

USE OF MULTIPLE OBJECTIVE ANALYSIS  
TECHNIQUES IN THE EVALUATION OF LOW  
VOLUME RURAL PENETRATION ROAD INVESTMENTS

by

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ABSTRACT

Traditionally benefit cost analysis relies on a monetized metric. Not all variables convert to this metric in a meaningful way. No single criterion appears sufficient to permit proper analysis, nor is it reasonable to assume that the decision makers perceive the decision in terms of a single criterion. This thesis reviews and develops several of the techniques available for the evaluation and, where appropriate, ranking of the many possible objectives of rural investments.

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## Chapter One

### INTRODUCTION

A large portion of the rural development budget is allocated to transportation, which is seen as a necessary, although not always sufficient, precondition to successful development. Traditionally, governments and international funding agencies (decision makers) select and rank transportation investments using economic efficiency criteria: benefit-cost ratios, net present worth, economic and internal rate of return. Each methodology depends upon vehicle operation cost savings to produce the bulk benefits. By definition, rural penetration roads have little or no traffic nor can traffic on other routes divert to them. A problem therefore exists: without current users, no traditional analysis can be performed.

To aid the decision maker, the analysts need a more relevant, flexible system to help identify, evaluate, and rank viable rural transportation investments.

The traditional benefit cost analysis referred to above relies on a monetary measure. Not all variables readily convert to this measure (e.g., access, health, education, political integration). No single criterion appears sufficient to permit proper analysis, nor is it reasonable to assume that the decision makers perceive the decision in terms of a single criterion.

In 1979 the Transport Research Review Panel(1) of the IBRD made the following observation on the subject of rural roads evaluation research:

"Although great attention is paid to qualitative factors, an important part of the research is devoted to the computation of money-expressed effects mainly related to the extension of the monetary exchange economy. However, an unusual amount of work has been devoted to analyze (and translate into money equivalents) the implications of the subsistence economy. Still more emphasis might be put on methods permitting the evaluation of qualitative effects by means of a ranking or ordinal approach."

This observation indicated that techniques were still needed to redress the problem voiced by the XV Congress of the PIARC(2) held in Mexico City in 1975:

"The use of Cost Benefit investigations to measure the value of new road projects has resulted in an exaggeration of the importance of the primary road system...[There] is a danger that financial resources may be diverted from the projects of rural systems which are directly linked to the promotion of social and economic development...."

To properly address these concerns, evaluation must take into account national and regional impacts, not just of

the traditional quantifiable economic effects, but of the "qualitative" effects as well: access, health, education, employment, and political integration.

The road investments discussed in this thesis serve little or no existing traffic, and during the analysis horizon may not exceed 50 to 100 AADT. User savings are not a source of benefits. The main monetary benefits will be agricultural or producer surplus generated in the zone of influence of the new road. Other benefits will come from the areas grouped under the heading "qualitative" as defined above.

The purpose of this thesis is to review and develop several of the techniques available for the evaluation and, when appropriate, the ranking of the many possible objectives of rural investments.

In order to achieve this purpose, the thesis presents a discussion of potential objectives (Chapter Two); reviews a series of techniques for evaluating them (Chapter Three); evaluates and ranks a hypothetical series of projects where consensus exists among decision makers as to inter-criteria weight (Chapter Four); and evaluates a case where no consensus exists (Chapter Five). Chapter Six summarizes the thesis and posits some issues for further research. The supporting appendices follow thereafter.

## Chapter Two

### OBJECTIVES USED IN EVALUATION OF RURAL ROAD INVESTMENTS

Rarely will the road investment be a sufficient condition for the zone of influence to shift from a traditional or subsistence economy status to a net producer of surplus cash crops for the economy. Additional investments will normally be needed to stimulate the population. These investments can be in the form of extra transport vehicles, medical services, educational facilities, and agricultural extension services, to name a few. To the extent that these investments are an identifiable incremental public spending as part of an integrated development program they will be included in the economic cost of the project; however, the perspective of a road investment will still be maintained.

The objectives listed in Table 1 and discussed below are not exhaustive. They are drawn from the literature and the author's experience in the field. National, regional, or local realities may often dictate that others be added by the host country decision makers. Special policy and lending guidelines may dictate other criteria to the lending agencies. To the extent that their actual objective functions concur or conflict will determine which of the later discussions apply.

Table 1

LIST OF POSSIBLE OBJECTIVE FUNCTIONS AND UNITS OF MEASURE

|   |                                     |
|---|-------------------------------------|
| Net National Income   | Monetary                            |
| Agricultural Surplus  | Monetary, food<br>value equivalents |
| Investment  | Monetary, units of<br>investment    |
| Net Regional Income   | Monetary                            |
| Foreign Exchange  | Monetary (usually<br>hard currency) |
| Benefit Distribution  | Variable                            |
| Employment  | Man years                           |
| Access (health, education,<br>services, etc.) National<br>Integration and National<br>Defense | Variable                            |
| Environmental Impact  | Variable                            |

Each of the objectives presented in this chapter will be summarized, the rationale explained, and possible metrics or measures discussed. Only economic impacts are addressed explicitly. The financial cost of the project, especially under budget constraints, can be relevant; however, it is not treated herein.

2.1 Net National Income Objective

The net national income objective is the traditional economic efficiency objective function described earlier. It is present here for completeness. To the extent that vehicle



operating costs or road maintenance costs are impacted, the effect will be accumulated in this category. The metric used is the currency of the country or the normative evaluation currency of the lending agency.

## 2.2 Agricultural Surplus Objective

The agricultural surplus objective tends to be the single most important objective in any evaluation. Also called producer surplus, it is the net increase in marketable crop production exported from the zone of influence. Extensive research has been conducted to better calculate this function. Theoretical studies, linear programming models, and empirical research by the IBRD and others continue in an effort to improve the predictive systems supporting this objective. The details are outside the scope of this thesis.

The incremental crop production, net of inputs, spoilage, seed reserves, and increased local consumption that can be sold to a market with a deficit is the agricultural surplus. The issue is complicated by the introduction of extension services bringing new seed and crops to farmers who wish to shift to a cash crop of higher value. Over time, farmers grow a smaller amount of food for consumption, buying the balance of their needs in the market. By this process farmers transfer from the traditional to the cash economy.

The preferred measure for this objective function is usually value added expressed in monetary terms. A secondary choice is food value units, e.g., equivalent tons of wheat, but this lacks the credibility that tonnage times market price less cost of inputs has. It is still possible, however, to define this objective in tonnage terms if one policy issue is to reach a production goal in a specific crop. The same tonnage might be monetized later under a different objective. A set of work sheets for the value added metric appear in Appendix C.

### 2.3 Investment Objective

In traditional analysis, the economic cost of the project is compared with the economic benefit (net national income objective) using a series of economic efficiency tests. In this thesis we treat direct public investment as a separate function, i.e., investment has an independent and positive aspect. One objective could be to maintain certain levels of investment in a country or to balance investments across political or development regions. In some cases, lending agencies might need to screen out projects whose cost is below a certain floor value. A government might screen out projects whose investments exceed a certain cost per kilometer or per person. The metric is monetary in this case also.

#### 2.4 Net Regional Income Objective

National development plans often contain a goal for balanced regional development. The metric for regional income is the same as for national income but the rules of accounting differ, primarily in the treatment of transfer payments.

From the viewpoint of net income to the nation, transfers are nonproductive economically and only serve objectives such as income redistribution (another potential objective). Taxing the income of a person in Region A to provide a health care unit in Region B is the same as using the tax to provide a health care unit in Region C or, for that matter, Region A. From the view point of Region A, however, the tax is a cost. If the health care unit were in Region B or C, it would be a pure benefit to that region, the same as any other costless investment. If the unit went to Region A, then the benefits would have to be netted against the costs.

A second example is the case of the recipient of a nationally funded welfare program. The welfare receipts represent income to the region in which he resides. Should this person become ineligible for welfare, the loss is a cost to the region. If the ineligibility resulted from his taking a job, then the effect on the net regional income will be the difference between the wage received and the foregone welfare payment (preferably a positive number).

Another example relates to the net agricultural surplus anticipated. The difference between the production plus delivery cost and the market price represents potential income nationally. To the region, the question is the distribution. Certainly the difference between the production cost and the farmgate price is a net increment to the regional income. Only to the extent that the transporter is a part of the regional economy (that is, he resides, consumes, and invests in the region) will the difference between transport price and cost be a regional income increment. Finally, the difference between wholesale cost and retail price represents income to the region only to the extent that middlemen and market women are part of the regional economy.

## 2.5 Foreign Exchange Objective

A major concern of government officials in most LDC's is the preservation of foreign exchange. It therefore follows that an important decision variable is the use of foreign exchange for any investment plan. High technology solutions are often positively correlated with increased foreign exchange components, regardless of whether the total economic cost is different. One example is the philosophy of zero maintenance pavements where the supply of maintenance is, at best, unreliable. Although the total transportation cost (construction plus maintenance plus vehicle operation

cost) will possibly be lower, the drain on foreign exchange reserves for the increased asphalt requirement may be unacceptable.

The foreign exchange objective function must be formulated in terms of the opportunity cost of the foreign exchange consumed. There are two classes: general reserves and "dedicated funds". Exchange in the general reserve yields an economic return to the country as a function of where it is invested. Obviously, the economic benefits from such consumer luxury goods as champagne and fancy cars is at the low end of the range, while necessary construction equipment and machine tools are at the high end. When health, education, and military material are competing against the rural transport sector for scarce dollars, these dollars should be weighted properly.

"Dedicated funds" are normally foreign exchange which "must" be spent on specific infrastructure investment. Exchange in this category would be lending agency sector loans or grants and legislatively established reserves from taxes or offshore revenues. These dollars cannot be spent on other sectors, but must still be competed for within the sector so long as there exists a scarcity of funds.

The metric for foreign exchange can be either the currency of the country or some other measure of "hard" currency (for example, dollars, francs, SDRs). If the chosen

metric is the national currency, the assumption is that the analyst has the knowledge or the authority to set the shadow price of foreign exchange for each use.

## 2.6 Benefit Distribution Objective

Neither agricultural surplus nor net regional income would be an adequate objective for lending agencies needing to know to whom the benefits would accrue. Explicit accounting of the distribution of economic benefits among project beneficiaries has long been recognized as an important aspect of the appraisal of feeder road projects. Alleviation of poverty among the poorest of the rural population has repeatedly been cited as a primary goal of rural development. Another critical element of project design is ensuring that the reduction in the cost of transport is not shared by only the middlemen and the transport owners. Regardless of how much money is saved, if the net farm income does not increase, there is no incentive to any farmer to increase production.

Often the distribution of this farmer surplus is further disaggregated to measure what USAID calls their "equity" objective. Prediction of the small farmers' share of increased agricultural production is, in practice, harder to make. Field surveys are required to determine tillable land ownership by income group. Use of this measure might entail certain restrictive assumptions, including, for example: (1)

economic conditions of perfect competition exist, (2) average productivity and crop choice of the land is uniform, and (3) share of economic benefits is proportional to land ownership. Nevertheless, this appears to be a reasonably reliable representation of the distribution of benefits to the target population. The use of this measure is illustrated by comparing the following two extreme cases.

In the first case, the project area of influence consists of a community of 500 persons, all of whom are presently existing on income levels below that of the target income level. Some 750 additional hectares of cultivatable land are to be opened up and planted, with the ownership to be distributed evenly among the population, resulting in a homogeneous distribution of the new output of agriculture. In the second case, the project area of influence consists of a community of 300 persons, of whom some 270 are peasants either farming at a traditional level of existence or working for the five relatively rich families of the community. Although induced agriculture production is expected to be large, because land tenure is not secure, the peasant group share is expected to be negligible since the five rich families will "own" almost all of the available new cultivatable land.

Within the context of the Benefit Distribution objective function, the use of a monetized metric would not provide

the necessary information. A better metric would be one which reflects the actual policy definition:

- how many hectares of arable land will be added to small farmer holdings?
- how many peasant farmers will reach the target income level?
- how many new farms of 0.75 to 2.0 hectares will result?

The latter could be particularly relevant if the results of empirical research show that small farm holdings must exceed 0.75 hectares before a shift to cash cropping will occur.

## 2.7 Employment Objective

Consideration of employment in project appraisal raises the question of whether employment should be treated as an end, or as a means to meeting other policy goals. Kessing(3) argues that employment must be treated as a separate objective as generation of employment does not emerge naturally from the process of pursuing traditional macro-economic objectives, while UNIDO(4) argues that it is a means associated with the redistribution objective. Additional arguments which consider employment as a separate measure include its service as an indicator of the mobility of labor, an important factor of production that needs to be



mobilized for productive purposes in many rural areas; as a measure of relative labor intensity among projects; as a measure of technology transfer (unskilled to semiskilled workers on construction or maintenance); as a measure of migration trend with respect to a new pattern of agricultural practice; and as a measure of local labor substituting for foreign capital.

Man-years of employment associated with or as a consequence of projected investment throughout the life of the project are suggested as a measure of employment. Included is employment generated as a direct result of construction and maintenance activities as well as that expected in conjunction with increased economic (primarily agricultural) activity.

Employment of extension workers and other government employees would not be included at the national level of employment analysis; if they would otherwise be employed elsewhere it could be relevant to a regional analysis. Although employment occurs over time, as do economic benefits and costs, its value, if expressed in a non-monetary metric, is assumed constant and no discounting is required. Possible refinement in this measure would incorporate a distinction between short- and long-term employment, checks on the expected availability of labor over time relative to its expected use, and unskilled versus skilled man-years generated.

Employment should not be monetized via wages, especially if the regional income objective is being used. It can, however, be normalized for zone population to aid comparison. Agricultural labor should be counted carefully to avoid overstating this objective where detrimental diversion occurs.

## 2.8 Access Objective

One of the most significant problems facing analysts has always been quantification of the benefits to the impacted population derived from improved access to social services such as schools, health care units (clinics, hospitals, etc.), dependable, clean water and electricity, and other government facilities. The basic cause of this problem results from demands of single-objective analysis, which require that a life saved or six extra years of education be monetized. This problem prompted analysts to place such objectives in the category of "unquantifiable" or "intangible."

The decision makers never establish as an investment goal one million dollars of life savings or one million dollars of education benefits. The goals are more frequently presented(5) as:

### Health Care

- One rural hospital bed per 1,000 population
- One rural health center per 50,000 people
- One dispensary per 10,000 people with a 10 kilometer access limit

## Education

- 100 percent primary school enrollment by 1977 with a school in each settlement or village

If the goal is access to a social service within  $x$  miles for everyone in the zone of impact, or access to one facility for each  $y$  thousand people, then this will define the metric for this objective. For the former, the scale would be the percent of population in the zone of influence having access to the social service unit; for the latter, the goal population,  $y$ , is divided into the population actually served. The definition of the population actually served, whether everyone in the zone or only those with real access, will depend on how the objective function is defined.

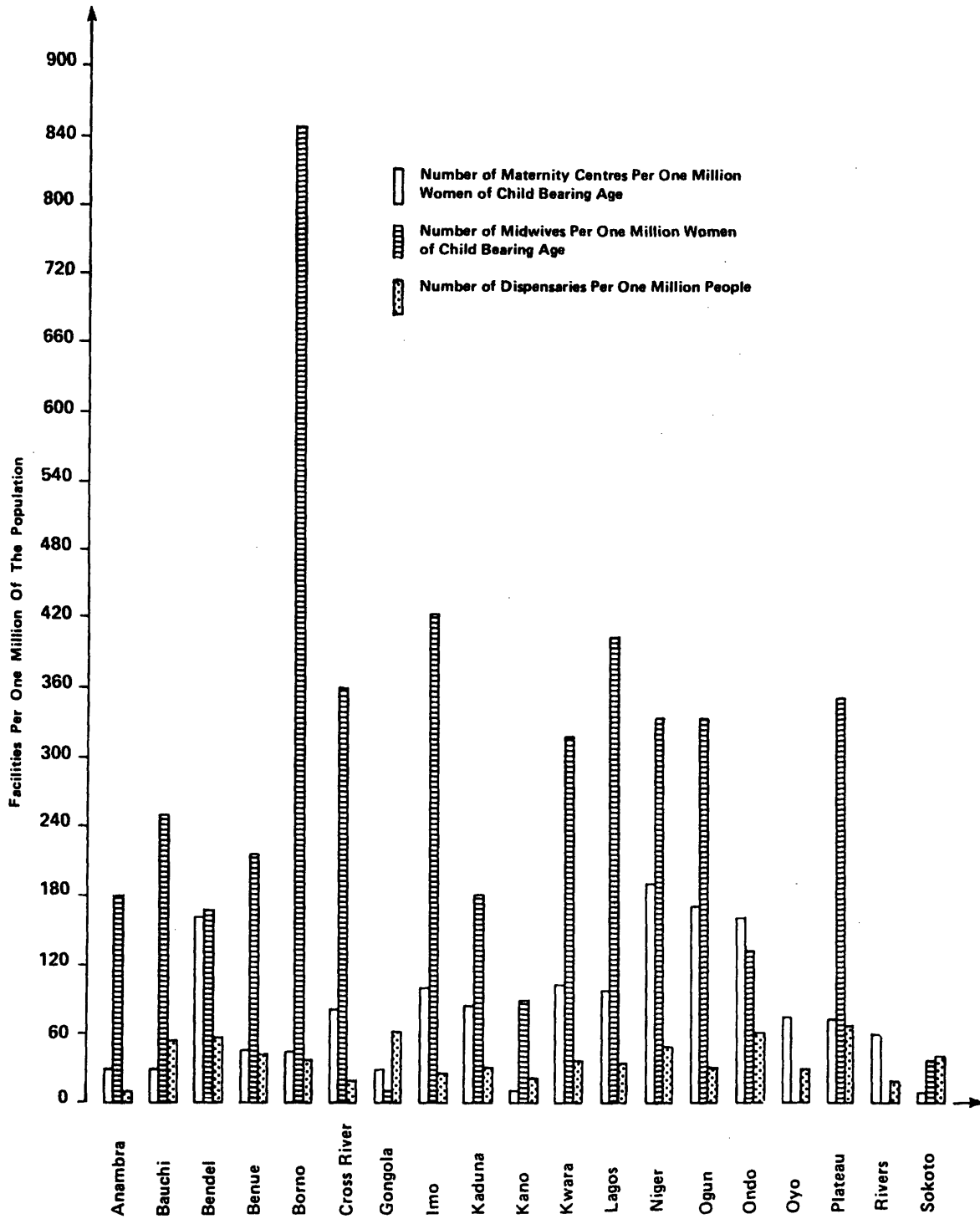
Figures 1 and 2 present such information for Nigeria.(6)

If several services are evaluated, a unified objective function could be designed, or each service kept as a separate function. Other services could include: rural electrification, visiting mobile clinic, post office, bank, piped water, telephone connection to the national network, and agricultural extension workers and pilot farm projects.

### 2.9 Environmental Impact Objective

This objective is mentioned because of the emphasis by some international lending agencies, primarily the U.S. Agency for International Development, although there is

Figure 1  
Health Care Facilities Per Million Population, Nigeria

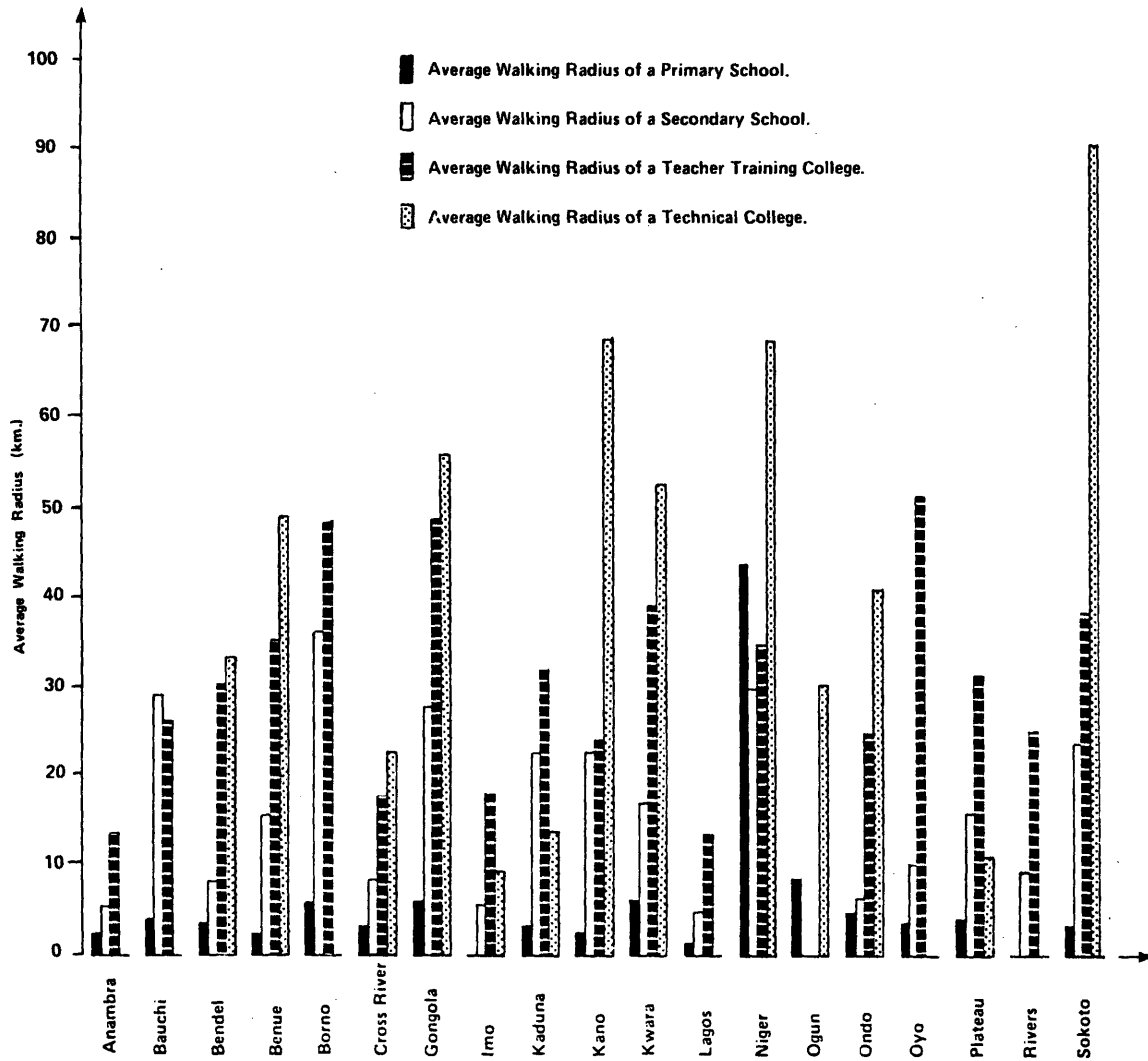


Source: Rural Infrastructures in Nigeria, Federal Department of Rural Development, Lagos, Nigeria 1981.

