Optimization of Outrigger Structures

by

Ali Lame

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Signature of Author: ______________________________
Department of Civil and Environmental Engineering
May 9, 2008

Certified by: ______________________________
Jerome J. Connor
Professor of Civil and Environmental Engineering
Thesis Supervisor

Accepted by: ______________________________
Daniel Veneziano
Chairman, Departmental Committee for Graduate Students

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Abstract

Designing a high rise building has its challenges. Different structural systems have been developed to control the lateral displacement of high rise buildings. One of these systems is called the outrigger which decreases both the horizontal movement of the structure and the moment on the foundation of the structure. However the location of the outriggers has an immense influence on the efficiency of the structure. Outrigger optimization is a significant challenge. The objective of this thesis is to give a better understanding of outrigger location optimization and the efficiency of each outrigger when several outriggers are used in the structure. An optimization is also performed on a certain configuration of an outrigger to a concrete core. Finally a program has been developed that enables the user to analyze a structure with up to 10 outriggers.

Thesis Supervisor: Jerome J. Connor

Title: Professor of Civil and Environmental Engineering

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Table of Contents

List of Figures

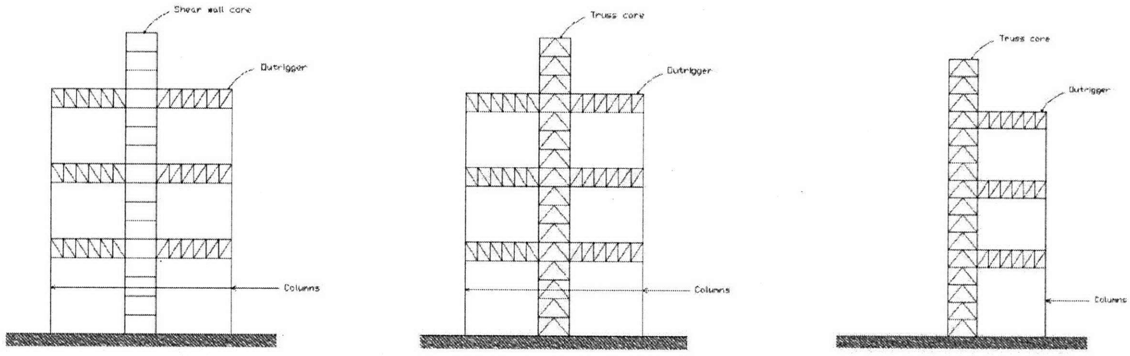
List of Tables

1.0.0 Title of Proposal

Optimization of outrigger structures

2.0.0 Introduction

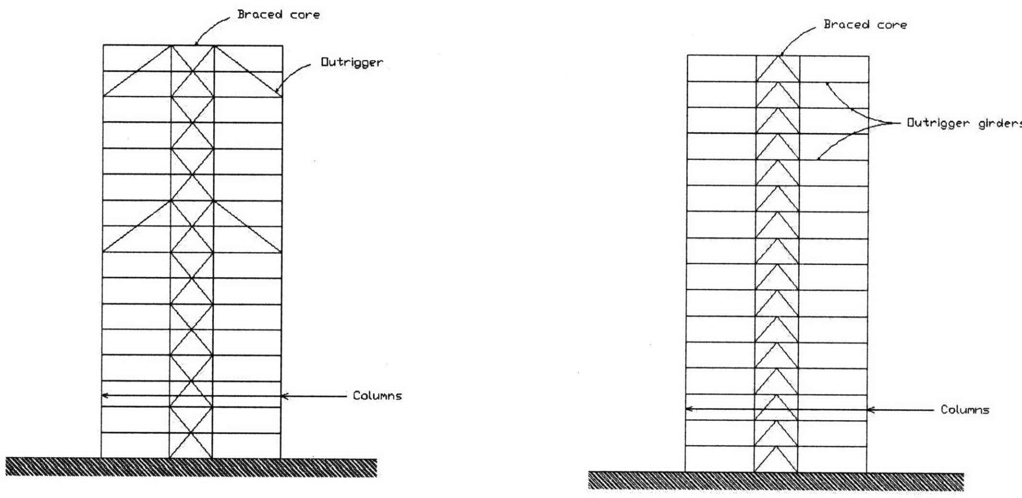
Civil engineers strive to build higher buildings. However, some constraints keep them from building infinitely tall buildings. One of these restrictions is that the lateral displacement of the building limits the height of the building. Structural engineers have come up with innovative designs throughout history to decrease the lateral movement of tall buildings by maintaining a reasonable tonnage of steel. One of these innovative designs is the bracing system. However as the building height exceed 30-40 stories this method becomes too expensive. Therefore engineers have developed other designs. One example is the belt truss system, also known as the core-outrigger system. Some of the common shapes of outriggers are shown in figure 1 and figure 2. In this method the structural engineers use a "hat" or "cap" truss to tie the core to the exterior columns. This method is mostly used against wind loading. One of the main advantages of the outrigger system is that it puts a limit on the lateral displacement of the building. In addition it will decrease the overturning moment of the building which will also decrease the cost of its columns and its foundation. Therefore this system is definitely an efficient way of material use. A structure with outriggers will have 30 to 40 percent less overturning moment in the core compared to a free cantilever in addition to having less drift. This structural system includes a core in the middle of the building (braced frames or shear walls) with horizontal trusses cantilevered from the central core. [1]



(a) (b) (c)

Figure 1- Various types of outrigger systems

One can see there are different types of outrigger systems. Tey can have a concrete core or a truss core. The other components of the outrigger system are also varied. They can be trusses, mega bracings or girders (the forces are carried by the moment connections to the core). The core is used to resist horizantal shear and the outriggers are used to transfer the vertical shear to the exterior columns. With proper placing of the outriggers, the flexural capacity of the building increases but the shear capacity remains the same, since the core mainly has to carry the shear.The outriggers can be on one or both sides of our structure. These mega braces are designed for tension and compresion since the load applied on them can be reveresed. In high siesmic zones a vierendeel system will provide the ductility needed. [1]



(a) (b)

Figure 2- Various types of outrigger systems

Under a lateral load the building wants to rotate. However, due to the outriggers resistant the rotation of the core and the displacement will be less than the free standing structure. (Figure 3)

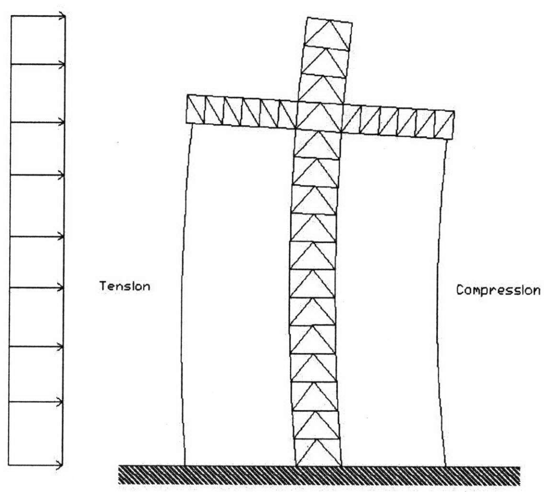


Figure 3-Schematic shape of exterior and interior columns exposed to wind

As it is illustrated in figure 3, tension will be induced in the exterior columns in the windward columns and compression in the leeward columns when a uniform loading is applied to the structure.

Outriggers must be stiff enough to be able to carry both the shear and moment. For this reason outriggers are often made one or two stories deep. This notably increases the flexural stiffness of the structure. However keep in mind that the shear resistance will not change (the shear is mainly carried by the core).

This thesis stars by studying the effect of a single outrigger while we vary its location in the structure.

3.0.0 Literature Review

3.1.0 History of Outriggers

Although outriggers have been used for approximately four decades, their existence as a structural member has a much longer history. Outriggers have been used in the sailing ship industry for many years. They are used to resist wind. The slender mast provides the use of outriggers. As a comparison the core can be related to the mast, the outriggers are like the spreaders and the exterior columns are like the shrouds or stays.

3.2.0 Outriggers Details

As previously mentioned, the exterior columns will cause a moment in the opposite direction of the moment in the core which is induced by the external loading. This results in a smaller moment in the core at the base than a free cantilever. For this system to perform better the outrigger arm is hinged to the exterior columns. This will develop the moment-resisting capacity of the core. (Figure 4)

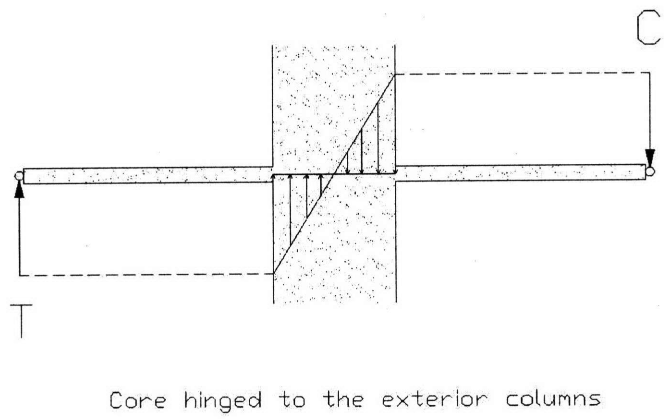


Figure 4- Detail of an outrigger connection to the columns

A detail of the connection of an outrigger to the concrete core has been analyzed with a software called ADINA and the optimized shape for that detail has been recommended.

3.3.0 Outrigger Existence

Recently many tall buildings incorporate a core in the middle of the structure to accommodate elevators. This requires the core to tolerate horizontal loading applied on the structure. This is inefficient after exceeding a certain amount of stories. The resistance that the core system provides to the overturning component of the drift decreases approximately with the cube of the height, showing that as the height of the structure increases the core system becomes more inefficient. Also the core structure alone can generate excessive uplift causing the foundation cost to exceed economic amounts. For example because of uplift the foundation has to be a mat or rock anchors instead of simple foundation alternatives. Another aspect of interest to architects is that they want space between the core and the exterior columns for free rentable space. These two reasons added up and made the structural engineers to create the outrigger solution which incorporates the exterior columns in the lateral structural system. [2]

3.4.0 Outriggers Performance

The outrigger acts like a rotational spring on top of the structure (the main purpose of an outrigger is to reduce the rotation of the core). By creating a resisting moment it decreases the moment in the core and the lateral movement of the core. The amount of reduction in the drift is a function of the building's property which are mentioned on the next page. The magnitude of this reduction is dependent on:

a) The flexural rigidity of the core

b) The flexural rigidity of the outriggers

c) The location of the outriggers up the height of the core

d) The axial force of the columns about the centroid of the core

[1,2,3,4]

It is clear that the outrigger system has a single degree of redundancy. Note that the outrigger is assumed to be rigid (infinite moment of inertia) and that the exterior columns will undergo compression and tension. Assuming B as the distance between the two exterior columns and α is the rotation of the core, $\alpha \times \frac{B}{2}$ is the axial deformation of the columns.

The axial force in the columns is:

$$F = \frac{AE}{L} \times \frac{B}{2} \times \alpha$$

where F is the axial load in the columns, A is the area of the columns, E is the modulus of elasticity, B is the distance between the exterior columns and L is the length of the vertical tie-downs. The resisting moment due to the forces in the exterior column is equal to:

$$M = F \times B = \frac{AE}{L} \times \frac{B^2}{2} \times \alpha$$

We express M as $M = K \times \alpha$ where $K = \frac{AE}{L} \times \frac{B^2}{2}$

Case 1: Take the situation in the figure 5 were the outrigger is assumed to be at the top of the structure.

We introduce the following definitions.

$\theta_w =$ Rotation of the structure at the top of the structure due to the lateral loading.

$\theta_s =$ Rotation due to the outrigger (rotational spring). Note that this rotation is in the opposite direction of the rotation of the structure.

$\theta_L =$ Actual rotation of the structure

So we can write: $\theta_w - \theta_s = \theta_L$ (I)

We know that if the structure is under a uniform load of P then $\theta_w = \frac{PH^3}{6EI}$ where H is the height of the structure, E is the modulus of elasticity and I is the moment of inertia. Assuming that the moment is M and the stiffness is K for the rotational spring we can write by substituting in the (I) equation:

$$\frac{PH^3}{6EI} - \frac{M_1 H}{EI} = \frac{M_1}{K_1} \rightarrow M_1 = \frac{\frac{PH^3}{6EI}}{\frac{1}{K_1} + \frac{H}{EI}}$$

For calculating the deflection at the top of the cantilever we can use the principle of superposition. The deflection at the top if the cantilever is equal to the displacement at the top of a cantilever due to the external uniform load minus the deflection due to outrigger (rotational spring) hence we can write:

$$\Delta_{top} = \Delta_{load} - \Delta_{spring} \rightarrow \frac{PH^4}{8EI} - \frac{M_1 H^2}{2EI} = \frac{H^2}{2EI}\left(\frac{PL^2}{4} - M_1\right)$$

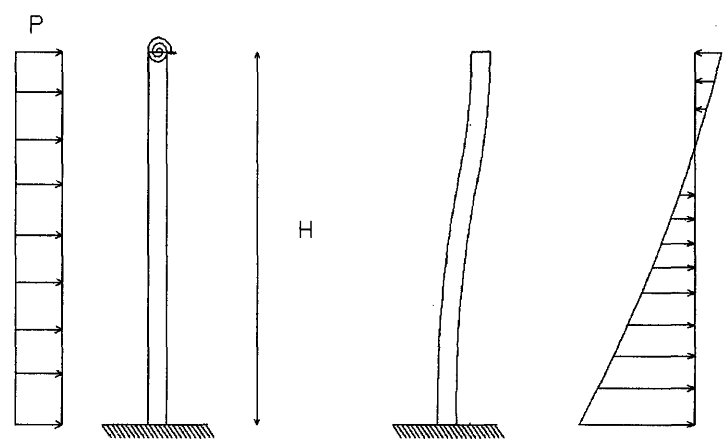


Figure 5-Single outrigger at the top of the structure

Case 2: Now assume that we lower the outrigger 25 percent of the total height (figure 6). We know that the rotation of the cantilever due to the uniform loading will be:

$$\theta = \frac{P}{6EI}\left(\frac{H^3}{64} - H^3\right) = \frac{63}{64} \times \frac{PH^3}{6EI}$$

Assuming M_2 and K_2 to be the moment and the stiffness of the rotational spring we can write:

$$\frac{63}{64} \times \frac{PH^3}{6EI} - \frac{M_2}{EI}\left(\frac{3H}{4}\right) = \frac{M_2}{K_2}$$

Since $K_2 = \frac{4K_1}{3}$ we can derive M_2's relationship with M_1.

$$M_2 = \frac{\frac{63}{64}}{\frac{3}{4}} \times \frac{\frac{PH^3}{6EI}}{\frac{1}{K_1} + \frac{H}{EI}} \rightarrow M_2 = 1.31M_1$$

The drift of the top of the cantilever can be calculated by:

$$\Delta_2 = \frac{PH^4}{8EI} - \frac{M_2 3H}{4EI}(H - \frac{3H}{8})$$

$$\Delta_2 = \frac{H^2}{2EI}(\frac{PH^2}{4} - 1.23M_1)$$

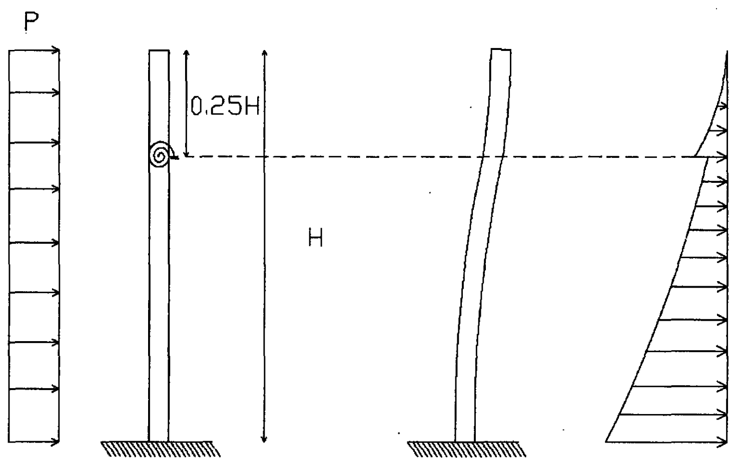


Figure 6- Single outrigger at 75 percent of the total height of the structure

Case 3: The third case is when we put the outrigger at the middle of the cantilever (figure 7). This time we will calculate the rotation and moment at the outrigger faster since we already have demonstrated the method twice.

$$\theta_w - \theta_s = \theta_L$$

$$\frac{7PH^3}{48EI} - \frac{M_3 H}{2EI} = \frac{M_3}{K_3}$$

We know that $K_3 = 2K_1$ therefore:

$$M_3 = \frac{7}{4} \times \frac{\frac{PH^3}{6EI}}{\frac{1}{K_1} + \frac{H}{EI}} \rightarrow M_3 = 1.75M_1$$

The drift of the top of the cantilever will be:

$$\Delta_3 = \frac{PH^4}{8EI} - \frac{M_3 H}{2EI}(H - \frac{H}{4})$$

$$\Delta_2 = \frac{H^2}{2EI}(\frac{PH^2}{4} - 1.31M_1)$$

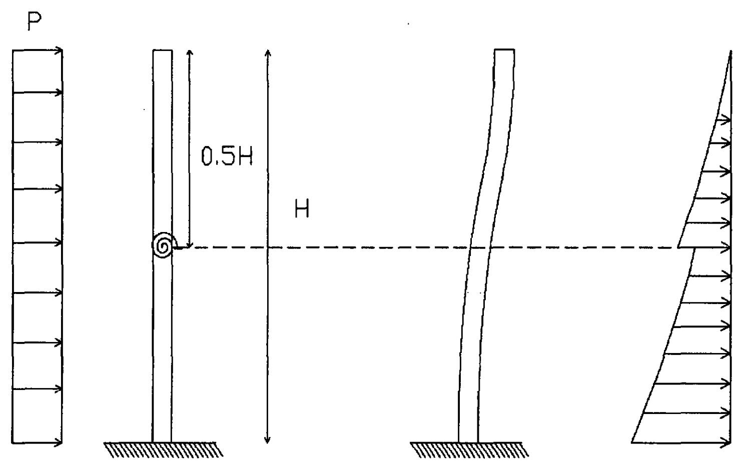


Figure 7- Single outrigger at the middle of the structure

Case 3: The fourth and the last case that we are going to analyze in this section is when the outrigger is at one fourth of the total height of the structure as illustrated in the figure 8.

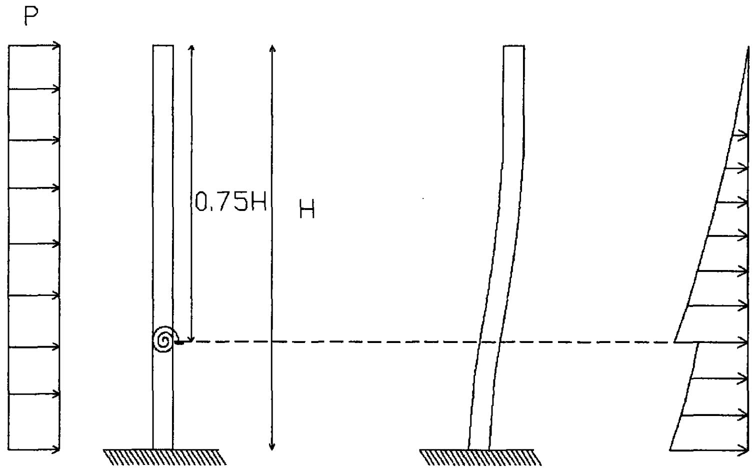


Figure 8- Single outrigger at 25 percent of the total height of the structure

$$\theta_w - \theta_s = \theta_L$$

$$\left(\frac{37}{64}\right)\frac{PH^3}{6EI} - \frac{M_4 H}{4EI} = \frac{M_4}{K_4}$$

We know that $K_3 = 4K_1$ therefore:

$$M_3 = \frac{37}{16} \times \frac{\frac{PH^3}{6EI}}{\frac{1}{K_1} + \frac{H}{EI}} \rightarrow M_4 = 2.31M_1$$

The drift of the top of the cantilever will be:

$$\Delta_3 = \frac{PH^4}{8EI} - \frac{M_4 H}{4EI}\left(H - \frac{H}{8}\right)$$

$$\Delta_2 = \frac{H^2}{2EI}\left(\frac{PH^2}{4} - M_1\right)$$

[1]

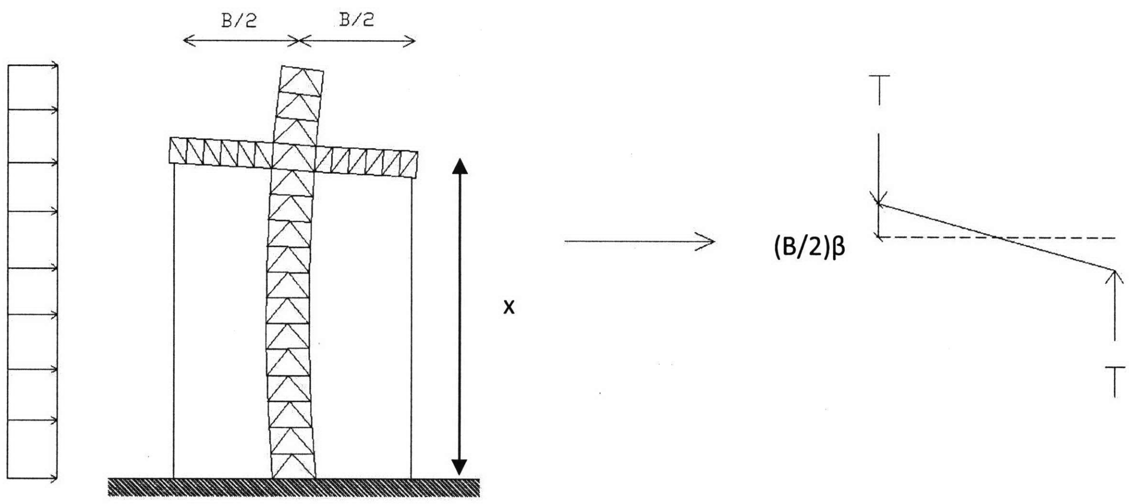


Figure 9- schematic shape of the forces induced in the outrigger

To solve for the stiffness of the spring we will apply a unit rotation at the place of the outrigger. This will cause the exterior columns to go under tension or compression as it's shown in the figure 9.

