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CONFIGURATION MIXING AND THE EFFECTS OF DISTRIBUTED NUCLEAR MAGNETIZATION ON HYPERFINE STRUCTURE IN ODD-A NUCLEI

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Configuration Mixing and the Effects of Distributed Nuclear Magnetization on Hyperfine Structure in Odd-A Nuclei*

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The theory of Blin-Stoyle and of Arima and Horie, in which the deviations of the nuclear magnetic moments from the single-particle model Schmidt limits are ascribed to configuration mixing, is used as a model to account quantitatively for the effects of the distribution of nuclear magnetization on hyperfine structure (Bohr-Weisskopf effect). A diffuse nuclear charge distribution, as approximated by the trapezoidal Hofstadter model, is used to calculate the required radial electron wave functions. A table of single-particle matrix elements of R^2 and R^4 in a Saxon-Woods type of potential well is included. Explicit formulas are derived to permit comparison with experiment. For all of the available data satisfactory agreement is found. The possibility of using hyperfine structure measurements sensitive to the distribution of nuclear magnetization in a semiphenomenological treatment in order to obtain information on nuclear configurations is indicated.

I. INTRODUCTION

IT is well known that the strict single-particle model fails in explaining most nuclear magnetic moments, even with quenching of the intrinsic spin or orbital g values of the nucleons.[1] On the other hand, reasonably successful theories have been developed by Blin-Stoyle,[2] and Arima and Horie,[3] to account for the departure of the magnetic moments of odd-A nuclei by configuration mixing calculations. This configurational mixing theory will be referred to as CMT. We investigate the application of such a configuration mixing theory to a closely related property of the nucleus—the distribution of its magnetization, as it is manifested in the hyperfine structure interaction of penetrating electrons.

Bohr and Weisskopf (BW) have calculated the hyperfine structure interaction of $s_{1/2}$ and $p_{1/2}$ electrons in the field of an extended distribution of nuclear charge and magnetism.[4] Two important conclusions may be drawn from their work. First, that the hfs for a finite nucleus is, in general, smaller than that to be expected for a hypothetical point nucleus. Second, that the isotopic variations of nuclear magnetic moments, combined with the different contributions to the hfs of the orbital and spin parts of the magnetization in the case of the extended nucleus, allow for relatively large isotopic variations in the departure from a point hfs interaction. The latter point is consistent with the experimental observation[5–11] that the ratio of the hfs constants for two isotopes may, in some cases, be different from the independently measured ratio of the magnetic moments. The discrepancy in these two ratios is commonly referred to as the "Bohr-Weisskopf effect" or "hfs anomaly."

Bohr[12] has treated this "hfs anomaly" within the framework of the collective or asymmetric model, and recently Reiner[13] has carried out calculations on the collective model, primarily in the region of the rare earths.

Most experimental data, however, lie in a region where the collective model is not ideally applicable. Furthermore the results of our experiments on the hfs of several Cs isotopes[10] (together with evidence for configuration mixing in the decay scheme study of

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Cs^{134} by Sunyar *et al.*[14]) pointed out the difficulty of accounting for the BW effect in them unless some detailed information about the nucleon configurations were included in the BW theory. We have therefore developed a formalism which considers configuration mixing effects, as used by Arima and Horie[3] and Noya *et al.*[15] and in turn makes possible the use of the BW effect in conjunction with magnetic moment data to give information on the admixed configurations. Modifications of the intrinsic nucleon g values can be introduced formally into the theory when such changes are expected to have a substantial effect, as is the case for the potassium isotopes.

II. EFFECT OF THE DISTRIBUTION OF CHARGE AND MAGNETIZATION ON HFS

Bohr and Weisskopf[4] have calculated expressions for the hfs interaction energy W of a nucleus of finite extent. For $s_{1/2}$ or $p_{1/2}$ electrons there will be an hfs doublet corresponding to the two values of the total angular momentum $F=j\pm\frac{1}{2}$, and they define W to be the energy by which the state $F=j+\frac{1}{2}$ is displaced. j is the nuclear spin. Alternatively, if $h\Delta\nu$ is the energy separation of the two states, then by the interval rule $W=jh\Delta\nu/(2j+1)$. They write $W=W_S+W_L$, where W_S and W_L are the contributions to W from spin and orbital magnetizations in the nucleus. For the spin part,

$$W_S=\pm\frac{16\pi e}{3}\int_N\sum_i d\tau_N\,\Psi_N^*(1\cdots i\cdots A)g_S^{(i)}\times\left[\mathbf{S}_Z^{(i)}\int_{R_i}^{\infty}FGdr+\mathbf{D}_Z^{(i)}\int_0^{R_i}\frac{r^3}{R_i^3}FGdr\right]\Psi_N. \quad (1)$$

The spin asymmetry operator in (1) is given by the tensor product (of rank 1)

$$\mathbf{D}=-\tfrac{1}{2}(10)^{\frac{1}{2}}[\mathbf{S}\times\mathbf{C}^2]^{(1)}, \quad (1a)$$

where $C_q^k=[4\pi/(2k+1)]^{\frac{1}{2}}Y_q^k(\theta,\phi)$, and Y is a spherical harmonic. It is equal to the bracket of Eq. (7) in BW as well as to the operator $-\langle\mathbf{S}_Z\rangle\zeta$, corresponding to Bohr's Eq. (2).[12] The orbital part of the interaction is

$$W_L=\pm\frac{16\pi e}{3}\int_N\sum_i d\tau_N\,\Psi_N^*g_L^{(i)}\mathbf{L}_Z^{(i)}\times\left[\int_{R_i}^{\infty}FGdr+\int_0^{R_i}\frac{r^3}{R_i^3}FGdr\right]\Psi_N. \quad (2)$$

The upper and lower signs in (1) and (2) refer to $s_{1/2}$ and $p_{1/2}$ electrons, respectively. The symbols are e, electron charge, $R(XYZ)$ and r, nuclear and electron coordinates, respectively, Ψ_N, nuclear wave function corresponding to the maximum z component of spin, F and G, Dirac electron wave functions for an extended nucleus, $g_S^{(i)}$ and $g_L^{(i)}$, spin and orbital g values of the ith nucleon, $\mathbf{S}$ and $\mathbf{L}$ nuclear spin and orbital angular momentum operators, A, mass number of the nucleus. By writing

$$W_{\text{extended}}\equiv W_{\text{point}}(1+\epsilon), \quad (3)$$

and noting that for a point nucleus the interaction energy is given by letting $R_i=0$ in the integral limits in (1) and (2), and replacing F and G by F_0 and G_0, their values for a point nucleus,

$$-\epsilon=\frac{1}{\mu\int_0^{\infty}F_0G_0dr}\left\{\int_N\sum_i d\tau_N\,\Psi_N^*\times\left[g_S^{(i)}\left(\mathbf{S}_Z^{(i)}\int_0^{R_i}FGdr-\mathbf{D}_Z^{(i)}\int_0^{R_i}\frac{FGr^3}{R_i^3}dr\right)+g_L^{(i)}\mathbf{L}_Z^{(i)}\int_0^{R_i}\left(1-\frac{r^3}{R_i^3}\right)FGdr\right]\Psi_N\right\}, \quad (4)$$

where μ is the nuclear magnetic moment. Equation (4) is the more general expression for ϵ which corresponds to BW Eq. (19) as modified by Bohr[12] [Eqs. (1) and (15)].

III. ELECTRON WAVE FUNCTIONS IN A HOFSTADTER-LIKE CHARGE DISTRIBUTION EVALUATION OF THE ELECTRON INTEGRALS

The functions F and G in (4) are to be calculated for a potential which corresponds to the actual nuclear charge distribution. This was approximated in BW by assuming a uniform distribution. We have found, however, that the electron integrals are noticeably sensitive to the model assumed for the distribution.[16] For this reason we obtained a series solution of the Dirac equation for a charge distribution which agrees better with the one indicated by high energy electron scattering[17] and other experimental data,[18] and therefore should correspond more closely to the actual nuclear charge distribution.

We found that the solution of the equations was very complicated to handle for any of the three forms of the charge distribution given in reference 17. It may be shown that it is simple only if the entire charge distribution can be represented by a polynomial in r. The solutions can then be carried out as in BW, relying on the validity of the approximations in the normalization of F, G, to F_0, G_0 as stated by Rosenthal

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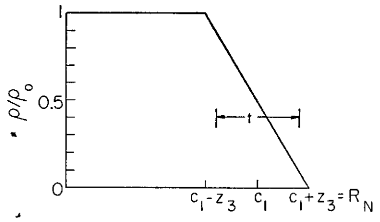

FIG. 1. Trapezoidal charge distribution of Hahn, Ravenhall, and Hofstadter.[17] (Our c_1 is their parameter c.)

and Breit.[19] We have therefore approximated the trapezoidal charge distribution ρ of reference 17 with the following polynomial in x ($x=r/R_N$, where $R_N=c_1+z_3$)

$$\rho=\rho_0+\rho_2x^2+\rho_3x^3+\rho_4x^4. \tag{5}$$

The dimensions c_1 and z_3 are shown in Fig. 1. The pertinent values used are $c_1=1.07A^{\frac{1}{3}}$ f, $t=1.60z_3=2.40$ f. The coefficients ρ_i were determined by demanding that ρ in Eq. (5) coincide with ρ of Fig. 1 at $r=0$, c_1-z_3, c_1, and R_N. In terms of the parameters of the trapezoidal distribution they are found to be

$$\begin{aligned}\rho_2&=\frac{\rho_0}{2}\frac{(c_1-z_3)(3c_1^2+3c_1z_3+z_3^2)}{z_3c_1^2},\\ \rho_3&=-\frac{\rho_0}{2}\frac{(5c_1+2z_3)R_N}{z_3c_1},\\ \rho_4&=\frac{\rho_0}{2}\frac{(2c_1+z_3)R_N^2}{z_3c_1^2}.\end{aligned} \tag{5a}$$

The nuclear charge, Ze, determines the central charge density,

$$\rho_0=\frac{420Zez_3c_1^2}{4\pi R_N^3(11c_1^3+45c_1^2z_3-34c_1z_3^2-12z_3^2)}. \tag{6}$$

A plot of ρ for $A\sim40$ and $A\sim200$ is given in Fig. 2. These distributions reproduce fairly well the trapezoidal one, and even the small central depression may be realistic.[18] From this charge distribution we obtain the potential

$$V(x)=\frac{Ze}{R_N}(K-a_2x^2-a_4x^4-a_5x^5-a_6x^6), \tag{7}$$

where

$$\begin{aligned}K&\equiv1+a_2+a_4+a_5+a_6,\\ a_2&=2\pi R_N^3\rho_0/3Ze,\\ a_4&=\pi R_N^3\rho_2/5Ze,\\ a_5&=2\pi R_N^3\rho_3/15Ze,\\ a_6&=2\pi R_N^3\rho_4/21Ze.\end{aligned} \tag{7a}$$

The solution of the Dirac equation for this potential, and the evaluation of the electron integrals of Eq. (4), are given in the Appendix. With these results [Eqs. (A.9)–(A.11)], Eq. (4) becomes

$$\begin{aligned}-\epsilon=(1/\mu)\bigg\{\int_N\sum_i d\tau_N\,\Psi_N^*\sum_n\frac{R_i^{2n}}{R_N^{2n}}&\\ \times[g_S^{(i)}(\mathbf{S}_Z^{(i)}(b_S)_{2n}-\mathbf{D}_Z^{(i)}(b_D)_{2n})&\\ +g_L^{(i)}\mathbf{L}_Z^{(i)}(b_L)_{2n}]\Psi_N\bigg\},&\quad(n=1,2).\end{aligned} \tag{8}$$

The sum over n results from the series solution of the Dirac equation. The values of the electron coefficients b_S and b_L (defined in the Appendix) are given in Table I for $s_{1/2}$ and $p_{1/2}$ electrons as a function of A and Z. Equation (A.12a) gives b_D in terms of b_S. A plot of these coefficients is shown in Fig. 3. For comparison we also show the results obtained for uniform and surface charge distributions.[16] It is interesting to note that the magnitudes of the b coefficients tend to decrease the more the nuclear charge is distributed at larger distances from the center, reflecting the corresponding changes in the electron binding. Figure 4 compares the b coefficients for the $s_{1/2}$ and $p_{1/2}$ states for the charge distribution of Eq. (5).

We have investigated the effect on these coefficients of a modification of the approximate representation of the charge distribution [Eq. (5)] in the form $\rho=\rho_0+\rho_2x^2+\rho_4x^4+\rho_6x^6$ [which in fact gives even a slightly better fit to the trapezoidal distribution than Eq. (5)]. We find that the b coefficients for these two representations agree to within 2.5% for $n=1$ and 2. The coefficients for $n>2$, which are small, are sensitive to such slight variations in ρ. Since at present there is no experimental evidence in favor of either one, these higher terms cannot be considered to have significance

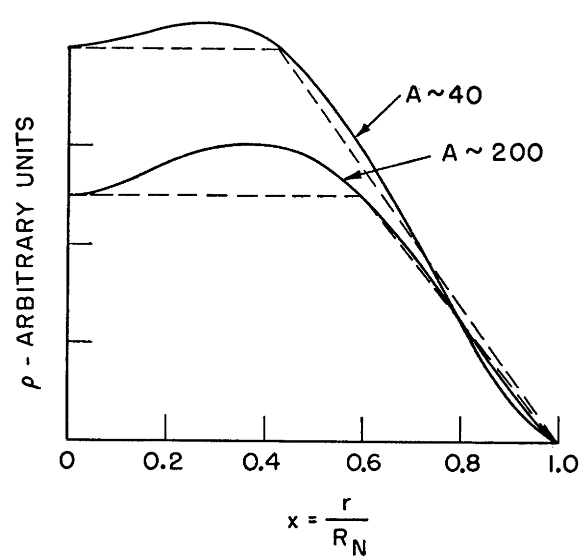

FIG. 2. Charge distribution as given by the representation of Eq. (5). The broken lines indicate the trapezoids used in the determination of the parameters.

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in the result. As we will show in Sec. V, the evaluation of the radial nuclear matrix elements involves $(R_0/R_N)^{2n}$, where $R_0=1.20A^{\frac{1}{3}}$ f and is the radial parameter involved in the nuclear potential well. If we take this factor into account, the $n>2$ coefficients may affect the value of ϵ to about five percent. We note, however, that in the comparison with experiment we take the difference of ϵ for two isotopes (see Sec. VI). Therefore if ϵ_1 and ϵ_2 are very similar, although their differences will be small, the effect of neglecting such higher terms will also be canceled to a large extent. On the other hand if the ϵ are very different, as they would be if the two isotopes have different spins, then the difference will be large, and again the terms $n>2$ will have relatively little effect. The actual extent of such cancellations will depend on the specific properties of the isotopes under consideration.

IV. EVALUATION OF THE NUCLEAR INTEGRALS

In Eq. (8) an expression is obtained for the quantity ϵ which involves calculating the expectation value of the operators $\mathbf{M}_n$, where

$$\mathbf{M}_n=\mathbf{M}_n{}^{SL}+\mathbf{M}_n{}^{D}, \tag{9}$$

and

$$\mathbf{M}_n{}^{SL}=\frac{1}{\mu}\sum_i \frac{R_i^{2n}}{R_N^{2n}}\big[g_S{}^{(i)}\mathbf{S}_Z{}^{(i)}(b_S)_{2n}+g_L{}^{(i)}\mathbf{L}_Z{}^{(i)}(b_L)_{2n}\big], \tag{10}$$

$$\mathbf{M}_n{}^{D}=\frac{1}{\mu}\sum_i \frac{R_i^{2n}}{R_N^{2n}}g_S{}^{(i)}\mathbf{D}_Z{}^{(i)}(b_D)_{2n}. \tag{11}$$

Explicitly for a nucleus of spin j, since the expectation value is to be taken with respect to a nuclear wave function having its maximum z component of spin, we require (writing only the angular terms in the following three parts)

$$M_n=C(j1j;j0)\langle j\|\mathbf{M}_n\|j\rangle, \tag{12}$$

where $\langle j\|\mathbf{M}_n\|j\rangle$ is the reduced martix element of $\mathbf{M}_n$. C is a Wigner coefficient.