Available Maternity and Dispensary Facilities Per One Million of the Population, by States, Nigeria, 1978/79-1980.

Figure 2

Average Access to Education Facilities in Nigeria



Source: Rural Infrastructures in Nigeria, Federal Department of Rural Development, Lagos, Nigeria 1981.

Accessibility of Educational Facilities, by States, Nigeria, 1978/79-1980.

little indication from aid recipients that this is a major concern to them.

Measuring environmental impacts, in the multiobjective sense, must necessarily be impact specific. If the impact is arable land loss due to road investment caused erosion, the metric could be arable hectarage. If, on the other hand, restorative steps were contemplated, the metric could be monetized, e.g., the cost of avoiding loss of hectarage. Similarly, for bacteriological health hazards in bodies of water, parts per million or restorative cost would be appropriate.

#### 2.10 Objective Utility Functions

Having chosen a set of objectives for the analysis and collected each data base in a relevant metric, the analyst must then seek the assistance of the decision makers to define each utility function. Various utility assessment techniques are available.

In the category technique a number of discrete categories are specified for a particular objective, and the decision maker is asked to assign each project to one of these categories on the basis of its contribution to the objective. Once this has been done for all projects, numerical worths can be determined for each category, the resultant value being rather approximate.

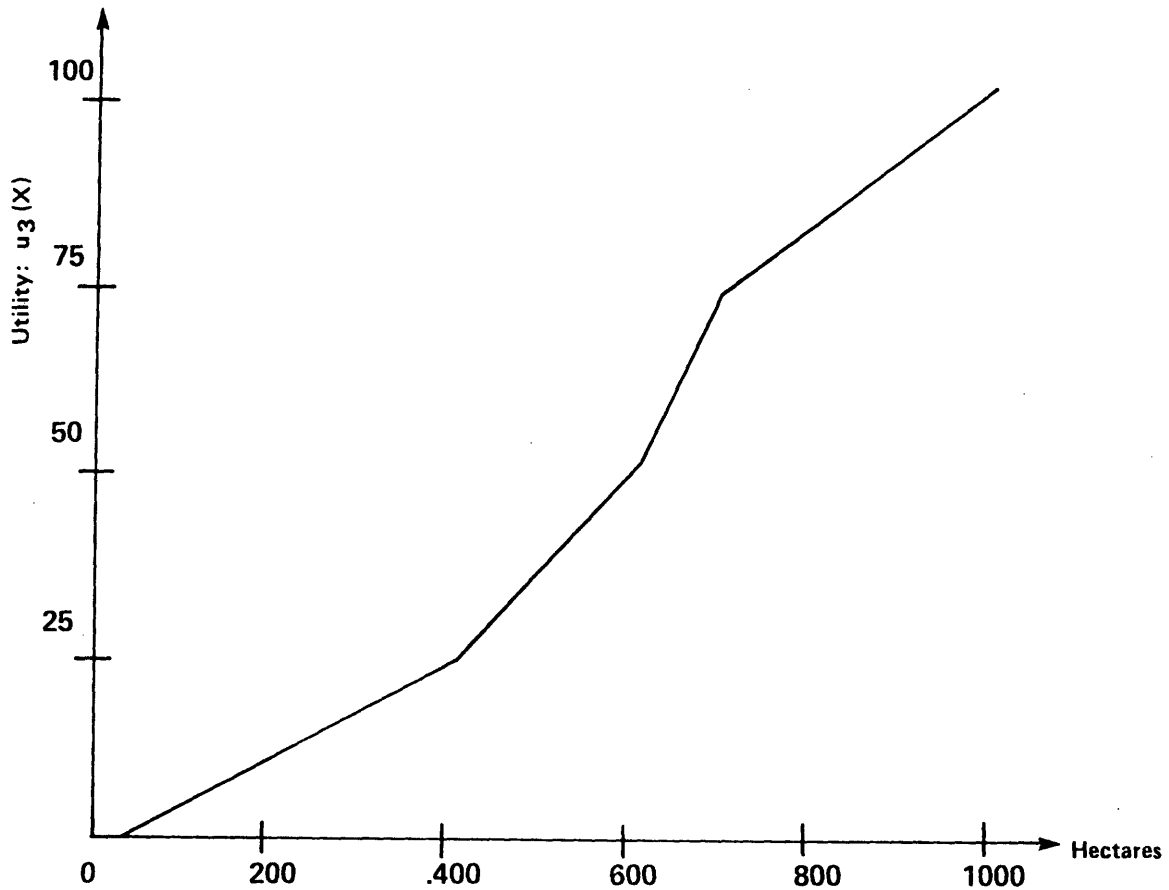
A second technique, the gamble, consists of lotteries constructed by varying the level of the measure or the

probabilities of occurrence until the decision maker is indifferent between the lottery and a certainty equivalent. This tends to be a somewhat complicated and confusing technique requiring quite a bit of time spent on the part of the analyst in educating the decision maker.

A third common approach, the direct technique, requires the decision maker to directly assign numerical values to the various levels of attainment of a particular measure. This technique can be accomplished in one of two ways: (1) anchor one extreme point of the measure and compare all other values of the measure with this anchor in assigning numerical values reflecting the utility; or (2) anchor the two extreme values of the measure along a scale of 0 to 100, specify a few convenient intermediate points such as the mid-, quarter-, and three-quarter points, and use linear interpolation to complete the preference function.

The direct technique is generally the most attractive and is used in the hypothetical testing of the appraisal framework in Chapter Four. A sample of its use in constructing the preference function for the benefit distribution objective is given in Figure 3. In actual practice, the final selection of the utility assessment technique depends very much on the preferences of the decision maker and the particular topic of the assessment.

Figure 3  
Distribution Preference (Utility) Function



The measure of distribution arising from all projects under consideration ranges from 20 hectares to 1000 hectares. 600 hectares has been anticipated as the "50" point, 400 as the "25" point, and 720 as the "75" point. The distribution preference function is therefore:

$$u(x_3) = \begin{cases} 0.0658 x_3 - 1.316 & 20 \leq x_3 \leq 400 \\ 0.125 x_3 - 25 & 400 \leq x_3 \leq 600 \\ 0.28 x_3 - 75 & 600 \leq x_3 \leq 720 \\ 0.0893 x_3 + 10.7 & 720 \leq x_3 \leq 1000 \end{cases}$$

Source: Chew (17)



Additional techniques can be developed should one of these three not seem appropriate. In some cases, the absolute value of the measure is sufficient and no utility function is needed. This occurs when a policy sets minimum values or cut-off points. An example of this would be a condition that no road would be considered where the investment per kilometer per capita exceeded \$200.

## Chapter Three

### TECHNIQUES FOR MULTI-OBJECTIVE EVALUATION

This chapter presents a series of methodologies for performing multi-objective analysis and evaluation. After a brief description of the current status of single objective analysis and its shortcomings as applied to development economics in the context of rural road investments, the chapter discusses the need for multi-objective analysis, the parameters of the analytical field, and a technique appropriate to each level of consensus/disparity within the decision making community.

#### 3.1 Introduction

Single objective analysis is the system currently in widest use by planners and lending agencies for project appraisal. In 1970, Israel (7) summarized the state of the art as follows:

"The methodologies used in feeder road appraisals fall into two groups: the social surplus methods, used in the quantification of road user savings, and the national income methods. In dealing with feeder roads, the practical possibilities of the national income approach are better than those of the social surplus approach. The operational difficulties of properly applying the latter when

major changes in income, income distribution, techniques, relative prices, and tastes are expected, are far greater than the theoretical limitations of the national income approach."

Clearly, the definition of a penetration road precludes any appraisal methodology relying on road user savings; however, a social surplus approach is not necessarily precluded. In a later Bank paper, Carnemark(8) argues the merits of a producer surplus methodology for estimating agricultural and forestry benefits "where existing levels of economic activity and traffic are insufficient for economic justification of the project."

Increased economic activity in the form of the export of surpluses from the region will show net financial benefits to everyone involved, from farmers to the middlemen and transporters to the markets. If not, they would not grow, buy, or transport it. This financial benefit may also represent a net economic benefit, thereby defending the investment in the road. However, the distribution of the net benefits accruing to the producers is the relevant issue in forecasting the production response, that is, the volume of surplus that will be grown.

If the farmer is assumed to be responsive to changes in his net income, the change in farmgate prices is required to predict the response. However, farmgate price change is only

used to predict producer activity, and Carnemark(8) still uses the net national income approach for overall evaluation.

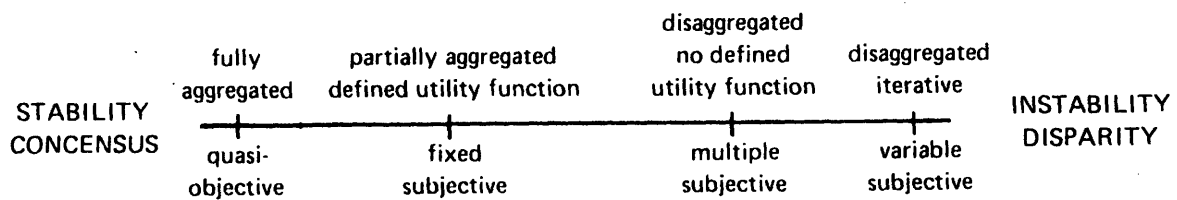
In fact, other than recognizing that the distribution of the benefits may have an influence on the level of response, there has been little progress in the application of multi-objective appraisal techniques for penetration roads over that defined in 1965 by Brown and Harral.(9)

### 3.2 Multi-Objective Analysis

Single objective techniques traditionally used in the analysis of road projects, like savings in user costs, producer and consumer surplus, and change in national income, are inadequate for a disaggregated analysis of the spectrum of objectives relevant to the rural development effort. Multi-objective analysis can incorporate both economic and non-economic objectives into the evaluation framework.

The techniques or methodologies available for multi-objective analysis reflect the degree of stability and consensus in the decision making environment. Where relationships and decision maker variables are well articulated and constant, the analysts can produce an optimal solution. As stability and consensus decline, the technician's solutions drop from optimal to "best" to pareto-optimal, to

what can only be termed a negotiated solution. The earlier in the analytical process the evaluation is truncated, the less aggregated the data to be presented the decision makers. This process is shown in Figure 4. The technique for presentation of the analysts "final" report must be appropriately chosen so as not to appear to be making decisions which were not delegated.



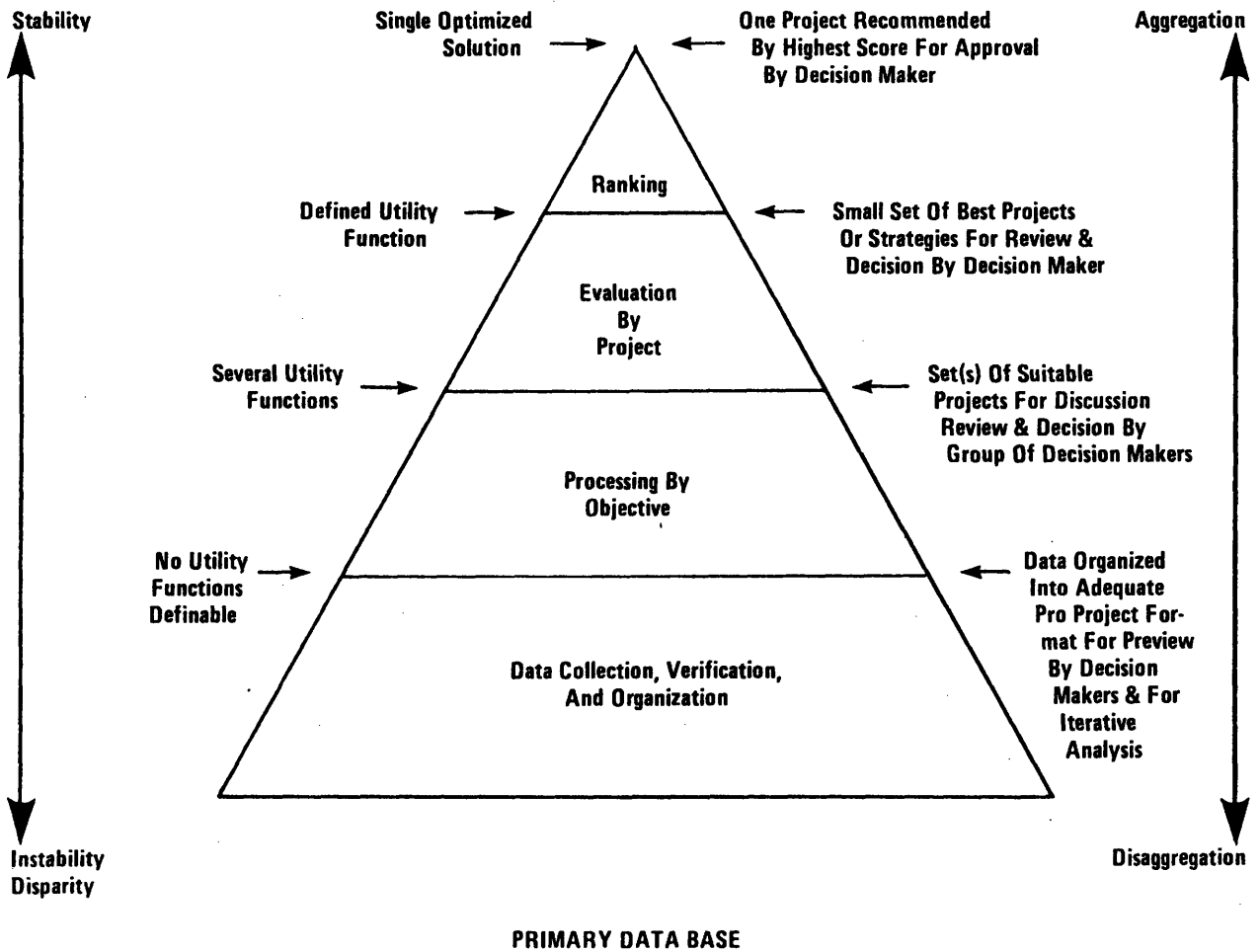
### 3.2.1 Ranking

At the apex of the analysis, each attribute is valued in terms of some common attribute, usually monetary. Thus, the many dimensions or attributes characterizing a given project may be collapsed into one dimension, and the value of the project is proportional to the total amount of the attribute. The project with the highest score is optimal.

The techniques corresponding to this level of analysis are those that are traditional to benefit-cost analysis methodologies; the unifying characteristic is the single numeraire, resulting in their being referred to as the aggregate method of multi-objective analysis. The UNIDO Guidelines(4), for example, use consumption measured in

Figure 4

THE EFFECT OF TRUNCATION ON THE PRESENTATION OF MULTIOBJECTIVE ANALYSIS RESULTS



domestic prices, while Little and Mirrlees(10) use a public income numeraire measured in world prices. Through the use of social pricing, more than a single objective can be implicitly considered: for example, the growth objective in the case of UNIDO(4) and the equity objective in that of Squire and van der Tak(11). Non-economic objectives may also be considered through the use of an appropriate metric, which is typically difficult to determine and next to impossible for planners and decision makers to agree on in practice.

### 3.2.2 Evaluation by Project

Often, the full set of attributes is not expressible in terms of a single numeraire, but sufficient consensus exists among decision makers that a utility function can be defined which expresses the level of satisfaction with each alternative project design. The project with the highest utility level is thus optimal.

At this stage of the analysis, the element of subjectivity still exists, but the value judgments are articulated explicitly by the appropriate elected or appointed official, as opposed to the implicit value judgments implied by the metric conversions that are generally carried out by rather arbitrary articulators in the course of the planning process. Keeney and Raiffa(12), in fact, have developed specialized techniques for determining the appropriate

mathematical form of the utility function depending on the type of independent relationships among the attributes.

### 3.2.3 Processing by Objective

In many situations the analysis cannot reach the project evaluation stage for one of two reasons: (1) there are several independent decision makers, each with a distinct utility function; or (2) an individual decision maker may have multiple objectives or measures of utility against which the project design must be compared, but he may be uncertain as to their relative importance. Therefore, a single utility function cannot generally be determined. The original list of alternatives might be narrowed to a set of efficient or non-inferior designs, with the final review and selection being made outside of this analytic framework.

The analyst's role, if analysis is to truncate at this stage, narrows to a search for the pareto-optimal set of alternatives (that set which is not dominated), and the final choice is likely to be heavily political. Location of the set of pareto-optimal alternatives is particularly relevant when a single alternative has to be selected, such as is the case in which the alternatives are all variants of the same project. It is equally relevant in the context of the feeder roads problem, where many projects are to be selected from an even larger set of potential projects, and the government and the lending agency are not optimizing



over the same objectives or each has a distinctly different inter-objective function.

#### 3.2.4 Data Collection, Verification and Organization

At the lowest level of truncation, an iterative analytic process between decision maker and analyst is used to arrive at the best compromise under a situation of multiple objectives or utility functions as follows: (1) the most efficient consequences of selected assumptions concerning the relative importance of each objective are presented; (2) a new set of assumptions is derived through inputs from the parties involved; (3) the associated efficient consequences are displayed; and (4) further iterations take place until a final decision is reached. This is essential if the decision maker is unwilling or unable to articulate the roles and, upon receiving the analysts' best effort, does not like the answer.

This iterative processing is often referred to as backing into the solution. The decision maker has received a directive - "Build the Amakalakala - Petchibun Road to class B-2 standards" and must now implement the political decision within the formal analytical structure of the Works Board or the rules of the lending agency. Often the decision maker is unable to state this fait accompli to the analyst, and must therefore shape the decision process by steps to ensure the "correct" answer is reached. The problem is not uncommon.

Turning to the problem at hand, feeder road appraisal can be visualized as a choice of many small projects where each project has several important selection criteria, each measure is expressed in its own units, and the set of projects to be implemented is selected from a much larger set. A good example is the Thailand Department of Highway's road study in 1980(13). The study reviewed 15,000 kilometers of national and 29,000 kilometers of provincial roads (some 2,100 links) and passed them through four screens, some with explicit utility functions (e.g., exclude roads upgraded in the last 3 years, or built in the last five, or scheduled for improvement in the next 5 year plan, or on a lending agency project list) and some iterative, in order to identify 1,500 kilometers of projects for the next five-year plan. Early in the study, it was observed that too many rural roads dropped out of the project pool. During an iterative stage and to control the loss of rural roads from the project set, 135 links failing the second screen were dubbed "developmental" roads and held for further testing. Still, only 12 of these links were suitable for further analysis. After a second iteration, a study called the Rural Roads One Program was commissioned into which the 123 remaining links were placed with some additional 3,000 kilometers of non-departmental roads. These roads were then analyzed under a completely new set of objective functions.

The situations characterized by this example are very realistic representations of scenarios in developing countries where there are numerous parties involved, each with its own interests and capabilities, and each desiring participation in the decision process. For the purpose of this thesis we will examine two situations: one where consensus exists, permitting the definition of a utility function, and one where several objective functions are clear but no consensus exists or was communicated to the analyst.

### 3.3 Evaluation Where Consensus Exists

Let us assume that there is a universal commitment to the achievement of certain accepted goals, making it unnecessary to model different preferences among different interest groups; that is, it is assumed that a single set of social preferences can be articulated with the help of the appropriate decision maker, who might be, for example, the Director of the road authority or the Minister of Public Works. In view of this assumption and of the characterization of the rural road situation given above, the rural road analysis must be truncated at the evaluation by project level.

For our example, five objectives are selected to be incorporated in the framework for evaluation of a set of rural road projects. These include: (1) net national income, (2) investment, (3) distribution, (4) accessibility to

social services, and (5) employment. Contributions to these objectives to be considered are those resulting from provision of the feeder road and its complementary investments. These represent just one possible set of objectives and are not intended to be a universal representation of the accounting of socioeconomic objectives of rural development activities. It is the ultimate decision maker in the particular case under study who must be satisfied that the set of objectives is sufficient. The appraisal framework, as structured here, is independent of changes in the objectives considered or in their number.

### 3.3.1 Equal Preference Alternative

Implicit in the no-preference alternative is the assumption that all objectives are, in terms of maximum likelihood, of equal importance, which can be demonstrated through the use of entropy arguments. Therefore, this case is actually a special subset of the complete information, cardinal weights case. Thus, the projects are ranked by the value of the average of the utilities over all objectives:

(3-1)

$$\text{RANK}_{\text{equal}} (P_j) = \frac{1}{n} \sum_{i=1}^n u_i(x_{i,j})$$

where:

$x_{i,j}$  is the score of the  $j^{\text{th}}$  project on the  $i^{\text{th}}$  objective

$u_i$  is the utility function for the  $i^{\text{th}}$  objective

$n$  is the number of objectives

$P_j$  denotes the  $j^{\text{th}}$  project

If the objectives are truly equal in importance, or if none can be determined to be more important than the others, according to the best of knowledge of or constraints upon the decision maker, the analysis can proceed directly using the above formulation without any further inputs from the decision maker.

### 3.3.2 Cardinal Preference Alternative

The cardinal weights, or complete-information, approach allows for differences in the relative importance of the various objectives and assumes that explicit weights can indeed be assigned to each. Projects, therefore, are ranked according to the weighted sum of the utilities over all objectives:

(3-2)

$$\text{RANK}_{\text{cardinal}}(P_j) = \left( \sum_{i=1}^n w_i \right)^{-1} \sum_{i=1}^n w_i u_i(x_{i,j})$$

where:

$w_i$  is the weight placed on the  $i^{\text{th}}$  objective and the other parameters are as before.

To complete the analysis using this formulation, articulation of the cardinal weights must be elicited from the appropriate decision maker. In actual practice, this often proves to be rather difficult due both to conceptual problems in explicitly assigning the correct social weights and to politically sensitive issues.

### 3.3.3 Ordinal Preference Alternative

In cases where the decision maker cannot or is unwilling to specify cardinal weights, the ordinal weights approach might be used in completing the analysis and ultimately ranking the projects. Application of this alternative requires the decision maker to designate an ordinal ranking of the objectives to reflect their relative importance. Given this relatively small amount of information, the analysis can be completed using either maximum likelihood or linear programming approaches. The linear programming formulation discussed below was initially developed by Cannon and Kmietowicz(14) for application to decision-making problems under uncertainty; further details of its derivation for its application here were performed by Brademeyer in 1980(15), and for completeness are given in

Appendix A. His maximum likelihood formulation is summarized in Appendix B.

The linear programming formulation states that given an ordered set of objectives, the set of utility functions of the various objectives, and a set of projects, an upper and lower bound on the weighted score of each project based on that order of objectives can be determined. That is, any vector of cardinal weights that obeys the stated ordering will have a weighted score of not more than this upper bound and not less than this lower bound for each project. From Appendix A, we can state that given a set of  $n$  ordered objectives, we can establish  $n$  sets of cardinal weights,  $k$ , obeying that order such that one set will produce the upper bound for any given project and one set will produce the lower bound.

