the rate of return on private investment, and  $g$  is the share of government consumption in aggregate output.

In our baseline parametrization, we take  $\alpha = 0.36$ ,  $\beta = 0.96$ , and  $\delta = 0.08$ ; these values are standard in the literature. For risk aversion, we take  $\gamma = 5$ , a value commonly used in the macro-finance literature to help generate plausible risk premia. For the elasticity of intertemporal substitution, we take  $\theta = 1$ , a value consistent with recent micro and macro estimates.<sup>16</sup> For the share of government, our baseline value is  $g = 25\%$  (as in the United

<sup>13</sup>A similar intuition underlies the steady-state supply of saving in Aiyagari (1994).

<sup>14</sup>In the Appendix we prove that the steady-state  $\phi$  is a decreasing function of the steady-state  $R$ , and hence an increasing function of  $g$ . It follows that, whenever the steady-state  $K$  is a non-monotonic function of  $g$ , it is a  $U$ -shaped function of  $g$ . Note, however, that a high enough  $\theta$  may suffice for  $\theta$  to be higher than  $\phi/(1-\phi)$  for *all* feasible levels of  $g$ , and hence for  $K$  to be a globally decreasing function of  $g$ .

<sup>15</sup> See, for example, Vissing-Jørgensen and Attanasio (2003), Mulligan (2002), and Gruber (2005). See also Guvenen (2006) and Angeletos (2007) for related discussions on the parametrization of the EIS.

<sup>16</sup>See the references in footnote 15.

States) and a higher alternative is  $g = 40\%$  (as in some European countries).

What remains is  $\sigma$ . Unfortunately, there is no direct measure of the rate-of-return risk faced by the “typical” investor in the US economy. However, there are various indications that investment risks are significant. For instance, the probability that a privately held firm survives five years after entry is less than 40%. Furthermore, even conditional on survival, the risks faced by entrepreneurs and private investors appear to be very large: as Moskowitz and Vissing-Jørgensen (2002) document, not only there is a dramatic cross-sectional variation in the returns to private equity, but also the volatility of the book value of a (value-weighted) index of private firms is twice as large as that of the index of public firms—one more indication that private equity is more risky than public equity. Note then that the standard deviation of annual returns is about 15% per annum for the entire pool of public firms; it is over 50% for a single public firm (which gives a measure of firm-specific risk); and it is about 40% for a portfolio of the smallest public firms (which are likely to be similar to large private firms).

Given this suggestive evidence, and in want of a better alternative, we take  $\sigma = 30\%$  for our baseline parameterization and consider  $\sigma = 20\%$  and  $\sigma = 40\%$  for sensitivity analysis. Although these numbers are somewhat arbitrary, the following observation is reassuring: the volatility of individual consumption generated by our model is comparable to its empirical counterpart. For instance, using the Consumer Expenditure Survey (CEX), Malloy, Moskowitz and Vissing-Jørgensen (2006) estimate the standard deviation of consumption growth to be about 8% for stockholders (and about 3% for non-stockholders). Similarly, using data that include consumption of luxury goods, Ait-Sahalia, Parker and Yogo (2001) get estimates between 6% and 15%. In our simulations, on the other hand, the standard deviation of individual consumption growth is less than 5% per annum (along the steady state).

Putting aside these qualifications about the parametrization of  $\sigma$ , we now examine the quantitative effects of government consumption on the steady state of the economy. Table 2.1 reports the per-cent reduction in the steady-state values of the capital-labor ratio ( $K/N$ ), labor productivity ( $Y/N$ ), and the saving rate ( $s$ ), relative to what their values would have been if  $g$  were 0.<sup>17</sup> Complete markets are indicated by CM and incomplete markets by IM.

	$K/N$		$Y/N$		$s$		$\tau_{equiv}^k$
	CM	IM	CM	IM	CM	IM	CM
baseline	<b>0</b>	<b>-10.02</b>	<b>0</b>	<b>-3.73</b>	<b>0</b>	<b>-1.14</b>	<b>17</b>
$\sigma = 40\%$	0	-12.18	0	-4.57	0	-1.21	20
$\sigma = 20\%$	0	-6.78	0	-2.5	0	-0.88	12
$g = 40\%$	0	-17.82	0	-6.82	0	-2.05	28

**Table 2.1:** *The steady-state effects of the size of government.*

<sup>17</sup>Here, since labor supply is exogenously fixed, the changes in  $K$  and  $Y$  coincide with those in  $K/N$  and  $Y/N$ ; this is not the case in the extensions with endogenous labor supply in the next section.

In our baseline parametrization, the capital-labor ratio is about 10% lower when  $g = 25\%$  than when  $g = 0$ . Similarly, productivity is about 4% lower and the saving rate is about 1 percentage point lower. These are significant effects. They are larger (in absolute value) than the steady-state effects of precautionary saving reported in Aiyagari (1994). They are equivalent to what would be the steady-state effects of a marginal tax on capital income equal to 17% in the complete-markets case. (The tax rate on capital income that would generate the same effects under complete markets is given in the last column of the table, as  $\tau_{equiv}^k$ .)

Not surprisingly, the effects are smaller if  $\sigma$  is lower (third row) or if  $\gamma$  is lower (not reported), because then risk matters less. On the other hand, the effects are larger when  $g = 40\%$  (final row): productivity is almost 18% lower, the saving rate is 2 percentage points lower, and the tax on capital income that would have generated the same effects under complete markets is 28%.

Table 2.2 turns from level to marginal effects: it reports the change in  $K/N$ ,  $Y/N$ , and  $s$  as we increase government spending by 1 percent, either from 25% to 26%, or from 40% to 41%. In the first case, productivity falls by 0.19%; in the second, by 0.26%. This is equivalent to what would have been under complete markets the effect of increasing the tax rate on capital income by about 0.75 percentage points in the first case, and about 0.8 percentage points in the second case.

	$K/N$		$Y/N$		$\tau_{equiv}^k$
	CM	IM	CM	IM	CM
$g = 25\% \rightarrow 26\%$	0	-0.52	0	-0.19	0.75
$g = 40\% \rightarrow 41\%$	0	-0.71	0	-0.26	0.8

**Table 2.2:** Long-run effects of a permanent 1% increase in government consumption.

## 2.5 Endogenous labor

In this section we endogenize labor supply in the economy. We consider three alternative specifications that achieve this goal without compromising the tractability of the model.

### 2.5.1 GHH preferences

One easy way to accommodate endogenous labor supply in the model is to assume preferences that rule out income effects on labor supply, as in Greenwood, Hercowitz and Huffman (1998). In particular, suppose that preferences are given by  $U_0 = \mathbb{E}_0 \int_0^\infty e^{-\beta t} u(c_t, l_t) dt$ , with

$$u(c_t, l_t) = \frac{1}{1-\gamma} [c_t + v(l_t)]^{1-\gamma}, \quad (2.22)$$

where  $l_t$  denotes leisure and  $v$  is a strictly concave, strictly increasing function.<sup>18</sup> The analysis can then proceed as in the benchmark model, with labor supply in period  $t$  given by  $N_t = 1 - l(\omega_t)$ , where  $l(\omega) \equiv \arg \max_l \{v(l) - \omega l\}$ .

This specification highlights an important difference between complete and incomplete markets with regard to the employment impact of fiscal shocks. Under incomplete markets, an increase in government spending can have a *negative* general-equilibrium effect on aggregate employment. This is never possible with complete markets, but it is possible with incomplete markets when an increase in  $g$  reduces the capital-labor ratio, and thereby the wage rate, which in turn discourages labor supply. Indeed, with GHH preferences,  $\theta > \frac{\phi}{1-\phi}$  suffices for both  $K/N$  and  $N$  to fall with  $g$  in both the short run and the long run.

Although it is unlikely that wealth effects on labor supply are zero in the long run, they may well be very weak in the short run. In the light of our results, one may then expect that after a positive shock to government consumption both employment and investment could drop on impact under incomplete markets.<sup>19</sup>

## 2.5.2 KPR preferences

A second tractable way to accommodate endogenous labor supply is to assume that agents have homothetic preferences over consumption and leisure, as in King, Plosser, and Rebelo (1988). In particular, suppose that preferences are given by  $U_0 = \mathbb{E}_0 \int e^{-\beta t} u(c_t, l_t) dt$ , with

$$u(c_t, l_t) = \frac{1}{1-\gamma} [c_t^{1-\psi} l_t^\psi]^{1-\gamma}, \quad (2.23)$$

where  $l_t$  denotes leisure and  $\psi \in (0, 1)$  is a scalar.<sup>20</sup>

The benefit of this specification is that it is standard in the literature (making our results comparable to previously reported results), while it also comes with zero cost in tractability.<sup>21</sup> The homotheticity of the household's optimization problem is then preserved and the equilibrium analysis proceeds in a similar fashion as in the benchmark model.<sup>22</sup> The only essential novelty is that aggregate employment is now given by  $N_t = 1 - L(\omega_t, C_t)$ , where

$$L(\omega_t, C_t) = \frac{\psi}{1-\psi} \frac{C_t}{\omega_t}.$$

The neoclassical effect of wealth on labor supply is now captured by the negative relationship between  $N_t$  and  $C_t$  (for given  $\omega_t$ ).

For the quantitative version of this economy, in line with King, Plosser, and Rebelo

---

<sup>18</sup>To allow for  $\theta \neq 1/\gamma$ , we let  $U_t = \mathbb{E}_t \int_t^\infty z(c_\tau + v(l_\tau), U_\tau) d\tau$ , with the function  $z$  defined as in condition (A.2).

<sup>19</sup>This discussion indicates that an interesting extension might be to consider a preference specification that allows for weak short-run but strong long-run wealth effects, as in Jaimovich and Rebelo (2006).

<sup>20</sup>To allow for  $\theta \neq 1/\gamma$ , we let  $U_t = \mathbb{E}_t \int_t^\infty z(c_\tau^\psi l_\tau^{1-\psi}, U_\tau) d\tau$ , with  $z$  defined as in (A.2).

<sup>21</sup>For convenience, we implicitly allow agents to trade leisure with one another, so that we can ignore corner solutions.

<sup>22</sup>The proofs are available upon request.

(1988) and Christiano and Eichenbaum (1992), we take  $\psi = 0.75$ . This value ensures that the steady-state fraction of available time worked approximately matches the US data. The rest of the parameters are as in the baseline specification of the benchmark model.

### 2.5.3 Hand-to-mouth workers

A third approach is to split the population into two groups. The first group consists of the households that have been modeled in the benchmark model; we will call this group the “investors”. The second group consists of households that supply labor but do not hold any assets, and simply consume their entire labor income at each point in time; we will call this group the “hand-to-mouth workers”. Their labor supply is given by

$$N_t^{htm} = \omega_t^{\epsilon_\omega} (C_t^{htm})^{\epsilon_c}, \quad (2.24)$$

where  $C_t^{htm}$  denotes the consumption of these agents,  $\epsilon_\omega > 0$  parameterizes the wage elasticity of labor supply, and  $\epsilon_c > 0$  parameterizes the wealth elasticity.<sup>23</sup>

This approach could be justified on its own merit. In the United States, a significant fraction of the population holds no assets, has limited ability to borrow, and sees its consumption tracking its income almost one-to-one. This fact calls for a richer model of heterogeneity than our benchmark model. But is unclear what the “right” model for these households is. Our specification with hand-to-mouth workers is a crude way of capturing this form of heterogeneity in the model while preserving its tractability.

A side benefit of this approach is that it also gives freedom in parameterizing the wage and wealth elasticities of labor supply. Whereas the KPR preference specification imposes  $\epsilon_\omega = -\epsilon_c = 1$ , the specification introduced above permits us to pick much lower elasticities, consistent with micro evidence. The point is not to argue which parametrization of the labor-supply elasticities is more appropriate for quantitative exercises within the neoclassical growth model; this is the subject of a long debate in the literature, to which we have nothing to add. The point here is rather to cover a broader spectrum of empirically plausible quantitative results.

For the quantitative version of this economy, we thus take  $\epsilon_\omega = 0.25$  and  $\epsilon_c = -0.25$ , which are in the middle of most micro estimates.<sup>24</sup> What then remains is the fraction of aggregate income absorbed by hand-to-mouth workers. As mentioned above, a significant fraction of the US population holds no assets. For example, using data from both the PSID and the SCF, Guvenen (2006) reports that the lower 80% of the wealth distribution owns only 12% of aggregate wealth and accounts for about 70% of aggregate consumption. Since some households may be able to smooth consumption even when their net worth is zero, 70% is likely to be an upper bound for the fraction of aggregate consumption accounted for by hand-to-mouth agents. We thus opt to calibrate the economy so that hand-to-mouth agents account for 50% of aggregate consumption. This is also the value of the relevant parameter

---

<sup>23</sup>Preferences that give rise to this labor supply are  $u_t = c_t^{\zeta_c} - n_t^{\zeta_n}$ , for appropriate  $\zeta_c, \zeta_n$ .

<sup>24</sup>See, for example, Hausman (1981), MaCurdy (1981), and Blundell and MaCurdy (1999).

that one would estimate if the model were to match US aggregate consumption data—we can deduce this from Campbell and Mankiw (1989).<sup>25</sup>

### 2.5.4 The long-run effects of government consumption with endogenous labor

Our main theoretical result (Proposition 4) continues to hold in all of the above variants of the benchmark model: in steady state, a higher rate  $g$  of government consumption necessarily reduces the interest rate  $R$ ; and it also reduces the capital-labor ratio  $K/N$ , labor productivity  $Y/N$ , and the wage rate  $\omega$  if and only if the elasticity of intertemporal substitution  $\theta$  is higher than  $\frac{\phi}{1-\phi}$ .<sup>26</sup>

What is not clear anymore is the effect of  $g$  on  $K$  and  $Y$ , because now  $N$  is not fixed. On the one hand, the reduction in wealth stimulates labor supply, thus contributing to an increase in  $N$ . This is the familiar neoclassical effect of government spending on labor supply. On the other hand, as long as  $\theta > \frac{\phi}{1-\phi}$ , the reduction in capital intensity depresses real wages, contributing towards a reduction in  $N$ . This is the novel general-equilibrium effect due to incomplete markets. The overall effect of government spending on aggregate employment is therefore ambiguous under incomplete markets, whereas it is unambiguously positive under complete markets.