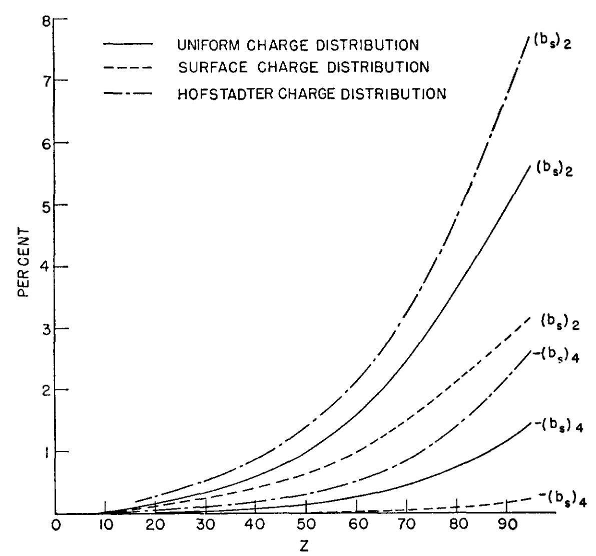

FIG. 3. Dependence on Z of the electron coefficients b_S for several nuclear charge distributions. The b's are defined in the Appendix.

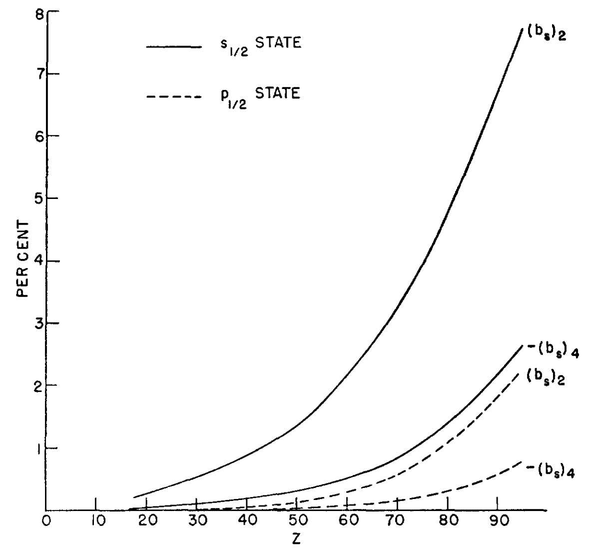

FIG. 4. Dependence on Z of the electron coefficients b_S for $s_{1/2}$ and $p_{1/2}$ states for an assumed Hofstadter type of nuclear charge distribution. The b's are defined in the Appendix.

In ignorance of the true nuclear wave function, some approximate or model wave function has to be used, and in view of the success of CMT in accounting for magnetic moments, this theory is also used in the following calculations. The basic idea is to write the nuclear wave function Ψ_N as

$$\Psi_N=\Psi_0+\textstyle\sum_{i\neq 0}\beta(i)\Psi_i, \tag{13}$$

where Ψ_0 (the zero-order state) represents a simple shell-model configuration and the Ψ_i represent admixed configurations characterized by the variable i. For small mixing coefficients $\beta(i)$, the main deviation of the expectation value of $\mathbf{M}_n$ from that given by the simple shell-model wave function will be that due to terms linear in $\beta(i)$ and the conditions that such contributions should occur is that Ψ_0 and Ψ_i must differ at most by one single-particle state. In addition for $\mathbf{M}_n{}^{SL}$ the orbital states must be the same ($\Delta l=0$), while for $\mathbf{M}_n{}^{D}$ states differing by $\Delta l=2$ may also be coupled.

We follow the classification and labeling of states suggested by Arima and Horie. Thus the zero-order state configuration is written as $j^p(J=j)$, where p is the number of odd particles in the state j and no indication is given of the even numbers of nucleons coupled to zero angular momentum. These latter nucleons, however, play a crucial role in the configuration admixtures considered here since these admixed states are those in which a nucleon is excited from or to these states. There are three types of

TABLE I. Electron coefficients b for a Hofstadter-type charge distribution. Values are in percentages.

		$s_{1/2}$ electrons				$p_{1/2}$ electrons			
Z	A	$(b_S)_2$	$(b_L)_2$	$-(b_S)_4$	$-(b_L)_4$	$(b_S)_2$	$(b_L)_2$	$-(b_S)_4$	$-(b_L)_4$
17	35	0.213	0.128	0.047	0.020	0.003	0.002	0.001	
	37	0.215	0.129	0.047	0.020	0.003	0.002	0.001	
18	37	0.230	0.138	0.051	0.022	0.003	0.002	0.001	
	39	0.233	0.140	0.051	0.022	0.003	0.002	0.001	
	41	0.235	0.141	0.051	0.022	0.003	0.002	0.001	
19	39	0.248	0.149	0.054	0.023	0.004	0.002	0.001	
	41	0.251	0.150	0.054	0.023	0.004	0.002	0.001	
	43	0.253	0.152	0.054	0.023	0.004	0.002	0.001	
20	41	0.267	0.160	0.058	0.025	0.005	0.003	0.001	
	43	0.269	0.162	0.058	0.025	0.005	0.003	0.001	
	45	0.272	0.163	0.058	0.025	0.005	0.003	0.001	
	47	0.274	0.164	0.058	0.025	0.005	0.003	0.001	
25	55	0.376	0.225	0.079	0.034	0.010	0.006	0.002	0.001
29	61	0.471	0.282	0.099	0.042	0.017	0.010	0.003	0.001
	63	0.474	0.284	0.099	0.042	0.017	0.010	0.003	0.001
	65	0.477	0.286	0.099	0.042	0.017	0.010	0.003	0.001
	67	0.480	0.288	0.099	0.042	0.017	0.010	0.003	0.001
30	63	0.498	0.299	0.105	0.045	0.019	0.011	0.004	0.002
	65	0.501	0.300	0.105	0.045	0.019	0.011	0.004	0.002
	67	0.504	0.302	0.105	0.045	0.019	0.012	0.004	0.002
	69	0.507	0.304	0.105	0.045	0.019	0.012	0.004	0.002
	71	0.510	0.306	0.105	0.045	0.019	0.012	0.004	0.002
31	65	0.526	0.315	0.111	0.047	0.021	0.013	0.004	0.002
	67	0.529	0.317	0.111	0.047	0.021	0.013	0.004	0.002
	69	0.532	0.319	0.111	0.047	0.021	0.013	0.004	0.002
	71	0.535	0.321	0.111	0.047	0.022	0.013	0.004	0.002
	73	0.538	0.323	0.111	0.047	0.022	0.013	0.004	0.002
33	71	0.588	0.353	0.123	0.053	0.027	0.016	0.005	0.002
	73	0.591	0.355	0.123	0.053	0.027	0.016	0.005	0.002
	75	0.595	0.357	0.124	0.053	0.027	0.016	0.005	0.002
	77	0.598	0.359	0.124	0.053	0.027	0.016	0.005	0.002
35	75	0.652	0.391	0.137	0.059	0.033	0.020	0.007	0.003
	77	0.656	0.393	0.138	0.059	0.033	0.020	0.007	0.003
	79	0.659	0.395	0.138	0.059	0.034	0.020	0.007	0.003
	81	0.662	0.397	0.138	0.059	0.034	0.020	0.007	0.003
	83	0.664	0.399	0.138	0.059	0.034	0.020	0.007	0.003
36	79	0.689	0.414	0.145	0.062	0.037	0.022	0.007	0.003
	81	0.693	0.416	0.145	0.062	0.037	0.022	0.007	0.003
	83	0.696	0.418	0.145	0.062	0.038	0.022	0.007	0.003
	85	0.700	0.420	0.145	0.062	0.038	0.023	0.007	0.003
37	81	0.725	0.435	0.153	0.066	0.041	0.025	0.008	0.004
	83	0.728	0.437	0.153	0.066	0.041	0.025	0.008	0.004
	85	0.732	0.439	0.153	0.066	0.041	0.025	0.008	0.004
	87	0.735	0.441	0.153	0.066	0.042	0.025	0.008	0.004
38	83	0.761	0.457	0.161	0.069	0.045	0.027	0.009	0.004
	85	0.765	0.459	0.161	0.069	0.046	0.027	0.009	0.004
	87	0.769	0.461	0.162	0.069	0.046	0.028	0.009	0.004
	89	0.772	0.463	0.162	0.069	0.046	0.028	0.009	0.004
40	91	0.847	0.508	0.179	0.077	0.056	0.033	0.011	0.005
42	95	0.931	0.559	0.199	0.085	0.067	0.040	0.014	0.006
	97	0.935	0.561	0.199	0.086	0.067	0.040	0.014	0.006
45	103	1.075	0.645	0.233	0.100	0.088	0.053	0.018	0.008
47	105	1.174	0.704	0.259	0.111	0.104	0.063	0.022	0.009
	107	1.179	0.707	0.259	0.111	0.105	0.063	0.022	0.009
	109	1.183	0.710	0.259	0.111	0.105	0.063	0.022	0.009
	111	1.187	0.712	0.259	0.111	0.106	0.063	0.022	0.009
	113	1.191	0.715	0.259	0.111	0.106	0.064	0.022	0.009

TABLE I.—*Continued.*

Z	A	$s_{1/2}$ electrons $(b_S)_2$	$(b_L)_2$	$-(b_S)_4$	$-(b_L)_4$	$p_{1/2}$ electrons $(b_S)_2$	$(b_L)_2$	$-(b_S)_4$	$-(b_L)_4$
48	105	1.224	0.734	0.272	0.117	0.113	0.068	0.024	0.010
	107	1.229	0.737	0.272	0.117	0.114	0.068	0.024	0.010
	109	1.233	0.740	0.272	0.117	0.114	0.068	0.024	0.010
	111	1.238	0.743	0.273	0.117	0.114	0.069	0.024	0.010
	113	1.242	0.745	0.273	0.117	0.115	0.069	0.024	0.010
	115	1.246	0.748	0.273	0.117	0.115	0.069	0.024	0.010
	117	1.251	0.750	0.273	0.117	0.116	0.070	0.024	0.010
49	109	1.286	0.771	0.287	0.123	0.124	0.074	0.026	0.011
	111	1.290	0.774	0.287	0.123	0.124	0.074	0.026	0.011
	113	1.295	0.777	0.287	0.123	0.124	0.075	0.026	0.011
	115	1.299	0.779	0.287	0.123	0.125	0.075	0.026	0.011
	117	1.304	0.782	0.288	0.123	0.125	0.075	0.026	0.011
	119	1.308	0.785	0.288	0.123	0.126	0.076	0.027	0.011
50	115	1.354	0.812	0.302	0.130	0.135	0.081	0.029	0.012
	117	1.358	0.815	0.303	0.130	0.136	0.081	0.029	0.012
	119	1.363	0.818	0.303	0.130	0.136	0.082	0.029	0.012
51	119	1.420	0.852	0.319	0.136	0.147	0.088	0.032	0.014
	121	1.425	0.855	0.319	0.137	0.148	0.088	0.032	0.014
	123	1.429	0.857	0.319	0.137	0.148	0.089	0.032	0.014
	125	1.434	0.860	0.319	0.137	0.149	0.089	0.032	0.014
52	123	1.489	0.893	0.336	0.144	0.160	0.096	0.035	0.015
	125	1.494	0.896	0.336	0.144	0.160	0.096	0.035	0.015
53	121	1.546	0.928	0.353	0.151	0.172	0.103	0.038	0.016
	123	1.551	0.931	0.353	0.151	0.172	0.103	0.038	0.016
	125	1.556	0.934	0.354	0.152	0.173	0.104	0.038	0.016
	127	1.561	0.936	0.354	0.152	0.173	0.104	0.038	0.016
	129	1.565	0.939	0.354	0.152	0.174	0.104	0.038	0.016
	131	1.570	0.942	0.354	0.152	0.175	0.105	0.038	0.016
54	123	1.616	0.969	0.372	0.159	0.186	0.111	0.041	0.018
	125	1.621	0.972	0.372	0.159	0.186	0.112	0.041	0.018
	127	1.625	0.975	0.372	0.160	0.187	0.112	0.041	0.018
	129	1.630	0.978	0.372	0.160	0.187	0.112	0.041	0.018
	131	1.635	0.981	0.373	0.160	0.188	0.113	0.041	0.018
	133	1.640	0.984	0.373	0.160	0.189	0.113	0.041	0.018
	135	1.645	0.987	0.373	0.160	0.189	0.114	0.041	0.018
55	125	1.688	1.013	0.391	0.168	0.200	0.120	0.045	0.019
	127	1.693	1.016	0.392	0.168	0.201	0.121	0.045	0.019
	129	1.698	1.019	0.392	0.168	0.202	0.121	0.045	0.019
	131	1.703	1.022	0.392	0.168	0.202	0.121	0.045	0.019
	133	1.708	1.025	0.392	0.168	0.203	0.122	0.045	0.019
	135	1.713	1.028	0.393	0.168	0.204	0.122	0.045	0.019
	137	1.717	1.030	0.393	0.168	0.204	0.123	0.045	0.019
56	129	1.768	1.061	0.412	0.177	0.217	0.130	0.049	0.021
	131	1.773	1.064	0.413	0.177	0.218	0.131	0.049	0.021
	133	1.778	1.067	0.413	0.177	0.218	0.131	0.049	0.021
	135	1.783	1.070	0.413	0.177	0.219	0.131	0.049	0.021
	137	1.788	1.073	0.413	0.177	0.220	0.132	0.049	0.021
	139	1.793	1.076	0.414	0.177	0.220	0.132	0.049	0.021
60	143	2.115	1.269	0.507	0.217	0.294	0.176	0.068	0.029
65	159	2.626	1.576	0.656	0.281	0.421	0.253	0.102	0.044
70	173	3.233	1.940	0.845	0.362	0.589	0.353	0.150	0.064
75	186	3.949	2.369	1.084	0.465	0.807	0.484	0.217	0.093
79	191	4.587	2.752	1.316	0.564	1.020	0.612	0.287	0.123
	193	4.594	2.756	1.316	0.564	1.021	0.613	0.287	0.123
	195	4.601	2.760	1.317	0.564	1.023	0.614	0.287	0.123
	197	4.608	2.765	1.317	0.564	1.025	0.615	0.287	0.123
	199	4.614	2.769	1.317	0.564	1.026	0.616	0.287	0.123
	201	4.621	2.773	1.318	0.565	1.028	0.617	0.287	0.123

TABLE I.—*Continued.*

		$s_{1/2}$ electrons				$p_{1/2}$ electrons			
Z	A	$(b_S)_2$	$(b_L)_2$	$-(b_S)_4$	$-(b_L)_4$	$(b_S)_2$	$(b_L)_2$	$-(b_S)_4$	$-(b_L)_4$
80	193	4.760	2.856	1.380	0.592	1.080	0.648	0.307	0.132
	195	4.767	2.860	1.381	0.592	1.081	0.649	0.307	0.132
	197	4.774	2.864	1.381	0.592	1.083	0.650	0.397	0.132
	199	4.781	2.869	1.381	0.592	1.085	0.651	0.307	0.132
	201	4.788	2.873	1.382	0.592	1.086	0.652	0.307	0.132
	203	4.795	2.877	1.382	0.592	1.088	0.653	0.307	0.132
81	197	4.945	2.967	1.447	0.620	1.144	0.686	0.328	0.141
	199	4.952	2.971	1.448	0.620	1.146	0.687	0.328	0.141
	201	4.959	2.976	1.448	0.621	1.147	0.688	0.328	0.141
	203	4.966	2.980	1.448	0.621	1.149	0.689	0.329	0.141
	205	4.973	2.984	1.449	0.621	1.151	0.690	0.329	0.141
85	214	5.727	3.436	1.741	0.746	1.427	0.856	0.426	0.183
90	228	6.736	4.042	2.159	0.925	1.826	1.096	0.576	0.247
95	242	7.716	4.630	2.609	1.118	2.254	1.352	0.751	0.322
100	256	8.476	5.086	3.025	1.296	2.643	1.586	0.931	0.399

excitation which need to be considered—referred to as types I, II, and III.

Type I Excitation

The zero-order configuration has p (odd) particles in state j, n_1 (even) in j_1 and n_2 (even) in j_2, the n_1 and n_2 particles being coupled separately to zero angular momentum so that the total angular momentum J of the state is equal to j. Thus, symbolically, the state can be written

$$\Psi_0=\Psi(j_1^{n_1}(0)j_2^{n_2}(0)j^p(j)J=j). \tag{14}$$

The admixed states of type I are then taken to be those in which a particle is excited from state j_1 to state j_2, each group coupling respectively to j_1 and j_2, and the j_1 and j_2 coupling together to J_1 which couples finally with j to give $J=j$. The nuclear state Ψ_N can therefore be written, on including one such admixture,

$$\begin{aligned}\Psi_N=\Psi(j_1^{n_1}(0)j_2^{n_2}(0)j^p(j)J=j)+\textstyle\sum_{J_1}\beta(J_1)\\ \times\Psi([j_1^{n_1-1}(j_1)j_2^{n_2+1}(j_2)](J_1)j^p(j)J=j).\end{aligned} \tag{15}$$

Of course, the states j_1 and j_2 are chosen so that the first-order matrix element of $\mathbf{M}_n$ is nonvanishing and so that the excitation involved is compatible with the exclusion principle.