We can formalize these  $n$  sets of weights for the  $n$  objectives in matrix form,  $w_{i,k}$ , as follows:

$$w_{i,k} = \begin{array}{c|cccccccc} & i & 1 & 2 & 3 & \dots & n & \\ \hline k & & & & & & & \end{array} \quad (3-3)$$

|             |   |     |     |     |     |     |
|-------------|---|-----|-----|-----|-----|-----|
| $w_{i,k} =$ | 1 | 1   | 0   | 0   | ... | 0   |
|             | 2 | 1/2 | 1/2 | 0   | ... | 0   |
|             | 3 | 1/3 | 1/3 | 1/3 | ... | 0   |
|             | . | .   | .   | .   | ... | .   |
|             | . | .   | .   | .   | ... | .   |
|             | . | .   | .   | .   | ... | .   |
|             | n | 1/n | 1/n | 1/n | ... | 1/n |

or, equivalently, as:

(3-4)

$$\begin{aligned} w_{i,k} &= 1/k & i &\leq k \\ w_{i,k} &= 0 & i &> k \end{aligned}$$

Therefore, from the linear programming formulation detailed in Appendix A, we can produce two decision rules for ranking the projects:

(3-5)

$$\text{RANK}_{\text{ordinal}}^{\text{max}} (P_j) = \text{Max}_k \left[ \sum_{i=1}^k \frac{u_i(x_{i,j})}{k} \right] \quad (k = 1, \dots, n)$$

in which the ranking is based on the highest score that each project may attain given the ordering of the objectives; and

(3-6)

$$\text{RANK}_{\text{ordinal}}^{\text{min}} (P_j) = \text{Min}_k \left[ \sum_{i=1}^k \frac{u_i(x_{i,j})}{k} \right] \quad (k = 1, \dots, n)$$

in which the ranking is based on the lowest score that each project may attain given the ordering of the objectives.

For the maximum-likelihood formulation, which is presented in Appendix B, the determination of the "most-likely" set of ordered weights has been designated the mode-ordinal rule. That analysis shows that given an ordering of the preferences on the objectives the most-likely set of weights will be given by:



$$w_i = 2^{-i} \quad (i = 1, 2, \dots, n-1) \quad (3-7)$$

$$w_n = w_{n-1}$$

Thus, the mode-ordinal rule produces a set of maximum-likelihood cardinal weights, and the projects can then be ranked according to this weighted sum of the utilities over all objectives:

$$\text{RANK}_{\text{ordinal}}(P_j) = \left( \sum_{i=1}^{n-1} 2^{-i} u_i(x_{i,j}) \right) + 2^{1-n} u_n(x_{n,j}) \quad (3-8)$$

#### 3.3.4 Partial Ordinal Preference Alternative

An alternative situation arises if the decision maker is only able to articulate a partial ordering of the weights; that is, he specifies his preference among independent subsets of the objectives, but cannot or is unwilling to articulate a preference among the objectives within each subset. This is a combination of the no information and ordinal information cases, and it is easily handled by the above ordinal decision rules. However, the sets of weights for the max-ordinal and min-ordinal rules are altered, as are the maximum-likelihood weights in the mode-ordinal rule.

To see this, consider a partition of the objectives into  $k$  mutually exclusive subsets,  $(S_k)$ , each containing  $n_k$  objectives and a stated preference order given by the index

k; that is,  $(S_k)$  is preferable to or as preferable as  $S_{k+1}$ . Then the weights for the linear programming rules become:

$$\begin{aligned} w_{i,k} &= 1 / \sum_{m=1}^k n_k & i &\leq \sum_{m=1}^k n_k \\ w_{i,k} &= 0 & i &> \sum_{m=1}^k n_k \end{aligned} \tag{3-9}$$

and the ranking rules in Equations (3-5) and (3-6) become:

$$\begin{aligned} \text{RANK}_{\text{ordinal}} \max (P_j) &= \text{Max}_k \cdot \frac{\sum_{i=1}^k \left( \sum_{m=1}^k n_k \right) u_i(x_{i,j})}{\sum_{m=1}^k n_k} \end{aligned} \tag{3-10}$$

$$\begin{aligned} \text{RANK}_{\text{ordinal}} \min (P_j) &= \text{Min}_k \cdot \frac{\sum_{i=1}^k \left( \sum_{m=1}^k n_k \right) u_i(x_{i,j})}{\sum_{m=1}^k n_k} \end{aligned} \tag{3-11}$$

This procedure is developed in detail in Appendix A.

For the mode-ordinal rule, the maximum-likelihood cardinal weights become:

$$\begin{aligned} w_i &= 2^{-m} & (i \in (S_m), m = 1, 2, \dots, k-1) \\ w_i &= 2^{1-k} & (i \in (S_k)) \end{aligned} \tag{3-12}$$

### 3.3.5 Implications of the Various Alternatives

It is imperative that the decision maker be properly informed of the various implications of these alternative

schemes for ranking rural penetration road projects. The appropriate procedure is obviously situation and case specific and is constrained by the type of value judgment the analyst can elicit from the decision maker. In order to demonstrate the various implications of these decision rules, consider the set of six projects to be evaluated under five objectives (assumed to be in preference order, as given in Table 2). The ranking scores under each of the non-cardinal decision rules are presented in Table 3.

These six projects all have the same total utility score and, hence, are indistinguishable under the equal weights decision rule. They were chosen to illustrate the implications of the ordinal decision rules since these are not intuitively obvious by any means.

The ranking of projects produced by the max-ordinal decision rule can be said to be of a less conservative/more aggressive nature. That is, if a situation arises in which the contribution to the preferred objective is exceptionally good relative to that of any of the other objectives (which might be exceptionally poor), the rule cannot take the latter into account. If this inability to account for an exceptionally poor objective measure is not a critical issue, as long as there exists a more preferred objective with an exceptionally good measure, use of the max-ordinal decision rule may be justified. This situation is illustrated by Project E, which is ranked relatively high in Table 3,

Table 2

A SET OF HYPOTHETICAL PROJECTS

Utility Scores on Various Objectives

| <u>Project</u> | <u>u<sub>1</sub></u> | <u>u<sub>2</sub></u> | <u>u<sub>3</sub></u> | <u>u<sub>4</sub></u> | <u>u<sub>5</sub></u> |
|----------------|----------------------|----------------------|----------------------|----------------------|----------------------|
| A              | 50                   | 0                    | 95                   | 55                   | 50                   |
| B              | 20                   | 80                   | 50                   | 20                   | 80                   |
| C              | 0                    | 25                   | 50                   | 75                   | 100                  |
| D              | 50                   | 100                  | 5                    | 45                   | 50                   |
| E              | 80                   | 20                   | 50                   | 80                   | 20                   |
| F              | 100                  | 75                   | 50                   | 25                   | 0                    |

Table 3

PROJECT RANKING UNDER VARIOUS DECISION RULES

| <u>Project</u> | <u>Ranking Score</u> |                    |                    |                     |
|----------------|----------------------|--------------------|--------------------|---------------------|
|                | <u>Equal</u>         | <u>Max Ordinal</u> | <u>Min Ordinal</u> | <u>Mode Ordinal</u> |
| A              | 50                   | 50                 | 25                 | 43                  |
| B              | 50                   | 50                 | 20                 | 42                  |
| C              | 50                   | 50                 | 0                  | 23                  |
| D              | 50                   | 75                 | 50                 | 56                  |
| E              | 50                   | 80                 | 50                 | 57                  |
| F              | 50                   | 100                | 50                 | 76                  |

| <u>Project</u> | <u>Project Ranking</u> |                    |                    |                     |
|----------------|------------------------|--------------------|--------------------|---------------------|
|                | <u>Equal</u>           | <u>Max Ordinal</u> | <u>Min Ordinal</u> | <u>Mode Ordinal</u> |
| A              | *                      | *                  | 4                  | 4                   |
| B              | *                      | *                  | 5                  | 5                   |
| C              | *                      | *                  | 6                  | 6                   |
| D              | *                      | 3                  | *                  | 3                   |
| E              | *                      | 2                  | *                  | 2                   |
| F              | *                      | 1                  | *                  | 1                   |

\*Denotes decision rule cannot identify rank positions for these projects.

although it scores poorly on the second objective. Additionally, as can be seen in Table 3, the max-ordinal rule tends to make little distinction between those projects appearing in the lower half of ranking; that is, it seeks more to identify "good" projects, according to the ordering, rather than "bad" projects.

The min-ordinal decision rule may, on the other hand, be described as more conservative/less aggressive in nature. That is, the occurrence of an exceptionally poor objective measure is taken into account by this decision rule, but it, in turn, is unable to reflect the occurrence of an exceptionally good objective measure. If the ability to account for a relatively poor objective measure is critical, as is illustrated by the analogy of "a chain is as strong as its weakest link," then use of the min-ordinal decision rule may be justified. This situation is illustrated by Project B, which is ranked relatively low in Table 3, although it scores well on the second objective. The min-ordinal rule tends to make little distinction between those projects appearing in the upper half of the ranking, as may be seen from Table 3; that is, it seeks more to identify (and eliminate) "bad" projects according to the ordering of the objectives rather than identify "good" projects.

The mode-ordinal rule may be regarded as "averagely" conservative/aggressive in nature. The contributions of all criterion measures are accounted for according to the most-likely preferences indicated by the ordering. As can be seen from Table 3, it may be regarded as a compromise between the max-ordinal and min-ordinal decision rules since it ranks the projects distinguished by either rule in a similar manner.

A further limitation of both the max-ordinal and min-ordinal decision rules, vis-a-vis the other rules, is that the set of projects is not ranked according to a single set of weights since those weights that maximize (minimize) the score of one project will, in general, not be the same as those maximizing (minimizing) another project's score. The above discussion of ordinal rule implications is, of course, directly applicable to the partial-ordinal decision rules.

Finally, it should be stated that if the information is available and believed reliable, use of the equal or cardinal weighting techniques for ranking projects may be most appropriate.

### 3.4 Processing Where No Consensus Exists

Often there are two or more decision makers who, having agreed on objectives, cannot agree on the ranking--either cardinal or ordinal. They wish, obviously, to be left room

to negotiate their own "unquantifiable" benefits and costs. In this situation, the responsibility of the analyst is the preparation and presentation of the evaluation in a manner that will aid the decision makers by informing them of the implications of each choice or set of choices. As Ato Hailu Shawel, General Manager of the Ethiopian Road Authority(16) said at 1975 conference on rural road evaluation,

"There might be, sometimes, non-scientific decisions to be made,...contrary to figures prepared by experts.... At least let us know what we are deciding when we take that position. If we decided to take Road C and it is, on our list, number 15 in priority, at least we knew what we were sacrificing when we took that field decision. But at the moment we take decisions on figures, but we do not know which item we picked actually."

Just as the social and political realities of a nation preclude the use of only one objective--national income--in choosing projects, the reality of budget or loan negotiations may require the flexibility of more information but no singular decision.

The ranking system given in Section 3.3 does not permit the competitive decision makers access to tradeoffs without referring back to the analyst. Furthermore, the analysis would typically be too technical for them to

utilize it knowledgeably. Various graphic presentations permit this to be done.

### 3.4.1 The Two Objective Space

Let us examine the project set J given in Table 4. Two objectives have been identified: Net Region A income and access to health care services. Net Region A income is in million currency units ranging from 2 million to 11 million. Health care access, the percentage of the 10,000 people within the geographic domain of the clinic who would now have access to the clinic, ranges from 20 to 100 percent. This information can also be displayed as shown in Figure 5.

Table 4

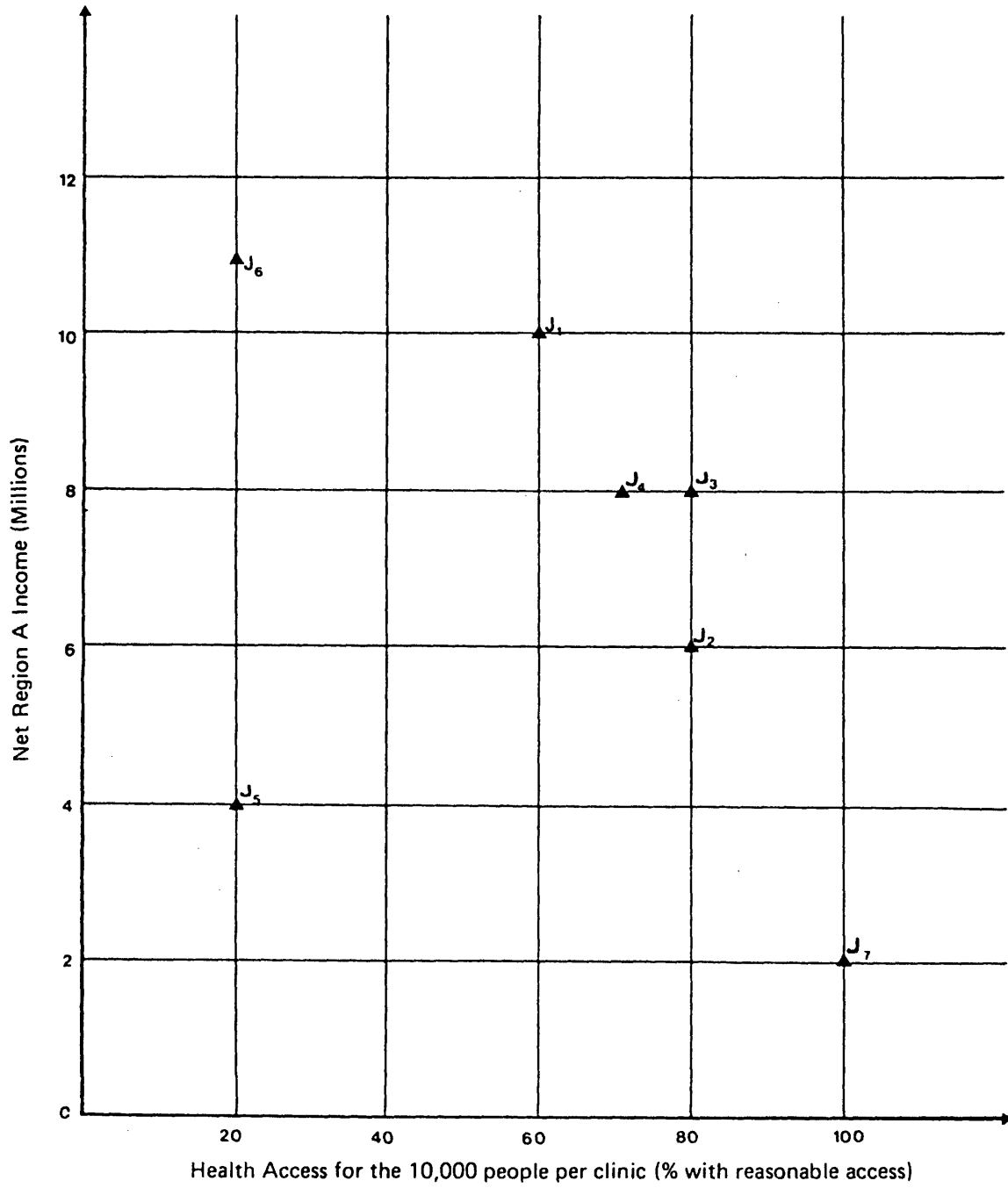
| <u>Project Solution</u> | <u>Net Region A Income</u> | <u>Health Access</u> |
|-------------------------|----------------------------|----------------------|
| J <sub>1</sub>          | 10                         | 60%                  |
| J <sub>2</sub>          | 6                          | 80%                  |
| J <sub>3</sub>          | 8                          | 80%                  |
| J <sub>4</sub>          | 8                          | 70%                  |
| J <sub>5</sub>          | 4                          | 20%                  |
| J <sub>6</sub>          | 11                         | 20%                  |
| J <sub>7</sub>          | 2                          | 100%                 |

Presented with either Table 4 or Figure 5, a decision maker, without attempting to aggregate regional income and lives, might observe:



Figure 5

TWO OBJECTIVE ANALYSIS SPACE



(1) J<sub>5</sub> and J<sub>6</sub> do not achieve a high enough health access

(2) J<sub>7</sub> does, but at the expense of regional income

(3) J<sub>2</sub> and J<sub>4</sub> are inferior to J<sub>3</sub>

The choice is therefore reduced to set J<sub>1</sub> and J<sub>3</sub>.

The final choice would depend on the perceived tradeoff between 2 million units of regional income and reasonable health access for 20 percent more people.

If, however, a constraint is present that states that all investments must obtain not less than 75 percent health access (line H-H on Figure 6), the decision is simplified since J<sub>1</sub> does not meet the constraint.

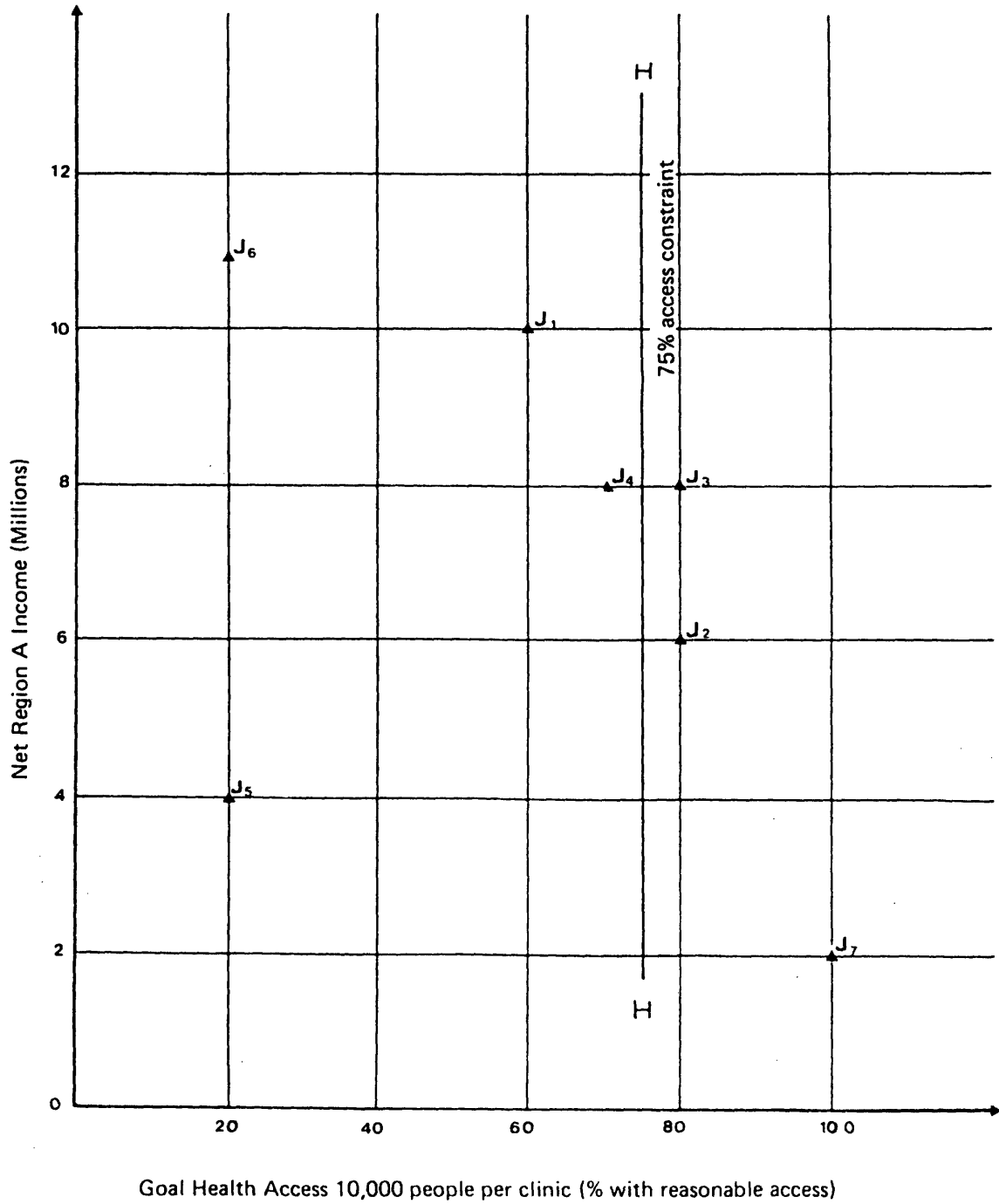
#### 3.4.2 The Three Objective Space

Evaluation rarely involves only two objectives, and a third could be the foreign exchange impact. If we add the present value of the net foreign exchange consumption (in millions) we have Table 5.

