Capacity Planning in a General Supply Chain with Multiple Contract Types

by

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Abstract

In this thesis, we study capacity planning in a general supply chain that contains multiple products, processes, and resources. We consider situations with demand uncertainty, outsourcing contracts, and option contracts. We develop efficient and practical algorithms to address the following three questions: which suppliers should the manufacturer select, which types of contracts should it use, and how much capacity should it reserve. Through the model and algorithms, we study the properties of, and draw managerial insights about the optimal capacity planning strategy.

First, we propose a model to study the single period capacity planning problem. We provide closed-form representations of the optimal capacity planning strategy for two special supply chain structures. We then develop a stochastic linear programming algorithm to solve the general single period problem and show that our algorithm outperforms the alternative algorithms by means of an empirical study. With the model and algorithm, we then study the effects of demand uncertainty, prices, common processes, and option contracts on the optimal capacity planning strategy. We conclude with a discussion on how to include lot size constraints into the model.

Second, we develop a decomposition method for the single period capacity planning problem under the assumption that each process has only one dedicated resource. The algorithm provides both a feasible solution and an upper bound on the profit of the capacity planning problem. We test the effectiveness of the feasible solution and the tightness of the upper bound in the single period problem through a series of randomly generated test cases. The result shows that the algorithm performs fairly well with an average error of 1.48% on a set of test cases.

Third, we extend the capacity planning model into a multi-period setting. We solve a special case of the multi-period problem by transforming it into a shortest-path problem. We use the algorithm for the single period problem, the decomposition method, and the result from the special case to develop an efficient heuristic algorithm to solve the general multi-period problem. The same algorithm also generates an upper bound of the problem. We then test the heuristic algorithm and upper bound through several sets of test cases. Each test case is a 12-period capacity planning problem with 7 products, 14 processes, 14 resources, and 4 contracts for each resource. We can solve these problems with an average error of 1.17%.

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Chapter 1

Introduction

In today's competitive economic environment, customers do not just prefer but demand manufacturers to provide quality products in a timely fashion at competitive prices. To satisfy this requirement, manufacturers need to plan necessary and sufficient capacity to meet market demands. However, capacity planning is a very challenging task for many manufacturers.

Demand Uncertainty. For most industries, it is very difficult to accurately forecast the demand for new products. In an emerging industry, manufacturers devote substantial efforts to studying the applications and benefits of new technologies. However, when a technology is new, firms have little information on the commercial uptake of new products and, therefore, have poor forecasts of the product demand. For example, GlobalStar, one of the key players in the emerging mobile satellite services industry during the 1990s, expected between 500,000 and 1,000,000 users in 1999, the first year of its operation; these numbers were confirmed by many other independent analysts. However, the actual number of users was only 100,000, which is significantly lower than the expectation.

Demand forecasts for new products can also be inaccurate in existing industries. Customers' tastes and preferences are hard to predict and will change over time. Therefore, the historical demand patterns for an existing product might not always be a good reference for the next generation of products. For example, when Mercedes-Benz first introduced its M-class cars in 1997, it forecasted its annual demand to be about 65,000 vehicles. This forecast was, in fact, too low and the firm expanded its capacity to 80,000 vehicles during 1998-1999, which was also insufficient to meet demand [34].

The cost of misplanning capacity can be very high for manufacturers. In the case of GlobalStar, because the demand forecast was overly optimistic, the company filed for bankruptcy protection with a debt of 3.34 billion dollars in 2002 after three years of operations [38]. Therefore, it is important for the manufacturers to take demand uncertainties into consideration when they are planning their capacity.

Large Scale. Manufacturers face the difficulty of planning resources for multiple products at the same time. Due to competition and the wide range of applications of a new technology, the manufacturer needs to produce a variety of generic or custom-made products to meet the requirements of its customers. Such variety in products adds complexity to a manufacturer's supply chain. Different products might share common manufacturing processes or use common components. Because of the linkage between the products, the manufacturer needs to plan its capacity for producing multiple products together. However, finding the right level of capacity for all products at the same time is a large scale problem. A manufacturer, therefore, would benefit from efficient and practical algorithms for solving large scale capacity planning problems.

Outsourcing Contracts. A manufacturer needs to incorporate outsourcing into its capacity planning strategy. Traditionally, a manufacturer acquires capacity by building in-house manufacturing facilities. However, this approach has several drawbacks. First, a manufacturer needs to bear the risk of the high fixed cost associated with building the facilities. Second, a manufacturer needs to manage the in-house facilities itself. Third, a manufacturer cannot take advantage of the technology developed by the contract suppliers. Fourth, the contract suppliers can usually provide the capacity at a lower cost by leveraging the benefits of economies of scale. Therefore, instead of building the capacity themselves, firms have started to outsource their manufacturing processes and "rent" capacity from the suppliers through capacity contracts.

Currently, outsourcing manufacturing is a common practice in some industries and expected to play an increasing role in providing capacity and expertise to manufacturers. For example, in the biopharmaceutical industry, a manufacturer can develop the formulation of a drug in-house, use a supplier to test the drug, and outsource the mass production of the drug to another supplier. A partial list of contract manufacturers in the biopharmaceutical industry is given in Appendix A.1. In another example, the electronic industry, a manufacturer can outsource the design and fabrication of the different components of a product to different suppliers and perform the final assembly and testing by itself. The top 10 electronic contract manufacturers in 2006 are listed in Appendix A.2, and their total revenue is 148,255 million dollars.

When a manufacturer outsources its manufacturing processes, it is important for the firm to secure the availability and price of the capacity. Some of the major manufacturers, such as Hewlett-Packard, Ford, Cisco, and Dell, have suffered serious consequences from lack of supply and volatile prices [28]. To assure the supply of capacity, a manufacturer can establish contracts with its suppliers to specify the price and amount of capacity that it will need. However, when the demand is uncertain and the structure of the supply chain is complex, it is not obvious how the manufacturer should specify these capacity contracts.

Moreover, planning capacity with outsourcing contracts has a different structure from that of traditional capacity planning. In the traditional approach, after the manufacturer acquires the capacity, it is a sunk cost and cannot be reserved. On the other hand, under outsourcing capacity contracts, the manufacturer can rent or reserve the capacity from its suppliers for certain time periods. Therefore, a manufacturer can temporarily increase or decrease its capacity by signing contracts with the right durations. For example, we can look at Li & Fung Limited, an export trading company in Hong Kong that manages supply chains and capacity for major brands and retailers worldwide. The company owns just a few production facilities, but has a network of nearly 10,000 international suppliers. To fulfill an order from its customer, Li & Fung reserves capacity beforehand from selected suppliers. The agreements between Li & Fung and its suppliers specify the starting time of the use of the capacity, the amount of capacity that is required, and the time to deliver [26].

The capacity planning problem with flexible outsourcing contracts like the ones used by Li & Fung has not received much attention in the literature.

Option Contracts. In addition to demand uncertainty, large problem size, and outsourcing contracts, manufacturers can also benefit from models and tools that can incorporate option contracts into capacity planning. A manufacturer might establish a fixed-price capacity contract with its suppliers to rent a fixed amount of capacity. The manufacturer needs to pay for the capacity whether or not it uses the capacity. In practice, the supplier's cost of capacity might have two components: a fixed cost and a variable cost. For example, equipment costs and the monthly salaries of workers are fixed costs, while power consumption and employee overtime payments are variable costs. An option contract separates these two types of costs. With option contracts, the manufacturer buys the rights to use a fixed amount of capacity with an upfront fixed payment. If it decides to execute its rights and use these capacities, it needs to pay an exercise price for each unit of capacity that it actually uses.

Option contracts have been in practice for a long time. The manufacturer will often make a deposit to its supplier once both sides agree on a contract. When the supplier delivers the products, the manufacturer will pay the remaining payment. If the manufacturer withdraws from the contract, the deposit will serve as the penalty cost. In these situations, the deposit is equivalent to the upfront payment in an option contract, and the difference between the full payment and deposit will be the exercise price.

There are several reasons why both manufacturers and suppliers might prefer an option contract, rather than a fixed-cost contract. For the manufacturer, option contracts can serve as a tool to reduce the risk of committing upfront to a certain amount of capacity at a fixed price. As discussed in the context of outsourcing contracts, the manufacturer might want to secure the availability and price of the capacity. However, when demand is lower than expected, committing to buying a fixed amount of capacity will result in excess capacity. Moreover, if the price of capacity falls, the manufacturer will pay more than its competitors to make the products. Using option contracts can reduce the risk of weak demand and price volatility. For example, Hewlett-Packard has implemented a Procurement Risk Management (PRM) system to utilize option contracts and has realized $425 million savings in cost over a six-year period [28].

From the other side, a supplier can secure higher revenue by taking advantage of option contracts. Since an option contract can serve as a hedging tool to protect the downside of its operation, the manufacturer might be willing to pay more for each unit of option capacity, which means that the reservation price plus the exercise price is higher than the fixed-price contract price. Moreover, since the manufacturer bears lower risk, it might purchase more capacity. As a result, the supplier can gain more revenue. Therefore, a method to incorporate option contracts into capacity planning will also be one of the manufacturers' primary interests.

In this thesis, we present a mathematical model and tools to help manufacturers plan their capacity under demand uncertainty for a general large scale supply chain structure. Moreover, we consider outsourcing contracts and option capacity contracts. We have developed efficient and practical algorithms to address the following three questions: which suppliers should the manufacturer select, which types of contracts should it use, and how much capacity should it reserve. Using the model and algorithms, we study the properties of, and draw managerial insights about, the optimal capacity planning strategy. Therefore, our research help managers to make these complex capacity planning decisions in a more systematic and effective way.

Structure of the Thesis. The thesis is organized as follows. In Chapter 2, we propose a framework to study the single period capacity planning problem. We derive closed-form solutions for two special supply chain strictures. We then compare five different algorithms for solving the general problem and show that the one we develop outperforms the others through a series of test cases. We also study the properties of the optimal capacity planning strategy. Finally, we consider a variant of the problem by adding constraints on the order size. In Chapter 3, we develop an efficient decomposition method that can provide both a feasible solution and an upper bound of the capacity planning problem. We examine the effectiveness of the feasible solution and tightness of the upper bound through a series of test cases.

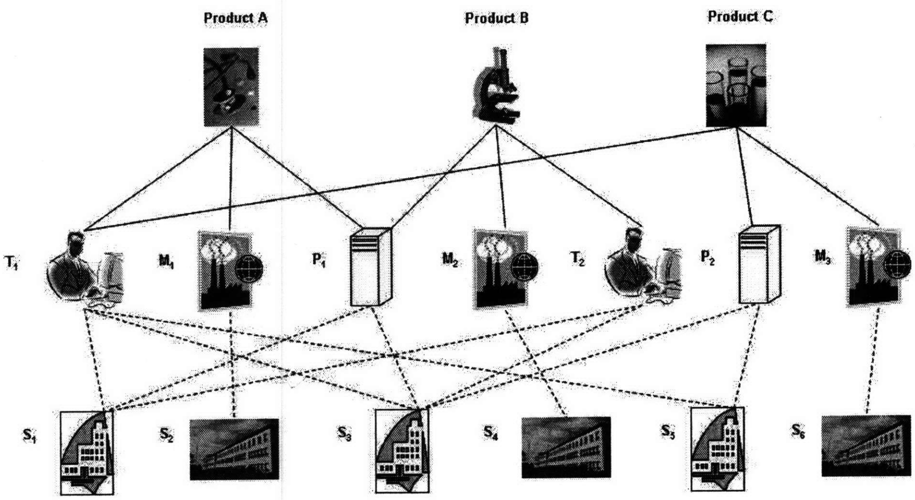

Figure 1-1: An example of capacity planning problem.

In Chapter 4, we extend the model to a multi-period setting and present an efficient heuristic algorithm to solve the multi-period problem. We then show that the heuristic algorithm performs fairly well through several sets of test cases. Finally, we discuss the future directions of our research and conclude the thesis in Chapter 5.

1.1 An Example

We illustrate the problems to be addressed in this thesis with the following example. A biomedical manufacturer has three major products: A, B, and C. Each product requires three processes: formulation, testing, and packaging. The manufacturer acquires capacity for each process through contracts with third-party suppliers. The structure of the supply chain of the manufacturer is given in Figure 1-1. Since the formulation processes of the products are different from each other, the company must use dedicated suppliers to provide the capacity for the product-specific formulation processes. However, there is some commonality between the testing and packaging processes: product A and B have the same packaging process, and product A and C have the same testing process. The company has three suppliers that can provide capacity for the testing and packaging processes. Because of the similarity in technology and cross-training of human capital, the capacity for testing and packaging processes is more flexible compared to that for the formulation process. As a result, the capacity from these suppliers can be used by multiple processes: Supplier S_5 can provide capacity for the testing and packaging processes of product C, supplier S_1 can provide capacity for the testing and packaging processes of products A and B, and supplier S_3 can provide capacity for all the testing and packaging processes. The manufacturer only has partial knowledge of the demands (e.g. the probability distribution) and it needs to plan its capacity for the next 12 months under the demand uncertainty. Each supplier submits a list of available contracts to the manufacturer. Given these inputs, how should the manufacturer choose the types and sizes of contracts to maximize its expected profit? The manufacturer also needs to have the ability to find the optimal capacity planning strategy in a timely fashion so that it can explore different configurations, when it designs its supply chain or when the environment changes. This thesis provides the mathematical foundation and efficient algorithms for manufacturers to achieve these goals.

1.2 Related Literature

The research in this thesis is related to the literature in four areas: Newsvendor Network and Assembly to Order (ATO) Systems, Traditional Capacity Planning, Option Contracts, and Stochastic Programming.

Newsvendor Network and Assembly to Order (ATO) Systems. Van Mieghem and Rudi [31] propose a newsvendor network that is closely related to the model that we use. In their model, the authors consider a supply chain that contains multiple products and multiple stocks. The manufacturer consumes the stocks to produce the products through activities. The stocks are subject to inventory constraints and the activities are subject to capacity constraints. They study a joint capacity investment and inventory management problem in their model. The capacity investment decision is made at the beginning of the planning horizon and remains in effect ever after. At the beginning of each period, the manufacturer will make the inventory procurement decisions depending on the stock level. The authors show that a stationary base-stock inventory policy is optimal for the lost sales case. They also show that the capacity planning problem is concave, and therefore concave optimization algorithms such as subgradient methods can be used to find the optimal capacity plan.

In contrast to their work, our model does not incorporate inventory but allows the manufacturer to establish different types of contracts with its suppliers. These contracts can be different in duration, price, and structure (such as fixed-cost contract and option contract). Therefore, our capacity planning problem has a very different structure in a multi-period setting. Moreover, their paper focuses on the structure of the optimal inventory replenishment policy, while we emphasize the algorithms for solving the capacity problem. For the single period problem, we discuss different concave optimization algorithms, which include the sub-gradient method suggested by Van Mieghem and Rudi. We show that the algorithm that we propose has a superior performance.

In terms of modelling the supply chain, the model that we propose in this thesis shares some commonality with the assemble-to-order (ATO) systems in the supply chain operation literature. An ATO system contains multiple products and multiple components. The system only keeps inventory on the component level. When demand arrives, it will assemble products using the necessary components. ATO systems capture some of the essential characteristics of a real life supply chain, such as common processes (e.g. Gerchak, et al. [15], Hillier [19], and Kulkarni, et al. [23]) and flexible resources (e.g. Fine and Freund [13], Van Mieghem [32], and Labro [25]). For a detailed survey and discussion of ATO systems, please refer to Song and Zipkin [30].

There are several major differences between ATO systems and our supply chain capacity model. First, our model has a multi-stage structure that allows both flexible resources and common processes. Second, we incorporate option contracts into the model. Third, our model focuses on capacity planning with different outsourcing contracts, while ATO systems mainly study inventory policies.

Traditional Capacity Planning. There are a wide variety of models used for multi-period capacity planning; these models vary depending on their assumptions on how capacity is acquired and how it can be modified over time.

We can divide the existing literature into two categories, depending on whether capacity can or cannot be reduced after the acquisition. In the first category, after the manufacturer acquires the capacity, the new capacity level remains effective until the end of the planning horizon. At the beginning of each period, the manufacturer will decide whether it wants to expand its capacity or not and how much it will expand if it decides to do so. Capacity expansion is an active research area. Van Mieghem [35] gives a survey of the literature on capacity expansion. Ahmed, et al. [1] study the capacity planning problem in a supply chain with a single product and multiple resources. They model the demand uncertainty as scenario trees. They propose a branch and bound algorithm to solve the problem. Zhang, et al. [40] consider a multi-product and multi-machine supply chain in the semiconductor industry. They assume that the demands have a certain structure and show that the problem can be solved as a max-flow min-cut problem. Ahmed, et al [2] apply a branch and bound method to solve a two-stage, multi-product, and multi-process capacity planning problem. Barahona et al. [5] study a tool purchasing problem in semiconductor manufacturing. Similar to Ahmed, et al [2], they consider a two-stage decision process: First, the manufacturer decides the tool purchasing schedule at the beginning of the planning horizon when the demand is uncertain. Second, the demand is realized and the manufacturer allocates tools to fabricate the products. The authors develop a heuristic stochastic integer programming algorithm to solve the problem and test it with a real life application at IBM. Shirodkar and Kempf [29] discuss how they apply a capacity planning model at Intel to make procurement decisions. Intel's assembly and test factories use different types of substrates to make the devices. The firm purchases the substrates from different suppliers. Each factory consumes multiple types of substrates, and each supplier can provide multiple types of substrates. The authors study and develop models to help Intel make the procurement decisions.

Our work adapts a two-stage decision model similar to the one used by Ahmed, et al [2] and Barahona et al. [5]. However, our model differs from the ones cited above in terms of the capacity acquisition method. In our model, instead of owning the capacity, the manufacturer gains the rights to use the capacity for certain periods from its suppliers through contracts of different durations and prices. When a contract finishes, the manufacturer does not need to pay for the capacity anymore.

In the second category, the manufacturer can reduce the capacity level at any time. Huh et al. [21] examine a capacity planning problem in the semiconductor industry where they allow tools to be retired if necessary. They propose a cluster-based heuristic algorithm to solve the problem. Angelus and Porteus [4] study a single product capacity planning and production management problem. In their model, capacity can be added or removed at each period. Under certain assumptions, they give an explicit solution for the capacity level.

In contrast, we require the manufacturer to determine the capacity contracts at the beginning of the planning horizon. As we have discussed, since the manufacturer does not own the capacity, reserving the capacity at the beginning to secure the supply and price is crucial to the firm. In the middle of a contract, both the manufacturer and supplier cannot change the terms such as the price and the quantity. In this thesis, we do not allow the manufacturer to modify the contracts after it has made the decisions at the beginning. We will discuss how one might relax this restriction in Chapter 5.

In addition to the flexibility of modifying the capacity plan, our model makes different assumptions on the inventory policy compared to those in the existing literature. Some of the existing literature on the multi-period capacity planning problem, such as Angelus and Porteus [4], Van Mieghem and Rudi [31], and Barahona et al. [5], takes inventory into consideration. In our model, the manufacturer does not build and store inventory. However, since the manufacturer can engage into a contract that spans multiple periods, the capacity decisions for different periods are related with each other.

Therefore, our model is significantly different from the traditional capacity planning problems in terms of the assumption on the flexibility of modifying the capacity, the assumption on inventory policy, and incorporating option contracts.

Option Contracts. The consideration of option contracts in supply chains is a more recent research topic. Cheng et al. [11] derive the optimal order decision for the manufacturer and the optimal pricing decision for the supplier in a single product, single supplier, and single period supply chain. Yazlali and Erhun [39] consider option contracts in a single product, dual supply, and multi-period problem. They use a two-stage decision process: first, the manufacturer reserves capacity for the whole planning horizon by signing a portfolio of contracts; second, it orders from the suppliers based on the contracts. Under certain assumptions on demands and prices, they show that for the second stage problem, a two-level modified base-stock policy is optimal, and, for the first stage, a reserve-up-to policy is optimal. Martinez-de-Albniz and Simchi-Levi [27] analyze the optimal option contract for a case of single product and multiple suppliers in the presence of a spot market. In their model, they also adapt a two-stage decision process. The manufacturer decides the quantity and portfolio of contracts at the beginning of the planning horizon. The duration of each contract is the whole planning horizon. The authors then study the optimal replenishment policy given the portfolio and conditions of the spot market. They show that the portfolio selection problem is a concave maximization problem. Fu, et al. [14] examine a single-period procurement problem with option contracts. Their model incorporates random spot price and demands. They show that option contracts can be very valuable for both the manufacturer and supplier. Nagali et al. [28] apply option contracts in HP's procurement risk management, and the system that they implemented has realized more than $425 million cost savings in a six year period. However, they do not provide details on the specific models that are used for evaluating these option contracts.

There are two major differences between our work and the existing literature studying option contracts: first, we incorporate contract durations into our model; second, we consider a more general supply chain structure that contains multiple products and multiple processes. However, our model takes the external market conditions as given and does not consider inventory. Finally, we see that our research can help to extend some of the existing models that contain a single supplier to a more general setting. In Chapter 3, we propose a decomposition method to separate a general supply chain into sub-problems; each sub-problem contains only one supplier.

After the decomposition, the results from the literature can be applied.

Stochastic Programming. Finally, our work is also related to the literature studying algorithms for stochastic linear and integer programming. Higle and Sen [17], [18] propose and summarize several stochastic linear programming algorithms to solve a general capacity planning problem. We adapt some of these techniques in our algorithm for solving our single period capacity planning problem. We show that the algorithm we propose has a better performance than the ones that Higle and Sen suggest through a series of randomly generated test cases. Higle and Sen [18] provide an excellent review of how to apply stochastic linear programming to solve large scale capacity planning problems. It is common that a capacity planning problem involves integral decisions. In these situations, the integral decisions can be modelled as integer decision variables. In [1], [3], and [5], the capacity expansion decisions are binary integer variables (e.g. variables can be either 1 or 0). In [2], [5], and [24], the capacity can only be purchased in integer units. In [2] and [5], the capacity can only be allocated in integer units. Stochastic integer programming is used in this literature to solve the capacity planning problems involving integer decision variables.

As contrasted with this literature, our model considers capacity contracts with different durations. In addition to deciding which contract the manufacturer should buy, we also need to decide the order of the contracts. Therefore, we cannot apply the traditional stochastic integer programming algorithms to our multi-period problem. As a result, we propose a new algorithm that takes advantages of the special structure of our problem.

Chapter 2

Single Period Capacity Planning Problem

In this chapter, we will study the single period capacity planning problem. The single period problem itself has significant applications. For example, in some situations, the capacity planning is an one-time event and therefore can be modelled as a single period problem. Moreover, as we will show in Chapter 4, the method that we develop for solving the single period case can be used in the algorithm to solve the multi-period capacity planning problem.

This chapter is organized as follows: In Section 2.1, we outline a mathematical model for the single period capacity planning problem. We then look at two special cases and derive closed-form solutions for the optimal strategies for these cases in Section 2.2. After that, in Section 2.3, we examine five algorithms to solve the general single period capacity problem and show that the one we develop has a better run time through a series of randomly generated test problems. In Section 2.4, we discuss the properties of optimal capacity planning strategies. Finally, in Section 2.5, we consider a variant of the single period problem: the capacity planning problem with constraints on the order size.

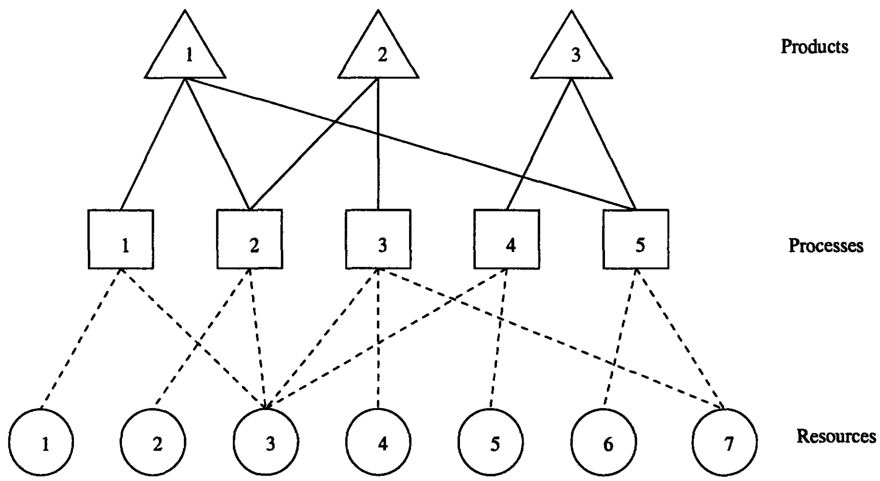

Figure 2-1: A supply chain network with 3 products, 5 processes, and 7 resources.

2.1 Model

2.1.1 Mathematical Model

We consider a multi-product and multi-stage supply chain consisting of M products, J processes, and K resources. A sample supply chain network with three products, five processes, and seven resources is given in Figure 2-1. The production of each product requires a certain amount (possibly zero) of each type of process. The solid links joining products and processes in Figure 2-1 signify this relationship. For example, product 1 requires processes 1, 2, and 5. In practice, a process can be either an operation such as assembly, testing, or packaging or a type of material or component or a sub-system that is required to produce the product. A resource provides capacity for one or more processes. The dashed links joining processes and resources in Figure 2-1 signify that the resource has the capability to deliver the process. For the network given in Figure 2-1, the firm can get capacity for process 1 from resource 1 or 3 and resource 3 can provide capacity to processes 1, 2, and 4. A resource might be an assembly line with the capability to assemble a single product type. A flexible resource might be an assembly line capable of assembling several different product types. We might also imagine a resource with capability to provide more than one type of process; for instance, a resource might do both assembly and test for a single product type. Without loss of generality, we assume that the production of one unit of product requires one unit of each of its required processes; we also assume that one unit of each process requires one unit of capacity from one of its resource options.

The supply chain structure that we propose for the single period problem is fairly general and can capture different types of interdependency between products, processes, and resources. First, to produce a product requires capacities from all of its processes. Therefore, the capacity levels of different processes of the same product are closely related to each other. Second, different products can share common processes and flexible resources can provide capacity to different processes. These common processes and flexible resources link the capacity planning decisions of different products together. One of our goals is to account for these interdependencies within capacity planning.

In addition to a general supply chain structure, we also consider two alternatives for procuring or reserving capacity for each resource: A firm can reserve capacity on a resource with a fixed-price capacity contract; alternatively a firm can reserve capacity on a resource with an option contract where there is a smaller upfront reservation price and then a variable exercise price for the use of this capacity. For instance, under a fixed-price capacity contract, the price for one unit of capacity is 1 dollar. Under an option contract, the firm might pay a fixed price of 30 cents initially to reserve one unit of the capacity. If the firm decides to use the capacity that it has reserved, it needs to pay another 80 cents per unit. Given these alternatives, the firm wants to determine the amount of each resource to use, as well as the contracts, so that the resulting supply chain can maximize the firm's expected profit.

We assume that any demand that cannot be filled is lost, and there is no penalty cost for not meeting demand. We also assume a two-stage sequential decision process. In the first stage, the firm determines the types and sizes of the contracts for each resource; in effect the firm decides its capacity plan. In the second stage, demand is realized and the firm decides how to allocate its production capacity to meet demand. To the extent that the firm employs options contracts, it will decide how much of each option to exercise. Also, the firm decides how to utilize the capacity of each flexible resource across the applicable processes.

For naming convention, we use bold letter to indicate a vector. For input parameters, we denote:

A An $J \times M$ matrix such that

$$A(j,m) = \begin{cases} 1, & \text{if product } m \text{ requires process } j; \\ 0, & \text{otherwise.} \end{cases}$$

B An $J \times JK$ matrix such that

$$B(j,(j,k)) = \begin{cases} 1, & \text{if resource } k \text{ can provide capacity to process } j; \\ 0, & \text{otherwise.} \end{cases}$$

H A $K \times JK$ matrix such that

$$H(k,(j,k)) = \begin{cases} 1, & \text{if resource } k \text{ can provide capacity to process } j; \\ 0, & \text{otherwise.} \end{cases}$$

$\boldsymbol{D}$ A vector of random variables, with probability density function, that represents the demand of products. (Vector of size M)

$\boldsymbol{d}$ A realization of random demand $\boldsymbol{D}$. (Vector of size M)

$\boldsymbol{r}$ Unit profit for filling product demand. (Vector of size M)

$\boldsymbol{p}$ Unit price of resources under fixed-price contract. (Vector of size K)

$\boldsymbol{q}$ Unit reservation price of resources under option contract. (Vector of size K)

$\boldsymbol{e}$ Unit exercise price of resources under option contract. (Vector of size K)

Without loss of generality, we assume that for each resource k, $p_k < q_k + e_k$ and $p_k > q_k$. If $p_k \geq q_k + e_k$, the manufacturer will not use any fixed-price capacity from resource k. Similarly, if $p_k \leq q_k$, the manufacturer will not reserve any option capacity. We also assume that the demand vector is non-negative, e.g. $\boldsymbol{D} \geq 0$.

For decision variables, we denote:

z_m Amount of product m that is produced and sold to meet demand. (Scalar)

$\boldsymbol{z}$ Amount of products that are produced and sold to meet demand. (Vector of size M)

x_{jk} Amount of resource k provided under a fixed-price capacity contract that is used to provide capacity to process j. (Scalar)

$\boldsymbol{x}$ The vector of x_{jk} . (Vector of size JK)

y_{jk} Amount of resource k provided under an option capacity contract that is used to provide capacity to process j. (Scalar)

$\boldsymbol{y}$ The vector of y_{jk} . (Vector of size JK)

$\boldsymbol{c}$ The amount of fixed-price capacity that the firm has reserved. (Vector of size K)

$\boldsymbol{g}$ The total amount of capacity, including fixed-price and option capacity, that the firm has reserved. (Vector of size K)

We now formulate the second stage problem as a single-period production planning problem with the objective to maximize the profit of the firm. We are given the demand realization $\boldsymbol{d}$ as well as $\boldsymbol{c}$, the amount of each resource reserved with fixed-price contract, and $\boldsymbol{g}$, the total amount of each resource reserved. We note that $g-c$ is the amount of each resource reserved with an option contract. We have the following linear optimization problem:

$$\begin{aligned} \pi(\boldsymbol{c}, \boldsymbol{g}, \boldsymbol{d}) = \max_{\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}} \quad & \pi(\boldsymbol{c}, \boldsymbol{g}, \boldsymbol{d}, \boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) = \boldsymbol{r}'\boldsymbol{z} - \boldsymbol{e}'H\boldsymbol{y} \\ s.t. \quad & \boldsymbol{z} \leq \boldsymbol{d} \\ & A\boldsymbol{z} \leq B(\boldsymbol{x}+\boldsymbol{y}) \\ & H\boldsymbol{x} \leq \boldsymbol{c} \\ & H(\boldsymbol{x}+\boldsymbol{y}) \leq \boldsymbol{g} \\ & \boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z} \geq \boldsymbol{0} \end{aligned} \tag{2.1}$$

The objective function of Problem (2.1) is the net revenue that the manufacturer will gain under given capacity level $\boldsymbol{c}$ and $\boldsymbol{g}$ and demand $\boldsymbol{d}$. For the second stage problem this is the revenue from selling $\boldsymbol{z}$, net of the additional cost from exercising the option contracts in the amount of $\boldsymbol{y}$. The first set of constraints restricts the amount of product sold to be less than the demand; we note that $\boldsymbol{d}-\boldsymbol{z}$ represents the amount of demand that is not met. The second set of constraints says that the amount of products produced can not exceed the total available capacity; the left hand side is the amount of process capacity required to produce $\boldsymbol{z}$ and the right hand side is the available process capacity given the allocation decisions $\boldsymbol{x}$ and $\boldsymbol{y}$. Finally, the third and fourth set of constraints assures that the resource availability is not exceeded. The left hand side of the third set represents the resource usage under the fixed-price contract, while the left hand side of the fourth set is the total resource usage for the allocation decisions.

By solving this optimization problem, we can find the revenue maximizing production level for a given demand realization and the given capacity planning decisions. Let $(\boldsymbol{x}^*, \boldsymbol{y}^*, \boldsymbol{z}^*)$ be an optimal solution of Problem (2.1); $(\boldsymbol{x}^*, \boldsymbol{y}^*, \boldsymbol{z}^*)$ is a function of $\boldsymbol{d}$, $\boldsymbol{c}$, and $\boldsymbol{g}$. The firm ultimately wants to find the optimal capacity planning strategy under demand uncertainty by solving the following first-stage problem:

$$\begin{aligned} \max_{\boldsymbol{c},\boldsymbol{g}} \quad & \Pi(\boldsymbol{c}, \boldsymbol{g}, \boldsymbol{D}) = E[\pi(\boldsymbol{c}, \boldsymbol{g}, \boldsymbol{D})] - \boldsymbol{p}'\boldsymbol{c} - \boldsymbol{q}'(\boldsymbol{g} - \boldsymbol{c}) \\ s.t. \quad & \boldsymbol{c} \leq \boldsymbol{g} \\ & \boldsymbol{c}, \boldsymbol{g} \geq \boldsymbol{0} \end{aligned} \tag{2.2}$$

The objective function of Problem (2.2) represents the expected total profit, which is equal to the expected total net revenue from the second stage, minus the first-stage reservation cost of the capacity. The first set of constraints ensures that the amount of fixed-price capacity reserved is no more than the amount of total capacity reserved.

Proposition 1 $\Pi(\boldsymbol{c}, \boldsymbol{g}, \boldsymbol{D})$ *is concave in* $(\boldsymbol{c}, \boldsymbol{g})$.

