15.053  Tuesday, March 9

- DEA: An interesting application area for Linear Programming
- No class on Thursday.
- Midterm from 7:30 to 9:30 (assigned rooms)
- no calculator, closed book
- bring IDs
- Extra review session. Wednesday 7:30-9:30
Goals of this lecture

- Introduce an interesting application area for linear programming called Data Envelopment Analysis (DEA)

- Turn it over to Ph.D. student Lincoln Chandler, who will talk about his research using DEA
Introduction to Data Envelopment Analysis

- The goal of DEA is to measure whether individual units (or firms) are “efficient”

- It contrasts efficiency in terms of seeing if they produce outputs effectively given their inputs.

- Developed by Charnes and Cooper in 1970’s
Measuring efficiency is difficult

- Suppose that one is measuring efficiency in a hospital.
  - Which is of more value: treating middle aged patients or older patients?
  - Which takes more resources?

- If we treat all patients as equal, then hospitals may have an incentive to treat healthier and younger patients because they make a hospital look efficient.
  - DEA avoids this issue by looking at each hospital in the best possible light.
## Hospital Data

<table>
<thead>
<tr>
<th>Hospitals</th>
<th>Inputs</th>
<th>Outputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>H1</td>
<td>5 14</td>
<td>9 4 16</td>
</tr>
<tr>
<td>H2</td>
<td>8 15</td>
<td>5 7 10</td>
</tr>
<tr>
<td>H3</td>
<td>7 12</td>
<td>4 9 13</td>
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</tbody>
</table>

**INPUTS:**
1. Hospital Beds
2. labor (1000s of hours)

**Outputs:** measured in 100s of patient days for patients
1. under age 14
2. ages 15 to 65
3. older than age 65
Measuring Efficiency

- Assume that input $i$ has a cost $w_i$.
- Assume that output $j$ has a value $t_j$.

\[
\text{Efficiency} = \frac{\text{Value of outputs}}{\text{Cost of inputs}}
\]

We will determine the $w$'s and $t$'s later.
More on efficiency

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<td></td>
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\[
E(\text{H1}) = \frac{9t_1 + 4t_2 + 16t_3}{5w_1 + 14w_2}
\]
Is Hospital 2 efficient?

This is interpreted as:

Can we assign positive costs to the inputs and positive values to the outputs in such a way that Hospital 2 is at least as efficient as the other hospitals. That is, can we assign costs and values so that

\[ E(H2) \geq E(H1) \quad \text{and} \quad E(H2) \geq E(H3)? \]

These will be non-linear constraints, but we can fix that.

\[
E(H2) = \frac{5t_1 + 7t_2 + 10t_3}{8w_1 + 15w_2} \quad \geq \quad E(H1) = \frac{9t_1 + 4t_2 + 16t_3}{5w_1 + 14w_2}
\]
Making the problem for H2 an LP

Step 1. (conceptual step). Multiply costs of inputs by a constant so that the cost of the inputs for H2 is exactly 1. That is, choose \( w_1 \) and \( w_2 \) so that

\[ 8w_1 + 15w_2 = 1 \]

Step 2. (conceptual step) Multiply through the values of the outputs by a constant so that \( E(H1) \leq 1 \) and \( E(H3) \leq 1 \).

Maximize the value of the outputs of H2

subject to the cost of the inputs for H2 is 1

\( E(H1) \leq 1 \) and \( E(H3) \leq 1 \).

costs and values are strictly positive
Making the problem for H2 an LP

Maximize \[ 5 t_1 + 7 t_2 + 10 t_3 \]
subject to \[ 8 w_1 + 15 w_2 = 1 \]
\[ 9 t_1 + 4 t_2 + 16 t_3 \leq 5 w_1 + 14 w_2 \]
\[ 4 t_1 + 9 t_2 + 13 t_3 \leq 7 w_1 + 12 w_2 \]
\[ t_1, t_2, t_3, w_1, w_2 \geq 0.00001 \]
On H2’s efficiency

- The optimal efficiency for H2 is .773
  - H2 is inefficient.

- The optimal efficiency for H1 and H3 is 1.
  - H1 is efficient and H3 is efficient
  - found by solving two more linear programs)
On the intuition for H2’s inefficiency.

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Consider a Hospital H4 by taking \(.3381 \times H1 + .8555 \times H2\)

H4

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<tr>
<td>Inputs</td>
<td>Outputs</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>6.759</td>
<td>15</td>
</tr>
<tr>
<td>2</td>
<td>6.465</td>
<td>9.052</td>
</tr>
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H2 uses more inputs than H4, and produces at most .773 of any input.
Summary

- Data Envelopment Analysis is a “hospital friendly” way of measuring efficiency of hospitals.
  - the hospital units can use any costs for inputs and any values for outputs
  - Is there some way of choosing costs and values so that the hospital is more efficient than every other hospital?