We know the moment due to the couple force of T is equal to B×T. Since the force in the columns can be written as T=$\frac{AE}{x}(\frac{B}{2})\beta$ therefore M= $K_i\,\beta = (\frac{AEB^2}{2x}) \times \beta$

All these computations demonstrate two major concepts. One is that the stiffness of system has an inverse relationship with the place of the outrigger. In other words, the stiffness is minimum when the outrigger is at the top and is maximum when at the bottom. Second is that the cantilever will move the most at the top under the uniform loading and zero at the bottom. These two criteria are opposing. This brings up the issue of outrigger optimization. We have to place the outriggers at a height so that the deformation of the cantilever will be the least and also the stiffness will be the maximum possible. [1]

In continue we will analyze the optimum place for one outrigger.

3.5.0 Advantages and Disadvantages of Outriggers

3.5.1 Advantages of Outriggers

By using outriggers at proper places one can gain the benefits below:

- Decreasing the overturning moment of the core. By using outriggers the effective width of the structure will increase from the core itself to almost the complete building.
- Decreasing the lateral movement of the structure by using the exterior columns for the lateral resisting system.
- Since the exterior framing can consist of simple beam and column framing without the necessity for rigid connections the cost of the structure can decrease significantly compared to a structure made with rigid connections.
- Significant reduction of uplift and reducing the cost of the foundation.
- In many of the cases and especially rectangular plan buildings the outriggers incorporate even the middle gravity columns into the lateral load resisting system. This will lead to a much more economic structure. [2]

3.5.2 Disadvantages of Outriggers

There are two main disadvantages in using outriggers in the structural system. First is that they consume a lot of rentable space. They might even be two stories deep. However this problem may be overcome by putting the outrigger in the mechanical level. Another problem is that usually the optimal erection of a building has a repetitive nature so that the construction staff work with a faster speed, however this is not true with outrigger systems. The outriggers have negative impact on the erection of the structure although this can be mitigated by providing clear erection guidelines for the construction staff. [2]

3.6.0 Range of Usage

It is clear that by using outriggers structural engineers have taken advantage of the material. This spurs the question: Over what ranges this system act efficiently? Nowadays this system is used in buildings from 40 to over 100 stories. However, by adding other levels of outriggers to the system the amount of efficiency of the resisting moment decreases and four or five outriggers seem to be the optimum number. We will now discuss the efficiency of outriggers and their optimum number in buildings. [3]

3.7.0 Single Truss Optimum Location

As mention in the previous section we have to calculate the optimum placement of the outrigger so that the stiffness of the outrigger is the maximum possible and the displacement of the cantilever is minimum. In order to derive the optimum solution we have to make some assumptions.

1- The area and the moment of inertia of the exterior columns are constant throughout the height.
2- The outrigger is assumed rigid. (So it only puts axial force in the exterior columns)
3- The core is fixed at the bottom and so has no displacement at the bottom.
4- The structure is linearly elastic.
5- We will neglect the rotation due to shear in the core.

We assume that the optimum placement for one outrigger is at distance X from top of the cantilever (figure 10).

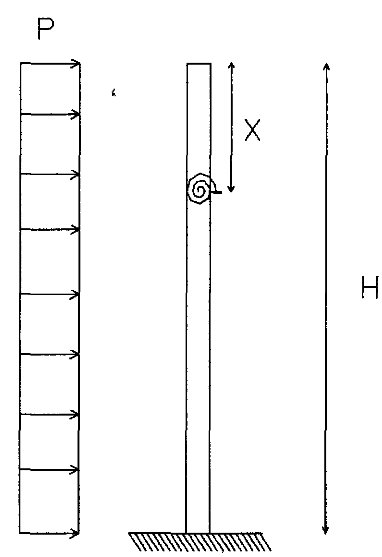


Figure 10-Schematic shape of an single outrigger building

The rotation at the distance X from top of the cantilever is:

$$\theta_X = \frac{P}{EI}(X^3 - H^3)$$

Assuming the restoring moment due to the outrigger is equal to M_{out} the rotation at the top of the cantilever is:

$$\theta_{top} = \frac{M_{out}}{EI}(H - X)$$

According to our previous equations we can write:

$$\frac{M_{out}}{K_{out}} = \frac{P}{EI}(X^3 - H^3) - \frac{M_{out}}{EI}(H - X)$$

Where $K_{out}$is equal to the stiffness of the outrigger. $K_{out} = \frac{AE}{(L-X)} \times \frac{d^2}{2}$

d is the distance between the exterior columns.

A is the area of the exterior columns.

E is the core modulus of elasticity.

[1]

3.8.0 Two Outrigger Levels

3.8.1 Outriggers Moment Resistance

To show how an N outrigger system will work we will now solve the two degree of freedom outrigger and then derive the equations for a N outrigger system.

By taking a look at the figure on the next page you will notice that a two outrigger system has two redundancies. As a result two compatibility equations are needed to solve the problem. The compatibility equations can be derived by setting the rotation of the core and the outriggers equal at each level. [3]

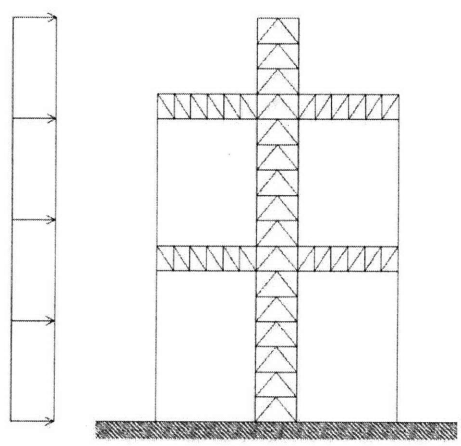


Figure 11-Schematic shape of two level outriggers

Figure 12 shows the analysis model and the related moment diagrams for a free standing cantilever and the moments induced in the system by each level of the outrigger and the final moment distribution.

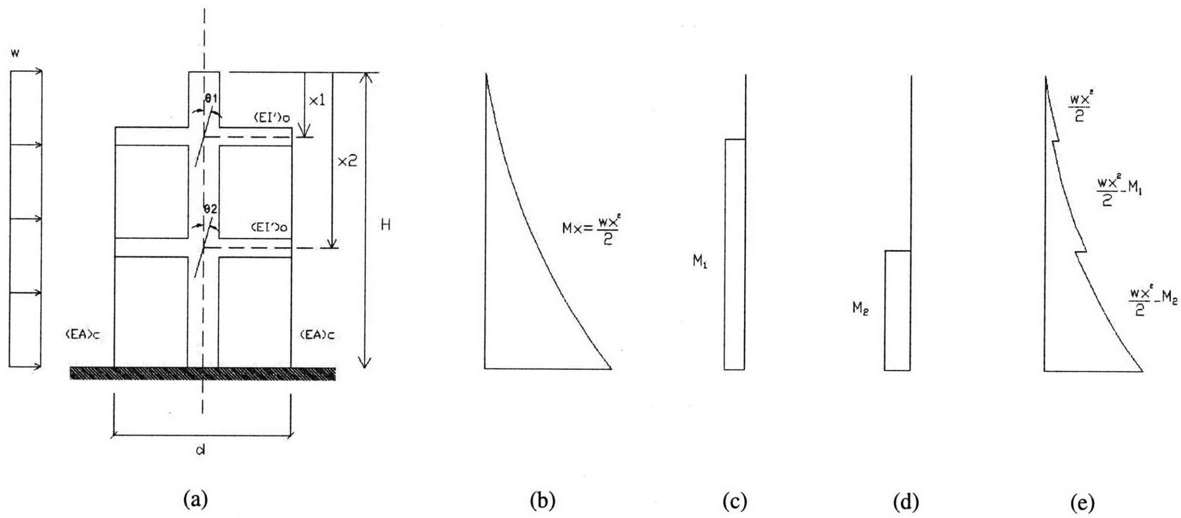


Figure 12- Two level outriggers and their moment diagrams

The rotation of the core at level one can be calculated as below:

$$\theta_1 = \frac{1}{EI}\int_{x_1}^{x_2}\left(\frac{wx^2}{2} - M_1\right)dx + \frac{1}{EI}\int_{x_2}^{H}\left(\frac{wx^2}{2} - M_1 - M_2\right)dx$$

Similarly the rotation of the core at level two can be written as:

$$\theta_2 = \frac{1}{EI}\int_{x_2}^{H}\left(\frac{wx^2}{2} - M_1 - M_2\right)dx$$

Where

EI is the flexural rigidity

H is the height of the structure

w is the uniform loading

x_1 and x_2 are the height of the outriggers from the top of the structure

M_1 and M_2 are the moments created at the level of the outriggers

The rotation of the outriggers can be written as the axial deformation of the columns and the bending of the outrigger. The rotation of the outrigger at level one and two can be expressed as below:

$$\theta_1 = \frac{2M_1(H-x_1)}{d^2(EA)_c} + \frac{2M_2(H-x_2)}{d^2(EA)_c} + \frac{M_1 d}{12(EI)_0}$$

$$\theta_2 = \frac{2(M_1+M_2)(H-x_2)}{d^2(EA)_c} + \frac{M_2 d}{12(EI)_0}$$

Where

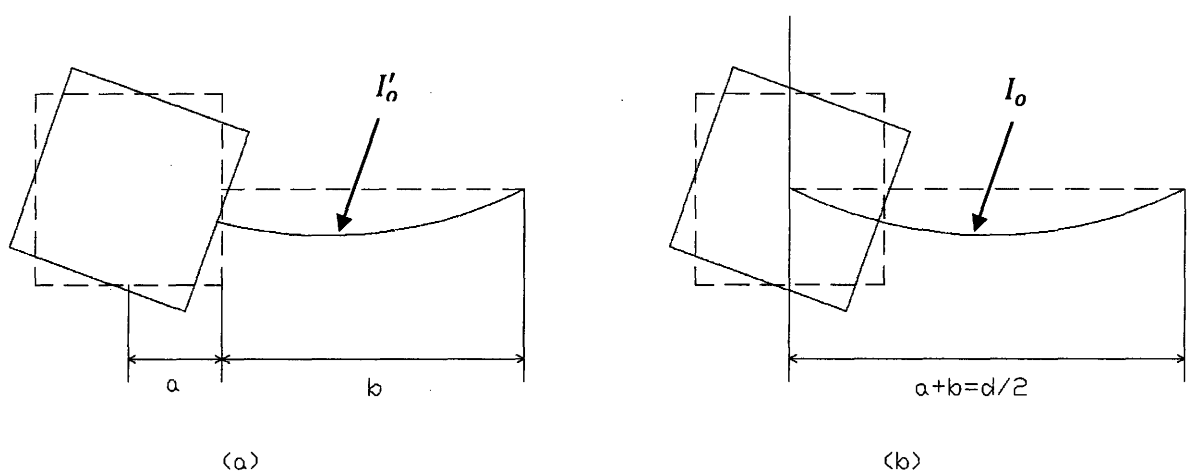
$(EA)_c$ is the axial rigidity of the column

d is the distance between the exterior columns

[3]

$(EI)_0$ is the effective flexural rigidity of the outrigger (see figure below) the effective flexural can be calculated from the actual flexural rigidity $(EI')_0$

$$(EI)_0 = (1+\frac{a}{b})^3 (EI')_0$$



Outrigger actual inertia I'_o Outrigger effective inertia I_o

Figure 13- Schematic shape of outrigger attachments to the edge and centroid of core

By equating the rotation of the core to the rotation of the outrigger at each level we will have:

$$\frac{2M_1(H-x_1)}{d^2(EA)_c}+\frac{2M_2(H-x_2)}{d^2(EA)_c}+\frac{M_1 d}{12(EI)_0}=$$

$$\frac{1}{EI}\int_{x_1}^{x_2}\left(\frac{wx^2}{2}-M_1\right)dx+\frac{1}{EI}\int_{x_2}^{H}\left(\frac{wx^2}{2}-M_1-M_2\right)dx$$

For the second level we can similarly say:

$$\frac{2(M_1+M_2)(H-x_2)}{d^2(EA)_c}+\frac{M_2 d}{12(EI)_0}=\frac{1}{EI}\int_{x_2}^{H}\left(\frac{wx^2}{2}-M_1-M_2\right)dx$$

The equations above can be rewritten as:

$$M_1[S_1+S(H-x_1)]+M_2S(H-x_2)=\frac{w}{6EI}(H^3-x_1^3)$$

And similarly

$$M_1S[(H-x_2)]+M_2[S_1+S(H-x_2)=\frac{w}{6EI}(H^3-x_2^3)$$

Where

$$S=\frac{1}{EI}+\frac{2}{d^2(EA)_c}$$

$$S_1=\frac{d}{12(EI)_0}$$

By solving the two simultaneous equations above we will have:

$$M_1=\frac{w}{6EI}\left[\frac{S_1(H^3-x_1^3)+S(H-x_2)(x_2^3-x_1^3)}{S_1^2+S_1S(2H-x_1-x_2)+S^2(H-x_2)(x_2-x_1)}\right]$$

And similarly

$$M_2=\frac{w}{6EI}\left[\frac{S_1(H^3-x_2^3)+S[(H-x_1)(H^3-x_2^3)-(H-x_2)(H^3-x_1^3)]}{S_1^2+S_1S(2H-x_1-x_2)+S^2(H-x_2)(x_2-x_1)}\right]$$

[3]

3.8.2 Moment in the Core

Having solved for the moments of the outriggers we can solve for the moment in the core by the equation below:

$$M_x = \begin{cases} \frac{wx^2}{2} & x < x_1 \\ \frac{wx^2}{2} - M_1 & x_1 < x < x_2 \\ \frac{wx^2}{2} - M_1 - M_2 & x_2 < x < H \end{cases}$$

[3]

3.8.3 Horizontal Deflection

The maximum drift occurs at the top of the structure. The displacement at the top can be written as:

$$\Delta = \frac{wH^4}{8EI} - \frac{1}{EI}[M_1(H^2 - x_1^2) + M_2(H^2 - x_2^2)]$$

where the first term on the left- hand side is the displacement of a free cantilever due to a uniform external loading. The second term is the decrease in the displacement at the top of the cantilever due to the two outriggers at x_1 and x_2. [3]

3.8.4 Optimum Location of Two Outriggers

As the structure becomes taller, motion based design will govern. We must minimize the deflection at the top of the structure. We will do so by maximizing the drift reduction previously mentioned (the second term in the `Δ equation). To achieve the optimum location of the outriggers for the two outrigger levels we have to calculate the derivative of Δ with respect to x_1 and x_2. The result is illustrated on the next page:

$$\frac{d\Delta}{dx_1} = (H^2 - x_1^2)\frac{dM_1}{dx_1} + (H^2 - x_2^2)\frac{dM_2}{dx_1} - 2x_1M_1 = 0$$

and

$$\frac{d\Delta}{dx_2} = (H^2 - x_1^2)\frac{dM_1}{dx_2} + (H^2 - x_2^2)\frac{dM_2}{dx_2} - 2x_2M_1 = 0$$

We can substitute for M_1 , M_2 , $\frac{dM_1}{dx_1}$, $\frac{dM_1}{dx_2}$, $\frac{dM_2}{dx_1}$ and $\frac{dM_2}{dx_2}$ from the equations mentioned earlier. However solving the two simultaneous equations will be tedious and requires a numerical method. In addition by defining new non-dimensional variables we can define the outrigger structure properties in a more concise and efficient way.

Therefore we define

$$\alpha = \frac{EI}{(EA)_c(d^2/2)}$$

and

$$\beta = \frac{EI}{(EI)_0} \times \frac{d}{H}$$

where α represents the rigidity of the core with respect to the rigidity of the column and β represents the rigidity of the core with respect to the rigidity of the outrigger. [3]

Below the graphs are presented for the different values of α and β for one to four level of outriggers (figure 14-(a) through figure 14-(d)). Note that for a constant value of α, as β increases the optimum place for the outriggers moves toward the bottom of the structure.