Table 5

| <u>Project Solution</u> | <u>Net Region<br/>A Income</u> | <u>Health<br/>Access</u> | <u>Net Foreign<br/>Exchange</u> |
|-------------------------|--------------------------------|--------------------------|---------------------------------|
| J <sub>1</sub>          | 10                             | 60%                      | 8                               |
| J <sub>2</sub>          | 6                              | 80%                      | 2                               |
| J <sub>3</sub>          | 8                              | 80%                      | 3                               |
| J <sub>4</sub>          | 8                              | 70%                      | 2                               |
| J <sub>5</sub>          | 4                              | 20%                      | 5                               |
| J <sub>6</sub>          | 11                             | 20%                      | 3                               |
| J <sub>7</sub>          | 2                              | 100%                     | 2                               |

Figure 6  
TWO OBJECTIVE ANALYSIS SPACE



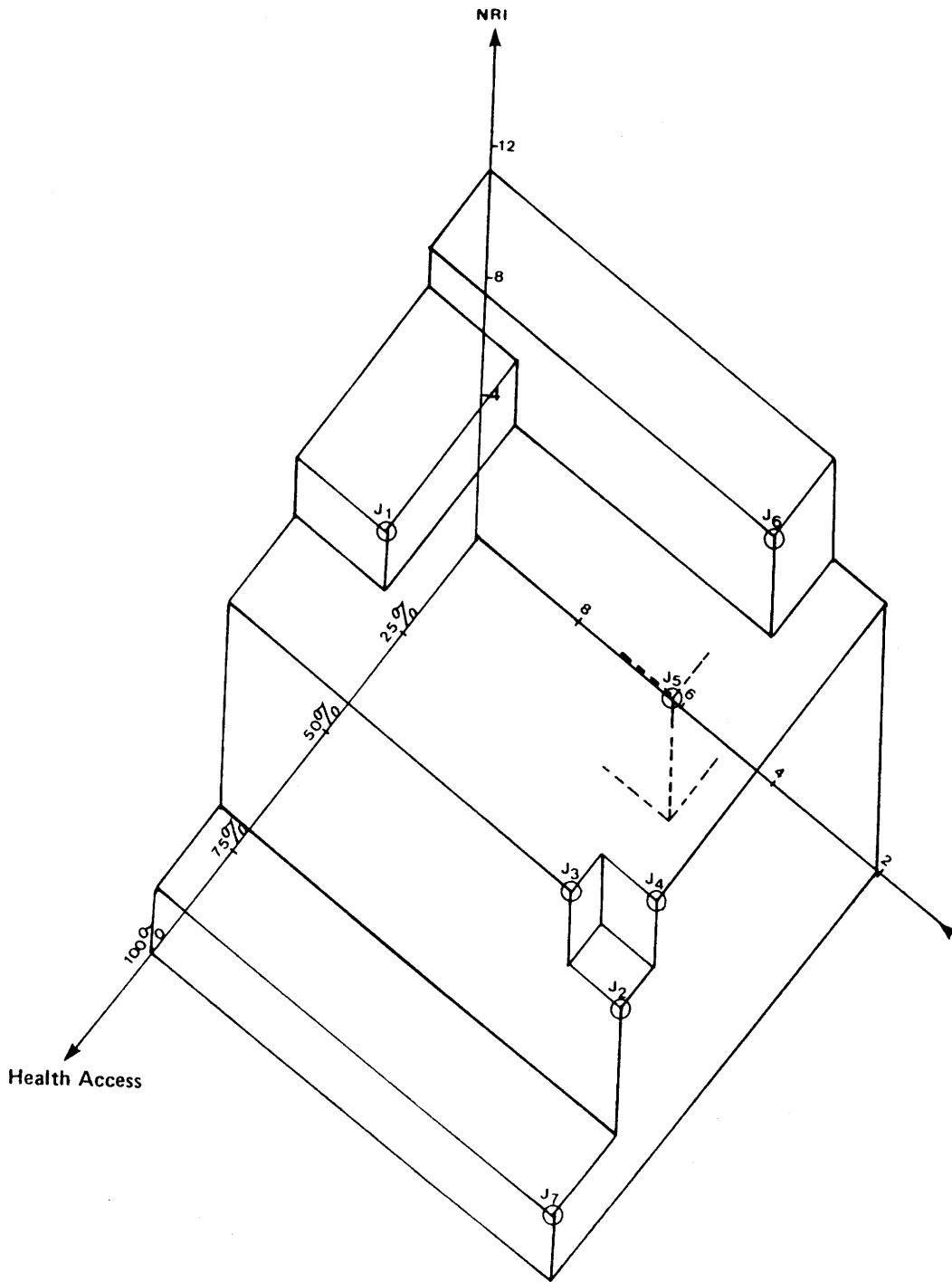
The evaluation of this information in tabular form is more difficult. Applying the 75 percent health access constraint, we are left with the following subset:

| <u>Project Solution</u> | <u>NRI</u> | <u>FX</u> |
|-------------------------|------------|-----------|
| J <sub>2</sub>          | 6          | 2         |
| J <sub>3</sub>          | 8          | 3         |
| J <sub>7</sub>          | 2          | 2         |

Certainly, J<sub>2</sub> seems preferable to J<sub>7</sub>. What about J<sub>2</sub> versus J<sub>3</sub>? How does one compare 2 million units of Region A income with 1 million of foreign exchange? If the desire is to minimize foreign exchange consumption, choose J<sub>2</sub> over J<sub>3</sub>. Yet if foreign exchange conservation is so important perhaps the 75 percent access cutoff may be too rigid and J<sub>4</sub> should be reconsidered. But what is the tradeoff between 1 million units of foreign exchange and access for 10 percent of the population? These data would better be displayed graphically.

It should be noted that consumption of foreign exchange is a negative concept. To facilitate the visual presentation by maintaining the "bigger is better" orientation, the scale for foreign exchange (FX) must be reversed. The resultant graph (Figure 7) makes analysis more straightforward. Project J<sub>5</sub> is clearly dominated under all conditions. If the subjective disposition of the decision maker is to go

Figure 7  
THREE OBJECTIVE ANALYSIS SPACE



for net regional income and health access, then  $J_1$ , is the clear choice; if income and conservation of exchange reserves, then  $J_6$ ; and if health access and conservation of reserves, the  $J_7$ . If, however, the decision maker feels that no pair of objectives dominates, then he needs to consider  $J_2$ ,  $J_3$ , and  $J_4$  from some other vantage point.

By passing plane H-H through the health access axis at 75 percent (Figure 8) we are left with  $J_2$ ,  $J_3$  and  $J_7$ . Both  $J_2$  and  $J_3$  are clearly superior to  $J_7$ , as concluded in the discussion of Table 5. The superiority of the graphical presentation over tabular is the retention of information even after the application of the cutoff for health access.

### 3.4.3 The Multi-Objective Space

A boon compared to tabular display, graphic analysis breaks down beyond three objectives (when displayed on a two dimensional medium). Evaluation of four or more objectives should use the strategy of minimum or maximum constraints (such as the minimum of 75 percent for health access) to reduce not only the set of solutions being evaluated but also the number of objectives remaining. Using the 75 percent health access constraint, all but  $J_2$ ,  $J_3$ , and  $J_7$  are eliminated. We now add the fourth objective, employment, in thousand labor years, as shown in Table 6.

Figure 8  
THREE OBJECTIVE ANALYSIS SPACE

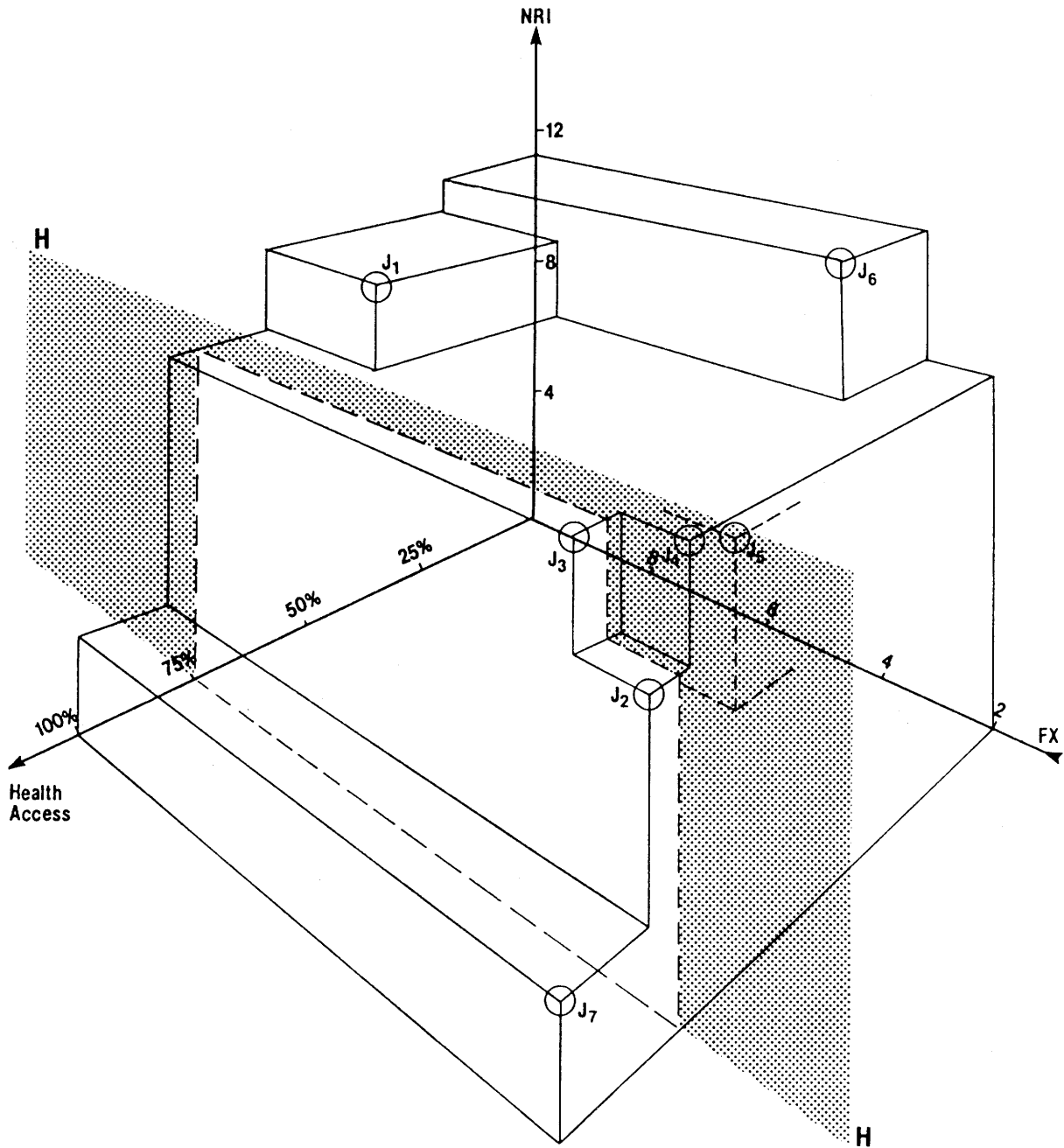


Table 6

| <u>Project Solution</u> | <u>NRI</u> | <u>FX</u> | <u>Work Days</u> | <u>Health Access</u> |
|-------------------------|------------|-----------|------------------|----------------------|
| J <sub>2</sub>          | 6          | 2         | 10               | 80%                  |
| J <sub>3</sub>          | 8          | 3         | 6                | 80%                  |
| J <sub>7</sub>          | 2          | 2         | 3                | 100%                 |

Figure 9 demonstrates that solution J<sub>7</sub> now clearly appears a subset of J<sub>2</sub>. J<sub>2</sub> and J<sub>3</sub> trade-off 2 million units of regional income for 40,000 labor years.

#### 3.4.4 Information Loss and Graphic Distortion

Note how J<sub>7</sub> is on one face of J<sub>2</sub> and therefore a subset of J<sub>2</sub>. This is called information loss. By setting a health access cutoff of 75 percent in order to display other objectives, we no longer can identify the 20 percent additional access achieved with solution J<sub>7</sub>. If the analyst provides the decision maker only with Figure 9, no tradeoff can be entertained on this 20 percent. This suggests both tabular presentation as well as multiple graphical displays, supported by caveats whenever information loss occurs.

A second cautionary note involves graphic distortion. Solution J<sub>2</sub> appears somewhat (though not excessively) better than solution J<sub>3</sub> in Figure 9; Figure 10 shows J<sub>2</sub> to be overwhelmingly better (bigger). The shifted axes and distorted scales also make J<sub>7</sub> appear competitive with J<sub>3</sub>.



Figure 9  
THREE OBJECTIVE ANALYSIS  
AFTER APPLICATION OF HEALTH ACCESS CUTOFF

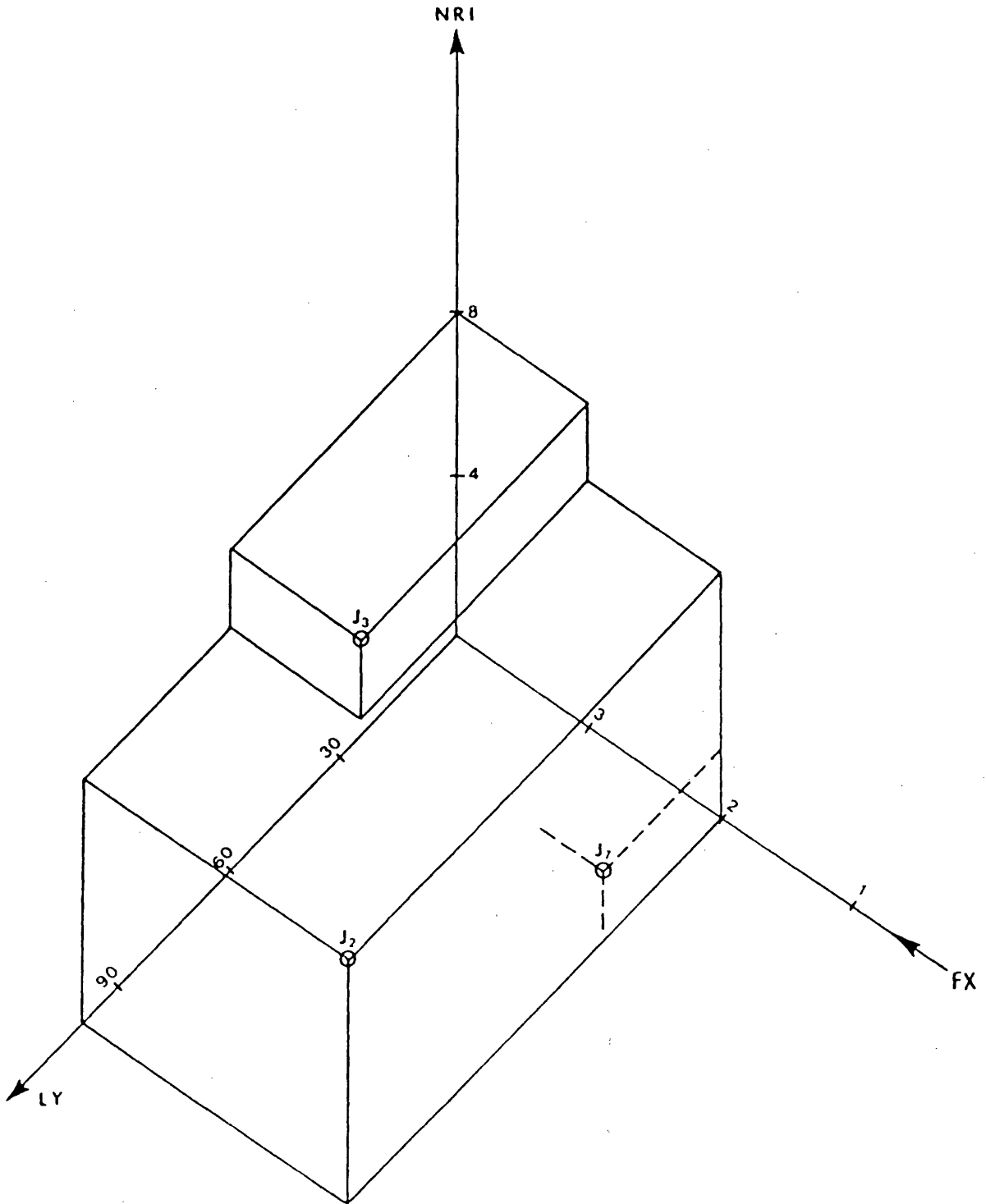
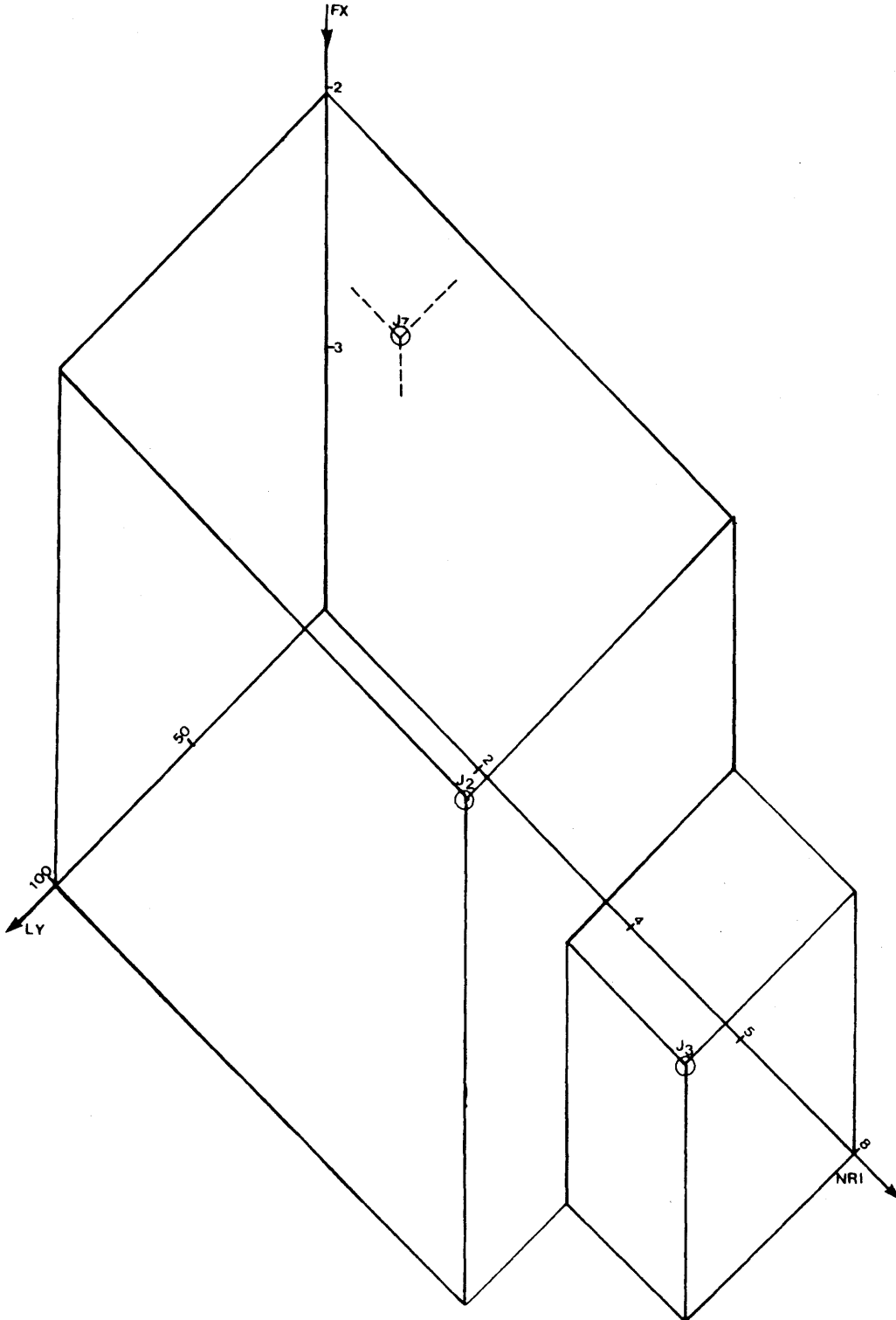


Figure 10  
THREE OBJECTIVE ANALYSIS FROM FIGURE 9  
WITH SHIFTED AXES AND ADJUSTED SCALES



### 3.4.5 Project Selection in Multi-Objective Space

The concept of rank ordering mutually exclusive investment options is a carry-over from single objective analysis. Ranking is basically a linear concept that is inappropriate in much of multi-objective analysis. It is impossible to rank in multi-objective space without explicitly weighting factors that relate not only national to regional to foreign exchange currency units, but also currency to health access to work days. If the decision maker is willing to set these weights for the decision-making process, the analyst should use the numerical methods discussed earlier.

It is this very difficulty, the eventual requirement of weighting and unification in order to effect ranking, that has in part kept multi-objective analysis from being used more fully. Faced with a linear ranking of investment choices, the decision maker can choose with some facility. Referring again to the quote from Ato Hailu Shawel (16), the decision maker can choose a project, know it is number 15, and accept the consequences with the justification of "other" objectives. This form of casual choice, where the "blame" rests on the technician, is not available in multi-objective graphic analysis. These "other" objectives are specifically identified by the analysts and evaluated for all projects and the results supplied to the decision maker for project selection. Selection may proceed in two ways.

a. Project Selection - by Project

Project selection can be performed in much the same manner as the solution evaluation methodology described above. This would essentially perform the function of removing inferior projects from the set, that is, projects wholly interior to or on the face of another project.

Of the projects remaining, should their total cost exceed the budget, elimination would proceed based on subjective trade-offs applied by the decision maker. If the trade-offs were quantifiable, they should have already been incorporated into the analysis procedure.

b. Project Selection - by Strategy

A second method of project selection, available at the analyst level, would involve grouping of projects into strategy sets. Assuming once again that the sum of all projects exceeds the budget, the analyst composes a series of strategies that meet the budget constraint. Within the budget constraint, however, many other objectives can be maximized. For instance, one strategy could emphasize balancing regional investment, another maximizing health access, another maximizing net national incomes, another maximizing net regional incomes, and another minimizing consumption of foreign exchange. Strategy identification proceeds as project evaluation.

Presenting strategies that emphasize each objective of interest to the decision maker while meeting the budget

constraint facilitates the decision process. If several objectives have primary importance (for example, budget constraint and balancing regional investment), a series of strategies could be defined that closely achieve both these objectives while emphasizing, in each different strategy, a third objective.

For each of these strategies, discussions of the effects of each optimization should be provided. When any one objective is maximized (minimized), the effect on all other objectives should be discussed. In addition, to show the relative extent of each maximization strategy, a fully optimized strategy should be presented that would represent the maximization of that objective function without the budgetary (or other primary) constraint assumption.

### 3.5 Summary

This chapter presented the multi-objective analytical problem in terms of the level of consensus/disparity amongst decision makers. For each of these levels, an appropriate technique for treatment of the multiple objectives was developed. These techniques fall into two broad categories depending upon the degree of consensus. Chapter 4 presents an application of the methodology where consensus exists; Chapter 5, where it does not.

## Chapter Four

### APPLICATION OF THE METHODOLOGY WHERE CONSENSUS EXISTS

Chapter 3 presented a set of methodologies for evaluating projects with multiple objectives. This chapter will demonstrate, using a hypothetical example, how to evaluate and rank 36 projects using five criteria and five decision rules. The criteria are economic benefits, economic costs, distribution, employment, and access to services. The decision rules are equal weights on the criteria, cardinal weights, ordinal ranking and two cases of partial ordinal ranking.

A hypothetical case study (17) involving some 36 alternative projects was designed for the purpose of demonstrating the use of the overall appraisal framework (details are given in Appendix C). A typical project might be as follows:

A 20-kilometer feeder road project is proposed to join a small agricultural community of some 600 persons to a small provincial market town served by a good secondary road. At present an earth trail, not passable by motor vehicles, exists and is mainly used for walking or transport by pack animals to the nearby town, where the peasants periodically go to sell agricultural surplus or purchase consumer goods. The community appears to have suitable environmental conditions for agricultural development.

As part of a regional development effort, a package of investment projects has been proposed by the design team, including upgrading the donkey trail to a gravel road; implementing agricultural extension services directed toward improving existing production, increasing the area of land under cultivation, and introducing a new crop, cocoa; establishing a health clinic in the community; and providing general primary education for the first time in the community.

The community has some 109 families, of which 5 are relatively rich and own between 45 and 50 hectares of land each, 34 families own 2 to 10 hectares each, and 70 are landless (50 of these rent a total of 100 hectares from the rich for subsistence farming, and the other 20 work for the rich families). Present production consists of cassava, rice, and maize and some livestock on 405 hectares of land; an additional 113 hectares of cultivatable land is currently idle. It is proposed to bring this land under cultivation with a new cash crop, cocoa, as well as an additional 70 hectares of nearby government land, the latter to be allotted to the 70 landless families. The target population is thus 104 families who currently own 278 hectares of land, which will be increased to 348 hectares by the project.

Given this sort of information, contributions to the various criteria were identified and measured in their appropriate units, as discussed in Chapter Two. This represents just one of the myriad possible scenarios for feeder road projects. Corresponding descriptions for the other 35 projects could be created. In the case at hand, a spectrum of plausible measures was simply developed. Using the utility assessment techniques described in Chapter Two, preference functions were developed for each criterion and utility values assessed for each project's contribution to each of the five criteria. The set of values developed and used in the case study is given in Table 7.