Using the results of Noya *et al.* specialized to our case, the following expression is obtained for the contribution of such a type I mixing to the reduced matrix element of $\mathbf{M}_n$ evaluated with respect to (15):

$$\begin{aligned}\langle j^p\, J=j\|\mathbf{M}_n\|j^p\, J=j\rangle_{\mathrm{I}}\\ =-(2j+1)^{\frac{1}{2}}C(j1j;\tfrac{1}{2}0)\\ \times[n_1(2j_2+1-n_2)/(2j_1+1)(2j_2+1)]\\ \times h_1^{lj}(l_1j_1,l_2j_2)\begin{Bmatrix}\tfrac{1}{2}(V_t-V_s)\\ -V_s\end{Bmatrix}I(j_1j_2j^2)/\Delta E,\end{aligned} \tag{16}$$

where[20]

$$\begin{aligned}h_1^{lj}(l_1j_1,l_2j_2)\\ =(2j_1+1)^{\frac{1}{2}}C(j_11j_2;\tfrac{1}{2}0)\langle j_1\|\mathbf{M}_n\|j_2\rangle(1\pm\theta),\end{aligned} \tag{17}$$

and

$$I(j_1j_2j^2)=\frac{1}{2}\int_0^\infty R_{j_1}(R)R_{j_2}(R)R_j^2(R)R^2dR. \tag{18}$$

The upper (lower) line in the bracket { } must be used when the excited nucleon in the orbit j_1 is different from (similar to) the nucleon in the orbit j. The quantity $\theta\equiv(-1)^{\frac{1}{2}+l-j}(j+\frac{1}{2})$ is to be taken with the $+$ sign for excitations with $\Delta l=0$, and $-$ sign for $\Delta l=2$ in Eq. (17).

In the above expressions, the admixture parameters $\beta(J_1)$ have been calculated by straightforward first-order perturbation theory using as the perturbing potential a delta-function interaction given by

$$\begin{aligned}V_{12}=[V_s(1-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)/4\\ +V_t(3+\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)/4]\delta(\mathbf{R}_1-\mathbf{R}_2),\end{aligned} \tag{19}$$

where V_s and V_t represent the singlet and triplet strengths of the internucleon interaction. ΔE is the energy needed to excite a particle from the state j_1 to the state j_2, and $\langle j_1\|\mathbf{M}_n\|j_2\rangle$ is the single-particle reduced matrix element of the operator $\mathbf{M}_n$. Now for $\mathbf{M}_n^{SL}$ the only nonvanishing reduced matrix element to be considered here[21] is that for which the particle excitation is from $j_1=l_1+\frac{1}{2}$ to $j_2=l_1-\frac{1}{2}$. However, for $\mathbf{M}_n^D$ we can have both $j_1=l_1+\frac{1}{2}$ to $j_2=l_1-\frac{1}{2}$ and also

[20] According to our calculations, Eq. (3.7) of Noya *et al.* is in error by a factor $(2j_2+1)^{\frac{1}{2}}$. With our choice of phase in the reduced matrix elements, we also differ in sign in this equation. Our θ is equal to their ϵ.

[21] There is also the possibility of an excitation to a state of the same j and l value but different n value. Such an excitation would be through essentially two oscillator shells and because of the associated large value of ΔE such excitations are neglected.

$j_1=l_1+\frac{1}{2}$ to $j_2=l_1+\frac{3}{2}$ or vice versa. The reduced matrix elements of $\mathbf{M}_n$ in each of these cases can be constructed easily from the single-particle reduced matrix elements of $\mathbf{S}_Z$, $\mathbf{L}_Z$, and $\mathbf{D}_Z$ given in Table II.

Using the foregoing relationships, we obtain finally for the contribution of type I admixtures to the matrix elements M_n the expressions given in Tables III and IV, where the radial matrix elements $\mathcal{G}_n(n_2,j_2,l_2;n_1,j_1,l_1)$ are given by

$$\mathcal{G}_n(n_2,j_2,l_2;n_1,j_1,l_1)=\int_0^\infty R_{n_2j_2l_2}(R)\frac{R^{2n}}{R_N^{2n}}R_{n_1j_1l_1}(R)R^2dR. \quad (20)$$

Here the radial functions are those describing the ground and excited states of the single particle involved in the type I excitation; the evaluation of the $\mathcal{G}_n$ and also the estimation of the ΔE will be discussed in Sec. V.

Type II Excitation

In this type of excitation, the orbit j_2 (of type I excitation) coincides with j. Thus the nuclear wave function, including a typical type II admixture, can now be written

$$\Psi_N=\Psi(j_1{}^n(0)j^p(j)J=j)+\textstyle\sum_{J_1}\beta(J_1)\Psi(j_1{}^{n-1}(j_1)j^{p+1}(J_1)J=j), \quad (21)$$

where p and n are the numbers of odd and even nucleons, respectively. Using the same interaction as in type I and specializing the results of Noya *et al.* to our case, we have

$$\langle j^pJ=j\|\mathbf{M}_n\|j^pJ=j\rangle_{\mathrm{II}}=-(2j+1)^{\frac{1}{2}}C(j1j;\tfrac{1}{2}0)[n/(2j_1+1)]\times[(2j-p)/(2j-1)]h_1{}^{lj}(l_1j_1,lj)\times(-V_s)I(j_1j^3)/\Delta E, \quad (22)$$

where the various components of this expression are defined as in Eqs. (17)–(19). The contributions of this type II admixture to the matrix elements M_n are given in Tables V and VI where the radial matrix elements $\mathcal{G}$ are defined as in Eq. (20).

Type III Excitation

Here the orbit j_1 coincides with the orbit j (of type I excitation). The nuclear wave function including an admixture of this type can now be written

$$\Psi_N=\Psi(j_1{}^n(0)j^p(j)J=j)+\textstyle\sum_{J_1}\beta(J_1)\Psi(j_1{}^{n+1}(j_1)j^{p-1}(J_1)J=j), \quad (23)$$

TABLE II. Reduced matrix elements $\langle j_1\|\mathbf{M}\|j_2\rangle$ of operators **S**, **L**, and **D**.

Operator **M**	l_1-l_2	j_1-j_2	$\langle j_1\|\mathbf{M}\|j_2\rangle$
S	0	1	$-\frac{1}{2}[(2j_1+1)(2j_2+1)/j_1]^{\frac{1}{2}}$
L	0	1	$\frac{1}{2}[(2j_1+1)(2j_2+1)/j_1]^{\frac{1}{2}}$
D	0	1	$-\frac{1}{8}[(2j_1+1)(2j_2+1)/j_1]^{\frac{1}{2}}$
S	0	-1	$\frac{1}{2}[(2j_1+1)(2j_2+1)/j_2]^{\frac{1}{2}}$
L	0	-1	$-\frac{1}{2}[(2j_1+1)(2j_2+1)/j_2]^{\frac{1}{2}}$
D	0	-1	$\frac{1}{8}[(2j_1+1)(2j_2+1)/j_2]^{\frac{1}{2}}$
D	2	1	$\frac{3}{2}[l_1(l_1-1)/2(2l_1-1)]^{\frac{1}{2}}$
D	-2	-1	$-\frac{3}{2}[l_2(l_2-1)/2(2l_2-1)]^{\frac{1}{2}}$

TABLE III. Contributions of type I admixtures to M_n; the excitation is one of an even number n_1 particles in orbit $j_1=l_1+\frac{1}{2}$ to orbit $j_2=l_1-\frac{1}{2}$ containing an even number n_2 particles. Note that for $(b_S)_{2n}=(b_L)_{2n}=\mathcal{G}_n=\mu=1$, $(b_D)_{2n}=0$, the values of M_n given by this table are just those obtained by Arima and Horie[3] for $\delta\mu_{\mathrm{I}}$.

Nucleus	j	$M_n\mu/[n_1(2j_2+1-n_2)/(2j_2+1)]$		Contribution from even numbers of
Odd proton (neutron)	$l+\frac{1}{2}$	$\dfrac{-(l+2)l_1[g_S((b_S)_{2n}-\frac{1}{4}(b_D)_{2n})-g_L(b_L)_{2n}]\mathcal{G}_n(n_1,l_1+\frac{1}{2},l_1;n_2,l_1-\frac{1}{2},l_1)}{(2l+3)(2l_1+1)}$	$-V_sI/\Delta E$	protons (neutrons)
			$\frac{1}{2}(V_t-V_s)I/\Delta E$	neutrons (protons)
	$l-\frac{1}{2}$	$\dfrac{(l-1)l_1[g_S((b_S)_{2n}-\frac{1}{4}(b_D)_{2n})-g_L(b_L)_{2n}]\mathcal{G}_n(n_1,l_1+\frac{1}{2},l_1;n_2,l_1-\frac{1}{2},l_1)}{(2l+1)(2l_1+1)}$	$-V_sI/\Delta E$	protons (neutrons)
			$\frac{1}{2}(V_t-V_s)I/\Delta E$	neutrons (protons)

TABLE IV. Contributions to $M_n{}^D$ for admixtures of type I with $\Delta l=2$. If $j_1<j_2$, l_2 is larger; for $j_1>j_2$, l_1 is larger. We denote the larger l by $l_>$.

Nucleus	j	$-\mu M_n{}^D/[n_1(2j_2+1-n_2)]$		Contribution from even numbers of
Odd proton (neutron)	$l+\frac{1}{2}$	$\dfrac{-(3/8)lg_S(b_D)_{2n}\mathcal{G}_n(n_1,j_1,l_1;n_2,j_2,l_2)}{(2l+3)(2l_>-1)}$	$-V_sI/\Delta E$	protons (neutrons)
			$\frac{1}{2}(V_t-V_s)I/\Delta E$	neutrons (protons)
	$l-\frac{1}{2}$	$\dfrac{(3/8)(l+1)g_S(b_D)_{2n}\mathcal{G}_n(n_1,j_1,l_1;n_2,j_2,l_2)}{(2l+1)(2l_>-1)}$	$-V_sI/\Delta E$	protons (neutrons)
			$\frac{1}{2}(V_t-V_s)I/\Delta E$	neutrons (protons)

TABLE V. Contributions of type II and III admixtures to M_n. Type II is the excitation of an even number n particles in orbit $j_1=l+\frac{1}{2}$ into the odd group $j=l-\frac{1}{2}$ containing p particles. Type III is the excitation of the p particles in the odd group $j=l+\frac{1}{2}$ into the orbit $j_1=l-\frac{1}{2}$ containing n particles. If we specialize as in Table III, M_n are again the results of Arima-Horie[3] for $\delta\mu_{\text{II}}$ and $\delta\mu_{\text{III}}$. For the latter, usually $n=0$.

Excitation type	$M_{n\mu}$
II	$\dfrac{n(2j-p)(l-1)l[g_S((b_S)_{2n}-\frac{1}{4}(b_D)_{2n})-g_L(b_L)_{2n}]\mathcal{J}_n(n,j_1=l+\frac{1}{2},l;n,j=l-\frac{1}{2},l)(-V_sI/\Delta E)}{(2j-1)(2l+1)^2}$
III	$\dfrac{-(p-1)(2j_1+1-n)(l+1)(l+2)[g_S((b_S)_{2n}-\frac{1}{4}(b_D)_{2n})-g_L(b_L)_{2n}]\mathcal{J}_n(n,j=l+\frac{1}{2},l;n,j_1=l-\frac{1}{2},l)(-V_sI/\Delta E)}{(2j_1+1)(2l+1)(2l+3)}$

and the appropriate reduced matrix element is

$$\begin{aligned}\langle j^pJ=j\|\mathbf{M}_n\|j^pJ=j\rangle_{\text{III}}&=-(2j+1)^{\frac{1}{2}}C(j1j;\tfrac{1}{2}0)[(2j_1+1-n)\\&\quad\times(p-1)/(2j_1+1)(2j-1)]h_1^{l_j}(l_1j_1,lj)\\&\quad\times(-V_s)I(j_1j^3)/\Delta E.\end{aligned}\tag{24}$$

The contributions to M_n resulting from this type of admixture are listed in Tables V and VII.

Zero-Order Term

Finally an expression has to be given for the reduced matrix element of $\mathbf{M}_n$ with respect to the zero-order function $\Psi(j^p(j)J=j)$. Only the odd (p) particles will contribute to this matrix element and we obtain in a straightforward fashion

$$\begin{aligned}M_n&=C(j1j;j0)\langle j^pJ=j\|\mathbf{M}_n\|j^pJ=j\rangle\\&=C(j1j;j0)\langle j\|\mathbf{M}_n\|j\rangle\end{aligned}$$

for p identical particles.[22,23]

Thus for $j=l+\frac{1}{2}$,

$$M_n=\frac{1}{\mu}\left\{g_L(b_L)_{2n}(j-\tfrac{1}{2})+\tfrac{1}{2}g_S\left[(b_S)_{2n}+\frac{2j-1}{4(j+1)}(b_D)_{2n}\right]\right\}\mathcal{J}_n(n,j,l;n,j,l);\tag{25}$$

TABLE VI. Contributions to $M_n{}^D$ for admixtures of type II with $\Delta l=2$.

j	$-\mu M_n{}^D/[n(2j-p)]$	
$l+\frac{1}{2}$	$\dfrac{-(3/8)(l+1)g_S(b_D)_{2n}\mathcal{J}_n(n_1,j_1,l_1;n,j,l)(-V_sI/\Delta E)}{(2l+3)^2}$	$j_1>j$
$l-\frac{1}{2}$	$\dfrac{(3/8)l(l+1)g_S(b_D)_{2n}\mathcal{J}_n(n_1,j_1,l_1;n,j,l)(-V_sI/\Delta E)}{(l-1)(4l^2-1)}$	$j_1<j$

[22] G. Racah, Phys. Rev. **63**, 367 (1943).
[23] C. Schwartz and A. de-Shalit, Phys. Rev. **94**, 1257 (1954).

and for $j=l-\frac{1}{2}$,

$$M_n=\left(\frac{1}{\mu}\right)j\left\{g_L(b_L)_{2n}\frac{2j+3}{2j+2}-(g_S)\frac{1}{2j+2}\times\left[(b_S)_{2n}+\frac{(2j+3)}{4j}(b_D)_{2n}\right]\right\}\mathcal{J}_n(n,j,l;n,j,l),\tag{26}$$

where in both expressions all the symbols have been defined previously.[24]

V. RADIAL MATRIX ELEMENTS AND NUCLEAR ENERGY LEVELS

Evaluation of $\mathcal{J}_n(n_2,j_2,l_2;n_1,j_1,l_1)$ and ΔE

In order to obtain values for the radial integrals $\mathcal{J}_n$ for two single particle states $n_2j_2l_2$ and $n_1j_1l_1$, the following approach was adopted. The relevant single-particle wave functions and energies were calculated for particle motion in a nuclear potential well of the Saxon-Woods type having the form

$$V(R)=\frac{|V_0|}{1+\exp[A_0(R-R_0)]}+\frac{\kappa\hbar^2}{4m^2c^2}\frac{A_0|V_0|\exp[A_0(R-R_0)]}{\{1+\exp[A_0(R-R_0)]\}^2R}\mathbf{L}\cdot\mathbf{S}.\tag{27}$$

Coulomb effects were taken into account by assuming that the protons also moved in the potential of a charge distribution $\rho(R)$ of the form

$$\rho(R)=\frac{\rho_0[1+R^2/R_c{}^2]}{1+\exp[A_1(R-R_c)]},\tag{28}$$

so normalized that the resulting Coulomb potential $V_c(R)$ satisfied

$$V_c(R)\to(Z-1)e^2/R\quad\text{for}\quad R\to\infty.$$

[24] The subscript n is used variously denoting in the nuclear radial integrals the principal quantum number, in the angular matrix elements, numbers of particles, and thirdly the terms arising from the series expansion of the Dirac equation. The particular meaning is obvious from the context.