Proof: Let $(\boldsymbol{c}^1, \boldsymbol{g}^1)$ and $(\boldsymbol{c}^2, \boldsymbol{g}^2)$ be two feasible capacity planning strategies. Let λ be a scalar that $0 < \lambda < 1$. Then, capacity planning strategy

$$(\boldsymbol{c}^3, \boldsymbol{g}^3) = (\lambda \boldsymbol{c}^1 + (1-\lambda)\boldsymbol{c}^2, \lambda \boldsymbol{g}^1 + (1-\lambda)\boldsymbol{g}^2)$$

is also feasible. For any demand realization $\boldsymbol{d}$, let $(\boldsymbol{x}^i, \boldsymbol{y}^i, \boldsymbol{z}^i)$ be an optimal solution of Problem (2.1) given capacity planning strategy $(\boldsymbol{c}^i, \boldsymbol{g}^i)$. Fix a scalar $\lambda \in [0, 1]$ and consider the production level

$$(\boldsymbol{x}^3, \boldsymbol{y}^3, \boldsymbol{z}^3) = (\lambda \boldsymbol{x}^1 + (1-\lambda)\boldsymbol{x}^2, \lambda \boldsymbol{y}^1 + (1-\lambda)\boldsymbol{y}^2, \lambda \boldsymbol{z}^1 + (1-\lambda)\boldsymbol{z}^2).$$

We can verify that $(\boldsymbol{x}^3, \boldsymbol{y}^3, \boldsymbol{z}^3)$ is a feasible solution of Problem (2.1) given demand realization $\boldsymbol{d}$ and strategy $(\boldsymbol{c}^3, \boldsymbol{g}^3)$. Therefore

$$\begin{aligned}
\pi(\boldsymbol{c}^3, \boldsymbol{g}^3, \boldsymbol{d}) &\geq \pi(\boldsymbol{c}^3, \boldsymbol{g}^3, d, \boldsymbol{x}^3, \boldsymbol{y}^3, \boldsymbol{z}^3) \\
&= \boldsymbol{r}'\boldsymbol{z}^3 - \boldsymbol{e}'H\boldsymbol{y}^3 \\
&= \lambda(\boldsymbol{r}'\boldsymbol{z}^1 - \boldsymbol{e}'H\boldsymbol{y}^1) + (1-\lambda)(\boldsymbol{r}'\boldsymbol{z}^2 - \boldsymbol{e}'H\boldsymbol{y}^2) \\
&= \lambda\pi(\boldsymbol{c}^1, \boldsymbol{g}^1, \boldsymbol{d}) + (1-\lambda)\pi(\boldsymbol{c}^2, \boldsymbol{g}^2, \boldsymbol{d})
\end{aligned}$$

Therefore, $\pi(\boldsymbol{c}, \boldsymbol{g}, \boldsymbol{d})$ is concave in $(\boldsymbol{c}, \boldsymbol{g})$ for any given $\boldsymbol{d}$. Since taking expectation will maintain concavity, $E[\pi(\boldsymbol{c}, \boldsymbol{g}, \boldsymbol{D})]$ is concave in $(\boldsymbol{c}, \boldsymbol{g})$. Therefore

$$\begin{aligned}
\Pi(\boldsymbol{c}^3, \boldsymbol{g}^3, \boldsymbol{D}) &= E[\pi(\boldsymbol{c}^3, \boldsymbol{g}^3, \boldsymbol{D})] - \boldsymbol{p}'\boldsymbol{c}^3 - \boldsymbol{q}'(\boldsymbol{g}^3 - \boldsymbol{c}^3) \\
&\geq \lambda\left(E[\pi(\boldsymbol{c}^1, \boldsymbol{g}^1, \boldsymbol{D})] - \boldsymbol{p}'\boldsymbol{c}^1 - \boldsymbol{q}'(\boldsymbol{g}^1 - \boldsymbol{c}^1)\right) \\
&\quad +(1-\lambda)\left(E[\pi(\boldsymbol{c}^2, \boldsymbol{g}^2, \boldsymbol{D})] - \boldsymbol{p}'\boldsymbol{c}^2 - \boldsymbol{q}'(\boldsymbol{g}^2 - \boldsymbol{c}^2)\right) \\
&= \lambda\Pi(\boldsymbol{c}^1, \boldsymbol{g}^1, \boldsymbol{D}) + (1-\lambda)\Pi(\boldsymbol{c}^2, \boldsymbol{g}^2, \boldsymbol{D}).
\end{aligned}$$

Therefore, $\Pi(\boldsymbol{c}, \boldsymbol{g}, \boldsymbol{D})$ is concave in $(\boldsymbol{c}, \boldsymbol{g})$. **Q.E.D.**

Proposition 1 guarantees that every local optimal solution for Problem (2.2) is a global optimal solution and that the algorithms given in Section 2.3 will converge.

2.1.2 An Example

We now conclude the model section with a numerical example. Let us consider that a computer manufacturer produces two types of laptop, namely A and B. Laptop A requires three manufacturing processes or inputs: the manufacture or procurement of chipset A, the manufacture or procurement of display A, and Assembly & Testing (A&T). Similarly, each laptop B requires chipset B, display B, and Assembly & Testing.

Laptop A is an entry level laptop selling at 700 dollars. Laptop B is a mid-range

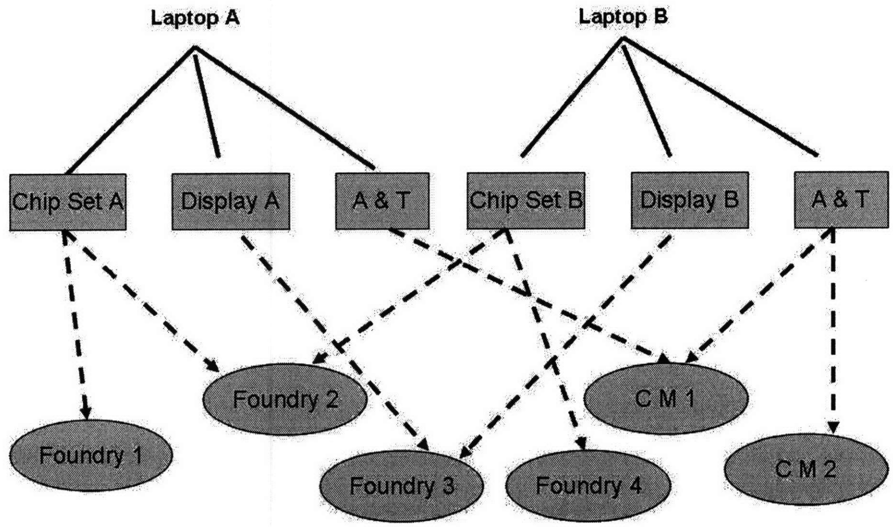

Figure 2-2: Single period numerical example: A manufacturer supply chain network containing two laptops, six processes, and six capacity providers.

	Laptop A	Laptop B
Price ($)	700	1000
Mean	2200	1000
STD	200	100

Table 2.1: Single period numerical example: table of product prices and demand information.

price laptop selling at 1000 dollars. The major difference between laptop A and B is that they use different chipsets. Chipset B is better than chipset A. The demand of both laptops follows a normal distribution with their mean and standard deviation given in Table 2.1.

The manufacturer uses contract suppliers to perform the manufacturing processes. It currently has six contract suppliers from which to choose: Foundry 1, 2, 3, 4 and Contract Manufacturer (CM) 1, 2. The capability of each supplier is given in Figure 2-2. For instance, contract manufacturer 2 (CM 2) is qualified to do the assembly and test for Laptop B, whereas contract manufacturer 1 (CM 1) is qualified to do assembly and test for both laptops. Similarly, Foundry 2 is flexible and can produce both chipsets, whereas Foundry 1 (Foundry 4) can only supply Chipset A (Chipset B).

	Fixed Unit Price	Unit Reservation Price		Unit Exercise Price	
		Case 1	Case 2	Case 1	Case 2
Foundry 1	90	85	10	10	85
Foundry 2	100	80	30	30	80
Foundry 3	200	160	50	50	160
Foundry 4	98	78	28	28	78
CM 1	115	100	25	25	100
CM 2	110	90	30	30	90

Table 2.2: Single period numerical example: table of capacity prices.

The manufacturer has two ways of contracting with each supplier. The price structure of each supplier for two different scenarios is given in Table 2.2. For Case 1, the unit reservation price is higher than the unit exercise price. The prices of the resources in Case 2 are the same as Case 1 except the unit reservation price and the unit exercise price are swapped.

The manufacturer can reserve capacity from each supplier with a fixed-price capacity contract. For instance, in Case 1, Foundry 1 quotes a fixed unit price of \$90. Thus, if the manufacturer were to reserve 200 units of capacity, it would pay Foundry 1 \$1800; Foundry 1 will then commit to provide the manufacturer with upto 200 units of Chipset A over the demand period. To keep things simple, we assume the only cost is the upfront fixed cost of \$1800.

Alternatively the manufacturer can reserve capacity from a supplier with an option contract where there is a smaller upfront fixed cost and then a variable cost for the use of this capacity. For instance, in Case 1, the manufacturer might purchase an option contract with Foundry 3 for 300 units of capacity. The manufacturer would pay Foundry 3 an upfront cost of $300 \times \$160 = \$48,000$ to reserve this capacity. Later, when it needs to make the actual procurement decisions, the manufacturer can decide how much of the capacity to use (up to 300 units) and for what mix of products (i.e., display A or display B). The manufacturer pays an additional \$50 per unit for each unit of capacity that it actually uses. We note that the fixed-price contract is effectively an option contract with a zero exercise price - as we don't require the manufacturer to use all of the fixed-capacity, and there is no additional cost for not using this capacity.

Given the demand distributions (Table 2.1), network structure (Figure 2-2), and cost structures of the suppliers (Table 2.2), the manufacturer wants to determine:

1. Which suppliers should it use?

2. What types of contract should it use for each supplier? Only fixed-price contract? Only option contract? Or Both.

3. How much capacity it should buy?

The firm needs to consider the trade-offs between different factors:

1. Demand is uncertain and the manufacturer will want to have enough process capacity to meet any demand outcome, up to some level.

2. To deliver a product the manufacturer must have sufficient capacity for all of its processes - having enough chipsets is not very useful if one is short of displays.

3. The resource options vary in terms of cost and flexibility. For instance, the capacity from Foundry 2 is more expensive relative to that from either Foundry 1 or 4; but the capacity at Foundry 2 is flexible as it can produce either display.

The model that we propose in this section and the algorithms that we will examine in the coming sections will help the manufacturer to answer these questions and understand the trade-offs.

For this example, the results of our algorithm are given in Table 2.3. For Case 1, the manufacturer should

1. use all six suppliers.

2. only use a fixed-price contract from Foundry 1, Foundry 2, and CM 1.

3. use both types of contract from the other suppliers.

A similar conclusion can be drawn for Case 2. This example also shows that for both cases the sums of total capacity for Foundry 1, 2, and 4, the total capacity for

	Fixed-Price Capacity		Option Capacity		Total Capacity	
	Case 1	Case 2	Case 1	Case 2	Case 1	Case 2
Foundry 1	1977	1667	0	491	1977	2158
Foundry 2	364	378	0	0	364	378
Foundry 4	757	823	79	0	836	823
Foundry 3	3023	2871	154	488	3177	3359
CM 1	2341	2115	0	370	2341	2485
CM 2	774	805	62	69	836	874

Table 2.3: Single period numerical example: results.

Foundry 3, and the sum of total capacity for CM 1 and CM 2 are equal to each other. Foundry 1, 2, and 4 provide capacity for the chipsets; Foundry 3 provides capacity for the display; CM 1 and 2 provide capacity for the A&T. Since to produce a product requires all three processes, the total capacity reserved for these processes are the same.

This example also illustrates the complexity of the optimal strategy. We expect that the manufacturer will reserve more capacity in Case 2, since the unit reservation prices of all resources are lower than Case 1. However, from the optimal solutions we can see that the manufacturer should not reserve any option capacity for Foundry 4 in Case 2, while in Case 1 it should reserve 79 units of option capacity. This is due to the interdependency between Foundry 1, 2, and 4. In Case 2, since the unit reservation price for Foundry 1 is much lower than the reservation price for Foundry 4, Foundry 1 has a more attractive option contract. Therefore, the manufacturer should buy more option contract from Foundry 1 and less option contract from Foundry 4. We will look at more examples to show the complexity of the optimal strategy in Section 2.4.

Finally, we can compare the optimal solutions with some alternative capacity plans. We consider the capacity strategies obtained from the following two plans

1. Ignoring option capacity.

2. Solving the capacity problem of each product separately.

	Case 1		Case 2	
Plan	Expected Profit	Improvement(%)	Expected Profit	Improvement(%)
Optimal	1,179,849	-	1,203,485	-
1	1,178,842	0.09%	1,178,842	2.09%
2	1,150,848	2.52%	1,189,287	1.19%
1 & 2	1,148,253	2.75%	1,148,253	4.81%

Table 2.4: Single period numerical example: comparisons with the other capacity plans.

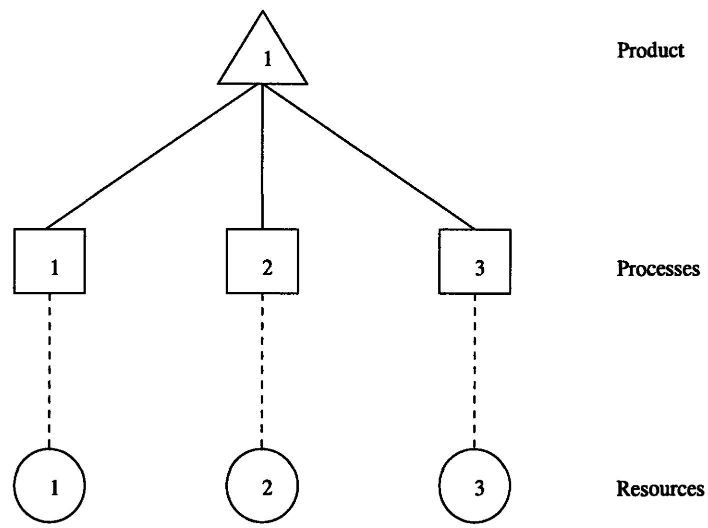

Figure 2-3: A supply chain network with single product and dedicated resources.

The expected profits that the manufacturer can get from using these capacity plans are given in Table 2.4.

2.2 Two Special Cases

Before we examine the algorithms to solve the general single period capacity planning problem, we will first study two special cases of the problem: single product with dedicated resources and single process with dedicated resource.

2.2.1 Special Case I: Single Product with Dedicated Resources

Let us consider the first special case where the network contains one product and dedicated resources. Figure 2-3 shows such a supply chain with a single product that requires three processes and each process has a dedicated resource to provide capacity for it. As there is exactly one resource for each process, we will view these as synonymous and will use the terms interchangeably. For this special case, we can derive a closed-form representation for the optimal capacity planning strategy. Without loss of generality, we assume that we number the resources such that

$$\frac{p_i - q_i}{e_i} \leq \frac{p_j - q_j}{e_j}, \text{ if } i < j \tag{2.3}$$

The ratio $\frac{p_i - q_i}{e_i}$ is non-negative and less than 1. The bigger the ratio, the more attractive the option contract would be.

Proposition 2 *Assume D is a vector of continuous random variables and $r > \sum_j e_j$. For a supply chain with single product and dedicated resources, a capacity planning strategy $(\boldsymbol{c}, \boldsymbol{g})$ is optimal iff there exists an integer $1 \leq \psi \leq J+1$ such that all of the following conditions are satisfied:*

$$g_j = g, \ \forall \ j \tag{2.4}$$

$$Pr(D > c_j) = \tfrac{p_j - q_j}{e_j}, \ \forall \ j \geq \psi \tag{2.5}$$

$$c_j = g, \ \forall \ j < \psi \tag{2.6}$$

$$Pr(D > g) = \frac{\sum_{j=1}^{\psi-1} p_j + \sum_{j=\psi}^{J} q_j}{r - \sum_{j=\psi}^{J} e_j} \tag{2.7}$$

$$\frac{p_\psi - q_\psi}{e_\psi} > \frac{\sum_{j=1}^{\psi-1} p_j + \sum_{j=\psi}^{J} q_j}{r - \sum_{j=\phi}^{J} e_j} \ \textit{if } \psi \leq J \tag{2.8}$$

Proof: By Proposition 1, $\Pi(\boldsymbol{c}, \boldsymbol{g}, D)$ is concave in both $\boldsymbol{c}$ and $\boldsymbol{g}$. Therefore, the first order necessary conditions will also be sufficient conditions for optimality. If $r > \sum_j e_j$, for given capacity plan $(\boldsymbol{c}, \boldsymbol{g})$ and product demand d, the maximal profit is as follows:

$$\pi(\boldsymbol{c}, \boldsymbol{g}, d) = r\min\{d, \min_j\{g_j\}\} - \sum_{j=1}^{J} [\min\{d, g_j\} - \min\{d, c_j\}]\, e_j \tag{2.9}$$

We can show by contradiction that under the optimal planning strategy g_j is the same for all j. Thus, we let

$$g_j = g, \; \forall \; j,$$

and we can rewrite Problem (2.2) as

$$\begin{array}{ll} \arg\min\limits_{\boldsymbol{c},g} & -\mathbf{E}\left[r\min\{D,g\} - \sum_{j=1}^{J}(c_j p_j + (g - c_j)q_j) - \sum_{j=1}^{J}[\min\{D,g\} - \min\{D,c_j\}]\, e_j\right] \\ s.t. & c_j \leq g, \; \forall \; j \end{array}$$

Since the constraints are linearly independent, the lagrange multipliers exist. Then, we can consider its Lagrange function

$$\begin{aligned} L(g, \boldsymbol{c}, \mu) \;\; = \;\; & -\mathbf{E}_D\left[r\min\{D,g\} - \sum_{j=1}^{J}(c_j p_j + (g - c_j)q_j) - \right. \\ & \left. \sum_{j=1}^{J}[\min\{D,g\} - \min\{D,c_j\}]\, e_j\right] + \sum_{j=1}^{J}\mu_j(c_j - g) \end{aligned} \tag{2.10}$$

By the first order necessary conditions, we have

$$\frac{\partial L}{\partial c_j} = p_j - q_j - Pr(D > c_j)e_j + \mu_j = 0 \tag{2.11}$$

$$\frac{\partial L}{\partial g} = \sum_{j=1}^{J} q_j + Pr(D > g)\left(\sum_{j=1}^{J} e_j - r\right) - \sum_{j=1}^{J}\mu_j = 0 \tag{2.12}$$

From Equation (2.11) we have

$$Pr(D > c_j) = \frac{p_j - q_j + \mu_j}{e_j}, \; \forall \; j$$

Then we will have two cases:

Case 1: There exists a process i such that $c_i < g$. Define i to be the process with the smallest index such that $c_i < g$. If $c_i < g$ then, $\mu_i = 0$. Therefore,

$$Pr(D > c_i) = \frac{p_i - q_i}{e_i}$$

Now, assume that these exist a $j > i$ such that $c_j = g$. Since

$$\frac{p_j - q_j}{e_j} \geq \frac{p_i - q_i}{e_i}$$

and $\mu_j \geq 0$,

$$Pr(D > c_j) = \frac{p_j - q_j + \mu_j}{e_j} \geq \frac{p_i - q_i}{e_i} = Pr(D > c_i)$$

This implies $c_j \leq c_i$. However,

$$g = c_j \leq c_i < g,$$

and this is a contradiction. Therefore,

$$c_j \begin{cases} < g, & \text{if } j \geq i; \\ = g, & \text{if } j < i. \end{cases} \tag{2.13}$$

We note that the second part of Equation (2.13) follows because we chose i to be the smallest index such that $c_i < g$. Moreover, for all j such that $c_j < g$, $\mu_j = 0$ and

$$Pr(D > c_j) = \frac{p_j - q_j}{e_j}$$

If we let $\psi = i$, we have shown that conditions (2.5) and (2.6) hold. Since $\mu_j = 0$ for all $j \geq i$, from Equation (2.12) we have

$$\sum_{j=1}^{J} q_j + Pr(D > g)\left(\sum_{j=1}^{J} e_j - r\right) - \sum_{j=1}^{\psi-1} \mu_j = 0. \tag{2.14}$$

From Equation (2.11) and (2.13), we have

$$\mu_j = -p_j + q_j + Pr(D > g)e_j, \ \forall\, j < \psi. \tag{2.15}$$

Then by Equation (2.14) and (2.15), we can re-express Equation (2.12) as

$$\sum_{j=1}^{J} q_j + Pr(D > g)\left(\sum_{j=1}^{J} e_j - r\right) + \sum_{j=1}^{\psi-1} (p_j - q_j - Pr(D > g)e_j) = 0$$

Simplifying the equation above we get,

$$\sum_{j=1}^{\psi-1} p_j + \sum_{j=\psi}^{J} q_j + Pr(D > g)\left(\sum_{j=\psi}^{J} e_j - r\right) = 0$$

Therefore,

$$Pr(D > g) = \frac{\sum_{j=1}^{\psi-1} p_j + \sum_{j=\psi}^{J} q_j}{r - \sum_{j=\psi}^{J} e_j}$$

which is condition (2.7). Finally, since $c_i < g$ for all $i \geq \psi$, then

$$\frac{p_i - q_i}{e_i} = Pr(D > c_i) > Pr(D > g) = \frac{\sum_{j=1}^{\psi-1} p_j + \sum_{j=\psi}^{J} q_j}{r - \sum_{j=\phi}^{J} e_j}$$

which shows condition (2.8) holds for $i = \psi$.

Case 2: $c_j = g$ for all j. By Equation (2.11) and (2.12) we have

$$\sum_{j=1}^{J} q_j + Pr(D > g)\left(\sum_{j=1}^{J} e_j - r\right) + \sum_{j=1}^{J} (p_j - q_j - Pr(D > g)e_j) = 0.$$

This implies

$$Pr(D > g) = \frac{\sum_{j=1}^{J} p_j}{r}.$$

For this case, $\psi = J + 1$ and we can be verify that all of the conditions are satisfied.

Q.E.D.

From Proposition 2, we can make a number of observations. First we see that all processes will reserve the same amount of total capacity, given by g. Since producing the product requires all processes, reserving more capacity for some but not all processes is a waste as the excess capacity can never be used.

Second, we see that we can interpret the optimal planning strategy in terms of the newsboy problem. To determine g, suppose we know how to partition the resources based on whether or not they will buy an option contract. Namely, we assume for resources $1, \cdots, \psi - 1$, we only invest in a fixed-price contract, while for resources $\psi, \cdots, J$, we invest in both a fixed-price contract and an option contract. Then in a newsboy context, we can see that the cost of overage is given by

$$C_o = \sum_{j=1}^{\psi-1} p_j + \sum_{j=\psi}^{J} q_j,$$

which equals the upfront investment to reserve the last unit of capacity. This is the incremental cost when demand falls below g. The underage cost is

$$C_u = r - \sum_{j=1}^{\psi-1} p_j - \sum_{j=\psi}^{J} (q_j + e_j),$$

which equals the incremental revenue net of the costs for all of the resources. This is the lost profit when demand exceeds g. Thus the critical ratio for determining g is given by (2.7), namely the traditional critical ratio for the newsboy:

$$Pr(D > g) = \frac{C_o}{C_o + C_u} = \frac{\sum_{j=1}^{\psi-1} p_j + \sum_{j=\psi}^{J} q_j}{r - \sum_{j=\psi}^{J} e_j},$$

where we have assumed that we are given the partition of resources.

Now to get insight into how to construct the partition, we consider each resource independently. Suppose we were to buy both a fixed-price contract and an option contract for resource j, subject to the fact that the total capacity is fixed at g. We wish to determine how much to buy of the fixed-price contract. For resource j, the overage cost is

$$C_{o,j} = p_j - q_j$$

as this represents the upfront premium that is paid for fixed-price contract relative to an option contract, and equals the amount that would be lost if this capacity is not needed. The underage cost is

$$C_{u,j} = q_j + e_j - p_j$$

which is equal to the cost premium to serve demand from the option contract relative to the fixed-price contract. Thus, the critical ratio for determining the size of the fixed-price contract for resource j is given by:

$$Pr(D > c_j) = \frac{C_{o,j}}{C_{o,j} + C_{u,j}} = \frac{p_j - q_j}{e_j}$$

which corresponds to (2.5). If this equation suggests buying more than g units of capacity, then we should not buy an option contract for resource j and we should reduce its fixed-price contract to g. In effect, this is what is enforced by Equation (2.6) and (2.8). Finally sorting the resources, as prescribed by (2.3), provides a simple way to find the partition.

Also, we see from condition (2.5) that for those processes that do buy an option contract, the optimal fixed-price capacity is independent of r, the price of the product. We also observe that from condition (2.7) that the optimal total capacity is independent of the prices of the fixed-price contract for the resources for which we buy option contracts.

For each process, the optimal strategy has a similar structure to that given by Martinez-de-Albeniz and Simchi-Levi in [27]. They study the replenishment policy and portfolio selection strategy for a single product that has a single process supply chain in the presence of a spot market. In their model, there are multiple option contracts available for the single process. For a single period model, they give a closed-form solution to the portfolio selection (capacity investment) problem. Our result of the optimal level of fixed-price capacity is similar to the result that they have for their single period problem. However, in our model, the manufacturer needs to acquire the capacity for multiple processes at the same time. Therefore, our results for the optimal level of total capacity differ from those in Martinez-de-Albeniz and Simchi-Levi, especially with regard to the partition property for separating the processes between those that use an option contract and those that do not.

A supply chain with a single product and dedicated resources is a very important case. Proposition 2 provides a closed-form solution for the optimal capacity planning strategy for this class of supply chain. This proposition not only reveals some interesting insights of the optimal strategy but also provides an effective way to find the optimal strategy. Moreover, it will also help us to develop an upper bound for both the single period problem (in Chapter 3) and the multi-period problem (in Chapter 4).

We can also extend the special case to a more general setting, where each process can have multiple dedicated resources. For each process j, define $\mathcal{K}_j$ to be

$$\mathcal{K}_j = \{k|B(j,(j,k)) = 1\}.$$

$\mathcal{K}_j$ is the set of resources that can provide capacity for process j. Note that for $k^1,\ k^2 \in \mathcal{K}_j$, if $p_{k^1} > p_{k^2}$, then the manufacturer will not reserve any fixed-price capacity from resource k^1. Therefore, we can assume that for each process, only one resource offers fixed-price capacity. We also note that for $k^1,\ k^2 \in \mathcal{K}_j$, if $e_{k^1} < e_{k^2}$ and $q_{k^1} \leq q_{k^2}$, then the manufacturer will not reserve any option capacity from resource k^2. Therefore, without loss of generality, we assume that for each process j,

$$e_{j,k^1} < e_{j,k^2} \text{ and } q_{j,k^1} > q_{j,k^2}, \text{ if } k^1 < k^2. \tag{2.16}$$

or equivalently

$$q_j(e_j) \text{ is a strictly decreasing function of } e_j.$$

Define $o_{j,k}, k = 1, \cdots, K_j$ to be the amount of option capacity reserved with resource k, which is associated with process j. Define $o_{j,0}$ to be the amount of fixed-price capacity reserved for the process j. Given $\boldsymbol{o}_j$, define

$$w_{j,k} = \sum_{i=0}^{k} o_{j,i}, k = 0, \cdots, K_j.$$

Set $w_{j,-1} = 0$ and $e_{j,0} = 0$. Let g_j be the total capacity that the manufacturer reserves from the resources for process j. By the same argument given in this section, g_j^* are the same for all j. The optimal capacity plan is given as

$$\begin{aligned} \arg\min_{\boldsymbol{O},g} \quad & \Pi(\boldsymbol{o}, g) = -E\left[r\min\{D,g\} - \sum_{j=1}^{J}\sum_{k=0}^{K_j} e_{j,k}\left[\min\{D - w_{j,k-1}, o_{j,k}\right]^+ - \sum_{j=1}^{J}\sum_{k=0}^{K_j} q_{j,k}o_{j,k}\right] \\ s.t. \quad & w_{j,K_j} \le g, j = 1, \cdots, J, \\ & w_{j,k-1} \le w_{j,k}, k = 1, \cdots, K_j, \forall\, j. \end{aligned} \tag{2.17}$$

Note that $\min\{D - w_{j,k-1}, o_{j,k}\} = \min\{D - w_{j,k-1}, w_{j,k} - w_{j,k-1}\} = \min\{D, w_{j,k}\} - w_{j,k-1}$. Therefore, we can rewrite Equation (2.17) as

$$\begin{aligned} \arg\min_{\boldsymbol{O},g} \quad & \Pi(\boldsymbol{o}, g) = -E\left[r\min\{D,g\} - \sum_{j=1}^{J}\sum_{k=0}^{K_j} e_{j,k}\left[\min\{D, w_{j,k}\} - w_{j,k-1}\right]^+ \right. \\ & \left. - \sum_{j=1}^{J}\sum_{k=0}^{K_j} q_{j,k}(w_{j,k} - w_{j,k-1})\right] \\ s.t. \quad & w_{j,K_j} \le g, j = 1, \cdots, J, \\ & w_{j,k-1} \le w_{j,k}, k = 1, \cdots, K_j, \forall\, j. \end{aligned} \tag{2.18}$$

The first question that we study is which resources the manufacturer should use. Let us fix all the decision variables except $w_{j,k}$, then

$$\frac{\partial \Pi(\boldsymbol{o}, g)}{\partial w_{j,k}} = q_{j,k+1} - q_{j,k} + Pr(D > w_{j,k})(e_{j,k+1} - e_{j,k}).$$

By setting $\partial \Pi(\boldsymbol{o}, g)/\partial w_{j,k}$ to 0, we get

$$Pr(D > w_{j,k}) = \frac{q_{j,k} - q_{j,k+1}}{e_{j,k+1} - e_{j,k}}. \tag{2.19}$$

Equation (2.19) specifies a threshold, $\xi_{j,k}$, between using option contract k and $k+1$.

For the demand below the threshold, it is better to use option contract k; for the demand above the threshold, it is better to use option contract $k+1$. The constraint $w_{j,k-1} \le w_{j,k}, k = 1, \cdots, K_j, \forall\, j$ in Equation (2.18) requires $\xi_{j,k-1} \le \xi_{j,k}$. By Equation (2.19), this requires that $q_j(e_j)$ is convex in e_j for each process j. If $q_j(e_j)$ is not convex in e_j, then the manufacturer will not use all the option contracts in the optimal capacity plan. To determine which option contracts the manufacturer should use, we have the following proposition.

Proposition 3 *Let $Q_j(e_j)$ be the convex envelop of $q_j(e_j)$, if the point $(e_{j,k}, q_{j,k}) \notin Q_j$, then $o^*_{j,k} = 0$.*

Proof: The proof follows directly from the definition of convex envelop. If $(e_{j,k}, q_{j,k}) \notin Q_j$, then we can construct a new option contract $(\bar{e}_{j,k}, \bar{q}_{j,k})$ that is a linear combination of option contracts $k-1$ and $k+1$ such that $\bar{e}_{j,k} = e_{j,k}$ and $\bar{q}_{j,k} < q_{j,k}$. Therefore, in the optimal capacity plan, the manufacturer will not use the option contract k and $o^*_{j,k} = 0$. **Q.E.D.**

Proposition 3 suggests an algorithm to find the optimal capacity plan given the total capacity g.

Algorithm 1:

Step 1: For each process j, rule out all the resources that is not on the convex envelop Q_j.

Step 2: For each process j, name the remaining resources according to Equation (2.16). Find the threshold $\xi_{j,k}$ for $k = 0, \cdots, K_j - 1$.

Step 3: For each process j, find the resource k_j such that $\xi_{j,k_j-1} < g \le \xi_{j,k_j}$. The optimal capacity plan given the total capacity g is

$$Pr(D > w^*_{j,i}(g)) = \frac{q_{j,i} - q_{j,i+1}}{e_{j,i+1} - e_{j,i}}, \text{ for } i = 0, \cdots, k_j - 1,$$

$$w^*_{j,i}(g) = g - w^*_{j,k_j-1}(g), \text{ for } i = k_j - 1, \cdots, K_j.$$

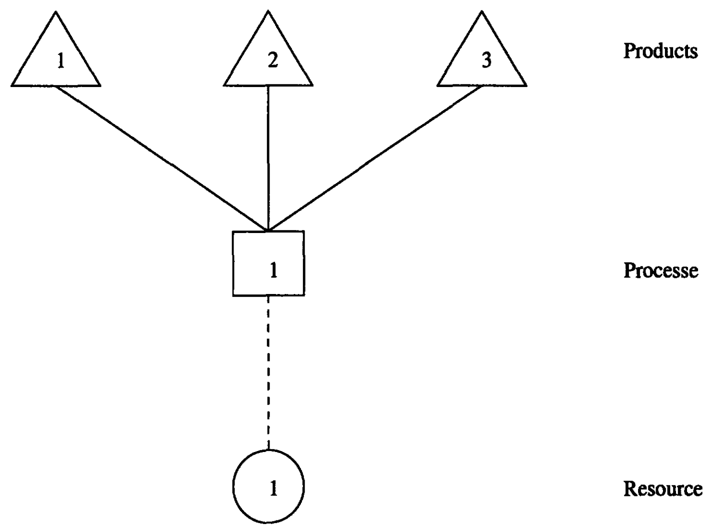

Figure 2-4: A supply chain network with single process and dedicated resource.

Since we can find the optimal capacity for given total capacity g using Algorithm 1, we can then apply convex search algorithms to find the optimal total capacity g^*.

2.2.2 Special Case II: Single Process with Dedicated Resource

We will now consider the second special case: a network with a single process and a single dedicated resource. An example of such a network is given in Figure 2-4. Without loss of generality, we assume $r_i \geq r_j$ if $i \leq j$. For this class of supply chains, we can also obtain a closed-form solution of the optimal capacity planning strategy, which is given in the following proposition:

Proposition 4 *Assume $\boldsymbol{D}$ is a continuous random vector and $r_i > e$ for all i. For a supply chain network with a single process and dedicated resource, a capacity planning strategy (c, g) is optimal iff it satisfies one of the following two sets of conditions:*

***Set 1*:**

$$Pr\left(\sum_{i=1}^{M} D_i > c\right) = \tfrac{p-q}{e} \tag{2.20}$$

$$q + ePr\left(\sum_{i=1}^{M} D_i > g\right) - \sum_{i=1}^{M}\left[r_i Pr\left(\sum_{j=1}^{i} D_j > g > \sum_{j=1}^{i-1} D_j\right)\right] = 0$$

$$ \tag{2.21} $$

$$c < g \tag{2.22}$$

***Set 2*:**

$$p - \sum_{i=1}^{M} \left[r_i Pr \left(\sum_{j=1}^{i} D_j > g > \sum_{j=1}^{i-1} D_j \right) \right] = 0 \tag{2.23}$$

$$c = g \tag{2.24}$$

Proof: By Proposition 1, it will be sufficient to show that one of the two sets of conditions is in fact the first order necessary condition for Problem (2.2). For any $(c, g, \boldsymbol{d})$, we can write

$$\begin{aligned} \pi(c, g, \boldsymbol{d}) &= \min\{d_1, g\} r_1 + \sum_{i=2}^{M} \left(\max\left\{0, g - \sum_{j=1}^{i-1} d_j \right\} r_i - \max\left\{0, g - \sum_{j=1}^{i} d_j \right\} r_i \right) \\ &\quad - \left[\min\left\{ \sum_{i=1}^{M} d_i, g \right\} - \min\left\{ \sum_{i=1}^{M} d_i, c \right\} \right] e. \end{aligned}$$

Therefore, we can write Problem (2.2) as follows

$$\begin{aligned} \arg\min_{c,g} \quad & -\mathbf{E}[\pi(c, g, \boldsymbol{D})] + pc + q(g - c) \\ \text{s.t.} \quad & c \leq g. \end{aligned}$$

Since lagrange multiplier exists, we can write the lagrange function

$$L(c, g, \boldsymbol{D}) = -\mathbf{E}[\pi(c, g, D)] + pc + q(g - c) + \mu(c - g).$$

From the first order necessary condition, we get

$$\frac{\partial L}{\partial c} = p - q - ePr \left(c < \sum_{i=1}^{M} D_i \right) + \mu = 0 \tag{2.25}$$

and

$$\frac{\partial L}{\partial g} = -r_1 Pr(g < D_1) + \sum_{i=2}^{M} \left(r_i \left(-Pr \left(g > \sum_{j=1}^{i-1} D_j \right) + Pr \left(g > \sum_{j=1}^{i} D_j \right) \right) \right) + q + ePr \left(g < \sum_{i=1}^{M} D_i \right) - \mu = 0 \quad (2.26)$$

Case 1: If $c < g$, then $\mu = 0$. Then, Equation (2.25) implies condition (2.20) and Equation (2.26) implies condition (2.21). Therefore, the first order necessary condition is equivalent to the first set of conditions.