A review of the figures in the table shows considerable variation in the contributions of the 36 projects to the five criteria. Patterns in these differences are difficult to establish because they depend on the specific circumstances of each particular project. An extreme project may have, for example, the best contribution with respect to one criterion and the worst with respect to another. Project 4 has such characteristics with regard to economic benefits and distribution, respectively. This is an example of a situation where the land in the project area of influence is owned by rich landlords who will be able to take advantage of the transport improvement, resulting in high economic benefits in the form of induced agricultural production but



Table 7

UTILITY OF THE CRITERIA MEASURES,  $u_i(x_{i,j})$ 

| Project Number | Economic Benefits | Economic Costs | Distribution | Employment | Accessibility to Social Services |
|----------------|-------------------|----------------|--------------|------------|----------------------------------|
| 1              | 34.09             | 61.82          | 21.58        | 83.04      | 42.79                            |
| 2              | 30.22             | 17.66          | 54.79        | 100.00     | 52.03                            |
| 3              | 15.69             | 37.91          | 20.13        | 13.71      | 33.28                            |
| 4              | 100.00            | 31.26          | 0.00         | 45.23      | 20.21                            |
| 5              | 90.43             | 72.67          | 0.99         | 24.48      | 10.21                            |
| 6              | 73.88             | 58.90          | 1.25         | 28.90      | 14.99                            |
| 7              | 41.35             | 55.67          | 27.63        | 88.02      | 31.26                            |
| 8              | 34.62             | 80.23          | 40.13        | 73.81      | 27.04                            |
| 9              | 20.77             | 81.81          | 56.46        | 46.26      | 15.61                            |
| 10             | 11.15             | 82.26          | 75.00        | 16.42      | 32.14                            |
| 11             | 30.22             | 77.86          | 12.04        | 43.94      | 24.53                            |
| 12             | 39.43             | 59.87          | 11.58        | 58.79      | 19.14                            |
| 13             | 41.35             | 38.40          | 82.14        | 89.39      | 71.45                            |
| 14             | 36.17             | 31.67          | 95.80        | 90.49      | 88.25                            |
| 15             | 29.44             | 44.46          | 76.07        | 58.27      | 52.56                            |
| 16             | 77.14             | 48.64          | 54.38        | 52.41      | 40.36                            |
| 17             | 79.93             | 2.76           | 29.00        | 33.07      | 27.01                            |
| 18             | 19.81             | 94.99          | 12.17        | 75.49      | 11.88                            |
| 19             | 3.30              | 95.85          | 5.46         | 0.00       | 5.13                             |
| 20             | 0.00              | 100.00         | 11.18        | 1.61       | 0.00                             |
| 21             | 44.54             | 84.08          | 0.33         | 31.85      | 14.50                            |
| 22             | 30.39             | 69.78          | 100.00       | 45.97      | 27.03                            |
| 23             | 63.84             | 0.00           | 52.50        | 68.12      | 45.21                            |
| 24             | 48.30             | 13.61          | 41.00        | 57.81      | 100.00                           |
| 25             | 21.16             | 50.00          | 70.83        | 9.68       | 17.80                            |
| 26             | 25.00             | 67.04          | 27.50        | 46.50      | 22.63                            |
| 27             | 53.93             | 38.55          | 50.00        | 67.18      | 38.75                            |
| 28             | 29.81             | 29.55          | 82.14        | 60.93      | 51.25                            |
| 29             | 6.73              | 67.06          | 87.68        | 31.85      | 27.00                            |
| 30             | 13.36             | 50.00          | 54.58        | 37.90      | 43.75                            |
| 31             | 60.31             | 47.73          | 40.38        | 56.25      | 45.00                            |
| 32             | 37.52             | 38.40          | 37.75        | 81.24      | 51.29                            |
| 33             | 32.98             | 8.14           | 11.97        | 46.37      | 57.50                            |
| 34             | 27.21             | 72.74          | 24.47        | 51.87      | 6.38                             |
| 35             | 30.77             | 34.93          | 15.26        | 58.12      | 45.38                            |
| 36             | 35.83             | 28.26          | 9.14         | 46.40      | 41.18                            |

Note: For all criteria except economic costs, the lowest attainment is assigned a utility of 0 and the highest 100; the situation is reversed for economic costs where the highest cost is assigned a utility of 0 and the lowest 100.

Source: Chew (17)

low in distribution effects in the sense that the poor farmer targets will reap only an insignificant amount of these benefits.

The various utility scores for each project were then combined and used in ranking the projects following the procedures outlined in Chapter Three. The mechanisms for the decision maker's articulation of preferences concerning the various criteria were tested under one or more sets of assumptions. Thus, the equal weights alternative, the ordinal weights alternative under max-, min- and mode-ordinal decision rules, the partial-ordinal alternative with the criteria in the same order for two different partial-specifications under max-and min-ordinal decision rules, and the cardinal weights alternative with the same order of criteria but three sets of weights were used for ranking the projects. Table 8 outlines the various ranking techniques employed, while Table 9 shows the alternative rankings of the set of projects achieved under these various assumptions as to a decision maker's level of information and preferences.

#### 4.1 Single-Rule Ranking of Projects

The ranking of projects by means of the equal-weights assumption (E) depends on their relative performance with regard to each criterion and in total. Projects ranked at the top tend to be uniformly good (for example, Project 16), or relatively good in two or more criteria and not too bad

Table 8

DECISION RULES USED IN THE CASE STUDY

- Equal weights on the criteria
- Cardinal weights on the criteria

| Rule | Weights, $w_i$ |       |       |       |       |
|------|----------------|-------|-------|-------|-------|
|      | $w_1$          | $w_2$ | $w_3$ | $w_4$ | $w_5$ |
| C1   | 50             | 20    | 15    | 10    | 5     |
| C2   | 22             | 21    | 20    | 19    | 18    |
| C3   | 90             | 4     | 3     | 2     | 1     |

- Ordinal ranking of criteria

| Rule         | Preferential Ordering of Criteria, $x_i$ |
|--------------|--|
| max-ordinal  |  |
| min-ordinal  | $x_1 > x_2 > x_3 > x_4 > x_5$            |
| mode-ordinal |  |

- Partial ordinal ranking of criteria

| Rule        | Preferential Ordering of Criteria, $x_i$ |
|-------------|--|
| max-ordinal |  |
| min-ordinal | $x_1 > (x_2 = x_3 = x_4 = x_5)$          |
| max-ordinal |  |
| min-ordinal | $(x_1 = x_2) > (x_3 = x_4 = x_5)$        |

where:  $>$  denotes "is preferred to"

- 1 - economic benefits
- 2 - economic costs
- i = 3 - distribution
- 4 - employment
- 5 - accessibility to social services

Source: Brademeyer (15)

TABLE 9  
A Summary Comparison of Project Rankings Using Different Decision Rules

| Rank | Equal Preference | Cardinal Preference |                |                | Ordinal Preference |     |      | Partial Ordinal Preference |                    |                    |                    |
|------|------------------|---------------------|----------------|----------------|--------------------|-----|------|----------------------------|--------------------|--------------------|--------------------|
|      | E                | C <sub>1</sub>      | C <sub>2</sub> | C <sub>3</sub> | Max                | Min | Mode | P-Max <sub>1</sub>         | P-Min <sub>1</sub> | P-Max <sub>2</sub> | P-Min <sub>2</sub> |
| 1    | 14               | 16                  | 14             | 4              | 4                  | 16  | 5    | 4                          | 16                 | 5                  | 16                 |
| 2    | 13               | 5                   | 13             | 5              | 5                  | 31  | 16   | 5                          | 31                 | 14                 | 8                  |
| 3    | 22               | 4                   | 16             | 16             | 17                 | 27  | 4    | 17                         | 27                 | 6                  | 22                 |
| 4    | 16               | 31                  | 22             | 17             | 16                 | 7   | 6    | 16                         | 24                 | 4                  | 31                 |
| 5    | 15               | 13                  | 15             | 6              | 6                  | 13  | 31   | 6                          | 23                 | 13                 | 7                  |
| 6    | 24               | 6                   | 8              | 23             | 14                 | 5   | 13   | 14                         | 13                 | 21                 | 1                  |
| 7    | 8                | 14                  | 24             | 31             | 22                 | 4   | 22   | 13                         | 7                  | 16                 | 27                 |
| 8    | 2                | 27                  | 31             | 27             | 13                 | 32  | 27   | 23                         | 5                  | 18                 | 9                  |
| 9    | 28               | 22                  | 28             | 24             | 21                 | 6   | 14   | 31                         | 4                  | 8                  | 10                 |
| 10   | 31               | 17                  | 27             | 21             | 23                 | 21  | 8    | 22                         | 12                 | 22                 | 18                 |
| 11   | 27               | 23                  | 2              | 13             | 31                 | 12  | 17   | 27                         | 32                 | 11                 | 13                 |
| 12   | 32               | 8                   | 7              | 7              | 8                  | 8   | 21   | 15                         | 14                 | 31                 | 5                  |
| 13   | 7                | 7                   | 32             | 12             | 18                 | 17  | 23   | 24                         | 6                  | 15                 | 4                  |
| 14   | 1                | 24                  | 1              | 14             | 10                 | 1   | 7    | 8                          | 21                 | 24                 | 32                 |
| 15   | 23               | 15                  | 23             | 32             | 11                 | 14  | 1    | 2                          | 8                  | 9                  | 12                 |
| 16   | 9                | 1                   | 9              | 8              | 27                 | 23  | 24   | 28                         | 17                 | 2                  | 26                 |
| 17   | 29               | 21                  | 29             | 1              | 29                 | 24  | 15   | 32                         | 1                  | 28                 | 11                 |
| 18   | 10               | 32                  | 10             | 36             | 7                  | 22  | 9    | 7                          | 36                 | 20                 | 15                 |
| 19   | 18               | 28                  | 18             | 22             | 9                  | 11  | 32   | 1                          | 33                 | 34                 | 29                 |
| 20   | 30               | 9                   | 5              | 15             | 15                 | 28  | 12   | 21                         | 35                 | 27                 | 34                 |
| 21   | 5                | 12                  | 4              | 28             | 24                 | 15  | 18   | 9                          | 22                 | 12                 | 6                  |
| 22   | 4                | 2                   | 30             | 2              | 2                  | 34  | 11   | 10                         | 2                  | 19                 | 21                 |
| 23   | 12               | 11                  | 12             | 33             | 28                 | 35  | 28   | 18                         | 11                 | 32                 | 17                 |
| 24   | 26               | 18                  | 11             | 11             | 1                  | 26  | 10   | 29                         | 28                 | 7                  | 14                 |
| 25   | 11               | 34                  | 26             | 35             | 20                 | 36  | 34   | 30                         | 15                 | 1                  | 25                 |
| 26   | 35               | 10                  | 34             | 34             | 34                 | 2   | 26   | 12                         | 34                 | 10                 | 35                 |
| 27   | 34               | 26                  | 6              | 26             | 12                 | 25  | 2    | 26                         | 26                 | 26                 | 36                 |
| 28   | 6                | 29                  | 35             | 9              | 19                 | 9   | 29   | 11                         | 25                 | 23                 | 23                 |
| 29   | 21               | 25                  | 21             | 18             | 32                 | 18  | 25   | 35                         | 9                  | 17                 | 30                 |
| 30   | 17               | 35                  | 17             | 25             | 25                 | 33  | 35   | 34                         | 18                 | 29                 | 24                 |
| 31   | 25               | 36                  | 25             | 30             | 26                 | 3   | 36   | 36                         | 3                  | 30                 | 28                 |
| 32   | 36               | 30                  | 36             | 3              | 30                 | 30  | 30   | 25                         | 30                 | 35                 | 3                  |
| 33   | 33               | 33                  | 33             | 10             | 35                 | 10  | 19   | 33                         | 10                 | 25                 | 2                  |
| 34   | 3                | 19                  | 3              | 29             | 36                 | 29  | 33   | 3                          | 29                 | 36                 | 20                 |
| 35   | 20               | 20                  | 20             | 19             | 33                 | 19  | 20   | 20                         | 19                 | 33                 | 19                 |
| 36   | 19               | 3                   | 19             | 20             | 3                  | 20  | 3    | 19                         | 20                 | 3                  | 33                 |

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Source: Koch (17)

in those remaining (for example, Projects 13, 14); those near the bottom tend to be uniformly poor (Project 3), or relatively poor in several criteria and maybe even quite good in one or two (Projects 19, 20). Such generalizations must be treated with some caution, however, because the ranking of projects is highly dependent on the particular projects in the set. It might also be noted that assigning some 36 separate rankings may be somewhat deceptive, in that certain of the projects may be rather close in terms of the numerical values underlying their rankings (for example, projects ranked 5 to 14 are within 7 percent) and may thus be relatively equally desirable, at least at first glance. In view of this, it is generally recommended that the values be looked at in conjunction with the rankings (see Chew (17)). Nevertheless, the strength of this appraisal framework is as a mechanism for sorting and ordering a large number of projects, and thereby selecting a group of potentially appropriate projects for further and more detailed inspection. These comments naturally pertain to all ranking approaches.

The ranking of a particular project, when cardinal weights,  $C$ , are specified, depends both on the relative weights on the individual criteria and on the project's performance relative to that of other projects in the set. Not surprisingly,  $C_2$ 's behavior is similar to that of  $E$  because the weights on the criteria are nearly uniform.  $C_1$

and  $C_3$  show different rankings, however, because both sets of weights are balanced in favor of the first criterion,  $C_1$  somewhat so and  $C_3$  90 percent. In the case of  $C_1$ , for example, projects with a reasonably high measure on the first one or two criteria and less high on the others tend to rank high (Projects 4, 5), while those with a reasonably low measure on the first criterion and still a relatively high measure on the others tend to rank low (Project 30). Distinct differences exist in the rankings obtained from  $C_1$  and  $C_3$ , as exemplified by Projects 17 and 22, with differences in the emphasis on the second criterion playing an important role here.

The ranking of projects by means of the ordinal weights assumption depends on the final decision rule (max-, min-, or mode-ordinal), the ordering of the criteria, and the relative performance of the projects in the set. The five top-ranked projects under the max-ordinal decision rule demonstrate its aggressive nature in that the contribution of the most preferred criterion, economic benefits, overshadows those of all other criteria; Project 4 is an extreme example. Once contributions to the less preferred criteria become larger than that to the most preferred criterion, however, these former measures begin to exert some influence, as in the case of Project 14. The more conservative nature of the min-ordinal decision rule is

evident in the lowering of Project 4's ranking and in the low ranks of Project 19 and 20. The observation that the ordinal rankings are similar is valid, but one cannot then proceed to assume that the rankings under other rules will also be similar, as is clearly demonstrated by the results in Table 9. Some similarities do exist, as in the top-ranked projects, but striking differences also exist, as in the case of Project 22. Figure 11 shows the upper and lower bounds for each project together with the scores under the equal-weights alternative.

There is, within the specification of cardinal weights, an infinite number of specifications that parallel the ordinal ranking of the criteria, but that result in different rankings for the projects. The mode-ordinal rule uses the "most likely" set of cardinal weights based on the ordering of the criteria, and is, in a sense, a "compromise" between the min-ordinal and max-ordinal decision rules in that it computes the most likely project score while the other rules compute lower and upper bounds. While not always literally true, this can be observed from the rankings of over half of all projects. Figure 12 shows the upper and lower bounds provided by the max-ordinal and min-ordinal decision rules, and the "averaging" effect of the mode-ordinal rule is clearly depicted.

The partial-ordinal decision rules represent a "weakening" of the max-ordinal and min-ordinal decision

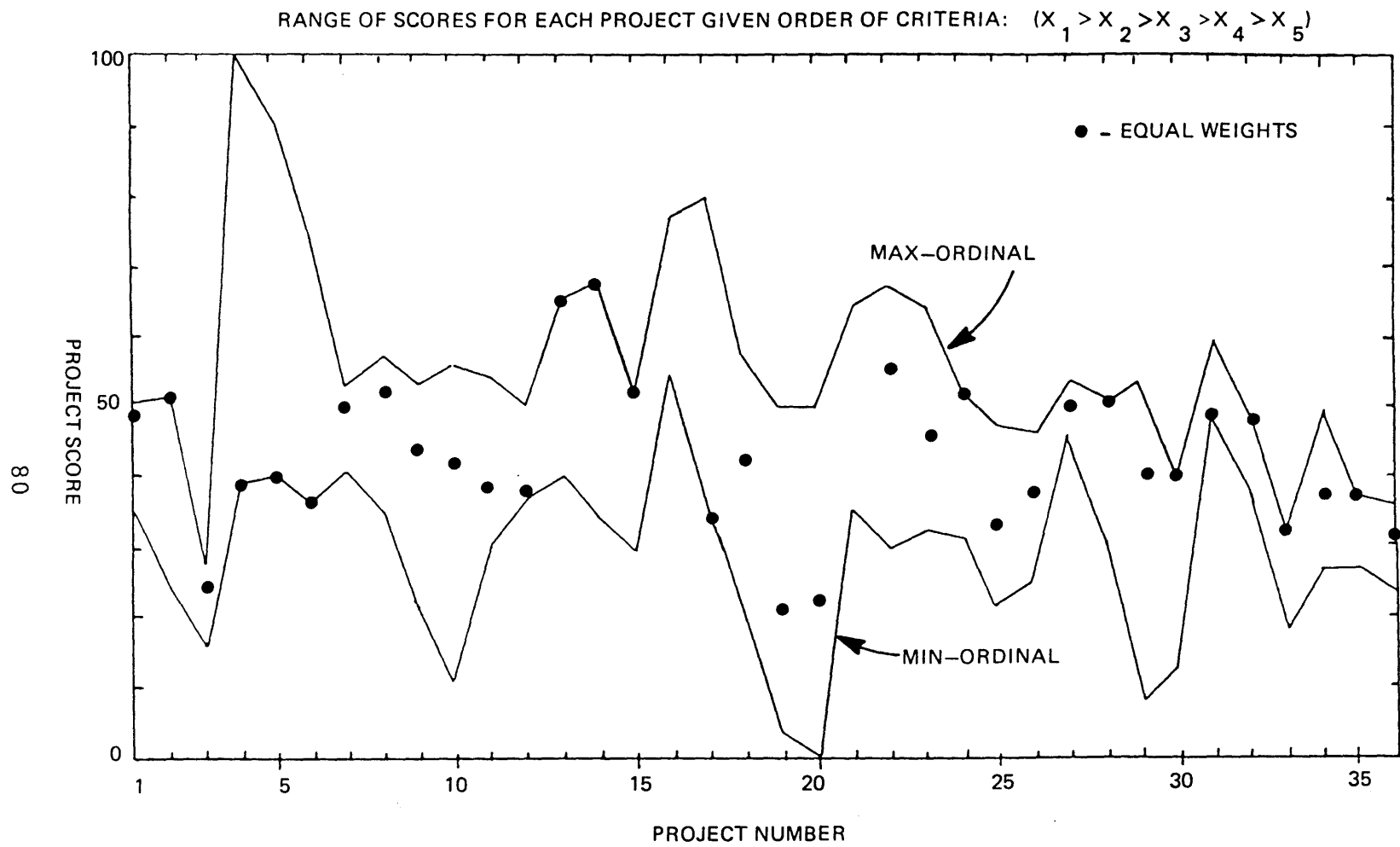


Figure 11. Upper and Lower Ordinal Bounds for the Projects, and Equal Weights Scores

Source: Brademeyer (15)



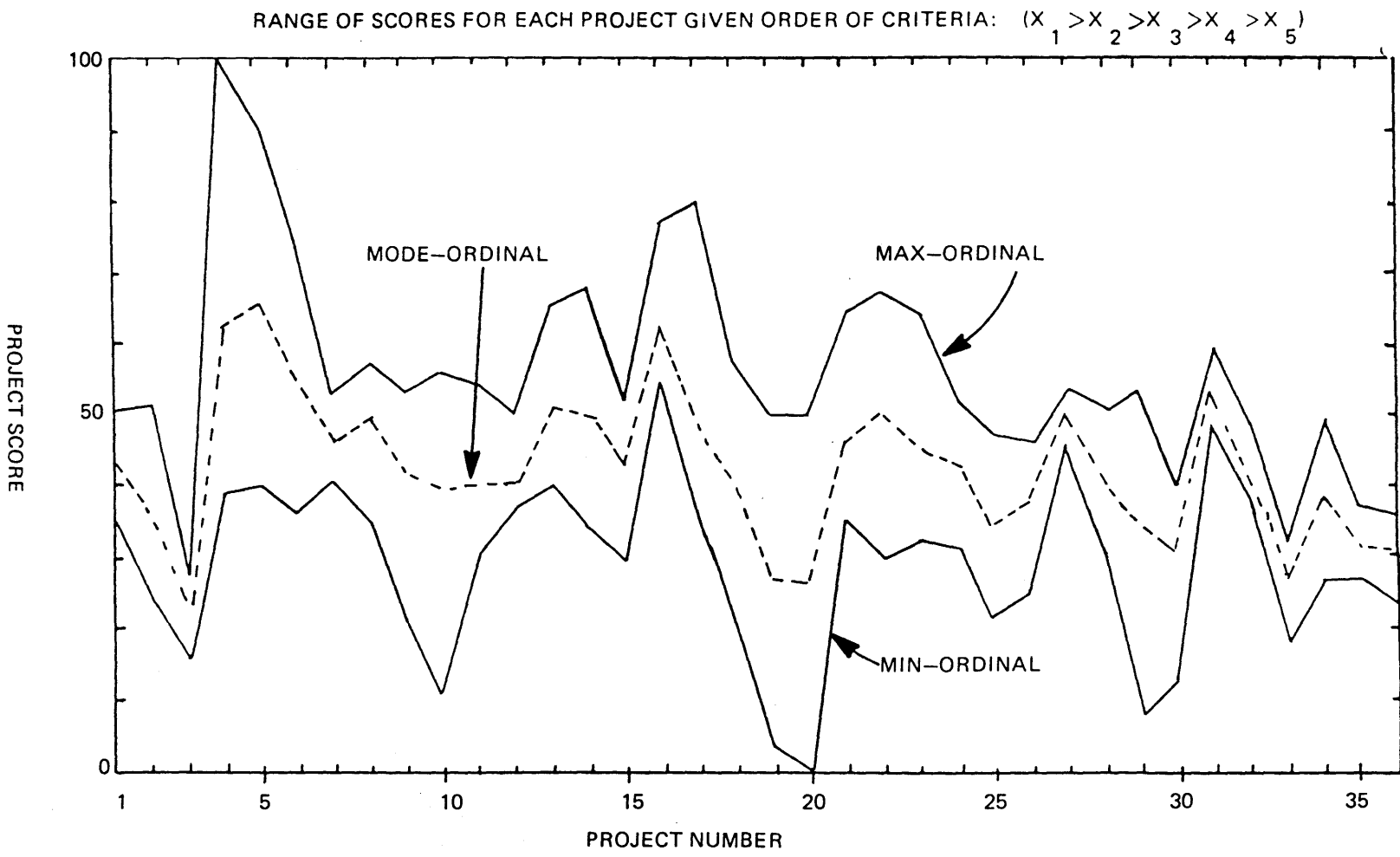


Figure 12. Averaging Effects of Mode-Ordinal Rule

Source: Brademeyer (15)

rules in that subsets of criteria are, in effect, averaged before they are combined. Figure 13 depicts the partial ordinal specification  $(x_1=x_2) > (x_3=x_4=x_5)$  upper and lower bounds for each project against the pure ordinal  $(x_1>x_2>x_3>x_4>x_5)$  bounds for each project, and the narrowing of the range of possible scores is clearly evident.