TABLE VII. Contributions to $M_n{}^D$ for admixtures of type III with $\Delta l=2$.

j	$-\mu M_n{}^D/[(2j_1+1-n)(p-1)]$	
$l+\frac{1}{2}$	$\dfrac{-(3/8)(l+1)g_S(b_D)_{2n}\mathcal{I}_n(n_1,j_1,l_1;n,j,l)(-V_sI/\Delta E)}{(2l+3)^2}$	$j_1>j$
$l-\frac{1}{2}$	$\dfrac{(3/8)l(l+1)g_S(b_D)_{2n}\mathcal{I}_n(n_1,j_1,l_1;n,j,l)(-V_sI/\Delta E)}{(l-1)(4l^2-1)}$	$j_1<j$

The well radii R_0 and R_c are defined by $R_0=r_0A^{\frac{1}{3}}$, $R_c=CR_0$, and the various values of the parameters used are as follows:

$V_0=64.5$ Mev for an odd proton, $\kappa=39.5$,

$V_0=50.0$ Mev for an odd neutron, $C=0.96$,

$r_0=1.20\times10^{-13}$ cm, $A_1=1.40\times10^{-13}$ cm^{-1}.

$A_0=1.40\times10^{-13}$ cm^{-1},

These values lead to an approximately correct ordering of the single particle neutron and proton energy levels.

We adopt the values of ΔE as given by Horie and Arima[25] who discuss their determination in detail. Our parameters are thus consistent with the ones used in their magnetic moment and electric quadrupole calculations. The pertinent energy denominators are reproduced in Table VIII.

The calculations of the wave functions, energies, and finally the radial matrix elements were carried out on the Mercury computer at Oxford using a program due to Dr. L. M. Delves.

The radial integrals required are of the form

$$\mathcal{I}_n=\frac{1}{R_N{}^{2n}}\int R_1(R)R^{2n+2}R_2(R)dR, \tag{29}$$

where R_N is the full radial extent of the trapezoidal charge distribution and is defined in Sec. III. In the machine calculations, the actual radial integrals calculated were

$$\mathcal{I}_n'=\frac{1}{R_0{}^{2n}}\int R_1(R)R^{2n+2}R_2(R)dR, \tag{30}$$

where R_0 is involved in expression (27) for the nuclear potential distribution. Thus $\mathcal{I}_n$ and $\mathcal{I}_n'$ are related by

$$\mathcal{I}_n=\left(\frac{R_0{}^2}{R_N{}^2}\right)^n\mathcal{I}_n'=\left(\frac{1.20A^{\frac{1}{3}}}{1.07A^{\frac{1}{3}}+1.50}\right)^{2n}\mathcal{I}_n', \tag{31}$$

where we have used the expression R_N given in Sec. III.

[25] H. Horie and A. Arima, Phys. Rev. **99**, 778 (1955).

TABLE VIII. Values in Mev of energy differences ΔE required for calculations of ϵ. These are obtained from the work of Arima and Horie (see references 3 and 25).

States	ΔE	States	ΔE	States	ΔE
$1d_{5/2}-1d_{3/2}$	5	$1g_{9/2}-1g_{7/2}$	2.5	$1h_{9/2}-2f_{7/2}$	0.5
$2s_{1/2}-1d_{3/2}$	4	$1g_{7/2}-2d_{5/2}$	0.5	$2f_{7/2}-2f_{5/2}$	1.5
$1f_{7/2}-1f_{5/2}$	3	$2d_{5/2}-2d_{3/2}$	1.5	$2f_{5/2}-3p_{3/2}$	0.75
$2p_{3/2}-2p_{1/2}$	1.5	$2d_{3/2}-3s_{1/2}$	0.25	$3p_{3/2}-3p_{1/2}$	0.5
$2p_{3/2}-1f_{5/2}$	0.5	$1h_{11/2}-1h_{9/2}$	2	$1i_{13/2}-1i_{11/2}$	2

In Table IX the final results for $\mathcal{I}_n'(n_1,l_1,j_1;n_2,l_2,j_2)$ are given, but it must be remembered that in using this table the relation of Eq. (31) must be used in order to obtain $\mathcal{I}_n(n_1,l_1,j_1;n_2,l_2,j_2)$. The program also printed out the radial wave functions, binding energies, $\mathcal{I}_3'$, and $\mathcal{I}_4'$.

Values of V_s, V_t, and I

In estimating the values of these three parameters, we follow the procedure of Arima and Horie and take $|V_t|\approx1.5|V_s|$. We further ignore the dependence of the integrals I [Eq. (18)] on the quantum numbers involved and only take into account the approximate mass dependence of I. The value of the product V_sI is related to pairing energy data and, following Arima and Horie, we take $V_sI=-25/A$ Mev.

VI. COMPARISON WITH EXPERIMENTAL RESULTS AND DISCUSSION

General Expression for ϵ

We now consider the general form taken by ϵ when many admixtures of different types are contributing. By Eq. (8),

$$-\epsilon=\sum_{n=1,2}\int\Psi_N{}^*\mathbf{M}_n\Psi_Nd\tau_N=\sum_{n=1,2}M_n, \tag{32}$$

where $\mathbf{M}_n$ is the operator defined in (9), and where Ψ_N may contain the three types of admixtures described. It must be remembered in this connection that there may be several different admixtures of each type contributing. Now it is of interest and of some practical use to write down in a semi-symbolic way the form taken by $-\epsilon$ taking into account all of the possible first-order admixtures investigated.[26] Referring to Tables III through VII, and Eqs. (25) and (26), it

[26] H. H. Stroke and R. J. Blin-Stoyle, *Proceedings of the International Conference on Nuclear Structure, Kingston*, edited by D. A. Bromley and E. W. Vogt (University of Toronto Press, Toronto, 1960), p. 518.

TABLE IX. Values of radial integrals $\mathcal{I}_n'$ between single-particle states $n_1l_1j_1$ and $n_2l_2j_2$ required for the calculation of hfs anomalies. For states which are unbound with the parameters indicated in the text, the program increases the well depth to give a binding energy $E=0$.

Element	Z	A	Proton states	$\mathcal{I}_1'$	$\mathcal{I}_2'$	Neutron states	$\mathcal{I}_1'$	$\mathcal{I}_2'$
Cl	17	35	$1d_{3/2}-1d_{3/2}$	0.686	0.641	$1d_{3/2}-1d_{5/2}$	0.800	0.876
			$1d_{3/2}-1d_{5/2}$	0.689	0.629			
			$1d_{3/2}-2s_{1/2}$	−0.616	−0.700			
		37	$1d_{3/2}-1d_{3/2}$	0.662	0.592			
			$1d_{3/2}-1d_{5/2}$	0.669	0.590			
			$1d_{3/2}-2s_{1/2}$	−0.590	−0.646			
Ar	18	37	$1d_{3/2}-1d_{5/2}$	0.670	0.592	$1d_{3/2}-1d_{3/2}$	0.788	0.874
			$1d_{3/2}-2s_{1/2}$	−0.592	−0.648			
		39	$1d_{3/2}-1d_{5/2}$	0.653	0.559	$1f_{7/2}-1f_{7/2}$	0.994	1.292
			$1d_{3/2}-2s_{1/2}$	−0.570	−0.603			
		41	$1d_{3/2}-1d_{5/2}$	0.638	0.531	$1f_{7/2}-1f_{7/2}$	0.962	1.199
			$1d_{3/2}-2s_{1/2}$	−0.551	−0.565	$1f_{5/2}-1f_{7/2}$	0.993	1.40
K	19	39	$1d_{3/2}-1d_{3/2}$	0.644	0.555			
		41	$1d_{3/2}-1d_{3/2}$	0.626	0.521	$1f_{5/2}-1f_{7/2}$	0.993	1.40
		43	$1d_{3/2}-1d_{3/2}$	0.610	0.492	$1f_{5/2}-1f_{7/2}$	0.974	1.32
Ca	20	41				$1f_{7/2}-1f_{7/2}$	0.962	1.199
		43				$1f_{7/2}-1f_{7/2}$	0.935	1.122
						$1f_{5/2}-1f_{7/2}$	0.974	1.32
		45				$1f_{7/2}-1f_{7/2}$	0.910	1.056
						$1f_{5/2}-1f_{7/2}$	0.952	1.26
		47				$1f_{7/2}-1f_{7/2}$	0.889	1.000
						$1f_{5/2}-1f_{7/2}$	0.932	1.20
Cu	29	61	$2p_{3/2}-2p_{3/2}$	0.721	0.804	$2p_{1/2}-2p_{3/2}$	0.892	1.250
			$1f_{5/2}-1f_{7/2}$	0.690	0.590	$1f_{5/2}-1f_{7/2}$	0.781	0.777
						$2p_{3/2}-1f_{5/2}$	−0.654	−0.902
		63	$2p_{3/2}-2p_{3/2}$	0.706	0.770	$2p_{1/2}-2p_{3/2}$	0.865	1.173
			$1f_{5/2}-1f_{7/2}$	0.680	0.571	$1f_{5/2}-1f_{7/2}$	0.767	0.745
						$2p_{3/2}-1f_{5/2}$	−0.631	−0.846
		65	$2p_{3/2}-2p_{3/2}$	0.692	0.740	$2p_{1/2}-2p_{3/2}$	0.840	1.106
			$1f_{5/2}-1f_{7/2}$	0.670	0.554	$1f_{5/2}-1f_{7/2}$	0.754	0.717
						$2p_{3/2}-1f_{5/2}$	−0.612	−0.798
		67	$2p_{3/2}-2p_{3/2}$	0.679	0.713	$2p_{1/2}-2p_{3/2}$	0.818	1.047
			$1f_{5/2}-1f_{7/2}$	0.661	0.538			
Zn	30	63	$2p_{1/2}-2p_{3/2}$	0.716	0.794	$1f_{5/2}-1f_{5/2}$	0.770	0.781
			$1f_{5/2}-1f_{7/2}$	0.680	0.572	$2p_{3/2}-2p_{3/2}$	0.839	1.098
			$2p_{3/2}-1f_{5/2}$	−0.513	−0.582	$2p_{1/2}-2p_{3/2}$	0.865	1.173
						$1f_{5/2}-1f_{7/2}$	0.767	0.745
						$2p_{3/2}-1f_{5/2}$	−0.631	−0.846
		65	$2p_{1/2}-2p_{3/2}$	0.700	0.758	$1f_{5/2}-1f_{5/2}$	0.753	0.739
			$1f_{5/2}-1f_{7/2}$	0.671	0.555	$2p_{3/2}-2p_{3/2}$	0.818	1.043
			$2p_{3/2}-1f_{5/2}$	−0.500	−0.557	$2p_{1/2}-2p_{3/2}$	0.840	1.106
						$1f_{5/2}-1f_{7/2}$	0.754	0.717
						$2p_{3/2}-1f_{5/2}$	−0.612	−0.798
		67	$2p_{1/2}-2p_{3/2}$	0.685	0.727	$1f_{5/2}-1f_{5/2}$	0.737	0.703
			$1f_{5/2}-1f_{7/2}$	0.662	0.539	$2p_{1/2}-2p_{3/2}$	0.818	1.047
			$2p_{3/2}-1f_{5/2}$	−0.489	−0.534			
		69	$2p_{1/2}-2p_{3/2}$	0.672	0.699	$2p_{1/2}-2p_{1/2}$	0.819	1.048
			$1f_{5/2}-1f_{7/2}$	0.654	0.525			
			$2p_{3/2}-1f_{5/2}$	−0.479	−0.514			
		71	$2p_{1/2}-2p_{3/2}$	0.659	0.673	$2p_{1/2}-2p_{1/2}$	0.798	0.993
			$1f_{5/2}-1f_{7/2}$	0.646	0.512	$1g_{9/2}-1g_{9/2}$	0.893	0.965
			$2p_{3/2}-1f_{5/2}$	−0.469	−0.497			

TABLE IX.—*Continued.*

Element	Z	A	Proton states	g_1'	g_2'	Neutron states	g_1'	g_2'
Ga	31	65	$2p_{3/2}-2p_{3/2}$	0.694	0.745	$2p_{1/2}-2p_{3/2}$	0.840	1.106
			$2p_{1/2}-2p_{3/2}$	0.701	0.761	$1f_{5/2}-1f_{7/2}$	0.754	0.717
			$1f_{5/2}-1f_{7/2}$	0.671	0.556	$2p_{3/2}-1f_{5/2}$	−0.612	−0.798
			$2p_{3/2}-1f_{5/2}$	−0.501	−0.558			
		67	$2p_{3/2}-2p_{3/2}$	0.681	0.717	$2p_{1/2}-2p_{3/2}$	0.818	1.047
			$2p_{1/2}-2p_{3/2}$	0.686	0.730	$1f_{5/2}-1f_{7/2}$	0.742	0.691
			$1f_{5/2}-1f_{7/2}$	0.663	0.541	$2p_{3/2}-1f_{5/2}$	−0.594	−0.756
			$2p_{3/2}-1f_{5/2}$	−0.490	−0.536			
		69	$2p_{3/2}-2p_{3/2}$	0.669	0.692	$2p_{1/2}-2p_{3/2}$	0.798	0.994
			$2p_{1/2}-2p_{3/2}$	0.673	0.701			
			$1f_{5/2}-1f_{7/2}$	0.654	0.526			
			$2p_{3/2}-1f_{5/2}$	−0.479	−0.516			
		71	$2p_{3/2}-2p_{3/2}$	0.658	0.669	$2p_{1/2}-2p_{3/2}$	0.780	0.948
			$2p_{1/2}-2p_{3/2}$	0.660	0.675	$1g_{7/2}-1g_{9/2}$	0.873	0.972
			$1f_{5/2}-1f_{7/2}$	0.647	0.513			
			$2p_{3/2}-1f_{5/2}$	−0.470	−0.498			
		73	$2p_{3/2}-2p_{3/2}$	0.647	0.648	$2p_{1/2}-2p_{3/2}$	0.763	0.907
			$2p_{1/2}-2p_{3/2}$	0.649	0.651	$1g_{7/2}-1g_{9/2}$	0.898	1.04
			$1f_{5/2}-1f_{7/2}$	0.640	0.501			
			$2p_{3/2}-1f_{5/2}$	−0.461	−0.482			
As	33	71	$2p_{3/2}-2p_{3/2}$	0.660	0.673	$2p_{1/2}-2p_{3/2}$	0.780	0.948
			$2p_{1/2}-2p_{3/2}$	0.662	0.680			
			$1f_{5/2}-1f_{7/2}$	0.648	0.515			
			$2p_{3/2}-1f_{5/2}$	−0.471	−0.501			
		73	$2p_{3/2}-2p_{3/2}$	0.649	0.652	$2p_{1/2}-2p_{3/2}$	0.763	0.907
			$2p_{1/2}-2p_{3/2}$	0.651	0.656	$1g_{7/2}-1g_{9/2}$	0.898	1.04
			$1f_{5/2}-1f_{7/2}$	0.641	0.503			
			$2p_{3/2}-1f_{5/2}$	−0.462	−0.484			
		75	$2p_{3/2}-2p_{3/2}$	0.639	0.632	$2p_{1/2}-2p_{3/2}$	0.747	0.869
			$2p_{1/2}-2p_{3/2}$	0.640	0.634	$1g_{7/2}-1g_{9/2}$	0.891	1.02
			$1f_{5/2}-1f_{7/2}$	0.634	0.491			
			$2p_{3/2}-1f_{5/2}$	−0.454	−0.469			
		77	$2p_{3/2}-2p_{3/2}$	0.630	0.614	$2p_{1/2}-2p_{3/2}$	0.732	0.834
			$2p_{1/2}-2p_{3/2}$	0.630	0.614	$1g_{7/2}-1g_{9/2}$	0.882	0.990
			$1f_{5/2}-1f_{7/2}$	0.628	0.481			
			$2p_{3/2}-1f_{5/2}$	−0.446	−0.455			
Br	35	75	$2p_{3/2}-2p_{3/2}$	0.640	0.635	$2p_{1/2}-2p_{3/2}$	0.747	0.869
			$2p_{1/2}-2p_{3/2}$	0.641	0.638			
			$1f_{5/2}-1f_{7/2}$	0.635	0.493			
			$2p_{3/2}-1f_{5/2}$	−0.454	−0.471			
			$1g_{7/2}-1g_{9/2}$	0.742	0.671			
		77	$2p_{3/2}-2p_{3/2}$	0.632	0.617	$2p_{1/2}-2p_{3/2}$	0.732	0.834
			$2p_{1/2}-2p_{3/2}$	0.631	0.618	$1g_{7/2}-1g_{9/2}$	0.882	0.990
			$1f_{5/2}-1f_{7/2}$	0.629	0.483			
			$2p_{3/2}-1f_{5/2}$	−0.447	−0.458			
			$1g_{7/2}-1g_{9/2}$	0.738	0.662			
		79	$2p_{3/2}-2p_{3/2}$	0.623	0.600	$2p_{1/2}-2p_{3/2}$	0.724	0.814
			$2p_{1/2}\ \ 2p_{3/2}$	0.622	0.599	$1g_{7/2}-1g_{9/2}$	0.847	0.902
			$1f_{5/2}-1f_{7/2}$	0.623	0.473			
			$2p_{3/2}-1f_{5/2}$	−0.440	−0.445			
			$1g_{7/2}-1g_{9/2}$	0.735	0.654			
		81	$2p_{3/2}-2p_{3/2}$	0.615	0.584	$2p_{1/2}-2p_{3/2}$	0.707	0.777
			$2p_{1/2}-2p_{3/2}$	0.613	0.582	$1g_{7/2}-1g_{9/2}$	0.840	0.886
			$1f_{5/2}-1f_{7/2}$	0.617	0.464			
			$2p_{3/2}-1f_{5/2}$	−0.434	−0.433			
			$1g_{7/2}-1g_{9/2}$	0.731	0.646			
		83	$2p_{3/2}-2p_{3/2}$	0.607	0.570	$2p_{1/2}-2p_{3/2}$	0.699	0.759
			$2p_{1/2}-2p_{3/2}$	0.604	0.566	$1g_{7/2}-1g_{9/2}$	0.834	0.870
			$1f_{5/2}-1f_{7/2}$	0.612	0.455			
			$2p_{3/2}-1f_{5/2}$	−0.428	−0.422			
			$1g_{7/2}-1g_{9/2}$	0.728	0.638			