Case 2: If $c = g$, then from Equation (2.25) and (2.26) we have

$$-r_1 Pr(g < D_1) + \sum_{i=2}^{M} \left(r_i \left(-Pr \left(g > \sum_{j=1}^{i-1} D_j \right) + Pr \left(g > \sum_{j=1}^{i} D_j \right) \right) \right) + q + ePr \left(g < \sum_{i=1}^{M} D_i \right) + p - q - ePr \left(g < \sum_{i=1}^{M} D_i \right) = 0$$

Simplifying the equation above will get condition (2.23). Therefore, the first order necessary condition is equivalent to the second set of conditions.

Q.E.D.

The first (second) set of conditions is the necessary and sufficient conditions for an optimal planning strategy with (without) the purchase of an option contract. If it is better for the manufacturer to reserve a positive amount of option capacity, the optimal policy will have similar structure to that found in Proposition 2 for the process using an option contract. Equation (2.20) is the same as for Proposition 2. For Equation (2.21), the first two terms are the incremental cost for increasing the size of the option contract; the third term is the incremental revenue from increasing the size of the option contract. The first order condition just equates the incremental cost with the incremental benefit, under the assumption that we use the option contract. Similarly, if it is better not to use an option contract, the incremental cost (revenue) for increasing the size of the fixed-price contract is given in the first (second) term of Equation (2.23). Equating them gives us the first order condition. Therefore, to find the optimal level of total capacity, we need to solve Equation (2.21) or Equation (2.23) depending on whether it is better for the manufacturer to reserve option capacity or not.

By examining the optimal strategy given in Proposition 4, we see that if it is optimal for the firm to reserve option capacity, the optimal fixed-price capacity is independent of the prices of the products and the optimal total capacity is independent of the price of fixed-price capacity. We have observed a similar property for the class of a supply chain with single product and dedicated resources. However, as we will discuss in Section 2.4, these properties are not true in a general supply chain network.

Finally, as we will discuss in Section 4.2, the multi-period extension of this special case will be an important component of the algorithm for solving the multi-period capacity planning problem.

2.3 Solving the Single Period Capacity Planning Problem

Unlike the special cases we have studied in the previous sections, it is very difficult to derive a closed-form solution for the optimal capacity planning strategy in a general supply chain setting. Therefore, in this section, we will study different algorithms for solving the general single period capacity planning problem (2.2) and compare their performances.

2.3.1 Sampling

Through the rest of this paper, we will use sampling to model demand uncertainty. Given any probability or empirical distribution of the demand, we randomly draw a set of demand realizations and denote this set by $\mathcal{S}$. In effect, we will model the given demand distribution by the sample; that is, we assume demand comes from a discrete distribution defined on the sample space, where each sample point is equally likely to occur. Let L be the size of the sample set. In this section, we will give some guidelines for picking the number L.

Let us assume that we have selected a set of demand samples $\mathcal{S}$ with size L. Denote $\pi_L(\boldsymbol{c}_L, \boldsymbol{g}_L, \boldsymbol{d}_L)$ to be the maximum objective function value of Problem (2.1) by replacing the expectation over the original demand distribution with the average over the L sample points and $\Pi_L(\boldsymbol{c}_L^*, \boldsymbol{g}_L^*)$ be the corresponding maximum objective function value of Problem (2.2). We would like to find a bound on the probability that $|\Pi(\boldsymbol{c}^*, \boldsymbol{g}^*) - \Pi_L(\boldsymbol{c}_L^*, \boldsymbol{g}_L^*)|$ is smaller than a positive scalar ϵ. We will give two bounds based on two different inequalities: Hoeffding inequality [20] and Chernoff inequality [41].

Bound Based on Hoeffding Inequality

We assume that for any given $\boldsymbol{c}$, $\boldsymbol{g}$, and $\boldsymbol{d}$, we can identify a lower and upper bound on the expected net revenue,

$$\pi_{min} \leq \pi(\boldsymbol{c}, \boldsymbol{g}, \boldsymbol{d}) \leq \pi_{max}.$$

In practice, π_{min} and π_{max} can be the minimum and maximum profit that the manufacturer can gain. By Hoeffding Inequality, we have

$$Pr(|\pi(\boldsymbol{c}, \boldsymbol{g}, \boldsymbol{D}) - \pi_L(\boldsymbol{c}, \boldsymbol{g}, \boldsymbol{D})| \geq \epsilon) \leq 2\exp\left(\frac{-L\epsilon^2}{2(\pi_{max} - \pi_{min})^2}\right), \quad \epsilon > 0. \tag{2.27}$$

Therefore,

$$Pr(|\pi(\boldsymbol{c}_L^*, \boldsymbol{g}_L^*, \boldsymbol{D}) - \pi_L(\boldsymbol{c}_L^*, \boldsymbol{g}_L^*, \boldsymbol{D})| \geq \epsilon) \leq 2\exp\left(\frac{-L\epsilon^2}{2(\pi_{max} - \pi_{min})^2}\right), \quad \epsilon > 0 \tag{2.28}$$

and

$$Pr(|\pi(\boldsymbol{c}^*, \boldsymbol{g}^*, \boldsymbol{D}) - \pi_L(\boldsymbol{c}^*, \boldsymbol{g}^*, \boldsymbol{D})| \geq \epsilon) \leq 2\exp\left(\frac{-L\epsilon^2}{2(\pi_{max} - \pi_{min})^2}\right), \quad \epsilon > 0. \tag{2.29}$$

Since $(\boldsymbol{c}^*, \boldsymbol{g}^*)$ is the optimal solution of problem Π and $(\boldsymbol{c}_L^*, \boldsymbol{g}_L^*)$ is the optimal solution of problem Π_L, we have

$$\Pi(\boldsymbol{c}^*, \boldsymbol{g}^*) \geq \Pi(\boldsymbol{c}_L^*, \boldsymbol{g}_L^*) \tag{2.30}$$

and

$$\Pi_L(\boldsymbol{c}_L^*, \boldsymbol{g}_L^*) \geq \Pi_L(\boldsymbol{c}^*, \boldsymbol{g}^*). \tag{2.31}$$

From Equation (2.31), we have

$$\Pi(\boldsymbol{c}^*, \boldsymbol{g}^*) - \Pi_L(\boldsymbol{c}_L^*, \boldsymbol{g}_L^*) \leq \Pi(\boldsymbol{c}^*, \boldsymbol{g}^*) - \Pi_L(\boldsymbol{c}^*, \boldsymbol{g}^*). \tag{2.32}$$

Similarly, from Equation (2.30), we have

$$\Pi(\boldsymbol{c}^*, \boldsymbol{g}^*) - \Pi_L(\boldsymbol{c}_L^*, \boldsymbol{g}_L^*) \geq \Pi(\boldsymbol{c}_L^*, \boldsymbol{g}_L^*) - \Pi_L(\boldsymbol{c}_L^*, \boldsymbol{g}_L^*). \tag{2.33}$$

By Eqs. (2.28), (2.29), (2.32) and (2.33), we get

$$\begin{aligned}
& Pr(|\Pi(\boldsymbol{c}^*, \boldsymbol{g}^*) - \Pi_L(\boldsymbol{c}_L^*, \boldsymbol{g}_L^*)| \leq \epsilon) \\
\geq\; & Pr(|\Pi(\boldsymbol{c}^*, \boldsymbol{g}^*) - \Pi_L(\boldsymbol{c}^*, \boldsymbol{g}^*)| \leq \epsilon \text{ AND } |\Pi(\boldsymbol{c}_L^*, \boldsymbol{g}_L^*) - \Pi_L(\boldsymbol{c}_L^*, \boldsymbol{g}_L^*)| \leq \epsilon) \\
=\; & Pr(|\Pi(\boldsymbol{c}^*, \boldsymbol{g}^*) - \Pi_L(\boldsymbol{c}^*, \boldsymbol{g}^*)| \leq \epsilon) + Pr(|\Pi(\boldsymbol{c}_L^*, \boldsymbol{g}_L^*) - \Pi_L(\boldsymbol{c}_L^*, \boldsymbol{g}_L^*)| \leq \epsilon) \\
& -Pr(|\Pi(\boldsymbol{c}^*, \boldsymbol{g}^*) - \Pi_L(\boldsymbol{c}^*, \boldsymbol{g}^*)| \leq \epsilon \text{ OR } |\Pi(\boldsymbol{c}_L^*, \boldsymbol{g}_L^*) - \Pi_L(\boldsymbol{c}_L^*, \boldsymbol{g}_L^*)| \leq \epsilon) \\
\geq\; & Pr(|\Pi(\boldsymbol{c}^*, \boldsymbol{g}^*) - \Pi_L(\boldsymbol{c}^*, \boldsymbol{g}^*)| \leq \epsilon) + Pr(|\Pi(\boldsymbol{c}_L^*, \boldsymbol{g}_L^*) - \Pi_L(\boldsymbol{c}_L^*, \boldsymbol{g}_L^*)| \leq \epsilon) - 1 \\
\geq\; & 2\left(1 - 2\exp\left(\frac{-L\epsilon^2}{2(\pi_{max} - \pi_{min})^2}\right)\right) - 1 \\
=\; & 1 - 4\exp\left(\frac{-L\epsilon^2}{2(\pi_{max} - \pi_{min})^2}\right)
\end{aligned} \tag{2.34}$$

Equation (2.34) suggests a guideline to pick a suitable sample size given the knowledge of the bound on the expected profit. For example, let's assume that the difference between maximum profit and minimum profit is \$35000. If the manufacturer uses 500 samples, the probability of having a sampling error that is greater than \$5000 is at most 0.0243.

Bound Based on Chernoff Inequality

If the manufacturer has an estimate of the maximum standard deviation, σ_π, of the expected profit, $\pi(\boldsymbol{c}, \boldsymbol{g}, \boldsymbol{D})$, and knows that the expected profit is bounded, it can bound the quantity $|\Pi(\boldsymbol{c}^*, \boldsymbol{g}^*) - \Pi_L(\boldsymbol{c}_L^*, \boldsymbol{g}_L^*)|$ using the Chernoff inequality.

By Chernoff inequality, we have

$$Pr(|\pi(\boldsymbol{c}_L^*, \boldsymbol{g}_L^*, \boldsymbol{D}) - \pi_L(\boldsymbol{c}_L^*, \boldsymbol{g}_L^*, \boldsymbol{D})| \geq \epsilon) \leq 2\exp\left(\frac{-L\epsilon^2}{4\sigma^2}\right), \quad \epsilon > 0$$

and

$$Pr(|\pi(\boldsymbol{c}^*, \boldsymbol{g}^*, \boldsymbol{D}) - \pi_L(\boldsymbol{c}^*, \boldsymbol{g}^*, \boldsymbol{D})| \geq \epsilon) \leq 2\exp\left(\frac{-L\epsilon^2}{4\sigma^2}\right), \quad \epsilon > 0.$$

Following a similar argument given above, we have

$$Pr(|\Pi(\boldsymbol{c}^*, \boldsymbol{g}^*) - \Pi_L(\boldsymbol{c}_L^*, \boldsymbol{g}_L^*)| \leq \epsilon) \geq 1 - 4\exp\left(\frac{-L\epsilon^2}{4\sigma^2}\right). \tag{2.35}$$

As an example, if the standard deviation of the expected profit is \$25000 and the manufacturer uses 500 samples, the probability of having a sampling error that is greater than \$5000 is at most 0.027.

Finally, because we use sampling to model the uncertainty, we do not make any assumption on the distribution of demand. In practice, the manufacturer can generate the demand samples from some probability distributions or from the demand history.

2.3.2 Linear Program Model

We now present and discuss the first algorithm to solve the general single period capacity planning problem. We can view the two-stage problem given by (2.1) and (2.2) as a stochastic optimization with recourse, and thus express it as one big deterministic linear program and solve it with standard linear program methods. Given the sample set $\mathcal{S}$, we can formulate the equivalent deterministic linear program of Problem (2.2) by substituting (2.1) into (2.2) and replacing expectation with the average of samples:

$$\begin{aligned} \max_{\boldsymbol{c},\boldsymbol{g}} \quad & \frac{1}{L}\sum_{\boldsymbol{d}\in\mathcal{S}} (\boldsymbol{r}'\boldsymbol{z_d} - \boldsymbol{e}'H\boldsymbol{y_d}) - \boldsymbol{p}'\boldsymbol{c} - \boldsymbol{q}'(\boldsymbol{g} - \boldsymbol{c}) \qquad (2.36) \\ s.t. \quad & \boldsymbol{z_d} \leq \boldsymbol{d}, \ \forall \ \boldsymbol{d} \in \mathcal{S} \\ & A\boldsymbol{z_d} \leq B(\boldsymbol{x_d} + \boldsymbol{y_d}), \ \forall \ \boldsymbol{d} \in \mathcal{S} \\ & H\boldsymbol{x_d} \leq \boldsymbol{c}, \ \forall \ \boldsymbol{d} \in \mathcal{S} \end{aligned}$$

$$H(\boldsymbol{x}_d + \boldsymbol{y}_d) \leq \boldsymbol{g}, \ \forall \ \boldsymbol{d} \in \mathcal{S}$$

$$\boldsymbol{c} \leq \boldsymbol{g}$$

$$\boldsymbol{x}_d, \boldsymbol{y}_d, \boldsymbol{z}_d \geq \boldsymbol{0}, \ \forall \ \boldsymbol{d} \in \mathcal{S}$$

$$\boldsymbol{c}, \boldsymbol{g} \geq \boldsymbol{0}$$

where $\boldsymbol{d}$ denotes a demand realization in the sample set $\mathcal{S}$ and the subscript indicates which demand realization that the production level decision variables are associated with.

The size of Problem (2.36) can be very large for a moderate size supply chain. Let N be the number of links between processes and resources. For any demand realization d, Problem (2.1) has $M + 2N$ variables and $M + J + 2K$ constraints. Therefore, the equivalent deterministic linear program (2.36) will have $L(M+2N)+2K$ variables and $L(M+J+2K)+K$ constraints. Consider a supply chain with $M = 10$ products, $J = 20$ processes, $K = 30$ resources, and $N = 40$ links. If the sample size is 500, then Problem (2.36) has 45,060 variables and 45,030 constraints. When the size of the problem is large, the run time of this algorithm is very slow. We will discuss its run time performance in Section 2.3.7. Moreover, as we increase the complexity of the supply chain structure and the size of the sample space, Problem (2.36) grows exponentially. Therefore, we need to develop other more efficient algorithms to solve the problem.

2.3.3 Sub-gradient Method

Van Meighem and Rudi [31] suggest a sub-gradient algorithm to solve a different but similar single period capacity planning problem. In their model, the firm can only reserve fixed-price capacity but not option capacity. The main purpose of their paper is to study the properties of optimal planning strategies. They proved the necessary and sufficient conditions of the optimal solution and briefly mention that the problem can be solved using a sub-gradient algorithm. Since our single period problem is similar, we can develop a similar sub-gradient algorithm to our model.

We first consider the sub-gradients of Problem (2.2). For each demand realization $\boldsymbol{d}$, let $\lambda(\boldsymbol{c}, \boldsymbol{g}, \boldsymbol{d})$ be the associated dual variables of constraints $H\boldsymbol{x} \leq \boldsymbol{c}$ and $\gamma(\boldsymbol{c}, \boldsymbol{g}, \boldsymbol{d})$ be the associated dual variables of constraints $H(\boldsymbol{x} + \boldsymbol{y}) \leq \boldsymbol{g}$ in Problem (2.1). Then the sub-gradients of the objective function of Problem (2.2) are

$$\nabla_{\boldsymbol{c}}\Pi = E[\lambda(\boldsymbol{c}, \boldsymbol{g}, \boldsymbol{D})] - \boldsymbol{p} + \boldsymbol{q}$$

and

$$\nabla_{\boldsymbol{g}}\Pi = E[\gamma(\boldsymbol{c}, \boldsymbol{g}, \boldsymbol{D})] - \boldsymbol{q}.$$

We omit the proof since it is very similar to the one given in [31]. By Proposition 1, $\Pi(\boldsymbol{c}, \boldsymbol{g}, \boldsymbol{D})$ is concave in $(\boldsymbol{c}, \boldsymbol{g})$. The first order conditions will also be the necessary conditions for optimality. Therefore, we can use a sub-gradient method to find the optimal solution. [31]

Sub-gradient Algorithm:

Step 0: Set $s = 0$. We start with a given initial feasible solution $(\boldsymbol{c}^0, \boldsymbol{g}^0)$.

Step 1: For capacity strategy $\boldsymbol{c}^s$ and $\boldsymbol{g}^s$, solve the linear program (2.1) and find the associated dual variables $\lambda(\boldsymbol{c}^s, \boldsymbol{g}^s, \boldsymbol{d})$ and $\gamma(\boldsymbol{c}^s, \boldsymbol{g}^s, \boldsymbol{d})$ numerically for each sample demand vector $\boldsymbol{d}$. Take the average of $\lambda(\boldsymbol{c}^s, \boldsymbol{g}^s, \boldsymbol{D})$ and $\gamma(\boldsymbol{c}^s, \boldsymbol{g}^s, \boldsymbol{D})$ over $\boldsymbol{D}$ as an unbiased estimate of $E\left[\lambda(\boldsymbol{c}^s, \boldsymbol{g}^s, \boldsymbol{D})\right]$ and $E\left[\gamma(\boldsymbol{c}^s, \boldsymbol{g}^s, \boldsymbol{D})\right]$, and use them to compute estimates of the sub-gradient $\nabla_{\boldsymbol{c}^s}\Pi$ and $\nabla_{\boldsymbol{g}^s}\Pi$.

Step 2: If $|\nabla_{\boldsymbol{c}^s}\Pi|$ and $\left|\nabla_{\boldsymbol{g}^s}\Pi\right|$ are smaller than some tolerance level, then stop. Otherwise, adjust capacity in the direction of the sub-gradients:

$$\boldsymbol{g}^{s+1} = \boldsymbol{g}^s + \xi \nabla_{\boldsymbol{g}^s}\Pi$$

and

$$\boldsymbol{c}^{s+1} = \min\left\{\boldsymbol{c}^s + \xi \nabla_{\boldsymbol{c}^s}\Pi, \boldsymbol{g}^{s+1}\right\}$$

where ξ is some step-size (or perform a line-search). Set $s = s + 1$ and return to step 1.

At each iteration, step 1 of the sub-gradient algorithm will solve L linear programs where L is the number of sample demand points that is used to estimate the sub-gradients. The computational requirements at each step can be very intensive depending upon the number of sample points. Therefore, if the sub-gradient method requires a large number of iterations to converge, the algorithm will take a long time to run. Unfortunately, the sub-gradient method can take a long time to converge, due to the following reasons:

1. *The convergence rate is constrained by the bottleneck processes.* To produce a product, the firm needs to plan the capacity of all processes for the product at the same time. If one of the processes is short of capacity, the production is constrained by the bottleneck process, which dictates the sub-gradient. Consider the following example: The firm produces a single product that requires two types of processes a and b. Resource 1 can provide fixed-price capacity to process a at a cost of 5 per unit and resource 2 can provide fixed-price capacity to process b at a cost of 4 per unit. The demand for the product follows a uniform distribution between 100 and 120. The price for the product is 12 per unit. The optimal capacity strategy will be $100 \leq c_1 = c_2 \leq 120$ for some value of $c_1 = c_2$. Now, suppose we start with initial point $c_1 = 10$ and $c_2 = 11$. Since $c_1 < c_2 < 100$, $\nabla_{c_1}\Pi = 12 - 5 = 7$ and $\nabla_{c_2}\Pi = 0 - 4 = -4$. The sub-gradient algorithm will adjust the capacity as follows:

$$[c_{1,new}, c_{2,new}] = [c_{1,old}, c_{2,old}] + \xi[7, -4]$$

   We also observe that when $c_2 < c_1 < 100$, the sign of the sub-gradient is reversed. Thus, depending upon how we set the step size, the sub-gradient algorithm can take a long time to converge as it will cycle back and forth between these two sub-gradients.

2. *The convergence rate is constrained by the non-uniqueness of the sub-gradient.* In a typical capacity planning problem, the number of processes is larger than the number of products and the number of resources is larger than the number

of processes. Therefore, for some capacity planning strategies $(\boldsymbol{c}, \boldsymbol{g})$ and demand $\boldsymbol{d}$, the solution of the dual problem of (2.1) is not unique. Therefore, the sub-gradient at some capacity strategies $(\boldsymbol{c}, \boldsymbol{g})$ is not unique. Following different sub-gradients will have very different convergence rates.

3. *The convergence rate depends heavily on the starting point.*

4. *The convergence rate depends heavily on the step size.* [17]

5. *Lack of good termination criterion.* [17]

Reason 3, 4, and 5 have been shown to be true in many different problem contexts. [17]. Even though the sub-gradient method might not be suitable for some instances of our problem especially when the structure of the supply chain is complicated, it gives an important insight of the problem: after evaluating the function $\Pi(\boldsymbol{c}, \boldsymbol{g}, \boldsymbol{D})$, we can get the sub-gradients of $\Pi(\boldsymbol{c}, \boldsymbol{g}, \boldsymbol{D})$ with small extra computational effort. This is because Problem (2.1) is a linear problem, therefore, the dual variables λ and γ of the problem are immediately available after we solve the problem [7]. Based on this observation, we suggest the following algorithms as possible improvement on the sub-gradient method.

2.3.4 Regular Supporting Hyperplane Algorithm

Another type of algorithm that uses the sub-gradient is the Supporting Hyperplane Algorithm suggested by Veinott [36]. Let us consider a new problem:

$$\begin{aligned} \min \quad & f \\ s.t. \quad & f + E[\pi(\boldsymbol{c}, \boldsymbol{g}, \boldsymbol{D})] - \boldsymbol{p}'\boldsymbol{c} - \boldsymbol{q}'(\boldsymbol{g} - \boldsymbol{c}) \geq \boldsymbol{0} \\ & \boldsymbol{c} \leq \boldsymbol{g} \end{aligned} \tag{2.37}$$

We can show that $(\boldsymbol{c}^*, \boldsymbol{g}^*)$ solves Problem (2.2) iff $(\boldsymbol{c}^*, \boldsymbol{g}^*, f^*)$ solves Problem (2.37) with

$$f^* + E[\pi(\boldsymbol{c}^*, \boldsymbol{g}^*, \boldsymbol{D})] - \boldsymbol{p}'\boldsymbol{c}^* - \boldsymbol{q}'(\boldsymbol{g}^* - \boldsymbol{c}^*) = \boldsymbol{0}.$$

The supporting hyperplane algorithm can be used to solve Problem (2.37).

We assume we can identify upper and lower bounds on f, $\boldsymbol{c}$, and $\boldsymbol{g}$. Let $\boldsymbol{c}_{upper}(\boldsymbol{c}_{lower})$ and $\boldsymbol{g}_{upper}(\boldsymbol{g}_{lower})$ be the upper (lower) bounds on the fixed-price and total capacities. Let $f_{upper}(f_{lower})$ be the upper (lower) bound of f. Let

$$V^0 = \left\{(\boldsymbol{c}, \boldsymbol{g}, f) : \boldsymbol{c} \in [\boldsymbol{c}_{lower}, \boldsymbol{c}_{upper}], \boldsymbol{g} \in [\boldsymbol{g}_{lower}, \boldsymbol{g}_{upper}], f \in [f_{lower}, f_{upper}]\right\}$$

Let $s = 0$, the algorithm consists of the following steps:

Regular Supporting Hyperplane Algorithm:

Step 1: Solve the linear program of minimizing f, subject to $(\boldsymbol{c}, \boldsymbol{g}, f) \in V^s$, and let $(\boldsymbol{c}^s, \boldsymbol{g}^s, f^s)$ be the optimal solution. If

$$f^s + E[\pi(\boldsymbol{c}^s, \boldsymbol{g}^s, \boldsymbol{D})] - \boldsymbol{p}'\boldsymbol{c}^s - \boldsymbol{q}'(\boldsymbol{g}^s - \boldsymbol{c}^s) \geq -\varepsilon.$$

where ε is a small positive number chosen by the user, stop. Otherwise, go to step 2.

Step 2: Use the simulation method given in the sub-gradient algorithm to calculate the sub-gradient $\nabla_{\boldsymbol{c}^s}\Pi$ and $\nabla_{\boldsymbol{g}^s}\Pi$. Add a linear constraint to the set V^s:

$$f + \Pi(\boldsymbol{c}^s, \boldsymbol{g}^s, \boldsymbol{D}) + [(\boldsymbol{c}, \boldsymbol{g}) - (\boldsymbol{c}^s, \boldsymbol{g}^s)]'(\nabla_{\boldsymbol{c}^s}\Pi, \nabla_{\boldsymbol{g}^s}\Pi) \geq 0 \qquad (2.38)$$

where $\Pi(\boldsymbol{c}^s, \boldsymbol{g}^s, \boldsymbol{D})$ is a constant, which equals $E[\pi(\boldsymbol{c}^s, \boldsymbol{g}^s, \boldsymbol{D})] - \boldsymbol{p}'\boldsymbol{c}^s - \boldsymbol{q}'(\boldsymbol{g}^s - \boldsymbol{c}^s)$. Let the new set be V^{s+1}. Set $s = s + 1$ and go to step 1.

Geometrically, the supporting hyperplane method approximates the function $\Pi(\boldsymbol{c}, \boldsymbol{g}, \boldsymbol{D})$ with hyperplanes. To construct the initial constraint set V^0, one can set $\boldsymbol{c}_{lower}$ and $\boldsymbol{g}_{lower}$ to be $\boldsymbol{0}$, $\boldsymbol{c}_{upper}$ and $\boldsymbol{g}_{upper}$ to be maximal capacity requirement to fill all demand, f_{lower} to be the objective value of any feasible strategy, and f_{upper} to be the maximal profit that the firm can achieve. At each step, the algorithm adds a new supporting hyperplane to the constraint set, based on the sub-gradient from the last solution (supporting point). It then uses all the sub-gradients that it has calculated so far to find the next supporting point. Since all previous calculated supporting hyperplanes will be used, the algorithm overcomes problems 1 and 2 of the sub-gradient algorithm. By the nature of the supporting hyperplane algorithm, it does not require a starting point or a step size. Finally, at each step $-f^s$ is an upper bound for $\Pi(\boldsymbol{c}^*, \boldsymbol{g}^*, \boldsymbol{D})$. Therefore, the ε in the stopping criterion in step 1 is an upper bound for $|\Pi(\boldsymbol{c}^s, \boldsymbol{g}^s, \boldsymbol{D}) - \Pi(\boldsymbol{c}^*, \boldsymbol{g}^*, \boldsymbol{D})|$.

Even though we expect the supporting hyperplane algorithm in general to have a better convergence rate compared to sub-gradient method, it suffers from the problem of slow start. Note that at each iteration, the algorithm needs to solve L linear programs to find the supporting hyperplane where L is the number of samples. At the beginning, the supporting point is likely to be far away from the optimal solution. It might not be necessary to construct an accurate supporting hyperplane at points that are far away from the optimum using all samples, since these hyperplanes are only used to find an approximate location of the next supporting point. As the algorithm proceeds, the supporting points get closer and closer to optimum, and we need more accurate supporting hyperplanes. Since the regular supporting hyperplane algorithm uses all samples regardless of which stage the algorithm is in, it wastes computational power at the beginning and therefore has a slow start problem. To overcome this problem, we can adapt a variation of the regular supporting hyperplane algorithm from large-scale stochastic linear programming to solve Problem (2.2) [17][18]. We will describe this algorithm in the next section.

2.3.5 Stochastic Supporting Hyperplane Algorithm

To address the slow start problem of the regular supporting hyperplane algorithm, we will adapt the technique suggested by Higle and Sen in [17] and [18]. In their algorithms, instead of using all samples at each step, they incrementally increase the number of sample points at each iteration.

Stochastic Supporting Hyperplane Algorithm:

Step 0: Set up the initial V^0 as for the regular supporting hyperplane algorithm.

Set $s = 0$ and the initial demand sample set $\mathcal{S}^0 = \emptyset$.

Step 1: Set $s = s + 1$. Randomly generate a demand observation ω^s independent of any previously generated observations. Let $\mathcal{S}^s = \mathcal{S}^{s-1} \bigcup \omega^s$. Construct the s^{th} supporting hyperplane using the same method given in the step 2 of the regular supporting hyperplane algorithm. Define the s^{th} supporting hyperplane at s^{th} iteration to be:

$$f + \alpha_s^s + (\beta_s^s)'\boldsymbol{c} + (\zeta_s^s)'\boldsymbol{g} \geq 0$$

where $\alpha_s^s = \Pi(\boldsymbol{c}^s, \boldsymbol{g}^s, \boldsymbol{D}) - (\boldsymbol{c}^s, \boldsymbol{g}^s)'(\nabla_{\boldsymbol{c}^s}\Pi, \nabla_{\boldsymbol{g}^s}\Pi)$, $\beta_s^s = \frac{1}{s}\sum_{\boldsymbol{d} \in \mathcal{S}^s} \lambda(\boldsymbol{c}^s, \boldsymbol{g}^s, \boldsymbol{d}) - \boldsymbol{p} + \boldsymbol{g}$, and $\zeta_s^s = \frac{1}{s}\sum_{\boldsymbol{d} \in \mathcal{S}^s} \gamma(\boldsymbol{c}^s, \boldsymbol{g}^s, \boldsymbol{d}) - \boldsymbol{q}$.

Step 2: Update the coefficients of all previously generated supporting hyperplane:

$$\alpha_t^s = \frac{s-1}{s}\alpha_t^{s-1} + \frac{1}{s}U,\ \beta_t^s = \frac{s-1}{s}\beta_t^{s-1} - \frac{\boldsymbol{p}}{s} + \frac{\boldsymbol{q}}{s},\ \zeta_t^s = \frac{s-1}{s}\zeta_t^{s-1} - \frac{\boldsymbol{q}}{s},$$

where U is an upper bound on $\pi(\boldsymbol{c}, \boldsymbol{g}, \boldsymbol{D})$; for $t = 1, \cdots, s-1$.

Step 3: Find the next supporting point using the same method given in the step 1 of regular supporting hyperplane algorithm. If the algorithm does not terminate, go to step 1.

We derive the update rules in step 2 in Appendix B. Note that the supporting hyperplane constructed at iteration s uses s samples. Therefore, the supporting hyperplanes from different iterations use different numbers of sample points. The updating rules in step 2 modify the previously generated supporting hyperplanes to incorporate this difference. For details of stochastic supporting hyperplane algorithm, such as its convergence property, please refer to [18]. The stochastic supporting hyperplane algorithm addresses the problem of slow start by incrementally increasing the size of the sample set by one at each iteration. Even though in general the algorithm will take more iterations to converge, the average computational effort required in each iteration is less than the regular supporting hyperplane algorithm. As a result, the performance of the algorithm increases significantly as we will see in Section 2.3.7. However, adding one demand sample at each step means that the algorithm needs to solve one more linear program for all future iterations. For our problem, the computational requirement increases very quickly as the number of iterations increases. Therefore, we have developed another algorithm based on the stochastic supporting hyperplane algorithm to solve Problem (2.2).

2.3.6 Stochastic Supporting Hyperplane Algorithm with Pre-solve Routine

The new algorithm contains two stages. We first choose a small subset of the sample set and use the regular supporting hyperplane method to construct an initial polyhedra V^0. We then use the stochastic hyperplane supporting algorithm to find the optimal solution. We now outline this algorithm:

Stochastic Supporting Hyperplane with Pre-solve Routine:

Stage I: Pick a subset $\bar{\mathcal{S}} \subset \mathcal{S}$, solve the problem with the regular supporting hyperplane algorithm described in Section 2.3.4. Let $\bar{V}$ be the final polyhedra of the master LP.

Stage II: Set $V^0 = \bar{V}$ and use the stochastic supporting hyperplane algorithm described in Section 2.3.5 to solve the capacity planning problem.

In stage I, the algorithm takes advantage of the fast convergence rate of regular supporting hyperplane algorithm but with a reduced computational requirement at each iteration by using a small sample size. We expect the solution from stage I to be close to the optimum. The algorithm then uses stochastic supporting hyperplane algorithm to refine the solution. Since the second stage problem starts with a good stating point and initial constraint set, we expect that the stochastic supporting hyperplane algorithm should converge faster compared to starting from scratch.

2.3.7 Algorithm Run Time Comparisons

After examining five different algorithms for solving the single period capacity planning problem, we now discuss their run time performances. We use a free linear program solver, GNU Linear Programming Kit (GLPK 4.11), for all of the test cases. This solver is slower than the commercial LP solver, CPLEX. However, the computational tests presented here show the relative performance comparison of the algorithms. The test machine is an IBM x40 laptop with a 1.29 GHz Intel Pentium M CPU and 760 MB of memory. All the tests were written in the C++ programming language and performed in a Windows XP environment.

We consider a supply chain with 15 products, 30 processes, and 30 resources. We generate random test cases according to the following rules. The demand of each product follows a normal distribution with mean uniformly distributed between 100 and 120 and standard deviation 10. The price of each product is uniformly distributed between 150 and 170. The price of fixed-price capacity, p_k, is uniformly distributed between 9 and 12. The cost of option capacity, q_k, is uniformly distributed between 1 and p_k. The exercise cost of option capacity is set to $p_k \times 1.1 - q_k$. A link joins a product and process with probability 0.2 (e.g. $Pr(A(j,m) = 1) = 0.2$) and a link joins a process and a resource with probability 0.2 (e.g. $Pr(B(j,(j,k)) = 1) = 0.2$). We write a routine to check whether the supply chain generated is connected or not. If not, we repeat the generation process until we have a connected supply chain. In each case, we set the sample size to be 500, and we use the same 500 sample demands for all algorithms.