In order to better understand the use and implications of the various decision rules and their associated preferences and information requirements, the behavior of three projects across these alternatives is traced, and an effort is made to account for this in terms of the project's particular characteristics and contributions to the various criteria. The movement of Project 4 is particularly interesting as a result of its extremes in attainment of the various criteria: it has the highest utility score for the economic benefits criterion and the lowest for the distributional one, with its score on the remaining criteria moderate to low. Thus, when equal weights (E) or nearly equal cardinal weights ( $C_2$ ) are specified, it ranks around number 21. When cardinal weights with relatively higher weight on economic benefits and lower on distributional effects ( $C_1$ ) are applied, Project 4 moves up to position 3, and up to a number 1 when the extremes in the weights are made greater yet ( $C_3$ ). The respectively aggressive and conservative natures of max-ordinal and min-ordinal decision

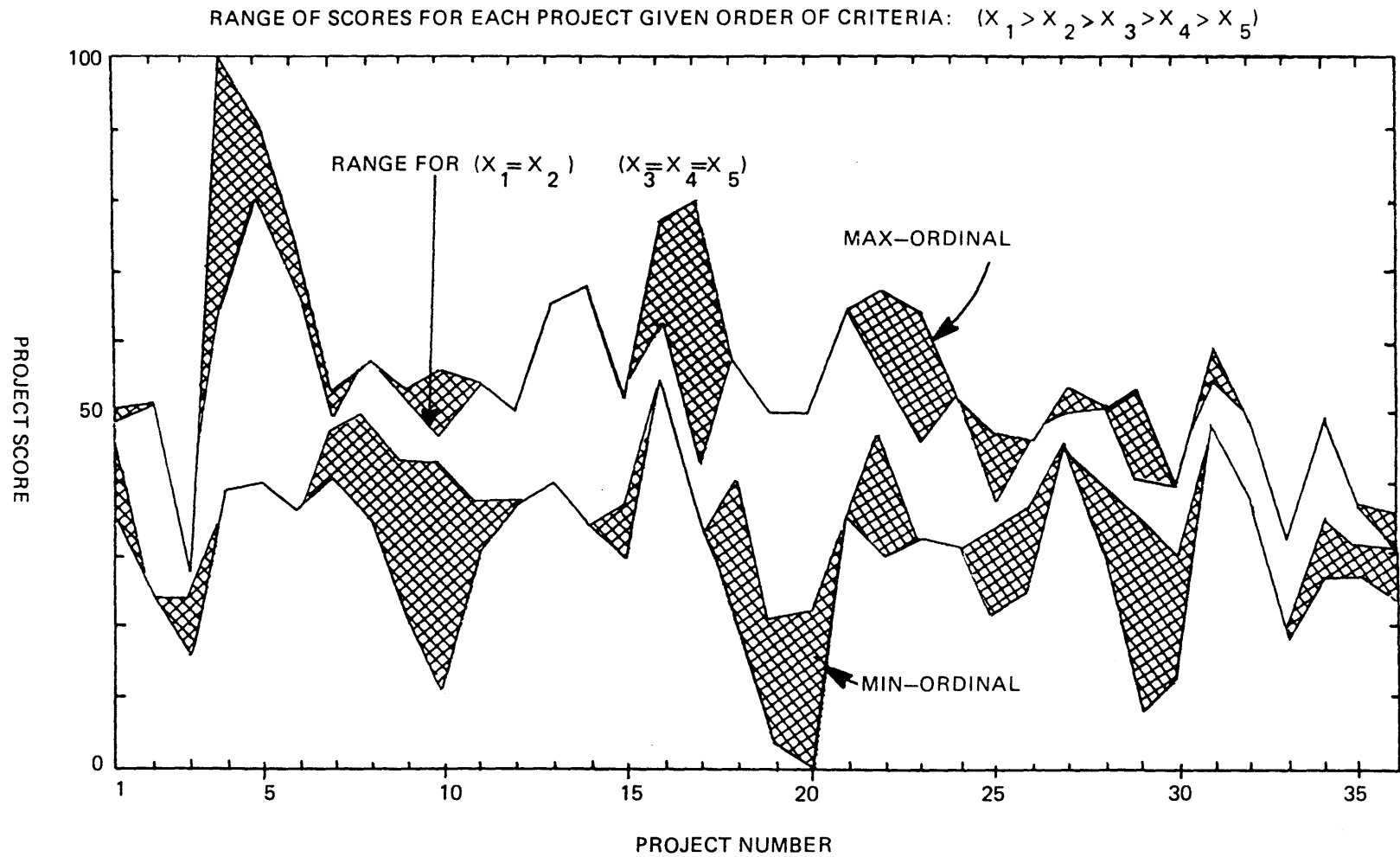


Figure 13. Restriction of Project Score Ranges due to Partial Ordinal Preference

rules are well depicted by Projects 4's behavior under the stated preferential orderings of the criteria. It ranks number 1 with the max-ordinal decision rule, since its highest score is attained on the most preferred criterion, and number 7 with the min-ordinal decision rule. The number 3 ranking under the mode-ordinal rule again shows its "averaging" effect. These relative rankings are present again in the partial-ordinal decision rules, with lower scores under P-Max<sub>2</sub> since that specification averages the scores on the first two criteria, or, in this case 100 and 31, which is still relatively good although not so much as when the first criterion stood alone.

The performance of Project 22 with respect to three out of five criteria is good (economic costs and employment) to excellent (distribution - top rated), and with respect to the remaining two it is relatively poor but not lowest. Its score on economic benefits is rather poor (30), and thus it ranks low, around 20, under C<sub>3</sub>, which puts nearly all its emphasis on this criterion, and under the min-ordinal rule, for which this criterion is most preferred. Its generally favorable performance with regard to the other criteria brings its rank up to 7 for the max-ordinal rule and up into the range of 2 to 9 for the equal, cardinal, and mode-ordinal schemes. Its performance under P-Max<sub>1</sub> and P-Min<sub>1</sub> parallels that of the max-ordinal and min-ordinal rankings,

being 3 lower in each case, because in this specification the last four criteria are unordered, whereas Project 22 has its scores generally increasing in the direction of preference. Under P-Max<sub>2</sub> it ranks identically as under P-Max<sub>1</sub> in that its highest score comes from the contributions of all criteria. However, it ranks number 3 under P-Min<sub>2</sub>, identical to its position under the equal weights alternative, due to its above average scores in three of the five criteria.

Projects 3's performance is relatively poor with regard to all the criteria, but most particularly employment, economic benefits, and distribution. Correspondingly, it ranks rather low for all decision rules, although it tends to rise a bit when the min-ordinal rules are used because of its uniformly poor performance without any extreme lows in its utility scores.

The results achieved in the form of ranking the projects in the hypothetical case study through the application of various decision rules within the overall appraisal framework naturally vary from one approach to the next because different value judgments and amounts of information have been provided in each case. It is not possible to suggest definitively which decision rule is the "best and only one" in the ranking of such a set of projects. Its selection is most appropriately made on a case-by-case basis taking into account, for example, the nature of the projects involved

and their expected contributions to development, the socio-political environment within which the planning is being done, and the type of value judgments the decision maker can and is willing to make. Adequate understanding by the analyst and proper education of the decision maker concerning the properties and implications of the various decision rules are thus essential to successful implementation of the "single-rule" framework for project appraisal.

#### 4.2 Ranking of Projects Under "Rule-Uncertainty"

Having presented a ranking of the projects under the various decision rules, based on the information provided by the decision maker, the situation may still arise wherein the decision maker is unsure about which rule to use. This is particularly likely in the ordinal-preference case and might be referred to as "rule uncertainty." While no one rule can be stated as superior, the implications of the various rules in terms of relative costs and benefits achieved can be further elaborated.

More concretely, imagine that our decision maker has stated an ordinal preference,  $x_1 > x_2 > x_3 > x_4 > x_5$ , that he wishes to implement for 9 of the possible 36 projects, but that he is not confident in his ability to choose the "best" list from Table 9. Based on this information, the rankings under the various ordinal rules can be produced, a recommended "best" list can be generated, and the

socioeconomic impacts of the various options can be summarized, as in Table 10.

Table 10 shows the 11 top-ranked projects under the equal weights, max-, min-, and mode-ordinal decision rules, along with a "recommended" ranking of the projects for implementation. The recommended list is intended to show the decision maker the "correlation," if any, between the various rankings, and thus the sensitivity/insensitivity of the project ranking to the rule actually chosen. In addition, it produces a new ranking of its own, which is in large part a "compromise" between the various decision rules.

The "recommended" ranking is done as follows: the top-ranked project is the one that appears highest in all but one of the decision-rule rankings; the second-ranked project is the one that appears next highest in all but one of the lists and so on. Ties are broken based on the total of the ranks in the top three lists. In the case at hand, Project 16 is recommended number one because it is ranked in the top four for all decision rules and no other project performs better. Projects 5 and 13 are ranked in the top six by three decision rules each; however, the total rank of Project 5 in its three lists is 9 ( $1 + 2 + 6$ ), while that of Project 13 is 13 ( $2 + 5 + 6$ ); so project 5 is ranked second and Project 13 third. Projects 4 and 22 rank at least seventh in three

Table 10

TOP NINE PROJECTS UNDER ORDINAL DECISION RULES

| Rank | Equal Weights | Ordinal Preference Rules |     |      | Recommended <sup>a</sup> |
|------|---------------|--------------------------|-----|------|--------------------------|
|      |               | Max                      | Min | Mode |                          |
| 1    | 14            | 4                        | 16  | 5    | 16                       |
| 2    | 13            | 5                        | 31  | 16   | 5                        |
| 3    | 22            | 17                       | 27  | 4    | 13                       |
| 4    | 16            | 16                       | 7   | 6    | 4                        |
| 5    | 15            | 6                        | 13  | 31   | 22                       |
| 6    | 24            | 14                       | 5   | 13   | 14                       |
| 7    | 8             | 22                       | 4   | 22   | 31                       |
| 8    | 2             | 13                       | 32  | 27   | 6                        |
| 9    | 28            | 21                       | 12  | 14   | 27                       |
| 10   | 31            | 23                       | 6   | 8    |                          |
| 11   | 27            | 31                       | 21  | 17   |                          |

Average utility scores over nine projects,  $\bar{u}_i(x_{i,j})^b$ 

| Rule          | $u_1$ | $u_2$ | $u_3$ | $u_4$ | $u_5$ |
|---------------|-------|-------|-------|-------|-------|
| Equal weights | 2     | -11   | 31    | 19    | 22    |
| Max-ordinal   | 26    | - 3   | 2     | - 2   | 0     |
| Min-ordinal   | 22    | - 4   | - 5   | 12    | 2     |
| Mode-ordinal  | 25    | - 3   | 9     | 5     | 5     |
| Recommended   | 25    | - 3   | 9     | 5     | 5     |

<sup>a</sup>Recommended list is ranked on order in which projects appear in at least three decision rule lists.

<sup>b</sup>Utility scores equal increase over average utility of each criterion over all projects.



lists, with Project 4 having the higher total rank (11 vs. 17), so they are ranked fourth and fifth, respectively. Project 14 is the only remaining project to rank at least ninth in three lists, so it is recommended as the sixth project. Projects 31 and 6 both appear at least tenth in three lists, with Project 31 having the higher total rank (17 vs. 19), so they rank seventh and eighth, respectively. Project 27 fills out the group of nine recommended projects.

This procedure produces an intersection of the various rankings, placing equal weight on each decision rule. More interesting than those included in the recommended list, however, are those omitted for each decision rule. For example, consider the max-ordinal ranking, which is aggressive as regards the interpretation of the ordinal preferences: eight of its top eleven projects are in the recommended list. Those omitted have the following utility scores on the ordered criteria:

|            |   |       |       |       |       |       |
|------------|---|-------|-------|-------|-------|-------|
| Project 17 | - | 79.93 | 2.76  | 29.00 | 33.07 | 27.01 |
| Project 21 | - | 44.54 | 84.08 | 0.33  | 31.85 | 14.50 |
| Project 23 | - | 63.84 | 0.00  | 52.50 | 68.12 | 45.21 |

Project 17, which ranks third under the max-ordinal rule, is rejected because it depends entirely on the first criteria, being poor to lowest on the others. Project 21, which was ranked ninth, does well only on the second criteria and thus is rejected, while Project 23, ranked

tenth, is omitted due to its extremely high costs (score 0 on the second criteria).

For the min-ordinal case, 3 of the top 10 projects are omitted; they had the following scores:

|            |   |       |       |       |       |       |
|------------|---|-------|-------|-------|-------|-------|
| Project 7  | - | 41.35 | 55.67 | 27.63 | 88.02 | 31.26 |
| Project 32 | - | 37.52 | 38.40 | 37.75 | 81.24 | 51.29 |
| Project 12 | - | 39.43 | 59.87 | 11.58 | 58.79 | 19.14 |

These projects, which are quite similar, were rejected because their scores are "uncorrelated" with the stated ordinal preference. For the mode-ordinal rule, all of its top nine projects are in the recommended list, indicating again its "averaging" effects as regards the max-ordinal and min-ordinal rules and that its ranking is based on the "most-likely" scores for the projects.

Five of the top 11 projects under the equal-weights alternative were omitted, although the top four were included. These projects had the following characteristics:

|            |   |       |       |       |        |        |
|------------|---|-------|-------|-------|--------|--------|
| Project 15 | - | 29.44 | 44.46 | 76.07 | 58.27  | 52.56  |
| Project 24 | - | 48.30 | 13.61 | 41.00 | 57.81  | 100.00 |
| Project 8  | - | 34.62 | 80.23 | 40.13 | 73.81  | 27.04  |
| Project 2  | - | 30.22 | 17.66 | 54.79 | 100.00 | 52.03  |
| Project 28 | - | 29.81 | 29.55 | 82.14 | 60.93  | 51.25  |

Of these, Project 8 is uncorrelated with the ordering, while the rest are negatively correlated; thus, their high average scores do not reflect the preferences stated by the decision maker.

The discussion so far has been of the inclusion/omission of projects in the production of the recommended ranking. It is also useful to present to the decision maker the cost/benefit impacts of those inclusions and omissions. Table 10 gives the difference between the average utility score for each criterion, considering the top nine projects under each decision rule, and the average utility score for that criterion considering all 36 projects.\* Thus, positive scores indicated points above average and negative scores indicated points below average.

From Table 10, the equal-weights alternative produces about average economic benefits (criterion one) and very good distribution, employment, and accessibility benefits (criteria 3, 4, and 5) at above average cost (negative score on criterion 2). The max-ordinal rule, on the other hand, produces very good economic benefits with about average performance on the other criteria. The min-ordinal rule produces slightly less economic benefits at slightly greater cost, with a small net increase in the other social benefit measures. The mode-ordinal rule produces almost the same economic benefits as the max-ordinal at the same cost, but provides above average social benefits as well. The recommended ranking in this case contains the same nine

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\*The average utility for each criterion over all projects was not 50, but rather the mean utilities were 38.07, 52.07, 38.70, 50.59, and 34.85 for the five criteria, respectively.

projects as the mode-ordinal ranking and hence has the same average utility scores.

While it would seem clear in this case that the recommended ranking should be the one implemented by the decision maker based on the relative utility scores and the stated ordinal preference, the actual choice still remains up to him. The types of information provided above are intended solely to help him make that decision and to feel confident about its soundness.

#### 4.3 Ranking of Projects Under "Ordinal-Uncertainty"

Suppose that we have through the above demonstrations gained the decision maker's confidence in the appraisal framework given a statement of his ordinal preferences. Furthermore, suppose that the decision maker, when asked to specify a "real" ordinal preference for the case at hand, expresses uncertainty (or equivalently, that there are several decision makers, each of whom has different preferences). Taking the latter interpretation, suppose there is a committee of three decision makers (call them A, B, and C) from whom we elicit the following statements of ordinal preference:

A: "I feel that economic benefits are most important and that costs must take precedence over other social objectives. I prefer income distribution to employment, with health and educational considerations being of lower priority."

B: "I agree that economic benefits are the most important, but I am unable to state a preference among the other criteria."

C: "Economic benefits must be balanced against costs, but they are of greater immediate need than the social objectives of income distribution, employment, and accessibility to social services, which are all equally desirable."

We would interpret these rather imprecise statements as the following ordinal preferences:

A:  $(x_1 > x_2 > x_3 > x_4 > x_5)$

B:  $x_1 > (x_2 = x_3 = x_4 = x_5)$

C:  $(x_1 = x_2) > (x_3 = x_4 = x_5)$

How do we reconcile these preferences so that the committee can reach a consensus, or compromise, on which projects to implement? The following procedure might suffice. First, for each decision maker, generate the project rankings under the various decision-rules along with a recommended ranking; second, with each decision maker, establish which ranking scheme "best" reflects his true preferences; and third, combine the three most-preferred lists into a final recommended list that the decision makers can then use to reach their consensus.

From the 36 projects in Table 7, we would produce for each decision maker the project rankings under the equal-weights, max-ordinal, and min-ordinal decision rules,\* along with a recommended "compromise" ranking and the socio-economic impacts of each. Assuming again that nine projects are to be implemented, we would have the results presented in Tables 10, 11, and 12 for decision makers A, B, and C, respectively.

The implications of A's preferences have been discussed in the previous section, and we will assume here that he has agreed that the recommended ranking in Table 10 reflects his preference satisfactorily.

Decision maker B (Table 11) has entered our discussion convinced that the aggressive max-ordinal rule should most nearly reflect his preferences. When shown that we recommend the omission of his third and fifth ranked projects, the inclusion of his tenth ranked, and the addition of Project 24, he needs to know the reasons. Our justification might proceed as follows given the characteristics of the projects in question in terms of their deviation from the average over all projects:

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\*The mode-ordinal decision-rule was omitted for decision makers B and C in that this rule is identical to the equal-weights case for their stated ordinal preferences. This is discussed in detail in Appendix B.

Table 11

TOP NINE PROJECTS UNDER ( $x_1 > (x_2 = x_3 = x_4 = x_5)$ )

| <u>Ordinal Preference</u> |         |                               |     |                          |
|---------------------------|---------|-------------------------------|-----|--------------------------|
| Rank                      | Equal   | <u>Ordinal Decision Rules</u> |     |                          |
|                           | Weights | Max                           | Min | Recommended <sup>a</sup> |
| 1                         | 14      | 4                             | 16  | 16                       |
| 2                         | 13      | 5                             | 51  | 14                       |
| 3                         | 22      | 17                            | 27  | 13                       |
| 4                         | 16      | 16                            | 24  | 24                       |
| 5                         | 15      | 6                             | 23  | 5                        |
| 6                         | 24      | 14                            | 13  | 23                       |
| 7                         | 8       | 13                            | 7   | 4                        |
| 8                         | 2       | 23                            | 5   | 31                       |
| 9                         | 28      | 31                            | 4   | 22                       |
| 10                        | 31      | 22                            | 12  |                          |

Average utility scores for each criterion,  $\bar{u}_i (x_{i,j})^b$ 

| Rule          | $u_1$ | $u_2$ | $u_3$ | $u_4$ | $u_5$ |
|---------------|-------|-------|-------|-------|-------|
| Equal weights | 2     | -11   | 31    | 10    | 22    |
| Max-ordinal   | 31    | -15   | 1     | 4     | 5     |
| Min-ordinal   | 15    | -14   | 0     | 10    | 10    |
| Recommended   | 23    | -10   | 14    | 9     | 8     |

<sup>a</sup>Recommended rank is based on order in which projects appear in at least two decision-rule lists.

<sup>b</sup>Utility scores represent increments over the average utility over all projects for each criterion.

|          |            |   |     |     |     |     |     |
|----------|------------|---|-----|-----|-----|-----|-----|
| omitted: | Project 17 | - | 42  | -49 | -10 | -18 | - 8 |
|          | Project 6  | - | 36  | 7   | -37 | -22 | -20 |
| added:   | Project 22 | - | - 8 | 18  | 61  | - 5 | - 8 |
|          | Project 24 | - | 10  | -38 | 2   | 7   | 65  |

Decision maker B has stated that economic benefits are his primary concern, while the other objectives are less important and equivalent with one another. However, he has not stated how much more important that criterion is or whether it is more important than all others combined. To clarify his true preferences, we note that the two projects that we recommend be omitted have a total of 78 points above average on the economic benefits criterion and a total of -157 points on all other criteria. The projects that we recommend be added have a total of 2 points above average on the economic benefits criterion and 102 points on all other criteria. Thus, we pose the following question to decision maker B: "Are 76 economic benefit points (78-2) of more importance to you than 259 (102-157) points for the other criteria?" For the remainder of this discussion, we will assume B answered negatively and accepted the recommended ranking. From his stated preferences, it is not expected that the min-ordinal or equal-weights rules will reflect B's interests, as can be seen from Table 11.

Decision maker C (Table 12) is more cautious than B and anticipates the min-ordinal rule as most reflective of his preferences. We would recommend to him the elimination of



Table 12

TOP NINE PROJECTS UNDER  $((x_1 = x_2) > (x_3 = x_4 = x_5))$

| Rank | Equal Weights | Ordinal Preference     |     |                          |
|------|---------------|------------------------|-----|--------------------------|
|      |               | Ordinal Decision Rules |     |                          |
|      |               | Max                    | Min | Recommended <sup>a</sup> |
| 1    | 14            | 5                      | 16  | 14                       |
| 2    | 13            | 14                     | 8   | 22                       |
| 3    | 22            | 6                      | 22  | 16                       |
| 4    | 16            | 4                      | 31  | 13                       |
| 5    | 15            | 13                     | 7   | 8                        |
| 6    | 24            | 21                     | 1   | 31                       |
| 7    | 8             | 16                     | 27  | 18                       |
| 8    | 2             | 18                     | 9   | 27                       |
| 9    | 28            | 8                      | 10  | 5                        |
| 10   | 31            | 22                     | 18  |                          |
| 11   | 27            | 31                     | 13  |                          |
| 12   | 32            | 15                     | 5   |                          |

Average utility scores for each criterion,  $\bar{u}_i (x_{i,j})^b$

| Rule          | $u_1$ | $u_2$ | $u_3$ | $u_4$ | $u_5$ |
|---------------|-------|-------|-------|-------|-------|
| Equal weights | 2     | -11   | 31    | 19    | 22    |
| Max-ordinal   | 19    | 8     | -7    | 6     | -2    |
| Min-ordinal   | 2     | 11    | 13    | 8     | -2    |
| Recommended   | 11    | 6     | 14    | 13    | 5     |

<sup>a</sup>Recommended rank is based on order in which projects appear in at least two decision-rule lists.