Other things equal, we expect the negative general-equilibrium effect to dominate, thus leading to a reduction in long-run employment after a permanent increase in government spending, if the wage elasticity of labor supply is sufficiently high relative to its income elasticity. This is clear in the GHH specification, where the wealth effect is zero. It can also be verified for the case of hand-to-mouth workers, where we have freedom in choosing these elasticities, but not in the case of KPR preferences, where both elasticities are restricted to equal one.

Given these theoretical ambiguities, we now seek to get a sense of empirically plausible quantitative effects. As already discussed, the GHH case (zero wealth effects on labor supply) is merely of pedagogical value. We thus focus on the parameterized versions of the other two cases, the economy with KPR (homothetic) preferences and the economy with hand-to-mouth workers.

Table 2.3 then presents the marginal effects on the steady-state levels of the capital-labor ratio, productivity, employment, and output for each of these two economies, as  $g$  increases from 25% to 26%, or from 40% to 41%.<sup>27</sup> The case of KPR preferences is indicated by KPR,

---

<sup>25</sup>Note that the specification of aggregate consumption considered in Campbell and Mankiw coincides with the one implied by our model. Therefore, if one were to run their regression on data generated by our model, one would correctly identify the fraction of aggregate consumption accounted for by hand-to-mouth workers in our model. This implies that it is indeed appropriate to calibrate our model's relevant parameter to Campbell and Mankiw's estimate.

<sup>26</sup>This is true as long as the steady state is unique, which seems to be the case but has not been proved as in the benchmark model. Also, in the variant with hand-to-mouth agents, we have to be cautious to interpret  $\phi$  as the ratio of private equity to effective wealth for the investor population alone.

<sup>27</sup>We henceforth focus on marginal rather than level effects just to economize on space.



while the case with hand-to-mouth workers is indicated by HTM. In either case, complete markets are indicated by CM and incomplete markets by IM.

		$K/N$		$Y/N$		$N$		$Y$		$\tau_{equiv}^k$
		CM	IM	CM	IM	CM	IM	CM	IM	CM
$g = 25\% \rightarrow 26\%$	KPR	0	-0.33	0	-0.12	1.4	1.27	1.4	1.15	0.52
	HTM	0	-0.3	0	-0.11	0.38	0.38	0.38	0.27	0.46
$g = 40\% \rightarrow 41\%$	KPR	0	-0.52	0	-0.19	1.76	1.53	1.76	1.34	0.68
	HTM	0	-0.36	0	-0.13	0.57	0.57	0.57	0.44	0.48

**Table 2.3:** Long-run effects with endogenous labor.

Regardless of specification, the marginal effects of higher government spending on capital intensity  $K/N$  and labor productivity  $Y/N$  are negative under incomplete markets (and are stronger the higher is  $g$ ), whereas they are zero under complete markets. As for aggregate employment  $N$ , the wealth effect of higher  $g$  turns out to dominate the effect of lower wages under incomplete markets, so that  $N$  increases with higher  $g$  under either complete or incomplete markets. However, the employment stimulus is weaker under incomplete markets, especially in the economy with hand-to-mouth workers. The same is true for aggregate output: it increases under either incomplete or complete markets, but less so under incomplete markets. Finally, the incomplete-markets effects are on average equivalent to what would have been the effect of increasing the tax rate on capital income by about 0.55% under complete markets.

## 2.6 Dynamic responses

The results so far indicate that the long-run effects of government consumption can be significantly affected by incomplete risk sharing. We now examine how incomplete risk sharing affects the entire impulse response of the economy to a fiscal shock.<sup>28</sup>

Starting from the steady state with  $g = 25\%$ , we hit the economy with a permanent 1% increase in government spending and trace its transition to the new steady state (the one with  $g = 26\%$ ). We conduct this experiment for both the economy with KPR preferences and the economy with hand-to-mouth workers, each parameterized as in the previous section; in either case, the transitional dynamics reduce to a simple system of two first-order ODE's in  $(K_t, H_t)$  when  $\theta = 1$ .<sup>29</sup>

<sup>28</sup>Note that the purpose of the quantitative exercises conducted here, and throughout the chapter, is not to assess the ability of the model to match the data. Rather, the purpose is to detect the potential quantitative significance of the particular deviation we took from the standard neoclassical growth model.

<sup>29</sup>Throughout, we focus on permanent shocks. Clearly, transitory shocks have no impact in the long run. As for their short-run impact, the difference between complete and incomplete markets is much smaller than in the case of permanent shocks. This is simply because transitory shocks have very weak wealth effects on

The results are presented in Figures 2.2 and 2.3. Time in years is on the horizontal axis, while deviations of the macro variables from their respective initial values are on the vertical axis. The interest rate and the investment rate are in simple differences, the rest of the variables are in log differences. The solid lines indicate incomplete markets, the dashed lines indicate complete markets.

As evident in these figures, the quantitative effects of a permanent fiscal shock can be quite different between complete and incomplete markets. The overall picture that emerges is that the employment and output stimulus of a permanent increase in government spending is weaker under incomplete markets than under complete markets. And whereas we already knew this for the long-run response of the economy, now we see that the same is true for its short-run response.

This picture holds for both the economy with KPR preferences and the one with hand-to-mouth workers. But there are also some interesting differences between the two. The mitigating effect of incomplete markets on the employment and output stimulus of government spending is much stronger in the economy with hand-to-mouth workers. As a result, whereas the short-run effects of higher government spending on the investment rate and the interest are positive under complete markets in both economies, and whereas these effects remain positive under incomplete markets in the economy with KPR preferences, they turn negative under incomplete markets in the economy with hand-to-mouth workers.

To understand this result, consider for a moment the benchmark model, where there are no hand-to-mouth workers and labor supply is completely inelastic. Under complete markets, a permanent change in government spending would be absorbed one-to-one in private consumption, leaving investment and interest rates completely unaffected in both the short- and the long-run. Under incomplete markets, instead, investment and the interest rate would fall on impact, as well as in the long run. Allowing labor supply to increase in response to the fiscal shock ensures that investment and the interest rate jump upwards under complete markets. However, as long as the response of labor supply is weak enough, the response of investment and the interest rate can remain negative under incomplete markets.

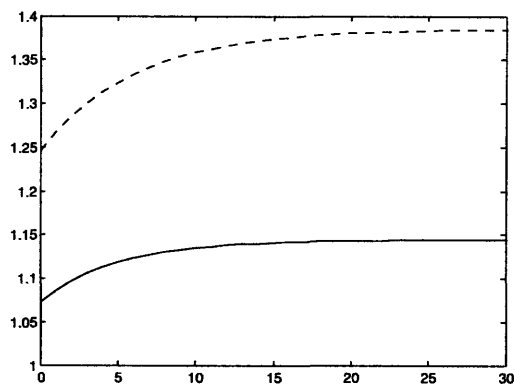
As a final point of interest, we calculate the welfare cost, in terms of consumption equivalent, associated with a permanent 1% increase in government spending. Under complete markets, welfare drops by 0.2%, whereas under incomplete markets it drops by 0.6%. In other words, the welfare cost of an increase in government spending is three times higher under incomplete markets than under complete markets.<sup>30</sup>

To recap, the quantitative results presented here indicate that a modest level of uninsured idiosyncratic investment risk can have a non-trivial impact on previously reported quantitative evaluations of fiscal policy. Note in particular that our quantitative economy with KPR preferences is directly comparable to two classics in the related literature, Aiya-

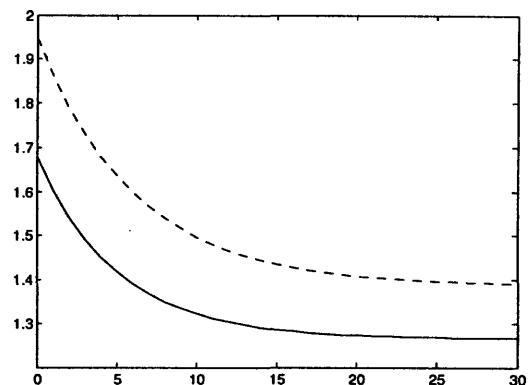
---

investment as long as agents can freely borrow and lend over time, which is the case in our model. However, we expect the difference between complete and incomplete markets to be larger once borrowing constraints are added to the model, for then investment will be sensitive to changes in current disposable income even if there is no change in present-value wealth.

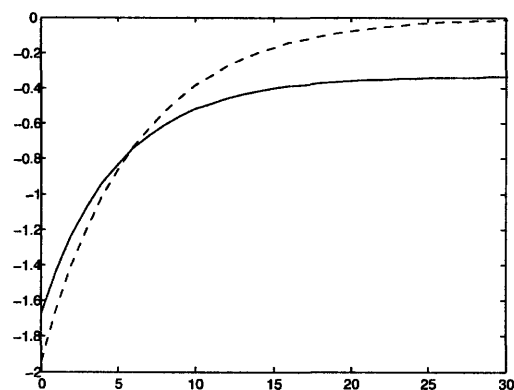
<sup>30</sup>Here we have assumed that government consumption has no welfare benefit, but this should not be taken literally: nothing changes if  $G_t$  enters separably in the utility of agents.



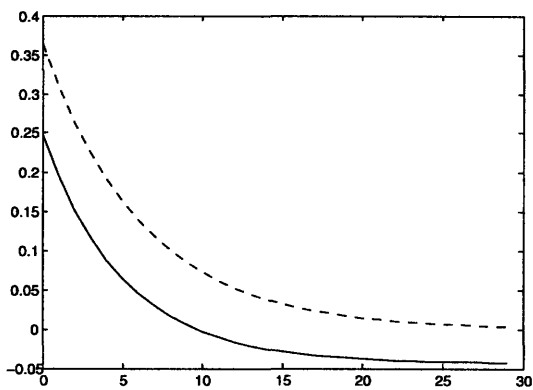
(a) Aggregate Output  $Y_t$



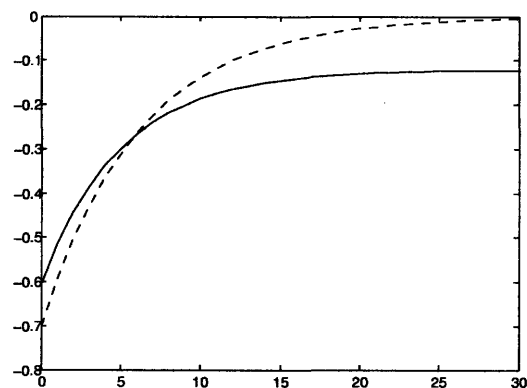
(b) Aggregate Employment  $N_t$



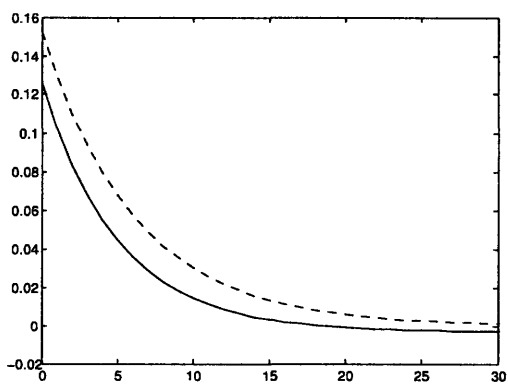
(c) Capital-Labor Ratio  $K_t/N_t$



(d) Investment Rate  $I_t/Y_t$

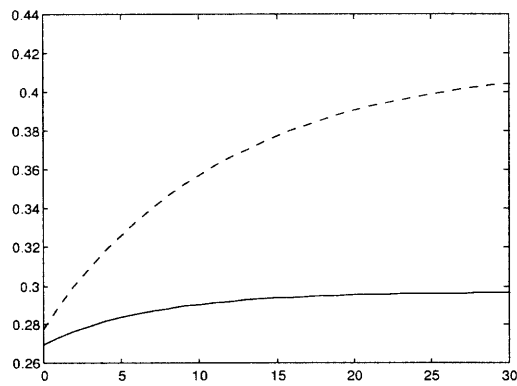


(e) Labor Productivity  $Y_t/N_t$

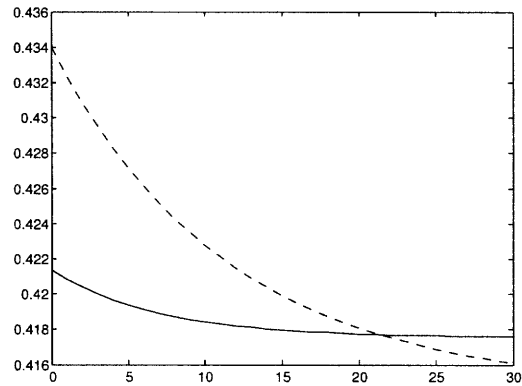


(f) Interest Rate  $R_t$

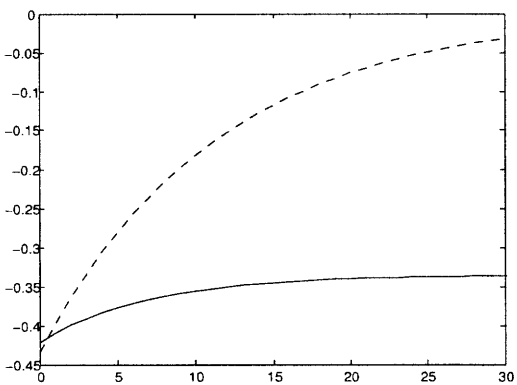
**Figure 2-2:** *Dynamic responses to a permanent shock with KPR preferences.*