We first test the linear program algorithm with a randomly generated test. The linear program has 182,560 variables and 197,530 constraints. It takes the algorithm 10 hours 36 minutes 12 seconds to find an optimal solution. As we will show later this run time is significantly slower than the other algorithms. For the sub-gradient algorithm, we have discussed its shortcomings in Section 2.3.3. Furthermore, Higle and Sen [18] show that the supporting hyperplance algorithm outperforms the sub-gradient algorithm for solving large scale stochastic linear problems. Therefore, we

Test Case	Regular	Stochastic	Pre-solve	Pre-solve/Stoc.	Pre-solve/Reg.
1	465	112	51	45.45%	10.97%
2	380	128	44	34.38%	11.58%
3	373	140	67	47.86%	17.96%
4	678	152	93	61.18%	13.72%
5	791	513	130	25.34%	16.43%
6	679	392	79	20.15%	11.63%
7	259	119	40	33.61%	15.44%
8	239	43	36	83.72%	15.06%
9	315	125	45	36.00%	14.29%
10	260	75	36	48.00%	13.85%
11	473	104	71	68.27%	15.01%
12	262	72	47	65.28%	17.94%
13	259	58	37	63.79%	14.29%
14	534	147	71	48.30%	13.30%
15	314	278	43	15.47%	13.69%
16	386	100	51	51.00%	13.21%
17	464	115	50	43.48%	10.78%
18	442	163	44	26.99%	9.95%
19	293	110	40	36.36%	13.65%
20	554	144	68	47.22%	12.27%
21	534	215	72	33.49%	13.48%
22	267	62	34	54.84%	12.73%
23	299	117	41	35.04%	13.71%
24	340	79	39	49.37%	11.47%
25	274	80	44	55.00%	16.06%
26	334	58	44	75.86%	13.17%
27	231	60	29	48.33%	12.55%
28	410	102	50	49.02%	12.20%
29	423	140	45	32.14%	10.64%
30	553	114	50	43.86%	9.04%
31	311	103	43	41.75%	13.83%
32	451	136	37	27.21%	8.20%
33	488	220	43	19.55%	8.81%
34	472	179	51	28.49%	10.81%
35	440	214	79	36.92%	17.95%
36	515	295	77	26.10%	14.95%
37	174	99	31	31.31%	17.82%
38	552	211	55	26.07%	9.96%
39	484	103	74	71.84%	15.29%
40	294	74	34	45.95%	11.56%

Table 2.5: Run time (in seconds) comparison of Regular Supporting Hyperplane algorithm, Stochastic Supporting Hyperplane algorithm, and Stochastic Supporting Hyperplane algorithm with Pre-solve Routine.

	Regular	Stochastic	Pre-solve	Pre-solve/Stoc.	Pre-solve/Reg.
Average	406.7	143.8	52.9	43.35%	13.23%
STD	138.5	93.2	20.1	16.14%	2.53%
Min	174.0	43.0	29.0	15.47%	8.20%
Max	794.0	513.0	130.0	83.72%	17.96%

Table 2.6: Run time comparison statistics.

will focus on comparing the performances of the three types of supporting hyperplane algorithms presented above.

For the three supporting hyperplane algorithms, we set the terminating error percentage to be less than 1%. We select a set of 100 samples randomly and use it in the stage I of the stochastic supporting hyperplane algorithm with pre-solve routine. For 40 randomly generated test cases, the results are given in Table 2.5.

We see that for all test cases, the algorithm using pre-solve routine has the best runtime. The statistics of the runtime comparisons are given in Table 2.6. The average runtime of the algorithm using pre-solve for these test cases is 13.23% of the average runtime of the regular supporting hyperplane algorithm and 43.35% of the average runtime of the stochastic supporting hyperplane algorithm. For the maximum improvement, the runtime of the algorithm with pre-solve routine is 8.20% of the runtime of the regular algorithm and 15.47% of the runtime of the stochastic algorithm.

2.4 Properties of Optimal Strategy

After examining the algorithms for solving the single period capacity planning problems, in this section we will study the properties of the optimal strategies.

2.4.1 Effects of Unit Profits and Unit Prices

We first study the effects of the unit profits and unit prices on the maximal profit that the manufacturer can obtain. We state the first result in the following proposition.

Proposition 5 *Let* $\Pi(\boldsymbol{c}^*, \boldsymbol{g}^*, \boldsymbol{D}, \boldsymbol{r}, \boldsymbol{p}, \boldsymbol{q}, \boldsymbol{e})$ *be the optimal total profit of capacity planning problem* $(\boldsymbol{D}, A, B, H, \boldsymbol{r}, \boldsymbol{p}, \boldsymbol{q}, \boldsymbol{e})$*, then the following statements are true:*

1. *If* $\widehat{\boldsymbol{r}} \geq \boldsymbol{r}$*, then* $\Pi(\widehat{\boldsymbol{c}}^*, \widehat{\boldsymbol{g}}^*, \boldsymbol{D}, \widehat{\boldsymbol{r}}, \boldsymbol{p}, \boldsymbol{q}, \boldsymbol{e}) \geq \Pi(\boldsymbol{c}^*, \boldsymbol{g}^*, \boldsymbol{D}, \boldsymbol{r}, \boldsymbol{p}, \boldsymbol{q}, \boldsymbol{e})$.
2. *If* $\widehat{\boldsymbol{p}} \geq \boldsymbol{p}$*, then* $\Pi(\widehat{\boldsymbol{c}}^*, \widehat{\boldsymbol{g}}^*, \boldsymbol{D}, \boldsymbol{r}, \widehat{\boldsymbol{p}}, \boldsymbol{q}, \boldsymbol{e}) \leq \Pi(\boldsymbol{c}^*, \boldsymbol{g}^*, \boldsymbol{D}, \boldsymbol{r}, \boldsymbol{p}, \boldsymbol{q}, \boldsymbol{e})$.
3. *If* $\widehat{\boldsymbol{q}} \geq \boldsymbol{q}$*, then* $\Pi(\widehat{\boldsymbol{c}}^*, \widehat{\boldsymbol{g}}^*, \boldsymbol{D}, \boldsymbol{r}, \boldsymbol{p}, \widehat{\boldsymbol{q}}, \boldsymbol{e}) \leq \Pi(\boldsymbol{c}^*, \boldsymbol{g}^*, \boldsymbol{D}, \boldsymbol{r}, \boldsymbol{p}, \boldsymbol{q}, \boldsymbol{e})$.
4. *If* $\widehat{\boldsymbol{e}} \geq \boldsymbol{e}$*, then* $\Pi(\widehat{\boldsymbol{c}}^*, \widehat{\boldsymbol{g}}^*, \boldsymbol{D}, \boldsymbol{r}, \boldsymbol{p}, \boldsymbol{q}, \widehat{\boldsymbol{e}}) \leq \Pi(\boldsymbol{c}^*, \boldsymbol{g}^*, \boldsymbol{D}, \boldsymbol{r}, \boldsymbol{p}, \boldsymbol{q}, \boldsymbol{e})$.

Proof: We will only show the proof of the first statement and the proofs of others are very similar. Let $(\boldsymbol{c}^*, \boldsymbol{g}^*)$ be the optimal capacity planning strategy for problem $(\boldsymbol{D}, A, B, H, \boldsymbol{r}, \boldsymbol{p}, \boldsymbol{q}, \boldsymbol{e})$. For a demand realization $\boldsymbol{d}$, let $(\boldsymbol{x}^*(\boldsymbol{d}), \boldsymbol{y}^*(\boldsymbol{d},), \boldsymbol{z}^*(\boldsymbol{d}))$ be an optimal production level of problem $(\boldsymbol{D}, A, B, H, \boldsymbol{r}, \boldsymbol{p}, \boldsymbol{q}, \boldsymbol{e})$ with optimal capacity planning strategy $(\boldsymbol{c}^*, \boldsymbol{g}^*)$. Clearly, $(\boldsymbol{c}^*, \boldsymbol{g}^*)$ is a feasible solution of problem $(\boldsymbol{D}, A, B, H, \widehat{\boldsymbol{r}}, \boldsymbol{p}, \boldsymbol{q}, \boldsymbol{e})$ and $(\boldsymbol{x}^*(\boldsymbol{d}), \boldsymbol{y}^*(\boldsymbol{d}), \boldsymbol{z}^*(\boldsymbol{d}))$ is a feasible production level of problem $(\boldsymbol{D}, A, B, H, \widehat{\boldsymbol{r}}, \boldsymbol{p}, \boldsymbol{q}, \boldsymbol{e})$ with capacity planning strategy $(\boldsymbol{c}^*, \boldsymbol{g}^*)$. For the same demand realization, $\boldsymbol{d}$, let $(\widehat{\boldsymbol{x}}^*(\boldsymbol{d}), \widehat{\boldsymbol{y}}^*(\boldsymbol{d},), \widehat{\boldsymbol{z}}^*(\boldsymbol{d}))$ be an optimal production level of problem $(\boldsymbol{D}, A, B, H, \widehat{\boldsymbol{r}}, \boldsymbol{p}, \boldsymbol{q}, \boldsymbol{e})$ with capacity planning strategy $(\boldsymbol{c}^*, \boldsymbol{g}^*)$. Since $\widehat{\boldsymbol{r}} \geq \boldsymbol{r}$,

$$
\begin{aligned}
\pi(\boldsymbol{c}^*, \boldsymbol{g}^*, \boldsymbol{d}, \widehat{\boldsymbol{x}}^*(\boldsymbol{d}), \widehat{\boldsymbol{y}}^*(\boldsymbol{d}), \widehat{\boldsymbol{z}}^*(\boldsymbol{d}), \widehat{\boldsymbol{r}}) &\geq \pi(\boldsymbol{c}^*, \boldsymbol{g}^*, \boldsymbol{d}, \boldsymbol{x}^*(\boldsymbol{d}), \boldsymbol{y}^*(\boldsymbol{d}), \boldsymbol{z}^*(\boldsymbol{d}), \widehat{\boldsymbol{r}}) \\
&\geq \pi(\boldsymbol{c}^*, \boldsymbol{g}^*, \boldsymbol{d}, \boldsymbol{x}^*(\boldsymbol{d}), \boldsymbol{y}^*(\boldsymbol{d}), \boldsymbol{z}^*(\boldsymbol{d}), \boldsymbol{r})
\end{aligned}
$$

Let $(\widehat{\boldsymbol{c}}^*, \widehat{\boldsymbol{g}}^*)$ be the optimal planning strategy for problem $(\boldsymbol{D}, A, B, H, \widehat{\boldsymbol{r}}, \boldsymbol{p}, \boldsymbol{q}, \boldsymbol{e})$. Then we have

$$
\begin{aligned}
\Pi(\widehat{\boldsymbol{c}}^*, \widehat{\boldsymbol{g}}^*, \boldsymbol{D}, \widehat{\boldsymbol{r}}, \boldsymbol{p}, \boldsymbol{q}, \boldsymbol{e}) &\geq \Pi(\boldsymbol{c}^*, \boldsymbol{g}^*, \boldsymbol{D}, \widehat{\boldsymbol{r}}, \boldsymbol{p}, \boldsymbol{q}, \boldsymbol{e}) \\
&= E[\pi(\boldsymbol{c}^*, \boldsymbol{g}^*, \boldsymbol{D}, \widehat{\boldsymbol{x}}^*, \widehat{\boldsymbol{y}}^*, \widehat{\boldsymbol{z}}^*, \widehat{\boldsymbol{r}})] - \boldsymbol{p}'\boldsymbol{c}^* - \boldsymbol{q}'(\boldsymbol{g}^* - \boldsymbol{c}^*) \\
&\geq E[\pi(\boldsymbol{c}^*, \boldsymbol{g}^*, \boldsymbol{D}, \boldsymbol{x}^*, \boldsymbol{y}^*, \boldsymbol{z}^*, \boldsymbol{r})] - \boldsymbol{p}'\boldsymbol{c}^* - \boldsymbol{q}'(\boldsymbol{g}^* - \boldsymbol{c}^*) \\
&= \Pi(\boldsymbol{c}^*, \boldsymbol{g}^*, \boldsymbol{D}, \boldsymbol{r}, \boldsymbol{p}, \boldsymbol{q}, \boldsymbol{e}).
\end{aligned}
$$

Q.E.D.

Proposition 5 shows the monotonic properties of the total profit on the unit profit and unit price. However, one assumption here is that when the unit profit and unit price change, the demands remain unchanged. This assumption might not hold in the reality. Therefore, if the demands change once the manufacturer adjusts the unit profit and unit price, the monotonic properties shown in Proposition 5 might not hold anymore.

Lemma 1 *For the following two types of supply chain:*

1. *a supply chain with a single product and dedicated resources and*

2. *a supply chain with a single process and dedicated resource,*

the following statements are true:

1. *For those resources that have an option contract in the optimal capacity plan, the optimal fixed-price capacity remains the same if the price of the product increases.*

2. *The optimal total capacity remains unchanged if the prices of the fixed-price contract for the resources having option contracts decrease.*

Lemma 1 follows immediately from Proposition 2 and Proposition 4. As we have discussed in Section 2.2, when the firm reserves a positive amount of the option capacity under the optimal plan, e.g. $\boldsymbol{g}^* > \boldsymbol{c}^*$, we determine the fixed-price capacity $\boldsymbol{c}^*$ so as to balance the expected marginal underage and overage costs, where the underage cost reflects the fact that we have option capacity available to meet excess demand. For the special cases listed in Lemma 1, neither the underage nor the overage cost depend on the unit profit $\boldsymbol{r}$. Therefore, as $\boldsymbol{r}$ increases, $\boldsymbol{c}^*$ remains the same. In a general case, however, these costs might change as $\boldsymbol{r}$ changes. Therefore, Lemma 1 might not hold anymore. To illustrate this, let's consider the following example which contains 5 products, 9 processes, and 9 resources. The structure of the supply chain is given in Figure 2-5. The demand for each product follows a normal distribution

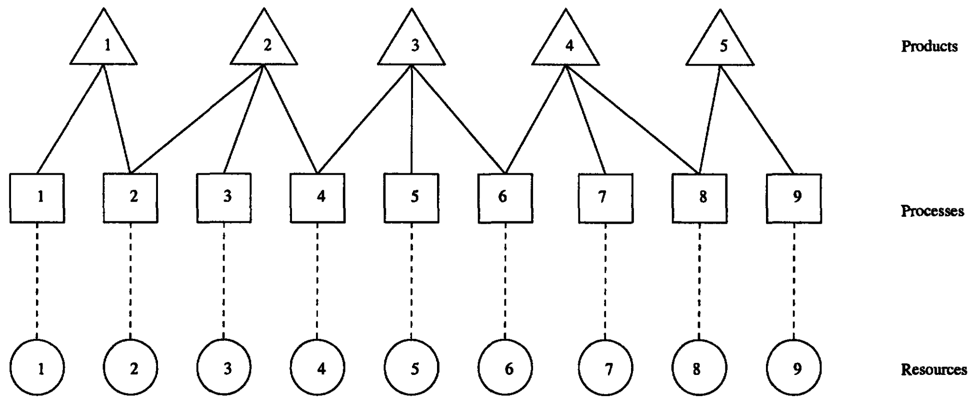

Figure 2-5: A supply chain with 5 products, 9 processes, and 9 resources to demonstrate the effects of unit profit and unit price on optimal capacity planning strategies.

$N(120, 10)$. We set

$$r = [30, 50, 46, 41, 25], \ p_k = 10 \ \forall \ k, \ q_k = 8 \ \forall \ k, \ \text{and} \ e_k = 3 \ \forall \ k.$$

We plot the change of optimal strategy for resource 1 as unit profit of product 1 increases in Figure 2-6. We can see that both the optimal total capacity and the fixed-price capacity increase as the unit profit increases and $\boldsymbol{g}^* > \boldsymbol{c}^*$. As the price of product 1 increases, the priority of product 1 in terms of order fulfillment increases. Therefore, the underage and overage costs change. As a result, both the optimal total capacity and the fixed-price capacity change.

2.4.2 Effect of Changes of Demands

The effect of changes of demands on the optimal capacity plan is complex. Let us consider the following counter intuitive example. Let $(\boldsymbol{c}^*, \boldsymbol{g}^*)$ be an optimal solution of the capacity planning problem $(\boldsymbol{D}, A, B, H, \boldsymbol{r}, \boldsymbol{p}, \boldsymbol{q}, \boldsymbol{e})$. Let $\widehat{\boldsymbol{D}}$ be another random demand vector that differs from $\boldsymbol{D}$ only in its first moments, that is $\widehat{\boldsymbol{D}} = \boldsymbol{D} + \Delta$, where Δ is a known positive deterministic vector. Let $(\widehat{\boldsymbol{c}}^*, \widehat{\boldsymbol{g}}^*)$ be the optimal solution of capacity planning problem $(\widehat{\boldsymbol{D}}, A, B, H, \boldsymbol{r}, \boldsymbol{p}, \boldsymbol{q}, \boldsymbol{e})$. The parameters of the two problems are the same except that in the second problem the manufacturer receives extra deterministic demand.

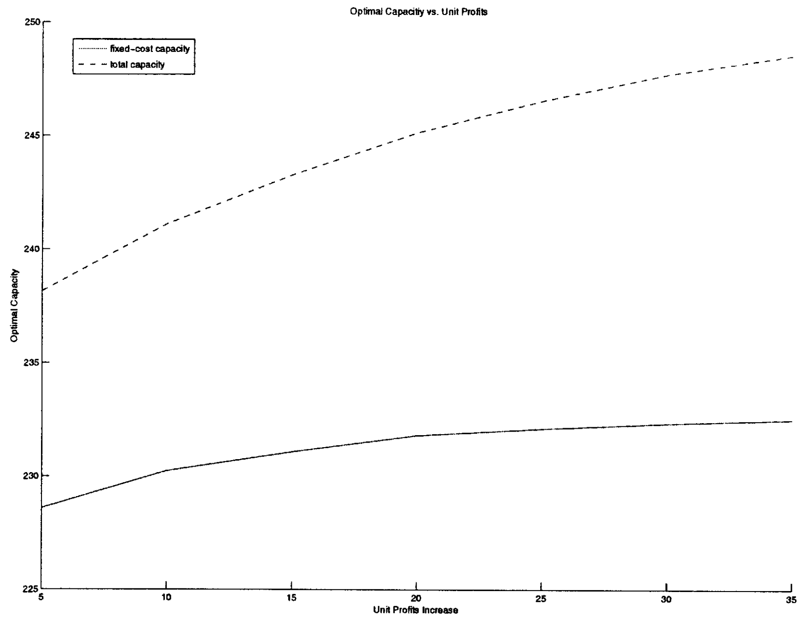

Figure 2-6: Effects of product profit, $\boldsymbol{r}$, on optimal capacity planning strategy.

One might expect that the optimal capacity plan for the second problem is first to adapt the optimal capacity plan for the first problem and then fill Δ using the cheapest way. Formally, if we let I be a $J \times K$ matrix such that

$$I(j,k) = \begin{cases} 1, & \text{if } p_k = \min\{p_n \mid B(j,(j,n)) = 1\}; \\ 0, & \text{otherwise.} \end{cases}$$

If $I(j,k) = 1$, it means that using resource k is the cheapest way to provide capacity for process j. One might expect that $\widehat{\boldsymbol{c}}^* = \boldsymbol{c}^* + I'A\Delta$ and $\widehat{\boldsymbol{g}}^* = \boldsymbol{g}^* + I'A\Delta$. However, this does not hold in general.

We consider the supply chain given in Figure 2-7 that consists of two products, one process, and one resource. The prices of the two products are: $r_1 = 1.5$ and $r_2 = 1.1$. Since there is only one process and one resource, we will view them as synonymous and use the terms interchangeably. The price of the fixed-price capacity

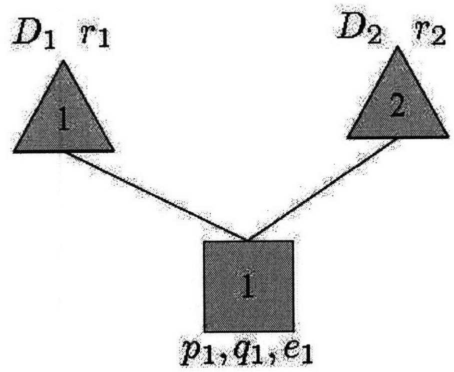

Figure 2-7: A supply chain with two products, one process, and one resource.

Scenario		$c_1 = 111$	$c_1 = 112$		
Prob.	Demand	Filled Demand	Filled Demand	Extra Cost	Extra Rev.
0.25	$d_1 = 50, d_2 = 102$	$z_1 = 50, z_2 = 51$	$z_1 = 50, z_2 = 52$	1	1.1
0.25	$d_1 = 50, d_2 = 101$	$z_1 = 50, z_2 = 51$	$z_1 = 50, z_2 = 52$	1	1.1
0.25	$d_1 = 10, d_2 = 102$	$z_1 = 10, z_2 = 101$	$z_1 = 10, z_2 = 102$	1	1.1
0.25	$d_1 = 10, d_2 = 101$	$z_1 = 10, z_2 = 101$	$z_1 = 10, z_2 = 101$	1	0

Table 2.7: Comparison between the capacity plan $c_1 = 111$ and $c_1 = 112$.

Scenario		$c_1 = 11$	$c_1 = 12$		
Prob.	Demand	Filled Demand	Filled Demand	Extra Cost	Extra Rev.
0.25	$d_1 = 50, d_2 = 2$	$z_1 = 11, z_2 = 0$	$z_1 = 12, z_2 = 0$	1	1.5
0.25	$d_1 = 50, d_2 = 1$	$z_1 = 11, z_2 = 0$	$z_1 = 12, z_2 = 0$	1	1.5
0.25	$d_1 = 10, d_2 = 2$	$z_1 = 10, z_2 = 1$	$z_1 = 10, z_2 = 2$	1	1.1
0.25	$d_1 = 10, d_2 = 1$	$z_1 = 10, z_2 = 1$	$z_1 = 10, z_2 = 1$	1	0

Table 2.8: Comparison between the capacity plan $c_1 = 11$ and $c_1 = 12$.

is $p_1 = 1$ and there is no option capacity. The demand for product 1 is either 50 or 10, and each occurs with probability 0.5. The demand for product 2 is either 2 or 1, and each occurs with probability 0.5. The optimal capacity plan is to reserve $c_1 = 12$ and the maximum expected net revenue is 5.325.

Now, we assume that the manufacturer receives extra 100 units of demand for product 2. With all the other parameters remain the same, the new optimal policy is $c_1 = 111$ but not 112. We compare the two capacity plans in Table 2.7. The capacity plan $c_1 = 112$ costs the manufacturer 1 dollar more and gains an extra expected net revenue of $0.25 \times 3.3 = 0.825$ dollar. Therefore, it is not optimal.

To gain insights into this example, we also need to compare the capacity plans $c_1 = 11$ and $c_1 = 12$ in the original problem. We give the comparison in Table 2.8. In this case, the capacity plan $c_1 = 12$ costs the manufacturer 1 dollar more but gains an extra expected net revenue of $0.25 \times 4.1 = 1.025$ dollar. Therefore, it is better than the plan $c_1 = 11$. From these two comparisons we see that after the manufacturer reserves more capacity in response to the increment in the demand, the allocation of this capacity depends on the prices and demands of the products. The manufacturer might use all of the new capacity to fill the extra demand. This dynamic complicates the decisions and, therefore fails our intuition.

2.4.3 Common Process and Option Capacity

In our model, there are three types of flexibility that the manufacturer can use to cope with the demand uncertainty: common processes, flexible resources, and option contracts. In this section, we will discuss the effects of using common processes and option contracts through a series of examples. Finally, we will draw some managerial insights into how to use these flexibilities.

Consider a supply chain given in Figure 2-8 that contains two products, four processes, and four resources. Each process has a dedicated resource and we will view them as synonymous and use the terms interchangeably.

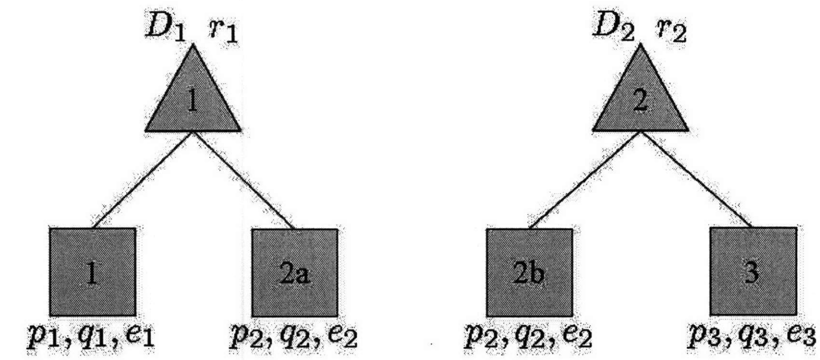

Figure 2-8: A supply chain with two products and four processes.

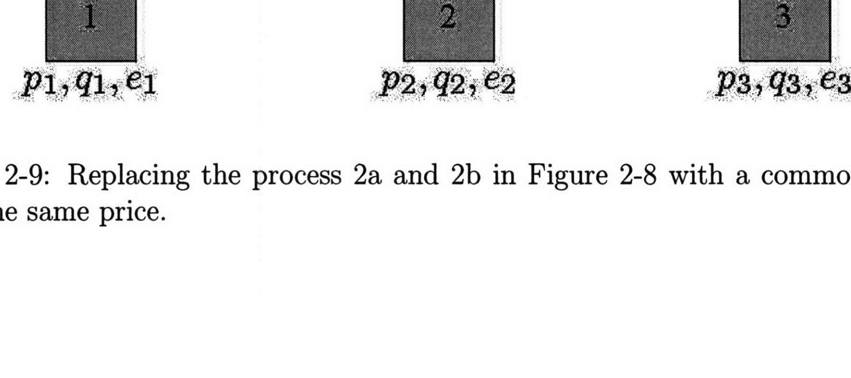

Figure 2-9: Replacing the process 2a and 2b in Figure 2-8 with a common process with the same price.

Both products have the same unit profit. The unit prices of the fixed-price capacity are

$$[p_1, p_{2a}, p_{2b}, p_3] = [10, 50, 50, 10].$$

The demand of each product follows normal distribution with mean and standard deviation:

$$E[D_1] = 502;\ \sigma(D_1) = 99;\ E[D_2] = 496;\ \sigma(D_2) = 99.$$

To study the effects of common processes and option contracts, we will consider the following four scenarios:

1. The optimal capacity strategy for the supply chain given in Figure 2-8.
2. The same problem in scenario 1 except that we replace processes 2a and 2b with a common process with the same price. The supply chain after the replacement is given in Figure 2-9.
3. The same problem in scenario 1 except that we add an option contract to process 2a and 2b. The option contract has a unit reservation price 5 and unit exercise price 50.
4. We combine scenario 2 and 3.

We will compare the change to the maximum expected profit in the four scenarios as we increase the unit profits for both products from 66 to 150. The results are given in Table 2.9. We use the maximum expected profit in scenario 1 as the reference point. We then quantify the benefit gained in the other scenarios as the percentage increase in profit compared to the reference. We have plotted the benefits versus profit margin in Figure 2-10. From this example, we have the following observations:

Profit	Margin	S1	S2	S3	S4	S2 vs. S1	S3 vs. S1	S4 vs. S1
66	1.54%	3,849	3,849	3,849	3,996	0.00%	0.00%	3.81%
67	3.08%	4,584	4,584	4,585	4,804	0.00%	0.04%	4.81%
68	4.62%	5,329	5,329	5,355	5,629	0.00%	0.49%	5.63%
69	6.15%	6,085	6,085	6,158	6,472	0.00%	1.20%	6.36%
70	7.69%	6,850	6,850	6,979	7,326	0.00%	1.87%	6.94%
80	23.08%	14,902	15,039	15,858	16,319	0.92%	6.41%	9.50%
90	38.46%	23,418	23,842	25,222	25,704	1.81%	7.70%	9.76%
100	53.85%	32,268	32,900	34,799	35,287	1.96%	7.84%	9.36%
110	69.23%	41,286	42,131	44,471	44,967	2.05%	7.71%	8.92%
150	130.77%	78,399	79,902	83,580	84,141	1.92%	6.61%	7.32%

Table 2.9: The benefits of using common processes and option contracts.

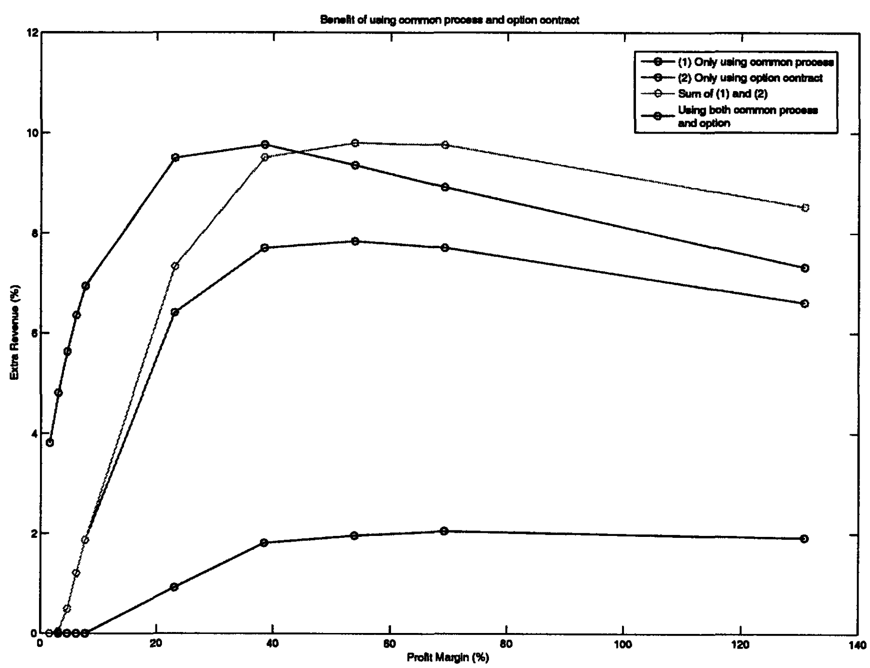

Figure 2-10: Comparing the benefits of using common process and option capacity.

1. *The benefits of using common process and option contract are small when the profit margin is low. The benefits increase and then decrease as the profit margin increases.*

   The benefit of common process comes from risk pooling. In this example, when the profit margin is very low, the loss from excess dedicated capacity is higher than the gain from the risk pooling on the common process. Therefore, the benefit of using common process is small.

   The benefit of using option contract is to reduce the overage cost (from the excess capacity) when the demand is low. When the profit margin is very low, the reduced overage cost is still too high compared to the underage cost (from the unfilled demands). Therefore, the option contract is also not very effective.

   When the profit margin is high, the underage cost is more significant compared to the overage cost. The manufacturer is willing to bear the cost of excess capacity as to not fill demands. Therefore, any savings from reducing common process capacity or reducing the cost of excess capacity become less significant when the profit margin is high.

2. *Using an option contract with small reservation price is more effective than using common process.*

   In Figure 2-10, the red line representing the benefit of using option contract is always above the blue line, which represents the benefit of using common process. We assume that the option contract has a 10% reservation price. The effective price of the option capacity, which is the sum of reservation price and exercise price, is 10% more than the price of the fixed-price capacity. In this case, option capacity can help the manufacturer to increase the expected profit by as high as 7.84% in the case where the profit margin is 53.83%. This example shows that an option contract with a small upfront price can be very effective. As the reservation price increases, the benefit of using an option contract decreases. For example, if we hold $q + e = 55$ and increase q from 5 to 25, the profit drops from 34,799 to 32,915. The increase in the expected profit

	c_1	c_{2a}	c_{2b}	c_3	g_1	g_{2a}	g_{2b}	g_3
Combined	437	760	-	426	437	863	-	426
Only option	406	378	367	397	406	406	397	397
Only common	384	763	-	378	384	763	-	378

Table 2.10: Studying the effects of common processes and option capacity: the optimal solutions of different strategies.

is 2.00%, which is slightly better than the 1.95% increase achieved from using a common process. If we further increase the reservation price to 40, the profit drops to 32,287. The percentage increase in profit is only 0.06%.

3. *When the profit margin is low, the strategy of using common process and an option contract with small reservation price together can gain extra benefits compared to using these two strategies separately.*

   Implementing both strategies, using common process and option contract, at the same time is better than just deploying one of them. Furthermore, there is a synergy in that the manufacturer gains an extra benefit by combining these two strategies at low profit margins. In Figure 2-10, the green line is the sum of the benefits of using common process and option contract separately. When the profit margin is lower than 40%, the benefit of combining two strategies, represented by the purple line, is higher than the green line. The gap between these two lines is the extra benefit that the manufacturer gets. Moreover, this extra benefit can be significant. For example, when the profit margin is 4.62%, using common process and option contract can increase the profit by 0% and 0.49% respectively. However, the combined strategy can achieve a 5.63% profit increase. To see the reasons behind this phenomenon, we can look at the optimal solution of the different strategies in Table 2.10. We note that in the combined strategy, the manufacturer first buys more capacity and second uses a larger portion of option capacity. The effectiveness of an option capacity depends on two factors: the price structure and the standard deviation of the demand. After replacing the process 2a and 2b with a common process, the standard deviation of the demand for the common process in larger than the standard deviation of the original dedicated process. Therefore, using common process amplifies the effectiveness of the option contract. On the other hand, the option contract makes the capacity for the common process more flexible. As a result, using the option contracts also amplifies the effectiveness of the risk pooling effect. Therefore, the combined strategy achieves a much higher percentage of profit increase.

2.5 Capacity Planning with Order Size Constraints

In practice, the capacity might only be procured or reserved in discrete or bulk units. This requires that the decision variables, $\boldsymbol{c}$ and $\boldsymbol{g}$, to be integer multiples of some base unit. Having integer decision variables will increase the difficulty of solving the problem. In this section, we will discuss how to find the optimal capacity planning strategies with order size constraints.

2.5.1 Algorithm

Let w_k be the order size of resource k. The firm can only reserve capacity from resource k in integer multiples of w_k. We define a componentwise product between two vectors as follows:

$$\boldsymbol{c} \odot \boldsymbol{w} = [c_1 w_1, \cdots, c_k w_k]' .$$

The optimal production level problem given capacity planning strategy $(\boldsymbol{c}, \boldsymbol{g})$ can be written as:

$$\begin{aligned} \max_{\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}} \quad & \bar{\pi}(\boldsymbol{c}, \boldsymbol{g}, \boldsymbol{d}, \boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) = \boldsymbol{r}'\boldsymbol{z} - \boldsymbol{e}'H\boldsymbol{y} \qquad (2.39) \\ s.t. \quad & \boldsymbol{z} \leq \boldsymbol{d} \\ & A\boldsymbol{z} \leq B(\boldsymbol{x} + \boldsymbol{y}) \end{aligned}$$

$$H\boldsymbol{x} \leq \boldsymbol{c} \odot \boldsymbol{w}$$

$$H(\boldsymbol{x} + \boldsymbol{y}) \leq \boldsymbol{g} \odot \boldsymbol{w}$$

$$\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z} \geq \boldsymbol{0}$$

And the firm will solve the following problem to find the optimal capacity planning strategy with order size constraints:

$$\begin{aligned} \max_{\boldsymbol{c},\boldsymbol{g}} \quad & E[\bar{\pi}(\boldsymbol{c}, \boldsymbol{g}, \boldsymbol{D}, \boldsymbol{x}^*, \boldsymbol{y}^*, \boldsymbol{z}^*)] - \boldsymbol{p}'(\boldsymbol{c} \odot \boldsymbol{w}) - \boldsymbol{q}'((\boldsymbol{g} - \boldsymbol{c}) \odot \boldsymbol{w}) \\ s.t. \quad & \boldsymbol{c} \leq \boldsymbol{g} \\ & c_k, g_k \text{ are non-negative integers for all } k. \end{aligned} \tag{2.40}$$

One way to solve the problem is to add integer constraints to linear problem (2.36) and apply the standard mixed integer algorithm such as branch and bound algorithm. However, as we have seen in Section 2.3, the linear problem approach to solve the general problem might take a long time itself. Adding integer constraints will further complicate the problem. We will now propose an algorithm that can take advantage of the stochastic supporting hyperplane algorithm that we have developed.