<sup>b</sup>Utility scores represent increments over the average utility over all projects for each criterion.

Projects 7, 1, 9, and 10 and the addition of Projects 18, 13, 5, and 14. The characteristics of these projects, in terms of deviation from the mean, are as follows:

|          |            |   |            |            |           |            |            |
|----------|------------|---|------------|------------|-----------|------------|------------|
| omitted: | Project 7  | - | 3          | 4          | -11       | 37         | - 4        |
|          | Project 1  | - | - 4        | 10         | -17       | 32         | 8          |
|          | Project 9  | - | -17        | 30         | 18        | - 4        | -19        |
|          | Project 10 | - | <u>-27</u> | <u>30</u>  | <u>36</u> | <u>-34</u> | <u>- 3</u> |
|          |            |   | 29         |            |           | 39         |            |
| added:   | Project 18 | - | -18        | 43         | -27       | 25         | -23        |
|          | Project 13 | - | 3          | -14        | 43        | 39         | 37         |
|          | Project 5  | - | 52         | 21         | -38       | -26        | -25        |
|          | Project 14 | - | <u>- 2</u> | <u>-20</u> | <u>57</u> | <u>40</u>  | <u>53</u>  |
|          |            |   | 65         |            |           | 155        |            |

Combining these scores for each group according to C's preferences,  $(x_1 + x_2)$  and  $(x_3 + x_4 + x_5)$ , we pose him the following question: "Are -36 points for economic costs and benefits (29-65) of more importance to you than 116 points for social objectives (155-39)?" In this case, we do know C's answer, since the recommended projects perform better on both considerations than those omitted from the min-ordinal ranking.

Decision maker C also accepts our recommended ranking of the projects.\*

Having satisfied each decision maker in turn as to the proper ranking of projects according to his preferences, we now must help them reach a consensus on the nine projects to be implemented. From the three recommended rankings, one for each decision maker, we produce a "final recommendation" as shown in Table 13. As can be seen from the rankings, some projects are omitted from each decision maker's list. We will discuss each in turn.

For decision maker A, we recommend he compromise by accepting his tenth ranked project (8) in place of his eighth ranked project (6). The two projects have the following characteristics in terms of deviations from the mean utilities:

|           |   |    |    |     |     |     |
|-----------|---|----|----|-----|-----|-----|
| Project 6 | - | 36 | 7  | -37 | -22 | -20 |
| Project 8 | - | -3 | 28 | 1   | 23  | -8  |

Decision maker A has a complex preference structure so that it may indeed be rather difficult for him to evaluate this comparison. The benefit picture of the overall implementation package, given one or the other of the above

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\*If C had favored the max-ordinal rule, we could develop the following question from the data in Table 12: "Are 10 points for economic costs and benefits (27-17) of more importance to you than 35 points for social objectives (32--3)?" Again, while we cannot provide C's answer, we assume he answers in the negative.

Table 13

TOP NINE PROJECTS UNDER ALL ORDINAL PREFERENCES

| <u>Recommended Rankings for Ordinal Preferences</u> |                               |                                 |                                   |                          |
|---|-------------------------------|---------------------------------|-----------------------------------|--------------------------|
| Rank  | $x_1 > x_2 > x_3 > x_4 > x_5$ | $x_1 > (x_2 = x_3 = x_4 = x_5)$ | $(x_1 = x_2) > (x_3 = x_4 = x_5)$ | Recommended <sup>a</sup> |
| 1   | 16                            | 16                              | 14                                | 16                       |
| 2   | 5                             | 14                              | 22                                | 14                       |
| 3   | 13                            | 13                              | 16                                | 13                       |
| 4   | 4                             | 24                              | 13                                | 5                        |
| 5   | 22                            | 5                               | 8                                 | 22                       |
| 6   | 14                            | 23                              | 31                                | 4                        |
| 7   | 31                            | 4                               | 18                                | 31                       |
| 8   | 6                             | 31                              | 27                                | 27                       |
| 9   | 27                            | 22                              | 5                                 | 8                        |
| 10  | 8                             | 27                              | 15                                |                          |

Average Utility Scores for Each Criterion,  $u_j(x_i, j)$ <sup>b</sup>

| Ordinal Preference              | $u_1$ | $u_2$ | $u_3$ | $u_4$ | $u_5$ |
|---------------------------------|-------|-------|-------|-------|-------|
| $x_1 > x_2 > x_3 > x_4 > x_5$   | 25    | -3    | 9     | 5     | 5     |
| $x_1 > (x_2 = x_3 = x_4 = x_5)$ | 23    | -10   | 14    | 9     | 8     |
| $x_1 = x_2 > x_3 = x_4 = x_5$   | 11    | 6     | 14    | 13    | 5     |
| Final Recommended               | 17    | 0     | 13    | 10    | 6     |

<sup>a</sup>Recommended rank is based on order in which projects appear in at least two recommended lists.

<sup>b</sup>Utility scores represent increments over the average utility over all projects for each criterion.

projects, may be easier for the decision maker to evaluate.

From Table 13, we would have:

|                |   |    |     |    |    |   |
|----------------|---|----|-----|----|----|---|
| with Project 6 | - | 25 | - 3 | 9  | 5  | 5 |
| with Project 8 | - | 17 | 0   | 13 | 10 | 6 |

Given this formulation, A might well agree to the compromise as consistent with his overall preferences, depending on his relative weight on the first (economic benefits) criterion.

Decision maker B has a "simple" preference function, but is asked to compromise on two of his projects, although one of the replacements was ranked as his tenth choice. Thus, we would have the following:

|                    |   |            |            |    |    |     |
|--------------------|---|------------|------------|----|----|-----|
| replace Project 24 | - | 10         | -38        | 2  | 7  | 65  |
| Project 23         | - | <u>26</u>  | <u>-52</u> | 14 | 18 | 10  |
|                    |   | 36         |            |    | 26 |     |
| with Project 27    | - | 16         | -14        | 11 | 17 | 4   |
| Project 8          | - | <u>- 3</u> | <u>28</u>  | 1  | 23 | - 8 |
|                    |   | 13         |            |    | 62 |     |

In other words, would he accept the substitution of 36 points on the last four criteria for 23 points on the most preferred, economic benefits criterion?

Decision maker C is being asked to accept the substitution of Project 4 for Project 18, and thus must compare the following:

|                    |   |            |           |            |     |            |
|--------------------|---|------------|-----------|------------|-----|------------|
| replace Project 18 | - | <u>-18</u> | <u>43</u> | <u>-27</u> | 25  | <u>-23</u> |
|                    |   | 25         |           |            | -25 |            |

|                |   |           |            |            |            |            |
|----------------|---|-----------|------------|------------|------------|------------|
| with Project 4 | - | <u>62</u> | <u>-21</u> | <u>-39</u> | <u>- 5</u> | <u>-15</u> |
|                |   | 41        |            |            | -59        |            |

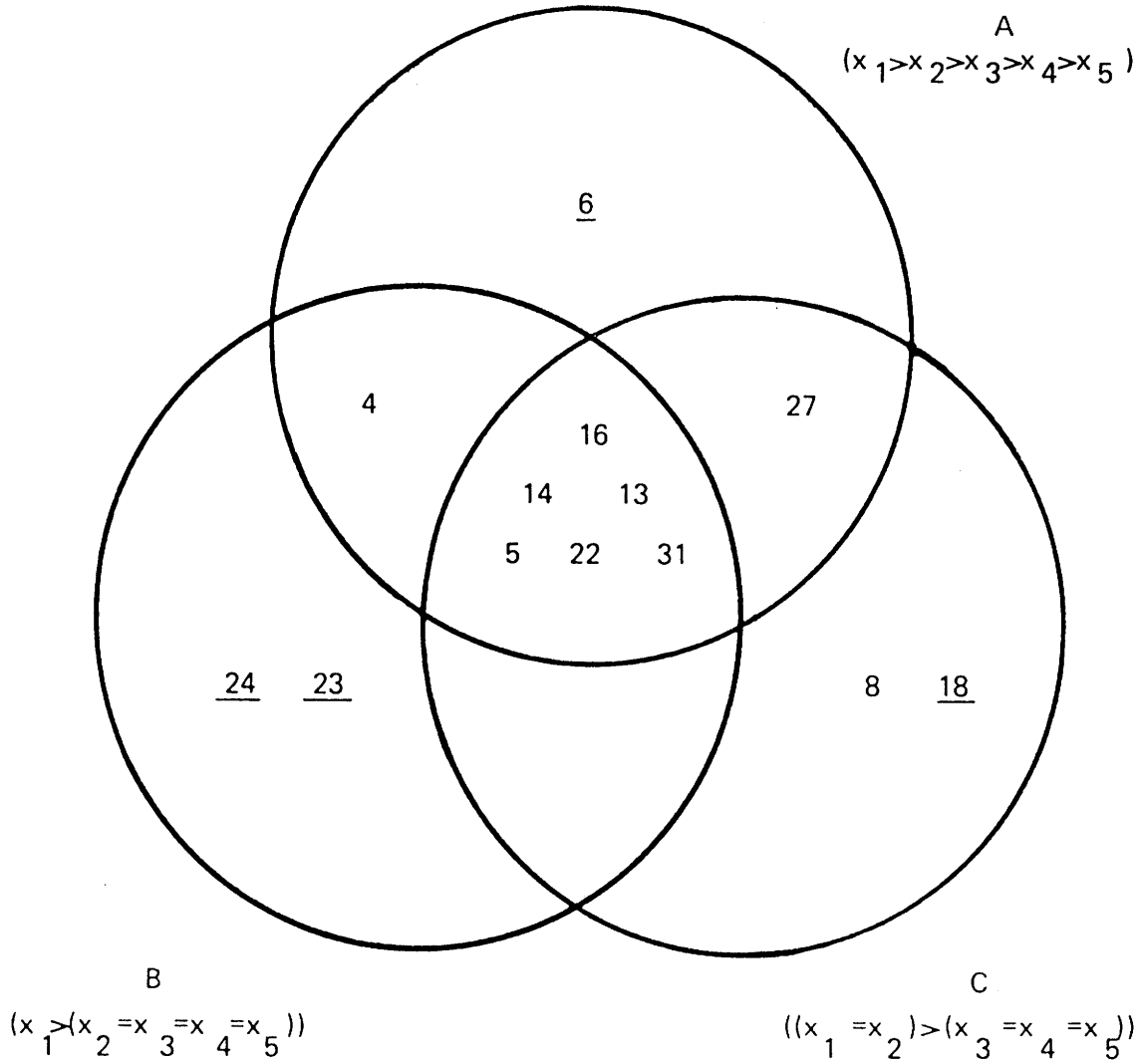
In other words, would C accept an increase of 16 economic costs and benefits points for a loss of 34 social objective points?

A few points should be clarified here. First, the application of the "recommended" rule in this case is truly a compromise situation since its application to the case of each decision maker has already produced the "best" ranking according to the preference of each; that is, it is likely that no one will "gain" from this exercise. Second, it is very difficult to compare projects one on one, or two on two if one's preferences are complex, due to the fluctuations of the scores from project to project on the various criteria. It may be much easier to reach a consensus if the entire package of projects to be implemented is compared to that preferred by each decision maker, rather than only those projects to be omitted/included. The scores for the recommended package might be acceptable to all if presented as in the lower portion of Table 13, where the "compromise" nature of the recommendations is clearly evident. Figure 14 shows this in Venn diagram form.

Finally, it should be pointed out that the projects accepted individually according to each decision maker's preferences were highly correlated with the projects finally recommended: 9 of the top 10 for A, 8 of the top 10 for B,

Figure 14

Compromise Among Top Nine Recommended Ordinal Preference Ranking




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NOTE: Underlined (\_\_\_) projects are not in the final nine recommended projects.

and 8 of the top 10 for C. In such cases, reaching a consensus should not be very difficult.

#### 4.4 Ranking of Projects Under "Cardinal-Uncertainty"

The procedures outlined in the previous section pertain also to the situation in which our committee of decision makers specifies various cardinal preferences. In this case, the "recommended" ranking is generated from the rankings under the equal-weights, mode-ordinal, and cardinal-weights decision rules, since these are the only cardinal rules available. Imagine that our committee has given the following cardinal preferences, with the criteria in the same order as before:

x: (50, 20, 15, 10, 5)

y: (22, 21, 20, 19, 18)

z: (90, 4, 3, 2, 1)

The recommended rankings are developed in Tables 14, 15, and 16 for decision makers x, y, and z, respectively. The final recommendation for their consensus is presented in Table 17. Figure 15 presents this compromise in Venn diagram form.

While the main features of the procedure have already been presented, and therefore will not be repeated here, some points concerning the cardinal-weights case should be examined. From the tables, we can see that the projects in



Table 14

TOP NINE PROJECTS UNDER (50, 20, 15, 10, 5)  
Cardinal Weights

| Rank | Equal Weights | Mode Ordinal | Cardinal Weights | Recommended <sup>a</sup> |
|------|---------------|--------------|------------------|--------------------------|
| 1    | 14            | 5            | 16               | 16                       |
| 2    | 13            | 16           | 5                | 5                        |
| 3    | 22            | 4            | 4                | 4                        |
| 4    | 16            | 6            | 31               | 13                       |
| 5    | 15            | 31           | 13               | 31                       |
| 6    | 24            | 13           | 6                | 6                        |
| 7    | 8             | 22           | 14               | 14                       |
| 8    | 2             | 27           | 27               | 22                       |
| 9    | 28            | 14           | 22               | 27                       |

Average utility scores for each criterion,  $\bar{u}_i (x_{i,j})^b$

| Rule             | $u_1$ | $u_2$ | $u_3$ | $u_4$ | $u_5$ |
|------------------|-------|-------|-------|-------|-------|
| Equal weights    | 2     | -11   | 31    | 19    | 22    |
| Mode-ordinal     | 25    | - 3   | 9     | 5     | 5     |
| Cardinal weights | 25    | - 3   | 9     | 5     | 5     |
| Recommended      | 25    | - 3   | 9     | 5     | 5     |

<sup>a</sup>Recommended rank is based on order in which projects appear in cardinal-weights list and at least one other decision-rule list.

<sup>b</sup>Utility scores represent increments over the average utility over all projects for each criterion.

Table 15

TOP NINE PROJECTS UNDER (22, 21, 20, 19, 18)  
Cardinal Weights

| Rank | Equal Weights | Mode Ordinal | Cardinal Weights | Recommended <sup>a</sup> |
|------|---------------|--------------|------------------|--------------------------|
| 1    | 14            | 5            | 14               | 14                       |
| 2    | 13            | 16           | 13               | 13                       |
| 3    | 22            | 4            | 16               | 16                       |
| 4    | 16            | 6            | 22               | 22                       |
| 5    | 15            | 31           | 15               | 15                       |
| 6    | 24            | 13           | 8                | 8                        |
| 7    | 8             | 22           | 24               | 24                       |
| 8    | 2             | 27           | 31               | 31                       |
| 9    | 28            | 14           | 28               | 28                       |

Average utility scores for each criterion,  $\bar{u}_i (x_{i,j})^b$

| Rule             | $u_1$ | $u_2$ | $u_3$ | $u_4$ | $u_5$ |
|------------------|-------|-------|-------|-------|-------|
| Equal weights    | 2     | -11   | 31    | 19    | 22    |
| Mode-ordinal     | 25    | - 3   | 9     | 5     | 5     |
| Cardinal weights | 5     | - 7   | 29    | 14    | 21    |
| Recommended      | 5     | - 7   | 29    | 14    | 21    |

<sup>a</sup>Recommended rank is based on order in which projects appear in cardinal-weights list and at least one other decision-rule list.

<sup>b</sup>Utility scores represent increments over the average utility score over all projects for each criterion.

Table 16

TOP NINE PROJECTS UNDER (90, 4, 3, 2, 1)  
Cardinal Weights

| Rank | Equal Weights | Mode Ordinal | Cardinal Weights | Recommended <sup>a</sup> |
|------|---------------|--------------|------------------|--------------------------|
| 1    | 14            | 5            | 4                | 5                        |
| 2    | 13            | 16           | 5                | 4                        |
| 3    | 22            | 4            | 16               | 16                       |
| 4    | 16            | 6            | 17               | 6                        |
| 5    | 15            | 31           | 6                | 31                       |
| 6    | 24            | 13           | 23               | 27                       |
| 7    | 8             | 22           | 31               | 24                       |
| 8    | 2             | 27           | 27               | 17                       |
| 9    | 28            | 14           | 24               | 13                       |
| 10   | 31            | 8            | 21               |                          |
| 11   | 27            | 17           | 13               |                          |

Average utility scores for each criterion,  $\bar{u}_i (x_{i,j})^b$

| Rule             | $u_1$ | $u_2$ | $u_3$ | $u_4$ | $u_5$ |
|------------------|-------|-------|-------|-------|-------|
| Equal weights    | 2     | -11   | 31    | 19    | 22    |
| Mode-ordinal     | 25    | - 3   | 9     | 5     | 5     |
| Cardinal weights | 34    | -17   | - 9   | - 2   | 3     |
| Recommended      | 31    | -13   | - 5   | 0     | 6     |

<sup>a</sup>Recommended rank is based on order in which projects appear in cardinal-weights list and at least one other decision-rule list.

<sup>b</sup>Utility scores represent increments over the average utility score over all projects for each criterion.

Table 17

TOP NINE PROJECTS UNDER ALL CARDINAL PREFERENCES

| Rank | <u>Recommended Rank for Cardinal Preferences</u> |                             |                         | Final                          |
|------|--|-----------------------------|-------------------------|--------------------------------|
|      | <u>(50, 20, 15, 10, 5)</u>                       | <u>(22, 21, 20, 19, 18)</u> | <u>(90, 4, 3, 2, 1)</u> | <u>Recommended<sup>a</sup></u> |
| 1    | 16   | 14                          | 5                       | 5                              |
| 2    | 5  | 13                          | 4                       | 16                             |
| 3    | 4  | 16                          | 16                      | 4                              |
| 4    | 13   | 22                          | 6                       | 13                             |
| 5    | 31   | 15                          | 31                      | 31                             |
| 6    | 6  | 8                           | 27                      | 6                              |
| 7    | 14   | 24                          | 24                      | 14                             |
| 8    | 22   | 31                          | 17                      | 24                             |
| 9    | 27   | 28                          | 13                      | 22                             |

Average Utility Scores for Each Criterion,  $\bar{u}_i(x_i, j)$ <sup>b</sup>

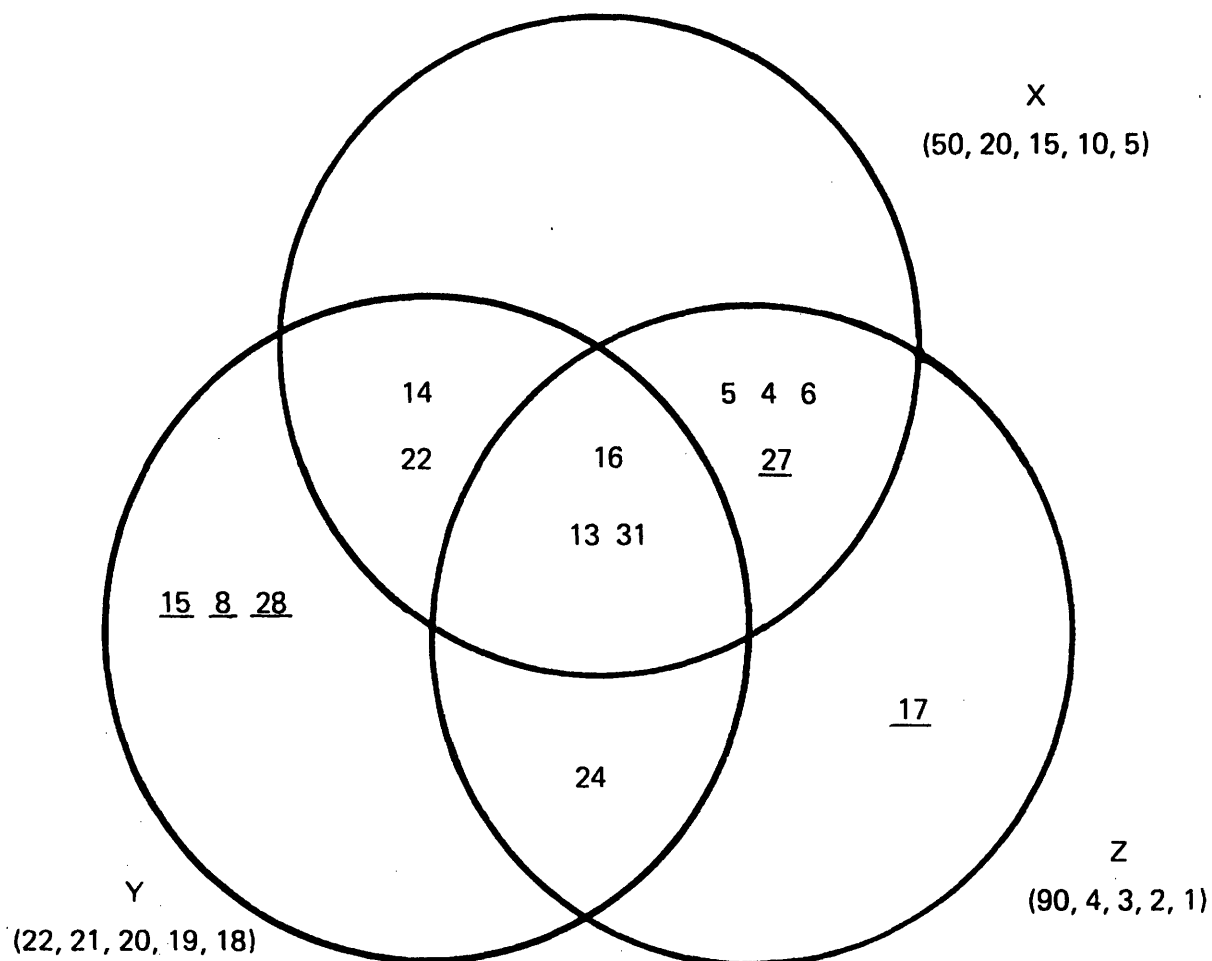
| Cardinal Weights     | $u_1$ | $u_2$ | $u_3$ | $u_4$ | $u_5$ |
|----------------------|-------|-------|-------|-------|-------|
| (50, 20, 15, 10, 5)  | 25    | -3    | 9     | 5     | 5     |
| (22, 21, 20, 19, 18) | 5     | -7    | 29    | 14    | 21    |
| (90, 4, 3, 2, 1)     | 31    | -13   | -5    | 0     | 6     |
| Final Recommended    | 24    | -6    | 8     | 4     | 12    |

<sup>a</sup>Recommended rank is based on order in which projects appear in at least two recommendation lists.