TABLE IX.—*Continued.*

Element	Z	A	Proton states	g_1'	g_2'	Neutron states	g_1'	g_2'
Kr	36	79	$2p_{1/2}-2p_{3/2}$	0.622	0.601	$2p_{1/2}-2p_{1/2}$	0.734	0.836
			$1f_{5/2}-1f_{7/2}$	0.624	0.474	$1g_{7/2}-1g_{9/2}$	0.847	0.902
			$2p_{3/2}-1f_{5/2}$	−0.441	−0.446	$2p_{1/2}-2p_{3/2}$	0.724	0.814
		81	$2p_{1/2}-2p_{3/2}$	0.614	0.583	$1g_{9/2}-1g_{9/2}$	0.842	0.847
			$1f_{5/2}-1f_{7/2}$	0.618	0.465	$1g_{7/2}-1g_{9/2}$	0.840	0.886
			$2p_{3/2}-1f_{5/2}$	−0.434	−0.434	$2p_{1/2}-2p_{3/2}$	0.707	0.777
		83	$2p_{1/2}-2p_{3/2}$	0.605	0.568	$1g_{9/2}-1g_{9/2}$	0.830	0.819
			$1f_{5/2}-1f_{7/2}$	0.613	0.456	$1g_{7/2}-1g_{9/2}$	0.834	0.870
			$2p_{3/2}-1f_{5/2}$	−0.428	−0.423	$2p_{1/2}-2p_{3/2}$	0.699	0.759
		85	$2p_{1/2}-2p_{3/2}$	0.597	0.553	$1g_{9/2}-1g_{9/2}$	0.821	0.800
			$1f_{5/2}-1f_{7/2}$	0.608	0.448	$1g_{7/2}-1g_{9/2}$	0.822	0.840
			$2p_{3/2}-1f_{5/2}$	−0.422	−0.413			
Rb	37	81	$2p_{3/2}-2p_{3/2}$	0.616	0.587	$1g_{7/2}-1g_{9/2}$	0.840	0.886
			$2p_{1/2}-2p_{3/2}$	0.614	0.585	$2p_{1/2}-2p_{3/2}$	0.707	0.777
		83	$1f_{5/2}-1f_{5/2}$	0.588	0.423	$1g_{7/2}-1g_{9/2}$	0.834	0.870
			$2p_{1/2}-2p_{3/2}$	0.606	0.569	$2p_{1/2}-2p_{3/2}$	0.699	0.759
		85	$1f_{5/2}-1f_{5/2}$	0.583	0.414	$1g_{7/2}-1g_{9/2}$	0.822	0.840
			$2p_{1/2}-2p_{3/2}$	0.598	0.554	$2p_{1/2}-2p_{3/2}$	0.684	0.726
		87	$2p_{3/2}-2p_{3/2}$	0.594	0.546	$1g_{7/2}-1g_{9/2}$	0.810	0.811
			$2p_{1/2}-2p_{3/2}$	0.590	0.540			
Sr	38	83	$2p_{1/2}-2p_{3/2}$	0.606	0.570	$1g_{9/2}-1g_{9/2}$	0.830	0.819
						$1g_{7/2}-1g_{9/2}$	0.834	0.870
						$2p_{1/2}-2p_{3/2}$	0.699	0.759
		85	$2p_{1/2}-2p_{3/2}$	0.598	0.556	$1g_{9/2}-1g_{9/2}$	0.821	0.800
						$1g_{7/2}-1g_{9/2}$	0.822	0.840
						$2p_{1/2}-2p_{3/2}$	0.684	0.726
		87	$2p_{1/2}-2p_{3/2}$	0.591	0.542	$1g_{9/2}-1g_{9/2}$	0.813	0.783
						$1g_{7/2}-1g_{9/2}$	0.810	0.811
		89	$2p_{1/2}-2p_{3/2}$	0.584	0.529	$2d_{5/2}-2d_{5/2}$	0.909	1.241
						$1g_{7/2}-1g_{9/2}$	0.799	0.786
Mo	42	95	$2p_{1/2}-2p_{3/2}$	0.567	0.499	$2d_{5/2}-2d_{5/2}$	0.865	1.11
			$1g_{7/2}-1g_{9/2}$	0.694	0.573	$2d_{3/2}-2d_{5/2}$	0.907	1.24
						$1g_{7/2}-1g_{9/2}$	0.774	0.729
						$2d_{5/2}-1g_{7/2}$	−0.567	−0.786
		97	$2p_{1/2}-2p_{3/2}$	0.561	0.489	$2d_{5/2}-2d_{5/2}$	0.850	1.07
			$1g_{7/2}-1g_{9/2}$	0.688	0.562	$2d_{3/2}-2d_{5/2}$	0.887	1.19
						$1g_{7/2}-1g_{9/2}$	0.766	0.711
						$2d_{5/2}-1g_{7/2}$	−0.546	−0.742
Ag	47	105	$2p_{1/2}-2p_{1/2}$	0.540	0.451	$1g_{7/2}-1g_{9/2}$	0.734	0.646
			$1g_{7/2}-1g_{9/2}$	0.669	0.530	$2d_{3/2}-2d_{5/2}$	0.815	0.996
						$2d_{5/2}-1g_{7/2}$	−0.501	−0.640
		107	$2p_{1/2}-2p_{1/2}$	0.535	0.443	$1g_{7/2}-1g_{9/2}$	0.727	0.633
			$1g_{7/2}-1g_{9/2}$	0.664	0.522	$2d_{3/2}-2d_{5/2}$	0.801	0.962
						$2d_{5/2}-1g_{7/2}$	−0.491	−0.620
		109	$2p_{1/2}-2p_{1/2}$	0.530	0.435	$1g_{7/2}-1g_{9/2}$	0.722	0.622
			$1g_{7/2}-1g_{9/2}$	0.660	0.514	$2d_{3/2}-2d_{5/2}$	0.788	0.930
						$2d_{5/2}-1g_{7/2}$	−0.483	−0.602
		111	$2p_{1/2}-2p_{1/2}$	0.526	0.428	$2d_{3/2}-2d_{5/2}$	0.776	0.901
			$1g_{7/2}-1g_{9/2}$	0.656	0.507			
		113	$2p_{1/2}-2p_{1/2}$	0.521	0.421	$2d_{3/2}-2d_{5/2}$	0.765	0.874
			$1g_{7/2}-1g_{9/2}$	0.652	0.500	$1h_{9/2}-1h_{11/2}$	0.805	0.780
Cd	48	105	$1g_{7/2}-1g_{9/2}$	0.670	0.531	$2d_{5/2}-2d_{5/2}$	0.791	0.930
			$2p_{1/2}-2p_{3/2}$	0.546	0.464	$2d_{3/2}-2d_{5/2}$	0.815	0.996
						$1g_{7/2}-1g_{9/2}$	0.734	0.646
						$2d_{5/2}-1g_{7/2}$	−0.501	−0.640

TABLE IX.—*Continued.*

Element	Z	A	Proton states	g_1'	g_2'	Neutron states	g_1'	g_2'
Cd	48	107	$1g_{7/2}-1g_{9/2}$	0.665	0.523	$2d_{5/2}-2d_{5/2}$	0.780	0.904
			$2p_{1/2}-2p_{3/2}$	0.541	0.456	$2d_{3/2}-2d_{5/2}$	0.801	0.962
						$1g_{7/2}-1g_{9/2}$	0.727	0.633
						$2d_{5/2}-1g_{7/2}$	−0.491	−0.620
		109	$1g_{7/2}-1g_{9/2}$	0.660	0.515	$2d_{5/2}-2d_{5/2}$	0.770	0.880
			$2p_{1/2}-2p_{3/2}$	0.537	0.449	$2d_{3/2}-2d_{5/2}$	0.788	0.930
						$1g_{7/2}-1g_{9/2}$	0.722	0.622
						$2d_{5/2}-1g_{7/2}$	−0.483	−0.602
		111	$1g_{7/2}-1g_{9/2}$	0.656	0.508	$3s_{1/2}-3s_{1/2}$	0.832	1.113
			$2p_{1/2}-2p_{3/2}$	0.532	0.442	$2d_{3/2}-2d_{5/2}$	0.776	0.901
						$1g_{7/2}-1g_{9/2}$	0.716	0.611
						$2d_{5/2}-1g_{7/2}$	−0.475	−0.585
		113	$1g_{7/2}-1g_{9/2}$	0.652	0.501	$3s_{1/2}-3s_{1/2}$	0.817	1.074
			$2p_{1/2}-2p_{3/2}$	0.528	0.435	$2d_{3/2}-2d_{5/2}$	0.765	0.874
		115	$1g_{7/2}-1g_{9/2}$	0.648	0.495	$3s_{1/2}-3s_{1/2}$	0.803	1.038
			$2p_{1/2}-2p_{3/2}$	0.524	0.428	$2d_{3/2}-2d_{5/2}$	0.754	0.849
						$1h_{9/2}-1h_{11/2}$	0.802	0.773
		117	$1g_{7/2}-1g_{9/2}$	0.644	0.488	$3s_{1/2}-3s_{1/2}$	0.789	1.004
			$2p_{1/2}-2p_{3/2}$	0.520	0.422	$2d_{3/2}-2d_{5/2}$	0.744	0.826
						$1h_{9/2}-1h_{11/2}$	0.801	0.766
In	49	109	$1g_{9/2}-1g_{9/2}$	0.700	0.565	$1g_{7/2}-1g_{9/2}$	0.722	0.622
			$1g_{7/2}-1g_{9/2}$	0.661	0.516	$2d_{3/2}-2d_{5/2}$	0.788	0.930
						$2d_{5/2}-1g_{7/2}$	−0.483	−0.602
		111	$1g_{9/2}-1g_{9/2}$	0.696	0.559	$1g_{7/2}-1g_{9/2}$	0.716	0.611
			$1g_{7/2}-1g_{9/2}$	0.657	0.509	$2d_{3/2}-2d_{5/2}$	0.776	0.901
						$2d_{5/2}-1g_{7/2}$	−0.475	−0.585
		113	$1g_{9/2}-1g_{9/2}$	0.693	0.553	$1g_{7/2}-1g_{9/2}$	0.710	0.601
			$1g_{7/2}-1g_{9/2}$	0.652	0.502	$2d_{3/2}-2d_{5/2}$	0.765	0.874
		115	$1g_{9/2}-1g_{9/2}$	0.689	0.547	$2d_{3/2}-2d_{5/2}$	0.754	0.849
			$1g_{7/2}-1g_{9/2}$	0.648	0.495	$1h_{9/2}-1h_{11/2}$	0.802	0.773
		117	$1g_{9/2}-1g_{9/2}$	0.686	0.541	$2d_{3/2}-2d_{5/2}$	0.744	0.826
			$1g_{7/2}-1g_{9/2}$	0.645	0.489	$1h_{9/2}-1h_{11/2}$	0.801	0.766
		119	$1g_{9/2}-1g_{9/2}$	0.683	0.536	$2d_{3/2}-2d_{5/2}$	0.735	0.805
			$1g_{7/2}-1g_{9/2}$	0.641	0.483	$1h_{9/2}-1h_{11/2}$	0.797	0.760
Sn	50	115	$1g_{7/2}-1g_{9/2}$	0.649	0.496	$3s_{1/2}-3s_{1/2}$	0.803	1.038
						$2d_{3/2}-2d_{5/2}$	0.754	0.849
		117	$1g_{7/2}-1g_{9/2}$	0.645	0.490	$3s_{1/2}-3s_{1/2}$	0.789	1.004
						$2d_{3/2}-2d_{5/2}$	0.744	0.826
						$1h_{9/2}-1h_{11/2}$	0.801	0.766
		119	$1g_{7/2}-1g_{9/2}$	0.642	0.484	$3s_{1/2}-3s_{1/2}$	0.777	0.973
						$2d_{3/2}-2d_{5/2}$	0.735	0.805
						$1h_{9/2}-1h_{11/2}$	0.797	0.760
Sb	51	119	$2d_{5/2}-2d_{5/2}$	0.645	0.612	$2d_{3/2}-2d_{5/2}$	0.735	0.805
			$1g_{7/2}-1g_{7/2}$	0.614	0.444	$1h_{9/2}-1h_{11/2}$	0.797	0.760
			$1g_{7/2}-1g_{9/2}$	0.642	0.485			
		121	$2d_{5/2}-2d_{5/2}$	0.639	0.601	$2d_{3/2}-2d_{5/2}$	0.726	0.785
			$1g_{7/2}-1g_{9/2}$	0.639	0.479	$1h_{9/2}-1h_{11/2}$	0.796	0.753
		123	$1g_{7/2}-1g_{7/2}$	0.607	0.433	$2d_{3/2}-2d_{5/2}$	0.717	0.766
			$1g_{7/2}-1g_{9/2}$	0.635	0.474	$1h_{9/2}-1h_{11/2}$	0.792	0.747
		125	$1g_{7/2}-1g_{7/2}$	0.604	0.428	$2d_{3/2}-2d_{5/2}$	0.709	0.748
			$2d_{5/2}-2d_{5/2}$	0.628	0.581	$1h_{9/2}-1h_{11/2}$	0.789	0.741
			$1g_{7/2}-1g_{9/2}$	0.632	0.469			
Te	52	123	$1g_{7/2}-1g_{9/2}$	0.636	0.474	$3s_{1/2}-3s_{1/2}$	0.754	0.918
			$2d_{3/2}-2d_{5/2}$	0.629	0.587	$2d_{3/2}-2d_{5/2}$	0.717	0.766
			$2d_{5/2}-1g_{7/2}$	−0.369	−0.394	$1h_{9/2}-1h_{11/2}$	0.792	0.747