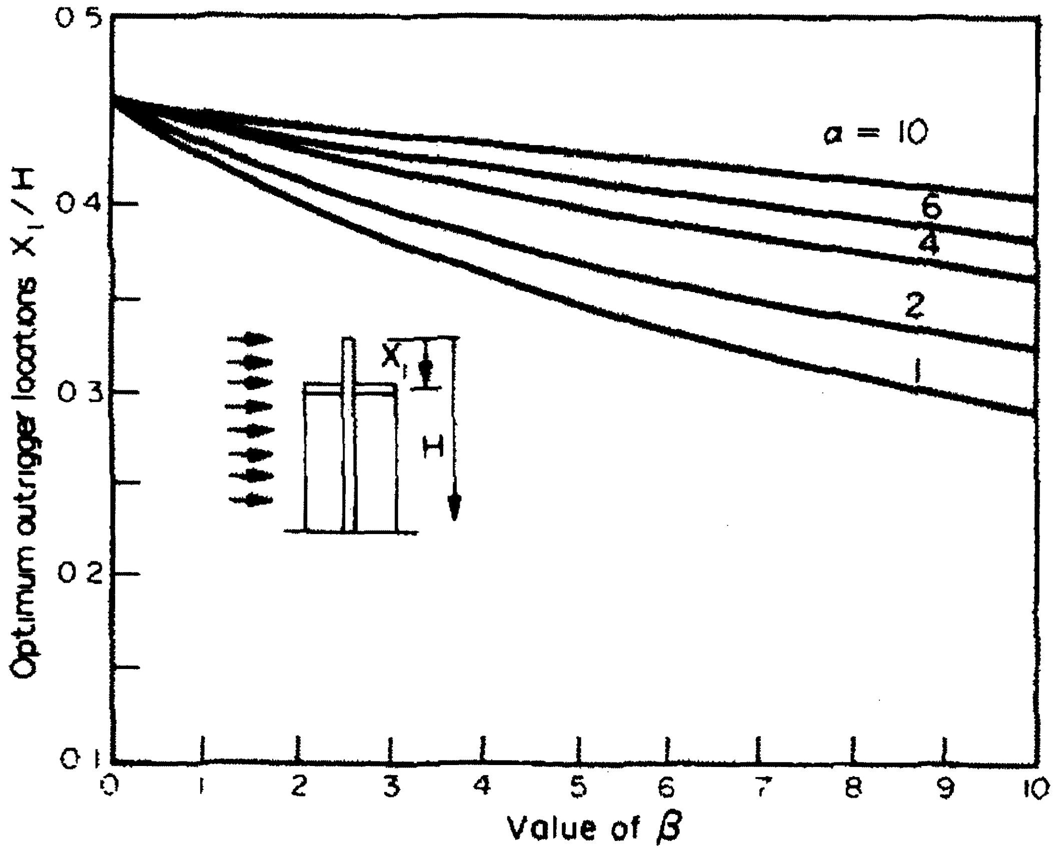


Figure 14-(a) Graph for the optimum location of a single outrigger system with respect to α and β [4]

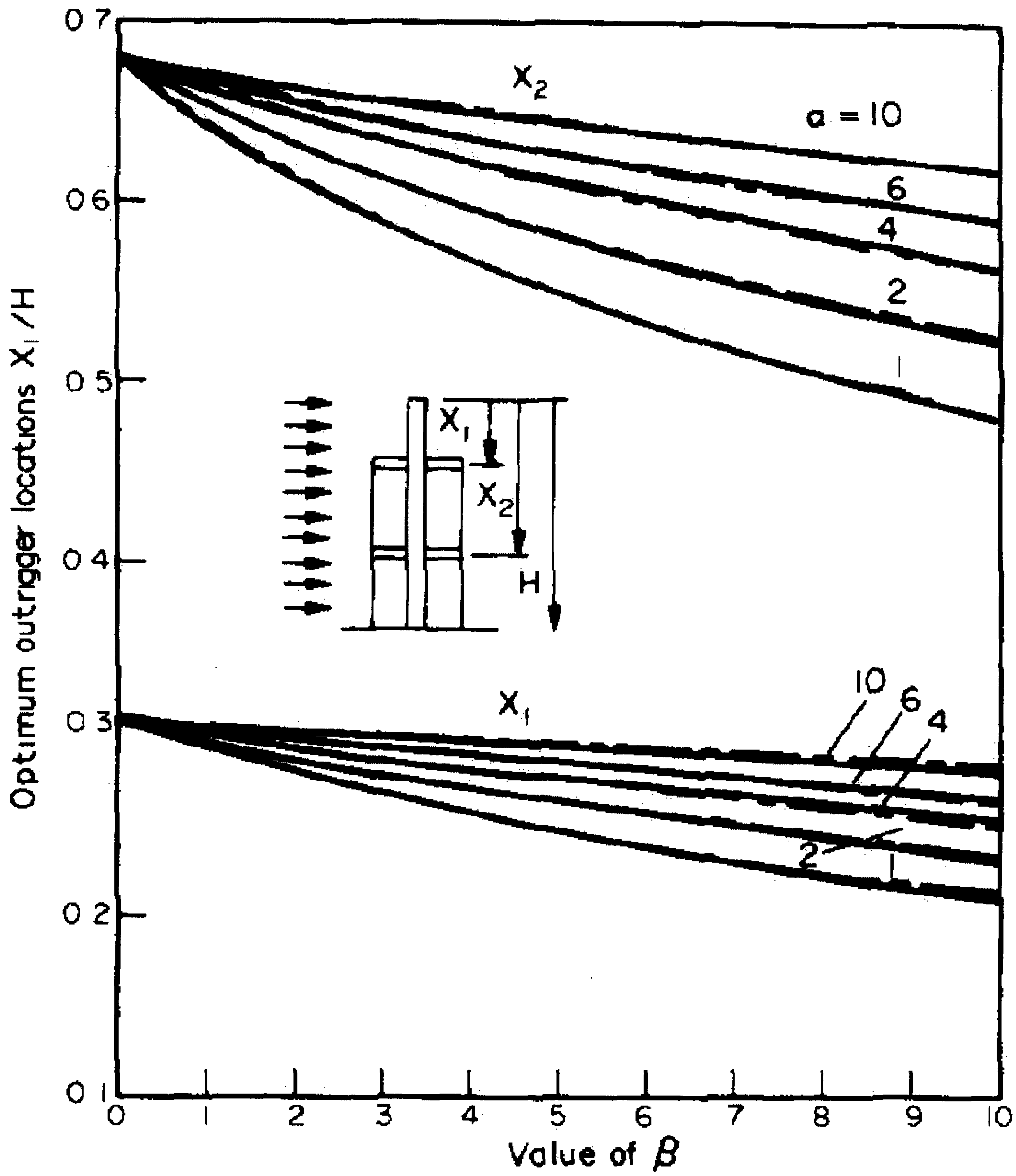


Figure 14-(b) Graph for the optimum locations of a double outrigger system with respect to α and β [4]

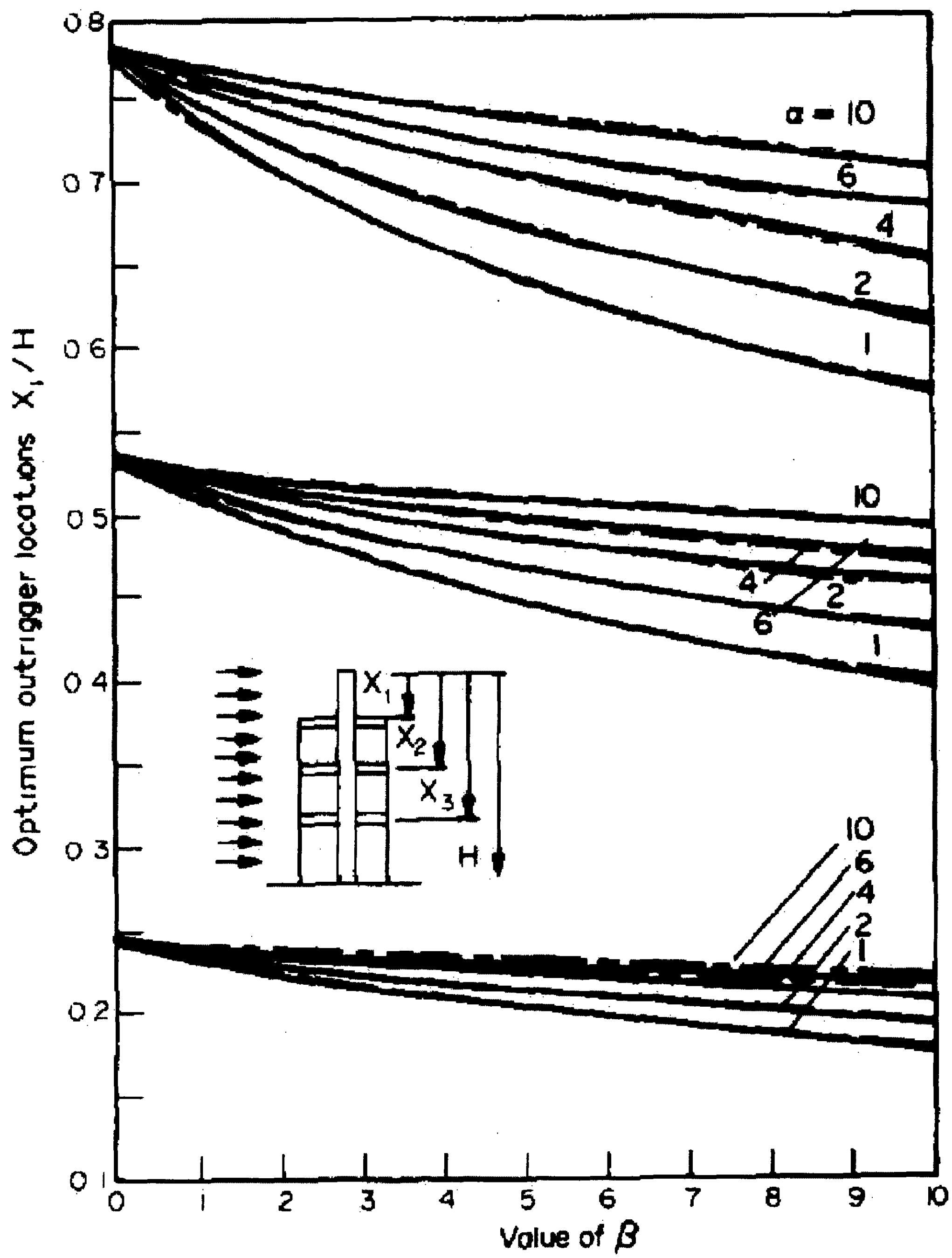


Figure 14-(c) Graph for the optimum locations of a triple outrigger system with respect to α and β [4]

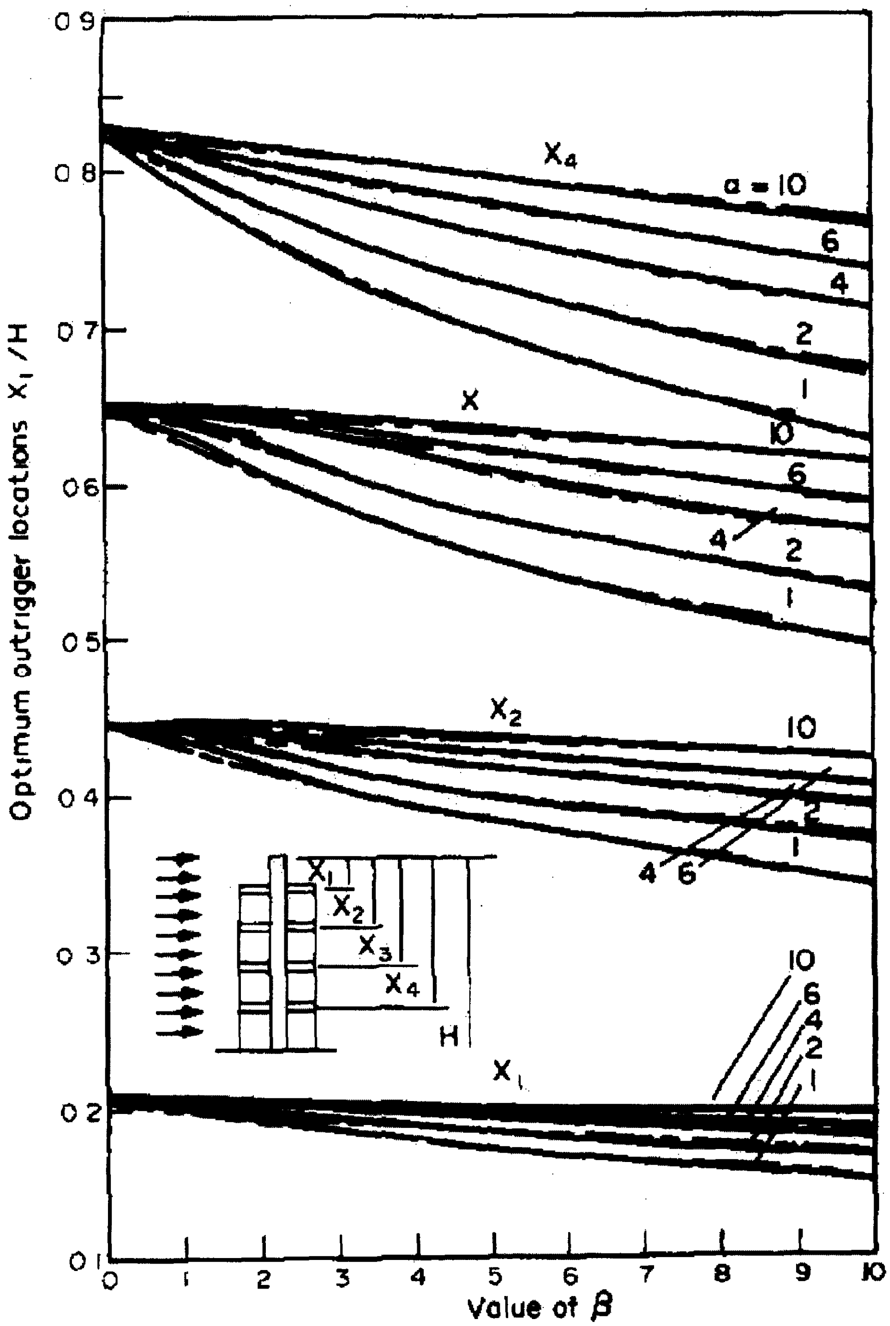


Figure 14-(d) Graph for the optimum locations of a four outrigger system with respect to α and β [4]

Now we want to show another way of analyzing the outriggers. By combining the two variables of α and β previously mentioned we get:

$$\omega = \frac{\beta}{12(1+\alpha)}$$

where ω is called the characteristic structural parameter. With the equation above it is clear that ω will decrease as the outriggers flexural stiffness is increased. In addition the equation shows that by increasing the axial columns stiffness ω will increase. (Note that $\omega = 0$ means that the outriggers have been taken to be rigid.) [3]

Given the structural properties, ω can be calculated and therefore the optimum placement of the two levels of the outrigger can be determined from the equations above. The result for the lowest drift has been shown in the figure 17-(a)

3.8.5 Two Level Outrigger (Flexibility Approach)

An example of a 46 story building has been analyzed and the result is given in the graph below. The building is 100 ft by 150 ft. The wind load was applied on the 150 ft. side of the building. The magnitude of the wind load varies from 20 psf. at the bottom to 26 psf. at the top. The core is at the center of the plan and is 34 ft by 50 ft. The schematic plan is shown below:

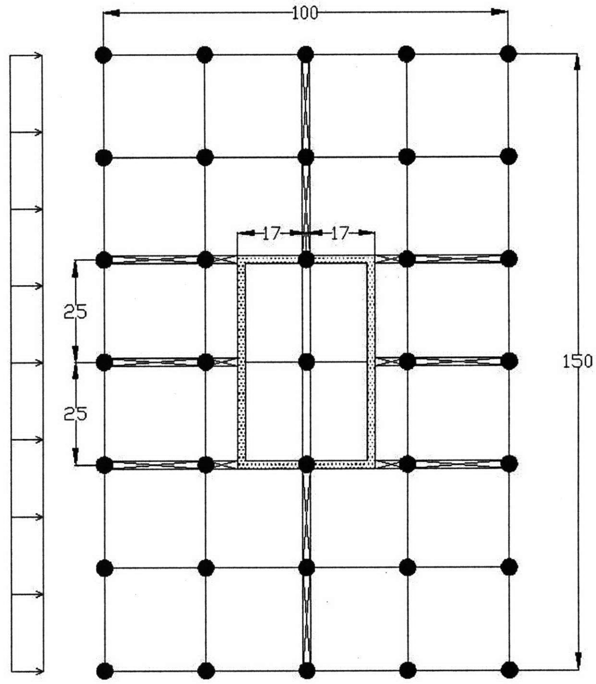


Figure 15- Schematic plan of the example

The columns are shown with a circle. The outriggers are shown with crosses and the core is the hatched area in the middle of the plan. [1]

The result of the analysis is shown in figure 16. The horizontal axis is the deflection of the top the structure divided by the deflection of the free standing cantilever. In other words, the vertical line at one (figure 16) shows the deflection of the free standing cantilever. The curve labeled "S" is for the deflection of a single outrigger divided by the deflection of a free cantilever. The distance XX^1 multiplied by the free cantilever will give the deflection of the top floor of the structure when the outrigger is at the 35th story. [1]

From the curves in the graph labled 4, 8, 12, …, 46 the top deflection of a structure with two outriggers can be achieved. The numbers on the vertical axis shows the location of the lower outrigger. The curves with the numbers present the location of the second outrigger. For example the HH^1show that the lower outrigger is located at the 15th story and the higher outrigger is at the 20th story.

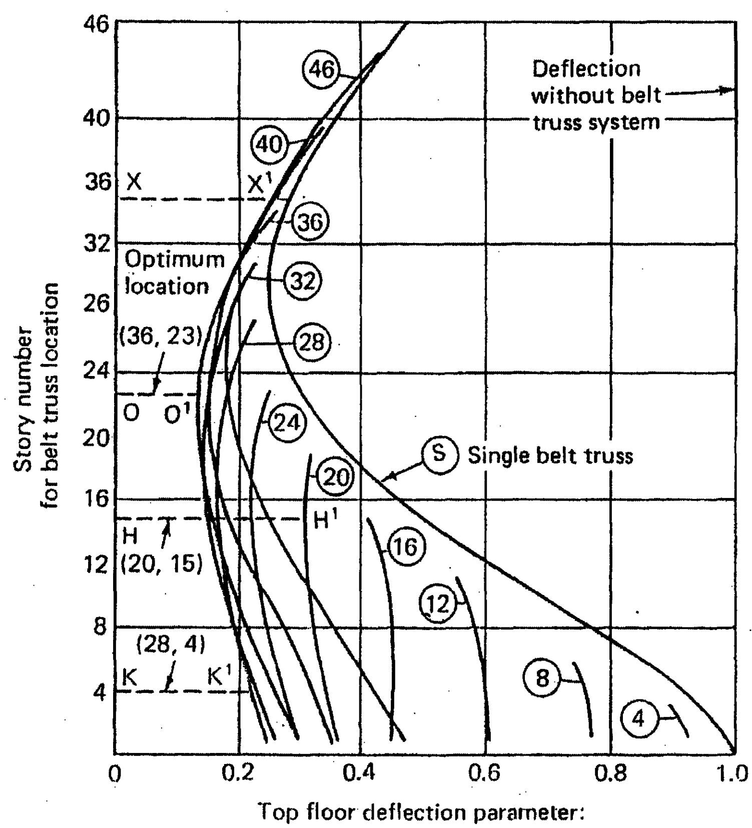


Figure 16- Optimum locations of the two levels of the outriggers in the example [1]

3.9.0 General Solution for N Level of Outriggers

3.9.1 Outriggers Moment Resistance

By adopting the same methods as illustrated for the two outriggers we can derive the general solution of the N level outriggers which is shown below:

The restraining moments can be calculated by solving for the matrix below:

$$\begin{bmatrix} M_1 \\ M_2 \\ \vdots \\ M_i \\ \vdots \\ M_n \end{bmatrix} = \frac{w}{6EI} \times \begin{bmatrix} S_1 + S(x - x_1) & S(H - x_2) & \dots & S(H - x_i) & \dots & S(H - x_n) \\ S(H - x_2) & S_1 + S(H - x_2) & \dots & S(H - x_i) & \dots & S(H - x_n) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ S(H - x_i) & S(H - x_i) & \dots & S_1 + S(H - x_i) & \dots & S(H - x_n) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ S(H - x_n) & S(H - x_n) & \dots & S(H - x_n) & \dots & S_1 + S(H - x_n) \end{bmatrix}^{-1} \times \begin{bmatrix} H^3 - x_1^3 \\ H^3 - x_2^3 \\ \vdots \\ H^3 - x_i^3 \\ \vdots \\ H^3 - x_n^3 \end{bmatrix}$$

[3]

3.9.2 Core's Moment

As a result the general expression for the moment in the core can be written as:

$$M_{core} = \frac{wx^2}{2} - \sum_{i=1}^{k} M_i$$

where k is the number of outriggers till the place in the core which we want to find the moment in the core, measured from top of the structure. Notice that the second term will be zero if we are looking at the top of the structure before reaching any outriggers. [3]

3.9.3 Horizontal Deflection

The top deflection of the structure will be equal to the equation below:

$$\Delta_0 = \frac{wH^4}{8EI} - \frac{1}{EI}\sum_{i=1}^{n} M_i(H^2 - x_i^2)$$

The figures to follow will show the optimum place for the one, two, three and four level of outrigger for having the smallest drift. [3]

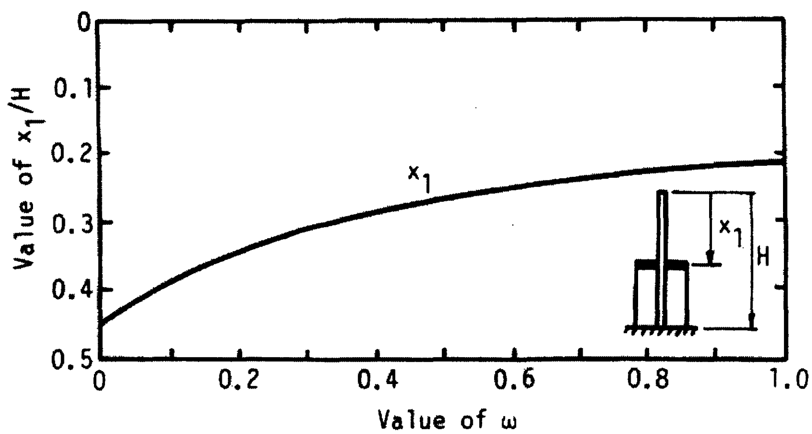


Figure 17-(a) Graph for the optimum location of a single outrigger with respect to ω [3]