- And now: Lincoln Chandler, who is doing his Ph.D. dissertation in the MIT Operations Research Center.
DEA in Practice: Accomplishment, Caveats, and Applications

Lincoln J. Chandler
Operations Research Center
15.053 Presentation
March 8, 2005
DEA in Practice: Bank Branches*

- The banking industry, like others, has experienced a great deal of consolidation over the last twenty years.

DEA in Practice: Bank Branches

- **Pre-Merger Expectations**
  - Increased profits from economies of scale
  - Broader service offerings / customer base
  - More competitive

- **Post-merger realities**
  - “Merged entities” still operate as if independent
  - Minimal increase in cost efficiency
  - 2 out of 3 bank mergers generate “disappointing” returns to shareholders
DEA in Practice: Bank Branches

- Despite the benefits of merging front-end operations, companies often postpone or forego this opportunity
  - Politics
  - Short-term economic targets
  - “Don’t fix what isn’t broken”

- A DEA study was used to gauge the savings loss from not merging operations
Outline of the Study

- DEA was used to compare the branch efficiency of four banks that had been merged into one (over 200 branches)

- Branches were compared within each parent bank, and then across the merged bank to determine efficiency loss, which relates to potential savings

- After four years, the merged bank had realized cost savings of about 8%; a comparison of savings estimates would help estimate the value of merging branch operations, a unrealized opportunity
The Bank Branch as a DEA Unit

- 2 inputs, 6 outputs
Results

- The analysis indicated substantial opportunities to achieve additional savings
  - When measured within the parent group, the average efficiency gap was about 7%
  - When measured across the system, the average efficiency gap was 23%, over three times higher!

- DEA was able to quantify a major source of unrealized shareholder value
More on DEA

- Given a set of business units, DEA provides a method of measuring relative efficiency, even for systems with multiple inputs and outputs.

- For each inefficient unit, DEA also provides a reference set of efficient units for guiding improvement strategy.

- The flexibility of DEA has led to its application in a wide variety of studies, including:
  - Capacity Management of Fishing Fleets (Denmark)
  - Hotel Room Utilization (United Kingdom)
  - Hospital Performance (Oman)
  - Brand Management of Business Schools (United States)
DEA in Public Schools

- My current research involves the use of DEA to measure relative efficiency in a public school district.

- My emphasis is on elementary schools located in large U.S. cities.

- My work is towards a refined DEA model that:
  - Recognizes the influence of leadership on school outcomes
  - Clarifies the relation between school effectiveness and efficiency
  - Facilitates improved strategies for management
Background

- There is some literature on DEA and school efficiency
  - Bessent and Bessent, 1980
  - Examined educational efficiency at the district, school, and individual level

- Few impacts of DEA studies on educational practice
  - DEA as a measurement, or sorting, tool…
  - Studies have not identified “best practice.”
  - DEA studies have not lead to improvements in system productivity
Recent Comments on Public Schools

- "America's high schools are obsolete" and are "ruining the lives of millions of Americans every year."
  - Bill Gates, Feb. 26, 2005*

- “Low-income families are also losing faith in these [public school] districts... they have to figure out how they create systems to take school reform to scale.”
  - Warren Simmons, Exec. Director, Annenberg Institute for School Reform**

*Remarks made during the National Governors Association Education Summit
Creating an DEA Model for Education

- My research
  - create a DEA model that compares the efficiency of elementary schools in the Chicago Public School system

- Next major step:
  - identify a good set of inputs and outputs for the analysis
    - Inputs: school resources
    - outputs: school outcomes
Creating the Model

- Determining which inputs to add into the analysis can be a complicated, and iterative, task

- Things to consider while creating the model
  - “Granularity” of Inputs/Outputs
  - Quality and Availability of Data
  - Fidelity of the Model

- Class exercise: brainstorm on plausible inputs and outputs
Summary of Main Research Ideas

- Schools are usually compared using 1-2 output measures

- DEA provides a richer method of comparison by considering inputs as well

- DEA can provide additional insights into reform strategy
  - We can identify specific peer schools to look for best practices
  - We can reveal inefficiencies in otherwise high performing schools
References on DEA

● Online
  – Anderson, Tim. “A DEA Home Page”
    • [http://www.emp.pdx.edu/dea/homedea.html](http://www.emp.pdx.edu/dea/homedea.html)
  – Emrouznejad, Ali. “DEA Homepage”
    • [http://www.deazone.com](http://www.deazone.com)

● Books (Available in Dewey)