TABLE IX.—*Continued.*

Element	Z	A	Proton states	g_1'	g_2'	Neutron states	g_1'	g_2'
Te	52	125	$1g_{7/2}-1g_{9/2}$	0.632	0.469	$3s_{1/2}-3s_{1/2}$	0.743	0.892
			$2d_{3/2}-2d_{5/2}$	0.623	0.576	$2d_{3/2}-2d_{5/2}$	0.709	0.748
			$2d_{5/2}-1g_{7/2}$	−0.365	−0.388	$1h_{9/2}-1h_{11/2}$	0.789	0.741
I	53	121	$1g_{7/2}-1g_{7/2}$	0.611	0.440	$2d_{3/2}-2d_{5/2}$	0.726	0.785
			$2d_{5/2}-2d_{5/2}$	0.640	0.604	$1h_{9/2}-1h_{11/2}$	0.796	0.753
			$1g_{7/2}-1g_{9/2}$	0.640	0.481			
			$2d_{3/2}-2d_{5/2}$	0.636	0.601			
			$2d_{5/2}-1g_{7/2}$	−0.373	−0.402			
		123	$2d_{5/2}-2d_{5/2}$	0.635	0.594	$2d_{3/2}-2d_{5/2}$	0.717	0.766
			$1g_{7/2}-1g_{9/2}$	0.636	0.475	$1h_{9/2}-1h_{11/2}$	0.792	0.747
			$2d_{3/2}-2d_{5/2}$	0.630	0.589			
			$2d_{5/2}-1g_{7/2}$	−0.369	−0.395			
		125	$2d_{5/2}-2d_{5/2}$	0.630	0.584	$2d_{3/2}-2d_{5/2}$	0.709	0.748
			$1g_{7/2}-1g_{9/2}$	0.633	0.470	$1h_{9/2}-1h_{11/2}$	0.789	0.741
			$2d_{3/2}-2d_{5/2}$	0.624	0.578			
			$2d_{5/2}-1g_{7/2}$	−0.366	−0.388			
		127	$2d_{5/2}-2d_{5/2}$	0.624	0.575	$2d_{3/2}-2d_{5/2}$	0.701	0.731
			$1g_{7/2}-1g_{9/2}$	0.630	0.465	$1h_{9/2}-1h_{11/2}$	0.789	0.736
			$2d_{3/2}-2d_{5/2}$	0.618	0.568			
			$2d_{5/2}-1g_{7/2}$	−0.362	−0.382			
		129	$1g_{7/2}-1g_{7/2}$	0.598	0.419	$2d_{3/2}-2d_{5/2}$	0.694	0.716
			$1g_{7/2}-1g_{9/2}$	0.627	0.460	$1h_{9/2}-1h_{11/2}$	0.785	0.730
			$2d_{3/2}-2d_{5/2}$	0.612	0.557			
			$2d_{5/2}-1g_{7/2}$	−0.359	−0.376			
		131	$1g_{7/2}-1g_{7/2}$	0.596	0.415	$2d_{3/2}-2d_{5/2}$	0.687	0.701
			$1g_{7/2}-1g_{9/2}$	0.624	0.456	$1h_{9/2}-1h_{11/2}$	0.782	0.725
			$2d_{3/2}-2d_{5/2}$	0.607	0.548			
			$2d_{5/2}-1g_{7/2}$	−0.356	−0.371			
Xe	54	123	$1g_{7/2}-1g_{9/2}$	0.637	0.476	$3s_{1/2}-3s_{1/2}$	0.754	0.918
			$2d_{5/2}-1g_{7/2}$	−0.370	−0.395	$2d_{3/2}-2d_{5/2}$	0.717	0.766
						$1h_{9/2}-1h_{11/2}$	0.792	0.747
		125	$1g_{7/2}-1g_{9/2}$	0.633	0.471	$3s_{1/2}-3s_{1/2}$	0.743	0.892
			$2d_{5/2}-1g_{7/2}$	−0.366	−0.389	$2d_{3/2}-2d_{5/2}$	0.709	0.748
						$1h_{9/2}-1h_{11/2}$	0.789	0.741
		127	$1g_{7/2}-1g_{9/2}$	0.630	0.466	$3s_{1/2}-3s_{1/2}$	0.733	0.869
			$2d_{5/2}-1g_{7/2}$	−0.362	−0.383	$2d_{3/2}-2d_{5/2}$	0.701	0.731
						$1h_{9/2}-1h_{11/2}$	0.789	0.736
		129	$1g_{7/2}-1g_{9/2}$	0.627	0.461	$3s_{1/2}-3s_{1/2}$	0.724	0.847
			$2d_{5/2}-1g_{7/2}$	−0.359	−0.377	$2d_{3/2}-2d_{5/2}$	0.694	0.716
						$1h_{9/2}-1h_{11/2}$	0.785	0.730
		131	$1g_{7/2}-1g_{9/2}$	0.624	0.457	$2d_{3/2}-2d_{3/2}$	0.695	0.715
			$2d_{5/2}-1g_{7/2}$	−0.356	−0.371	$2d_{3/2}-2d_{5/2}$	0.687	0.701
						$1h_{9/2}-1h_{11/2}$	0.782	0.725
						$3s_{1/2}-2d_{3/2}$	−0.659	−0.749
		133	$1g_{7/2}-1g_{9/2}$	0.622	0.452	$2d_{3/2}-2d_{3/2}$	0.687	0.699
			$2d_{5/2}-1g_{7/2}$	−0.353	−0.366	$2d_{3/2}-2d_{5/2}$	0.680	0.687
						$1h_{9/2}-1h_{11/2}$	0.777	0.714
						$3s_{1/2}-2d_{3/2}$	−0.651	−0.732
		135	$1g_{7/2}-1g_{9/2}$	0.619	0.448	$2d_{3/2}-2d_{3/2}$	0.680	0.684
			$2d_{5/2}-1g_{7/2}$	−0.350	−0.361	$1h_{9/2}-1h_{11/2}$	0.772	0.702
						$3s_{1/2}-2d_{3/2}$	−0.643	−0.715
Cs	55	125	$3s_{1/2}-3s_{1/2}$	0.638	0.665	$2d_{3/2}-2d_{5/2}$	0.709	0.748
			$2d_{5/2}-2d_{5/2}$	0.631	0.586	$1h_{9/2}-1h_{11/2}$	0.789	0.741
			$1g_{7/2}-1g_{7/2}$	0.430	0.355			
			$1g_{7/2}-1g_{9/2}$	0.634	0.472			
			$2d_{3/2}-2d_{5/2}$	0.625	0.581			
			$2d_{5/2}-1g_{7/2}$	−0.366	−0.390			
		127	$3s_{1/2}-3s_{1/2}$	0.631	0.651	$2d_{3/2}-2d_{5/2}$	0.701	0.731
			$1g_{7/2}-1g_{9/2}$	0.631	0.467	$1h_{9/2}-1h_{11/2}$	0.789	0.736
			$2d_{3/2}-2d_{5/2}$	0.619	0.570			
			$2d_{5/2}-1g_{7/2}$	−0.363	−0.384			

TABLE IX.—*Continued.*

Element	Z	A	Proton states	g_1'	g_2'	Neutron states	g_1'	g_2'
Cs	55	129	$3s_{1/2}-3s_{1/2}$	0.624	0.637	$2d_{3/2}-2d_{5/2}$	0.694	0.716
			$1g_{7/2}-1g_{9/2}$	0.628	0.462	$1h_{9/2}-1h_{11/2}$	0.785	0.730
			$2d_{3/2}-2d_{5/2}$	0.614	0.560			
			$2d_{5/2}-1g_{7/2}$	−0.359	−0.378			
		131	$2d_{5/2}-2d_{5/2}$	0.616	0.560	$2d_{3/2}-2d_{5/2}$	0.687	0.701
			$1g_{7/2}-1g_{9/2}$	0.625	0.457	$1h_{9/2}-1h_{11/2}$	0.782	0.725
			$2d_{3/2}-2d_{5/2}$	0.609	0.551			
			$2d_{5/2}-1g_{7/2}$	−0.356	−0.372			
		133	$1g_{7/2}-1g_{7/2}$	0.594	0.412	$2d_{3/2}-2d_{5/2}$	0.680	0.687
			$1g_{7/2}-1g_{9/2}$	0.622	0.453	$1h_{9/2}-1h_{11/2}$	0.777	0.714
			$2d_{3/2}-2d_{5/2}$	0.604	0.542			
			$2d_{5/2}-1g_{7/2}$	−0.353	−0.367			
		135	$1g_{7/2}-1g_{7/2}$	0.591	0.408	$2d_{3/2}-2d_{5/2}$	0.673	0.674
			$1g_{7/2}-1g_{9/2}$	0.619	0.449	$1h_{9/2}-1h_{11/2}$	0.772	0.702
			$2d_{3/2}-2d_{5/2}$	0.599	0.533			
			$2d_{5/2}-1g_{7/2}$	−0.350	−0.362			
		137	$1g_{7/2}-1g_{7/2}$	0.588	0.404	$1h_{9/2}-1h_{11/2}$	0.766	0.691
			$1g_{7/2}-1g_{9/2}$	0.617	0.445			
			$2d_{3/2}-2d_{5/2}$	0.594	0.525			
			$2d_{5/2}-1g_{7/2}$	−0.347	−0.357			
Ba	56	129	$1g_{7/2}-1g_{9/2}$	0.628	0.462	$3s_{1/2}-3s_{1/2}$	0.724	0.847
			$2d_{3/2}-2d_{5/2}$	0.614	0.562	$2d_{3/2}-2d_{5/2}$	0.694	0.716
			$2d_{5/2}-1g_{7/2}$	−0.360	−0.378	$1h_{9/2}-1h_{11/2}$	0.785	0.730
		131	$1g_{7/2}-1g_{9/2}$	0.625	0.458	$3s_{1/2}-3s_{1/2}$	0.715	0.827
			$2d_{3/2}-2d_{5/2}$	0.609	0.552	$2d_{3/2}-2d_{5/2}$	0.687	0.701
			$2d_{5/2}-1g_{7/2}$	−0.356	−0.373	$1h_{9/2}-1h_{11/2}$	0.782	0.725
		133	$1g_{7/2}-1g_{9/2}$	0.622	0.454	$3s_{1/2}-3s_{1/2}$	0.706	0.807
			$2d_{3/2}-2d_{5/2}$	0.604	0.543	$2d_{3/2}-2d_{5/2}$	0.680	0.687
			$2d_{5/2}-1g_{7/2}$	−0.353	−0.367	$1h_{9/2}-1h_{11/2}$	0.777	0.714
		135	$1g_{7/2}-1g_{9/2}$	0.620	0.449	$2d_{3/2}-2d_{3/2}$	0.680	0.684
			$2d_{3/2}-2d_{5/2}$	0.599	0.534	$2d_{3/2}-2d_{5/2}$	0.673	0.674
			$2d_{5/2}-1g_{7/2}$	−0.351	−0.362	$1h_{9/2}-1h_{11/2}$	0.772	0.702
						$3s_{1/2}-2d_{3/2}$	−0.643	−0.715
		137	$1g_{7/2}-1g_{9/2}$	0.617	0.445	$2d_{3/2}-2d_{3/2}$	0.673	0.669
			$2d_{3/2}-2d_{5/2}$	0.595	0.526	$1h_{9/2}-1h_{11/2}$	0.766	0.691
			$2d_{5/2}-1g_{7/2}$	−0.347	−0.357			
		139	$1g_{7/2}-1g_{9/2}$	0.615	0.441	$2f_{7/2}-2f_{7/2}$	0.848	1.029
			$2d_{3/2}-2d_{5/2}$	0.590	0.518	$1h_{9/2}-1h_{11/2}$	0.761	0.680
			$2d_{5/2}-1g_{7/2}$	−0.345	−0.353			
Au	79	191	$2d_{3/2}-2d_{3/2}$	0.518	0.395	$3p_{1/2}-3p_{3/2}$	0.731	0.844
			$2d_{3/2}-2d_{5/2}$	0.522	0.406	$1i_{11/2}-1i_{13/2}$	0.756	0.655
			$1h_{9/2}-1h_{11/2}$	0.638	0.467			
			$3s_{1/2}-2d_{3/2}$	−0.470	−0.400			
		193	$2d_{3/2}-2d_{3/2}$	0.515	0.392	$3p_{1/2}-3p_{3/2}$	0.724	0.828
			$2d_{3/2}-2d_{5/2}$	0.520	0.403	$1i_{11/2}-1i_{13/2}$	0.755	0.653
			$1h_{9/2}-1h_{11/2}$	0.636	0.464			
			$3s_{1/2}-2d_{3/2}$	−0.467	−0.396			
		195	$2d_{3/2}-2d_{3/2}$	0.513	0.388	$3p_{1/2}-3p_{3/2}$	0.718	0.814
			$2d_{3/2}-2d_{5/2}$	0.518	0.400	$1i_{11/2}-1i_{13/2}$	0.752	0.648
			$1h_{9/2}-1h_{11/2}$	0.635	0.461			
			$3s_{1/2}-2d_{3/2}$	−0.465	−0.393			
		197	$2d_{3/2}-2d_{3/2}$	0.511	0.385	$3p_{1/2}-3p_{3/2}$	0.711	0.800
			$2d_{3/2}-2d_{5/2}$	0.516	0.397	$1i_{11/2}-1i_{13/2}$	0.749	0.642
			$1h_{9/2}-1h_{11/2}$	0.633	0.459			
			$3s_{1/2}-2d_{3/2}$	−0.463	−0.390			
		199	$2d_{3/2}-2d_{3/2}$	0.509	0.382	$3p_{1/2}-3p_{3/2}$	0.705	0.786
			$2d_{3/2}-2d_{5/2}$	0.514	0.394	$1i_{11/2}-1i_{13/2}$	0.745	0.636
			$1h_{9/2}-1h_{11/2}$	0.632	0.456			
			$3s_{1/2}-2d_{3/2}$	−0.461	−0.387			

TABLE IX.—*Continued.*

Element	Z	A	Proton states	$\mathscr{g}_1'$	$\mathscr{g}_2'$	Neutron states	$\mathscr{g}_1'$	$\mathscr{g}_2'$
Au	79	201	$2d_{3/2}-2d_{3/2}$	0.507	0.379	$3p_{1/2}-3p_{3/2}$	0.699	0.773
			$2d_{3/2}-2d_{5/2}$	0.512	0.391	$1i_{11/2}-1i_{13/2}$	0.742	0.630
			$1h_{9/2}-1h_{11/2}$	0.630	0.454			
			$3s_{1/2}-2d_{3/2}$	−0.459	−0.383			
Hg	80	193	$1h_{9/2}-1h_{11/2}$	0.637	0.464	$3p_{1/2}-3p_{1/2}$	0.735	0.850
						$3p_{3/2}-3p_{3/2}$	0.715	0.809
						$3p_{1/2}-3p_{3/2}$	0.724	0.828
						$1i_{11/2}-1i_{13/2}$	0.755	0.653
						$3p_{3/2}-2f_{5/2}$	−0.604	−0.691
		195	$1h_{9/2}-1h_{11/2}$	0.635	0.462	$3p_{1/2}-3p_{1/2}$	0.728	0.834
						$3p_{3/2}-3p_{3/2}$	0.709	0.796
						$3p_{1/2}-3p_{3/2}$	0.718	0.814
						$1i_{11/2}-1i_{13/2}$	0.752	0.648
						$3p_{3/2}-2f_{5/2}$	−0.598	−0.680
		197	$1h_{9/2}-1h_{11/2}$	0.634	0.459	$3p_{1/2}-3p_{1/2}$	0.721	0.818
						$1i_{11/2}-1i_{13/2}$	0.749	0.642
						$3p_{3/2}-2f_{5/2}$	−0.593	−0.669
		199	$1h_{9/2}-1h_{11/2}$	0.632	0.457	$3p_{1/2}-3p_{1/2}$	0.714	0.804
						$1i_{11/2}-1i_{13/2}$	0.745	0.636
						$3p_{3/2}-2f_{5/2}$	−0.588	−0.658
		201	$1h_{9/2}-1h_{11/2}$	0.630	0.455	$3p_{3/2}-3p_{3/2}$	0.693	0.760
						$3p_{1/2}-3p_{3/2}$	0.699	0.773
						$1i_{11/2}-1i_{13/2}$	0.742	0.630
						$3p_{3/2}-2f_{5/2}$	−0.583	−0.649
		203	$1h_{9/2}-1h_{11/2}$	0.629	0.452	$3p_{1/2}-3p_{1/2}$	0.702	0.775
						$3p_{3/2}-3p_{3/2}$	0.687	0.748
						$3p_{1/2}-3p_{3/2}$	0.693	0.760
						$1i_{11/2}-1i_{13/2}$	0.739	0.625
						$3p_{3/2}-2f_{5/2}$	−0.578	−0.639
Tl	81	197	$3s_{1/2}-3s_{1/2}$	0.505	0.427	$3p_{1/2}-3p_{3/2}$	0.711	0.800
			$1h_{9/2}-1h_{11/2}$	0.634	0.460	$1i_{11/2}-1i_{13/2}$	0.749	0.642
		199	$3s_{1/2}-3s_{1/2}$	0.503	0.424	$3p_{1/2}-3p_{3/2}$	0.705	0.786
			$1h_{9/2}-1h_{11/2}$	0.632	0.458	$1i_{11/2}-1i_{13/2}$	0.745	0.636
		201	$3s_{1/2}-3s_{1/2}$	0.501	0.420	$3p_{1/2}-3p_{3/2}$	0.699	0.773
			$1h_{9/2}-1h_{11/2}$	0.631	0.455	$1i_{11/2}-1i_{13/2}$	0.742	0.630
		203	$3s_{1/2}-3s_{1/2}$	0.498	0.417	$3p_{1/2}-3p_{3/2}$	0.693	0.760
			$1h_{9/2}-1h_{11/2}$	0.629	0.453	$1i_{11/2}-1i_{13/2}$	0.739	0.625
		205	$3s_{1/2}-3s_{1/2}$	0.496	0.413	$3p_{1/2}-3p_{3/2}$	0.688	0.749
			$1h_{9/2}-1h_{11/2}$	0.628	0.450	$1i_{11/2}-1i_{13/2}$	0.736	0.620