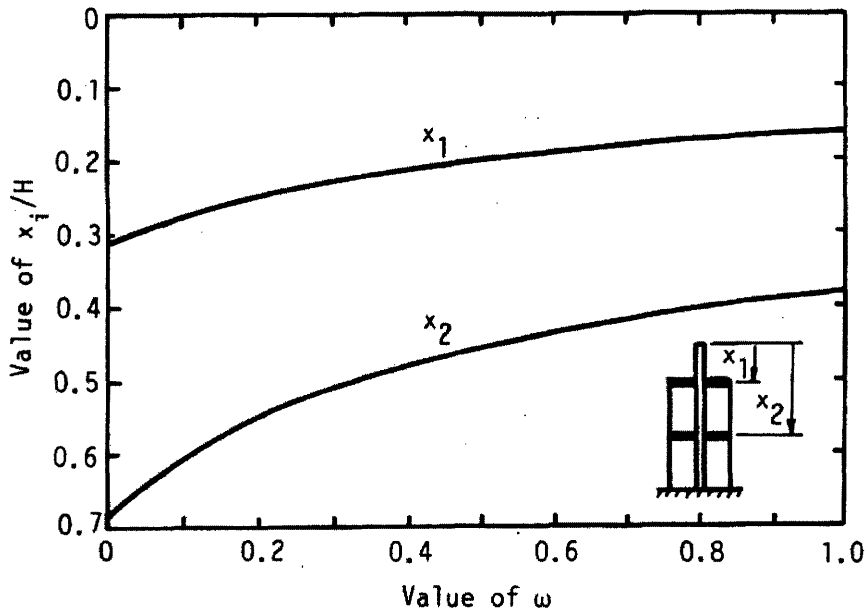


Figure 17-(b) Graph for the optimum locations of two outriggers with respect to ω [3]

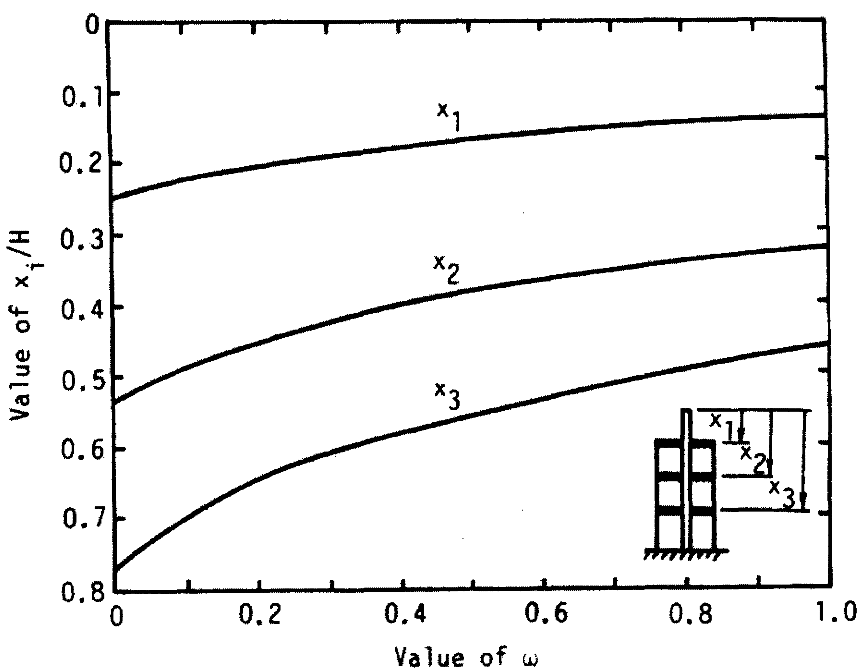


Figure 17-(c) Graph for the optimum locations of three outriggers with respect to ω [3]

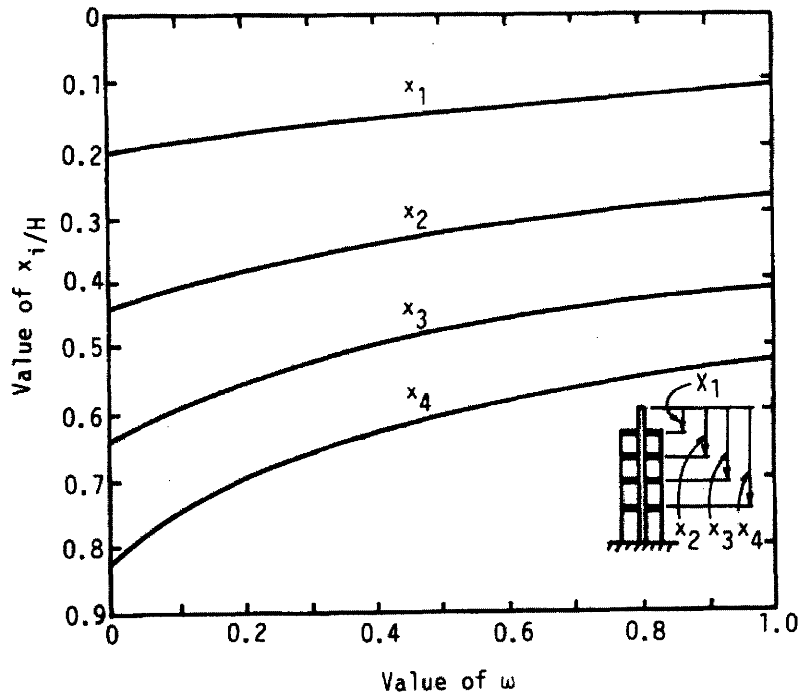


Figure 17-(d) Graph for the optimum locations of four outriggers with respect to ω [3]

By taking a closer look at the previous graphs, one will notice that assuming the outriggers act rigidly; the minimum drift in a single outrigger is when the outrigger is placed approximately at the middle of the structure. For the two-system outrigger structure the outriggers are installed approximately at one third and two thirds of the structure. Similarly, in a three level outrigger structure the outriggers are approximately placed at one fourth, half and three fourth of the height of the structure. Finally, it is clear that for a four level outrigger the optimal solution for the rigid outriggers will be at one fifths, two fifths, three fifths and four fifths of the total height of the structure. As a rule of thumb, the place for the outriggers that minimize the drift can be obtained from the numbers below. (Keep in mind that the outriggers are assumed to be rigid.) [3]

$\frac{1}{n+1}, \frac{2}{n+1}, \frac{3}{n+1}, \ldots, \frac{n}{n+1}$ where n is the number of the level of the outriggers.

Notice that as ω increases (outriggers become more flexible) the outriggers should be placed higher in order to keep the drift limitations below the allowable limits. However, the intervals between the outriggers (the distance between the curves in each graph) remain the same. [3]

3.10.0 Outrigger Performance and Efficiency

Is there a specific number of outriggers for which a structure will work most efficiently? In order to discuss this issue we need to discuss some issues about the performance of the outriggers. [3]

By now it seems clear that by putting an outrigger at the top of the structure we will have the least amount of efficiency in the drift reduction. Indeed it has been shown that n optimal outriggers are almost equivalent in drift for the same structure with a single outrigger at the top of the structure. [3]

Also the lowest outrigger creates the biggest resisting moment and as we move up the structure the other outriggers will induce less moment in the system. [3]

In the following figures (figures 18-(a) and figure 18-(b))the efficiency on the number of outriggers have been evaluated by observing the amount that each case reduces the moment at the bottom of the core and reduces the displacement at the top of the structure. [3]

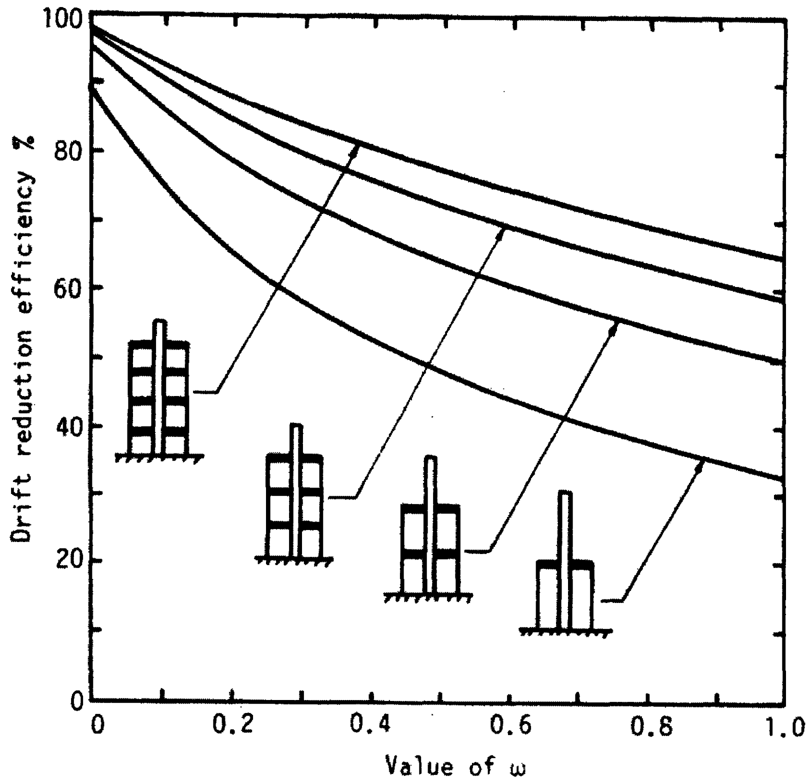


Figure 18-(a) Efficiency of number of outriggers with respect to drift reduction [3]

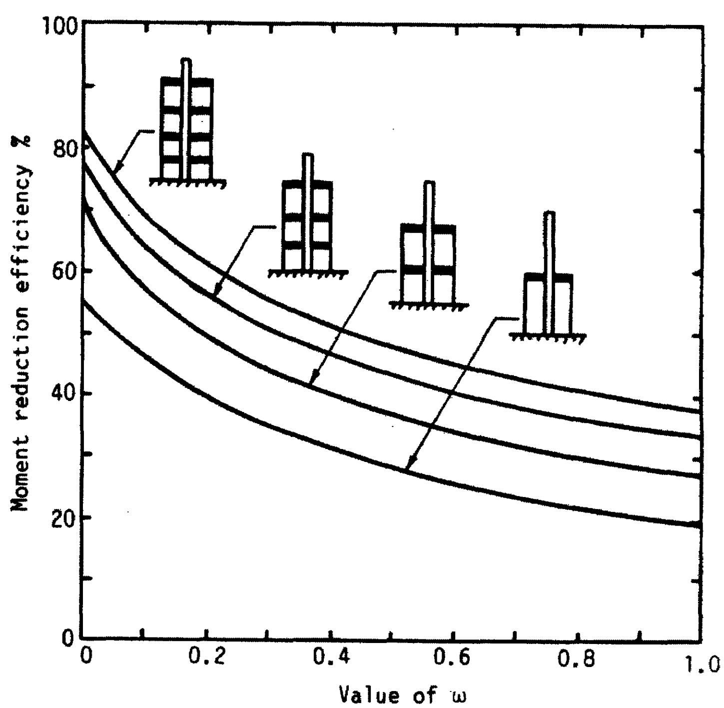


Figure 18-(b) Efficiency of number of outriggers with respect to moment reduction [3]

Notice that in the graph that shows the drift reduction versus ω (figure 18-(a)) if we assume the outriggers act rigidly it is clear that the drift reduction efficiency is increased. However, the difference between three and four levels of outriggers is almost negligible. Therefore it can be concluded that using four levels of outriggers is reasonable since the amount of horizontal resistance gained by more than four outriggers is not substantial. [3]

While designing a structure the structural engineer must first determine which of the two cases, drift or core moment is governing. If the moment of the core needs to be reduced, the engineer can place the outriggers at a higher level than the optimal solution. If drift governs the engineer can place the outriggers lower than the optimal solution. [3]

3.11.0 Examples of Outriggers in Practice

3.11.1 The First Wisconsin Center

The First Wisconsin Center is shown in the figure below. The building is 42 storeis and 1.3 million square feet (120,770 meter square). The height of the building is 601 ft (183 m). The building is used as a bank and an office space. It has three belt trusses which are located at the bottom, middle and top of the building. The belt truss located at the bottom of the structure acts as a transfer truss but the belt trusses located at the top and the middle of the structure act as outriggers. The mechanical equipment is located at the outrigger floor. The architectural design was done by the Chicago office of Skidmore, Owings & Merrill. [1]

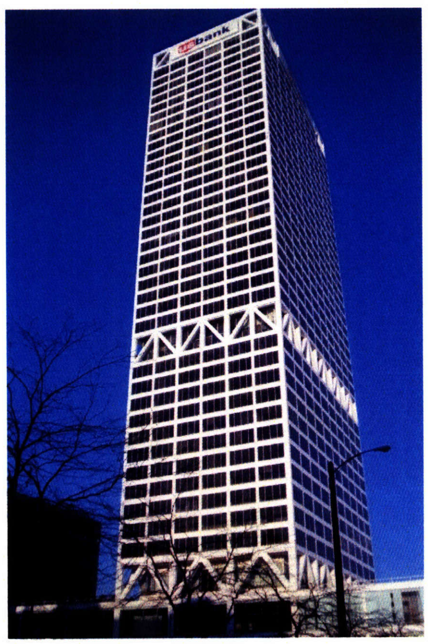


Figure 19-First Wisconsin Center (Milwaukee) [5]

3.11.2 One Houston Center

This building is located in Houston Texas. It has 48-strories. The total height of the building is 681 ft (207.5 m). It has a 2-story outrigger between the 33^{rd} story and 35^{th} story. Because of the façade the outriggers cannot be seen, however a picture of the K-braced outrigger is shown in figure 21. [1]

Figure 20-One Houston Center [6]

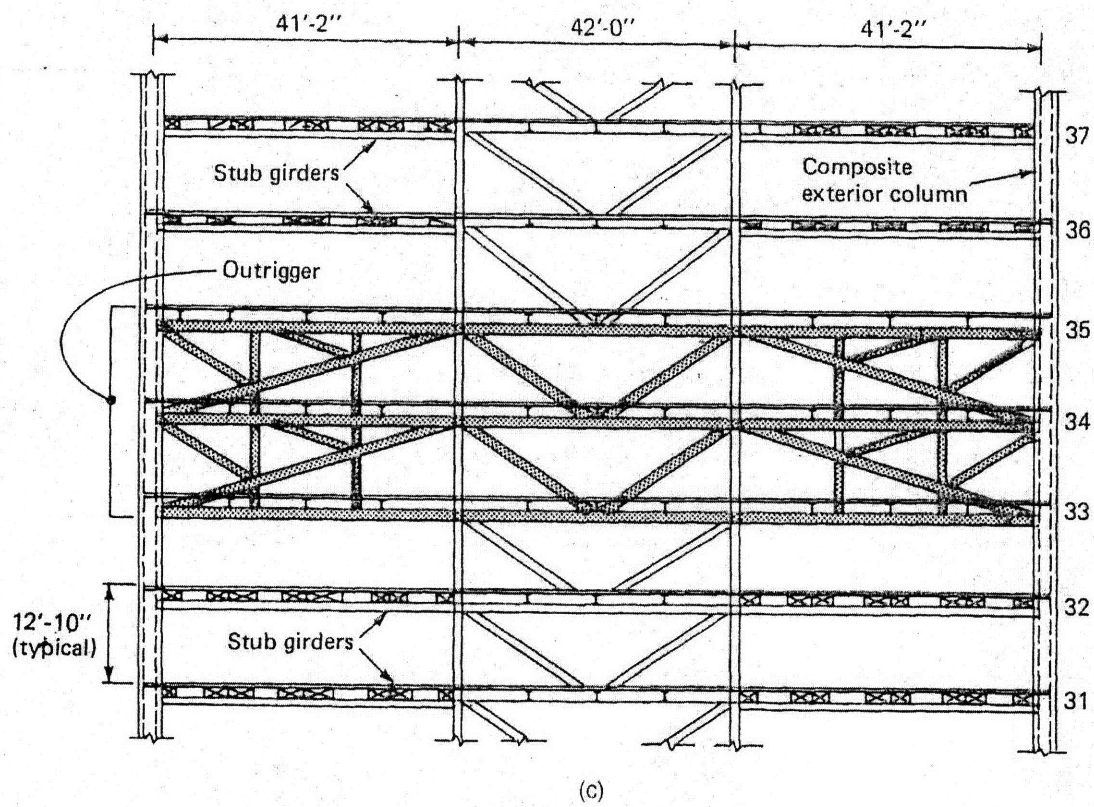


Figure 21- Schematic elevation view of the One Houston Center

The use of outriggers has enabled the engineers to decrease the drift to $\frac{H}{460}$ where H is the total height of the structure. [1]

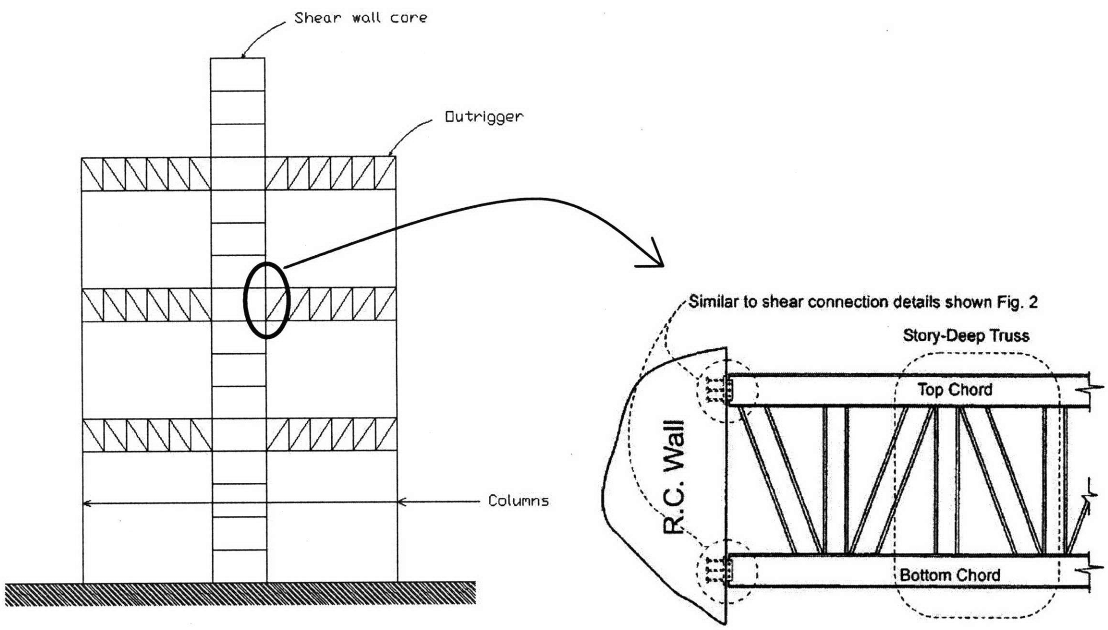
3.11.3 Place Victoria

The Place Victoria is located in Montreal. It's a 47-story building and it was built in 1964. The height of the structure is 623 ft (190 m). This building is special in the sense that it was the first concrete structure to integrate outriggers. It has four levels of X-braced outriggers that connect the four mega corner columns to the core. [2]

Figure 22-Place Victoria in Montreal [7]

3.12.0 Analysis of an Outrigger Connection

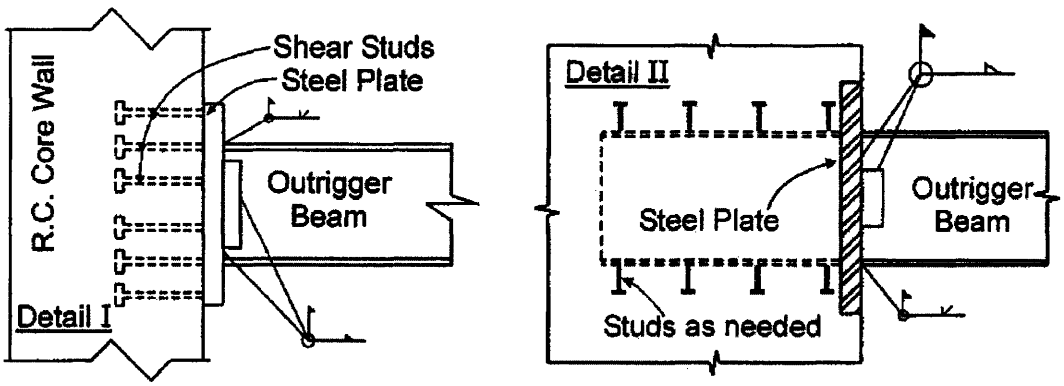
. Figure 23-(a) demonstrates a three level of outrigger attached to a core.



(a) (b) [8]

Figure 23- Outrigger general shape

One of the places that the outrigger has a problem is at the location where the outrigger is connected to the core. According to Bahram M. Shahrooz and etal, two of the common options for connecting the outrigger to the core are shown in figure 24-(a) and figure 24-(b).