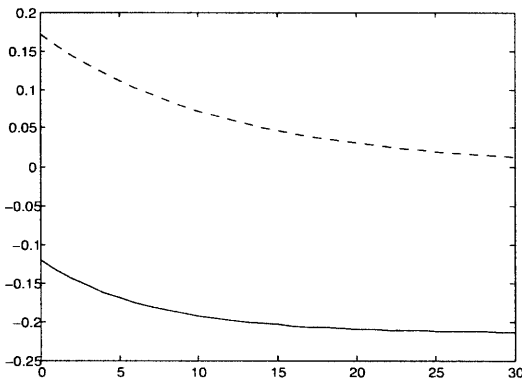
(a) Aggregate Output  $Y_t$



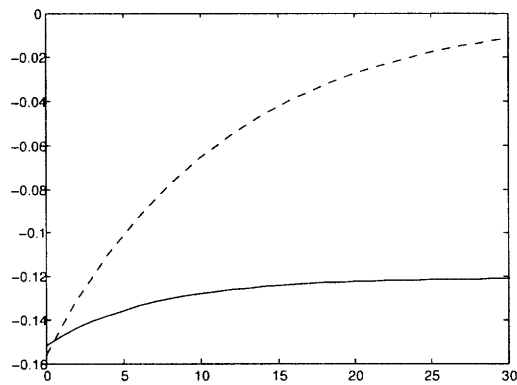
(b) Aggregate Employment  $N_t$



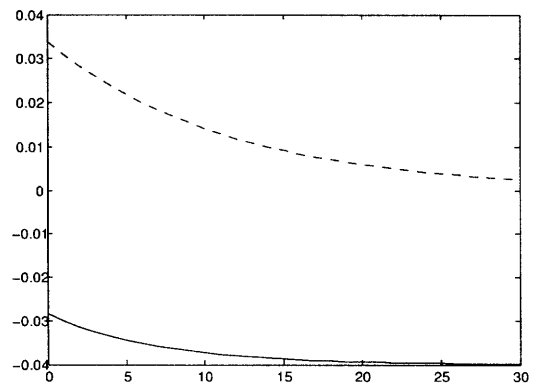
(c) Capital-Labor Ratio  $K_t/N_t$



(d) Investment Rate  $I_t/Y_t$



(e) Labor Productivity  $Y_t/N_t$



(f) Interest Rate  $R_t$

**Figure 2-3:** *Dynamic responses to a permanent shock with hand-to-mouth agents.*

gari, Christiano and Eichenbaum (1992) and Baxter and King (1993). Therefore, further investigating the macroeconomic effects of fiscal shocks in richer quantitative models with financial frictions appears to be a promising direction for future research.

## 2.7 Conclusion

This chapter revisits the macroeconomic effects of government consumption in an incomplete-markets version of the neoclassical growth model. Incomplete markets make individual investment sensitive to individual wealth for given prices. It follows that an increase in government spending can crowd-out private investment simply by reducing disposable income. As a result, market incompleteness can seriously upset the supply-side effects of fiscal shocks: an increase in government consumption, even if financed with lump-sum taxation, tends to reduce capital intensity, labor productivity, and wages in both the short-run and the long-run. For plausible parameterizations of the model, these results appear to have not only qualitative but also quantitative content.

These results might, or might not, be bad news for the ability of the neoclassical paradigm to explain the available evidence regarding the macroeconomic effects of fiscal shocks.<sup>31</sup> However, the goal of this chapter was not study whether our model could match the data. Rather, the goal was to identify an important mechanism through which incomplete markets modify the response of the economy to fiscal shocks: wealth effects on investment.

In our model, these wealth effects originated from uninsured idiosyncratic investment risk combined with diminishing absolute risk aversion. Borrowing constraints could lead to similar sensitivity of investment to wealth (or cash flow).<sup>32</sup> Also, this mechanism need not depend on whether prices are flexible (as in the neoclassical paradigm) or sticky (as in the Keynesian paradigm). The key insights of this chapter are thus clearly more general than the specific model we employed—but the quantitative importance of these insights within richer models of the macroeconomy is, of course, a widely open question.

An important aspect left outside our analysis is the optimal financing of government expenditures. In this chapter, as in much of the related literature, we assumed that the increase in government spending is financed with lump-sum taxation, only because we wished to isolate wealth effects from tax distortions. Suppose, however, that the government has access to two tax instruments, a lump-sum tax and a proportional income tax.<sup>33</sup> Suppose further that the government chooses taxes so as to maximize ex ante utility (equivalently, a utilitarian welfare criterion) subject to its budget constraint. Clearly, with complete markets (and no inequality) it would be optimal to finance any exogenous increase in government spending with only lump-sum taxes. With incomplete markets, however, it is likely that an

---

<sup>31</sup>Whether the evidence is consistent with the neoclassical paradigm is still debatable. For example, using structural VARs with different identification assumptions, Ramey and Shapiro (1998) and Ramey (2006) find that private consumption falls in response to a positive shock to government consumption, as predicted by the neoclassical paradigm, while Blanchard and Perotti (2002) and Perotti (2007) find the opposite result.

<sup>32</sup>On this point, see also Challe and Ragot (2007).

<sup>33</sup>As in Werning (2006), this might be a good proxy for more general non-linear tax schemes.

increase in government spending is financed with a mixture of both instruments: while using only the lump-sum tax would disproportionately affect the utility of poor agents, using both instruments permits the government to trade off less efficiency for more equality. Further exploring these issues is left for future research.

## Chapter 3

# Consumption Behavior in Greece: Alternative Explanations, Identification and Interpretation

### Abstract

This chapter uses annual Greek data to test the validity of the Permanent Income Hypothesis (PIH) versus the “Keynesian” or excess sensitivity view that consumption responds to current income. The PIH is rejected by all tests, and so is the simple excess sensitivity hypothesis with a constant marginal propensity to consume. Possible causes for the failure of the PIH are then examined, such as liquidity constraints as proxied by unemployment, the amount of new loans, and private interest rate spreads. Finally, the relevance of a more sophisticated excess sensitivity version, with a time-varying marginal propensity to consume, is determined. It is shown that liquidity constraints, in the form of the spread between private interest rates on loans and deposits, negatively affect the marginal propensity to consume out of current disposable labor income. However, this result disappears when total disposable income is considered instead. This finding is interpreted as evidence that consumers are able to self-insure, and to buffer the shocks to their labor income, by investing in non-human assets.

### 3.1 Introduction

This chapter examines the relevance of the permanent income hypothesis (PIH) versus the alternative null of excess sensitivity of consumption to current income, using annual Greek data for the period 1960-2004. According to the permanent income hypothesis, the time pattern of income over the life-cycle should influence only the pattern of saving, and not the timing of consumption. Hence, consumption should not respond to transitory changes in income. The PIH has been subject to much research in the theoretical and the empirical

literature, because it provides appealing explanations for many important features of consumption behavior. For example, it explains why temporary tax cuts appear to have much smaller effects than permanent ones. Yet there are also aspects of consumption behavior that seem to be inconsistent with the PIH, such as the evidence that consumption responds to predictable changes in income. This “Keynesian” version of consumption behavior is known as the excess sensitivity of consumption to current income.

The PIH consists of three joint assumptions. First, agents’ expectations are rational. Second, desired consumption is determined by permanent income. Third, capital markets are perfect, and agents can freely borrow and lend against their future expected labor income at the same interest rate. Most empirical studies have rejected the PIH, concluding that consumption is more sensitive to current income than is compatible with the theory, but they have not attempted to explain which of the components of the joint hypothesis is responsible for the rejection.

As in Campbell and Deaton (1989), and Flavin (1993), the saving of rational agents will be crucial for econometrically identifying the variance of the revisions in the unobservable permanent income, both under the null of the PIH, and under the null of excess sensitivity. A bivariate VAR on income and saving rejects both the PIH, and the excess sensitivity of consumption with a constant marginal propensity to consume. However, the orthogonality restrictions are also rejected, implying that consumption responds to news about current income, which seems to be inconsistent with the rejection of excess sensitivity. Subsequently, it is examined which of two assumptions might be responsible for the failure of the PIH: agents’ myopic behavior, in the sense of a positive marginal propensity to consume out of transitory income, or capital market imperfections, in the form of liquidity constraints. To this end, single equation tests are performed for the specification of the excess sensitivity consumption equation.

It is shown that the estimate for the marginal propensity to consume out of transitory income is not dramatically affected by the inclusion of the overall unemployment rate as a proxy for liquidity constraints. In addition, the unemployment rate does not seem to have a direct impact on consumption. Furthermore, the failure of the PIH is not due to liquidity constraints as captured by the amount of new loans. The tests suggest that the failure of the PIH is probably due to liquidity constraints in the form of the spread between the private borrowing and lending interest rates, which makes it harder for agents to borrow against their future labor income when they face an adverse shock. But when consumers use their non-human wealth to buffer their labor-income shocks, these liquidity constraints do not influence their consumption behavior anymore.

This chapter is organized as follows. Section 2 presents the theoretical implications of a simple model for the behavior of consumption under the two alternative null hypotheses, permanent income versus excess sensitivity. Section 3 presents the implications of the restrictions imposed by the two hypotheses for a bivariate VAR on income and saving. Section 4 tests these sets of restrictions. Section 5 examines possible causes of misspecification of the consumption equation, and proposes a more sophisticated version of the simple Keynesian view. Section 6 concludes. The appendix describes the data.



## 3.2 Consumption and the variance of consumption innovations

According to the permanent income hypothesis (PIH), the time pattern of income over the life-cycle should influence only the pattern of saving, and not the timing of consumption. Hence, consumption should not respond to transitory changes in income. In other words, an agent will only revise his desired consumption plan if he receives news that permanently affect his income in the future. Permanent income,  $y_t^P$ , is defined as the annuity value of an agent's human and non-human wealth:

$$y_t^P = \left( \frac{r}{1+r} \right) \left( A_t + \sum_{\tau=0}^{\infty} \delta^\tau E_t y_{t+\tau} \right),$$

where  $\delta$  is the discount factor,  $r$  is the interest rate, which is assumed constant,  $y_t$  is labor income or human wealth, and  $A_t$  is assets or non-human wealth. If the agent consumes exactly his permanent income each period, then permanent income is a martingale. Since the interest rate is constant, only future labor income is subject to exogenous shocks, and hence:

$$\Delta y_t^P = \left( \frac{r}{1+r} \right) \sum_{\tau=0}^{\infty} \delta^\tau (E_t - E_{t-1}) y_{t+\tau} \equiv \varepsilon_{ypt},$$

where  $\varepsilon_{ypt}$  is the expectational innovation in the annuity value of labor income.

Under the Permanent Income Hypothesis (PIH), consumption,  $c_t$ , should follow a martingale process, with innovations equal to the innovations in permanent income:

$$\text{var}(\varepsilon_{c_t}) = \text{var}(\Delta c_t) = \text{var}(\Delta y_t^P) = \text{var}(\varepsilon_{ypt}).$$

If, on the other hand, consumption exhibits excess sensitivity to current income, then:

$$c_t = \beta y_t^T + y_t^P,$$

where  $\beta$  is the marginal propensity to consume out of transitory income,  $y_t^T$ , and where transitory income is defined as the residual between current total net worth and permanent income. The implications of the excess sensitivity hypothesis for saving,  $s_t$ , and consumption,  $c_t$ , respectively are:

$$s_t = (1 - \beta) \left( y_t - \left( \frac{r}{1+r} \right) \sum_{\tau=0}^{\infty} \delta^\tau E_t y_{t+\tau} \right), \quad (3.1)$$

$$\Delta c_t = \beta \Delta y_t + (1 - \beta) \varepsilon_{ypt}. \quad (3.2)$$

Hence the reduced form consumption disturbance is a weighted average of the reduced form income disturbance and the expectational revision in permanent income.

### 3.3 Bivariate VAR on income and saving

Although permanent income is not observable, under the null of the PIH, agents' optimal behavior implies that saving summarizes the information used by them but unobserved by the econometrician. In other words, the agents' behavior reveals a sufficient statistic measure of omitted information, in the form of an endogenous signaling variable, saving. For example, if saving increases, the econometrician can infer that the agent's expectation of the discounted value of his future labor income has fallen, and this helps control for the agent's superior information when predicting income. This projection argument has been used by several authors, including Campbell and Deaton (1989), to finesse the omitted information problem. As will be discussed below, the same reasoning is also valid for the null of excess sensitivity. Hence, under both the permanent income and the excess sensitivity hypotheses, it is possible to identify the expectational innovations in permanent income.

Following the literature, the restrictions implied by the permanent income and excess sensitivity hypotheses will first be tested in the context of a bivariate VAR on income and saving. In addition, an orthogonality assumption will be tested, namely that consumption behavior is orthogonal to current and lagged information, which should be the case under the PIH. This test will be a more efficient version of Hall's (1988) orthogonality test.

The unrestricted VAR on income,  $y_t$ , and saving,  $s_t$ , has the form:

$$\begin{pmatrix} y_t \\ s_t \end{pmatrix} = \begin{pmatrix} a_{11}(L) & a_{12}(L) \\ a_{21}(L) & a_{22}(L) \end{pmatrix} \begin{pmatrix} y_t \\ s_t \end{pmatrix} + \begin{pmatrix} \varepsilon_{yt} \\ \varepsilon_{st} \end{pmatrix} .$$

Since  $s_t = \Delta y_t + (1+r)s_{t-1} - \Delta c_t$ , the restriction  $\Delta c_t = \varepsilon_{ypt}$  imposed by the PIH transforms the VAR as follows:

$$\begin{pmatrix} y_t \\ s_t \\ \Delta c_t \end{pmatrix} = \begin{pmatrix} a_{11}(L) & a_{12}(L) \\ a_{11}(L) - L & a_{12}(L) + (1+r)L \\ 0 & 0 \end{pmatrix} \begin{pmatrix} y_t \\ s_t \end{pmatrix} + \begin{pmatrix} \varepsilon_{yt} \\ \varepsilon_{yt} - \varepsilon_{ypt} \\ \varepsilon_{ypt} \end{pmatrix} .$$

The equation for consumption has been incorporated here, in order to test the orthogonality assumption. The restrictions imposed by orthogonality are:

$$a_{21}(L) = a_{11}(L) - L \quad \text{and} \quad a_{22}(L) = a_{12}(L) + (1+r)L .$$

Under the null of the PIH, the expectational revision in permanent income inferred by the econometrician on the basis of the bivariate VAR,  $e_{ypt}$ , coincides with the true expectational revision in permanent income with respect to the agent's superior information set, i.e.  $e_{ypt} = \varepsilon_{ypt} = \varepsilon_{ct}$ . Therefore, the variance of  $\varepsilon_{ypt}$  can be correctly inferred from the VAR, namely  $\text{var}(e_{ypt}) = \text{var}(\varepsilon_{ypt})$ . The restrictions imposed by the PIH are:

$$a_{21}(\delta) = a_{11}(\delta) - \delta \quad \text{and} \quad a_{22}(\delta) = a_{12}(\delta) + (1+r)\delta = a_{12}(\delta) + 1 ,$$

where  $L = \delta = (1+r)^{-1}$ . Note that the orthogonality restrictions imply the PIH restrictions,

but not the other way round.