follows that we can formally write [using Eq. (A.12a)]

$$-\epsilon=\frac{1}{\mu}\Big\{\alpha_{S\,\mathrm{s.p.}}g_S\Big[(b_S)_2\Big[1+\Big(\frac{2}{5}\Big)\zeta\Big]\mathscr{g}_1(\mathrm{s.p.})+(b_S)_4\Big[1+\Big(\frac{4}{7}\Big)\zeta\Big]\mathscr{g}_2(\mathrm{s.p.})\Big]+\alpha_{L\,\mathrm{s.p.}}g_L[(b_L)_2\mathscr{g}_1(\mathrm{s.p.})+(b_L)_4\mathscr{g}_2(\mathrm{s.p.})]$$

$$+\sum_i\alpha_0^{(i)}\Big[\Big((b_S)_2\Big(\frac{9}{10}\Big)\mathscr{g}_1(i)+(b_S)_4\Big(\frac{6}{7}\Big)\mathscr{g}_2(i)\Big)g_S^{(i)}-((b_L)_2\mathscr{g}_1(i)+(b_L)_4\mathscr{g}_2(i))g_L^{(i)}\Big]$$

$$+\sum_i\alpha_2^{(i)}\Big[\Big(\frac{2}{5}\Big)(b_S)_2\mathscr{g}_1(i)+\Big(\frac{4}{7}\Big)(b_S)_4\mathscr{g}_2(i)\Big]g_S^{(i)}\Big\},\qquad(33)$$

where by comparison with Eqs. (25) and (26)

$$\alpha_{S\,\text{s.p.}}=\tfrac{1}{2},\quad \alpha_{L\,\text{s.p.}}=(j-\tfrac{1}{2}),\quad \zeta=\frac{2j-1}{4(j+1)}\quad \text{for } j=l+\tfrac{1}{2}, \tag{34a}$$

$$\alpha_{S\,\text{s.p.}}=\frac{-j}{2j+2},\quad \alpha_{L\,\text{s.p.}}=\frac{j(2j+3)}{2j+2},\quad \zeta=\frac{2j+3}{4j}\quad \text{for } j=l-\tfrac{1}{2}, \tag{34b}$$

and

$$\mathfrak{g}_n(\text{s.p.})=\frac{1}{R_N^{2n}}\int R_{\text{s.p.}}(R)R^{2n+2}R_{\text{s.p.}}(R)dR. \tag{35}$$

Here the suffix s.p. stands for "single particle" since the contribution to $-\epsilon$ from these terms alone is just that which would be obtained for a single-particle shell-model description.

The $\alpha_0{}^i$ and $\alpha_2{}^i$ refer to $\Delta l=0$ and $\Delta l=2$ excitations respectively, the label i designating a particular admixture. Their values could be written down explicitly by referring to Tables III through VII but this will not be done here. Finally the $\mathfrak{g}_n(i)$ are the relevant radial matrix elements [Eq. (20)] for the ith admixture and $g_S{}^{(i)}$ and $g_L{}^{(i)}$ are the g factors for the excited particle in this admixture.

It is to be noticed that in terms of the parameters α, the theoretical value for the magnetic moment resulting from admixtures of the above type is

$$\mu_{\text{th}}=\alpha_{S\,\text{s.p.}}g_S+\alpha_{L\,\text{s.p.}}g_L+\textstyle\sum_i \alpha_0{}^{(i)}(g_S{}^{(i)}-g_L{}^{(i)}). \tag{36}$$

Now, as can be calculated, the $\Delta l=2$ contributions are generally small. Thus if, for example, there is only one likely $\Delta l=0$ admixture, $i=k$ (say), then $\alpha_0{}^{(k)}$ could be determined empirically by requiring that μ_{th} of (36) agrees with the experimental value of μ. The $\alpha_0{}^{(k)}$ so determined could then be used in (33) to obtain an empirical value for $-\epsilon$. Alternately, if there are *two* likely admixtures, we can use the magnetic moment and the "hfs anomaly" data for the determination of their contributions. Both of these methods and the direct computation of ϵ will be used in the following investigation of the experimental cases.

Experimental Data

The comparison of the theoretical value of ϵ with that obtained experimentally is usually not made directly through the relation of Eq. (3). This is because W_{point} would have to be calculated to a precision of better than 0.1% in order to compare it meaningfully with the experimental result, W_{extended}. In practice this is not achieved except in light nuclei, which we do not consider here, and we compare therefore the ratio of the measured values of single electron magnetic interaction constants for two isotopes with the independently-measured ratio of the nuclear g values. The latter would correspond to the ratio of the point interactions (since these measurements are performed in a uniform magnetic field, and are therefore insensitive to any departure from a point magnetic moment), in most cases to a degree of accuracy much better than is required for the above comparison. In view of this, only the part of the Rosenthal-Breit-Crawford-Schawlow correction[27] which affects the Bohr-Weisskopf effect through the variations of the charge distribution between isotopes is included. This is obtained formally by using in the calculation of ϵ electron coefficients b which are functions not only of Z and a value of A which corresponds, for example, to the most stable isotope, but actually $b(Z,A)$. In the case where the magnetic moments are very nearly equal and the spins identical for the two isotopes, the Breit-Rosenthal point-magnetic moment correction may however still predominate.[28] Consequently, for one-electron spectra, using the relationship between W and $h\Delta\nu$ (the hfs separation energy between the two states $F_+=j+\frac{1}{2}$ and $F_-=j-\frac{1}{2}$, with the electron angular momentum $J=\frac{1}{2}$), we find[10] for two isotopes 1 and 2, using Eq. (3),

$$\frac{\Delta\nu_1}{\Delta\nu_2}=\frac{g_1(2j_1+1)(1+\epsilon_1)}{g_2(2j_2+1)(1+\epsilon_2)}, \tag{37}$$

or as $\Delta\nu=aF_+$, where a is the magnetic dipole interaction constant in the Hamiltonian, and neglecting terms other than linear in ϵ,

$$\frac{a_1g_2}{a_2g_1}-1\approx\epsilon_1-\epsilon_2\equiv\Delta_{12}. \tag{38}$$

The comparison with experiment is therefore via Eq. (38). It is clear that if we deal with a spectrum of more than one electron, the contribution of the single $s_{1/2}$ or $p_{1/2}$ electron first has to be separated out properly from the measured magnetic interaction constant. Schwartz[29] has pointed out that in the case of p electrons a number of important corrections have to be applied before a value of Δ can be obtained. These involve screening effects as well as configuration interaction influences. In particular he shows that such configuration interactions can lead to hfs anomalies for a $p_{3/2}$, and in fact any electron. Thus for other than s electrons, in view of these possible ambiguities, the comparison of the experimental data with our calculations may be subject to significant modifications. The experimental results are given in Table X.

Discussion of the Experimental Cases

In discussing the various isotopes we indicate only the groups of nucleons which contribute in zero-order

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TABLE X. Experimental data of magnetic moments (μ), g-value and hfs interaction constant (a) ratios, and $\Delta_{\exp}=(a_1g_2/a_2g_1)-1$; this is the quantity which is compared to the theoretical calculation, $\Delta_{\text{th}}=\epsilon_1-\epsilon_2$. The atomic state in which the hfs was measured is also given. In the cases of spectra of more than one electron the a-value ratios indicated may not be equal to those of single s or p electrons and reference should be made to the literature for a proper interpretation. Consideration should also be given to electronic perturbation effects (see text) in the case of p states. For a review of the experimental techniques, as well as that of the Bohr-Weisskopf effect and our early work see J. Eisinger and V. Jaccarino, Revs. Modern Phys. **30**, 528 (1958).

Isotope	I	μ (nm)	g_1/g_2	Atomic state in which hfs measured	a_1/a_2	Δ (percent)	References μ	References hfs
$_{17}Cl^{35}$	3/2	0.8211	1.20132±5	$p_{1/2}$	1.20136±1	0.0033±43	a, b, c	d
$_{17}Cl^{37}$	3/2	0.6835		$p_{3/2}$	1.2013078±3	−0.0010±42		e
$_{19}K^{39}$	3/2	0.3909	1.82189±17	$s_{1/2}$	1.81767±4	−0.232±10	f, g	h, i
$_{19}K^{41}$	3/2	0.2145						
$_{29}Cu^{63}$	3/2	2.2206	0.933424±19	$s_{1/2}$	0.933567±2	0.015±2	j	k
$_{29}Cu^{65}$	3/2	2.3790						
$_{31}Ga^{69}$	3/2	2.0108	0.7870148±13	$p_{1/2}$	0.7870196±6	0.00062±23	j, l	m, nn
$_{31}Ga^{71}$	3/2	2.5549		$p_{3/2}$	0.7869949±9	−0.00252±32		n
$_{35}Br^{79}$	3/2	2.0990	0.927691±16	$p_{3/2}$	0.927697±20	0.00065±280	j	o
$_{35}Br^{81}$	3/2	2.2626						
$_{37}Rb^{85}$	5/2	1.3482	0.2950740±12	$s_{1/2}$	0.2961101±2	0.3511±4	p, q	r
$_{37}Rb^{87}$	3/2	2.7414		$p_{1/2}$	0.295±4	−0.02±136		s
				$p_{3/2}$	0.295±3	−0.02±102		s
$_{47}Ag^{107}$	1/2	−0.1130	0.86985±1	$s_{1/2}$	0.866268±27	−0.412±6	f, t	u
$_{47}Ag^{109}$	1/2	−0.1300						
$_{48}Cd^{111}$	1/2	−0.5923	0.955947±3	$^3P_1$	0.955945±6	−0.0002±7	v, w	x
$_{48}Cd^{113}$	1/2	−0.6196		$^3P_2$	0.9559612±6	0.0016±3		y
$_{49}In^{113}$	9/2	5.4960	0.9978609±12	$p_{1/2}$	0.99786844±25	0.00075±13	a, c, z	aa
$_{49}In^{115}$	9/2	5.5077		$p_{3/2}$	0.99783716±26	−0.00238±13		bb
$_{51}Sb^{121}$	5/2	3.3600	1.84661±1	$^4S_{3/2}$	1.840763±55	−0.317±3	cc	dd
$_{51}Sb^{123}$	7/2	2.5484		Paramagnetic resonance	1.84012±9	−0.352±5		cc
$_{55}Cs^{133}$	7/2	2.5789	0.945001±8	$s_{1/2}$	0.9453527±15	0.037±9	j, ee	ee, ff
$_{55}Cs^{135}$	7/2	2.7290						
$_{55}Cs^{135}$	7/2	2.7290	0.961492±8	$s_{1/2}$	0.9612967±21	−0.020±9	ee	ee
$_{55}Cs^{137}$	7/2	2.8382						
$_{80}Hg^{199}$	1/2	0.5041	−2.70902±3	$^3P_1$	−2.705039±48	−0.1746±89	j, gg	hh, ii, jj
$_{80}Hg^{201}$	3/2	−0.5582		$^3P_2$	−2.704764±1	−0.1636±27		kk
				Knight shift	−2.708925±73	−0.16±10		ll
$_{81}Tl^{203}$	1/2	1.5962	0.9902578±10	$p_{1/2}$	0.9903622±5	0.01050±15	w, mm	nn
$_{81}Tl^{205}$	1/2	1.6118		$p_{3/2}$	0.9886498±5	−0.00162±62		oo

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and through excitation (to or from them) to the magnetic moment and to the hyperfine structure anomaly. The uncertainties indicated in Δ include only those which arise from some 5 or 6% variations in ϵ which may result from neglected terms in the series expansion of the Dirac equation as was discussed in Sec. III.

Atoms in $s_{1/2}$ States

Potassium. K^{39} has $(1d_{3/2})^3$ protons with no possible admixtures. Therefore we would expect this isotope to have the extreme single-particle moment of 0.124 nm. Actually this is not the case, and the discrepancy may be attributed to a quenched g factor[30] for the $d_{3/2}$ proton; we find, by demanding agreement with the experimental value of μ in K^{39}, g_S(effective)$=4.7$. In K^{41} we have, in addition to the $(d_{3/2})^3$ protons, the contribution from excitations of the $(1f_{7/2})^2$ neutrons. From the magnetic moment of K^{41}, and using the proton g_S value found in K^{39}, we determine the mixing coefficient of these neutrons. The value of Δ which we obtain is $-0.25\pm\approx0.03\%$. This is in excellent agreement with experiment. If, on the other hand, one does not consider configuration mixing in K^{41} but tries to fit the moment entirely with a different g_S(effective), the result is -0.36%. Similarly if in K^{41} we use g_S(free) and determine the $1f$ neutron admixture empirically, we find $\Delta=-0.17\%$.

Copper. We have for the protons $2p_{3/2}(1f_{7/2})^8$. In Cu^{63} the neutron contributions are $(1f_{5/2})^2(2p_{3/2})^4$, and in Cu^{65} $(1f_{5/2})^4(2p_{3/2})^4$. The calculated magnetic moments are 2.17 nm and 2.30 nm for Cu^{63} and Cu^{65}, and Δ is approximately zero with an estimated error of about 0.015%.

Rubidium. The pair of isotopes Rb^{85} and Rb^{87} is particularly interesting as the addition of two neutrons changes the nuclear spin and hence causes a substantial difference in the distribution of magnetization. (In fact this was the first experimental observation of the "hfs anomaly.") For Rb^{85} we have $(1f_{5/2})^5(2p_{3/2})^4$ protons and $(1g_{9/2})^8$ neutrons. The contributions in Rb^{87} are protons: $(2p_{3/2})^3$, neutrons: $(1g_{9/2})^{10}$. The calculated magnetic moments are 1.32 nm and 2.79 nm for Rb^{85} and Rb^{87}, and $\Delta=0.332\pm\approx0.016\%$, which is in good agreement with the experimental value. We also calculate $\Delta=0.019\pm0.002\%$ for the $p_{1/2}$ hfs.

Silver. For Ag^{107} we have $2p_{1/2}(1g_{9/2})^8$ protons and $(2d_{5/2})^2$ neutrons. In Ag^{109} we have the same protons and $(2d_{5/2})^4$ neutrons. The δ-function interaction does not permit admixtures if the odd nucleon is in a $p_{1/2}$ state. We therefore take the semiphenomenological approach. By admixing either the g proton or d neutron excitation, we obtain $\Delta\approx-0.42\pm\approx0.30\%$. The large uncertainty reflects the fact that for these silver isotopes the values of ϵ are large. As a consequence it is not possible to determine the two admixtures individually. If we attribute the entire deviation from the single-particle magnetic moment to the g-proton excitations, we find [through the use of Eq. (4), reference 3] that this requires a mixing coefficient of 0.014 in the wave function.

Cesium. At $Z=55$ there is competition between the $1g_{7/2}$ and $2d_{5/2}$ proton levels. We might have therefore $(1g_{7/2})^5$, $(1g_{7/2})^3(2d_{5/2})^2$, or $1g_{7/2}(2d_{5/2})^4$. In the 50–82 neutron region the $1g_{7/2}$ and $2d_{5/2}$ levels lie lowest, with the $3s_{1/2}$, $1h_{11/2}$, and $2d_{3/2}$ states on the top. The program used for calculating the radial integrals $\mathcal{I}'$, gave binding energies of about 10.3, 9.6, and 9.4 Mev for the $3s_{1/2}$, $1h_{11/2}$, and $2d_{3/2}$ neutrons, respectively. This order of filling the neutron levels also leads to the best agreement in the magnetic moments. We should remark, however, that the same magnetic moment corrections are obtained in the three Cs isotopes which we consider if the $3s_{1/2}$ states get filled after the $1h_{11/2}$ neutrons. Thus for Cs^{133} the neutron contributions are $(1h_{11/2})^{12}$, $(2d_{5/2})^6$; for the protons a mixture of $1g_{7/2}(2d_{5/2})^4$ and $(1g_{7/2})^3(2d_{5/2})^2$ leads to agreement with the experimental magnetic moment. In Cs^{135} the neutron contributions are $(1h_{11/2})^{12}(2d_{3/2})^2$, while for the protons we have a similar mixture as in Cs^{133}. Finally in Cs^{137} the best agreement in the magnetic moment ($\mu=2.67$ nm) is obtained with $1g_{7/2}(2d_{5/2})^4$ protons, and of course we have only the $(1h_{11/2})^{12}$ neutron contribution. The anomalies which we calculate are $\Delta_{133-135}=+0.068\%$ and $\Delta_{135-137}=-0.026\%$, both $\pm\approx0.025$. Thus it is indeed possible to obtain a reversal in the sign of Δ in going from the $Cs^{133}-Cs^{135}$ pair to $Cs^{135}-Cs^{137}$, and this we were not able to do with purely effective moment calculations.