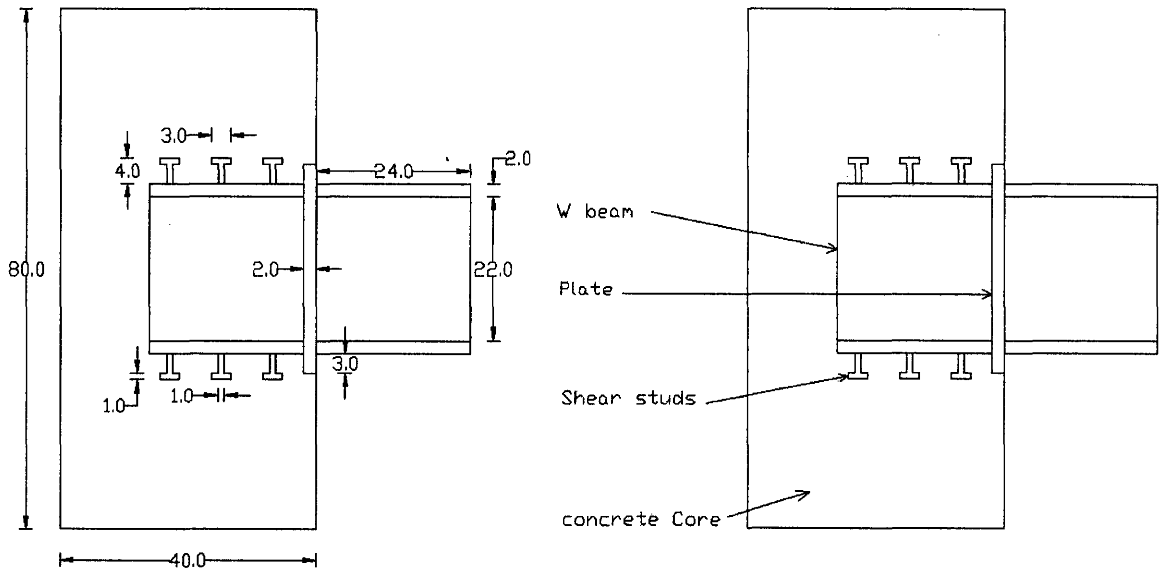
(a) (b)

Figure 24 – Connection Detail [8]

3.12.1 Defining the Problem

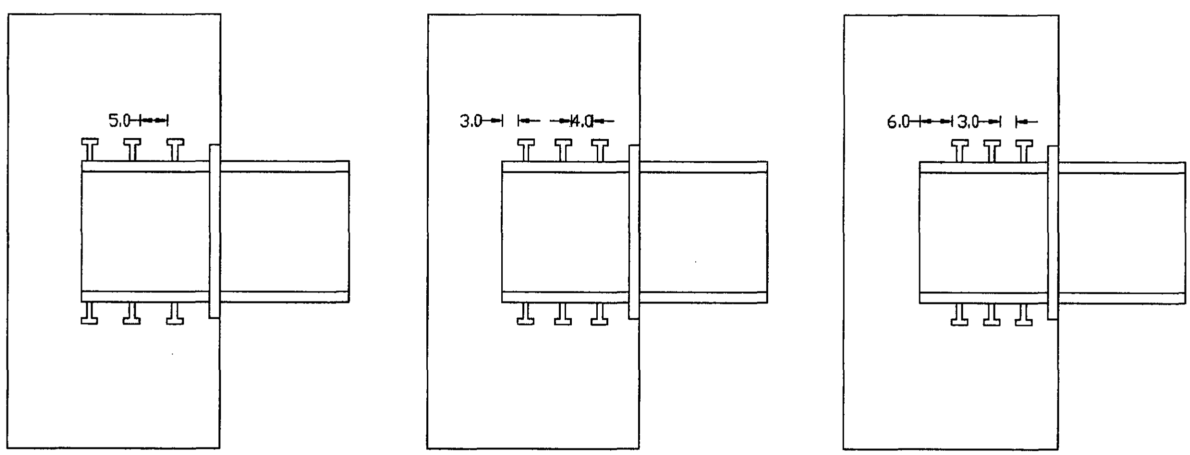
With the brief introduction given for the outrigger system, now we focus on the connection of the outrigger to the concrete core. Two of the common shapes of the connection are shown in figure 24. In this project we have analyzed the shape in figure 24 (b) – for more information on figure 24 (a) see reference 10.

Figure 24-(b) has been modeled with three different configurations. The outrigger beam, steel plate, concrete and the size of the shear studs have been fixed and the distance between the shear studs has been varied. Figure 26 shows the three different configurations and the dimensions between the shear studs. Figure 25-(a) shows the general dimension of the connection and in figure 25-(b) different parts in the connection are introduced.



(a) (b)

Figure 25- Connection Dimensions



(a) (b) (c)

Figure 26 – Three connection configuration

Note: The geometry of the W-beam, plate and the shear studs is shown in figure 27.

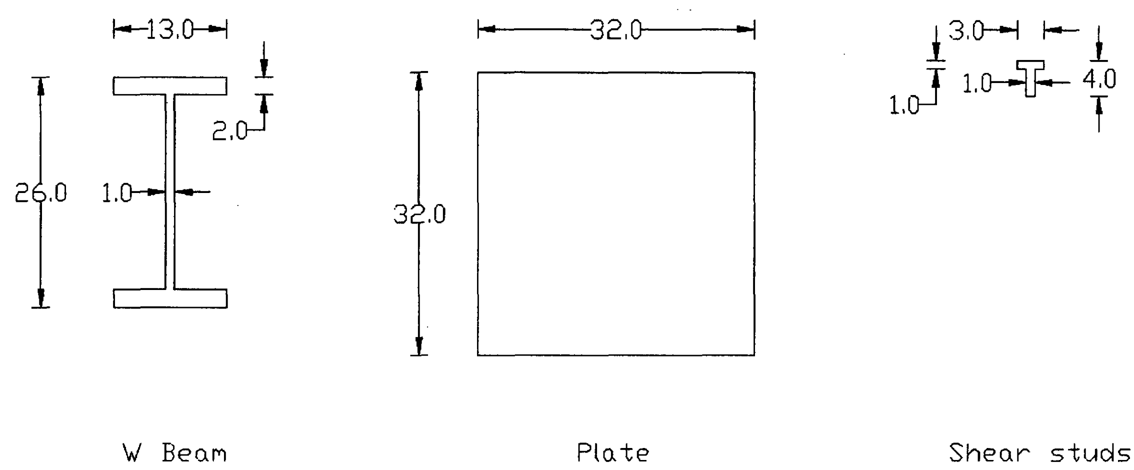


Figure 27 – Element dimension

The problem has been solved as a 2D problem however the thickness of each section has been considered as above. Adina has the power to take into account different thicknesses for each group of elements.

3.12.2 Boundary Conditions

Initially the boundary conditions were taken to be fixed all around the concrete. However, this does not seem valid since the core can move horizontally. Therefore the boundary conditions were modified to allow lateral motion. Figure 28-(a) shows the boundary conditions that were used as the final boundary conditions of the concrete core.

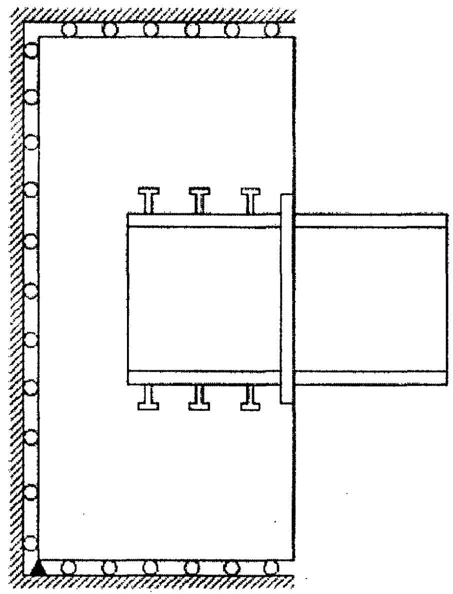


Figure 28 – Boundary conditions

Note that there is a hinge in figure 28-(a) at the left bottom corner to restrict the rigid body motion.

3.12.3 Loading

Usually both a moment and a shear force act on the outrigger connection. Here for simplicity and because we understand the load paths we will only take the moment into account. To apply the moment we apply a uniform distributed load equal on the flanges. These forces represent a moment acting clockwise on the outrigger connection. Figure 29 shows the loading on the connection.

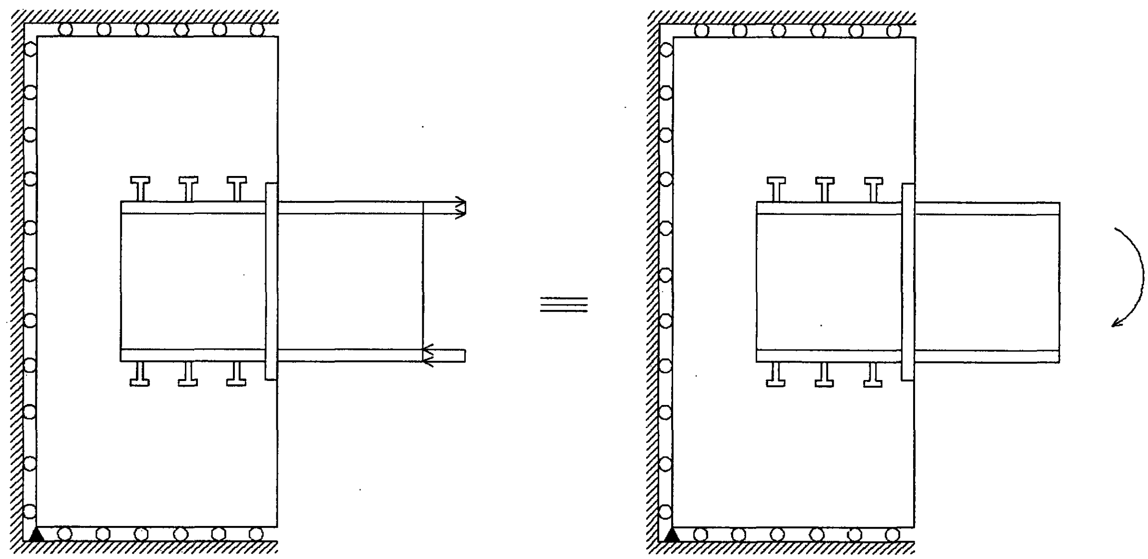

(a) (b)

Figure 29 – Loading model

Figure 29-(a) shows the model analyzed with Adina. It can be shown that the moment is carried mostly by the flanges. This is the reason for placing the coupled forces instead of the moment. (figure 29-(b))

3.12.4 Result of the Finite Element Analysis

As previously mentioned, three different configurations of the connection have been established and analyzed. The results are illustrated in Figure 30.

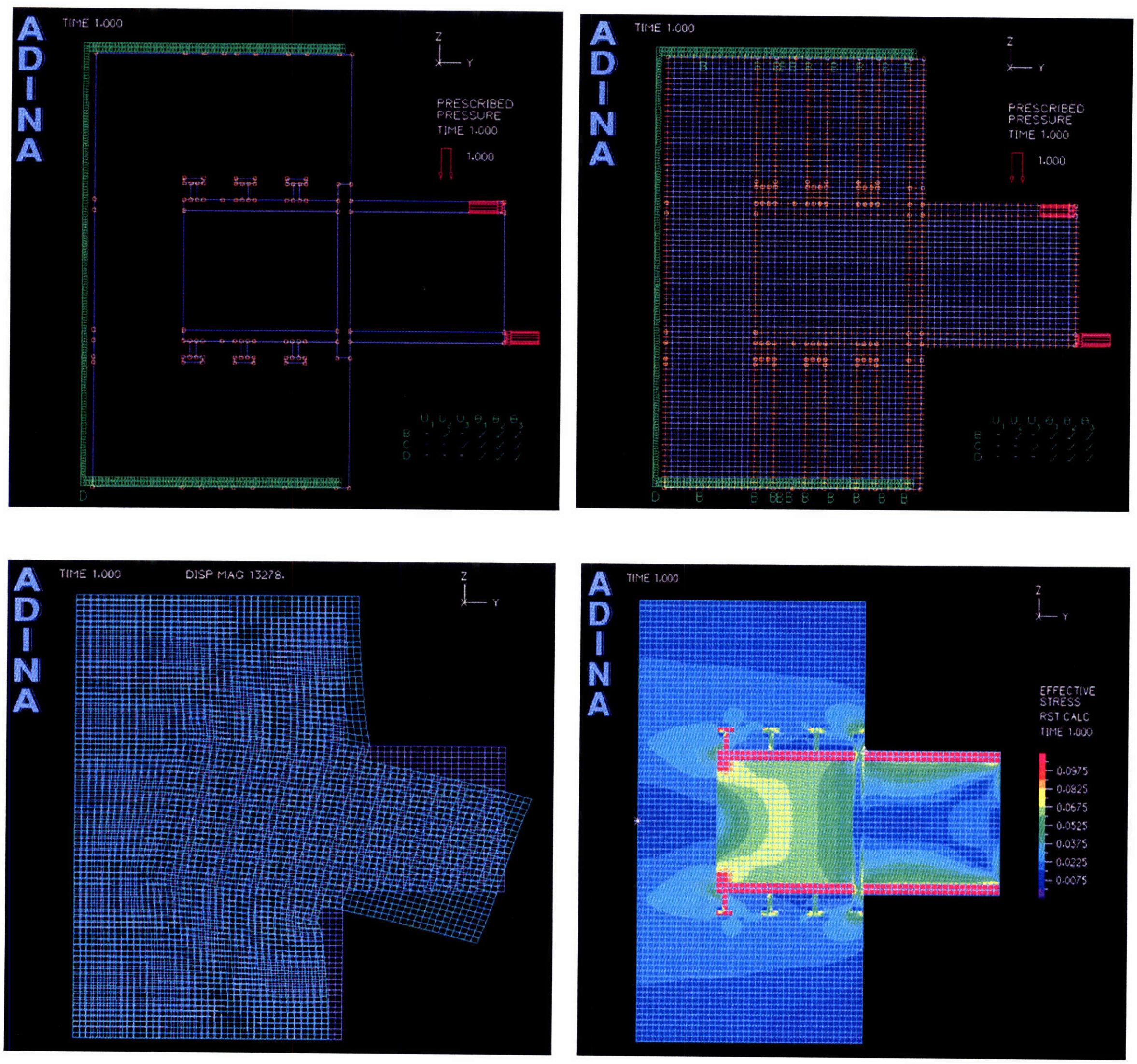


Figure 30 – Case 1 stress pattern

Figure 30 shows the result for configuration 1. (Figure 26-(a))

The result of the next configuration is shown in figure 31.

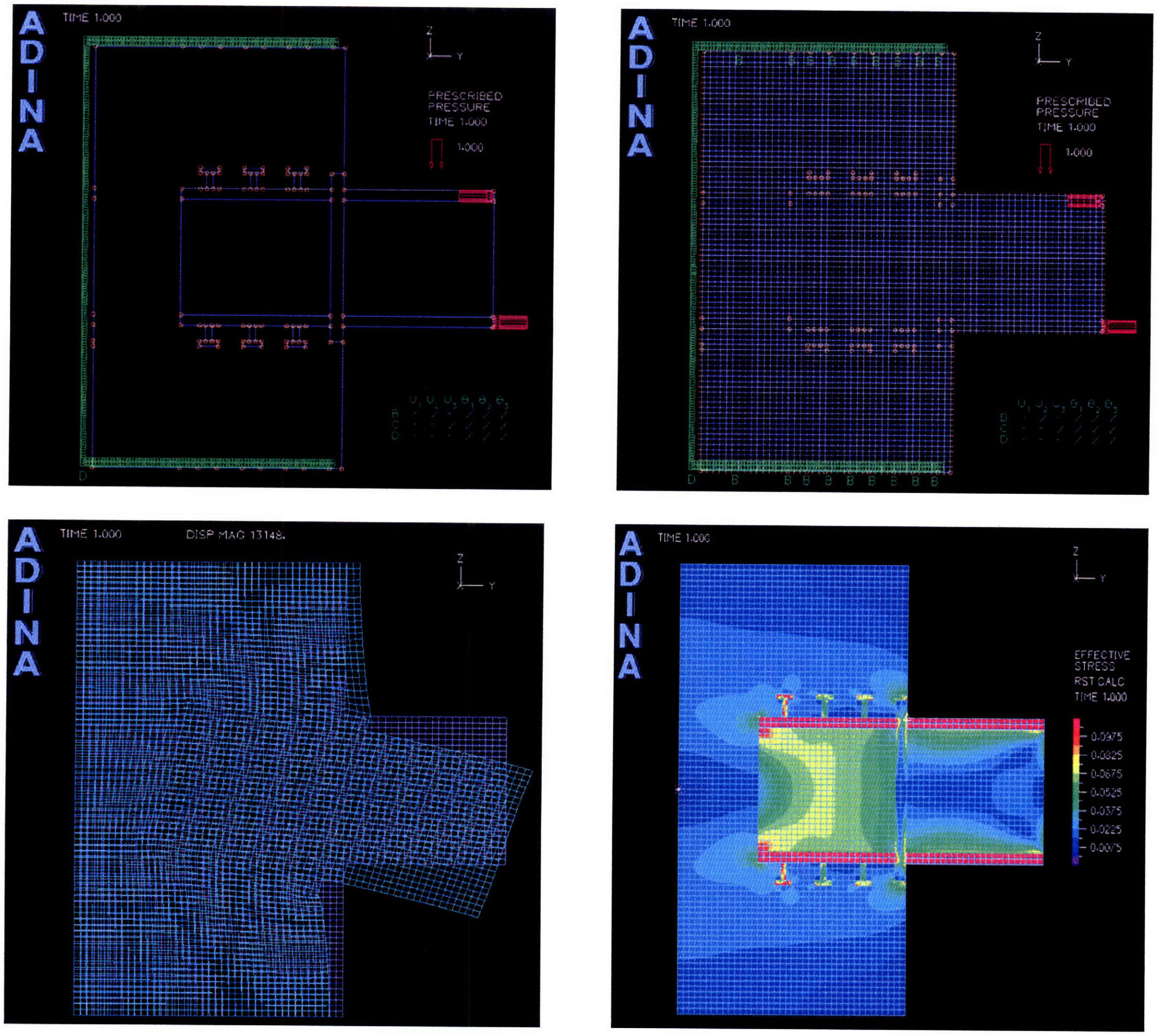


Figure 31 – Case 2 stress pattern

The dimension of this configuration has been illustrated in figure 26-(b)

Finally, the last configuration result is shown in figure 32. The dimensions for this shape are demonstrated in figure 26-(c)

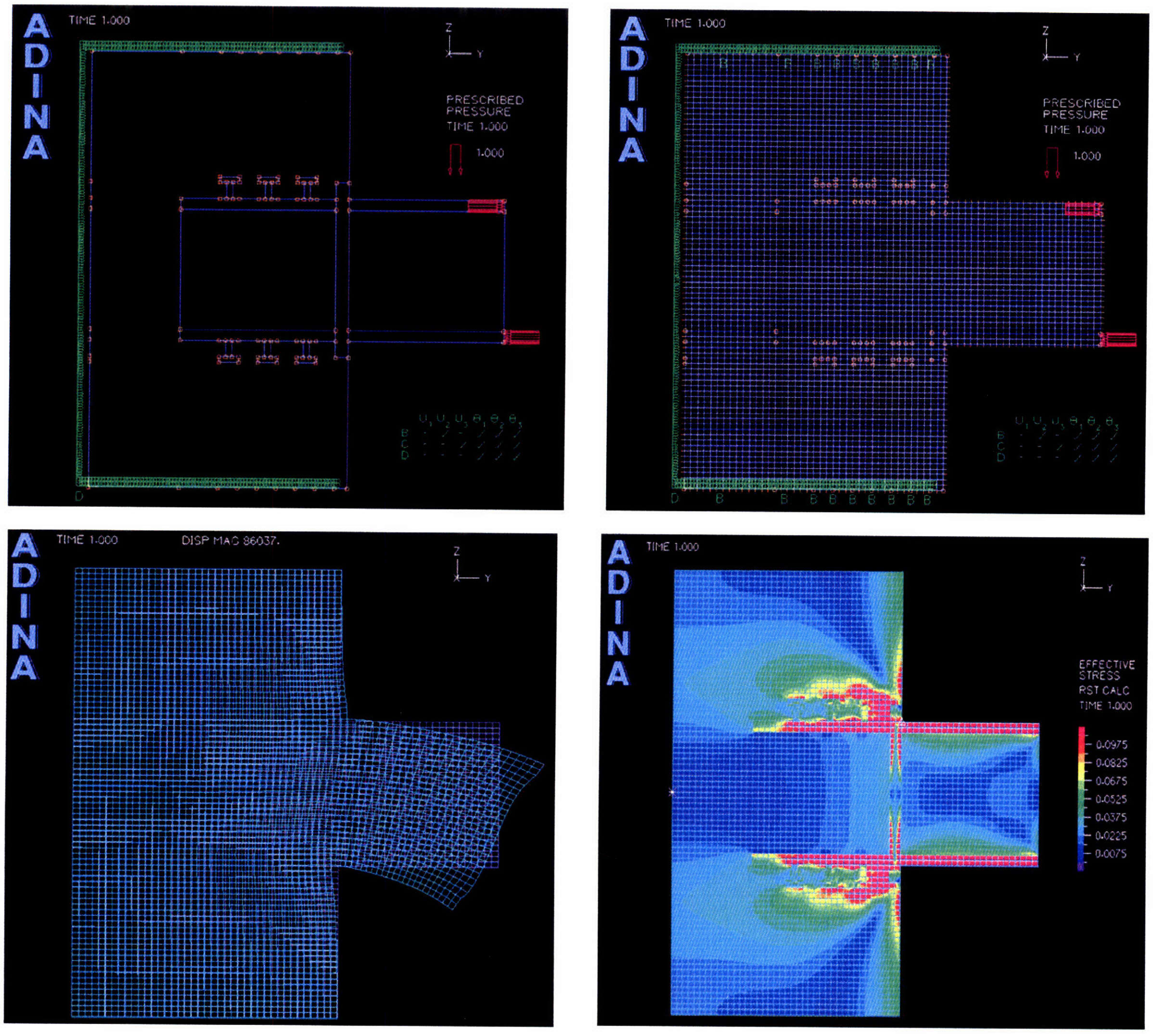


Figure 32 - Case 3 stress pattern

3.12.5 Time Step Analysis

One task that can be performed in ADINA is a time step analysis. This type of analysis shows how the load flows according to time steps that have been specified. For this project the load was applied by the following four time steps as written below:

$@t_1 \quad F = 0.1 \times P$

$@t_2 \quad F = 0.3 \times P$

$@t_3 \quad F = 0.6 \times P$

$@t_4 \quad F = 1 \times P$

Where P is the magnitude of the force applied which is one for this project and F is the magnitude of the force at time t

The time step function has been illustrated in figure 33.