If the PIH were to fail in an arbitrary way, saving might lose the property of incorporating all the information available to the agent, in which case the variance of revisions to permanent income would not be identified. However, Flavin (1993) proves that if the data is generated by the null of excess sensitivity, this problem does not arise. As seen by (3.1), the intuition for this result is that saving under excess sensitivity is just a rescaled version of the PIH saving, and hence the informational content of saving is not destroyed. So again here,  $var(e_{ypt}) = var(\varepsilon_{ypt})$ . The restrictions imposed by excess sensitivity are:

$$a_{21}(L) = (1 - \beta)(a_{11}(L) - L) \quad \text{and} \quad a_{22}(L) = a_{12}(L) + (1 + r)L.$$

### 3.4 Bivariate VAR hypothesis tests

The three sets of restrictions (orthogonality, PIH, and excess sensitivity) will be tested using the bivariate VAR in second order. The order of the VAR was chosen through diagnostic tests for the number of lags included. Table 3.1 reports the results of the modified likelihood ratio tests (LR)<sup>1</sup>. The modified LR test has the same asymptotic distribution as the usual LR test, but is less likely to reject the null in small samples.

LR for Hypothesis Tested	Second Order Bivariate VAR
Orthogonality	$\chi^2(4) = 19.78^{**}$
Permanent Income	$\chi^2(2) = 17.16^{**}$
Excess Sensitivity	$\chi^2(4) = 13.42^{**}$
Value of $\beta$ Under Excess Sensitivity	0.112 (0.089)
** indicates rejection at the 5% significance level	
Standard errors in parenthesis	

**Table 3.1:** *Likelihood Ratio (LR) Tests.*

The over-identifying restrictions are rejected for all three hypotheses at the 1% and 5% significance levels. Hence the data does not support the PIH. The failure of the orthogonality condition implies that consumption responds to lagged information. This would seem to be evidence in favor of excess sensitivity, but such in fact is not the case, since the restrictions imposed by the simple excess sensitivity hypothesis are also rejected. When testing for excess sensitivity, the point estimate of the marginal propensity to consume turns out to be 0.11, and is precise (standard error less than 0.1), but not significant. In a similar context, Flavin (1993) rejects the PIH, but not the simple excess sensitivity hypothesis.

As a further test, it is noted that, under the PIH, saving should Granger-cause labor income. A Granger causality test with two or more lags cannot reject the null that saving does not Granger-cause labor income. For example, with two lags, the value of the F-statistic

<sup>1</sup>The annual interest rate is assumed throughout to be 5%.

for the null that saving does not Granger-cause labor income is 1.05, which is less than the critical values at the 1% and 5% significance levels. This is interpreted as further evidence against the PIH. It should not however be taken as indicating that saving is exogenous in any way, or that there is a force of causation from saving to income.

In order to further deal with small sample bias, simulations have been performed to find the critical values of the likelihood ratio, without assuming any particular distribution for the simulated residuals. The results presented above do not change qualitatively.

## 3.5 Single equation tests and interpretations

So far, the bivariate VAR on income and saving rejected both the null of the PIH, and the alternative null of excess sensitivity of consumption to current income with a constant marginal propensity to consume. However, the orthogonality restrictions were also rejected, with the implication that consumption responds to news about current income. This section examines possible causes of misspecification of the consumption equation (3.2), and proposes a more sophisticated version of the simple Keynesian view, which will turn out to better characterize the data.

### 3.5.1 A single equation test of the PIH

The PIH predicts that consumption only responds to changes in permanent income. In order to test this implication, a dummy variable,  $D_t$ , is constructed, taking the value 1 in years when Greek consumers received news that would lead them to revise upward their expectations about their future labor income. Such years are for example the year when Greece applied for entry into the European Economic Community, the year that Greece was accepted, and so on. Consumer confidence has been reported to peak during each one of these years. The assumption here is that this correctly captures the structure of the agents' expectations. Nonetheless, the same qualitative results obtain when lags or persistence in the expectations are allowed. According to the PIH, the coefficient,  $h$ , on this dummy should be significant:

$$\Delta c_t = a + h D_t .$$

The coefficient  $h$  is  $-53$ , hence it does not have the correct sign, but it has a standard error of 55, so it is not significant in this or for that matter in any other specification of the consumption equation. Hence, the PIH is rejected.

### 3.5.2 Unemployment as proxy for liquidity constraints

This section examines possible reasons for the failure of the PIH, in a single-consumption-equation context. The focus will be on potential misspecification of equation (3.2) arising from the absence of liquidity constraints as a determinant of consumption. If consumption

is constrained by current income, actual consumption and transitory income will be positively correlated. As a proxy for the prevalence of liquidity constraints, the empirical work focuses on the rate of unemployment. The assumption here is that a significant fraction of the population has net worth insufficient to insulate them from negative transitory labor income shocks. The modified Keynesian consumption function, including the specification instrument unemployment,  $z_t$ , is:

$$\Delta c_t = \beta \Delta y_t + \gamma \Delta z_t + (1 - \beta) \Delta y_t^p ,$$

where permanent income has been delegated to the error term. The parameters of interest can be consistently estimated by instrumental variables, provided the instruments are uncorrelated with the error term. Since permanent income and its revisions are uncorrelated with any lagged information, lagged values of  $\Delta y_t$  and  $\Delta z_t$  are valid instruments. The coefficient  $\gamma$  of the unemployment rate is expected to be negative if liquidity constraints affect consumption. Misspecification occurs if  $\gamma$  is nonzero. In particular, the consumption equation estimated has the form:

$$\Delta c_t = a + \beta_0 \Delta y_t + \beta_1 \Delta y_{t-1} + \beta_2 \Delta y_{t-2} + \gamma_0 \Delta z_t + \gamma_1 \Delta z_{t-1} + \gamma_2 \Delta z_{t-2} + \omega_t , \quad (3.3)$$

where  $\omega_t = (1 - \beta_0) \Delta y_t^p$ . Since the informational content of  $y_t$  and  $z_t$  is not of interest, it is sufficient to perform single equation IV, instead of embedding (3.3) in a system with an explicit income forecasting model. Besides, one can prove that the fully unconstrained system of a bivariate VAR on income and unemployment, along with an unconstrained equation for consumption, is exactly identified. Hence, FIML on that system would give estimates for the  $\beta_i$ 's and the  $\gamma_i$ 's which are numerically identical to the estimates obtained by IV on (3.3). However, the instrumental variables technique runs across the problem of weak instruments.

### 3.5.3 The weak instruments problem

The correlation between first and second differences of disposable labor income is 0.224 and not statistically significant. Actually, the best predictor for the first difference in labor income turns out to be the lagged first difference in consumption. Hence  $\Delta y_{t-1}$  is probably a weak instrument for  $\Delta y_t$ , all the more because the number of observations is relatively small. This will bias the coefficient of the marginal propensity to consume upwards, i.e. TSLS is biased in the direction of OLS, and this bias might be significant given that the sample is small. To verify that  $\Delta y_{t-1}$  is indeed a weak instrument, a Hausman-Hahn (2004) test is performed, using an algorithm that incorporates their explicit formulas for the test statistic. Their statistic has an asymptotically standard normal distribution. Lagged first differences of income and consumption are used as instruments for the first difference in income. The critical value of the statistic is 2, hence the null of strong instruments is rejected at the 5% significance level. In an attempt to correct the problem, the bias-corrected estimator is also calculated. It turns out however that this estimator has a higher value than both the TSLS and the OLS estimator. This (unusual at best) result is interpreted to mean that, in lack of

strong instruments, it is preferable to estimate the versions of equation (3.3) using OLS.

### 3.5.4 Estimation of the single consumption equation (3.3)

When estimating (3.3), two lags are used, in order to compare the results to Flavin's (1985). Flavin rejects the PIH, but finds some evidence for the simple Keynesian view that consumption responds to current income with a constant marginal propensity. Table 3.2 presents the results for the various tests performed. When the  $\gamma_i$ 's are all constrained to be zero,  $\beta_0$ , which is interpreted as the marginal propensity to consume out of current income, is positive, significant and precise. Introducing the lags for unemployment increases the marginal propensity to consume, but also the standard errors, so the result for  $\beta_0$  remains mainly unchanged. None of the unemployment coefficients are significant, though they have the expected negative signs. When the  $\beta_i$ 's are restricted to be zero, the qualitative results for the unemployment coefficients continue to hold.

Constraints	Estimates						F-statistic
	$\beta_0$	$\beta_1$	$\beta_2$	$\gamma_0$	$\gamma_1$	$\gamma_2$	
None	0.388** (0.048)	0.073 (0.057)	-0.092 (0.053)	32.058 (23.997)	-7.630 (25.926)	-15.370 (24.57)	
$\gamma_i = 0$	0.380** (0.043)	0.065 (0.046)	-0.079 (0.047)				0.864
$\beta_i = 0$				-14.714 (35.926)	-54.271 (38.155)	-40.872 (35.424)	23.764
$\beta_i = \gamma_i = 0$							15.366

\*\* indicates rejection at the 1% significance level  
Standard errors in parentheses

**Table 3.2:** *The Role of Unemployment.*

The null of no excess sensitivity of consumption ( $\beta_i = 0$ ) is rejected at the 1% and the 5% significance levels. The null of the PIH ( $\beta_i = \gamma_i = 0$ ) is rejected at the 1% and the 5% levels. The null that  $\gamma_i = 0$  cannot be rejected. These results further support the bivariate VAR findings. Since the  $\gamma_i$ 's are not statistically different from zero, there is no misspecification of the simple consumption equation due to liquidity constraints as proxied by unemployment. These facts would lead to the conclusion that consumer myopia, in the sense of a positive marginal propensity to consume out of current income, is a possible reason for the failure of the PIH. But this possibility has already been examined and rejected in the VAR approach. However, equation (3.3) provides an asymptotically valid test of excess sensitivity because all the regressors are stationary, which is not the case in the VAR.

Since, in contrast to most of the empirical literature, unemployment as a proxy for liquidity constraints does not seem to explain the failure of the PIH, the relevance of an additional variable as proxy for liquidity constraints will next be examined. This variable is

the total amount of liquidity available in the economy in the form of new loans extended from banks to private businesses. This is a valid proxy for three reasons. First, up until recently, consumption loans and housing loans have not been widespread among Greek consumers. Second, over 85% of all private businesses in Greece are family businesses, i.e. they have a small number of employees and they are not incorporated in the stock market. Third, the percentage of people who declare they are self-employed reaches about 70%. The regression is (3.3), where  $z_t$  is now the liquidity available in the economy. Since fewer lags are preferable in the presence of the weak instruments problem, one lag is used for each differenced variable. Now  $\Delta z_{t-1}$  can be used as an instrument for  $\Delta z_t$ , because the correlation between them is high at 0.6, with a standard error of 0.14. The marginal propensity to consume is expected to be higher, the higher is liquidity. Table 3.3 presents the results.

Income Measure	Estimates	
	$\beta_0$	$\gamma_0$
Labor	0.328 (0.19)	0.306 (0.19)
Total disposable	0.25** (0.05)	0.17 (0.145)
** indicates rejection at the 5% significance level Standard errors in parentheses		

**Table 3.3:** *The Role of Liquidity.*

The adjusted R-square for the regression with labor income is 0.47, and it increases to 0.65 for the regression with total income. The coefficient  $\gamma_0$  has the anticipated sign, but is not significant. Therefore, it appears that liquidity constraints are not the reason for the failure of the PIH.

### 3.5.5 A more sophisticated version of the Keynesian view

The analysis so far provides sufficient evidence to conclude that consumption behavior in Greece is not described by either the PIH or the simple Keynesian view with a constant marginal propensity to consume. In light of these results, it could then be the case that the marginal propensity to consume is time-varying. This section examines the case where the marginal propensity to consume depends on the spread between the interest rate on bank saving accounts and the interest rate on short-run or long-run bank loans to private businesses. If this spread affects the marginal propensity to consume out of current labor income, one reason for the failure of the PIH will have been identified. Other specifications that make the marginal propensity to consume time-varying, for example using unemployment, have been examined, but none of them yields statistically significant results. The relevant equation is then:

$$\Delta c_t = a + \beta_0 \Delta y_t + \beta_1 \Delta y_t \cdot spread_t + \omega_t, \quad (3.4)$$

where the time-varying effective marginal propensity to consume is  $\beta_0 + \beta_1 \cdot spread_t$ . Equation (3.4) is estimated using both disposable labor income and total disposable income. Lagged spread is used as an instrument for spread (correlation 0.87). Table 3.4 presents the results for the case of the long-term bank loans. The results for the short-term loans are qualitatively similar.

Income Measure	Estimates		
	$\beta_0$	$\beta_1$	$\beta_0 + \beta_1 \cdot spread$
Labor	0.942** (0.202)	-0.084** (0.039)	0.60**
Total disposable	0.219** (0.057)	0.030 (0.018)	0.30**
** indicates rejection at the 5% significance level			
Standard errors in parentheses			
<i>spread</i> denotes the mean of the interest rate spread over the sample period			

**Table 3.4:** *Time-Varying Marginal Propensity to Consume.*

When income is taken to be labor income, the coefficient  $\beta_0$  is significant and positive, as predicted by the theory, while the coefficient  $\beta_1$  is significant and negative. Indeed, given that  $\beta_0$  is positive, the only way that the marginal propensity to consume could fall when liquidity constraints become more severe is if  $\beta_1$  is negative. The intuition is that an increase in the interest rate spread increases liquidity constraints, and therefore increases the incentive to save for a rainy day, thus decreasing the marginal propensity to consume out of labor income. The effective marginal propensity coefficient is approximately 0.6 and statistically significant. If we estimate the same equation using total income, which includes non-labor income, then  $\beta_1$  is not statistically significant any more, and the overall marginal propensity coefficient drops to 0.3.