Atoms in p States

Chlorine. For the protons in both Cl^{35} and Cl^{37} we have $1d_{3/2}(2s_{1/2})^2$. We have neutron contributions only in Cl^{35}, i.e., $(1d_{3/2})^2$. Here we adopt the modified values of the interaction strengths as used by Arima and Horie[3] so that $I(1d,1d):I(2d,2d)=31:20$, with $V_sI(2d,2d)$ having the standard value $-25/A$ Mev. The resulting magnetic moments are 0.710 and 0.582 for Cl^{35} and Cl^{37}. The hyperfine structure anomaly calculated for the $p_{1/2}$ electron is zero, in agreement with experiment.

Gallium. There are two possibilities for the contributing protons—$(2p_{3/2})^3(1f_{7/2})^8$ or $2p_{3/2}(1f_{5/2})^2(1f_{7/2})^8$. There is a neutron contribution only in the Ga^{69} isotope, i.e., $(2p_{3/2})^4$. Arima and Horie[3] suggest that the first choice is more likely on the basis of the positive quadrupole moments. The magnetic moments for Ga^{69} and Ga^{71} are 1.58 nm and 1.82 nm for the first choice in the proton configuration, and 2.85 nm and 3.05 nm for the second one, with the experimental values

[30] I. Talmi and A. de-Shalit (private communication). Good agreement between theory and experiment is obtained with such an effective g factor for magnetic moments of the potassium isotopes. See also S. D. Drell and J. D. Walecka, Phys. Rev. **120**, 1069 (1960).

lying in between. Since we are dealing with a p-electron hfs in relatively light isotopes, the anomaly is expected to be very small in either case. Indeed we find $\Delta=0\pm\approx 0.0005\%$ with the first proton choice, and $-0.001\pm\approx 0.001\%$ for the second. The Breit-Rosenthal correction is also relatively important here.

Bromine. We have two alternatives for the protons: $(2p_{3/2})^3(1f_{5/2})^4$ and $(2p_{3/2})^3(1f_{5/2})^2(1g_{9/2})^2$. The neutrons are $(1g_{9/2})^4$ and $(1g_{9/2})^6$ for Br^{79} and Br^{81}. The first proton configuration leads to $\mu^{79}=2.56$ nm and $\mu^{81}=2.53$ nm while the second one gives $\mu^{79}=1.92$ nm and $\mu^{81}=1.90$ nm. As both give moments which are nearly identical for the two isotopes and the hfs is of a p electron state, we again expect a very small anomaly. We calculate $\Delta=-0.001\pm\approx 0.001\%$ for the first proton configuration.

Indium. The contributing protons are $(1g_{9/2})^9$. In In^{113} we have $(2d_{5/2})^6$ neutrons, and for In^{115} in addition $(1h_{11/2})^2$ neutrons. We find $\mu^{113}=5.62$ nm and $\mu^{115}=5.59$ nm, numerically close to the experimental values but with wrong relative sizes. Similar electronic and other correction considerations as in Ga apply. We find $\Delta=0\pm\approx 0.004\%$.

Thallium. The proton contributions are $3s_{1/2}(1h_{11/2})^{12}$. For the neutrons, the program fills the 126 shell in the order $1i_{13/2}$, $3p_{3/2}$, $3p_{1/2}$. Thus for Tl^{203} the neutron contributions are $(1i_{13/2})^{14}(3p_{3/2})^2$ and in $Tl^{205}(1i_{13/2})^{14}(3p_{3/2})^4$. This yields $\mu^{203}=1.36$ nm, $\mu^{205}=1.21$ nm, and $\Delta=-0.041\pm\approx 0.017\%$, in poor agreement with experiment. We note also that experimentally μ^{205} is larger than μ^{203}. Somewhat better agreement can be obtained if we assume the $1i_{13/2}$ states to be filled last, but this is more unlikely from the point of view of pairing energy. For this case we have $(1i_{13/2})^{10}$ and $(1i_{13/2})^{12}$ neutrons in Tl^{203} and Tl^{205}, with resulting magnetic moments of 1.58 nm and 1.56 nm, and $\Delta=-0.011\pm\approx 0.023\%$. As we pointed out earlier, the Breit-Rosenthal correction and electronic perturbations are significant here.

Other Cases

Cadmium. The protons contribute $(1g_{9/2})^8$. For the neutrons Cd^{111} has $3s_{1/2}(1g_{7/2})^6(2d_{5/2})^6$, and Cd^{113} $3s_{1/2}(1g_{7/2})^8(2d_{5/2})^6$. The calculated magnetic moments are -0.49 nm and -0.77 nm for Cd^{111} and Cd^{113}, and $\Delta=0.018\pm\approx 0.006\%$. In view of the small observed anomaly, the electronic and Breit-Rosenthal corrections are important and we do not draw any definite conclusions.

Antimony. The pair of isotopes Sb^{121} and Sb^{123}, similarly to the rubidium isotopes, change spin with the addition of two neutrons. Thus for Sb^{121} we have $2d_{5/2}(1g_{9/2})^{10}$ protons and $(1h_{11/2})^6(2d_{5/2})^6$ neutrons, while in Sb^{123} we have $1g_{7/2}$ proton and $(1h_{11/2})^8(2d_{5/2})^6$ neutrons. The resulting magnetic moments are 3.49 nm and 2.49 nm and the anomaly $-0.421\pm 0.033\%$. If we fill the neutron levels on the basis of the spins of odd neutron nuclei in this region rather than on that of pairing energies, we obtain $\mu^{121}=3.55$ nm, $\mu^{123}=2.46$ nm, and $\Delta=-0.439\%$, in somewhat worse agreement with experiment.

Mercury. Hg^{199} has an odd $3p_{1/2}$ neutron and therefore again we do not have any corrections with the δ-function interaction. Thus we adopt the semiempirical approach for Hg. We assume that the $2d_{3/2}$ protons close the 82 shell and that the $(1h_{11/2})^{12}$ and $(2d_{3/2})^2$ protons contribute the major part of the deviation from the single-particle value of μ. In Hg^{201} the odd neutron is in the $3p_{3/2}$ orbit. With this choice the magnetic moments and the hfs anomaly can be fitted with reasonable admixture coefficients, i.e., in Hg^{199} $\alpha(h)=-0.135$ and $\alpha(d)=0.248$, and in Hg^{201} $\alpha(h)=0.172$ and $\alpha(d)=0.144$. Here we made use again of Eq. (4), reference 3. If we try to admix the $1i_{13/2}$ or $3p$ neutrons instead of one of the proton groups, or substitute both neutron excitations for the two proton excitations, the required admixture coefficients become unreasonably large.

Conclusion

The configuration mixing theory accounts satisfactorily for a great number of magnetic moments of odd-A nuclei. We have extended this theory to permit the calculation of the effects of the distribution of nuclear magnetization as manifested by hyperfine structure anomalies. From a comparison of the theory with experiments performed up to date, reasonable agreement is obtained. In view of this success, more experiments of such a nature would appear fruitful.

We also note that the δ-function interaction does not allow any admixtures if the odd nucleon is in a $p_{1/2}$ state. In this case, as well as for nuclei where there may be only two important admixtures, the semi-phenomenological approach has been found useful: it permits the determination of these two configuration admixtures by making use of the hfs anomaly data in conjunction with the values of the magnetic moments, while only one such admixture could be determined from a knowledge of the magnetic moment alone.

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APPENDIX

Evaluation of Electron Integrals in Eq. (4)

Letting $\chi_1=rF$, $\chi_2=rG$, where χ_1 and χ_2 are the small and large components, respectively, of a Dirac wave function, and neglecting the binding energy of the electron compared to its rest mass, i.e., taking $E\approx mc^2$, the Dirac equation for the electron in the potential of Eq. (7) becomes

$$\begin{aligned}\frac{d\chi_1}{dx}\pm\frac{\chi_1}{x}&=-\gamma(K-a_2x^2-a_4x^4-a_5x^5-a_6x^6),\\ \frac{d\chi_2}{dx}\mp\frac{\chi_2}{x}&=\gamma(2\epsilon_A+K-a_2x^2-a_4x^4-a_5x^5-a_6x^6).\end{aligned}\tag{A.1}$$

The upper and lower signs above and in several expressions below, are to be taken for $s_{1/2}$ and $p_{1/2}$ electrons, respectively. Here $\gamma=Z\alpha$, where $\alpha=e^2/\hbar c$ is the fine structure constant, $\epsilon_A\equiv mcR_N/\gamma\hbar$, m is the electron mass, c the velocity of light, and $\hbar=h/2\pi$, where h is Planck's constant.

We obtain series solutions of Eq. (A.1) which are well behaved at $x=0$ in the form[31]

$$\begin{aligned}\chi_1&=\sum_{n=1}^{\infty} l_nx^{n+1},\\ \chi_2&=\sum_{n=0}^{\infty} q_nx^{n+1},\end{aligned}\tag{A.2}$$

for the $s_{1/2}$ electron. It is found that $l_0=0$; q_0 is the normalization factor. Similarly for the $p_{1/2}$ electron

$$\begin{aligned}\chi_1&=\sum_{n=0}^{\infty} u_nx^{n+1},\\ \chi_2&=\sum_{n=1}^{\infty} v_nx^{n+1},\end{aligned}\tag{A.3}$$

where now $v_0=0$, and u_0 is determined by the normalization. By inserting (A.2) in (A.1) we obtain the recursion formulas for the coefficients in the series for the $s_{1/2}$ electron:

$$\begin{aligned}l_n(n+2)&\\ &=\gamma(-Kq_{n-1}+a_2q_{n-3}+a_4q_{n-5}+a_5q_{n-6}+a_6q_{n-7}),\\ q_nn&=\gamma[(2\epsilon_A+K)l_{n-1}-a_2l_{n-3}-a_4l_{n-5}-a_5l_{n-6}-a_6l_{n-7}].\end{aligned}\tag{A.4}$$

The $p_{1/2}$ electron recursion formulas are similarly obtained by inserting (A.3) in (A.1). The result is

$$\begin{aligned}u_nn&=\gamma(-Kv_{n-1}+a_2v_{n-3}+a_4v_{n-5}+a_5v_{n-6}+a_6v_{n-7}),\\ v_n(n+2)&\\ &=\gamma[(2\epsilon_A+K)u_{n-1}-a_2u_{n-3}-a_4u_{n-5}\\ &\qquad-a_5u_{n-6}-a_6u_{n-7}].\end{aligned}\tag{A.5}$$

Although explicit expressions for the above coefficients can be obtained easily, in practice it is simpler to use the recursion formulas numerically. The functions are, for the $s_{1/2}$ state

$$\begin{aligned}F&=\frac{r}{R_N^2}\left[l_1+l_3\left(\frac{r^2}{R_N^2}\right)+l_5\left(\frac{r^4}{R_N^4}\right)+\cdots\right],\\ G&=\frac{1}{R_N}\left[q_0+q_2\left(\frac{r^2}{R_N^2}\right)+q_4\left(\frac{r^4}{R_N^4}\right)+\cdots\right],\end{aligned}\tag{A.6}$$

and for the $p_{1/2}$ state

$$\begin{aligned}F&=\frac{1}{R_N}\left[u_0+u_2\left(\frac{r^2}{R_N^2}\right)+u_4\left(\frac{r^4}{R_N^4}\right)+\cdots\right],\\ G&=\frac{r}{R_N^2}\left[v_1+v_3\left(\frac{r^2}{R_N^2}\right)+v_5\left(\frac{r^4}{R_N^4}\right)+\cdots\right].\end{aligned}\tag{A.7}$$

The integrals in the numerator of Eq. (4) are now evaluated. For the $s_{1/2}$ state we find

$$\int_0^R FGdr=\frac{1}{R_N}\left[\tfrac{1}{2}l_1q_0\left(\frac{R^2}{R_N^2}\right)+\tfrac{1}{4}(l_3q_0+l_1q_2)\left(\frac{R^4}{R_N^4}\right)+\cdots\right].\tag{A.8}$$

The $p_{1/2}$ integral is identical to (A.8) if we replace q by u, and l by v. The terms in the remaining integrals of Eq. (4) are related to those of (A.8) by numerical factors and will be given below. We can write for the electron factor of the spin contribution to ϵ in (4)

$$\int_0^R FGdr\Big/\int_0^\infty F_0G_0dr=(b_S)_2\left(\frac{R^2}{R_N^2}\right)+(b_S)_4\left(\frac{R^4}{R_N^4}\right)+\cdots,\tag{A.9}$$

[31] See, for example, H. A. Bethe and E. E. Salpeter, *Quantum Mechanics of One- and Two-Electron Atoms* (Academic Press, Inc., New York, 1957), Chap. Ib.

where the coefficients b_S are defined by comparison of (A.9) with (A.8). The factor of the asymmetrical spin contribution, **D**, in (4) is written similarly

$$\int_0^R FG\frac{r^3}{R^3}dr \bigg/ \int_0^\infty F_0G_0dr = (b_D)_2\left(\frac{R^2}{R_N^2}\right)+(b_D)_4\left(\frac{R^4}{R_N^4}\right)+\cdots, \quad \text{(A.10)}$$

and that of the orbital contribution,

$$\int_0^R \left(1-\frac{r^3}{R^3}\right)FGdr \bigg/ \int_0^\infty F_0G_0dr = (b_L)_2\left(\frac{R^2}{R_N^2}\right)+(b_L)_4\left(\frac{R^4}{R_N^4}\right)+\cdots. \quad \text{(A.11)}$$

We find the simple relations

$$(b_D)_2=(2/5)(b_S)_2, \quad (b_D)_4=(4/7)(b_S)_4, \quad \text{(A.12a)}$$

also

$$b_L=b_S-b_D. \quad \text{(A.12b)}$$

For $r>R_N$, the necessary Coulomb wave functions ($V=-Ze^2/r$) are obtained from the Dirac equation:

$$\chi_1=C_1J_{2\rho}(2(2\gamma y)^{\frac{1}{2}})+C_2J_{-2\rho}(2(2\gamma y)^{\frac{1}{2}}),$$

$$\chi_2=(1/\gamma)\{C_1[(\mp1-\rho)J_{2\rho}(2(2\gamma y)^{\frac{1}{2}}) +(2\gamma y)^{\frac{1}{2}}J_{2\rho+1}(2(2\gamma y)^{\frac{1}{2}})]\mp C_2[(1\pm\rho)J_{-2\rho}(2(2\gamma y)^{\frac{1}{2}}) \pm(2\gamma y)^{\frac{1}{2}}J_{-(2\rho+1)}(2(2\gamma y)^{\frac{1}{2}})]\}. \quad \text{(A.13)}$$

J are Bessel functions, $\rho=(1-\gamma^2)^{\frac{1}{2}}$, $y=r/\lambda\!\!\!^{-}_c$ [$\lambda\!\!\!^{-}_c=\hbar/mc=(1/2\pi)\times$Compton wavelength]. The constants C_1 and C_2 are determined by matching (A.13) to the interior functions (A.6) and (A.7). Using the approximate expressions of the Bessel functions for small arguments, $J_\rho(x)\approx x^\rho/2^\rho\rho!$ and $J_{-\rho}(x)\approx 2^\rho x^{-\rho}/(-\rho)!$, we find

$$C_1\approx\mp(2\rho-1)!L^{-2\rho} \times[(1\mp\rho)\chi_1(x=1)\pm\gamma\chi_2(x=1)], \quad \text{(A.14)}$$

where $L=(2\gamma R_N/\lambda\!\!\!^{-}_c)^{\frac{1}{2}}$. For well-behaved point wave functions, we must take $C_2=0$, and we assume, to adequate precision, that C_1 of the point wave function, equals C_1 of (A.14) as in Rosenthal and Breit.[19] Using the integration formulas for the Bessel functions,[32] we obtain

$$\int_0^\infty F_0G_0dr=\frac{C_1^2}{\lambda\!\!\!^{-}_c\rho(4\rho^2-1)}\times\begin{cases}-3 & (s_{1/2})\\ +1 & (p_{1/2}).\end{cases} \quad \text{(A.15)}$$

The b coefficients in Table I were calculated with these formulas, together with Eqs. (5a), (6), (7a), and the appropriate values of c_1 and z_3.

[32] G. N. Watson, *A Treatise on the Theory of Bessel Functions* (Cambridge University Press, New York, 1952), 2nd. ed., p. 403.