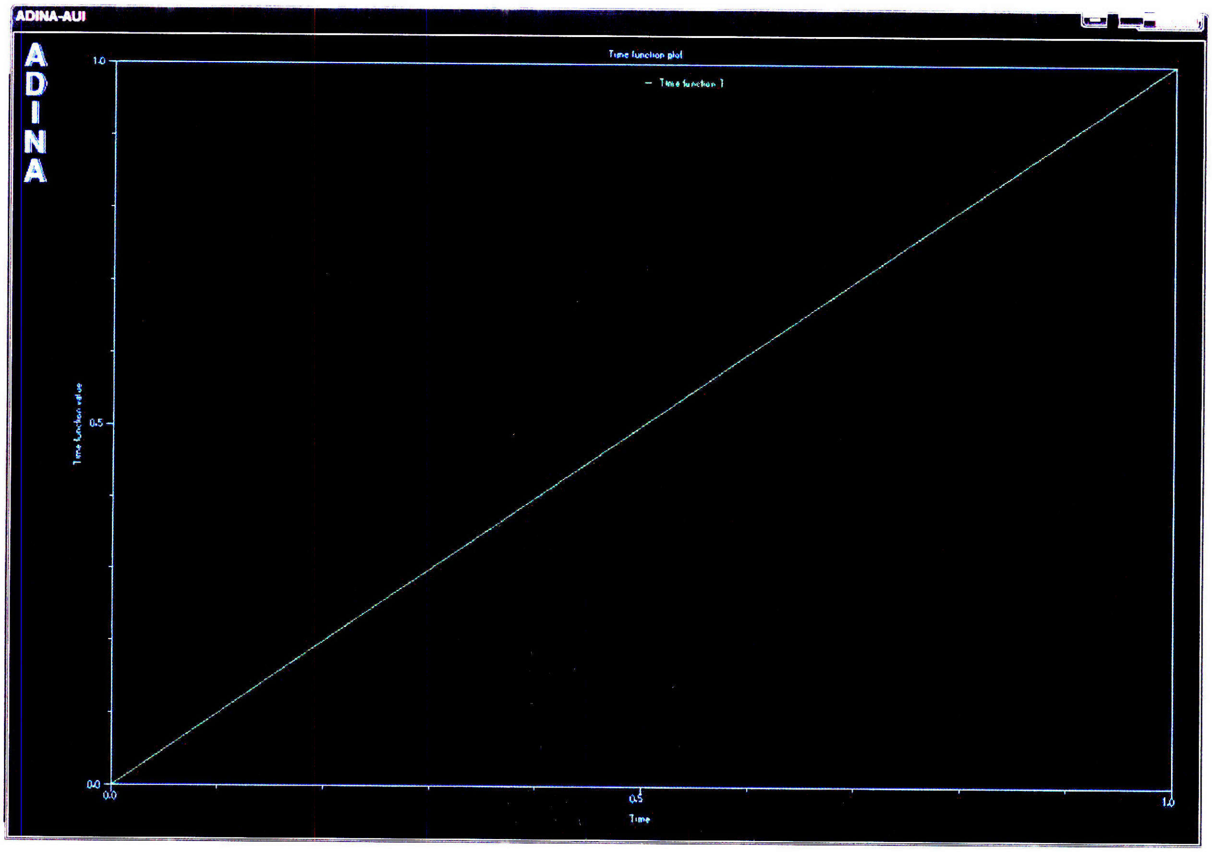


Figure 33 – Time function

The results of the three configurations are shown below, one after another, with the same order as before shown in figure 26-(a), (b) and (c)

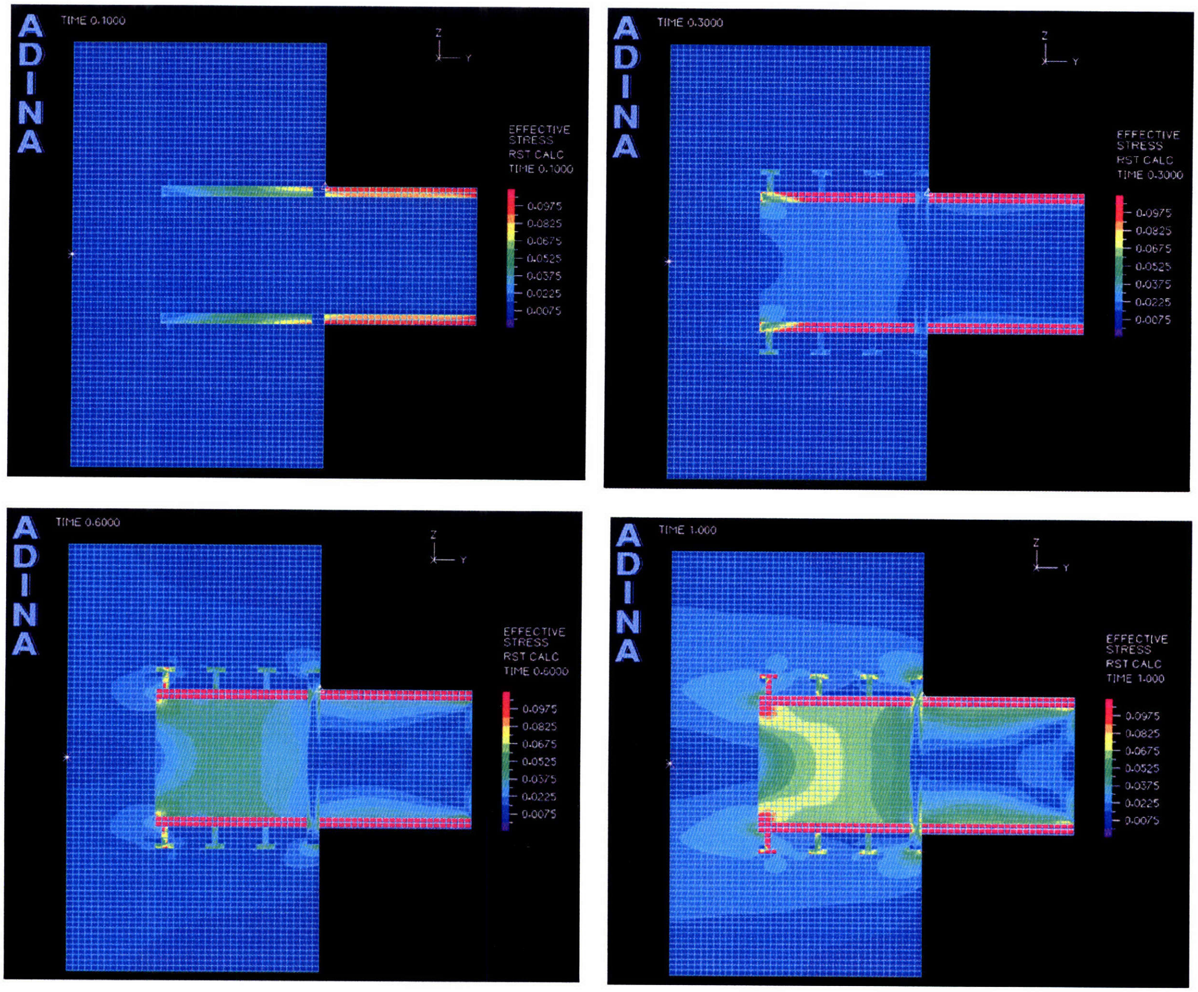


Figure 34 – Time analysis of case 1

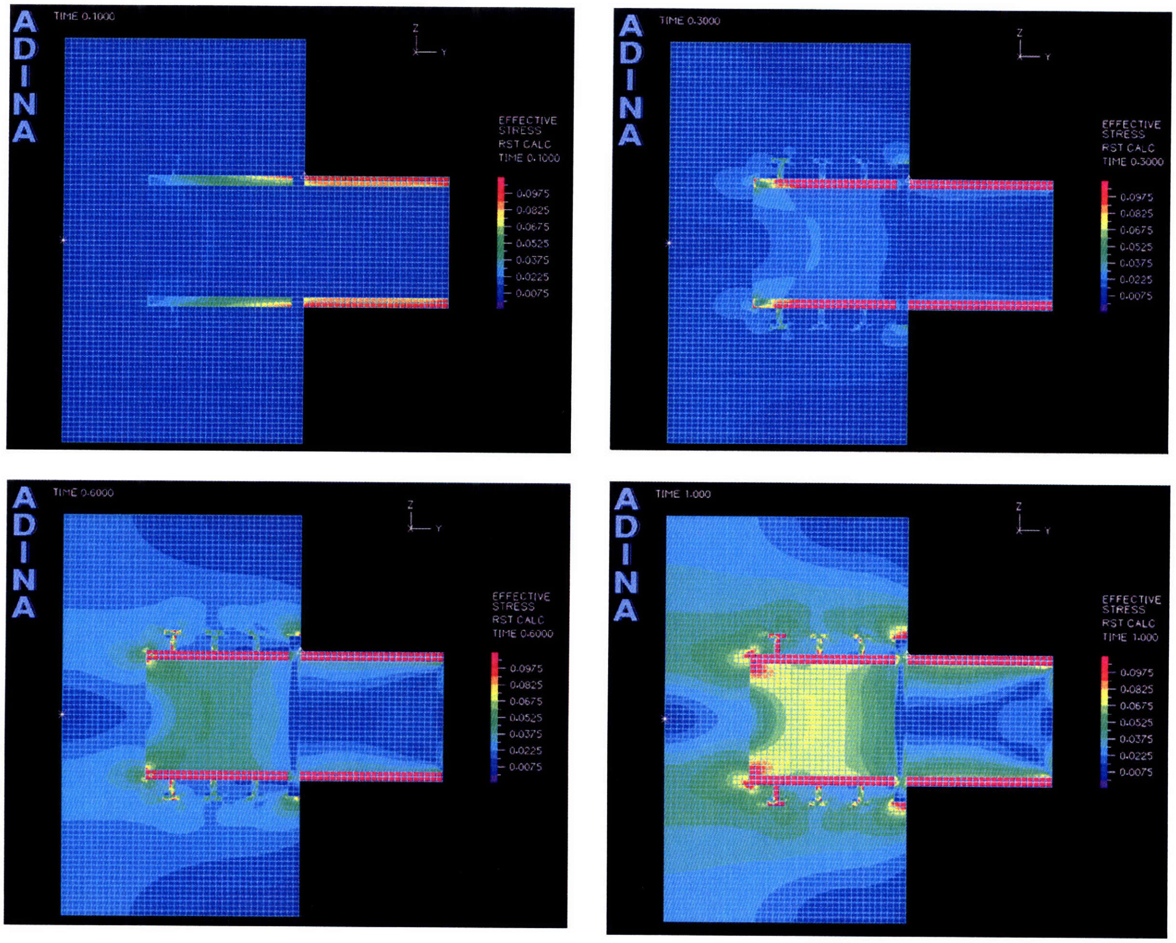


Figure 35 - Time analysis of case 2

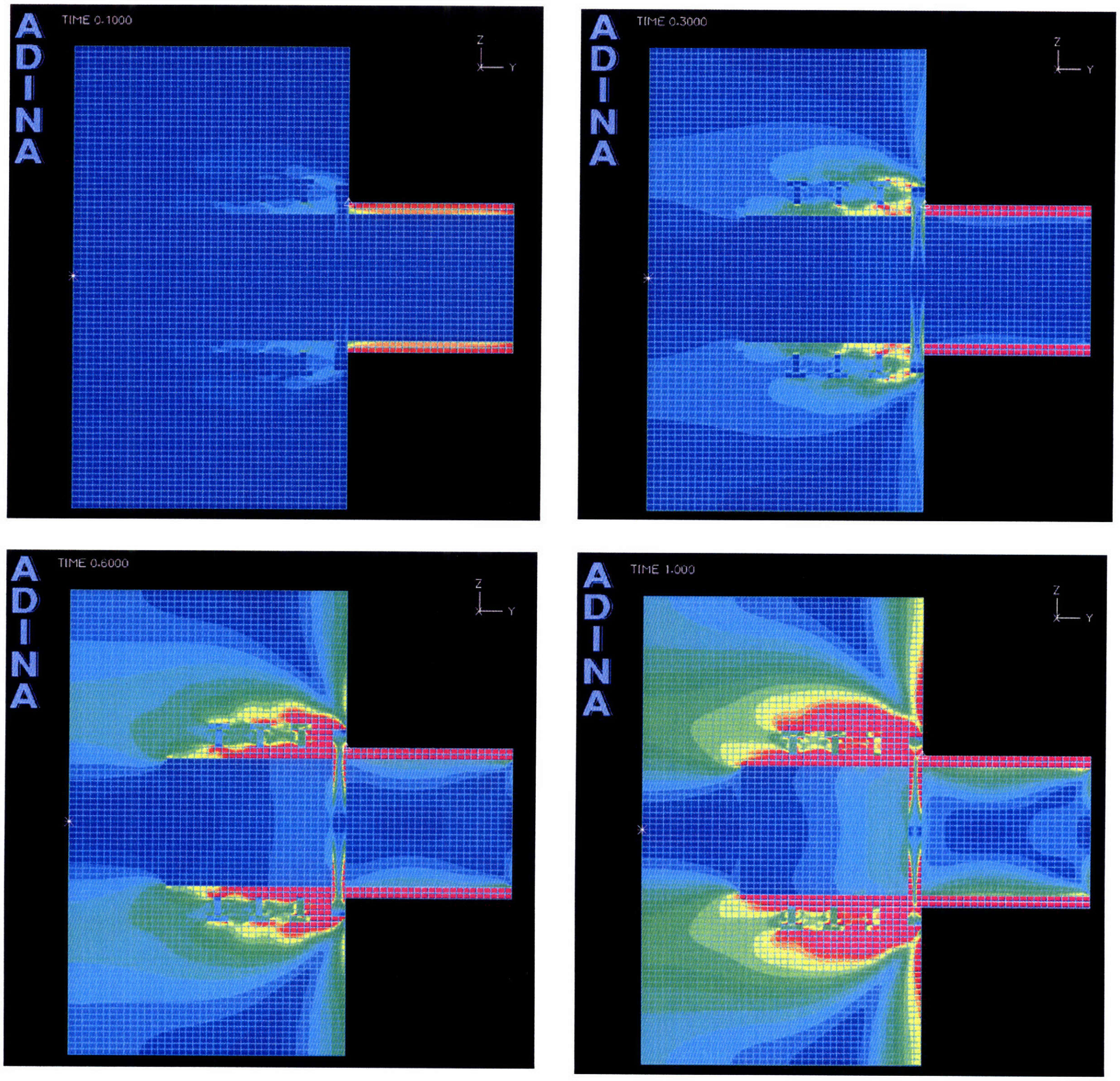


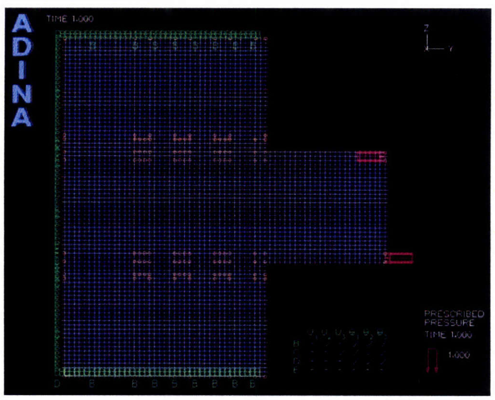
Figure 36 - Time analysis of case 3

3.12.6 Accuracy of the model

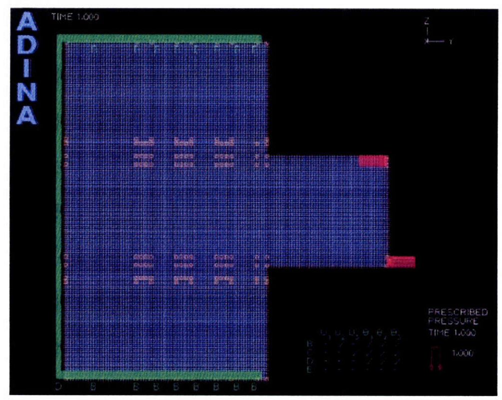
To check the accuracy of the mesh, six different types of mesh were constructed and the results have been compared. Table 1 shows the resulting strain energy for each case. Figure 37 shows the different meshes.