These results can be interpreted as follows. Liquidity constraints in the form of interest rate spreads play an important role in decreasing the marginal propensity to consume out of current disposable labor income. When the agent faces a higher spread, his difficulty in borrowing against his future labor income increases. Therefore, he is more likely to increase his saving and decrease his propensity to consume. However, such liquidity constraints are not important for the behavior of consumption out of total disposable income, which includes for example rents from land. This means that, by accumulating other assets, agents have more or less managed to self-insure and to counteract the adverse effects of liquidity constraints and capital-market imperfections on their consumption.

Finally, it is noted that if the lagged first difference in consumption is included in (3.4), then the overall fit is improved and the relevant coefficient is positive and significant. This



might imply either that consumption responds with a lag, for instance if it takes time to get used to a higher income, or it might be an indication of habit formation.

## 3.6 Conclusions

The above analysis is subject to the problems of weak instruments and of the upward bias of the coefficients, since the single consumption equation is estimated by OLS. Issues of aggregation have been avoided by assuming a representative agent throughout. Even if there were Greek data at the micro level running many years, which is unfortunately not the case, aggregating across heterogeneous consumers would require non-trivial restrictions on their information sets to ensure that no information is lost from the individual to the aggregate level.

The rationale for talking interchangeably about liquidity constrained consumers and consumers with a precautionary saving motive is as follows. Caballero (1989) and Weil (1993) have shown that the PIH is compatible with a precautionary saving motive, resulting in consumption being equal to permanent income minus a term induced by precautionary saving. Carroll (2001) shows that there is an observational equivalence between financially constrained consumers without a precautionary saving motive on the one hand, and impatient consumers with a precautionary saving motive on the other. The intuition is that both types of consumers want to save in advance, either because they are afraid they will hit a constraint in the future or because they are afraid they might become unemployed. Hence, precautionary saving considerations are relevant in the context of both the permanent income and the Keynesian hypothesis.

A bivariate VAR on income and saving rejects both the null of the permanent income hypothesis as well as the alternative null of excess sensitivity of consumption to current income with a constant marginal propensity to consume. However, the orthogonality restrictions are also rejected. To identify possible reasons for the failure of the PIH, single equation tests are performed. It is shown that the marginal propensity to consume out of transitory income is not dramatically affected by the inclusion of the overall unemployment rate as a proxy for liquidity constraints. Furthermore, the failure of the PIH is not due to liquidity constraints as captured by the amount of new loans. The tests suggest that the failure of the PIH is probably due to liquidity constraints in the form of the spread between the private borrowing and lending interest rates, which makes it harder for agents to borrow against their future labor income. But when consumers use their non-human wealth to buffer their labor-income shocks, these liquidity constraints do not influence their consumption behavior anymore. With a slight abuse of phraseology it might then be said that Greek consumers behave like permanent income consumers out of their total (human plus non-human) disposable income.



# Appendix A

## Appendix to Chapter 1

**Lemma 2.** *Let preferences be described by:*

$$J_t = \{(1 - e^{-\beta\Delta t})(c_t^{1-\psi} n_t^\psi)^{1-1/\theta} + e^{-\beta\Delta t}(E_t[J_{t+\Delta t}^{1-\gamma}])^{\frac{1-1/\theta}{1-\gamma}}\}^{\frac{1}{1-1/\theta}},$$

where  $c$  is consumption and  $n$  is leisure. Then, given the processes for  $c$  and  $n$ , the utility process is defined as the solution to the following integral equation:

$$U_t = E_t \int_t^\infty z(c_s, U_s) ds, \quad (\text{A.1})$$

where

$$z(c, U) \equiv \frac{\beta}{1 - 1/\theta} \left[ \frac{(c_t^{1-\psi} n_t^\psi)^{1-1/\theta}}{((1-\gamma)U)^{\frac{-1/\theta+\gamma}{1-\gamma}}} - (1-\gamma)U \right]. \quad (\text{A.2})$$

**Proof of Lemma 2.** Define the functions

$$g(x) = \frac{((1-\gamma)x)^{\frac{1-1/\theta}{1-\gamma}}}{1 - 1/\theta}$$

$$U_t = \frac{J_t^{1-\gamma}}{1-\gamma}.$$

Then:

$$g(U_t) = \frac{J_t^{1-1/\theta}}{1 - 1/\theta} = (1 - e^{-\beta\Delta t}) \frac{(c_t^{1-\psi} n_t^\psi)^{1-1/\theta}}{1 - 1/\theta} + e^{-\beta\Delta t} g(E_t[U_{t+\Delta t}]).$$

Take a first order Taylor expansion in  $\Delta t$  :

$$g(U_t) = g(U_t) + \beta \frac{(c_t^{1-\psi} n_t^\psi)^{1-1/\theta}}{1 - 1/\theta} \Delta t - \beta g(U_t) \Delta t + g'(U_t) E_t[\Delta U_t].$$

Then

$$E_t[\Delta U_t] = -\frac{\beta \frac{(c_t^{1-\psi} n_t^\psi)^{1-1/\theta}}{1-1/\theta} - \beta g(U_t)}{g'(U_t)} \Delta t,$$

where

$$\frac{g(U_t)}{g'(U_t)} = \frac{(1-\gamma)U_t}{(1-1/\theta)}.$$

Hence

$$E_t[\Delta U_t] = -z(c_t, n_t, U_t) \Delta t,$$

where

$$z(c_t, n_t, U_t) \equiv \frac{\beta}{1-1/\theta} \left[ \frac{(c_t^{1-\psi} n_t^\psi)^{1-1/\theta}}{((1-\gamma)U_t)^{\frac{-1/\theta+\gamma}{1-\gamma}}} - (1-\gamma)U_t \right].$$

For a more general proof of the above and for a proof of existence and uniqueness of the solution to the integral equation (A.1) see Duffie and Epstein (1992).

**Proof of Proposition 7.** Because of the CRRA/CEIS specification of preferences, guess that the value function for an entrepreneur is

$$J(w^E, t) = B_t^E \frac{w^{E 1-\gamma}}{1-\gamma},$$

where the term  $B_t^E$  captures the time dimension. The Bellman equation for an entrepreneur is

$$\begin{aligned} 0 = & \max_{c^E, n^E, \phi} z(c^E, n^E, J^E(w^E, t)) + \frac{\partial J^E}{\partial w^E}(w^E, t) [(\phi(1-\tau_t^K)r_t + (1-\phi)(1-\tau_t^K)R_t)w^E - c^E - (1-\tau_t^L)\omega_t n^E] \\ & + \frac{\partial J^E}{\partial t}(w^E, t) + \frac{1}{2} \frac{\partial^2 J^E}{\partial w^E 2}(w^E, t) \sigma^2 (1-\tau_t^K)^2 \phi^2 w^2 + p_{EL} [J(w^L, t) - J(w^E, t)], \end{aligned}$$

where the function  $z$  is given by (A.2), and where the last term shows that the entrepreneur might switch into being a worker with probability  $p_{EL}$ . Because of the homogeneity of  $J^E$  in  $w^E$ , the marginal propensity to consume and the portfolio choice will be the same for all entrepreneurs. The first order condition for the optimal portfolio allocation gives the condition for  $\phi_t$  in (1.16). Combining the first order conditions for consumption and leisure we get the optimal leisure choice:

$$n_t^i = (\psi/(1-\psi))(1/(1-\tau_t^L)\omega_t)/c_t^i. \quad (\text{A.3})$$

From the envelope condition we get

$$m^E \equiv B^{E \frac{1-\theta}{1-\gamma}} \left( \frac{\psi}{1-\psi} \frac{1}{(1-\tau^L)\omega} \right)^{-\psi(1-\theta)} (1-\psi)^\theta \beta^\theta.$$

Similarly, guess that the value function for a laborer is

$$J(w^L, t) = B_t^L \frac{w^{L^{1-\gamma}}}{1-\gamma},$$

The Bellman equation for a laborer is

$$\begin{aligned} 0 = \max_{c^L, n^L} & z(c^L, n^L, J^L(w^L, t)) + \frac{\partial J^L}{\partial w^L}(w^L, t)[R_t w^L - c^L - (1 - \tau_t^L)\omega_t n^L] \\ & + \frac{\partial J^L}{\partial t}(w^L, t) + p_{LE}[J(w^E, t) - J(w^L, t)]. \end{aligned}$$

The consumption and leisure choices for the laborer are made in the exact same way as for the entrepreneur, and from the envelope condition we get:

$$m^L \equiv B_t^{L \frac{1-\theta}{1-\gamma}} \left( \frac{\psi}{1-\psi} \frac{1}{(1-\tau^L)\omega} \right)^{-\psi(1-\theta)} (1-\psi)^\theta \beta^\theta.$$

It follows that

$$\frac{B^E}{B^L} = \left( \frac{m^E}{m^L} \right)^{\frac{1-\gamma}{1-\theta}}.$$

Using this, the first order conditions, the envelope conditions, and plugging back into the Bellman equation we get (1.18) and (1.19).

**Proof of Proposition 2.** Let  $\tilde{R}_t$  be the effective risk-free rate. The human wealth for each individual  $i = E, L$  in the economy is  $h_t^i = \int_t^\infty e^{-\int_t^s \tilde{R}_j dj} ((1 - \tau_s^L)\omega_s + T_s) ds$ . The human wealth of the measure- $\chi_t$  group of entrepreneurs is  $H_t^E = \chi_t \int_t^\infty e^{-\int_t^s \tilde{R}_j dj} ((1 - \tau_s^L)\omega_s + T_s) ds$ , and the human wealth of the measure- $(1 - \chi_t)$  group of laborers is  $H_t^L = (1 - \chi_t) \int_t^\infty e^{-\int_t^s \tilde{R}_j dj} ((1 - \tau_s^L)\omega_s + T_s) ds$ . Hence total human wealth is  $H_t = H_t^E + H_t^L = \int_t^\infty e^{-\int_t^s \tilde{R}_j dj} ((1 - \tau_s^L)\omega_s + T_s) ds = h_t^i$ . Using the Leibniz rule, and substituting in from the government budget constraint (1.7), we get that the evolution of total human wealth is described by (1.23). Since only entrepreneurs invest in capital, the aggregate capital stock in the economy is given by  $K_t = \phi_t W_t^E$ . For an agent in the E and L group respectively  $b_t^E + h_t^E = (1 - \phi_t)w_t^E$  and  $b_t^L + h_t^L = w_t^L$ . Aggregating over each group we get  $B_t^E + \chi_t H_t = (1 - \phi_t)W_t^E$  and  $B_t^L + (1 - \chi_t)H_t = W_t^L$ . Adding up and using the fact that  $B_t^E + B_t^L = 0$  we get  $H_t = (1 - \phi_t)W_t^E + W_t^L$ . Now using  $W_t = W_t^E + W_t^L$  and  $K_t = \phi_t W_t^E$  we get  $W_t = K_t + H_t$ . Combining  $H_t = (1 - \phi_t)W_t^E + W_t^L$ ,  $K_t = \phi_t W_t^E$ , and  $\lambda_t = W_t^E/W_t$  we get (1.24). Aggregating across leisure choices we get  $\frac{\psi}{1-\psi} \frac{1}{(1-\tau_t^L)\omega_t} C_t + L_t = 1$ , where  $C_t = m_t^E W_t^E + m_t^L W_t^L$ ,  $W_t^L = W_t - W_t^E$ , and  $W_t = K_t + H_t$ . Aggregating across (1.12) and (1.13), and adding up, using  $B_t^E + B_t^L = 0$ ,  $H_t = H_t^E + H_t^L$ , and labor market clearing, we get:

$$W_t = [(1 - \tau_t^K)r_t K_t + (1 - \tau_t^K)R_t H_t - \frac{1}{1-\psi} C_t] dt.$$

Using  $H_t = (1 - \phi_t)W_t^E + W_t^L$ ,  $K_t = \phi_t W_t^E$ ,  $\mu_t = (1 - \tau_t^K)r_t - (1 - \tau_t^K)R_t$ , and  $C_t = m_t^E W_t^E + m_t^L W_t^L$ , and dividing through with  $W_t$  we get

$$\frac{\dot{W}_t}{W_t} = (1 - \tau_t^K)r_t \phi_t \mu_t + (1 - \tau_t^K)R_t - \frac{1}{1 - \psi}(\lambda_t m_t^E + (1 - \lambda_t)m_t^L),$$

which gives (1.21) when we use  $\rho_t = \phi_t \mu_t + (1 - \tau_t^K)R_t$ . Aggregating across (1.12), and subtracting from (1.21), we get (1.22).

**Proof of Proposition 3.** The proof starts for the case of exogenous labor and  $\theta = 1$ . In steady state  $\dot{W}/W = 0$ , which from (1.21) yields  $\lambda = (\beta - (1 - \tau^K)R)/(\phi\mu)$ . Combining this with (1.22) in steady state gives (1.25), which verifies that  $\lambda < 1$ . Differentiating with respect to  $R$  we get that  $\lambda'(R) < 0$ . Plugging this back into  $\dot{W}/W = 0$  we get

$$\phi\mu = \frac{(\beta - (1 - \tau^K)R)(\beta - (1 - \tau^K)R + p_{LE} + p_{EL})}{(\beta - (1 - \tau^K)R + p_{LE})},$$

from which we get (1.26) if we use the definition of  $\mu$ . Differentiating this with respect to  $R$  we get that  $\mu'(R) < 0$  and  $\phi'(R) < 0$  in steady state. Finally, combining  $\dot{H} = 0$  from (1.23) and bond market clearing (1.24) we get (1.27).