We note that Problem (2.39) is the same as the original optimal production level Problem (2.1). However, Problem (2.40) is not concave in $\boldsymbol{c}$ or $\boldsymbol{g}$ because of the integer constraints. Therefore, we can not directly apply the supporting hyperplane algorithm. To overcome this problem, we have the following algorithm:

Algorithm for Solving the Capacity Planning Problem with Order Size Constraints

Step 1: Solve Problem (2.2) without integer constraints using stochastic supporting hyperplane algorithm and let V^0 be the resulting constraint set of Problem (2.37) and $(\boldsymbol{c}^*, \boldsymbol{g}^*)$ be the optimal solution. Set $s = 0$.

Step 2: Solve the problem,

$$\begin{aligned} \min_{\boldsymbol{c},\boldsymbol{g},f} \quad & f \\ s.t. \quad & (\boldsymbol{c}, \boldsymbol{g}, f) \in V^s \\ & \boldsymbol{c} = \boldsymbol{a} \odot \boldsymbol{w} \text{ for some vectors } \boldsymbol{a} \text{ with all components being} \\ & \text{ non-negative integers} \\ & \boldsymbol{g} = \boldsymbol{b} \odot \boldsymbol{w} \text{ for some vectors } \boldsymbol{b} \text{ with all components being} \\ & \text{non-negative integers} \end{aligned} \tag{2.41}$$

and let $(\boldsymbol{c}^s, \boldsymbol{g}^s, f^s)$ be the optimal solution. If $-f^s - \Pi(\boldsymbol{c}^s, \boldsymbol{g}^s, \boldsymbol{D}) < \varepsilon$, where ε is the error bound given by user, then stop.

Step 3: Add a new supporting hyperplane with supporting point $(\boldsymbol{c}^s, \boldsymbol{g}^s)$ to the set V^s and let the new constraint set be V^{s+1}. Set $s = s + 1$ and go to Step 2.

2.5.2 Error Bound

Let $(\boldsymbol{c}^*, \boldsymbol{g}^*)$ be the optimal solution of the mixed integer problem (2.40) and $(\boldsymbol{c}', \boldsymbol{g}', f')$ be the solution of our algorithm. Then we have the following proposition.

Proposition 6

$$\Pi(\boldsymbol{c}^*, \boldsymbol{g}^*, \mathrm{D}) - \Pi(\boldsymbol{c}', \boldsymbol{g}', \boldsymbol{D}) \leq -f' - \Pi(\boldsymbol{c}', \boldsymbol{g}', \boldsymbol{D}). \tag{2.42}$$

Proof: Let $(\boldsymbol{c}^*, \boldsymbol{g}^*)$ be the optimal solution of the mixed integer problem (2.40) and $(\boldsymbol{c}', \boldsymbol{g}', f')$ be the solution returned by the algorithm for solving the capacity planning problem with order size constraints.

Let V' be the set of supporting hyperplanes returned from the algorithm. Let f^* be the optimal solution of the following problem:

$$\min_{f} \quad f \tag{2.43}$$

$$s.t. \qquad (\boldsymbol{c}^*, \boldsymbol{g}^*, f) \in V'$$

Therefore, $-f^* \geq \Pi(\mathbf{c}^*, \mathbf{g}^*, \boldsymbol{D})$. Since $(\boldsymbol{c}^*, \boldsymbol{g}^*, f^*)$ is a feasible solution of Problem (2.41),

$$-f' \geq -f^*.$$

Therefore,

$$\begin{aligned} & \Pi(\boldsymbol{c}^*, \boldsymbol{g}^*, \boldsymbol{D}) - \Pi(\boldsymbol{c}', \boldsymbol{g}') \\ \leq \; & -f^* - \Pi(\boldsymbol{c}', \boldsymbol{g}', \boldsymbol{D}) \\ \leq \; & -f' - \Pi(\boldsymbol{c}', \boldsymbol{g}', \boldsymbol{D}) \end{aligned}$$

Q.E.D.

In the algorithm, if the user picks ε as the terminating error bound, Proposition 6 guarantees that $\Pi(\boldsymbol{c}^*, \boldsymbol{g}^*, \boldsymbol{D}) - \Pi(\boldsymbol{c}', \boldsymbol{g}', \boldsymbol{D})$ is less than ε.

Chapter 3

A Decomposition Method

In Chapter 2, we developed algorithms to solve the single period capacity planning problem. We now present a decomposition method that can separate the capacity planning problem into multiple subproblems. After the decomposition, the new optimization problem will provide an upper bound and a feasible solution and lower bound to the original problem.

This decomposition method is important in both the single period and multi-period setting. First, even though we have proposed an efficient algorithm to solve the single period problem, when the size of the problem is large, finding the optimal capacity strategy still requires a considerable amount of computational power. In these cases, the decomposition method provides a good feasible solution that we can calculate efficiently. Moreover, the upper bound generated by the method provides a criterion to evaluate the quality of the feasible solution.

Second, as we will see in Chapter 4, finding an optimal solution for the multi-period capacity planning problem is very difficult. The decomposition method proposed in this chapter is a crucial step in the approximation algorithm that we use to solve the multi-period problem. Moreover, as in the single period case, the method also provides an upper bound to check the accuracy of the approximation algorithm for the multi-period problem.

We will illustrate the method in the single period case in this chapter and will extend it to the multi-period case in chapter 4. Through the rest of this chapter,

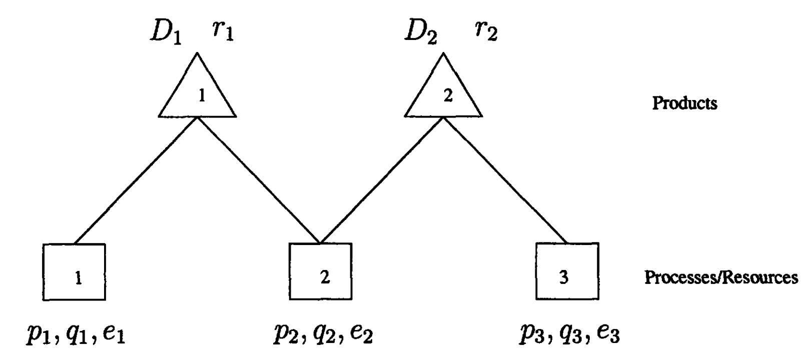

Figure 3-1: A supply chain with two products and three processes/resources.

we will use the following assumption. The decomposition method we propose in this chapter requires this assumption.

Assumption 1 *All processes have only dedicated resources.*

3.1 A Decomposition Method Leading to a Feasible Solution and an Upper Bound

The original single period capacity planning problem with Assumption 1 is not separable because different products might share the same processes. One possible intuitive approach to decompose the problem is to ignore the risk pooling effect on the shared processes. A shared process might be separated into multiple identical (in terms of price structure) but independent processes such that each one of them is used by one and only one product. However, such a relaxation can not provide an upper bound of the problem. Moreover, as we will see in Chapter 4, since different processes of the same product are still linked together after the decomposition, this method does not help to resolve or simplify the contract selection of different processes in the multi-period case. Therefore, we will propose another way to transform the problem.

To illustrate the decomposition method, we consider the following example. Figure 3-1 shows a supply chain with two products and three processes. In this supply chain, to produce product 1, we require both process 1 and 2 and to produce product 2, we require both process 2 and 3. Now, we relax the constraint that requires both

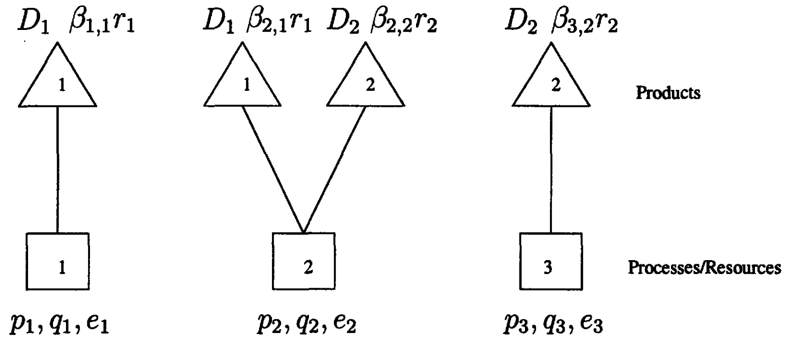

Figure 3-2: An upper bound to the supply chain given in Figure 3-1.

process 1 and 2 to produce product 1 to obtain the revenue from product 1. Rather we permit these processes to operate independently, with each being rewarded with a fraction of the revenue for product 1. Similarly, we allow process 2 and process 3 to produce product 2 independently. The supply chain after the relaxation is given in Figure 3-2. Now, we can independently use process 1 and 2 to produce product 1. However, the product 1 produced from process 1 has a new price $\beta_{1,1}r_1$ and the product 1 produced from process 2 now has a new price $\beta_{2,1}r_1$. Similarly, $\beta_{2,2}r_2$ and $\beta_{3,2}r_2$ are the prices for the new product 2 after the relaxation.

We first notice that solving the relaxed problem gives a feasible capacity plan for the original problem. Furthermore, we will show that if we set

$$\beta_{1,1} + \beta_{2,1} = 1 \text{ and } \beta_{2,2} + \beta_{3,2} = 1,$$

the solution to the relaxation provides an upper bound to the original problem.

Because of Assumption 1, the relaxation decomposes the original problem into subproblems such that we have one subproblem for each process.

We now formalize the decomposition method described above. Let us recall the problem for single period capacity planning:

$$\begin{aligned} \max_{\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}} \quad & \pi(\boldsymbol{c}, \boldsymbol{g}, \boldsymbol{d}, \boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) = \boldsymbol{r}'\boldsymbol{z} - \boldsymbol{e}'H\boldsymbol{y} \\ s.t. \quad & \boldsymbol{z} \le \boldsymbol{d} \\ & A\boldsymbol{z} \le B(\boldsymbol{x} + \boldsymbol{y}) \end{aligned}$$

$$\boldsymbol{H x} \leq \boldsymbol{c}$$
$$\boldsymbol{H}(\boldsymbol{x}+\boldsymbol{y}) \leq \boldsymbol{g}$$
$$\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z} \geq \boldsymbol{0}$$

and

$$\begin{aligned} \max_{\boldsymbol{c},\boldsymbol{g}} \quad & \Pi(\boldsymbol{c}, \boldsymbol{g}, \boldsymbol{D}) = E[\pi(\boldsymbol{c}, \boldsymbol{g}, \boldsymbol{D}, \boldsymbol{x}^*, \boldsymbol{y}^*, \boldsymbol{z}^*)] - \boldsymbol{p}'\boldsymbol{c} - \boldsymbol{q}'(\boldsymbol{g} - \boldsymbol{c}) \\ s.t. \quad & \boldsymbol{c} \leq \boldsymbol{g} \\ & \boldsymbol{c}, \boldsymbol{g} \geq \boldsymbol{0} \end{aligned}$$

For each product m, define

$$\mathcal{J}_m = \{j \mid A(j, m) = 1\}$$

Therefore, $\mathcal{J}_m$ is the set of processes that product m requires. Let $J_m = |\mathcal{J}_m|$. Let $\{1, \cdots, J_m\}$ be the indices of the processes in $\mathcal{J}_m$. Therefore, p_j where $j \in \mathcal{J}_m$ refers to the price of fixed capacity for the j^{th} process in $\mathcal{J}_m$. To simplify the presentation, we will not always identify the process by its product, i.e. $j \in \mathcal{J}_m$, as this should be clear from the context. Similarly, for each process j, define

$$\mathcal{M}_j = \{m \mid A(j, m) = 1\}$$

$\mathcal{M}_j$ is the set of products that require process j.

For each product m and process j, let $\beta_{j,m}$ be a fixed real number. Then, for each process j, we consider the following optimization problem:

$$\begin{aligned} \max_{x_j, y_j, z_m : m \in \mathcal{M}_j, \beta_j} \quad & \theta_j(c_j, g_j, \boldsymbol{d}, x_j, y_j, z_{m:m\in\mathcal{M}_j}, \beta_j) = \sum_{m \in \mathcal{M}_j} \beta_{j,m} r_m z_m - e_j y_j \quad (3.1) \\ s.t. \quad & z_m \leq d_m, \ \forall\, m \in \mathcal{M}_j \\ & \sum_{m \in \mathcal{M}_j} z_m \leq x_j + y_j \\ & x_j \leq c_j \end{aligned}$$

$$
\begin{aligned}
& x_j + y_j \le g_j \\
& x_j, y_j \ge 0 \\
& z_m \ge 0,\ \forall\, m \in \mathcal{M}_j
\end{aligned}
$$

and

$$
\begin{aligned}
\max_{c_j, g_j} \quad & \Theta_j(c_j, g_j, \boldsymbol{D}, \beta_j) = E[\theta_j(c_j, g_j, \boldsymbol{D}, x_j^*, y_j^*, z^*_{m:m\in\mathcal{M}_j}, \beta_j)] - p_j c_j - q_j(g_j - c_j) \\
s.t. \quad & c_j \le g_j \qquad (3.2) \\
& c_j, g_j \ge 0.
\end{aligned}
$$

Define

$$
\theta(\boldsymbol{c}, \boldsymbol{g}, \boldsymbol{d}, \boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}, \beta) = \sum_{j=1}^{J} \theta_j(c_j, g_j, \boldsymbol{d}, x_j, y_j, z_{m:m\in\mathcal{M}_j}, \beta_j). \qquad (3.3)
$$

We now consider a new optimization problem

$$
\begin{aligned}
\max_{\boldsymbol{c}, \boldsymbol{g}} \quad & \Theta(\boldsymbol{c}, \boldsymbol{g}, \boldsymbol{D}, \beta) = E[\theta(\boldsymbol{c}, \boldsymbol{g}, \boldsymbol{D}, \boldsymbol{x}^*, \boldsymbol{y}^*, \boldsymbol{z}^*, \beta)] - \boldsymbol{p}'\boldsymbol{c} - \boldsymbol{q}'(\boldsymbol{g} - \boldsymbol{c}) \qquad (3.4) \\
s.t. \quad & \boldsymbol{c} \le \boldsymbol{g} \\
& \boldsymbol{c}, \boldsymbol{g} \ge \boldsymbol{0}
\end{aligned}
$$

We can separate Problem (3.4) into J independent subproblems:

$$
\Theta(\boldsymbol{c}, \boldsymbol{g}, \boldsymbol{D}, \beta) = \sum_{j=1}^{J} \Theta_j(c_j, g_j, \boldsymbol{D}, \beta_j)
$$

Moreover, each subproblem (3.2) is essentially the same as the special case we presented in Section 2.2.2 which has a closed form solution. Therefore, we can solve Problem (3.4) effectively. We now show that if we choose β properly, Problem (3.4) provides an upper bound for Problem (2.2).

Proposition 7 *Let $(\overline{\boldsymbol{c}}^*, \overline{\boldsymbol{g}}^*)$ be the optimal capacity planning strategy for Problem (3.4). If for each product m,*

$$
\sum_{j\in\mathcal{J}_m} \beta_{j,m} = 1, \qquad (3.5)
$$

then $\Theta(\overline{c}^*, \overline{g}^*, D, \beta) \geq \Pi(c^*, g^*, D)$, *where* (c^*, g^*) *is the optimal solution to Problem (2.2).*

Proof: Let (c^*, g^*) be an optimal solution of Problem (2.2) given demand distribution D. Since $(\overline{c}^*, \overline{g}^*)$ is a feasible solution of Problem (2.2), we have

$$\Pi(c^*, g^*, D) \geq \Pi(\overline{c}^*, \overline{g}^*, D).$$

Let $(c_j, g_j, d, \overline{x}_j^*, \overline{y}_j^*, \overline{z}_{m:m\in\mathcal{M}_j}^*)$ be an optimal solution of Problem (3.1) under capacity planning strategy (c_j, g_j) and $\mathcal{C}_j$ be the corresponding constraint set. Let $\mathcal{C} = \bigcup_j \mathcal{C}_j$. Since Problem (3.3) is separable and each subproblem is equivalent to Problem (3.1), $\mathcal{C}$ is the constraint set of Problem (3.3). Let $(c, g, d, x^*(d), y^*(d), z^*(d))$ be an optimal solution of Problem (2.1) given (c, g, d). Since we have relaxed the constraint that to produce a product requires all of its processes, we can show that any feasible solution to Problem (2.1) is also a feasible solution to Problem (3.3). Therefore, for any given realization d, the constraint set of Problem (2.1) is a subset of $\mathcal{C}$. As a result, for a given demand realization d, $(x^*(d), y^*(d), z^*(d))$ is a feasible solution of Problem (3.3) and $(x_j^*(d), y_j^*(d), z_{m:m\in\mathcal{M}_j}^*(d))$ is a feasible solution of Problem (3.1). Then, given d we have

$$\theta_j(c_j, g_j, d, \overline{x}_j^*, \overline{y}_j^*, \overline{z}_{m:m\in\mathcal{M}_j}^*, \beta_j) \geq \sum_{m\in\mathcal{M}_j} \beta_{j,m} r_m z_m^* - e_j y_j^*. \tag{3.6}$$

Because Equation (3.6) holds for all d, we have

$$E[\theta(c, g, D, \overline{x}^*, \overline{y}^*, \overline{z}^*, \beta] \geq E\left[\sum_{j=1}^{J}\left(\sum_{m\in\mathcal{M}_j} \beta_{j,m} r_m z_m^* - e_j y_j^*\right)\right]. \tag{3.7}$$

If

$$\sum_{j\in\mathcal{J}_m} \beta_{j,m} = 1,$$

then

$$E[\pi(c, g, D, x^*, y^*, z^*)] \quad = \quad E\left[\sum_{m=1}^{M} r_m z_m^* - \sum_{j=1}^{J} e_j y_j^*\right] \tag{3.8}$$

$$
\begin{aligned}
&= E\left[\sum_{m=1}^{M}\left(\sum_{j\in\mathcal{J}_m}\beta_{j,m}r_m z_m^*\right)-\sum_{j=1}^{J}e_j y_j^*\right] \\
&= E\left[\sum_{j=1}^{J}\left(\sum_{m\in\mathcal{M}_j}\beta_{j,m}r_m z_m^*-e_j y_j^*\right)\right].
\end{aligned}
$$

From Equation (3.7) and (3.8), we have

$$
E[\theta(\boldsymbol{c},\boldsymbol{g},\boldsymbol{D},\overline{\boldsymbol{x}}^*,\overline{\boldsymbol{y}}^*,\overline{\boldsymbol{z}}^*,\beta)] \geq E[\pi(\boldsymbol{c},\boldsymbol{g},\boldsymbol{D},\boldsymbol{x}^*,\boldsymbol{y}^*,\boldsymbol{z}^*)]. \tag{3.9}
$$

By Equation (3.9), we have

$$
E[\theta(\boldsymbol{c},\boldsymbol{g},\boldsymbol{D},\overline{\boldsymbol{x}}^*,\overline{\boldsymbol{y}}^*,\overline{\boldsymbol{z}}^*,\beta)]-\boldsymbol{p}'\boldsymbol{c}-\boldsymbol{q}'(\boldsymbol{g}-\boldsymbol{c}) \geq E[\pi(\boldsymbol{c},\boldsymbol{g},\boldsymbol{D},\boldsymbol{x}^*,\boldsymbol{y}^*,\boldsymbol{z}^*)]-\boldsymbol{p}'\boldsymbol{c}-\boldsymbol{q}'(\boldsymbol{g}-\boldsymbol{c}).
$$

Therefore,

$$
\Theta(\boldsymbol{c}^*,\boldsymbol{g}^*,\boldsymbol{D},\beta) \geq \Pi(\boldsymbol{c}^*,\boldsymbol{g}^*,\boldsymbol{D}). \tag{3.10}
$$

Since $(\overline{\boldsymbol{c}}^*,\overline{\boldsymbol{g}}^*)$ is the optimal solution of Problem (2.2),

$$
\Theta(\overline{\boldsymbol{c}}^*,\overline{\boldsymbol{g}}^*,\boldsymbol{D},\beta) \geq \Theta(\boldsymbol{c}^*,\boldsymbol{g}^*,\boldsymbol{D},\beta). \tag{3.11}
$$

Therefore, combining Equation (3.10) and (3.11), we have

$$
\Theta(\overline{\boldsymbol{c}}^*,\overline{\boldsymbol{g}}^*,\boldsymbol{D},\beta) \geq \Pi(\boldsymbol{c}^*,\boldsymbol{g}^*,\boldsymbol{D}).
$$

Q.E.D.

Proposition 7 says that if we choose $\beta's$ that satisfy Equation (3.5), we will get an upper bound of the original problem after the decomposition. However, there are infinitely many choices of β that satisfy Equation (3.5) and some βs will give tighter upper bounds than the others. The next problem that we will address is how we should pick the βs.

The analysis given in the section is related to the resource directive decomposition [37] method in deterministic linear programming. Different from the traditional resource directive decomposition method, we apply a decomposition method to a stochastic linear problem. Therefore, the decomposition is a relaxation and does not guarantee the optimality after the decomposition. Moreover, in the following sections, we suggest an efficient algorithm to pick the weight factors. The algorithm takes advantage of the special structure of our problem.

3.2 Picking the Weight Factors

In this section, we will propose a heuristic algorithm to find a good β. We call a β optimal, if $\Theta(\overline{\boldsymbol{c}}^*, \overline{\boldsymbol{g}}^*, \boldsymbol{D}, \beta)$ is the least upper bound of $\Pi(\boldsymbol{c}^*, \boldsymbol{g}^*, \boldsymbol{D})$. The algorithm given in this section cannot guarantee the optimal choice of β. However, as we will show in Section 3.2.2, it provides both a good feasible solution and an good upper bound for the original capacity planning problem.

3.2.1 Algorithm

We have derived the closed-form solution for the single period capacity planning problem that contains a single product and multiple processes in Section 2.2.1. Can we find the weight factor β such that for this special case, the original problem and the problem after the decomposition have the same solution? If so, this provides a heuristic method to pick the weight factor. There two benefits for using this heuristic method: first, the upper bound generated by this heuristic method is tight for the special case with a single product and multiple processes and we will prove this in Proposition 9; second, the β given by this heuristic method has a closed-form representation.

We will use an example to illustrate this. We consider a simple supply chain given in Figure 3-3 that consists of one product with unit price 50. To produce the product, it needs both process 1 and 2. Process 1 has price structure $(p_1, q_1, e_1) = (10, 9, 2)$ and process 2 has price structure $(p_2, q_2, e_2) = (9, 8, 2)$. After the decomposition, we have two products, $1a$ and $1b$, and they have the same demand. The unit price for product $1a$ is βr and unit price for product $1b$ is $(1 - \beta)r$. We plot the maximum expected profit, as a function of β, of the problem after the decomposition in Figure

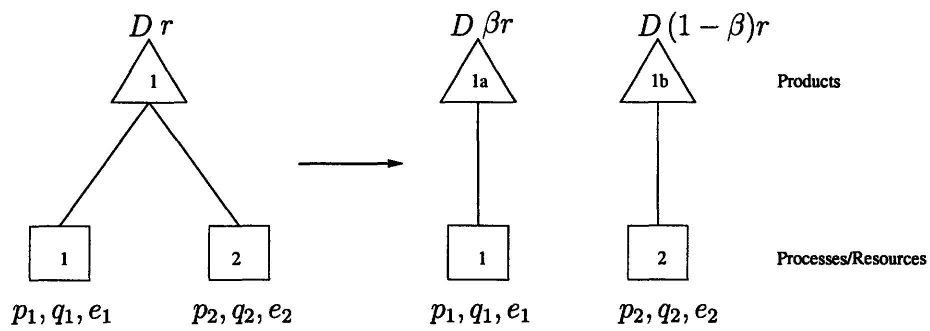

Figure 3-3: An example of decomposition to illustrate the effect of β.

3-4. We can see that for all β the maximal expected profit of the supply chain after the decomposition is an upper bound on the maximal expected profit of the original supply chain. When $\beta = 0.527$, the problem after the decomposition and the original problem have the same solution and, therefore, the upper bound is tight. We now show how to find the closed-form representation of the optimal β for the special case that contains one product and dedicated processes.

There are two important insights that we can draw from this example. First, β should be proportional to the price of the process. Process 1 is slightly more expensive than Process 2. Therefore, the price of product $1a$ is slightly higher than the price of product $1b$ and, therefore, the optimal β is slightly bigger than 0.5. Second, the optimal total profit is convex in β. Therefore, effective search algorithms for convex optimization problems can be applied to find the optimal β. We will devote the rest of this section to formalize the method that we have described and discuss how to search for the optimal β in Section 3.3.

Without loss of generality, we assume that for all $i, j \in \mathcal{J}_m$

$$\frac{p_i - q_i}{e_i} \geq \frac{p_j - q_j}{e_j}, \text{ if } i \geq j.$$

For each product m, define

$$\psi_m = \begin{cases} J_m + 1, & \text{if } \frac{p_{J_m} - q_{J_m}}{e_{J_m}} \leq \frac{\sum_{j=1}^{J_m} p_j}{r}; \\ \min\left\{ i \mid \frac{p_i - q_i}{e_i} > \frac{\sum_{j=1}^{i-1} p_j + \sum_{j=i}^{J_m} q_j}{r - \sum_{j=i}^{J_m} e_j} \right\}, & \text{otherwise.} \end{cases} \tag{3.12}$$

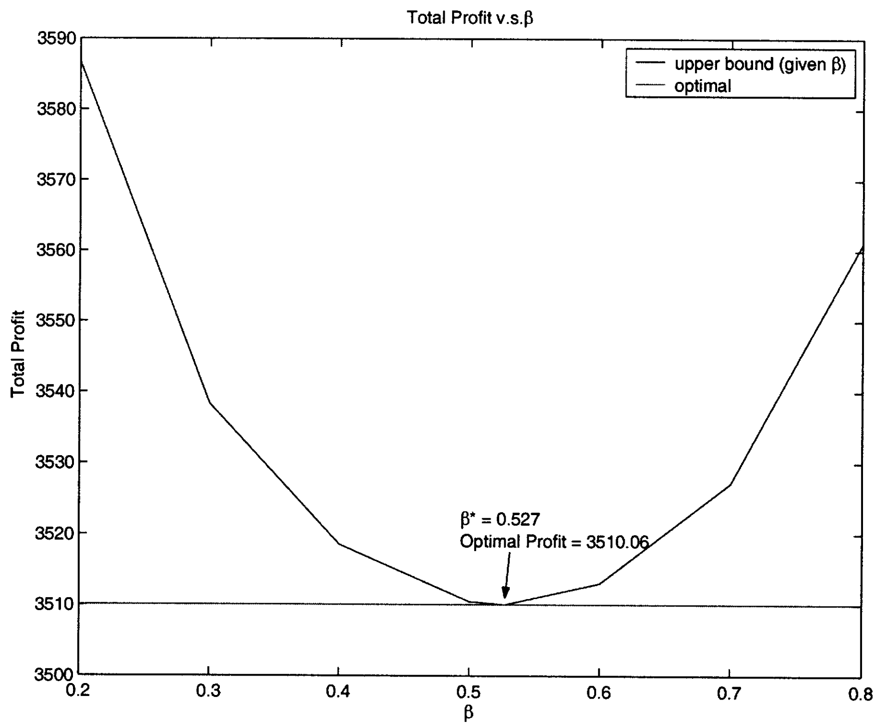

Figure 3-4: Different β will have different upper bound. When $\beta = 0.527$, the upper bound is tight.

Also, for each product m, we define the following ratio

$$\alpha_m = \frac{\sum_{j=1}^{\psi_m - 1} p_j + \sum_{j=\psi_m}^{J_m} q_j}{r_m - \sum_{j=\psi_m}^{J} e_j} \tag{3.13}$$

Proposition 8 *If for each j and m such that $A(j,m) = 1$, we set*

$$\beta_{j,m} = \begin{cases} \frac{q_j}{\alpha_m r_m} + \frac{e_j}{r_m}, & \textit{if } j \geq \psi_m; \\ \frac{p_j}{\alpha_m r_m}, & \textit{otherwise.} \end{cases} \tag{3.14}$$

then

$$\Theta(\overline{\boldsymbol{c}}^*, \overline{\boldsymbol{g}}^*, \boldsymbol{D}, \beta) \geq \Pi(\boldsymbol{c}^*, \boldsymbol{g}^*, \boldsymbol{D})$$

Proof: If for each j and m such that $A(j,m) = 1$, we set

$$\beta_{j,m} = \begin{cases} \frac{q_j}{\alpha_m r_m} + \frac{e_j}{r_m}, & \text{if } j \geq \psi_m; \\ \frac{p_j}{\alpha_m r_m}, & \text{otherwise.} \end{cases}$$

then

$$\begin{aligned} \sum_{j \in \mathcal{J}_m} \beta_{j,m} &= \frac{\sum_{j=1}^{\psi_m - 1} p_j}{\alpha_m r_m} + \frac{\sum_{j=\psi_m}^{J_m} q_j}{\alpha_m r_m} + \frac{\sum_{j=\psi_m}^{J_m} e_j}{r_m} \\ &= \frac{\left(r_m - \sum_{j=\psi_m}^{J_m} e_j\right) \sum_{j=1}^{\psi_m - 1} p_j}{r_m \left(\sum_{j=1}^{\psi_m - 1} p_j + \sum_{j=\psi_m}^{J_m} q_j\right)} + \frac{\left(r_m - \sum_{j=\psi_m}^{J_m} e_j\right) \sum_{j=\psi_m}^{J_m} q_j}{r_m \left(\sum_{j=1}^{\psi_m - 1} p_j + \sum_{j=\psi_m}^{J_m} q_j\right)} + \frac{\sum_{j=\psi_m}^{J_m} e_j}{r_m} \\ &= \frac{\left(r_m - \sum_{j=\psi_m}^{J_m} e_j\right) \left(\sum_{j=1}^{\psi_m - 1} p_j + \sum_{j=\psi_m}^{J_m} q_j\right)}{r_m \left(\sum_{j=1}^{\psi_m - 1} p_j + \sum_{j=\psi_m}^{J_m} q_j\right)} + \frac{\sum_{j=\psi_m}^{J_m} e_j}{r_m} \\ &= \frac{r_m - \sum_{j=\psi_m}^{J_m} e_j}{r_m} + \frac{\sum_{j=\psi_m}^{J_m} e_j}{r_m} \\ &= 1. \end{aligned}$$

Therefore, setting

$$\beta_{j,m} = \begin{cases} \frac{q_j}{\alpha_m r_m} + \frac{e_j}{r_m}, & \text{if } j \geq \psi_m; \\ \frac{p_j}{\alpha_m r_m}, & \text{otherwise.} \end{cases}$$

for each j and m such that $A(j,m) = 1$ will provide an upper bound of the problem.

Q.E.D.

Equation (3.14) suggests a heuristic method to pick the βs. Even though this method cannot guarantee the optimality of β, it provides a good upper bound for the original capacity problem. We will study the tightness and scalability of the upper bound with this β in the next section.

3.2.2 The Tightness and Scalability of the Upper Bound

Proposition 8 suggests that if we choose βs satisfying Equation (3.14), we will get an upper bound. We will now verify that this indeed is a good upper bound. We see that according to Equation (3.14), β is proportional to the price of the process: more profit will be assigned to the process with a higher cost. This is consistent with the first insight that we get from the previous example. We now prove that the β given in Proposition 8 is optimal for the special case with a single product and dedicated processes.

Proposition 9 *If we set βs using Equation (3.14), then for the single period supply chain that contains one product and dedicated processes,*

$$\Theta(\overline{\boldsymbol{c}}^*, \overline{\boldsymbol{g}}^*, \boldsymbol{D}, \beta) = \Pi(\boldsymbol{c}^*, \boldsymbol{g}^*, \boldsymbol{D}).$$

Proof: Note that if we apply the decomposition algorithm with the βs given in Equation (3.14) to a supply chain contains a single product and dedicated resources, each sub-problem is a supply chain with one product, one process, and one resource. We can apply Proposition 2 from Chapter 2 to find the optimal solution of the sub-problem. Since there is only one product, we omit the subscript m. We consider the subproblem associated with process k. Let $(\hat{c}_k, \hat{g}_k)$ and (c_k^*, g_k^*) be the optimal solution of the subproblem and the optimal solution of the original problem. There are two cases:

Case 1: If $k \geq \psi$, then the new price, r_k, of the product associated with process k

after the decomposition is

$$r_k = \left(\frac{q_k}{\alpha r} + \frac{e_k}{r}\right) r = \frac{q_k}{\alpha} + e_k.$$

Therefore,

$$\frac{q_k}{r_k - e_k} = \alpha. \tag{3.15}$$

Since $k \geq \psi$, by Equation (3.12) and (3.13)

$$\frac{p_k - q_k}{e_k} \geq \frac{p_\psi - q_\psi}{e_\psi} > \frac{\sum_{j=1}^{\psi-1} p_j + \sum_{j=\psi}^{J_m} q_j}{r - \sum_{j=\psi}^{J_m} e_j} = \alpha. \tag{3.16}$$

From Equation (3.15) and (3.16), we have

$$\frac{p_k - q_k}{e_k} > \frac{q_k}{r_k - e_k} \tag{3.17}$$

We now have a single product and single process capacity planning problem. By Proposition 2, we have a closed-form solution for this problem. Based on Proposition 2, Equation (3.17) implies that the optimal capacity plan will use option capacity, and the solution is given as:

$$\hat{g}_k > \hat{c}_k,$$

$$Pr(D > \hat{c}_k) = \frac{p_k - q_k}{e_k},$$

and

$$Pr(D > \hat{g}_k) = \frac{q_k}{r_k - e_k} = \alpha = \frac{\sum_{j=1}^{\psi-1} p_j + \sum_{j=\psi}^{J} q_j}{r - \sum_{j=\psi}^{J} e_j}.$$

This, in fact, is the optimal capacity plan for process k in the original capacity planning problem: $c_k^* = \hat{c}_k$ and $g_k^* = \hat{g}_k$.