Mesh Type / Element Nodes	Finest Mesh	Fine Mesh	Coarse Mesh
4 Node Elements	0.5409E-03	0.5419E-03	0.5388E-03
9 Node Elements	0.5421E-03	0.5424E-03	0.5414E-03

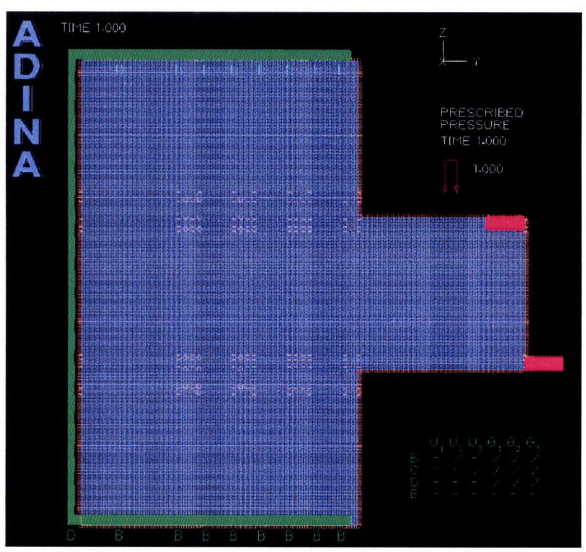
Table 1- strain energies of different meshes



(a) Coarse Mesh



(b) Fine Mesh



(c) Finest Mesh

Figure 37 – Different meshes

		E	E (Exact)	E2-E1	Log(E2-E1)	Log(h)
	1x1	0.0005389	0.0005424	3.513E-06	-5.454333	0
4 nodes	0.5x0.5	0.000541	0.0005424	1.453E-06	-5.837713	-0.30103
	0.25x0.25	0.0005419	0.0005424	5.275E-07	-6.277739	-0.60206
	1x1	0.0005414	0.0005424	9.64E-07	-6.015944	0
9 nodes	0.5x0.5	0.0005421	0.0005424	2.955E-07	-6.529376	-0.30103
	0.25x0.25	0.0005424	0.0005424	4.354E-08	-7.36108	-0.60206

Table2 – Convergence

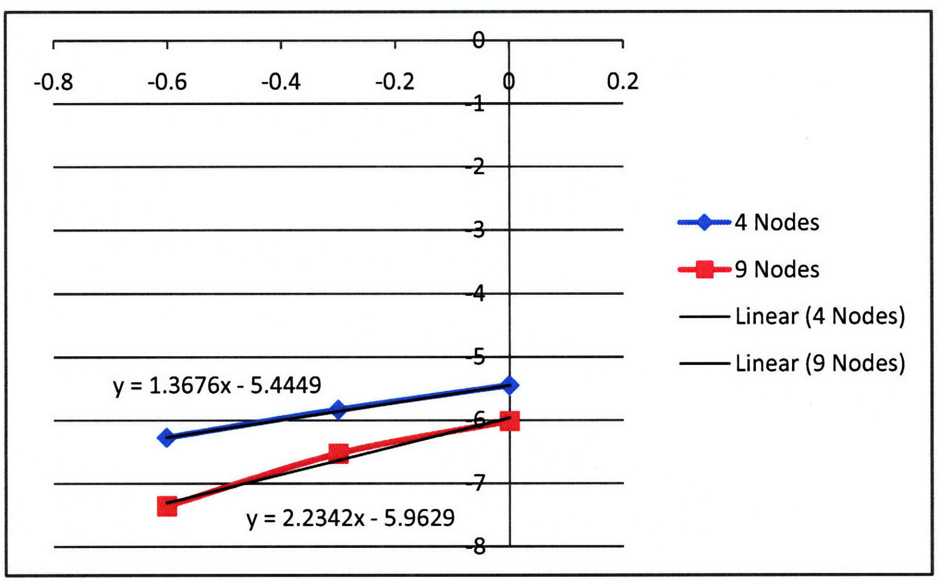


Figure 38- Convergence lines

As it can be seen the rate of convergence for a nine node element is almost double the amount of a four node element. (The convergence rate is the slope of the lines in figure 38)

For this project the nine node element was found appropriate. As shown in table 1 the nine node element with a coarse mesh will have sufficient accuracy to give reliable results with the least amount of elements.

3.12.7 Force Graph

Another function available in ADINA is the ability to plot force and moment graphs. As an example the force on the vertical line was plotted and checked with the applied force to show the reliability of the model. The reaction in the Y direction is as figure 39

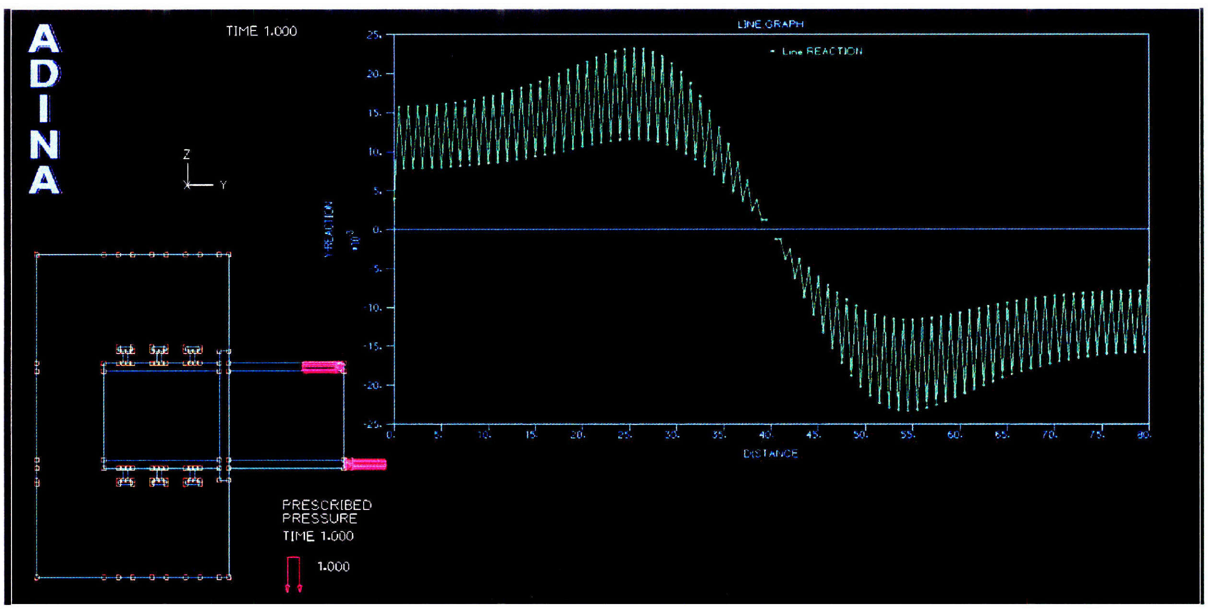
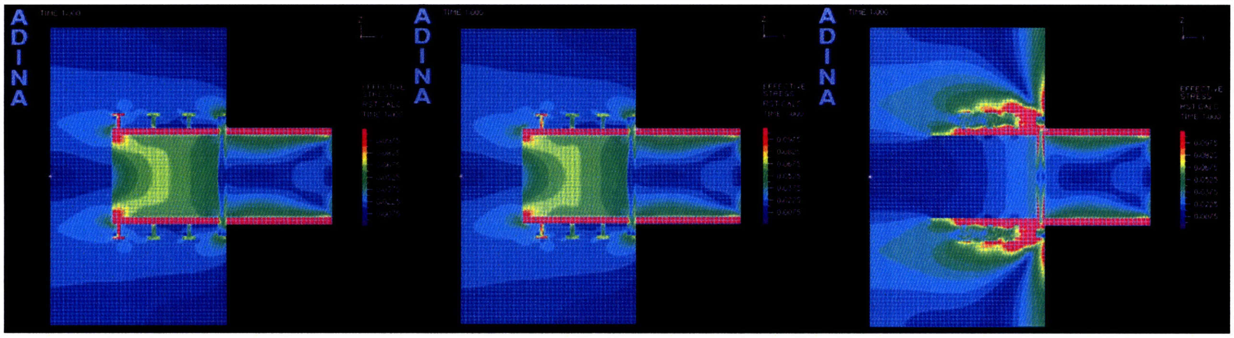


Figure 39 – Graph of force on the vertical boundary conditions

As it can be seen in figure 39 each side of the graph (from 40 which is located in the middle) has 78 points. By taking 12.5 as the average value for the Y-reaction value we get the amount of 0.975 which is close to 1 (the moment applied to the connection).

3.12.7 Interpretation of Result

For conclusion we can compare the stress pattern for each of the three outrigger connection detail configurations. To do so observe the three stress patterns below:



(a) (b) (c)

Figure 40 – Stress patterns for the three configurations

In figure 40 as we move from figure 40-(a) towards figure 40-(c) the distance between the shear studs decrease. The forces are carried by the shear studs as long as they have a certain amount of distance from the plate which connects the two beams. However, if the shear studs are too close to the plate then the force will be carried out more by the concrete core which does not seem appropriate. Therefore Configuration (a) and (b) seem more reasonable to use. For more advice on these connections more analysis is needed which was found out of the scope of this project.

3.13.0 Visual Basic Program

3.13.1 Introduction

Outriggers help to decrease the deflection of the structure. In addition, they help to decrease the moment of the core. These issues were discussed in-depth in section 3.9.0. A program was written to calculate the top deflection of the outrigger structure and the moment at the base. Theoretically the algorithm used in the program can be used for infinite number of outriggers. However, the user interface limits the actual number of outriggers. The author decided to allow the user to analyze a structure with up to 10 outriggers. This number is more than two times the amount of outriggers that act most efficiently as determined in section 3.11.0. However, one should be aware that by changing the stiffnesses of columns, outriggers, and the core, the efficient number of outriggers will change. There may be some limitation to this interface.

Using this program is very easy and it has been explained below.

3.13.2 Instructions

The interface of the program can be seen in figure 41.

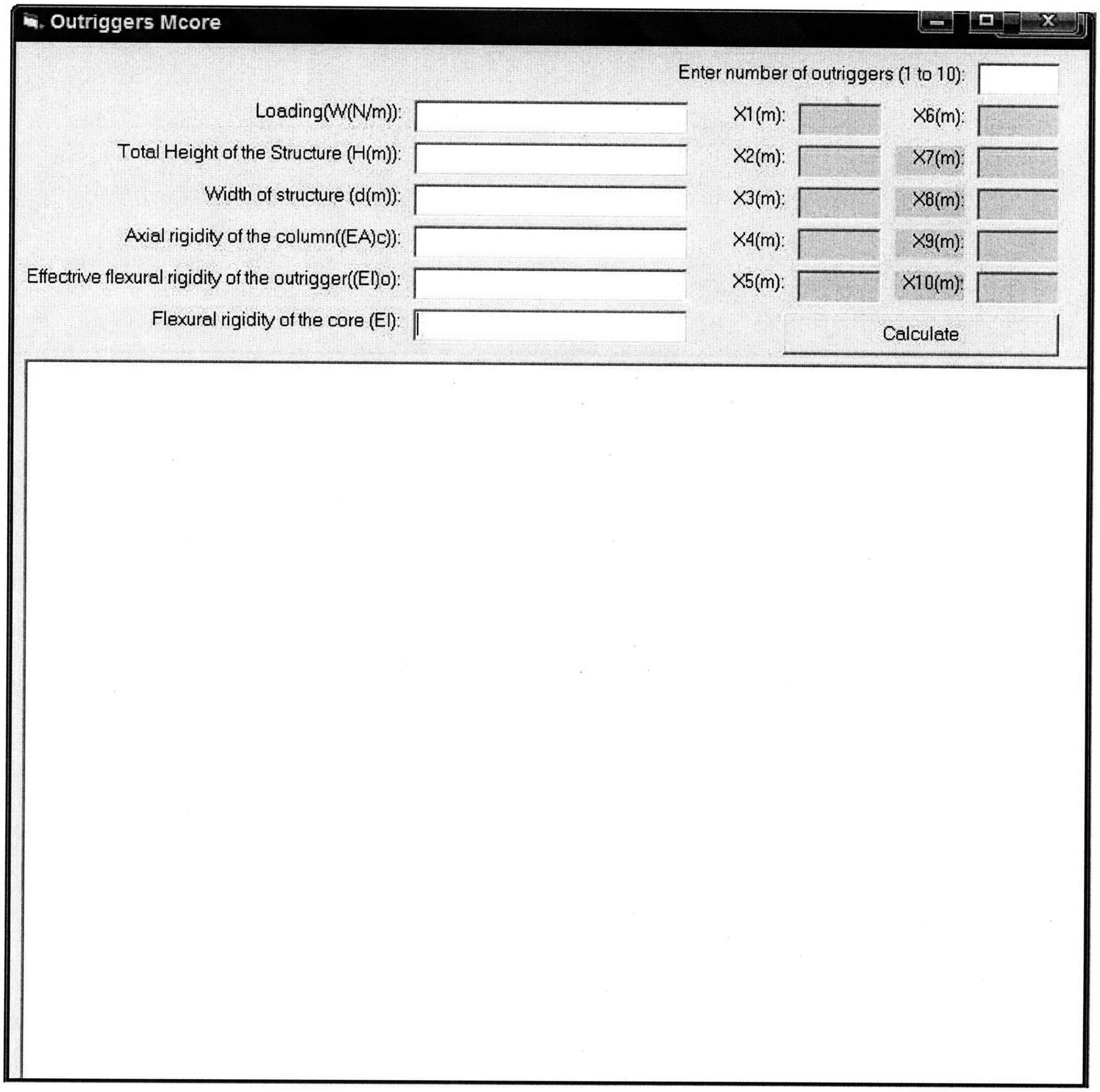


Figure 41- Visual Basic Program Interface

The program is introduced in the steps below.

<u>Step 1</u>- Enter the data in the appropriate place located in front of them.

- First enter the loading which is assumed to be a uniform loading with the magnitude of "w" with the units of newton per meter.
- Second enter the total height of the structure. (The units are in meters)
- Third you can enter the width of your structure. (This is the length of the outriggers on both sides of the core plus the core width- the units are in meters.)
- In the three remaining spots enter the axial rigidity of the columns, the flexural rigidity of the outriggers and the core flexural rigidity.
- Tell the program how many outriggers are needed and their locations. The number of outriggers can be specified in the place which says "number of outriggers (1 to 10)". After defining the number of outriggers the program will allow the user to enter the location of the same amount of outriggers. Notice that the locations are defined from the top of the structure. Figure 42 shows the configuration for a two level outrigger system.

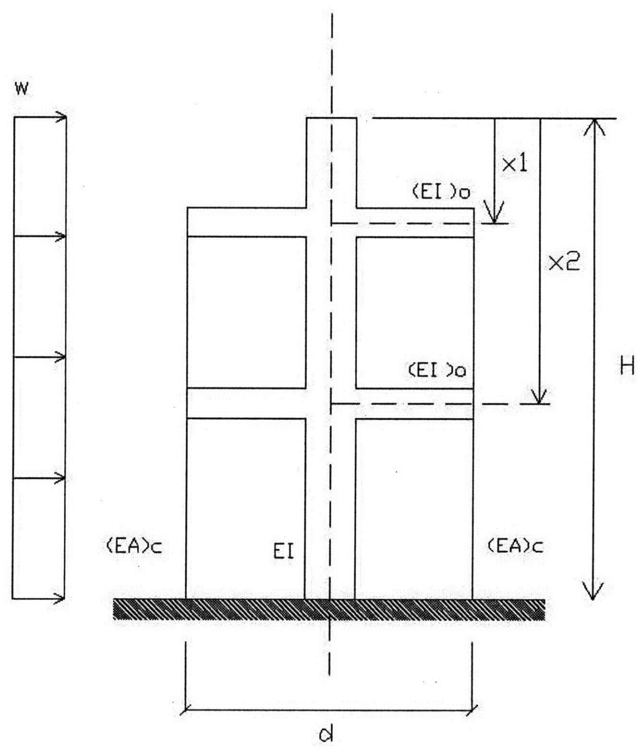


Figure 42- Program data configuration

Step 2- Press the "calculate" button and the results will be shown in the white space below. More details about how to interpret the output will be given through an example.

3.13.3 Example

Assume the properties of a building are:

n (number of outriggers) = 4

W = 600 N/m

H = 100 m

d = 26 m

(EA)c = 2E11 N

(EI)o = 1E25N.m^2

EI = 14E12 N.m^2

X1 = 20 m

X2 = 40 m

X3 = 50 m

X4 = 70 m

Enter as displayed in figure 43. Finally, press the calculate button and the result of analyzing the structure is given in the white area below the place that we entered the properties of the structure.

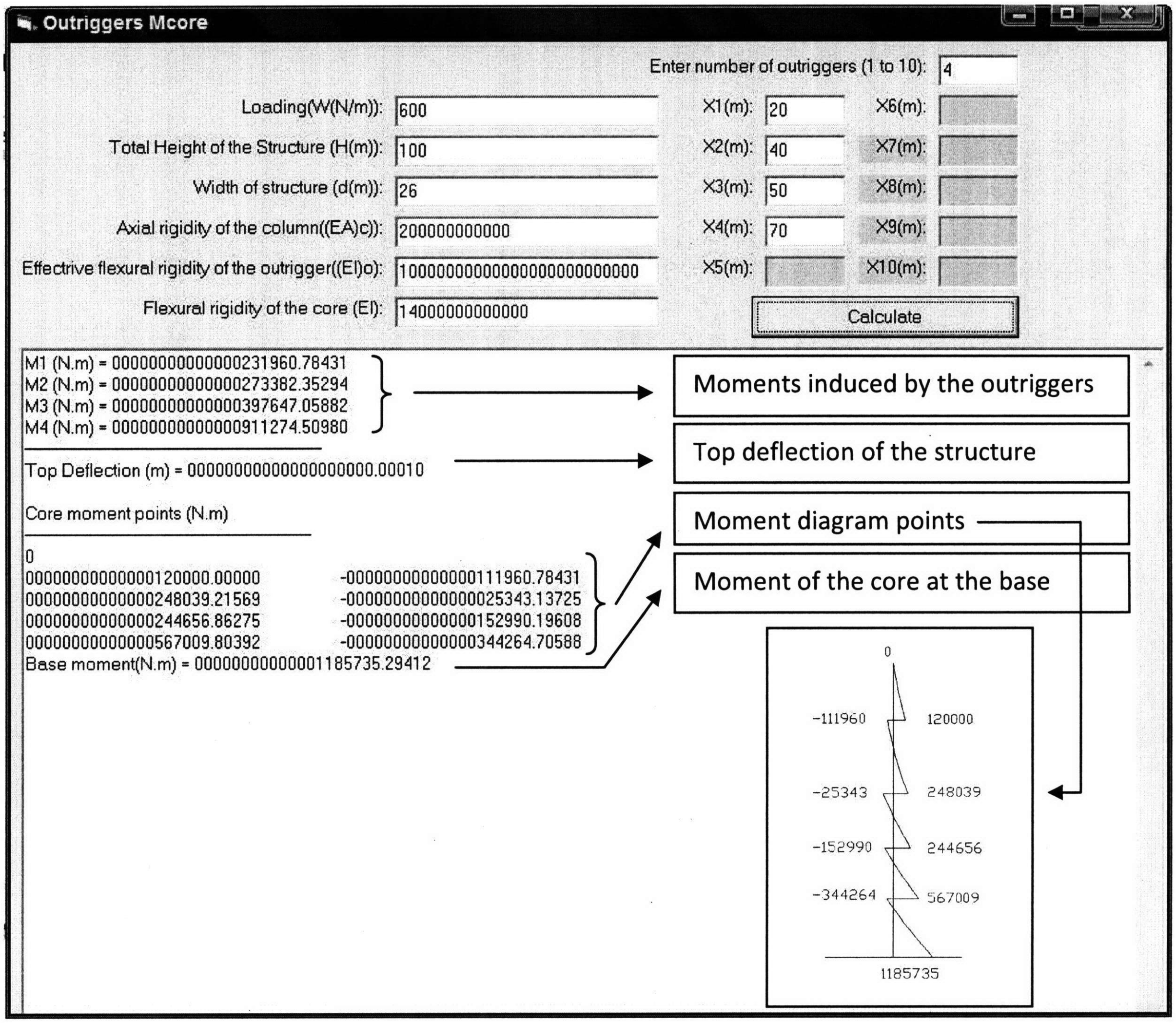


Figure 43 – Program result

4.0.0 Conclusion

Outriggers are with no doubt an efficient structural system against lateral loading and wind in specific. In general they reduce the drift of the structure and reduce the moment in the core. There can be several layers of outriggers in a structure. There optimum placement depends on a multitude of structural factors such as the location of the outriggers, the axial rigidity of the columns, the flexural rigidity of the core and the outriggers. However it is important to keep in mind that sometimes there are additional factors other than structural criteria. For example, the location of the outrigger might be changed from its optimal structural location either for architectural or mechanical issues. These constraints must also be integrated in the design. Cost is another decisive factor. The economics of a building have a great influence on the design procedure. Ultimately the final design decision will be the one that satisfies the majority most of these elements. Optimization depends on the time, place and each individual structure. [1,3]

5.0.0 References

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6-http://www.skyscrapercity.com/showthread.php?t=69136, retrieved April 1st 2008

7- www.arch.mcgill.ca/.../concrete/timeline.html, retrieved April 1st 2008

8- Bahram M. Shahrooz; Jeremy T. Deason; and Gokhan Tunc, Outrigger Beam–Wall Connections. I: Component Testing and Development of Design Model, JOURNAL OF STRUCTURAL ENGINEERING © ASCE / FEBRUARY 2004.

9- K.J. Bathe, *Finite Element Procedures*, Prentice Hall, Englewood Cliffs, NJ, 1996

10- Renard Gamaliel Meng 2008, Frequency based response of the damped outrigger system for tall buildings

6.0.0 Appendix

6.1.0 Cantilever bending and displacement under uniform loading

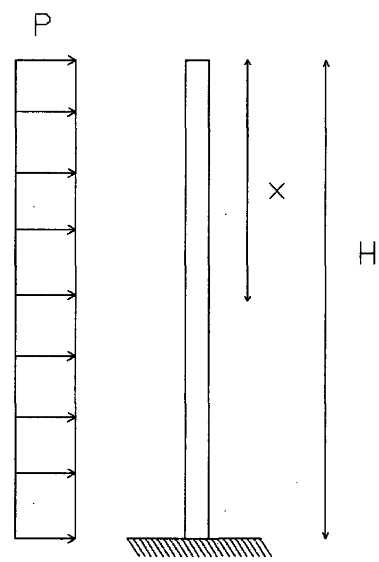

Figure 44-A cantilever with a uniform loading

$$U(x) = \frac{P}{24EI}(x^4 - 4H^3x + 3H^4)$$

$$\theta(x) = \frac{P}{6EI}(x^3 - H^3)$$

Where:

U(x) is the transverse displacement at point x for a cantilever beam with a uniform load on it.