For uniqueness notice first that we can write (1.26) as

$$\mu(R) = \sqrt{\frac{1}{\lambda(R)}\gamma\sigma^2(1 - \tau^K)^2(\beta - (1 - \tau^K)R)} \quad ,$$

from which we get

$$K(R) = \left(\frac{\mu(R) + (1 - \tau^K)R}{\alpha(1 - \tau^K)}\right)^{\frac{1}{\alpha-1}}.$$

Define the ratio of the net foreign asset position to domestic capital for an open economy that faces an exogenously given interest rate:

$$D(R) \equiv \frac{(1 - g\tau^L)(1 - \alpha)K(R)^{\alpha-1} + (1 - g)\tau^K f_K(R)}{((1 - \tau^K)R + v)} - \frac{1}{\phi(R)\lambda(R)} + 1.$$

Assume  $\tau^K \simeq 0$  for simplicity. For existence and uniqueness of the steady state for the closed economy it suffices to prove that there exists a unique  $R$  solving  $D(R) = 0$ , where  $R \in (-v, \beta)$ . From (1.25) we have  $\lambda(-v) = (\beta + v + p_{LE})/(\beta + v + p_{LE} + p_{EL})$  finite positive, hence so is  $\mu(-v)$ ,  $\phi(-v)$ , and also  $K(-v)$  as long as  $\mu(-v) \geq v$ . Next,  $\lambda(\beta)$  is finite positive, so  $\mu(\beta) = 0$ , clearly  $K(\beta)$  is the capital stock under complete markets, which is finite and positive, and  $f_K(\beta) = R = \beta$  when markets are complete. Finally, note that  $1 - g\tau^L > 0$ . Then

$$\lim_{R \rightarrow -v^+} D(R) = [(1 - g\tau^L)(1 - \alpha)K(-v)^{\alpha-1} + (1 - g)\tau^K f_K(-v)] \lim_{R \rightarrow -v^+} \frac{1}{R + v} - \frac{1}{\phi(-v)\lambda(-v)} + 1 = +\infty.$$

$$\lim_{R \rightarrow \beta^-} D(R) = [(1 - g\tau^L)(1 - \alpha)K(\beta)^{\alpha-1} + (1 - g)\tau^K\beta] \frac{1}{\beta + v} - \frac{1}{\lambda(\beta)} \lim_{R \rightarrow \beta^-} \frac{1}{\phi(R)} + 1 = -\infty.$$

Hence the steady state always exists.

When labor is endogenous and  $\theta = 1$ , then  $m^E = m^L = (1 - \psi)\beta$ , and the proofs above carry through the same way, with  $f_K(K/L)$ , and  $\omega(K/L)$ . So for characterization of the steady state we need to add the labor market clearing condition, and the steady state system will be in  $K, L, R$ . In particular, labor market clearing, combined with  $C = (1 - \psi)\beta W$ ,  $\lambda = W^E/W$  and  $W^E = K/\phi$  gives

$$L = \left( \frac{\psi\beta}{(1 - \tau^L)\omega(K, L)} \frac{1}{\lambda(R)} \frac{K/L}{\phi(K, L, R)} + 1 \right)^{-1}.$$

Finally, when labor is endogenous and  $\theta \neq 1$  then

$$\lambda = \frac{\frac{1}{1-\psi}m^L - (1 - \tau^K)R + p_{LE}}{\frac{1}{1-\psi}m^L - (1 - \tau^K)R + p_{LE} + p_{EL}},$$

and

$$\mu = \sqrt{\frac{\gamma\sigma^2(1 - \tau)^2}{\lambda} \left( \frac{1}{1 - \psi} (m^E\lambda + m^L(1 - \lambda)) - (1 - \tau^K)R \right)}.$$

Here we need to add two more equations to characterize the steady state, namely the Euler conditions for the marginal propensities to consume. This will be a system of two equations in two unknowns to be solved as a function of steady state prices. The conditions needed for establishing that  $\lambda > 0$  are satisfied in simulations.

**Proof of Proposition 4.** Let  $d_t$  be the indicator function, where  $d_t = 1$  for entrepreneurs and  $d_t = 0$  for laborers. The dynamic system for the state vector  $(\xi_t^i, d_t)$  is:

$$\begin{aligned} \dot{\xi}_t^i &= \mu(\xi_t^i, d_t) + \sigma(\xi_t^i, d_t) dz_t^i - (\xi_t^i - 1)N_t^1 \\ \dot{d}_t &= s(d_t) dN_t^2, \end{aligned}$$

where  $dN_t^1$  is the Poisson process denoting death with arrival rate  $vdt$ , and where  $dN_t^2$  is the Poisson switching process with arrival intensity  $p(I) dt$ :

$$\begin{aligned} p(d) &= p_{LE} \quad \text{if } d = 0 \\ p(d) &= p_{EL} \quad \text{if } d = 1, \end{aligned}$$

where:

$$\begin{aligned} s(d) &= 1 \quad \text{if } d = 0 \\ s(d) &= -1 \quad \text{if } d = 1, \end{aligned}$$

and:

$$\begin{aligned}
\mu(\xi_t, 1) &= \left[ \frac{1}{1-\psi}(\bar{m}_t - m_t^E) + \phi_t(1-\lambda_t)((1-\tau_t^K)r_t - (1-\tau_t^K)R_t) \right] \xi_t \\
\mu(\xi_t, 0) &= \left[ \frac{1}{1-\psi}(\bar{m}_t - m_t^L) - \lambda_t \phi_t((1-\tau_t^K)r_t - (1-\tau_t^K)R_t) \right] \xi_t \\
\sigma(\xi_t, 1) &= \phi_t \sigma_t (1 - \tau_t^K) \xi_t \\
\sigma(\xi_t, 0) &= 0.
\end{aligned}$$

Let  $\Phi_E \equiv \Phi(\xi, 1)$  and  $\Phi_L \equiv \Phi(\xi, 0)$  be the conditional distributions for entrepreneurs and laborers respectively. Let the newborn household get a weighted average  $aW_t + (1-a)w_t^i$ , where  $0 < a < 1$ , upon birth. In steady state the conditional distribution  $\Phi_L$  satisfies the forward Kolmogorov equation:

$$0 = -\frac{\partial(\mu(\xi, 0)\Phi_L)}{\partial\xi} - p(0)\Phi_L + (p\Phi_L)(\xi, 0 - \eta(0)) - v\Phi_L + \frac{v}{1-a}\Phi_L\left(\frac{\xi-a}{1-a}\right),$$

and the conditional distribution  $\Phi_E$  satisfies the forward Kolmogorov equation:

$$0 = \frac{1}{2}\frac{\partial^2(\sigma(\xi, d)^2\Phi_E)}{\partial\xi^2} - \frac{\partial(\mu(\xi, 1)\Phi_E)}{\partial\xi} - p(1)\Phi_E + (p\Phi_E)(\xi, 1 - \eta(1)) - v\Phi_E + \frac{v}{1-a}\Phi_E\left(\frac{\xi-a}{1-a}\right).$$

In the two equations above we need to calculate:

$$(p\Phi)(\xi, d - \eta(d)) = p(d - \eta(d))\Phi(\xi, d - \eta(d)).$$

To that end, let the old state be  $d$ , and the new state be  $d'$ . They are related through  $d' = d + s(d)$ , and we need to compute  $\eta(d') = s(d)$ . For  $d = 0$ , we have  $d' = 0 + s(0) = 0 + 1 = 1$ , and  $\eta(d') = \eta(1) = s(0) = 1$ , hence  $\eta(1) = 1$ . For  $d = 1$ , we have  $d' = 1 + s(1) = 1 - 1 = 0$ , and  $\eta(d') = \eta(0) = s(1) = -1$ , hence  $\eta(0) = -1$ . Therefore:

$$p(0 - \eta(0))\Phi(\xi, 0 - \eta(0)) = p(1)\Phi(\xi, 1) = p_{EL}\Phi_E,$$

and:

$$p(1 - \eta(1))\Phi(\xi, 1 - \eta(1)) = p(0)\Phi(\xi, 0) = p_{LE}\Phi_L.$$

Substituting for  $\mu(\xi_t, 0)$ ,  $\mu(\xi_t, 1)$ ,  $\sigma(\xi_t, 0)$ ,  $\sigma(\xi_t, 1)$  and using the above, we can write the system of the two Kolmogorov equations as:

$$\begin{aligned}
0 &= c_1 \xi^2 \frac{\partial^2 \Phi_E}{\partial \xi^2} + c_2 \xi \frac{\partial \Phi_E}{\partial \xi} + c_3 \Phi_E + p_{LE} \Phi_L + \frac{v}{1-a} \Phi_E \left( \frac{\xi-a}{1-a} \right) \\
0 &= c_4 \xi \frac{\partial \Phi_L}{\partial \xi} + c_5 \Phi_L + p_{EL} \Phi_E + \frac{v}{1-a} \Phi_L \left( \frac{\xi-a}{1-a} \right),
\end{aligned}$$



where:

$$\begin{aligned}
c_1 &= \frac{\phi^2 \sigma^2 (1 - \tau)^2}{2} \\
c_2 &= 2\phi^2 \sigma^2 (1 - \tau)^2 - \left[ \frac{1}{1 - \psi} (\bar{m} - m^E) + \phi \mu (1 - \lambda) \right] \\
c_3 &= \phi^2 \sigma^2 (1 - \tau)^2 - \left[ \frac{1}{1 - \psi} (\bar{m} - m^E) + \phi \mu (1 - \lambda) \right] - p_{EL} - v \\
c_4 &= \lambda \phi \mu - \frac{1}{1 - \psi} (\bar{m} - m^L) \\
c_5 &= \lambda \phi \mu - \frac{1}{1 - \psi} (\bar{m} - m^L) - p_{LE} - v
\end{aligned}$$

Now, the Laplace transform for any variable  $y$  is defined as:

$$Y(s) = \int_0^{\infty} e^{-st} y(t) dt.$$

and therefore:

$$Y'(s) = - \int_0^{\infty} e^{-st} t y(t) dt = -L[ty(t)] \Rightarrow L[ty] = -\frac{d}{ds} Y(s).$$

and:

$$Y'(s) = - \int_0^{\infty} e^{-st} t y(t) dt \Rightarrow Y''(s) = \int_0^{\infty} e^{-st} t^2 y(t) dt = L[t^2 y(t)].$$

Hence we find that:

$$L[ty'] = \int_0^{\infty} e^{-st} t y' dt = -sY'(s) - Y(s),$$

and:

$$L[t^2 y''] = \int_0^{\infty} e^{-st} t^2 y'' dt = s^2 Y''(s) + 4sY'(s) + 2Y(s).$$

Let  $c = \frac{1}{1-a}$ ,  $k = \frac{a}{1-a}$ , and  $\tau = ct - k$ , then  $d\tau = c dt$  and  $t = \frac{\tau+k}{c}$ . So we have:

$$L\left[y\left(\frac{t-a}{1-a}\right)\right] = \int_{k/c}^{\infty} e^{-st} y(ct - k) dt = \frac{1}{c} e^{-s \frac{k}{c}} L[y(t)]_{s \rightarrow s/c} = (1-a) e^{-k(1-a)} L[y(t)]_{s \rightarrow s(1-a)}.$$

Hence when  $a = 1$ :

$$L\left[y\left(\frac{t-a}{1-a}\right)\right] = (1-1) e^{-k(1-1)} \int_0^{\infty} y(t) dt = 0 \cdot 1 \cdot 1,$$

if  $y$  is a probability density function. Hence the last term in both Kolmogorov equations will drop out when  $a = 1$ .

After changing variables to  $\xi = e^x$ , and defining  $\partial\Phi_E/\partial x \equiv \Phi_2$  we get:

$$\begin{pmatrix} \Phi'_L \\ \Phi'_E \\ \Phi'_2 \end{pmatrix} = \begin{pmatrix} 0 & c_5/c_4 - 1 & p_{EL}/c_4 \\ 1 & 0 & 0 \\ c_2/c_1 - 3c_1 & -p_{LE}/c_1 & -2 + c_2/c_1 - c_3/c_1 \end{pmatrix} \begin{pmatrix} \Phi_L \\ \Phi_E \\ \Phi_2 \end{pmatrix}$$

Since all coefficients are constant, and  $\xi$  is bounded, a Lipschitz condition is satisfied, hence the solution to the system exists and is unique. The conditional densities can be recovered by inverting the Laplace transforms.

**Proof of Proposition 5.** Consider the open economy, i.e.  $R_t = R$  for all  $t$ , with entrepreneurs only, so that  $\lambda = 1$ , and assume that  $v = 0$ . Then the equations describing the evolution of the marginal propensity to consume and of total effective wealth ((1.18) and (1.21)) become:

$$\frac{\dot{W}_t}{W_t} = \rho_t - m_t = \phi_t \mu_t + (1 - \tau_t^K)R - m_t \quad (\text{A.4})$$

$$\frac{\dot{m}_t}{m_t} = m_t - \theta\beta + (\theta - 1)\hat{\rho}_t \quad (\text{A.5})$$

In steady state, equation (A.4), using  $\hat{\rho}_t = 1/2 \phi_t \mu_t + (1 - \tau_t^K)R$ , gives:

$$m_t = \theta\beta - (\theta - 1)\left(\frac{1}{2}\phi_t \mu_t + (1 - \tau_t^K)R\right)$$

Plugging this into the steady state version of (A.5), we get:

$$0 = \frac{\theta + 1}{2}\phi_t \mu_t + \theta(1 - \tau_K)R - \theta\beta$$

where  $\phi\mu = (F_{K_t} - \delta - R)^2/\gamma\sigma^2$ . Therefore, an increase in  $\tau_t^K$  reduces the second term on the right-hand-side. Since the interest rate is constant, the only way stationarity can be restored is if  $K_t$  falls (this is the only way  $\phi_t \mu_t$  can increase). Hence, at the new steady state of the open economy with a higher capital-income tax, steady-state capital is unambiguously lower.