Case 2: If $k < \psi$, then the new price, r_k, of the product associated with process k after the decomposition is

$$r_k = \frac{p_k}{\alpha r} r = \frac{p_k}{\alpha}.$$

Since $k \leq \psi - 1$,

$$\frac{p_k - q_k}{e_k} \leq \frac{p_{\psi-1} - q_{\psi-1}}{e_{\psi-1}}. \tag{3.18}$$

Now, by the definition of ψ, since $\psi - 1 < \psi$,

$$\frac{p_{\psi-1} - q_{\psi-1}}{e_{\psi-1}} \leq \frac{\sum_{j=1}^{(\psi-1)-1} p_j + \sum_{j=\psi-1}^{J_m} q_j}{r - \sum_{j=\psi-1}^{J_m} e_j}.$$

This implies that

$$\frac{p_{\psi-1} - q_{\psi-1}}{e_{\psi-1}} \leq \frac{\sum_{j=1}^{(\psi-1)-1} p_j + \sum_{j=\psi-1}^{J_m} q_j + p_{\psi-1} - q_{\psi-1}}{r - \sum_{j=\psi-1}^{J_m} e_j + e_{\psi-1}} = \frac{\sum_{j=1}^{\psi-1} p_j + \sum_{j=\psi}^{J_m} q_j}{r - \sum_{j=\psi}^{J_m} e_j} = \alpha \tag{3.19}$$

Combing Equation (3.18) and (3.19), we have

$$\frac{p_k - q_k}{e_k} \leq \alpha \tag{3.20}$$

By Equation (3.20), we get

$$q_k \geq p_k - \alpha e_k. \tag{3.21}$$

Therefore, by Equation (3.21),

$$\frac{q_k}{r_k - e_k} = \frac{q_k}{\frac{p_k}{\alpha} - e_k} = \frac{q_k}{p_k - \alpha e_k}\alpha \geq \alpha. \tag{3.22}$$

Combing Equation (3.18) and (3.22), we have

$$\frac{p_k - q_k}{e_k} \leq \frac{q_k}{r_k - e_k}. \tag{3.23}$$

Then, similar to Case 1, Equation (3.23) implies that the optimal capacity plan for the capacity planning problem for process k after the decomposition will not reserve any option capacity. By Proposition 2,

$$\hat{c}_k = \hat{g}_k,$$

$$Pr(D > \hat{g}_k) = \frac{p_k}{r_k} = \alpha = \frac{\sum_{j=1}^{\psi-1} p_j + \sum_{j=\psi}^{J} q_j}{r - \sum_{j=\psi}^{J} e_j},$$

and

$$c_j^* = \hat{c}_j, \; g_j^* = \hat{g}_j.$$

Since in both cases, $c_j^* = \hat{c}_j$ and $g_j^* = \hat{g}_j$,

$$\Theta(\overline{\boldsymbol{c}}^*, \overline{\boldsymbol{g}}^*, \boldsymbol{D}, \beta) = \Pi(\boldsymbol{c}^*, \boldsymbol{g}^*, \boldsymbol{D}).$$

Q.E.D.

We will examine the tightness of the upper bound for a general supply chain through a computational experiment. Table 3.1 lists the results of 40 randomly generated test cases. The settings of the parameters of these test cases are the same as the ones given in Section 2.3.7 except that each process uses only one dedicated resource (e.g. Assumption 1 holds). We can see that the maximal percentage error of the upper bound is 2.66% and the average percentage error is 1.48%. Therefore, from these test cases, we see that the gap between the upper bound and optimal value is small. Table 3.1 also lists the total expected profits of the feasible strategy generated by the decomposition method, which is a lower bound. The maximal percentage error and average percentage error of this approximate solution is 1.62% and 0.81%. This suggests that this sub-optimal capacity planning strategy is indeed a good approximation of the optimal strategy.

Another question that we are interested in is how the upper bound algorithm performs as the size of the problem increases. To answer this question, we will consider the following example which is given in Figure 3-5. The supply chain has n identical products and $n+1$ identical processes. Product $j-1$ and j share process j, for each $j = 1, 2, \cdots, n$. The prices of the products are 55, and the price structures of the resources are: $p = 10,\ q = 8,$ and $e = 3$. The demand of each product is a normally distributed random variables with mean 120 and standard deviation 10. We increase the size of the supply chain by adding more products and processes while maintaining the same structure. The results are given in Table 3.2. We see that even though the

Test Csae	Lower Bound	Optimal	Upper Bound	L. B. Err. (%)	U. B. Err. (%)
1	146723	147704	149600	0.66%	1.28%
2	152326	153901	156566	1.02%	1.73%
3	139513	140971	143637	1.03%	1.89%
4	148558	149928	152283	0.91%	1.57%
5	147784	149152	151368	0.92%	1.49%
6	157781	158884	160946	0.69%	1.30%
7	167504	168639	170341	0.67%	1.01%
8	157834	158723	160634	0.56%	1.20%
9	157625	158676	160874	0.66%	1.39%
10	157281	158367	160448	0.69%	1.31%
11	143344	144521	146961	0.81%	1.69%
12	142045	143201	145604	0.81%	1.68%
13	148853	149779	151824	0.62%	1.37%
14	153720	155252	157982	0.99%	1.76%
15	147932	149023	151143	0.73%	1.42%
16	161809	162710	164526	0.55%	1.12%
17	155365	156453	158485	0.70%	1.30%
18	149283	150417	152541	0.75%	1.41%
19	142271	143644	146388	0.96%	1.91%
20	160772	161976	164184	0.74%	1.36%
21	123531	125550	128606	1.61%	2.43%
22	154571	156040	157960	0.94%	1.23%
23	161914	162991	164790	0.66%	1.10%
24	157914	158858	160899	0.59%	1.28%
25	153325	154388	156154	0.69%	1.14%
26	147893	149188	151793	0.87%	1.75%
27	159611	160924	163206	0.82%	1.42%
28	172441	173563	175228	0.65%	0.96%
29	135962	137940	141097	1.43%	2.29%
30	153495	154364	156212	0.56%	1.20%
31	159224	160191	162177	0.60%	1.24%
32	147988	149177	151325	0.80%	1.44%
33	119238	121049	124266	1.50%	2.66%
34	129972	131481	134512	1.15%	2.31%
35	161692	162368	163755	0.42%	0.85%
36	116893	118052	120425	0.98%	2.01%
37	175952	176964	178616	0.57%	0.93%
38	162712	163840	165786	0.69%	1.19%
39	148083	149291	151685	0.81%	1.60%
40	180917	181767	183507	0.47%	0.96%

Table 3.1: Test Results: lower bound and upper bound for single period capacity planning problem and their percentage errors.

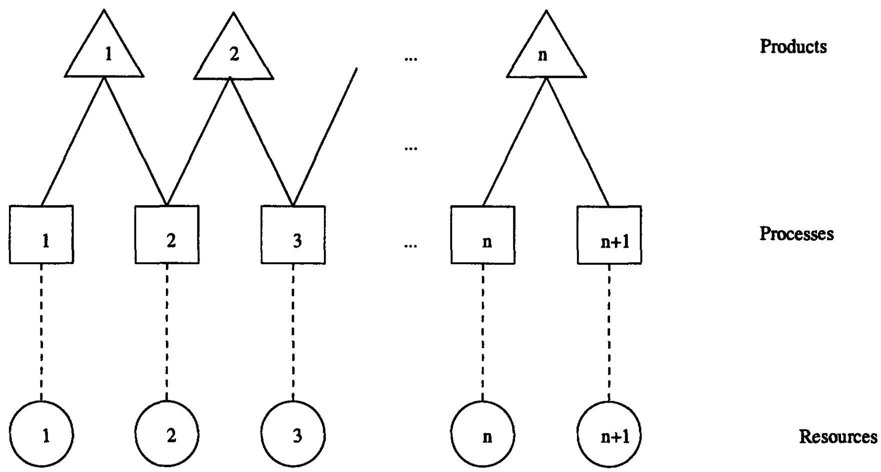

Figure 3-5: An example to illustrate the error of the upper bound as the size of supply chain increases.

# of Products	Lower B.	Optimal	Upper B.	L.B. Error	U.B. Error
2	7,979.58	7,982.38	8,032.28	0.04%	0.63%
3	12,031.65	12,038.20	12,110.51	0.05%	0.60%
4	16,049.78	16,059.32	16,153.84	0.06%	0.59%
5	20,074.93	20,084.96	20,199.45	0.05%	0.57%
6	24,077.34	24,093.88	24,232.68	0.07%	0.58%
7	28,105.63	28,122.84	28,289.74	0.06%	0.59%
8	32,180.95	32,200.25	32,392.10	0.06%	0.60%
9	36,203.02	36,228.38	36,437.85	0.07%	0.58%
10	40,235.19	40,260.34	40,491.64	0.06%	0.57%

Table 3.2: Test result: percentage error as the size of problem increases.

absolute error increases as the size of problem increases (as shown in Figure 3-6), but the error percentage remains stable (as shown in Figure 3-7). Therefore, these test cases provide some evidence that the algorithm is scalable in terms of problem size.

3.3 Searching for the Weight Factors

The final problem that we want to address is how we can further improve the upper bound by finding a better β. We consider the following proposition:

Proposition 10 *$\Theta(\boldsymbol{c}^*, \boldsymbol{g}^*, \boldsymbol{D}, \beta)$ is convex in β, for β that satisfies Equation (3.5).*

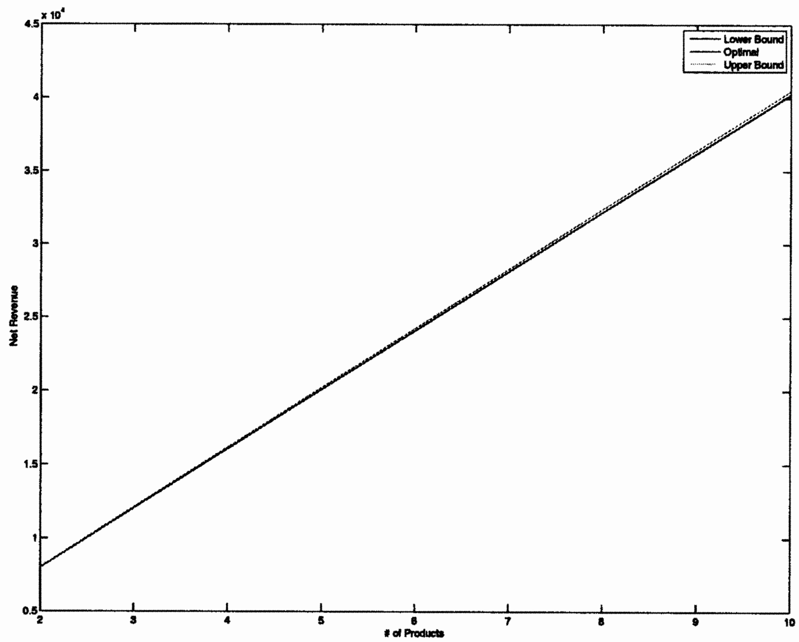

Figure 3-6: As number of products and processes increases, the absolute error increases.

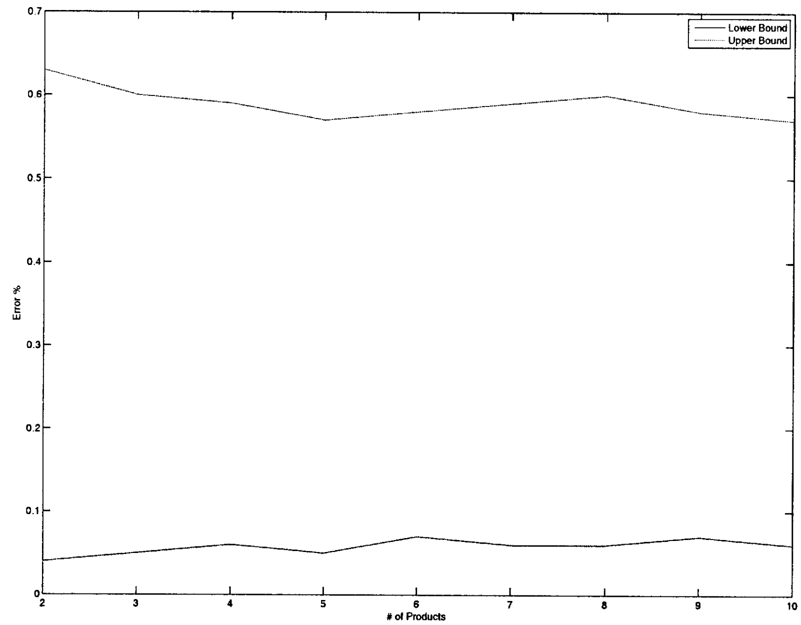

Figure 3-7: As number of products and processes increases, the error percentage remains at the same level.

Proof: Let β^1 and β^2 be two matrices satisfying Equation (3.5) and λ be a scalar $\in [0,1]$. Let $\beta^3 = \lambda\beta^1 + (1-\lambda)\beta^2$. Then, for each m,

$$\begin{aligned}\sum_{j\in\mathcal{J}_m} \beta^3_{j,m} &= \sum_{j\in\mathcal{J}_m} \lambda\beta^1_{j,m} + (1-\lambda)\beta^2_{j,m} \\ &= \lambda \sum_{j\in\mathcal{J}_m} \beta^1_{j,m} + (1-\lambda)\sum_{j\in\mathcal{J}_m} \beta^2_{j,m} \\ &= \lambda + (1-\lambda) \\ &= 1.\end{aligned}$$

Therefore, β^3 also satisfies Equation (3.5). Now, let $(\boldsymbol{c}^1, \boldsymbol{g}^1)$, $(\boldsymbol{c}^2, \boldsymbol{g}^2)$, and $(\boldsymbol{c}^3, \boldsymbol{g}^3)$ be the optimal solutions of Problem (3.4) for given β^1, β^2, and β^3. Since $\beta^3 = \lambda\beta^1 + (1-\lambda)\beta^2$, we have

$$\Theta(\boldsymbol{c}^3, \boldsymbol{g}^3, \boldsymbol{D}, \beta^3) = E[\theta(\boldsymbol{c}, \boldsymbol{g}, \boldsymbol{D}, \boldsymbol{x}^*, \boldsymbol{y}^*, \boldsymbol{z}^*, \beta^3)] - \boldsymbol{p}'\boldsymbol{c}^3 - \boldsymbol{q}'(\boldsymbol{g}^3 - \boldsymbol{c}^3)$$

$$
\begin{aligned}
&= E\left[\sum_j\left(\sum_{m\in\mathcal{M}_j}\beta^3_{j,m}r_m z_m - e_j y_j\right)\right] - \boldsymbol{p}'\boldsymbol{c}^3 - \boldsymbol{q}'(\boldsymbol{g}^3 - \boldsymbol{c}^3)\\
&= \lambda\left(E\left[\sum_j\left(\sum_{m\in\mathcal{M}_j}\beta^1_{j,m}r_m z_m - e_j y_j\right)\right] - \boldsymbol{p}'\boldsymbol{c}^3 - \boldsymbol{q}'(\boldsymbol{g}^3 - \boldsymbol{c}^3)\right) + \\
&\quad (1-\lambda)\left(E\left[\sum_j\left(\sum_{m\in\mathcal{M}_j}\beta^2_{j,m}r_m z_m - e_j y_j\right)\right] - \boldsymbol{p}'\boldsymbol{c}^3 - \boldsymbol{q}'(\boldsymbol{g}^3 - \boldsymbol{c}^3)\right)\\
&= \lambda\Theta(\boldsymbol{c}^3,\boldsymbol{g}^3,\boldsymbol{D},\beta^1) + (1-\lambda)\Theta(\boldsymbol{c}^3,\boldsymbol{g}^3,\boldsymbol{D},\beta^2)\\
&\le \lambda\Theta(\boldsymbol{c}^1,\boldsymbol{g}^1,\boldsymbol{D},\beta^1) + (1-\lambda)\Theta(\boldsymbol{c}^2,\boldsymbol{g}^2,\boldsymbol{D},\beta^2).
\end{aligned}
$$

Q.E.D.

By proposition 10, all stationary points of $\Theta(\boldsymbol{c}^*,\boldsymbol{g}^*,\boldsymbol{D},\beta)$ will be a global minima. As we have discussed before, $\Theta(\boldsymbol{c}^*,\boldsymbol{g}^*,\boldsymbol{D},\beta)$ is separable and we have developed efficient methods for solving each of the subproblems. Therefore, we have an efficient algorithm to evaluate Θ for any given β. Thus, we can use an effective algorithm for convex optimization to find the optimal β that minimize $\Theta(\boldsymbol{c},\boldsymbol{g},\boldsymbol{D},\beta)$.

We will discuss one of these algorithms: *Block Coordinate Descent* method. The β satisfying Equation (3.5) has a block structure: for each product m, $\sum_{j\in\mathcal{J}_m}\beta_{j,m} = 1$. We rewrite β in terms of blocks $(\beta_1,\cdots,\beta_M)$ where $\beta_m = \{\beta_{j,m} \mid j \in \mathcal{J}_m\}$. Therefore, we can apply the following algorithm to find the optimal β:

Block Coordinate Descent Method for Searching β:

Step 1: We start with a given β^0. Set $s = 0$.

Step 2: For $m = 1$ to M,

$$
\begin{aligned}
\beta^{s+1}_m &= \arg\min_{\omega}\Theta\left(\boldsymbol{c}^s,\boldsymbol{g}^s,\boldsymbol{D},(\beta^s_1,\cdots,\beta^s_{m-1},\omega,\beta^s_{m+1},\cdots,\beta^s_M)\right)\\
s.t. &\quad \sum_{j\in\mathcal{J}_m}\omega_j = 1
\end{aligned}
$$

Step 3: Set $\beta^{s+1} = (\beta^{s+1}_1,\cdots,\beta^{s+1}_m,\cdots,\beta^{s+1}_M)$.

If $\Theta(\boldsymbol{c}^s,\boldsymbol{g}^s,\boldsymbol{D},\beta^s) = \Theta(\boldsymbol{c}^{s+1},\boldsymbol{g}^{s+1},\boldsymbol{D},\beta^{s+1})$, stop. Otherwise, set $s = s+1$ and go to step 2.

Case	U.B.	Opt. U.B.	L.B.	Optimal	U.B. Imprv.	U.B. Gap	L.B. Gap
1	51,919	51,862	50,726	51,079	0.11%	2.34%	0.69%
2	240,851	240,560	237,167	238,552	0.12%	1.54%	0.58%
3	167,802	167,591	165,903	166,498	0.13%	1.14%	0.36%
4	333,460	333,312	330,784	331,452	0.04%	0.81%	0.20%
5	210,881	210,751	209,307	209,669	0.06%	0.75%	0.17%
6	184,413	184,148	182,380	182,739	0.14%	1.11%	0.20%
7	67,495	67,458	66,430	66,809	0.06%	1.59%	0.57%
8	235,379	235,248	233,729	234,282	0.06%	0.70%	0.24%
9	275,597	275,390	273,201	273,585	0.08%	0.88%	0.14%
10	239,552	239,336	235,491	237,007	0.09%	1.71%	0.64%
11	167,580	167,517	165,770	166,139	0.04%	1.09%	0.22%
12	154,500	154,378	152,034	152,701	0.08%	1.61%	0.44%
13	165,112	165,014	163,496	164,078	0.06%	0.98%	0.35%
14	190,372	190,117	187,536	188,467	0.13%	1.50%	0.49%
15	179,718	179,533	177,712	178,266	0.10%	1.12%	0.31%
16	313,616	313,583	311,539	311,603	0.01%	0.67%	0.02%
17	145,682	145,585	143,881	144,249	0.07%	1.25%	0.26%
18	136,057	135,801	133,583	134,326	0.19%	1.84%	0.55%
19	71,308	71,193	70,880	70,951	0.16%	0.60%	0.10%
20	58,513	58,466	56,916	57,521	0.08%	2.78%	1.05%

Table 3.3: Performance of the Block Coordinate Descent method.

The algorithm finds the best β for one product while holding the βs associated with the other products constant. For an analysis of Block Coordinate Descent Method, please refer to [6]. Finally, Proposition 8 suggests a good starting point for the search algorithms.

We test the performance of the Block Coordinate Descent method through a series of randomly generated test cases. We consider a supply chain with 7 products, 14 processes, and 14 resources. We generate random test cases according to the following rules. The demand of a product in each period is a normal random variable with randomly generated mean and standard deviation 10. The formula for generating the demands is given as follows:

$$N(0,1) \times 10 + U[100,120] \times (U[0,2] + 0.5),$$

where $N(0,1)$ is the standard normal distribution and $U[0,2]$ is the uniform distribution between 0 and 2. The price of each product is uniformly distributed between 150 and 300. The price of fixed-price capacity, p_k, is uniformly distributed between 9 and 20. The cost of option capacity, q_k, is uniformly distributed between 1 and p_k. The exercise cost of option capacity is set to $p_k \times 1.1 - q_k$. A link joins a product and process with probability 0.3 (e.g. $Pr(A(j,m) = 1) = 0.3$) and a link joins a process and a resource with probability 0.3 (e.g. $Pr(B(j,(j,k)) = 1) = 0.3$). In each case, we set the sample size to be 500. The termination error percentage is 0.5%. The results are given in Table 3.3. The column "U.B." and "L.B." record the upper bound and feasible solution returned from our algorithm. The column "Opt. U.B." records the upper bound after applying the Block Coordinate Descent Method. The column "Optima" records the optimal solution of the capacity planning problem. Finally,

$$\text{U.B. Imprv.} = \frac{\text{U.B. - Opt. U.B.}}{\text{Optimal}},$$

$$\text{U.B. Gap} = \frac{\text{U.B. - Optimal}}{\text{Optimal}},$$

and

$$\text{L.B. Gap} = \frac{\text{Optimal - L.B.}}{\text{Optimal}}.$$

These test cases illustrate two points:

1. The gap between the upper bound given by the decomposition method and the optimal upper bound is small for these cases.

2. The upper bound generated by the decomposition method is not tight for these cases. For each test case, the upper bound error % is greater than the % improvement obtained from the Block Coordinate Descent method.

A manufacturer might use the approximation algorithm and upper bound when the size of the supply chain and demand samples are large. In these situations, finding the optimal capacity planning strategy is computational infeasible. The manufacturer can use the approximation algorithm to find a sub-optimal capacity planning strategy and then use the upper bound to check whether the strategy is indeed a good one or not.

Finally, the decomposition method presented in this section also suggests that capacity planning can be done locally under the condition that the sub-optimal strategy described above provides a good approximation to the optimal capacity planning strategy. The manufacturer can first calculate the βs using Equation (3.14). Given the βs, the manager of each process can plan the capacity without knowing the capacity decisions for the other processes. When the demand and/or price of a product changes, the manufacturer only needs to recalculate the capacity plans for the processes that are required to produce the product by solving the capacity planning problems (e.g. (3.2)) for these processes. When the cost structure of a process changes, the manufacturer only needs to adjust the capacity plans for all the processes that share some products with this process; the capacity plans of the others can remain unchanged. The performance of these local planning methods depends on the performance of the approximation algorithm, which can be verified using the upper bound presented in this section.

Using local planning can save some overhead costs of changing the capacity strategy of all the processes and can respond quickly to the change of environment. The approximation algorithm suggests an effective method to perform local planning. The manufacturer can evaluate the benefits and costs of using local planning through the upper bound that we have proposed and then decide whether it is a suitable strategy for the firm or not.

Chapter 4

Multi-Period Capacity Planning Problem

In the previous chapters, we have studied the single period capacity planning problem. We now discuss how to extend the single period model to a multi-period setting. In practice, a contract will have a duration. In the existing literature that studies capacity contracts, there are two different ways to model the duration of a contract. If the contracts require a long term commitment, after the firm signs the contract to acquire capacity from its supplier, the firms reserve or buy the same amount of capacity in each period until the end of the planning horizon. On the other hand, if the contracts are short term, the firm can reserve different amounts of capacity for different periods. For example, Huang, et al. [24], Barahona et al. [5], and Martinez-de-Albniz and Simchi-Levi [27] consider long term contracts while Yazlali and Erhun [39] use one-period short term contract.

In the context of the design of a new supply chain, the firm does not own the capacity itself but reserves capacity from its suppliers. The contract does not need to be for either the short term such as one period or the long term such as to the end of the planning horizon. The firm and its suppliers can reach agreement on a duration that is beneficial to both parties. For instance, a supplier might want to offer a contract with median duration and better price to encourage the firm to commit. For the firm, signing a long term contract might be too risky; on the other hand, short term contracts might be too expensive. In this chapter, we will study how the firm should plan its capacity when it has the flexibility to choose the durations of the contracts.

4.1 Model

4.1.1 Mathematical Model

In the single period problem, we can specify each contract with three terms: per-period unit price of the fixed-price capacity, per-period unit price to reserve the option capacity, and per-period unit exercise price of the option capacity. In a multi-period setting, we will add another specification, which is the contract duration. For example, a supplier quotes a three-month contract with fixed-price \$50, option reservation price \$5, and option exercise price \$50 to the manufacturer. The manufacturer decides to reserve 100 units of fixed-price capacity and 20 units of option capacity under this contract. It must pay the price of 100 units fixed-price capacity ($\$50 \times 100 = \$5000$) and 20 units option capacity ($\$5 \times 20 = \$100$) in each of the three consecutive months starting with the first month of the contract. The manufacturer then has 100 units of fixed-price capacity and 20 units of option capacity for each of the three consecutive months.

The prices of the contract can depend on the duration. To encourage a longer commitment, the prices might decrease as the duration of the contract increases. In these situations, the multi-period capacity planning problem involves another type of tradeoff between the flexibility (or duration) of the contract and its price. Contracts with shorter duration have more flexibility while contracts with longer duration offer lower prices.

Let T be the length of the planning horizon. Resource k offers contracts with durations in the set $T_k = \{T_{k,1}, \cdots, T_{k,i}, \cdots\}$. To simplify the notation, we assume that for any resource all contracts have different durations. This assumption can be relaxed and all the results still follow. Without loss of generality, we assume that

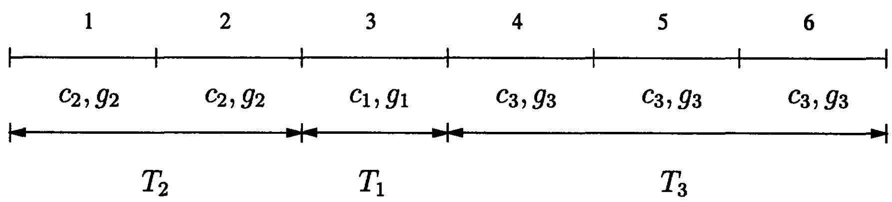

Figure 4-1: Using three capacity contracts with duration 2, 1, and 3 periods to cover a horizon of six periods.

$T_{k,i} < T_{k,j}$ if $i < j$. Therefore, we specify the set of contracts that resource k offers as $\{(p_k(T_{k,i}), q_k(T_{k,i}), e_k(T_{k,i})) \mid T_{k,i} \in T_k\}$.

Given the contracts that each resource offers, we assume that the firm will choose for each resource a sequence of contracts $\mathcal{T}_k = \{\mathcal{T}_{k,1}, \cdots, \mathcal{T}_{k,i}, \cdots\}$ that satisfies the following conditions:

1. Contract $\mathcal{T}_{k,i}$ has duration $t_{k,i}$ and it covers from period $\sum_{j=1}^{i-1} t_{k,j} + 1$ to period $\sum_{j=1}^{i} t_{k,j}$.

2. $\sum_i t_{k,i} = T$ for all k.

The first condition says a contract starts after the previous contract finishes. Condition 2 specifies that the manufacturer does not reserve capacity beyond the planning horizon. We call a sequence feasible if it satisfies these two conditions. One implicit assumption here is that for each period, we have only one contract active for each resource. In addition to deciding the sequence of the contracts for each resource, the manufacturer needs to decide the corresponding sizes: $\{c_{k,1}, \cdots, c_{k,i}, \cdots\}$ and $\{g_{k,1}, \cdots, g_{k,i}, \cdots\}$. We note that we permit zero capacity contracts at zero cost, which allows the firm to not use a resource for any subset of periods. Figure 4-1 gives an example of a valid sequence of contracts to cover a horizon of six periods. The first contract will cover the first two periods. Since the first two periods are cover by the same contract, the fixed-price and total capacity reserved for each of these two periods are the same, which are c_2 and g_2. Similarly, a contract with duration 1 period is used to covered period 3 and a contract with duration 3 periods is used to cover the rest of the horizon.

To simplify the notation and the representation of the multi-period capacity planning problem, we will write a feasible sequence of contracts for resource k as follows:

$$\mathcal{T}_k = \{\mathcal{T}_{k,1}, \cdots, \mathcal{T}_{k,i}, \cdots\}, \text{ where } \mathcal{T}_{k,i} \text{ has duration } t_{k,i} \text{ and } \sum_i t_{k,i} = T$$
$$\{c_{k,1}, \cdots, c_{k,T}\} \text{ and } c_{k,i} = c_{k,j} \text{ if } \exists\, a \text{ such that } i,j \in \left[\sum_{l=1}^{a-1} t_{k,l} + 1, \sum_{l=1}^{a} t_{k,l}\right]$$
$$\{g_{k,1}, \cdots, g_{k,T}\} \text{ and } g_{k,i} = g_{k,j} \text{ if } \exists\, a \text{ such that } i,j \in \left[\sum_{l=1}^{a-1} t_{k,l} + 1, \sum_{l=1}^{a} t_{k,l}\right]$$

We use superscript to indicate time period. Given that the firm has decided its capacity planning strategy, the sequence and sizes of the contracts for each resource, and given a multi-period demand realization vector $\boldsymbol{d}$, we can write the multi-period production planning problem as:

$$\begin{aligned} \max_{\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}} \quad & \hat{\pi}(\mathcal{T}, \boldsymbol{c}, \boldsymbol{g}, \boldsymbol{d}, \boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) = \boldsymbol{r}' \sum_{i=1}^{T} \boldsymbol{z}^i - \sum_{i=1}^{T} (\boldsymbol{e}^i)' H \boldsymbol{y}^i \qquad (4.1) \\ s.t. \quad & \boldsymbol{z}^i \leq \boldsymbol{d}^i, \ \forall\, i \\ & A\boldsymbol{z}^i \leq B(\boldsymbol{x}^i + \boldsymbol{y}^i), \ \forall\, i \\ & H\boldsymbol{x}^i \leq \boldsymbol{c}^i, \ \forall\, i \\ & H(\boldsymbol{x}^i + \boldsymbol{y}^i) \leq \boldsymbol{g}^i, \ \forall\, i \\ & \boldsymbol{x}^i, \boldsymbol{y}^i, \boldsymbol{z}^i \geq \boldsymbol{0}, \ \forall\, i \end{aligned}$$

Similar to the single period case, in a multi-period setting, the firm's ultimate purpose is to choose the strategy to maximize its expected profit with expectation taken over the distribution of the multi-period demand random vector:

$$\begin{aligned} \max_{\mathcal{T},\boldsymbol{c},\boldsymbol{g}} \quad & \hat{\Pi}(\mathcal{T}, \boldsymbol{c}, \boldsymbol{g}, \boldsymbol{D}) = E\left[\hat{\pi}(\mathcal{T}, \boldsymbol{c}, \boldsymbol{g}, \boldsymbol{D}, \boldsymbol{x}^*, \boldsymbol{y}^*, \boldsymbol{z}^*)\right] - \sum_{i=1}^{T} (\boldsymbol{p}^i)' \boldsymbol{c}^i - \sum_{i=1}^{T} (\boldsymbol{q}^i)' (\boldsymbol{g}^i - \boldsymbol{c}^i) \\ s.t. \quad & \boldsymbol{c}^i \leq \boldsymbol{g}^i, \forall\, i \qquad (4.2) \\ & \mathcal{T}_k \text{ are feasible for all } k. \end{aligned}$$

We assume that unfilled demands are lost and unused capacity cannot be saved for future usage. We also assume that the manufacturer will not use any unused capacity to build and store inventory. Even though we do not allow inventory, the multi-period capacity planning problem is not separable since the firm can use a contract to cover multiple periods.

We assume that the manufacturer needs to decide the sequence and sizes of the contracts for each resource at the beginning of the planning horizon. To this extent, we also assume that it has a demand forecast for each period at the beginning of the first period. In practice, capacity decisions usually need to be made with a much longer lead time than the planning horizon. In these situations, our two-stage decision process matches with the reality. Moreover, as we have discussed in the introduction, since the manufacturer doesn't own the capacity, it is important for it to secure the price and supply of the capacity by signing contracts at an early stage. However, this is a restrictive assumption and it would be interesting to study the capacity planning problem in a dynamic setting. We will discuss the dynamic capacity planning problems in Chapter 5.

A strategy in multi-period problem contains two types of decisions: the sequence of contracts to be used and the amount of capacity to acquire after choosing the sequence of contracts. There are an exponential number of combinations of contracts that the manufacturer can choose from. To evaluate one strategy, the firm needs to solve a large scale stochastic linear program, e.g. Problem (4.1), to find the optimal contract sizes. Therefore, the multi-period problem is much more complex than the single period problem.

In the following sections, we will develop an efficient heuristic algorithm that can find a good capacity plan for the multi-period problem under Assumption 1. The same heuristic algorithm will also provide a good upper bound to verify the effectiveness of the capacity plan.

4.1.2 An Example

To illustrate the multi-period capacity planning problem, we consider the supply chain given in Figure 4-2. Since each process has only a dedicated resource, we view process and resource as synonymous. The manufacturer needs to plan its capacity for the next 12 months. The expectations of the demands during the planning horizon

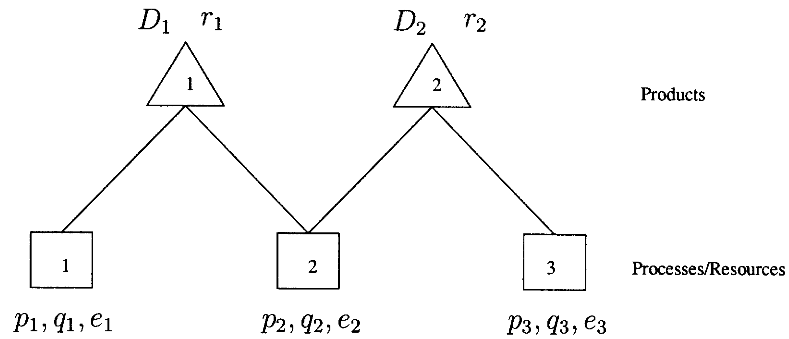

Figure 4-2: A supply chain with two products and three processes/resources.