And $\theta(x)$ is the rotation of the cantilever beam with the uniform load on it at point x.

6.2.0. Cantilever bending and displacement under concentrated load and moment

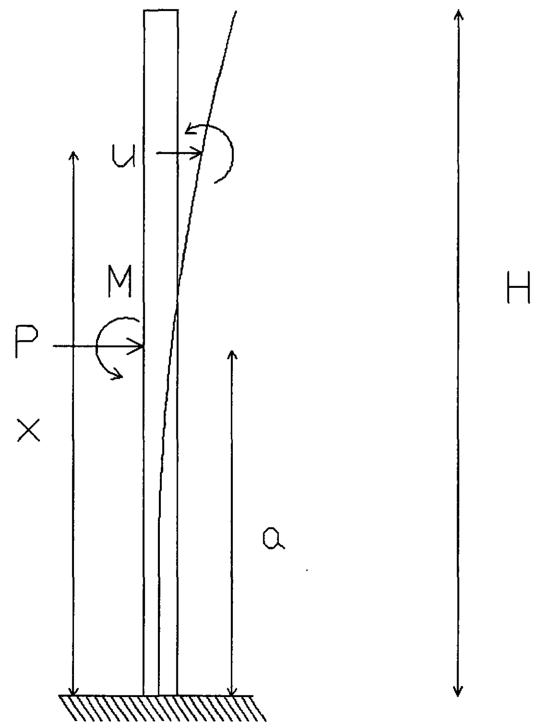

Figure 45- A cantilever with a concentrated load and concentrated moment

$$U(x) = \frac{P}{6EI}x^2(3a - x) - \frac{M}{2EI}x^2 \qquad x \le a$$

$$\theta(x) = -\frac{P}{2EI}x(2a - x) + \frac{M}{EI}x \qquad x \le a$$

$$U(x) = \frac{P}{6EI}a^2(3x - a) - \frac{M}{2EI}a(2x - a) \qquad x \ge a$$

$$\theta(x) = -\frac{P}{2EI}a^2 + \frac{M}{EI}a \qquad x \ge a$$

6.3.0. Program Code

The following are listing from the metablites.bas File:

'Define Global variables

'Matrix dimensions are set to Max 10x10 for the interface needs, but can be increased here to whatever

Public Const MAX_DIM = 10

Public System_DIM As Integer 'Current Matrix [A] dimensions

Public Matrix_A(1 To MAX_DIM, 1 To MAX_DIM) As Double

Public Operations_Matrix(1 To MAX_DIM, 1 To 2 * MAX_DIM) As Double 'Matrix where the calculations are done

Public Inverse_Matrix(1 To MAX_DIM, 1 To MAX_DIM) As Double 'Matrix with the Inverse of [A]

Public tempArray(1 To MAX_DIM) As Double ' used for:H^3-X1^3, H^3-X2^3...H^3-Xn^3

Public mArray(1 To MAX_DIM) As Double ' used for M1 to Mn values

Public S_ As Double

Public S1_ As Double

Public W_Divide_6EI_ As Double ' W / 6 * EI

Public DeltaO_ As Double ' delta=w*H^4/8*EI - 1/EI * (M1*(H^2 - X1^2) + M2*(H^2 - X2^2) + ... + Mn*(H^2 - Xn^2))

Public mCore_(1 To MAX_DIM, 1 To 2) As Double

Public mCoreLast_ As Double

Public Solution_Problem As Boolean 'Determines whether the inverse was found or not

The following are from Form1b.frm file:

```
' Author   Ali Almeh

' Date     05/01/2008

'Calculates the Inverse of a Rectangular Matrix [A] (Dimensions N x N) using the Gauss
elimination method,


Private Sub cmdCore_Click()

 Me.Text3.Text = ""

 If checkAll = False Then Exit Sub

 ' compute S = 1/EI + 2/d^2 (EAc)

 S_ = 1 / CDbl(Me.txtEI.Text) + 2 / (CDbl(Me.txtWD.Text) ^ 2 * CDbl(Me.txtEAc.Text))

 ' compute s1 = d / 12(EIo)

 S1_ = CDbl(Me.txtWD.Text) / (12 * CDbl(Me.txtEIo.Text))

 ' build matrix 1 to n

 W_Divide_6EI_ = CDbl(Me.txtWT.Text) / (6 * CDbl(Me.txtEI.Text))

 Build_Matrix

 Calculate_Inverse

 loadTempArray

 multiply_Matrix

 Compute_DeltaO

 Compute_Mcore

 display_result


End Sub
```

```
Sub display_result()
 Dim msg As String
 Dim n As Integer, m As Integer
 Const FORMAT_STRING As String = "#00000000000000000000.00000"

' msg = "S = " & S_ & Space(20) & "S1 = " & S1_ & vbCrLf & vbCrLf
' msg = msg & " Matrix " & vbCrLf & "========" & vbCrLf
' For m = 1 To System_DIM
'   For n = 1 To System_DIM
'     msg = msg & Matrix_A(n, m) & Space(20)
'   Next
'   msg = msg & vbCrLf & vbCrLf
' Next
' msg = msg & " Inverse Matrix " & vbCrLf & "================" & vbCrLf
' For m = 1 To System_DIM
'   For n = 1 To System_DIM
'     msg = msg & Inverse_Matrix(n, m) & Space(20)
'   Next
'   msg = msg & vbCrLf & vbCrLf
' Next

 For n = 1 To System_DIM
   msg = msg & "M" & n & " (N.m) = " & Format(mArray(n), FORMAT_STRING) & vbCrLf
 Next
 msg = msg & "--------------------------------------------------------" & vbCrLf
```

```
 msg = msg & "Top Deflection (m) = " & Format(DeltaO_, FORMAT_STRING) & vbCrLf &
vbCrLf

 msg = msg & "Core moment points (N.m)" & vbCrLf & "--------------------------------------------
---------" & vbCrLf

 msg = msg & "0" & vbCrLf


 For n = 1 To System_DIM

   msg = msg & Format(mCore_(n, 1), FORMAT_STRING) & Space(20) & Format(mCore_(n,
2), FORMAT_STRING) & vbCrLf

 Next

 msg = msg & "Base moment(N.m) = " & Format(mCoreLast_, FORMAT_STRING)

 Me.Text3.Text = msg

End Sub


Sub Build_Matrix()

 ' S1+S(H-X1) S(H-X2) S(H-X3) ... S(H-Xn)

 ' S(H-X1) S1+S(H-X2) S(H-X3) ... S(H-Xn)

 ' S(H-X1) S(H-X2) S1+S(H-X3) ... S(H-Xn)

 ' ....

 ' ....

 ' S(H-X1) S(H-X2) S(H-X3) ... S1+S(H-Xn)

 Dim n As Integer

 Dim m As Integer


 For n = 1 To System_DIM

  For m = 1 To System_DIM
```

```
    If n = m Then
      Matrix_A(n, m) = S1_ + S_ * (CDbl(Me.txtHT.Text) - CDbl(Me.txtXM(m - 1).Text))
    ElseIf n > m Then
      Matrix_A(n, m) = S_ * (CDbl(Me.txtHT.Text) - CDbl(Me.txtXM(n - 1).Text))
    Else
      Matrix_A(n, m) = S_ * (CDbl(Me.txtHT.Text) - CDbl(Me.txtXM(m - 1).Text))
    End If
  Next m
 Next n

End Sub
Sub loadTempArray()
 ' H^3 - X1^3
 ' H^3 - X2^3
 ' H^3 - X3^3
 '........
 '........
 ' H^3 - Xn^3
 Dim n As Integer

 For n = 1 To System_DIM
   tempArray(n) = CDbl(Me.txtHT.Text) ^ 3 - CDbl(Me.txtXM(n - 1).Text) ^ 3
 Next n

End Sub
```

```
Sub multiply_Matrix()
 Dim n As Integer
 Dim m As Integer
 Dim temp As Double
 For n = 1 To System_DIM
  temp = 0
  For m = 1 To System_DIM
   temp = temp + (tempArray(m) * Inverse_Matrix(n, m))
  Next m
  mArray(n) = W_Divide_6EI_ * temp
 Next n
End Sub
Sub Compute_DeltaO()
 Dim n As Integer
 Dim temp As Double
 DeltaO_ = (CDbl(Me.txtWT.Text) * CDbl(Me.txtHT.Text) ^ 4) / (8 * CDbl(Me.txtEI.Text))
 temp = 0
 For n = 1 To System_DIM
  temp = temp + mArray(n) * (Me.txtHT.Text ^ 2 - CDbl(Me.txtXM(n - 1).Text) ^ 2)
 Next
 DeltaO_ = DeltaO_ - (1 / (2 * CDbl(Me.txtEI.Text))) * temp
End Sub
Sub Compute_Mcore()
 Dim n As Integer, m As Integer
```

```
For n = 1 To System_DIM
 mCore_(n, 1) = 0
 mCore_(n, 2) = 0
Next
' mCore_(1, 1) = 0
' mCore_(1, 2) = mArray(1)
' mCore_(2, 1) = mArray(1)
' mCore_(2, 2) = mArray(1) + mArray(2)
' mCore_(3, 1) = mArray(1) + mArray(2)
' mCore_(3, 2) = mArray(1) + mArray(2) + mArray(3)
' mCore_(4, 1) = mArray(1) + mArray(2) + mArray(3)
' mCore_(4, 2) = mArray(1) + mArray(2) + mArray(3) + mArray(4)
' mCore_(5, 1) = mArray(1) + mArray(2) + mArray(3) + mArray(4)
' mCore_(5, 2) = mArray(1) + mArray(2) + mArray(3) + mArray(4) + mArray(5)
' mCore_(6, 1) = mArray(1) + mArray(2) + mArray(3) + mArray(4) + mArray(5)
' mCore_(6, 2) = mArray(1) + mArray(2) + mArray(3) + mArray(4) + mArray(5) + mArray(6)
' mCore_(7, 1) = mArray(1) + mArray(2) + mArray(3) + mArray(4) + mArray(5) + mArray(6)
' mCore_(7, 2) = mArray(1) + mArray(2) + mArray(3) + mArray(4) + mArray(5) + mArray(6)
+ mArray(7)
' mCore_(8, 1) = mArray(1) + mArray(2) + mArray(3) + mArray(4) + mArray(5) + mArray(6)
+ mArray(7)
' mCore_(8, 2) = mArray(1) + mArray(2) + mArray(3) + mArray(4) + mArray(5) + mArray(6)
+ mArray(7) + mArray(8)
' mCore_(9, 1) = mArray(1) + mArray(2) + mArray(3) + mArray(4) + mArray(5) + mArray(6)
+ mArray(7) + mArray(8)
```

```
 ' mCore_(9, 2) = mArray(1) + mArray(2) + mArray(3) + mArray(4) + mArray(5) + mArray(6)
+ mArray(7) + mArray(8) + mArray(9)

 ' mCore_(10, 1) = mArray(1) + mArray(2) + mArray(3) + mArray(4) + mArray(5) +
mArray(6) + mArray(7) + mArray(8) + mArray(9)

 ' mCore_(10, 2) = mArray(1) + mArray(2) + mArray(3) + mArray(4) + mArray(5) +
mArray(6) + mArray(7) + mArray(8) + mArray(9) + mArray(10)


 For n = 1 To MAX_DIM

  For m = 1 To n - 1

   mCore_(n, 1) = mCore_(n, 1) + mArray(m)

  Next

  For m = 1 To n

   mCore_(n, 2) = mCore_(n, 2) + mArray(m)

  Next

 Next


 For n = 1 To System_DIM

  mCore_(n, 1) = ((CDbl(Me.txtWT.Text) * (CDbl(Me.txtXM(n - 1).Text) ^ 2)) / 2) -
mCore_(n, 1)

  mCore_(n, 2) = ((CDbl(Me.txtWT.Text) * (CDbl(Me.txtXM(n - 1).Text) ^ 2)) / 2) -
mCore_(n, 2)

 Next

 mCoreLast_ = ((CDbl(Me.txtWT.Text) * (CDbl(Me.txtHT.Text) ^ 2)) / 2) - mCore_(10, 2)

End Sub



Sub Calculate_Inverse()
```

```
'Uses Gauss elimination method in order to calculate the inverse matrix [A]-1
'Method: Puts matrix [A] at the left and the singular matrix [I] at the right:
'[ a11 a12 a13 I 1 0 0 ]
'[ a21 a22 a23 I 0 1 0 ]
'[ a31 a32 a33 I 0 0 1 ]
'Then using line operations, we try to build the singular matrix [I] at the left.
'After we have finished, the inverse matrix [A]-1 (bij) will be at the right:
'[ 1 0 0 I b11 b12 b13 ]
'[ 0 1 0 I b21 b22 b23 ]
'[ 0 0 1 I b31 b32 b33 ]

On Error GoTo errhandler 'In case the inverse cannot be found (Determinant = 0)

Solution_Problem = False

'Assign values from matrix [A] at the left
For n = 1 To System_DIM
   For m = 1 To System_DIM
      Operations_Matrix(m, n) = Matrix_A(m, n)
   Next
Next

'Assign values from singular matrix [I] at the right
For n = 1 To System_DIM
   For m = 1 To System_DIM
```

```
      If n = m Then
        Operations_Matrix(m, n + System_DIM) = 1
      Else
        Operations_Matrix(m, n + System_DIM) = 0
      End If
    Next
Next

'Build the Singular matrix [I] at the left
For k = 1 To System_DIM
  'Bring a non-zero element first by changes lines if necessary
  If Operations_Matrix(k, k) = 0 Then
    For n = k To System_DIM
      If Operations_Matrix(n, k) <> 0 Then line_1 = n: Exit For 'Finds line_1 with non-zero
element
    Next n
    'Change line k with line_1
    For m = k To System_DIM * 2
      temporary_1 = Operations_Matrix(k, m)
      Operations_Matrix(k, m) = Operations_Matrix(line_1, m)
      Operations_Matrix(line_1, m) = temporary_1
    Next m
  End If

  elem1 = Operations_Matrix(k, k)
```

```
For n = k To 2 * System_DIM
 Operations_Matrix(k, n) = Operations_Matrix(k, n) / elem1
Next n

'For other lines, make a zero element by using:
'Ai1=Aij-A11*(Aij/A11)
'and change all the line using the same formula for other elements
For n = 1 To System_DIM
    If n = k And n = MAX_DIM Then Exit For 'Finished
    If n = k And n < MAX_DIM Then n = n + 1 'Do not change that element (already equals to 1), go for next one
  If Operations_Matrix(n, k) <> 0 Then 'if it is zero, stays as it is
    multiplier_1 = Operations_Matrix(n, k) / Operations_Matrix(k, k)
    For m = k To 2 * System_DIM
      Operations_Matrix(n, m) = Operations_Matrix(n, m) - Operations_Matrix(k, m) * multiplier_1
    Next m
  End If
Next n
Next k

'Assign the right part to the Inverse_Matrix
For n = 1 To System_DIM
  For k = 1 To System_DIM
    Inverse_Matrix(n, k) = Operations_Matrix(n, System_DIM + k)
  Next k
```

```
Next n

Exit Sub

errhandler:
message$ = "An error occured during the calculation process. Determinant of Matrix [A] is probably equal to zero."
response = MsgBox(message$, vbCritical)
Solution_Problem = True

End Sub

Private Sub txtEAc_Change()
 If Me.txtEAc.Text = "" Then Exit Sub
 If IsNumeric(Me.txtEAc.Text) = False Then
  MsgBox ("Please enter a numeric value for the axial rigidity of the column.")
  Me.txtEAc.Text = ""
  Exit Sub
 End If
End Sub

Private Sub txtEI_Change()
 If Me.txtEI.Text = "" Then Exit Sub
 If IsNumeric(Me.txtEI.Text) = False Then
  MsgBox ("Please enter a numeric value for the flexural rigidity of the core.")
```

```
    Me.txtEI.Text = ""
    Exit Sub
  End If
End Sub

Private Sub txtEIo_Change()
  If Me.txtEIo.Text = "" Then Exit Sub
  If IsNumeric(Me.txtEIo.Text) = False Then
    MsgBox ("Please enter a numeric value for the effective flexural rigidity of the outrigger.")
    Me.txtEIo.Text = ""
    Exit Sub
  End If
End Sub

Private Sub txtHT_Change()
  If Me.txtHT.Text = "" Then Exit Sub
  If IsNumeric(Me.txtHT.Text) = False Then
    MsgBox ("Please enter a numeric value for the total height of the structure.")
    Me.txtWT.Text = ""
    Exit Sub
  End If
End Sub

Private Sub txtN_Change()
  If Me.txtN.Text = "" Then Exit Sub
```

```
 If IsNumeric(Me.txtN.Text) = False Then
  MsgBox ("Please enter a number between 1 and 10")
  Me.txtN.Text = ""
  Exit Sub
 ElseIf Me.txtN.Text < 1 Or Me.txtN.Text > 10 Then
  MsgBox ("Please enter a number between 1 and 10")
  txtN.Text = ""
  Exit Sub
 Else
  System_DIM = Val(Me.txtN.Text) 'Matrix [A] dimensions
  hide_show
 End If
End Sub
Sub hide_show()
 Dim i As Integer
 If Me.txtN.Text = "" Then System_DIM = 0
 For i = 0 To System_DIM - 1
  Me.txtXM(i).Enabled = True
  Me.txtXM(i).BackColor = &HFFFFFF
 Next
 For i = System_DIM To MAX_DIM - 1
  Me.txtXM(i).Text = ""
  Me.txtXM(i).Enabled = False
  Me.txtXM(i).BackColor = &HE0E0E0
 Next
```

```
End Sub

Private Sub txtN_KeyPress(KeyAscii As Integer)
 If KeyAscii = 13 Then Me.txtWT.SetFocus
End Sub

Private Sub txtN_LostFocus()
 If txtN.Text = "" Then System_DIM = 0
 hide_show
End Sub

Private Sub txtWD_Change()
 If txtWD.Text = "" Then Exit Sub
 If IsNumeric(txtWD.Text) = False Then
  MsgBox ("Please enter a numeric value for the width of the structure.")
  txtWD.Text = ""
  Exit Sub
 End If

End Sub

Private Sub txtWT_Change()
 If Me.txtWT.Text = "" Then Exit Sub
 If IsNumeric(Me.txtWT.Text) = False Then
  MsgBox ("Please enter a numeric value for the loading.")
```

```
  Me.txtWT.Text = ""
  Exit Sub
 End If
End Sub
Function checkAll() As Boolean
 Dim i As Integer
 If Me.txtN.Text = "" Then
  MsgBox ("Please enter a value.")
  Me.txtN.SetFocus
  checkAll = False
  Exit Function
 End If
 If Me.txtWT.Text = "" Then
  MsgBox ("Please enter a value.")
  Me.txtWT.SetFocus
  checkAll = False
  Exit Function
 End If
 If Me.txtHT.Text = "" Then
  MsgBox ("Please enter a value.")
  Me.txtHT.SetFocus
  checkAll = False
  Exit Function
 End If
 If Me.txtWD.Text = "" Then
```

```
 MsgBox ("Please enter a value.")
 Me.txtWD.SetFocus
 checkAll = False
 Exit Function
End If
If Me.txtEAc.Text = "" Then
 MsgBox ("Please enter a value.")
 Me.txtEAc.SetFocus
 checkAll = False
 Exit Function
End If
If Me.txtEIo.Text = "" Then
 MsgBox ("Please enter a value.")
 Me.txtEIo.SetFocus
 checkAll = False
 Exit Function
End If
If Me.txtEI.Text = "" Then
 MsgBox ("Please enter a value.")
 Me.txtEI.SetFocus
 checkAll = False
 Exit Function
End If
For i = 0 To System_DIM - 1
 If Me.txtXM(i).Text = "" Then
```

```
    MsgBox ("Please enter a value.")
    Me.txtXM(i).SetFocus
    checkAll = False
    Exit Function
  End If
Next
checkAll = True

    End Function
```