**Proof of Proposition 6.** Take the open economy with  $\lambda = 1$  and  $v = 0$ . Then, using (A.4) and (A.5) in steady state, using the definition of  $\hat{\rho}$ , and taking the total differential with respect to  $K$  and  $R$  gives:

$$\frac{\partial K}{\partial R} = \frac{\phi - \theta(1 - \phi)}{\phi(\theta + 1)} \frac{1}{F_{KK}},$$

which proves that:

$$\frac{\partial K}{\partial R} > 0 \Leftrightarrow \theta > \frac{\phi}{1 - \phi}.$$

**Lemma 3.** *The steady state measure of entrepreneurs is given by  $p_{LE}/(p_{LE} + p_{EL})$ .*

**Proof of Lemma 3.** Call  $\chi$  the measure of entrepreneurs today, and  $\chi'$  their measure tomorrow. Then  $\chi' = \chi(1 - p_{EL}) + (1 - \chi)p_{LE}$ . But in steady state  $\chi = \chi'$  hence  $\chi = p_{LE}/(p_{LE} + p_{EL})$ .



# Appendix B

## Appendix to Chapter 2

**Proof of Proposition 7.** Let  $J(w, t)$  denote the value function for the household's problem. The value function depends on time  $t$  because of discounting as well as because the price sequence  $\{\omega_t, R_t\}_{t \in [0, \infty)}$  need not be stationary. However, the value function does not depend on  $i$ , because households have identical preferences, they have access to the same technology, and they face the same sequence of prices and the same stochastic process for idiosyncratic risk. The Bellman equation that characterizes the value function is given by

$$0 = \max_{m, \phi} \left\{ z(mw, J(w, t)) + \frac{\partial J}{\partial t}(w, t) + \frac{\partial J}{\partial w}(w, t)[\phi \bar{r}_t + (1 - \phi)R_t - m]w + \frac{1}{2} \frac{\partial^2 J}{\partial w^2}(w, t) \phi^2 w^2 \sigma^2 \right\}. \quad (\text{B.1})$$

The first term of the Bellman equation (B.1) captures utility from current-period consumption; the second term takes care of discounting and the non-stationarity in prices; the third term captures the impact of the mean growth in wealth; and the last term (Itô's term) captures the impact of risk.

Because of the CRRA/CEIS specification of preferences, an educated guess is that there exists a deterministic process  $B_t$  such that

$$J(w, t) = B_t \frac{w^{1-\gamma}}{1-\gamma}. \quad (\text{B.2})$$

Because of the homogeneity of  $J$  in  $w$ , the Bellman equation then reduces to

$$0 = \max_{m, \phi} \left\{ z(m, J(1, t)) + \frac{\partial J}{\partial t}(1, t) + \frac{\partial J}{\partial w}(1, t)[\phi \bar{r}_t + (1 - \phi)R_t - m] + \frac{1}{2} \frac{\partial^2 J}{\partial w^2}(1, t) \phi^2 \sigma^2 \right\}, \quad (\text{B.3})$$

so that the optimal  $m$  and  $\phi$  are independent of  $w$ . Using (A.2) and (B.2), the above becomes

$$0 = \max_{m, \phi} \left\{ \frac{\beta}{1 - 1/\theta} [B_t^{\frac{1/\theta - 1}{1-\gamma}} m^{1-1/\theta} - 1] + \frac{\dot{B}_t/B_t}{1-\gamma} + [\phi \bar{r}_t + (1 - \phi)R_t - m] - \frac{1}{2} \gamma \phi^2 \sigma^2 \right\}. \quad (\text{B.4})$$

The first order condition for  $\phi$  gives

$$\phi_t = \frac{\bar{r}_t - R_t}{\gamma\sigma^2}, \quad (\text{B.5})$$

while the the first order condition for  $m$  gives

$$m_t = \beta^\theta B_t^{\frac{1-\theta}{1-\gamma}}. \quad (\text{B.6})$$

Substituting this into (B.4), using the definition of  $\hat{\rho}_t$ , and rearranging, we get

$$0 = \frac{\beta^\theta B_t^{\frac{1-\theta}{1-\gamma}} - \theta\beta}{\theta - 1} + \frac{\dot{B}_t/B_t}{1 - \gamma} + \hat{\rho}_t.$$

This ODE, together with the relevant transversality condition, determines the process for  $B_t$ . Using (B.6), this is equivalent to

$$\frac{\dot{m}_t}{m_t} = m_t + (\theta - 1)\hat{\rho}_t - \theta\beta,$$

which is the Euler condition (13).

**Proof of Proposition 8.** Since aggregate labor demand is  $\int_i n_t^i = \bar{n}(\omega_t)K_t$  and aggregate labor supply is 1, the labor market clears if and only if  $\bar{n}(\omega_t)K_t = 1$ . It follows that the equilibrium wage satisfies  $\omega_t = F_L(K_t, 1)$  and, similarly, the equilibrium mean return to capital satisfies  $\bar{r}_t = F_K(K_t, 1) - \delta$ . The bond market, on the other hand, clears if and only if  $0 = (1 - \phi_t)W_t + H_t$ . Combining this with  $K_t = \phi_t W_t$  gives condition (17).

Combining the intertemporal government budget with the definition of human wealth, we get

$$H_t = h_t = \int_t^\infty e^{-\int_t^s R_j dj} (\omega_s - G_s) ds. \quad (\text{B.7})$$

Expressing this in recursive form gives condition (16).

Combining the latter with  $K_t + H_t = W_t$  and  $\dot{W}_t = \rho W_t - C_t$ , we have  $\dot{K}_t = \dot{W}_t - \dot{H}_t = (\rho_t W_t - C_t) - (R_t H_t - \omega_t + G_t)$ . Using  $\rho_t W_t = \bar{r}_t \phi_t W_t + R_t (1 - \phi_t) W_t = \bar{r}_t K_t + R_t H_t$ , we get  $\dot{K}_t = \bar{r}_t K_t + \omega_t - C_t - G_t$ . Together with the fact, in equilibrium,  $\bar{r}_t K_t + \omega_t = F(K_t, 1) - \delta K_t$ , this gives condition (14), the resource constraint.

Finally, using  $C_t = m_t W_t$ , and therefore  $\dot{C}_t/C_t = \dot{m}_t/m_t + \dot{W}_t/W_t$  together with  $\dot{W}_t = \rho_t W_t - C_t = (\rho_t - m_t) W_t$  and (2.13), gives condition (15), the aggregate Euler condition.

**Proof of Proposition 9.** First, we derive the two equations characterizing the steady state  $K$  and  $R$ . In steady state, the Euler condition gives

$$0 = \theta(\rho - \beta) - (\theta - 1)\frac{1}{2}\gamma\sigma^2\phi^2,$$

where

$$\rho = R + \frac{[f'(K) - \delta - R]^2}{\gamma\sigma^2} \quad \text{and} \quad \phi = \frac{f'(K) - \delta - R}{\gamma\sigma^2}.$$

Combining and solving for  $f'(K)$  gives condition (2.18). Condition (2.19), on the other hand, follows directly from (16) and (17).

Next, we prove existence and uniqueness of the steady state. Let  $\mu(R)$  and  $\phi(R)$  denote, respectively, the risk premium and the fraction of effective wealth held in capital, when  $K$  is given by (2.18):

$$\mu(R) \equiv \sqrt{\frac{2\theta\gamma\sigma^2}{1+\theta}}(\beta - R) \quad \text{and} \quad \phi(R) \equiv \sqrt{\frac{2\theta}{\gamma\sigma^2(1+\theta)}}(\beta - R).$$

Note that  $\mu'(R) < 0$  and  $\phi'(R) < 0$ . Next, let  $K(R)$  denote the solution to (2.18), or equivalently

$$K(R) = \left[ \frac{\mu(R) + \delta + R}{\alpha} \right]^{\frac{1}{\alpha-1}}. \quad (\text{B.8})$$

Finally, let

$$D(R; g) \equiv (1 - \alpha - g) \frac{K(R)^{\alpha-1}}{R} - \frac{1 - \phi(R)}{\phi(R)}. \quad (\text{B.9})$$

This represents the ratio of the net foreign asset position to domestic capital of an open economy that faces an exogenous interest rate  $R \in (0, \beta)$ . (Note that we have used  $\omega = (1 - \alpha)Y$ ,  $G = gY$ , and  $Y = f(K) = K^\alpha$ , where  $\alpha > 0$ ,  $g \geq 0$ , and  $\alpha + g < 1$ .) To establish existence and uniqueness of the steady state (for the *closed* economy), it suffices to show that there exists a unique  $R$  that solves  $D(R; g) = 0$ .

Fix  $g$  henceforth, and consider the limits of  $D$  as  $R \rightarrow 0^+$  and  $R \rightarrow \beta^-$ . Note that  $\mu(0) = (\frac{2\theta\gamma\sigma^2}{1+\theta}\beta)^{1/2}$  is finite and hence both  $\phi(0)$  and  $K(0)$  are finite. It follows that

$$\lim_{R \rightarrow 0^+} D(R; g) = (1 - \alpha - g)K(0)^{\alpha-1} \lim_{R \rightarrow 0^+} \frac{1}{R} - \frac{1}{\phi(0)} + 1 = +\infty.$$

Furthermore,  $\mu(\beta) = 0$ , implying  $\phi(\beta) = 0$  and  $K(\beta) = K_{\text{compl}} \equiv (f')^{-1}(\beta)$  is finite. It follows

$$\lim_{R \rightarrow \beta^-} D(R; g) = (1 - \alpha - g)K(\beta)^{\alpha-1} \frac{1}{\beta} - \lim_{R \rightarrow \beta^-} \frac{1}{\phi(R)} + 1 = -\infty.$$

These properties, together with the continuity of  $D(R)$  in  $R$ , ensure the existence of an  $R \in (0, \beta)$  such that  $D(R) = 0$ .

If we now show that  $D(R; g)$  is strictly decreasing in  $R$ , then we also have uniqueness. To show this, note that from (B.9),

$$\frac{\partial D}{\partial R} = (1 - \alpha - g) \frac{K(R)^{\alpha-1}}{R^2} \left[ (\alpha - 1)R \frac{K'(R)}{K(R)} - 1 \right] + \frac{\phi'(R)}{\phi(R)^2}. \quad (\text{B.10})$$

Now note that

$$K^{\alpha-1} = \frac{f'(K)}{\alpha}, \quad \frac{K'}{K} = \frac{1}{\alpha-1} \frac{\mu' + 1}{f'(K)}, \quad \text{and} \quad \frac{\phi'}{\phi^2} = \frac{\gamma\sigma^2\mu'}{\mu^2},$$

where we suppress the dependence of  $K$ ,  $\mu$ , and  $\phi$  on  $R$  for notational simplicity. It follows that

$$\begin{aligned} \frac{\partial D}{\partial R} &= \frac{1-\alpha-g}{\alpha} \frac{f'(K)}{R^2} \left[ R \frac{\mu'+1}{f'(K)} - 1 \right] + \frac{\gamma\sigma^2\mu'}{\mu^2} = \\ &= \frac{1-\alpha-g}{\alpha} \frac{R\mu'+R-f'(K)}{R^2} + \frac{\gamma\sigma^2\mu'}{\mu^2}. \end{aligned}$$

Since  $\mu'(R) < 0$  and  $R < f'(K(R))$  for all  $R \in (0, \beta)$ , we have that  $\partial D/\partial R < 0$  for all  $R \in (0, \beta)$ , which completes the argument.

**Proof of Lemma I.** Recall that (2.18) is equivalent to

$$\theta(\rho - \beta) - (\theta - 1)\gamma\phi^2\sigma^2 = 0,$$

where  $\rho = \phi f'(K) + (1 - \phi)R$  and  $\phi = (f'(K) - R)/\gamma\sigma^2$ . Applying the implicit function theorem, we get

$$\left. \frac{\partial K}{\partial R} \right|_{(2.18)} = \frac{\phi - \theta(1 - \phi)}{\phi(\theta + 1)} \frac{1}{f''},$$

which proves that:

$$\frac{\partial K_1}{\partial R} < 0 \Leftrightarrow \theta < \frac{\phi}{1 - \phi}.$$

On the other hand, from (2.19), using  $\phi = (f'(K) - R)/\gamma\sigma^2$ ,  $f'(K) = \alpha K^{\alpha-1}$  and  $\omega - G = (1 - \alpha - g)K^\alpha$ , we get

$$\left. \frac{\partial K}{\partial R} \right|_{(2.19)} = -\frac{(1-\alpha)\frac{K^{\alpha-1}}{R^2} + \frac{\gamma\sigma^2}{\mu^2}}{\frac{(1-\alpha)^2}{R}K^{\alpha-2} + \frac{\gamma\sigma^2}{\mu^2}(1-\alpha)\frac{f'}{K}} < 0,$$

which proves that  $\partial K_2/\partial R < 0$  always.

**Proof of Proposition 10.** From (B.9), we have that  $\partial D/\partial g < 0$ . Together with the property that  $\partial D/\partial R < 0$ , this implies that the steady-state  $R$  necessarily decreases with  $g$ . The impact of  $g$  on the steady-state  $K$  then follows from the fact that  $K_1(R)$ , defined by (2.18), does not depend on  $g$  and is increasing/decreasing in  $R$  if and only if  $\theta$  is higher/lower than  $\phi/(1 - \phi)$ .