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
$E[D_1]$	70	100	180	210	240	240	230	180	100	70	60	50
$E[D_2]$	240	230	180	100	70	60	50	70	100	180	210	240

Table 4.1: Multi-period example: demand information.

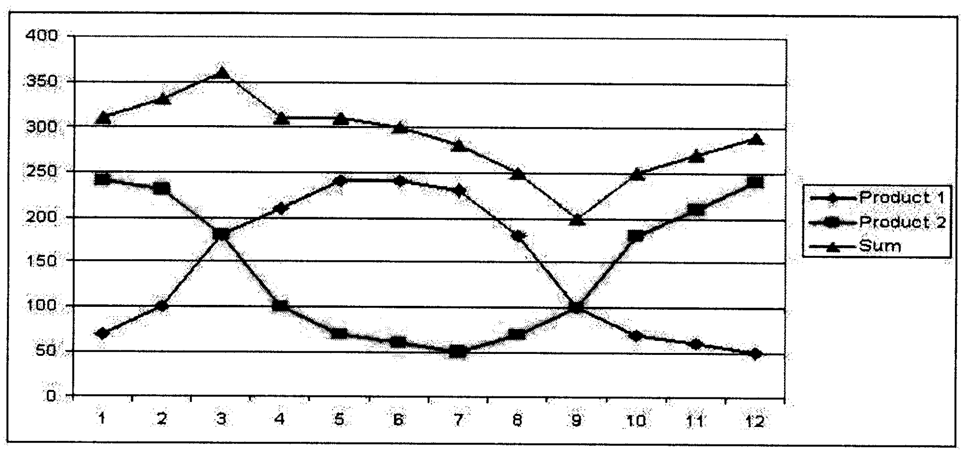

Figure 4-3: Multi-period example – demand patterns.

Duration	1	3	6	12
Fixed Price	10	9.5	9	8.5
Reservation Price	8	7.5	7.25	6.75
Exercise Price	3	2.5	2.25	2.25

Table 4.2: Multi-period example – contracts' durations and prices.

are given in Table 4.1. We also plot the demand trends in Figure 4-3. Product 1 is introduced to the market at the beginning of the first month. Its demand grows with time and reaches its peak at the fifth month. After that, the market is saturated and the demand starts to drop. Product 2, on the other hand, is a mature product at the beginning and as time passes by, it phases out. At the seventh month, the manufacturer introduces a new version of product 2 and it starts to gain more demand from then on. The standard deviations of the demands of both products at each period are 10.

Both products are sold at $65. All processes have the same price structure. Each process offers contracts in four different durations: 1 month, 3 months, 6 months, and 12 months. The corresponding prices of the fixed-price and option contract are given in Table 4.2. The contracts with longer duration have lower per-period prices.

Given the supply chain structure (Figure 4-2), demand information (Table 4.1), and contract information (Table 4.2), the manufacturer needs to make the following decisions:

1. what sequence of contracts that it should use for each process,

2. what types of contract (fixed-price and option) that it should use, and

3. how much capacity it should reserve or buy for each type of contract.

Decision 2 and 3 are the same as in the single period case while decision 1 is unique to the multi-period problem. Since the example only contains dedicated resources, the manufacturer does not need to choose suppliers. However, similar to the single period problem, the firm still faces the other trade-offs that involve demand uncertainty, common process, coordination among the processes of the same product, and option capacity. Moreover, the manufacturer needs to consider the trade-off between contract flexibility and prices. Should it use shorter contracts to match the demand or should it take advantage of lower prices by using longer contracts?

For this example, the sequences of the contracts for the processes suggested by our algorithm are given in Table 4.3:

1. For process 1, the manufacturer should use two 1-month contracts to cover the first two periods. It can then obtain a 6-month contract to cover month 3 to month 8. Following another 1-month contract in month 9, it should get a 3-month contract to cover the rest of the planning horizon.

2. For process 2, the manufacturer should take full advantage of the low price from a longer contract and secure the capacity for 12 months with the 12-month contract.

3. For process 3, the manufacturer should use a 3-month contract to cover month 5, 6, and 7. For the other months, it should use 1-month contracts.

The quantity of the fixed-price and option contract for each process are given in Figure 4-4, 4-5, and 4-6. We see that the contracts reserved for process 3 vary to match the demands. On the other hand, the contract reserved for process 2 is fixed over the horizon and doesn't fluctuate with the demand. We also notice that for the contracts with a long duration, the option capacity component is significant. We will discuss this in Section 4.5.

Finding the right level of flexibility, in terms of shorter contracts and/or in the use of option contracts, is a complex problem that needs to consider demand variability, product profits, contract durations, and contract prices. In the remainder of this chapter, we will look at an efficient algorithm that can help the manufacturer to make these decisions.

	1	2	3	4	5	6	7	8	9	10	11	12
Process 1	69	99	247	247	247	247	247	247	105	72	72	72
Process 2	325	325	325	325	325	325	325	325	325	325	325	325
Process 3	242	236	166	115	74	74	74	83	110	199	217	248

Table 4.3: Multi-period example – the sequences of contracts and capacity strategies for all processes suggested by our algorithm.

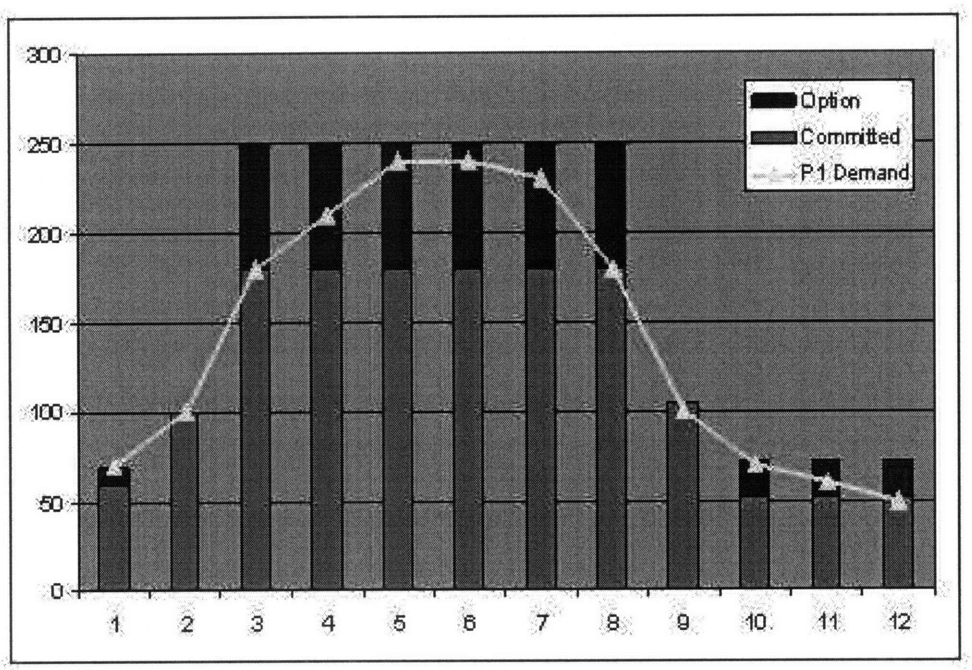

Figure 4-4: Multi-period example – the sequence of contracts and capacity strategy for process 1 suggested by our algorithm.

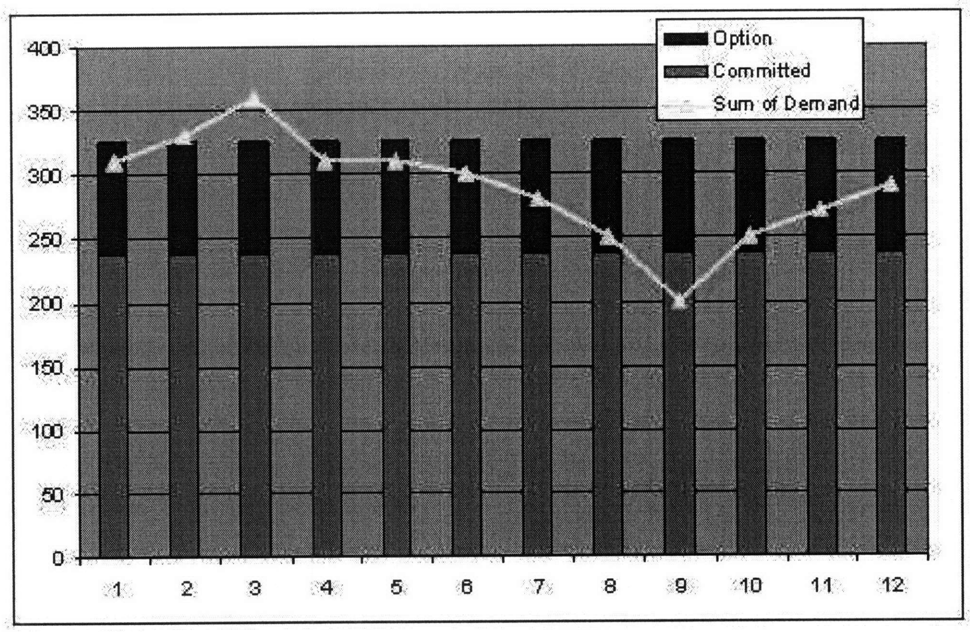

Figure 4-5: Multi-period example – the sequence of contracts and capacity strategy for process 2 suggested by our algorithm.

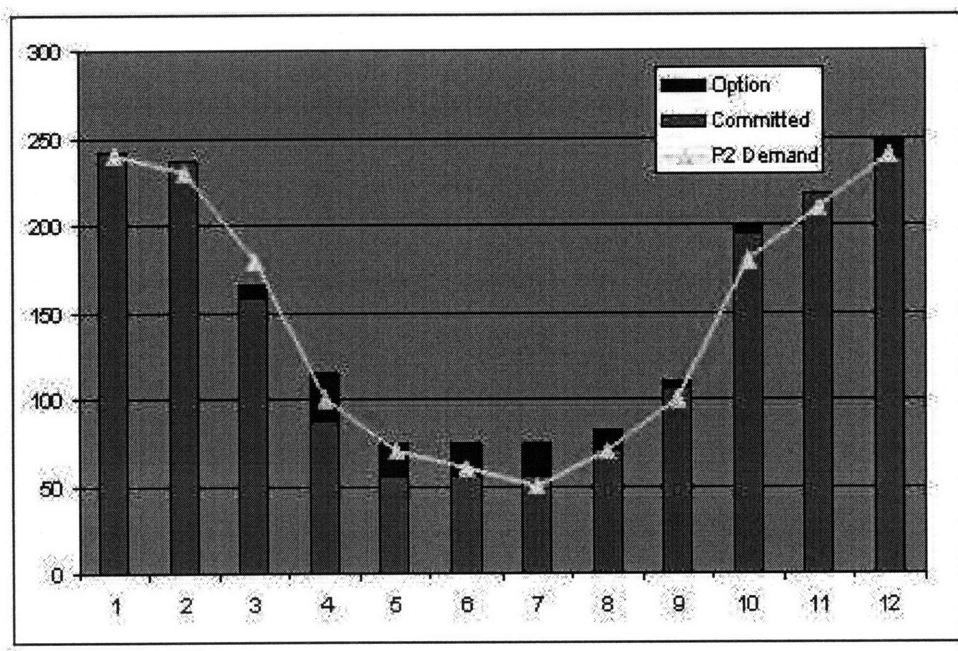

Figure 4-6: Multi-period example: the sequence of contracts and capacity strategy for process 3 suggested by our algorithm.

4.2 Special Case: Multiple Products and One Process.

We first consider a special case where there are multiple products, one process, and one resource. In this section, we will present an efficient algorithm to solve this special case. Since there is only one process and one resource, we will drop the subscript j. The capacity planning problem is then given as:

$$\begin{aligned} \max_{\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}_{m\in\mathcal{M}}} \quad & \hat{\pi}(\mathcal{T}, \boldsymbol{c}, \boldsymbol{g}, \boldsymbol{d}, \boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}_{m\in\mathcal{M}}) = r\sum_{i=1}^{T}\sum_{m\in\mathcal{M}} z_m^i - \sum_{i=1}^{T} e^i y^i \qquad (4.3) \\ s.t. \quad & z_m^i \leq d_m^i, \ \forall\, i, m \\ & \sum_{m\in\mathcal{M}} z_m^i \leq x^i + y^i, \ \forall\, i \\ & x^i \leq c^i, \ \forall\, i \\ & x^i + y^i \leq g^i, \ \forall\, i \\ & x^i, y^i, z^i \geq 0, \ \forall\, i \end{aligned}$$

and

$$\begin{aligned} \max_{\mathcal{T}} \quad & \hat{\Pi}(\mathcal{T}, \boldsymbol{c}, \boldsymbol{g}, \boldsymbol{D}) = E\left[\hat{\pi}(\mathcal{T}, \boldsymbol{c}, \boldsymbol{g}, \boldsymbol{D}, \boldsymbol{x}^*, \boldsymbol{y}^*, \boldsymbol{z}^*)\right] - \sum_{i=1}^{T} p^i c^i - \sum_{i=1}^{T} q^i (g^i - c^i) \qquad (4.4) \\ s.t. \quad & c^i \leq g^i, \ \forall\, i \\ & \mathcal{T} \text{ is feasible.} \end{aligned}$$

We can transform Problem (4.4) into a directed shortest path problem. For each period i, we denote a vertex v_i. Denote v_0 to be the vertex representing period 0. Let $\mathcal{A} = \{v_0, \cdots, v_T\}$ be the set of vertices. Let $|\mathcal{T}|$ be the cardinality of set $\mathcal{T}$. There are $|\mathcal{T}|$ types of contracts and each of them has a different duration. At vertex v_i, for each contract $\mathcal{T}_l$ such that $t_l + i \leq T$, where t_l is the duration of contract $\mathcal{T}_l$, we add a link joining vertex v_i and v_{i+t_l}. Let $\mathcal{E}$ be the set of links.

We now show how to find the cost for each link. We consider a link joining vertex v_i and v_j, where $i < j$. Let $\mathcal{T}_l$ be the corresponding contract for the link. We consider the following optimization problems:

$$\begin{aligned}
\max_{\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}} \quad & \lambda_l(c, g, \boldsymbol{d}) = \sum_{s=i+1}^{j} \sum_{m \in \mathcal{M}} r_m z_m^s - e_l \sum_{s=i+1}^{j} y^s \\
s.t. \quad & z_m^s \leq d_m^s, \ s = i+1, \cdots, j \text{ and } m \in \mathcal{M} \\
& \sum_{m \in \mathcal{M}} z_m^s \leq x^s + y^s, \ s = i+1, \cdots, j \\
& x^s \leq c, \ s = i+1, \cdots, j \\
& x^s + y^s \leq g, \ s = i+1, \cdots, j \\
& x^s, y^s, z_m^s \geq 0, \ s = i+1, \cdots, j
\end{aligned} \tag{4.5}$$

and

$$\begin{aligned}
\max_{c,g} \quad & \Lambda_l(c, g) = E\left[\lambda_l(c, g, \boldsymbol{D})\right] - c t_l p_l - (g - c) t_l q_l \\
s.t. \quad & c \leq g, \\
& c, g \geq 0.
\end{aligned} \tag{4.6}$$

Problem (4.6) is a linear program. Moreover, similar to the single period case, Problem (4.6) is concave and only has two decision variables, (c, g). Therefore, we can use the algorithm that we have proposed in Section 2.3 for the single period problem to solve Problem (4.6).

Let $a_{i,j}$ be the optimal objective value of Problem (4.6). We denote the cost of the link joining v_i and v_j to be $-a_{i,j}$. We will then have the following proposition.

Proposition 11 *Let $(\mathcal{A}, \mathcal{E})$ be a directed graph that is constructed as above. Let $\mathcal{P}$ be a shortest path from v_0 to v_T. For the l^{th} link on the shortest path joining vertices v_i and v_j, define a contract $\mathcal{T}_l$ with duration $j - i$ that covers from period $i + 1$ to j. We set the arc cost $a_{i,j}$ to be equal to $-\Lambda_l(c_l^*, g_l^*)$, $\Lambda_l(c_l^*, g_l^*)$ is the objective value of Problem (4.6), where (c_l^*, g_l^*) is an optimal choice of (c_l, g_l). Then the capacity planning strategy $\{(\mathcal{T}_1, c_1^*, g_1^*), \cdots, (\mathcal{T}_l, c_l^*, g_l^*), \cdots\}$ is an optimal solution of Problem (4.4).*

The correctness of Proposition 11 follows from the way that we construct the shortest path problem. By this proposition, we can solve the shortest path problem on $(\mathcal{A}, \mathcal{E})$ to find an optimal solution of Problem (4.4). This special case together with the decomposition idea presented in Chapter 3 provide the building blocks to solve the general multi-period problem.

4.3 Solving the General Multi-period Problem

The main difference from the single period case is that the multi-period capacity planning problem needs to decide the sequence of the contracts. The amount of capacity that needs to be reserved depends on the contract sequence that the firm has chosen. If we fix the sequence for each process, finding the optimal contract sizes is a stochastic linear programming problem that is very similar to the single period capacity planning problem, which we can solve using the algorithm that we proposed in Section 2.3.

The difficulty of solving the multi-period problem lies in the fact that there are a large number of combinations of contract sequences that the firm can choose from. The algorithm that we proposed for the single period problem is effective, but it still requires a considerable amount of computational power. Therefore, in this section we will develop an efficient heuristic algorithm for the general multi-period capacity planning problem under the assumption that each process only has one dedicated resource (e.g. Assumption 1 from Chapter 3 holds).

The idea is to separate the decision of choosing the contract sequence from finding the optimal contract sizes. The algorithm consists of the following steps:

1. We use the decomposition method proposed in Chapter 3 to separate the original multi-period capacity planning problem into independent sub-problems, with one multi-period problem for each process.

2. We solve each multi-period sub-problem to find a feasible contract sequence for each process. This provides an initial feasible solution.

3. We fix the contract sequence for each process and then find the optimal contract

sizes. This provides an improvement to the initial solution.

We now formalize the algorithm. Recall that in the decomposition algorithm presented in Chapter 3, we distribute the revenue of each product into each process based on the prices of the contracts for the process. We will use the same method to separate the multi-period problem. However, in the multi-period problem, each process has multiple sets of prices, with one for each contract duration. Therefore, for each process, we will use the average prices over all the contract durations in the decomposition method. Let

$$\bar{p}_j = \frac{1}{|T_j|}\sum_{T_{j,i}\in T_j} p_j(T_{j,i}),\ \bar{q}_j = \frac{1}{|T_j|}\sum_{T_{j,i}\in T_j} q_j(T_{j,i}),\ \bar{e}_j = \frac{1}{|T_j|}\sum_{T_{j,i}\in T_j} e_j(T_{j,i}).$$

$\bar{p}_j$, $\bar{q}_j$, and $\bar{e}_j$ are the average unit price of fixed-price capacity, average unit option reservation price, and average unit option exercise price for process j. Without loss of generality, we assume that for all $i, j \in \mathcal{J}_m$

$$\frac{\bar{p}_i - \bar{q}_i}{\bar{e}_i} \geq \frac{\bar{p}_j - \bar{q}_j}{\bar{e}_j}, \text{ if } i \geq j.$$

For each product m, we define

$$\overline{\psi}_m = \begin{cases} J_m + 1, & \text{if } \frac{\bar{p}_{J_m}-\bar{q}_{J_m}}{\bar{e}_{J_m}} \leq \frac{\sum_{j=1}^{J_m}\bar{p}_j}{r_m}; \\ \min\left\{ i \mid \frac{\bar{p}_i-\bar{q}_i}{\bar{e}_i} > \frac{\sum_{j=1}^{i-1}\bar{p}_j+\sum_{j=i}^{J_m}\bar{q}_j}{r_m-\sum_{j=i}^{J_m}\bar{e}_j} \right\}, & \text{otherwise.} \end{cases}$$

For each product m, we define the following ratio

$$\bar{\alpha}_m = \frac{\sum_{j=1}^{\bar{\psi}_m-1}\bar{p}_j + \sum_{j=\bar{\psi}_m}^{J_m}\bar{q}_j}{r_m - \sum_{j=\bar{\psi}_m}^{J}\bar{e}_j}.$$

For each product m, $\overline{\psi}_m$ and $\overline{\alpha}_m$ are the multi-period counterparts of ψ_m and α_m in the decomposition algorithm in Chapter 3. The difference is that we use the average prices of the processes in $\overline{\psi}_m$ and $\overline{\alpha}_m$.

Given $\overline{\psi}_m$ and $\overline{\alpha}_m$ for all products, we will have the following heuristic algorithm for solving the general multi-period capacity planning problem under Assumption 1.

Heuristic Algorithm for Solving Multi-period Problem:

Stage I: (Finding the sequences of the contracts and a feasible solution) For each process j, do the following steps:

Step 1: For each $m \in \mathcal{M}_j$, let

$$r_{m,j} = \frac{\bar{q}_j}{\bar{\alpha}_m} + \bar{e}_j.$$

Step 2: Solve the multiple products and single process multi-period capacity planning problem (4.4) using the algorithm developed in Section 4.2. Let $(\mathcal{T}_j^*, \boldsymbol{c}_j^*, \boldsymbol{g}_j^*)$ be an optimal solution.

Stage II: (Improving the feasible solution) Fix $\mathcal{T} = \left\{\mathcal{T}_j^* \mid j = 1, \cdots, J\right\}$. Solve Problem (4.2) for given $\mathcal{T}$ with the stochastic supporting hyperplane algorithm with pre-solve routine for the single period problem.

In stage I, $r_{m,j}$ is the ratio of the revenue of product m that is assigned to process j. In stage II, after we fix the contract sequences, the optimization problem of finding optimal contract sizes is a stochastic linear program that is similar to the single period capacity planning problem. In particular, it is a two-stage optimization problem. For each capacity plan, $(\boldsymbol{c}, \boldsymbol{g})$, and demand realization, $\boldsymbol{d}$, finding the production levels to maximize the profit is a linear optimization problem. The first stage problem, which is finding the optimal capacity sizes to maximize the expected profit over multi-period random vector $\boldsymbol{D}$, is a concave optimization problem. Since for each process, the contract sequences are fixed, we know which contract will be used in each period. For the periods that are covered by the same contract, we use one set of decision variables, (c, g), to enforce that the same capacity will be chosen in each period. This, however, will not affect the algorithm to solve the problem.

After stage I of the algorithm, we will have a feasible solution, $(\mathcal{T}^*, \boldsymbol{c}^*, \boldsymbol{g}^*)$. In stage II, the algorithm fixes the sequences, $\mathcal{T}^*$, and finds the optimal sizes of the contracts. Since the sequences found in stage I of the algorithm might not be optimal, the algorithm can not guarantee optimality. To access its performance, we need to derive an upper bound of the multi-period problem.

4.4 An Upper Bound

We have shown that the decomposition method we have proposed in Chapter 3 provides not only a feasible solution but an upper bound for the single period capacity planning problem. In this section, we will extend the method to a multi-period setting. In fact, we use the decomposition method in stage I of the proposed heuristic algorithm to solve the general multi-period capacity planning problem in Section 4.3. Therefore, after stage I of the heuristic algorithm, we have not only a feasible solution but also an upper bound of the problem.

We now provide a mathematical justification that the method indeed generates an upper bound in the multi-period case. For product m, process j, and period i, we define $\beta^i_{j,m}$ to be a fixed real number. We consider the following optimization problems:

$$\begin{aligned}
\max_{\boldsymbol{x}_j, \boldsymbol{y}_j, \boldsymbol{z}_m : m\in\mathcal{M}_j} \quad & \hat{\theta}_j(\mathcal{T}_j, \boldsymbol{c}_j, \boldsymbol{g}_j, \boldsymbol{d}, \boldsymbol{x}_j, \boldsymbol{y}_j, \boldsymbol{z}_{m:m\in\mathcal{M}_j}, \beta_j) = \sum_{i=1}^{T}\left(\sum_{m\in\mathcal{M}_j} \beta^i_{j,m} r_m z^i_m - e^i_j y^i_j\right) \\
s.t. \quad & z^i_m \le d^i_m, \ \forall\, m \in \mathcal{M}_j, \ i = 1, \cdots, T \qquad (4.7) \\
& \sum_{m\in\mathcal{M}_j} z^i_m \le x^i_j + y^i_j, \ i = 1, \cdots, T \\
& x^i_j \le c^i_j, \ i = 1, \cdots, T \\
& x^i_j + y^i_j \le g^i_j, \ i = 1, \cdots, T \\
& x^i_j, y^i_j, \ i = 1, \cdots, T \ge 0 \\
& z^i_m \ge 0, \ \forall\, m \in \mathcal{M}_j, \ i = 1, \cdots, T
\end{aligned}$$

and

$$\max_{\mathcal{T}_j, \boldsymbol{c}_j, \boldsymbol{g}_j} \quad \hat{\Theta}_j(\mathcal{T}_j, \boldsymbol{c}_j, \boldsymbol{g}_j, \boldsymbol{D}, \beta) =$$

$$
\begin{aligned}
& E\left[\hat{\theta}_j(\mathcal{T}_j, \boldsymbol{c}_j, \boldsymbol{g}_j, \boldsymbol{D}, \boldsymbol{x}_j^*, \boldsymbol{y}_j^*, \boldsymbol{z}_{m:m\in\mathcal{M}_j}^*, \beta_j)\right] - \sum_{i=1}^{T} p_j^i c_j^i - \sum_{i=1}^{T} q_j^i (g_j^i - c_j^i) \\
s.t.\quad & c_j^i \leq g_j^i,\ i = 1, \cdots, T \qquad (4.8)\\
& c_j^i, g_j^i \geq 0,\ i = 1, \cdots, T \\
& \mathcal{T}_j \text{ is feasible.}
\end{aligned}
$$

Equation (4.7) and (4.8) are the multi-period counterparts of Equation (3.1) and (3.2). They are the optimization problems to find the optimal contract sequence and capacity sizes for a sub-problem after the decomposition.

Similarly, we define the multi-period counterparts of Equation (3.3) and (3.4) as

$$
\hat{\theta}(\mathcal{T}, \boldsymbol{c}, \boldsymbol{g}, \boldsymbol{d}, \boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}, \beta) = \sum_{j=1}^{J} \hat{\theta}_j(\mathcal{T}_j, \boldsymbol{c}_j, \boldsymbol{g}_j, \boldsymbol{d}_j, \boldsymbol{x}_j, \boldsymbol{y}_j, \boldsymbol{z}_{m:m\in\mathcal{M}_j}, \beta_j) \qquad (4.9)
$$

and

$$
\hat{\Theta}(\mathcal{T}, \boldsymbol{c}, \boldsymbol{g}, \boldsymbol{D}, \beta) = \sum_{j=1}^{J} \hat{\Theta}_j(\mathcal{T}_j, \boldsymbol{c}_j, \boldsymbol{g}_j, \boldsymbol{D}, \beta_j). \qquad (4.10)
$$

These are the optimization problems after the decomposition. We then can extend Proposition 7, 8, and 10 into a multi-period setting.

Proposition 12 *If for each product m*

$$
\sum_{j\in\mathcal{J}_m} \beta_{j,m}^i = 1,\ \forall\ i = 1, \cdots, T \qquad (4.11)
$$

then $\hat{\Theta}(\overline{\mathcal{T}}^*, \overline{\boldsymbol{c}}^*, \overline{\boldsymbol{g}}^*, \boldsymbol{D}, \beta) \geq \hat{\Pi}(\mathcal{T}^*, \boldsymbol{c}^*, \boldsymbol{g}^*, \boldsymbol{D})$, where $(\overline{\mathcal{T}}^*, \overline{\boldsymbol{c}}^*, \overline{\boldsymbol{g}}^*)$ is the optimal solution of Problem (4.10).

For each process j, suppose we define p_j, q_j, and e_j as positive real numbers such that

$$
q_j < p_j < q_j + e_j.
$$

p_j, q_j, and e_j are the dummy or arbitrary prices for process j. One choice can be using the average prices over all the contracts as we did in the heuristic algorithm given in Section 4.3. Without loss of generality, we assume that for all $i, j \in \mathcal{J}_m$

$$\frac{p_i - q_i}{e_i} \geq \frac{p_j - q_j}{e_j}, \text{ if } i \geq j.$$

For each product m, define

$$\psi_m = \begin{cases} J_m + 1, & \text{if } \frac{p_{J_m} - q_{J_m}}{e_{J_m}} \leq \frac{\sum_{j=1}^{J_m} p_j}{r}; \\ \min\left\{ i \mid \frac{p_i - q_i}{e_i} > \frac{\sum_{j=1}^{i-1} p_j + \sum_{j=i}^{J_m} q_j}{r - \sum_{j=i}^{J_m} e_j} \right\}, & \text{otherwise.} \end{cases} \tag{4.12}$$

Also, for each product m, we define the following ratio

$$\alpha_m = \frac{\sum_{j=1}^{\psi_m - 1} p_j + \sum_{j=\psi_m}^{J_m} q_j}{r_m - \sum_{j=\psi_m}^{J} e_j}. \tag{4.13}$$

We then have the following proposition:

Proposition 13 *For each process j and m such that $A(j, m) = 1$, if we set*

$$\beta_{j,m} = \frac{q_j}{\alpha_m r_m} + \frac{e_j}{r_m}, \tag{4.14}$$

then $\hat{\Theta}(\overline{\mathcal{T}}^*, \overline{\boldsymbol{c}}^*, \overline{\boldsymbol{g}}^*, \boldsymbol{D}, \beta) \geq \hat{\Pi}(\mathcal{T}^*, \boldsymbol{c}^*, \boldsymbol{g}^*, \boldsymbol{D})$.

Technically, one can use different dummy prices and β's in different periods. As long as these dummy prices β's satisfy Equation (4.12), (4.13), and (4.14), the heuristic algorithm will provide an upper bound to the problem. However, through a series of test cases, we will show that using the same β for all periods provides a good upper bound.

Finally, we have

Proposition 14 $\hat{\Theta}(\overline{\mathcal{T}}^*, \overline{\boldsymbol{c}}^*, \overline{\boldsymbol{g}}^*, \boldsymbol{D}, \beta)$ *is convex in β for β that satisfies Equation (4.14).*

We skip the proofs of these propositions since they are very similar to their counterparts in the single period problem.

During the stage I of the heuristic algorithm given in Section 4.3, we set

$$\beta^i_{j,m} = \frac{\bar{q}_j}{\bar{\alpha}_m r_m} + \frac{\bar{e}_j}{r_m} \;\forall\; i = 1, \cdots, T.$$

where $\bar{p}_j$, $\bar{q}_j$, and $\bar{e}_j$ are the average price of fixed-price contract, average option reservation price, and average option exercise price for process j. These β's satisfy Equation (4.14) and, therefore, by Proposition 12, the heuristic algorithm provides an upper bound to the multi-period capacity planning problem.

4.5 Simulation Results

In the last section of studying the multi-period capacity planning problem, we will access the effectiveness of the feasible solution and the tightness of the upper bound generated by our heuristic algorithm through several sets of test cases.

4.5.1 Test Cases Set I

We first test the heuristic algorithm and the upper bound with a series of randomly generated test cases. The purpose of this test case is to see whether the algorithm can handle an arbitrary randomly generated test case. The supply chain contains 7 products, 14 processes, and 14 resources. The planning horizon is 12 periods. A link joins a product and process with probability 0.3 (e.g. $Pr(A(j,m) = 1) = 0.3$). Therefore, on average, each product has 4.2 processes. Each process has one dedicated resource. (e.g. Assumption 1 holds)

The price of each product is drawn from a uniform distribution between 90 and 300, $U[90, 300]$, and does not change during the planning horizon. The demand of a product in each period is a normal random variable with randomly mean generated using the equation

$$U[100, 120] \times (U[0, 2] + 0.5),$$

where $N(0, 1)$ is the standard normal distribution and $U[0, 2]$ is the uniform distribution between 0 and 2, and standard deviation 10. The random mean is to make the

Test Case	Upper Bound	L.B. (Stage I)	L.B. (Stage II)	Error 1	Error 2
1	2,483,546.46	2,468,392.44	2,465,094.85	0.61%	0.61%
2	2,394,320.75	2,357,421.35	2,375,587.45	1.55%	0.79%
3	2,043,868.69	2,003,346.63	2,022,023.93	2.00%	1.08%
4	1,993,805.29	1,971,762.12	1,979,577.58	1.11%	0.72%
5	2,166,160.90	2,128,124.32	2,145,013.19	1.77%	0.99%
6	2,211,052.61	2,199,631.34	2,195,965.66	0.52%	0.52%
7	2,123,398.37	2,084,819.65	2,104,994.97	1.83%	0.87%
8	2,190,897.34	2,153,455.11	2,171,868.80	1.72%	0.88%
9	1,701,365.48	1,656,352.74	1,671,653.21	2.69%	1.78%
10	1,829,860.87	1,791,223.88	1,809,531.30	2.14%	1.12%
11	2,640,927.51	2,628,109.55	2,622,219.53	0.49%	0.49%
12	2,985,632.64	2,968,501.49	2,963,897.74	0.58%	0.58%
13	2,188,717.78	2,148,047.04	2,167,121.27	1.88%	1.00%
14	2,136,158.83	2,103,736.04	2,120,432.48	1.53%	0.74%
15	2,174,388.97	2,149,120.58	2,156,085.91	1.17%	0.85%
16	2,113,909.16	2,046,332.41	2,083,342.59	3.24%	1.47%
17	2,347,750.33	2,320,086.51	2,328,738.35	1.19%	0.82%
18	2,498,375.49	2,466,197.72	2,480,468.00	1.30%	0.72%
19	2,604,573.37	2,580,315.64	2,586,975.22	0.94%	0.68%
20	2,314,434.66	2,289,079.17	2,295,462.86	1.10%	0.83%
21	1,734,822.69	1,708,874.70	1,719,531.13	1.51%	0.89%
22	1,924,675.12	1,901,229.18	1,899,456.05	1.23%	1.23%
23	1,655,698.55	1,632,984.93	1,642,122.71	1.38%	0.83%
24	2,280,561.56	2,269,747.72	2,263,592.38	0.48%	0.48%
25	2,112,209.18	2,080,822.34	2,079,590.78	1.51%	1.51%
26	2,366,356.78	2,348,906.54	2,349,139.19	0.74%	0.73%
27	2,462,377.79	2,432,675.38	2,444,430.41	1.22%	0.73%
28	1,975,885.64	1,958,265.54	1,961,730.67	0.90%	0.72%
29	2,250,434.37	2,194,936.68	2,219,381.51	2.50%	1.40%
30	2,207,470.52	2,191,356.12	2,191,129.98	0.74%	0.74%
31	1,891,005.39	1,850,118.67	1,870,039.09	2.19%	1.12%
32	2,608,564.41	2,589,269.32	2,589,360.76	0.75%	0.74%
33	1,777,993.68	1,744,348.93	1,760,864.34	1.91%	0.97%
34	2,053,558.49	2,033,223.98	2,035,520.57	1.00%	0.89%
35	2,257,567.80	2,215,874.55	2,230,411.49	1.87%	1.22%
36	2,736,309.07	2,719,452.35	2,718,618.07	0.62%	0.62%
37	1,940,960.31	1,900,610.28	1,920,815.59	2.10%	1.05%
38	2,391,125.96	2,370,158.24	2,374,307.29	0.88%	0.71%
39	2,462,064.01	2,448,751.88	2,446,685.36	0.54%	0.54%
40	2,535,908.13	2,518,775.94	2,519,704.52	0.68%	0.64%

Table 4.4: Multi-period test set I results

demands not stationary. Therefore, the formula for generating the demand is given as follows:

$$N(0,1) \times 10 + U[100,120] \times (U[0,2] + 0.5). \tag{4.15}$$

Each process has four contract durations: 1-period, 3-period, 6-period, and 12-period. The unit price of the fixed-price 1-period capacity, $p_{1-period}$, is uniformly distributed between 9 and 12, $U[9,12]$. The price of the option capacity, $q_{1-period}$, is uniformly distributed between 1 and $p_{1-period}$. The price to exercise the option capacity is $p_{1-period} \times 1.1 - q_{1-period}$. The price of the fixed-price capacity, the price to reserve the option capacity, and the price to exercise the option capacity with 3-period duration are 90% of their 1-period counterparts. Similarly, the prices of the capacity with 6-period duration are 90% of their 3-period counterparts and the prices of the capacity with 12-period duration are 90% of their 6-period counterparts.

We randomly generate 40 test cases and in each case we use 500 sample demands. The terminating error used by the stochastic supporting hyperplance algorithm in stage II of the heuristic algorithm is 0.5%.

We define Error 1 as

$$\text{Error 1} = \frac{\text{Upper Bound - Lower Bound (Stage I)}}{\text{Lower Bound (Stage II)}} \times 100\%$$

and it signifies the maximal gap between the upper bound and the feasible solution obtained at stage I of the heuristic algorithm. Similarly, we define Error 2 as

$$\text{Error 2} = \frac{\text{Upper Bound - Lower Bound (Stage II)}}{\text{Lower Bound (Stage II)}} \times 100\%$$

and it signifies the maximal gap between the upper bound and the feasible solution obtained at stage II of the algorithm. The results of these test cases are given in Table 4.4. We summarize the statistics of Error 1 and Error 2 in Table 4.5.

From these results, we see that the feasible solution obtained at stage II is at least as good as the feasible solution obtained at stage I. This is because at stage II we improve the feasible solution from stage I by finding better contract sizes. Finally, we

	Average	STD	Minimum	Maximum
Error 1	1.35%	0.67%	0.48%	3.24%
Error 2	0.88%	0.29%	0.48%	1.78%

Table 4.5: Multi-period test set I statistics

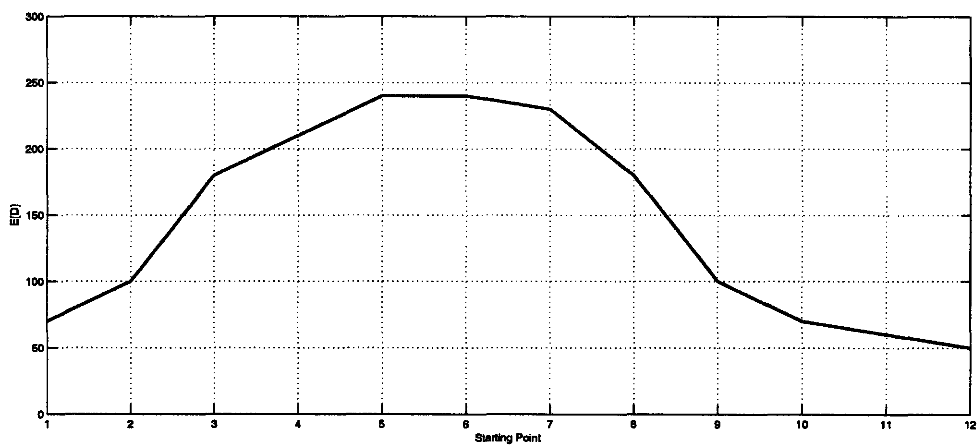

Figure 4-7: Test set II demand pattern.

see that both feasible solutions perform fairly well in these test cases. In particular, the best feasible solution from our algorithm (which is the one obtained at stage II) has a maximal error less than 1.78% for these test problems.

4.5.2 Test Cases Set II

In test set I, the demands in different periods are independent of each other. In practice, the demand of a product might follow a pattern. We will incorporate this into the second set of test cases. We consider a basic demand pattern given in Figure 4-7. The figure plots the demand expectations over time. In test set II, the expectation of the demand of a product follows this basic pattern but with a randomly selected starting time. The starting time of a product is an integer generated from the uniform distribution between 1 and 12. Once time reaches point 12, the pattern will continue at point 1 again. For example, if the product demand pattern starts at point 3, the demand expectations for the 12-period horizon are given in the Figure

Test Case	Upper Bound	L.B. (Stage I)	L.B. (Stage II)	Error 1	Error 2
1	2,220,875.05	2,197,374.22	2,204,298.55	1.07%	0.75%
2	1,820,467.28	1,791,086.83	1,804,414.39	1.63%	0.89%
3	1,849,816.70	1,811,127.16	1,826,780.58	2.12%	1.26%
4	1,862,508.07	1,818,627.52	1,836,907.26	2.39%	1.39%
5	1,501,790.50	1,425,630.41	1,462,517.52	5.21%	2.69%
6	1,860,200.26	1,805,789.10	1,833,393.48	2.97%	1.46%
7	1,907,669.79	1,850,650.33	1,884,928.64	3.03%	1.21%
8	1,980,639.40	1,922,467.02	1,954,121.66	2.98%	1.36%
9	1,564,783.21	1,533,766.24	1,543,009.10	2.01%	1.41%
10	1,928,277.42	1,873,433.61	1,900,714.43	2.89%	1.45%
11	1,661,543.47	1,631,649.59	1,645,069.79	1.82%	1.00%
12	1,469,706.39	1,431,471.32	1,453,741.71	2.63%	1.10%
13	1,989,994.06	1,966,449.57	1,975,519.32	1.19%	0.73%
14	2,250,694.49	2,221,790.46	2,232,433.86	1.29%	0.82%
15	1,673,039.09	1,629,135.14	1,650,760.61	2.66%	1.35%
16	1,824,120.62	1,776,410.14	1,809,271.19	2.64%	0.82%
17	1,889,576.04	1,862,799.62	1,872,648.47	1.43%	0.90%
18	1,750,001.04	1,685,781.88	1,717,299.01	3.74%	1.90%
19	1,894,197.21	1,873,691.20	1,880,176.19	1.09%	0.75%
20	1,686,882.06	1,647,184.35	1,673,620.82	2.37%	0.79%
21	1,070,772.54	1,011,322.94	1,038,815.64	5.72%	3.08%
22	1,843,727.08	1,771,433.65	1,801,964.11	4.01%	2.32%
23	1,834,977.92	1,756,590.34	1,791,352.93	4.38%	2.44%
24	2,074,227.25	2,022,467.66	2,051,490.02	2.52%	1.11%
25	1,910,700.69	1,883,052.87	1,897,355.71	1.46%	0.70%
26	1,803,831.21	1,768,291.24	1,784,339.64	1.99%	1.09%
27	1,749,538.08	1,713,476.69	1,732,218.06	2.08%	1.00%
28	2,106,467.52	2,078,877.06	2,091,720.24	1.32%	0.71%
29	1,919,851.20	1,908,141.86	1,905,516.58	0.61%	0.61%
30	1,686,164.71	1,639,476.53	1,661,109.66	2.81%	1.51%
31	1,621,859.54	1,599,760.29	1,609,819.08	1.37%	0.75%
32	1,825,858.76	1,807,307.94	1,812,429.17	1.02%	0.74%
33	1,736,249.83	1,708,086.65	1,720,048.47	1.64%	0.94%
34	2,079,572.83	2,025,598.70	2,056,253.08	2.62%	1.13%
35	2,269,876.88	2,240,109.84	2,253,511.47	1.32%	0.73%
36	1,919,502.95	1,904,279.64	1,906,073.12	0.80%	0.70%
37	1,653,764.42	1,635,748.11	1,638,013.21	1.10%	0.96%
38	2,034,174.32	2,017,160.11	2,020,050.48	0.84%	0.70%
39	1,956,769.55	1,927,348.78	1,942,119.64	1.51%	0.75%
40	2,097,396.83	2,067,656.05	2,081,768.37	1.43%	0.75%

Table 4.6: Multi-period test set II results

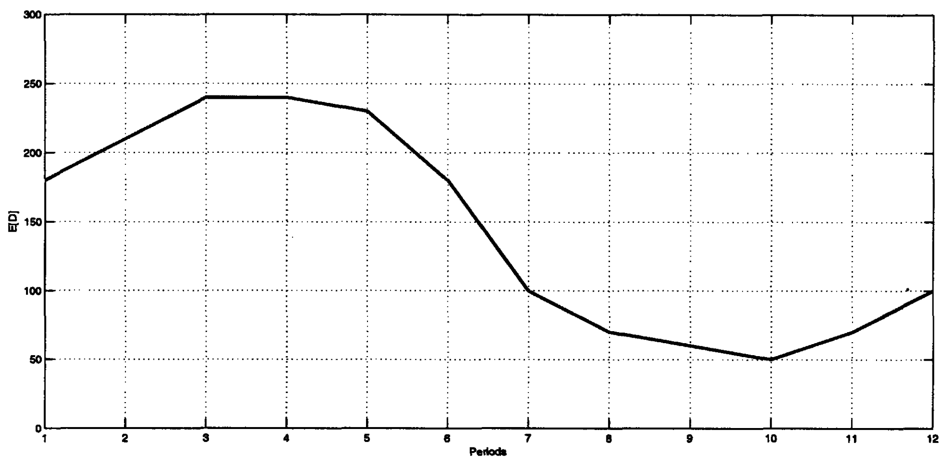

Figure 4-8: Test set II – The demands of a product that starts at point 3.

	Average	STD	Minimum	Maximum
Error 1	2.19%	1.18%	0.61%	5.72%
Error 2	1.17%	0.58%	0.61%	3.08%

Table 4.7: Multi-period test set II statistics

4-8. We use this method to simulate products at different stages of their life cycles. The standard deviation of the demand for each product in each period is 10. The other settings for the parameters in test set II are the same as the ones in test set I.

The results and statistics of the test set II are given in Table 4.6 and Table 4.7. Similar to test set I, we see that the feasible solution obtained at stage II is superior to the feasible solution from stage I. Moreover, this test case also shows that the heuristic algorithm performs well: the average gap between the solution that the algorithm generated and the optimal solution is less than 1.17% and the maximal gap is less than 3.08%.

4.5.3 Test Cases Set III

Test set III is designed to show the effect of option contracts on contract durations. We take the setting of test set II and make the following modifications:

Scenario 1: Fix the price of each product to be 150 and set the prices of the 1-period contracts to be: $p_{1-period} = 10$, $q_{1-period} = 8$, and $e_{1-period} = 3$.

Scenario 2: Fix the price of each product to be 150 and set the prices of the 1-period contracts to be: $p_{1-period} = 10$, $q_{1-period} = 3$, and $e_{1-period} = 8$.

The prices of the capacity with 3-period duration, 6-period duration, and 12-period duration are 90% of their 1-period, 3-period, and 6-period counterparts. For each test case, we generate the random supply chain structure and the demands of the products. We then solve the case for both scenarios. For each test case, we define the following measurements:

$$Count_{x,y} = \text{Number of contracts in scenario x with duration y },$$

$$Ratio_{x,y} = \frac{\text{Option contract sizes in scenario x with duration y}}{\text{Total contract sizes in scenario x with duration y}}.$$

For example, the first test case in scenario I uses 18 contracts with duration 1, 4 contracts with duration 3, 13 contracts with duration 6, and 5 contracts with duration 12. Therefore, $Count_{1,1} = 18$, $Count_{1,3} = 4$, $Count_{1,6} = 13$, and $Count_{1,4} = 5$. Among the 18 contracts with duration 1, 22.15% of the capacity are option capacity. Therefore, $Ratio_{1,1} = 22.15\%$.

We also define the following aggregate measurements for each scenario:

$$\%Count_{x,y} = \frac{\text{Number of periods in scenario x covered by the contracts with duration y}}{\text{Total number of periods}},$$

$$\overline{Ratio}_{x,y} = \text{Average of } Ratio_{x,y} \text{ over 40 test cases.}$$

The statistics for the heuristic algorithm in both scenarios are given in Table 4.8. We note that the heuristic algorithm performs better in scenario 2 than scenario 1. This is because both Error 1 and Error 2 depend on the cost of deviating from the upper bound. Since the prices of the products are the same in both scenarios, if the deviation from the optimal solution results in a lost sale, the penalty is the same for both scenarios. On the other hand, since the manufacturer pays less to reserve

	Average	STD	Minimum	Maximum
Scenario 1: Error 1	4.22%	1.52%	2.01%	7.89%
Scenario 1: Error 2	2.34%	0.74%	1.32%	4.26%
Scenario 2: Error 1	1.21%	0.35%	0.58%	2.05%
Scenario 2: Error 2	0.79%	0.11%	0.58%	1.09%

Table 4.8: Multi-period test set III statistics

Scenario	$\%Count_{x,1}$	$\%Count_{x,3}$	$\%Count_{x,6}$	$\%Count_{x,12}$
1	16.07%	14.82%	31.96%	37.14%
2	2.95%	4.55%	13.04%	79.46%
	$Ratio_{x,1}$	$Ratio_{x,3}$	$Ratio_{x,3}$	$Ratio_{x,12}$
1	13.99%	18.89%	22.09%	27.19%
2	27.23%	48.28%	33.42%	56.59%

Table 4.9: Multi-period test set III contract usage statistics.

the option capacity in scenario 2, if the deviation from the optimal solution results in excess capacity, the cost is lower in scenario 2 than scenario 1. Therefore, the heuristic algorithm performs better in scenario 2 than scenario 1.

We now discuss the effect of option contracts on contract durations. We list the contract usage of scenario 1 and 2 in Table 4.10 and 4.11 and summarize the statistics in Table 4.9. From these results, we have two observations:

1. *Scenario 2 uses more option contracts than scenario 1.* For all y, $ratio_{1,y} < ratio_{2,y}$. Since the manufacturer pays less to reserve the option capacity in scenario 2, the cost of reserving excess option capacity is lower. Therefore, the manufacturer tends to buy more option capacity.

2. *Scenario 2 uses more contracts with long duration than scenario 1.* In particular, scenario 2 covers 79.46% of the periods with the 12-period contract and 56.59% of this capacity is option capacity. Option contract can be used to reduce the risk from a long term contract. Therefore, when the reservation price of the option capacity is low, the manufacturer can sign a long term contract with a significant amount of option capacity to take advantage of the lower cost and

Case	$Count_{1,1}$	$Count_{1,3}$	$Count_{1,6}$	$Count_{1,12}$	$Ratio_{1,1}$	$Ratio_{1,3}$	$Ratio_{1,6}$	$Ratio_{1,12}$
1	18	4	13	5	22.15%	30.23%	27.97%	29.69%
2	24	8	8	6	25.98%	20.81%	23.34%	28.95%
3	51	7	10	3	0.00%	25.74%	29.31%	33.56%
4	21	5	10	6	13.76%	31.40%	8.74%	26.89%
5	30	6	8	6	25.51%	32.59%	17.56%	26.74%
6	39	9	13	2	5.05%	24.84%	10.09%	11.07%
7	18	8	9	6	22.43%	13.91%	11.11%	31.80%
8	36	6	9	5	27.67%	22.93%	19.92%	35.67%
9	30	16	5	5	20.69%	1.50%	44.05%	17.39%
10	15	5	7	8	22.30%	15.90%	17.07%	36.36%
11	18	12	17	1	6.44%	7.71%	10.68%	18.40%
12	45	11	13	1	26.32%	22.68%	12.62%	41.47%
13	30	8	11	4	39.50%	11.90%	16.68%	29.08%
14	24	6	5	8	0.00%	16.68%	32.90%	26.30%
15	12	8	2	10	0.00%	20.76%	26.76%	20.49%
16	18	8	11	5	18.78%	30.71%	29.66%	8.51%
17	21	7	11	5	0.00%	24.78%	24.20%	20.27%
18	15	5	11	6	4.57%	31.45%	15.46%	18.16%
19	12	4	6	9	6.66%	27.70%	10.93%	23.91%
20	33	7	11	4	8.95%	24.96%	10.33%	29.48%
21	21	5	6	8	20.82%	36.72%	27.36%	18.59%
22	30	6	8	6	23.15%	16.78%	42.98%	42.98%
23	24	10	5	7	0.00%	15.97%	34.96%	25.99%
24	21	3	5	9	4.71%	35.42%	24.73%	24.70%
25	30	8	11	4	10.19%	14.59%	29.64%	36.36%
26	24	6	15	3	9.70%	21.55%	27.79%	23.70%
27	6	2	8	9	1.61%	23.12%	34.19%	26.46%
28	36	8	12	3	12.76%	7.67%	11.28%	22.46%
29	30	10	10	4	0.00%	15.69%	30.75%	26.35%
30	42	6	14	2	21.23%	13.20%	22.22%	25.24%
31	33	17	4	5	14.65%	17.94%	28.80%	35.12%
32	27	11	6	6	24.04%	8.09%	25.93%	12.71%
33	39	13	7	4	0.00%	23.36%	31.42%	12.93%
34	33	13	10	3	23.46%	24.70%	22.93%	28.00%
35	39	15	8	3	29.97%	35.15%	30.35%	32.88%
36	3	1	9	9	2.40%	0.00%	27.08%	40.21%
37	33	9	10	4	26.78%	28.02%	20.31%	29.67%
38	24	14	7	5	0.00%	12.33%	17.95%	26.24%
39	39	15	4	5	2.91%	0.00%	28.38%	43.45%
40	36	10	9	4	0.00%	20.06%	19.07%	39.94%

Table 4.10: Multi-period test set III scenario 1 contract usage.

Case	$Count_{2,1}$	$Count_{2,3}$	$Count_{2,6}$	$Count_{2,12}$	$Ratio_{2,1}$	$Ratio_{2,3}$	$Ratio_{2,6}$	$Ratio_{2,12}$
1	0	0	4	12	-	-	50.74%	78.14%
2	15	5	5	9	3.80%	36.32%	21.54%	48.94%
3	3	9	5	9	31.52%	44.74%	22.80%	78.79%
4	6	2	4	11	31.64%	47.84%	36.72%	30.31%
5	9	3	3	11	33.13%	51.23%	41.17%	39.62%
6	12	4	6	9	45.60%	53.91%	24.63%	72.67%
7	3	1	1	13	37.81%	50.71%	36.19%	42.48%
8	9	7	5	9	28.00%	59.82%	20.47%	55.09%
9	3	1	1	13	36.69%	52.64%	40.33%	78.92%
10	3	1	1	13	37.08%	51.77%	28.78%	50.58%
11	0	0	10	9	-	-	42.99%	80.15%
12	24	8	8	6	15.05%	42.52%	25.63%	75.46%
13	3	3	4	11	23.58%	52.63%	25.38%	44.64%
14	3	3	2	12	40.29%	40.97%	40.39%	39.52%
15	3	1	1	13	35.76%	45.28%	39.95%	39.46%
16	12	4	4	10	16.18%	36.28%	23.48%	59.75%
17	9	3	3	11	22.02%	44.39%	38.43%	70.62%
18	3	1	5	11	36.70%	48.17%	40.24%	64.91%
19	3	1	1	13	35.09%	53.34%	40.45%	34.12%
20	12	4	6	9	47.27%	46.81%	34.29%	73.37%
21	0	2	1	13	-	55.75%	34.57%	76.76%
22	3	7	4	10	45.79%	57.16%	36.31%	41.99%
23	3	1	1	13	34.68%	45.03%	38.72%	49.18%
24	3	1	1	13	34.23%	53.04%	42.60%	55.87%
25	3	3	6	10	37.17%	54.91%	27.87%	79.84%
26	3	3	8	9	32.63%	55.74%	26.12%	39.71%
27	0	0	6	11	-	-	47.88%	34.01%
28	6	4	5	10	36.96%	41.07%	38.66%	50.34%
29	9	3	3	11	27.73%	31.05%	34.85%	80.11%
30	6	4	7	9	21.82%	53.25%	37.85%	63.19%
31	0	2	3	12	-	58.89%	37.53%	67.09%
32	0	0	0	14	-	-	-	25.05%
33	3	1	1	13	22.82%	51.41%	27.49%	78.46%
34	3	3	6	10	37.17%	54.91%	27.87%	79.84%
35	3	1	1	13	16.76%	28.73%	37.52%	77.89%
36	0	0	6	11	-	-	39.91%	43.31%
37	3	1	3	12	7.16%	50.32%	39.51%	20.35%
38	6	2	2	12	33.96%	55.36%	30.91%	61.17%
39	0	0	0	14	-	-	-	48.82%
40	9	3	3	11	19.00%	44.26%	20.78%	64.03%

Table 4.11: Multi-period test set III scenario 2 contract usage.

does not need to bear a large risk. We have observed the same behavior in the example given in Section 4.1.2.

Chapter 5

Conclusions and Future Directions

5.1 Conclusions

In this thesis, we present a model to study capacity planning in a general supply chain that contains multiple products, multiple processes, and multiple resources. The model incorporates rental like capacity contracts and option contracts that can have different price structures and durations.

We first consider the capacity planning problem in a single period setting. We derive closed-form optimal capacity plans for two special cases of supply chains. For a general single period capacity planning problem, we propose an efficient algorithm to find the optimal capacity plan and test its performance with other existing algorithms empirically. We then study the properties of the optimal capacity plan and see how the capacity plan changes as the parameters of the supply chain or the structures of the contracts change. We also discuss how to incorporate order size constraints into the model.

We then propose a decomposition method that can separate the original capacity planning problem into sub-problems, under the assumption that each process has only one dedicated resource. Each sub-problem in the decomposition contains only one process. The decomposition method provides both a feasible solution and an upper bound to the original capacity planning problem. We examine the effectiveness of the feasible solution and the tightness of the upper bound through a series of test cases.

Finally, we study the multi-period capacity planning problem under the assumption that the manufacturer does not store inventory. For the case where each process has only one dedicated resource, we present a heuristic algorithm to solve the problem. We also propose a method to find an upper bound of the problem. We then test the heuristic algorithm and upper bound using several sets of test cases.

5.2 Future Directions

The work in this thesis opens the door to many research directions.

Capacity Planning with Demand Forecast Updates. One of the major assumptions that we have made in this thesis is that the manufacturer follows a two-stage decision process: first, it decides the contract sequences and sizes for the whole planning horizon based on the initial demand forecasts; second, it learns the demands for the entire horizon and allocates capacity to fill the demand at each period. However, in practice, the manufacturer might have the opportunity to improve the demand forecasts over time and revise the contracts that have not started using the new demand forecasts. In these situations , if we assume that the lead time to commit a contract is $\mathcal{H}$, the actual decision process at each period would be as follows:

- At the beginning of the period, the manufacturer decides and signs all the contracts that will start in $\mathcal{H}$ periods using the available information. This information can include the current forecasts of the future demand, the demand progression information, the demand history, etc.
- The manufacturer observes the demand and other information revealed during the period. It then updates the information set to include the new data.

This is a multi-stage dynamic capacity planning problem with demand forecast updates.

The heuristic algorithm that we have proposed for the two-stage multi-period capacity problem in this thesis can serve as a heuristic algorithm for the multi-stage dynamic problem. The specific steps are as follows:

- At the beginning of each period, based on the current demand forecasts, the manufacturer uses the heuristic algorithm to find a capacity plan. It then commits all the contracts in the plan that will start in $\mathcal{H}$ periods.

- The manufacturer observes the demand and other information revealed during the period. It then updates the demand forecasts.

This algorithm might not provide an optimal strategy for the dynamic capacity planning problem. It is an interesting research problem to examine the performance of this algorithm and develop new algorithms to solve the multi-stage dynamic capacity planning problem with forecast updates.

Capacity Planning with a Rolling Horizon. In this thesis, we assume there is a fixed planning horizon T. In practice, the capacity planning is a rolling process:after each period is over, a new period will be appended to the end of the current planning horizon. We can add the rolling process to the two-stage model studied in this thesis or the multi-stage dynamic model mentioned above. In either case, our heuristic algorithm given in Chapter 4 provides a heuristic algorithm to solve the problem with a rolling process. Studying the capacity planning problem with a rolling horizon is another interesting research direction that one might study.

Capacity Planning with Initial Setup Costs. Another assumption that we have made in the thesis is that there is no setup cost for each contract. If this assumption does not hold, the manufacturer needs to pay a fixed cost for using a contract. In these situations, the single period capacity planning problem will be a stochastic mixed integer program rather than a stochastic linear program. It will be an interesting research problem to develop algorithms to take the initial contract setup costs into consideration.

Combining the Decomposition Method with other Models. We have discussed in the literature survey section that people have proposed models to solve problems that contain a single product or process. For example, Martinez-de-Albniz and Simchi-Levi [27] and Yazlali and Erhun [39]. These models have more complex demand and price structures and allow the manufacturer to store inventory. On the other hand, one of the contributions of our work is introducing a decomposition method to divide the capacity planning problem that contains multi-product and multi-process supply chain into sub-problems where each sub-problem has only one process. It will be beneficial to study how to use our decomposition method to expand the existing models to a more general setting.

Industrial Study. Finally, we are looking for industrial case studies to validate and improve our model. Through these studies, we can test our algorithms in real applications.

Appendix A

List of Contract Manufacturers

A.1 A Partial List of Contract Manufacturers in the Biopharmaceutial Industry

1. Albemarle Corporation

2. Avecia, Inc.

3. Bachem Holding Ag

4. Baxter Pharmaceutical Solutions LLC

5. Ben Venue Laboratories

6. Bioreliance Corporation

7. Biovectra DCL

8. Boehringer-Ingelheim

9. Cambrex Corporation

10. Cardinal Health Contract Manufacturing

11. Chesapeake Biological Laboratories

12. DPT Laboratories, Ltd.
13. DSM Pharmaceuticals, Inc.
14. Degussa AG
15. Dowpharma
16. Draxis Health, Inc.
17. Genzyme Pharmaceuticals
18. Girindus AG
19. Glatt Air Techniques, Inc.
20. HollisterStier Contract Manufacturing
21. Hospira One 2 One
22. Laureate Pharma
23. Lonza Group Ltd
24. Lyne Laboratories
25. Mallinckrodt Pharmaceuticals
26. Patheon, Inc
27. Rhodia Pharma Solutions
28. Wellspring Pharmaceutical

A.2 Top 10 Electronics Manufacturing Services (EMS) Companies in 2006

Rank	Name	Total Revenue 2006 ($ M)
1	Foxconn	40,527
2	Flextronics	17,708
3	Asustek	17,196
4	Quanta Computer	16,503
5	Solectron	11,200
6	Sanmina-SCI	10,955
7	Jabil	10,300
8	Celestica	8,800
9	Inventec	7,890
10	TPV Technology	7,176

Appendix B

Deriving the Updates Rules in the Stochastic Supporting Hyperplane Algorithm

In this appendix, we look at the update rules in step 2 of the stochastic supporting hyperplane algorithm given in Chapter 2. The derivation is based on the method given by Higle and Sen in [18].

In step s, if we use all the sample points in the set $\mathcal{S}^s$ to construct the supporting hyperplane at $(\boldsymbol{c}^{s-1}, \boldsymbol{g}^{s-1})$, the constraint to be added as given in (2.38) is

$$f + \frac{1}{s} \sum_{\boldsymbol{d} \in \mathcal{S}^s} \pi(\boldsymbol{c}^{s-1}, \boldsymbol{g}^{s-1}, \boldsymbol{d}) - \boldsymbol{p}' \boldsymbol{c}^{s-1} - \boldsymbol{q}'(\boldsymbol{g}^{s-1} - \boldsymbol{c}^{s-1})$$
$$+ \left[(\boldsymbol{c}, \boldsymbol{g}) - (\boldsymbol{c}^{s-1}, \boldsymbol{g}^{s-1})\right]' \left(\frac{1}{s} \sum_{\boldsymbol{d} \in \mathcal{S}^s} \lambda(\boldsymbol{c}^{s-1}, \boldsymbol{g}^{s-1}, \boldsymbol{d}) - \boldsymbol{p} + \boldsymbol{q}, \frac{1}{s} \sum_{\boldsymbol{d} \in \mathcal{S}^s} \gamma(\boldsymbol{c}^{s-1}, \boldsymbol{g}^{s-1}, \boldsymbol{d}) - \boldsymbol{q} \right) \geq 0$$

Simplifying the equation above, we get

$$f + \frac{1}{s} \sum_{\boldsymbol{d} \in \mathcal{S}^s} \pi(\boldsymbol{c}^{s-1}, \boldsymbol{g}^{s-1}, \boldsymbol{d})$$
$$-(\boldsymbol{c}^{s-1}, \boldsymbol{g}^{s-1})' \left(\frac{1}{s} \sum_{\boldsymbol{d} \in \mathcal{S}^s} \lambda(\boldsymbol{c}^{s-1}, \boldsymbol{g}^{s-1}, \boldsymbol{d}), \frac{1}{s} \sum_{\boldsymbol{d} \in \mathcal{S}^s} \gamma(\boldsymbol{c}^{s-1}, \boldsymbol{g}^{s-1}, \boldsymbol{d}) \right)$$

$$+(\boldsymbol{c},\boldsymbol{g})'\left(\frac{1}{s}\sum_{\boldsymbol{d}\in\mathcal{S}^s}\lambda(\boldsymbol{c}^{s-1},\boldsymbol{g}^{s-1},\boldsymbol{d})-\boldsymbol{p}+\boldsymbol{q},\frac{1}{s}\sum_{\boldsymbol{d}\in\mathcal{S}^s}\gamma(\boldsymbol{c}^{s-1},\boldsymbol{g}^{s-1},\boldsymbol{d})-\boldsymbol{q}\right)\geq 0$$

If we separate the terms associated with ω^s, which is the demand sample generated in step s, we get

$$\begin{aligned}
f+\frac{s-1}{s}\Bigg[\frac{1}{s-1}\sum_{\boldsymbol{d}\in\mathcal{S}^{s-1}}\pi(\boldsymbol{c}^{s-1},\boldsymbol{g}^{s-1},\boldsymbol{d})\\
-(\boldsymbol{c}^{s-1},\boldsymbol{g}^{s-1})'\left(\frac{1}{s-1}\sum_{\boldsymbol{d}\in\mathcal{S}^{s-1}}\lambda(\boldsymbol{c}^{s-1},\boldsymbol{g}^{s-1},\boldsymbol{d}),\frac{1}{s-1}\sum_{\boldsymbol{d}\in\mathcal{S}^{s-1}}\gamma(\boldsymbol{c}^{s-1},\boldsymbol{g}^{s-1},\boldsymbol{d})\right)\\
+(\boldsymbol{c},\boldsymbol{g})'\left(\frac{1}{s-1}\sum_{\boldsymbol{d}\in\mathcal{S}^{s-1}}\lambda(\boldsymbol{c}^{s-1},\boldsymbol{g}^{s-1},\boldsymbol{d})-\boldsymbol{p}+\boldsymbol{q},\frac{1}{s-1}\sum_{\boldsymbol{d}\in\mathcal{S}^{s-1}}\gamma(\boldsymbol{c}^{s-1},\boldsymbol{g}^{s-1},\boldsymbol{d})-\boldsymbol{q}\right)\Bigg]\\
+\frac{1}{s}\Big[\pi(\boldsymbol{c}^{s-1},\boldsymbol{g}^{s-1},\omega^s)-(\boldsymbol{c}^{s-1},\boldsymbol{g}^{s-1})'(\lambda(\boldsymbol{c}^{s-1},\boldsymbol{g}^{s-1},\omega^s),\gamma(\boldsymbol{c}^{s-1},\boldsymbol{g}^{s-1},\omega^s))\\
+(\boldsymbol{c},\boldsymbol{g})'\left(\lambda(\boldsymbol{c}^{s-1},\boldsymbol{g}^{s-1},\omega^s)-\boldsymbol{p}+\boldsymbol{q},\gamma(\boldsymbol{c}^{s-1},\boldsymbol{g}^{s-1},\omega^s)-\boldsymbol{q}\right)\Big]\geq 0
\end{aligned}\tag{B.1}$$

By the definitions of α, β, and ζ, Equation (B.1) can be written as

$$\begin{aligned}
f+\frac{s-1}{s}\left[\alpha_{s-1}^{s-1}+(\boldsymbol{c},\boldsymbol{g})'(\beta_{s-1}^{s-1},\zeta_{s-1}^{s-1})\right]+\frac{1}{s}(c,g)'(-\boldsymbol{p}+\boldsymbol{q},-\boldsymbol{q})\\
+\frac{1}{s}\Big[\pi(\boldsymbol{c}^{s-1},\boldsymbol{g}^{s-1},\omega^s)-(\boldsymbol{c}^{s-1},\boldsymbol{g}^{s-1})'(\lambda(\boldsymbol{c}^{s-1},\boldsymbol{g}^{s-1},\omega^s),\gamma(\boldsymbol{c}^{s-1},\boldsymbol{g}^{s-1},\omega^s))\\
+(\boldsymbol{c},\boldsymbol{g})'\left(\lambda(\boldsymbol{c}^{s-1},\boldsymbol{g}^{s-1},\omega^s),\gamma(\boldsymbol{c}^{s-1},\boldsymbol{g}^{s-1},\omega^s)\right)\Big]\geq 0
\end{aligned}\tag{B.2}$$

Now if we set $(\boldsymbol{c},\boldsymbol{g})=(\boldsymbol{c}^{s-1},\boldsymbol{g}^{s-1})$, the last three terms of the Equation (B.2) will be

$$\frac{1}{s}\left[\pi(\boldsymbol{c}^{s-1},\boldsymbol{g}^{s-1},\omega^s)\right]$$

If we replace this with its upper bound U, we get

$$f+\frac{s-1}{s}\left[\alpha_{s-1}^{s-1}+(\boldsymbol{c},\boldsymbol{g})'(\beta_{s-1}^{s-1},\zeta_{s-1}^{s-1})\right]+\frac{1}{s}(c,g)'(-\boldsymbol{p}+\boldsymbol{q},-\boldsymbol{q})+\frac{1}{s}U\geq 0\tag{B.3}$$

Equation (B.3) uses the hyperplane at $(\boldsymbol{c}^{s-1},\boldsymbol{g}^{s-1})$ in step $s-1$ to construct a relaxation of the supporting hyperplane at the same point in step s. A similar relaxation is applied to all the hyperplanes generated before step s.

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