# Appendix C

## Appendix to Chapter 3

### Data Description

Sample period: 1960 – 2004

Sources: Bank of Greece, Ministry of Finance, National Statistical Services

$y_t$  = disposable labor income, real, per capita

$y_t$  = wages + current transfer receipts - direct taxes - current transfer payments

$yt_t$  = disposable labor income, real, per capita

$yt_t$  = wages + non-labor income + current transfer receipts - direct taxes - current transfer payments

$c_t$  = total national private consumption, real, per capita

$s_t = yt_t - c_t$  = national private savings, real, per capita

All series deflated by total population and by a consumer price deflator

Private consumption includes durables, non-durables, and services



# Bibliography

- [1] Ahsan, S. M. (1974), "Taxation in a Two-Period Temporal Model of Consumption and Portfolio Allocation," *Journal of Public Economics* 5, 337-352.
- [2] Ait-Sahalia, Y., Parker, J. A., and M. Yogo (2001), "Luxury Goods and the Equity Premium," NBER, Working Paper 8417.
- [3] Aiyagari, S. R. (1994), "Uninsured Idiosyncratic Risk and Aggregate Saving," *Quarterly Journal of Economics* 109, 659-684.
- [4] Aiyagari, S. R. (1995), "Optimal Capital Income Taxation with Incomplete Markets, Borrowing Constraints, and Constant Discounting," *Journal of Political Economy* 103, 1158-1175.
- [5] Albanesi, S. (2006), "Optimal Taxation of Entrepreneurial Capital with Private Information," NBER Working Paper 12419.
- [6] Altig, D., Auerbach, A. J., Kotlikoff, L. J., Smetters, K. A., and J. Walliser (2001), "Simulating Fundamental Tax Reform in the United States," *American Economic Review* 91, 574-595.
- [7] Angeletos, G.-M. (2007), "Uninsured Idiosyncratic Investment Risk and Aggregate Saving," *Review of Economic Dynamics* 10, 1-30.
- [8] Angeletos, G.-M., and L.-E. Calvet (2006), "Idiosyncratic Production Risk, Growth, and the Business Cycle," *Journal of Monetary Economics* 53, 1095-1115.
- [9] Angeletos, G.-M. , and V. Panousi (2007), "Revisiting the Supply-Side Effects of Government Spending Under Incomplete Markets," NBER Working Paper 13136.
- [10] Angeletos, G.-M. , and I. Werning (2006), "Pecuniary Externality and Efficiency with Uninsurable Investment Risk," MIT mimeo.
- [11] Atkeson, A., Chari, V. V., and P. J. Kehoe (1999), "Taxing Capital Income: A Bad Idea ," *Federal Reserve Bank of Minneapolis Quarterly Review* 23, 3-17.
- [12] Attanasio, O. P., and G. Weber (1993), "Consumption Growth, Interest Rate, and Aggregation," *Review of Economic Studies* 60, 631-649.

- [13] Barro, R. J. (1981), "Output Effects of Government Purchases," *Journal of Political Economy* 89, 1086-1121.
- [14] Barro, R. J. (1989), "The Ricardian Approach to Budget Deficits," *Journal of Economic Perspectives* 3, 37-54.
- [15] Barsky, R. B., Juster, F. T., Kimball, M. S., and M.D. Shapiro (1997), "Preference Parameters and Behavioral Heterogeneity: An Experimental Approach in the Health and Retirement Study," *Quarterly Journal of Economics* 112, 537-579.
- [16] Baxter, M., and R. G. King (1993), "Fiscal Policy in General Equilibrium," *American Economic Review* 83, 315-334.
- [17] Benabou, R. (2002), "Tax and Education Policy in a Heterogeneous-Agent Economy: What Levels of Redistribution Maximize Growth and Efficiency," *Econometrica* 70, 481-517.
- [18] Blanchard, O., and R. Perotti (1999), "An Empirical Characterization of the Dynamic Effects of Changes in Government Spending and Taxes on Output," NBER Working Paper 7269.
- [19] Blundell, R., and T. MaCurdy (1999), "Labour Supply: A Review of Alternative Approaches," in O. Ashenfelter and D. Card (eds.), *Handbook of Labor Economics* 3.
- [20] Buera, F., and T. Shin (1999), "Financial Frictions and the Persistence of History: a Quantitative Exploration," Northwestern University mimeo.
- [21] Caballero, R. (1989), "Consumption Puzzles and Precautionary Saving," *Journal of Monetary Economics* 25, 113-136.
- [22] Cagetti M., and M. De Nardi (2004), "Taxation, Entrepreneurship, and Wealth," Federal Reserve Bank of Minneapolis, Research Department Staff Report 340.
- [23] Cagetti M., and M. De Nardi (2006), "Entrepreneurship, Frictions and Wealth," *Journal of Political Economy* 114, 835-870.
- [24] Campbell, J. Y. (1987), "Does Saving Anticipate Declining Labor Income? An Alternative Test of the Permanent Income Hypothesis," *Econometrica* 55, 1249-1273.
- [25] Campbell, J. Y. (1996), "Understanding Risk and Return," *Journal of Political Economy* 104, 298-345.
- [26] Campbell, J. Y. and A. S. Deaton (1989), "Why is Consumption so Smooth?," *Review of Economic Studies* 56, 357-373.
- [27] Campbell, J. Y. and N. G. Mankiw (1990), "Permanent Income, Current Income, and Consumption," *Journal of Business and Economic Statistics* 8, 265-279.

- [28] Carroll, C. D. (2001), "A Theory of the Consumption Function, With and Without Liquidity Constraints," *NBER Working Paper* July, 8387.
- [29] Campbell, J. Y., Lettau, M., Malkiel, B. G., and Y. Xu (2001), "Have Individual Stocks Become More Volatile? An Empirical Exploration of Idiosyncratic Risk," *Journal of Finance* LVI, 1-41.
- [30] Campbell, J., and N.G. Mankiw (1989), "Permanent Income, Current Income, and Consumption," *NBER Macroeconomics Annual 1989*.
- [31] Card, D. (1990), "Intertemporal Labor Supply: An Assessment," Princeton University mimeo.
- [32] Carroll, C. (2000), "Portfolios of the Rich," NBER Working Paper 430.  
Challe, Edouard, and Xavier Ragot (2007), "The Dynamic Effects of Fiscal Shocks in a Liquidity-Constrained Economy," University of Paris-Dauphine/Paris School of Economics mimeo.
- [33] Chamley C. (1986), "Optimal Taxation of Capital Income in General Equilibrium with Infinite Lives," *Econometrica* 54, 607-622.
- [34] Christiano, L. J., and M. Eichenbaum (1992), "Current Real-Business-Cycle Theories and Aggregate Labor Market Fluctuations," *American Economic Review* 82, 430-450.
- [35] Cohen, A., and L. Einav (2005), "Estimating Risk Preferences from Deductible Choice," NBER Working Paper 11461.
- [36] Covas, F. (2006), "Uninsured Idiosyncratic Production Risk with Borrowing Constraints," *Journal of Economic Dynamics and Control* 30, 2167-2190.
- [37] Davila, J., Hong, J. H., Krusell, P., and J.-V. Ríos-Rull (2007), "Constrained Efficiency in the Neoclassical Growth Model with Uninsurable Idiosyncratic Shocks".
- [38] Deaton, A. S. (1992), *Understanding Consumption*, Oxford Clarendon Press.
- [39] Dohmen, T., Falk, A., Huffman, D., Sunde, U., Schupp, J., and G. G. Wagner (2005), "Individual Risk Attitudes: New Evidence from a Large, Representative, Experimentally-Validated Survey," IZA, Discussion Paper 1730.
- [40] Davis, S. J., Haltiwanger, J., Jarmin, R., and J. Miranda (2006), "Volatility and Dispersion in Business Growth Rates: Publicly Traded versus Privately Held Firms," NBER Working Paper 12354.
- [41] Domar, E. D., and A. M. Musgrave (1944), "Proportional Income Taxation and Risk Taking," *Quarterly Journal of Economics* 58, 388-422.

- [42] Duffie, D., and L. G. Epstein (1992), "Stochastic Differential Utility," *Econometrica* 60, 353-394.
- [43] Erosa, A., and T. Koreshkova (2007), "Progressive Taxation in a Dynastic Model of Human Capital," *Journal of Monetary Economics* 54, 667-685.
- [44] Flavin, M. A. (1985), "Excess Sensitivity of Consumption to Current Income: Liquidity Constraints or Myopia?," *Canadian Journal of Economics* 18, 117-136.
- [45] Flavin, M. A. (1993), "The Excess Smoothness of Consumption: Identification and Interpretation," *Review of Economic Studies* 60, 651-666.
- [46] Gentry, W. M., and R. G. Hubbard (2000), "Entrepreneurship and Household Saving," NBER Working Paper 7894.
- [47] Greenwood, J., Hercowitz, Z., and G. Huffman (1988), "Investment, Capacity Utilization and the Real Business Cycle," *American Economic Review* 78, 402-417.
- [48] Gruber, J. (2005), "A Tax-Based Estimate of the Elasticity of Intertemporal Substitution," NBER Working Paper 11945.
- [49] Guvenen, F. (2006), "Reconciling Conflicting Evidence on the Elasticity of Intertemporal Substitution: A Macroeconomic Perspective," *Journal of Monetary Economics* 53, 1451-1472.
- [50] Hall, R. (1980), "Stabilization Policy and Capital Formation," *American Economic Review* 70, 156-63.
- [51] Hall, R. (1988), "Intertemporal Substitution in Consumption," *Journal of Political Economy* 96, 339-357.
- [52] Hall, R. E. (1988), "Stochastic Implications of the Life Cycle-Permanent Income Hypothesis: Theory and Evidence," *Journal of Political Economy* 86, 971-987.
- [53] Hausman, J. (1981), "Labor Supply: How taxes affect Economic Behavior," in H. Aaron and J. Pechman (eds.), *Tax and the Economy*, Brookings Institute, Washington DC.
- [54] Hausman, J., Stock, J. H., and M. Yogo (2004), "Asymptotic Properties for the Hausman-Hahn for Weak Instruments," January, Working Paper.
- [55] Heathcote, J. (2005), "Fiscal Policy with Heterogeneous Agents and Incomplete Markets," *Review of Economic Studies* 72, 161-188.
- [56] Himmelberg, C. P., Hubbard, R. G., and I. Love (2002), "Investor Protection, Ownership, and the Cost of Capital," Working Paper.
- [57] Huggett, M. (1997), "The One-Sector Growth Model With Idiosyncratic Shocks," *Journal of Monetary Economics* 39, 385-403.

- [58] Jaimovich, N., and S. Rebelo (2006), "Can News About the Future Drive the Business Cycle?," NBER Working Paper 12537.
- [59] Jokisch, S. and L. J. Kotlikoff (2006), "Simulating the Dynamic Macroeconomic and Microeconomic Effects of the Fair Tax".
- [60] Judd, K. L. (1985), "Redistributive Taxation in a Simple Perfect Foresight Model," *Journal of Public Economics* 28, 59-83.
- [61] King, R. G., and S. Rebelo (1990), "Public Policy and Economic Growth: Developing Neoclassical Implications," *Journal of Political Economy* 98, S126-S150.
- [62] King, R. G., Plosser, C., and S. Rebelo (1988), "Production, Growth and Business Cycles: I. the Basic Neoclassical Model," *Journal of Monetary Economics* 21, 195-232.
- [63] Krusell, P., and A.A. Smith (1998), "Income and Wealth Heterogeneity in the Macroeconomy," *Journal of Political Economy* 106, 867-896.
- [64] Li, W. (2002), "Entrepreneurship and Government Subsidies: A General Equilibrium Analysis," *Journal of Economic Dynamics and Control* 26, 1815-1844.
- [65] MaCurdy, T. (1981), "An Empirical Model of Labor Supply in a Life-Cycle Setting," *Journal of Political Economy* 86, 1059-1085.
- [66] Malloy, C. J., Moskowitz, T. J., and A. Vissing-Jørgensen (2006), "Long Run Stockholder Consumption Risk and Asset Returns," mimeo.
- [67] Mankiw, G. N., Rotemberg, J., and L. H. Summers (1985), "Intertemporal Substitution in Macroeconomics," *Quarterly Journal of Economics* 100, 1059-1085.
- [68] Mankiw, G. N., and S. P. Zeldes (1991), "The Consumption of Stockholders and Non-stockholders," *Journal of Financial Economics* 29, 97-112.
- [69] McGrattan, E., and L. E. Ohanian (1999), "The Macroeconomic Effects of Big Fiscal Shocks: The Case of World War II," Federal Reserve Bank of Minneapolis Working Paper 599.
- [70] McGrattan, E., and L. E. Ohanian (2006), "Does Neoclassical Theory Account for the Effects of Big Fiscal Shocks: Evidence from World War II," Federal Reserve Bank of Minneapolis Research Department Staff Report 315.
- [71] Meh, C.A. (2005), "Entrepreneurship, Wealth Inequality, and Taxation," *Review of Economic Dynamics* 8, 688-719.
- [72] Meh, C., and V. Quadrini (2006), "Endogenous Market Incompleteness with Investment Risks," *Journal of Economic Dynamics and Control* 30, 2143-2165.

- [73] Moskowitz, T.J., and A. Vissing-Jørgensen (2002), "The Returns to Entrepreneurial Investment: A Private Equity Premium Puzzle?," *American Economic Review* 92, 745-778.
- [74] Mulligan, C. (2002), "Capital, Interest, and Aggregate Intertemporal Substitution," NBER Working Paper 9373.
- [75] Obstfeld, M. (1994), "Risk-Taking, Global Diversification, and Growth," *American Economic Review* 84, 1310-1329.
- [76] Perotti, R. (2007), "In search of the transmission mechanism of fiscal policy," *NBER Macroeconomics Annual 2007*.
- [77] Quadrini, V. (2000), "Entrepreneurship, Saving, and Social Mobility," *Review of Economic Dynamics* 3, 1-40.
- [78] Ramey, V.A. (2006), "Identifying government spending shocks: It's all in the timing," UCSD mimeo.
- [79] Ramey, V.A. and Shapiro, M.D. (1997), "Costly capital reallocations and the effects of government spending," *Carnegie Rochester Conference on Public Policy* 48, 145-194.
- [80] Stiglitz, J. E. (1969), "The Effect of Income, Wealth, and Capital Gains Taxation on Risk-Taking," *Quarterly Journal of Economics* 83, 263-283.
- [81] Vissing-Jørgensen, A. (2002), "Limited Asset Market Participation and the Elasticity of Intertemporal Substitution," *Journal of Political Economy* 110, 825-853.
- [82] Vissing-Jørgensen, A., and O. P. Attanasio (2003), "Stock Market Participation, Intertemporal Substitution, and Risk Aversion," *American Economic Review* 93, 383-391.
- [83] Weil, P. (1993), "Precautionary Savings and the Permanent Income Hypothesis," *Review of Economic Studies* 60, 367-383.
- [84] Werning, I. (2006), "Optimal Fiscal Policy with Redistribution," *Quarterly Journal of Economics*, forthcoming.