<sup>b</sup>Utility scores represent increments over the average utility score over all projects for each criterion.

Figure 15

Compromise Among Top Nine Recommended Cardinal Preference Rankings




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NOTE: Underlined (—) projects are not in the final nine recommended projects.

the recommended list are the same as those in the cardinal-weights ranking using the decision makers' original preferences, although the orders are slightly perturbed. The only exception is Project 23 for decision maker z, who provided an "extreme " preference function. The procedure recommended replacement of this project with Project 13, which has superior performance in all but the first criterion. That is, the procedure acts to suppress the skewed effects of an extreme cardinal specification. However, the decision maker may still choose whichever ranking he judges most consistent with his own preferences.

The method presented for assisting the decision maker in making and evaluating choices is not dependent on a consensus on the order of the criteria. However, as is easily anticipated, reaching the final compromise will in such cases be much more difficult.

Finally, it may be useful to state the "recommended" ranking procedure in a more formal fashion. If the projects are being considered under k possible decision rules, the following information is needed:

$R_{k,j}$  - the lowest rank\* achieved by the  $j^{\text{th}}$  project  
under any decision rule

$R_{k-1,j}$  - the second lowest rank for the  $j^{\text{th}}$  project

---

\*Here high ranks denote good projects, with the highest rank being 1.

$T_j$  - the sum of the highest  $k-1$  ranks for the  $j^{\text{th}}$  project:

$$\sum_{i=1}^{k-1} R_{i,j}$$

The recommended ranking is then as follows:

$$\text{RANK}_{\text{recommended}}(P_j) = \text{RANK}(R_{k-1,j})$$

That is, the projects are ranked according to their second-worst performance considering all of the decision-rules. If two or more projects are tied under this scheme, the precedence is determined by the total of all but their worst ranks:

if  $\text{RANK}(P_i) = \text{RANK}(P_j)$  for some  $i, j$ ,

$\text{RANK}(P_i) > \text{RANK}(P_j)$  if  $T_i < T_j$

If the projects are still tied after this operation, the worst performances are taken into account:

$\text{RANK}(P_i) > \text{RANK}(P_j)$  if  $T_i + R_{k,i} < T_j + R_{k,j}$

If the projects are still tied, it is probably not important to distinguish between them.

The above formulation is for the ordinal preference case. For the cardinal preference situation, projects are ranked as follows:

cardinal

$$\begin{aligned} \text{RANK}_{\text{recommended}}(P_j) &= \text{RANK}_C(P_j) \text{ if } \text{RANK}_C \leq \text{RANK}^* \\ &= \text{RANK}^*(P_j) \text{ if } \text{RANK}_C > \text{RANK}^* \end{aligned}$$

where  $\text{RANK}_C(P_j)$  is the cardinal weights rank for the  $j^{\text{th}}$  project

$RANK^*(P_j)$  is the highest rank attained by the

$j$ th project under any non-cardinal decision rule

In this case, ties are broken by the value of  $RANK_C$ , that is the ranking of the projects under the cardinal weights rule. This scheme places greater emphasis on the actual cardinal weights articulated by the decision maker than on the other decision rules.

Again, if  $RANK_C(P_j) = 4$  and  $RANK^*(P_j) = 8$ ,  $RANK_C > RANK^*$ , since 1 is the "highest" project ranking. These rules are developed in detail in Appendix D.

#### 4.5 Summary

Chapter Four presented a hypothetical case study requiring analysis of 36 rural road projects in a multi-objective analysis space where the decision makers were in agreement as to the evaluation criteria and the relative position of each criterion with respect to the others. It was shown that the absence of a common metric, i.e. monetary, was not a constraint to ranking. It was further shown that ranking could be performed when all criteria were given equal weight, when they were given known relative weights, when they were given a preferred order, and when the preferred order is only partially known. Under the last rule, two studies were made, the first where one criterion was more important than the other four, amongst which the decision makers were indifferent; and a second case where



two criteria, between which the decision makers were indifferent, were more important than the other three, amongst which there was indifference.

## Chapter Five

### APPLICATION OF THE METHODOLOGY WHERE NO CONSENSUS EXISTS

The previous chapter discussed analysis where consensus exists among decision makers. This chapter will present the more difficult problem of applying a technique from Chapter 3 for situations where consensus among decision makers cannot be reached, or where the decision rules are not disclosed. As in the previous chapter, a case study is hypotheticized and analyzed. The technique employed is based on a system of tables and graphs which are then analyzed iteratively with the decision makers until a compromise is reached.

#### 5.1 The Scenario

The President of the country of Tap has decided to invest in pure rural development. He does not demand that the stream of monetary benefits exceed the costs, in fact he is not interested in the monetary benefit stream at all. He wants to gamble on the idea that lack of access is the controlling factor constraining rural citizens from joining the cash economy. A budget of four million tapas (T 4,000,000) has been established for this program.

In explaining the rural roads program to the Minister of Works, the President cites the following national policies:

- balancing public works investment in the three regions of the country: x,y, and z

- increased agricultural output
- improved primary health services; and
- increasing non-agricultural employment opportunities

He states to the Minister that it is on these factors that the selection should be made, without regard to the normal monetized benefit to cost comparisons.

On his return to the Ministry of Works, the Minister calls in the Director of Planning and explains the situation to her. Together they work out the following guidelines for the analyses.

- a. Each of the ten states will have one project to be evaluated, each project should be from the most rural area of the state
- b. No project should exceed T 1.0 Million to simplify the regional investment balancing issue; and each region must get one project
- c. No project should be less than 30 kilometers to ensure the chosen projects would have good political visibility
- d. Because production and yield statistics are unreliable, increased agricultural output would be estimated using hectares of arable land within ten kilometers of the road which are not now being farmed

- e. Improved health care would be estimated using access time reductions for the average person in the area
- f. For employment generation, only the worker demand generated by the road would be considered

His parting remark to the Director was, "Now remember, we won't be able to build one in each state so make sure you give me enough information to formulate a recommendation the politicians can accept".

## 5.2 Development of the Program Objectives

From past experience, the Director had learned a basic flaw in the definition of all objective functions constructed to support multi-objective analysis--the population bias. Variables tended to be set in terms of total population served, total income generated, total people with new access to water, electricity, banking facilities, or a mid-wife. This bias defeated the objective of moving away from urban project evaluation tools towards a methodology which favors rural development. Urban projects dominate investment allocations because of high population concentrations. If the rural analysis objective functions are tied to population they will also favor investment in the most urbanized parts of the rural areas.

## 5.3 The Project Set - Cost and Length

After considerable research the Director of Planning came up with the ten projects in Table 18. Each project is

Table 18

PROJECTS FOR EVALUATION UNDER THE RURAL ROADS PROGRAM of TAP

| <u>Project Name</u>  | <u>State</u> | <u>Region</u> | <u>Length</u> | <u>Estimated Cost*</u><br>(T 000's) |
|----------------------|--------------|---------------|---------------|-------------------------------------|
| AX                   | A            | X             | 35            | 735                                 |
| BX                   | B            | X             | 31            | 651                                 |
| CX                   | C            | X             | 42            | 882                                 |
| Regions x Subtotals  |              | 113           | 2373          |                                     |
| DY                   | D            | Y             | 30            | 730                                 |
| EY                   | E            | Y             | 35            | 910                                 |
| FY                   | F            | Y             | 32            | 832                                 |
| GY                   | G            | Y             | 38            | 988                                 |
| Regions Y Subtotals  |              | 135           | 3510          |                                     |
| HZ                   | H            | Z             | 47            | 705                                 |
| JZ                   | J            | Z             | 66            | 990                                 |
| KZ                   | K            | Z             | 35            | 525                                 |
| Regions Z Subtotals  |              | 148           | 2220          |                                     |
| TOTAL KILOMETERS     |              |               | 396           |                                     |
| TOTAL ESTIMATED COST |              |               |               | T 8,103,000                         |

\*Cost per kilometer were estimated at:

Region X = T 21,000 per kilometer  
Region Y = T 26,000 per kilometer  
Region Z = T 15,000 per kilometer

identified by the state name (A through K) and the region name (x,y, or z). The total cost of the ten proposed roads is double the budget available. (For argument sake we assume the projects are not amenable to being built shorter, i.e. are not divisible.)

The next step is to combine projects into strategies (Table 19) which satisfy a series of preliminary criteria within the budget constraint and with at least one project per region:

- a. Maximize length
- b. Maximize projects
- c. Best distribution of length
- d. Best distribution of investment

The best solution strategy for maximum length is unacceptable because it exceeds the budget constraint. Substituting AX for GY satisfies the budget constraint, but fails because the strategy excludes Region Y. Therefore, FY must be taken to make strategy ST-1, which is given in Table 19 (A).

There are no 6 projects which meet the budget constraint, so criterion b cannot be met.

To examine the best distribution of lengths, it is necessary to solve for the nominal length per region:

Table 19

STRATEGY DEFINITION USING THE BUDGET CONSTRAINT  
PLUS ONE OTHER CRITERION

A. Maximum Length

|    | <u>Length</u> | <u>Cost</u> |        |
|----|---------------|-------------|--------|
| CX | 42            | 882         |        |
| FY | 32            | 832         | (ST-1) |
| HZ | 47            | 705         |        |
| JZ | 66            | 990         |        |
| KZ | <u>35</u>     | <u>525</u>  |        |
|    | 222           | 3934        |        |

B. Best Distribution of Length and of Investment

|     |    |           |       |            |        |        |
|-----|----|-----------|-------|------------|--------|--------|
| (1) | CX | 42        |       | 882        |        |        |
|     | FY | 32        |       | 832        |        |        |
|     | GY | 38        | > 70  | 988        | > 1820 | (ST-4) |
|     | HZ | 47        |       | 705        |        |        |
|     | KZ | <u>35</u> | > 82  | <u>525</u> | > 1230 |        |
|     |    | 194       |       | 3932       |        |        |
| (2) | AX | 35        |       | 735        |        |        |
|     | BX | 31        | > 66  | 651        | > 1386 | (ST-5) |
|     | GY | 38        |       | 988        |        |        |
|     | JZ | 66        |       | 990        |        |        |
|     | KZ | <u>35</u> | > 101 | <u>525</u> | > 1515 |        |
|     |    | 205       |       | 3889       |        |        |
| (3) | AX | 35        |       | 735        |        |        |
|     | BX | 31        | > 66  | 651        | > 1386 |        |
|     | DY | 30        |       | 870        |        |        |
|     | FY | 32        | > 62  | 832        | > 1612 | (ST-2) |
|     | JZ | <u>66</u> |       | <u>990</u> |        |        |
|     |    | 194       |       | 3988       |        |        |

$$\text{Budget} = \text{MXC}_x + \text{MYC}_y + \text{MZC}_z \quad (5-1)$$

where Budget = 4,000,000

M = Kilometerage in the region

C = Cost per kilometer in the region

If  $\text{MX} = \text{MY} = \text{MZ}$ , then

$$\text{Budget} = M (\text{C}_x + \text{C}_y + \text{C}_z) \quad (5-2)$$

Solving for M, average length per region,

$$M = 64.51 \text{ kilometers per region}$$

The following set satisfies this criterion quite well:

|    |           |      |            |        |
|----|-----------|------|------------|--------|
| AX | 35        |      | 735        |        |
|    |           | > 66 |            |        |
| BY | 31        |      | 651        |        |
| DY | 30        |      | 780        |        |
|    |           | > 62 |            |        |
| FY | 32        |      | 832        | (ST-2) |
| JZ | <u>66</u> |      | <u>990</u> |        |
|    | 194       |      | 3988       |        |

If a weighting by state (3-4-3) is required, then the kilometers per state would be 18.87, and the regional goals would be:

$$X = 56.6$$

$$Y = 75.5$$

$$Z = 56.6$$

The following strategy attempts this distribution but doesn't come as close as did ST-2 to the regional goal.



|    |           |      |            |        |
|----|-----------|------|------------|--------|
| AX | 35        |      | 735        |        |
| BX | 31        | > 66 | 651        |        |
| EY | 35        |      | 910        | (ST-3) |
| GY | 38        | > 73 | 988        |        |
| HZ | <u>47</u> |      | <u>705</u> |        |
|    | 186       |      | 3989       |        |

This strategy appears inferior to ST-2 in that for essentially the same cost it generates 8 kilometers less road and a seemingly less equitable distribution regionally. Admittedly, unlike ST-2, no one state will get twice the road length of another, or even equal to or more than any two other states combined.

Best Distribution of Investment

1. Even distribution of investment would put approximately T 1,333,333 in each region. The nearest solution to this using the regional unit rates is X = 63, Y = 51, Z = 89.

|    |           |      |            |        |
|----|-----------|------|------------|--------|
| AX | 35        |      | 735        |        |
| BX | 31        | > 66 | 651        | > 1386 |
| GY | 38        |      | 988        |        |
| HZ | 47        |      | 705        |        |
| KZ | <u>35</u> | > 82 | <u>525</u> | > 1230 |
|    | 186       |      | 3594       |        |

More of the budget could be utilized by using any of the strategies in Table 19 (B). However, none of these

four strategies is quite satisfactory in achieving a good distribution of investment.

2. Even distribution of investment could also be interpreted as equal weight per state, in which case the regional goals would be:

|          |             |    |       |
|----------|-------------|----|-------|
| Region Z | T 1,200,000 | or | 57 km |
| Region Y | T 1,600,000 | or | 62 km |
| Region Z | T 1,200,000 | or | 80 km |

The best solution here is ST-4 which is an optimization of the following set:

|    |                |  |            |   |      |
|----|----------------|--|------------|---|------|
| CX | 42             |  | 882        |   |      |
| DY | 30             |  | 780        |   |      |
| FY | 32 > 62        |  | 832        | > | 1612 |
| HZ | 47             |  | 705        |   |      |
| KZ | <u>35</u> > 82 |  | <u>525</u> | > | 1230 |
|    | 186            |  | 3724       |   |      |

#### 5.4 The Project Set - Arable Land Available

The Director then collected the data in Table 20 concerning the area of influence, current development, and development potential. The available arable land ranges from 18,000 hectares for project DY, to 95,040 for project JZ. The average per project is 39,000 hectares; the average per region is 130,000. Using the same preliminary criteria to evaluate this objective as were used in 5.2.1, yields the sets in Table 21.

Table 20

RAW AGRICULTURAL POTENTIAL BY PROJECT (HECTARES)

| <u>Project Name</u> | <u>Length of Road (Km)</u> | <u>Zone of Influence (Hectares) (000)</u> | <u>Cultivation of Current<sup>2</sup> (%)</u> | <u>Land Potential (%)</u> | <u>Available Land (Ha) (000)</u> |
|---------------------|----------------------------|---|---|---------------------------|----------------------------------|
| AX                  | 35                         | 70  | 43%   | 90%                       | 33                               |
| BX                  | 31                         | 62  | 48%   | 90%                       | 26                               |
| CX                  | 42                         | 84  | 36%   | 90%                       | 45                               |
| DY                  | 30                         | 60  | 50%   | 80%                       | 18                               |
| EY                  | 35                         | 70  | 43%   | 80%                       | 25                               |
| FY                  | 32                         | 64  | 47%   | 80%                       | 21                               |
| GY                  | 38                         | 76  | 39%   | 80%                       | 31                               |
| HZ                  | 47                         | 94  | 32  | 95%                       | 59                               |
| JZ                  | 66                         | 132                                       | 23%   | 95%                       | 95                               |
| KZ                  | 35                         | 70  | 43%   | 95%                       | 36                               |

## Notes

1. Calculated as 10 kilometers either side of the road
2. Calculated as an area of 20 kms x 10 kms along the connecting road plus pockets of subsistive farming, i.e. about 30,000 Ha.

Table 21

STRATEGY DEFINITION FOR AGRICULTURE USING  
BUDGET CONSTRAINT AND ONE OTHER CRITERION

## A. Maximize New Hectarage

| <u>Project</u> | <u>New Hectarage</u><br>(000) | <u>Road Cost</u><br>(000) |        |
|----------------|-------------------------------|---------------------------|--------|
| JZ             | 95                            | 990                       |        |
| HZ             | 59                            | 705                       |        |
| CX             | 45                            | 880                       | (ST-1) |
| KZ             | 36                            | 525                       |        |
| FY             | <u>21</u>                     | <u>832</u>                |        |
|                | 256                           | 3934                      |        |

## B. Best Distribution of Arable Land

## - Regional Basis

|    |           |      |            |        |
|----|-----------|------|------------|--------|
| AX | 33        | > 59 | 735        |        |
| BX | 26        |      | 651        |        |
| EY | 26        | > 57 | 910        | (ST-3) |
| GY | 31        |      | 988        |        |
| HZ | <u>59</u> |      | <u>705</u> |        |
|    | 175       |      | 3989       |        |

## - State Basis

|    |           |            |    |           |            |
|----|-----------|------------|----|-----------|------------|
| AX | 33        | 735        | CY | 45        | 882        |
| DY | 18        | 780        | DY | 18        | 780        |
| EY | 26 > 65   | 910        | EY | 26 > 65   | 910        |
| FY | 21        | 832        | FY | 21        | 832        |
| HZ | <u>59</u> | <u>705</u> | KZ | <u>36</u> | <u>525</u> |
|    | 157       | 3962       |    | 146       | 3929       |

## C. Distribution by Investment Balance Criterion

| <u>Project</u> | <u>Land</u> | <u>Cost</u> |        |
|----------------|-------------|-------------|--------|
| AX             | 33          | 735         |        |
| BX             | 26 > 59     | 651 > 1386  |        |
| DY             | 18          | 780         | (ST-2) |
| FY             | 21 > 39     | 832 > 1612  |        |
| JZ             | <u>95</u>   | <u>990</u>  |        |
|                | 193         | 3988        |        |

Arable land available is so closely correlated to road length that the same strategy, ST-1, satisfies the maximum hectarage criterion as satisfied the maximum length criterion.

To determine the best distribution of new arable land by region,  $D_R$ , solve:

$$D_R = \left( \frac{\text{Budget}}{\text{Total Cost}} \right) \left( \sum_{i=1}^P X_i \right) R^{-1} \quad (5-3)$$

where,  $X$  is the objective value

$P$  is the member of projects

$R$  is the number of regions

$i$  is the specific project

$$D_R = 64.17$$

The solution is strategy ST-3, the same strategy which satisfied the length distribution criterion for the state based distribution. For the state based distribution on the land objective, substitute  $S$  for  $R$  in Equation 5-3, where  $S$  is the number of states. This gives as the distribution goals:

$$\text{Region X} = 57.76$$

$$\text{Region Y} = 77.01$$

$$\text{Region Z} = 57.76$$

The criterion would be well met if project DY or FY could be added to ST-3 without exceeding the budget constraint. Alternately, strategies using the lowest three

projects from Region Y and one each from X and Z were developed. These appear in Table 21(B). Neither strategy seems competitive to ST-2.

The equal investment criterion used in 5.2.1 is used for this objective as well, while trying to maximize and equalize distribution of arable land. Strategy ST-2, the preferred strategy for the length objective, appears to be the best choice for this criterion.

#### 5.5 The Project Set - Health Care Access

The decision to use basic health care access reflects the Director's readings in the World Bank's Health Sector Policy Paper.(18)

"It is estimated that cholera patients who arrive at a hospital within three hours of the onset of symptoms run no risk of death; that those who arrive after 3 to 6 hours have a 10% fatality rate; and that after 6 hours the fatality rate is 30%."

A metric for health access was proposed which reflected this concept of access time to a medical facility. A reduction in access time from over 6 hours to 3 to 6, and from 3 to 6 to less than 3 hours represents potential lives saved. In the former case 20 lives are saved per 100 and in the latter, 10 lives per 100. To remove the population bias, the potential lives saved will be normalized by the total population served. Obviously cholera is not the only disease

in rural Tap, it doesn't strike everyone, and the rural health units provide services beyond rehydration; nevertheless this system is a meaningful proxy for the benefits of improved access to primary health services. Table 22 presents the relevant data by population.

(The low population figures in all the project zones raise another interesting issue. Examining project EY shows a population density of about 100 people per kilometer of new road. The large increase in arable land (Table 20) will require a large influx of new farmers to make the land productive. This could form the basis for another objective, if resettlement is an important goal.)

Table 23 shows the standardization of the reduction in fatality measure for population, i.e. reduction per 1000 inhabitants, based on the previously stated guidelines. The preliminary evaluation of this objective is presented as Table 24. The health maximization strategy ST-6 should have included project DY but for the budget constraint.

After the best four projects, sufficient funds were only available for Project KZ. No other combination of projects provides a higher health access score within the budget.

Table 22

HEALTH ACCESS IMPROVEMENT DATA (POPULATION 000's)

| <u>Road Project</u> | <u>Total Popula-<br/>tion</u> | <u>POPULATION GROUPED BY ACCESS TIME</u> |             |                    |             |                     |             |
|---------------------|-------------------------------|--|-------------|--------------------|-------------|---------------------|-------------|
|                     |                               | <u>0 - 3 Hours</u>                       |             | <u>3 - 6 Hours</u> |             | <u>Over 6 Hours</u> |             |
|                     |                               | <u>without</u>                           | <u>with</u> | <u>without</u>     | <u>with</u> | <u>without</u>      | <u>with</u> |
| AX                  | 3.6                           | 0.3                                      | 2.8         | 1.5                | 0.6         | 1.8                 | 0.2         |
| BX                  | 2.2                           | 0.1                                      | 2.1         | 0.3                | 0.0         | 1.8                 | 0.1         |
| CX                  | 4.3                           | 0.1                                      | 1.1         | 0.8                | 2.0         | 3.4                 | 1.2         |
| DY                  | 3.4                           | 0.2                                      | 1.4         | 0.5                | 1.5         | 2.7                 | 0.5         |
| EY                  | 3.5                           | 0.2                                      | 2.4         | 0.7                | 0.3         | 2.4                 | 0.8         |
| FY                  | 1.8                           | 0.1                                      | 1.6         | 0.5                | 0.2         | 1.2                 | 0.0         |
| GY                  | 2.7                           | 0.0                                      | 2.2         | 0.2                | 0.5         | 2.5                 | 0.0         |
| HZ                  | 2.3                           | 0.1                                      | 0.4         | 0.0                | 1.3         | 2.2                 | 0.6         |
| JZ                  | 1.1                           | 0.0                                      | 1.1         | 0.0                | 0.0         | 1.1                 | 0.0         |
| KZ                  | 3.9                           | 0.3                                      | 1.9         | 0.9                | 1.1         | 2.7                 | 0.9         |



Table 23

CONVERSION OF HEALTH ACCESS PROXY TO  
HEALTH ACCESS BENEFIT MEASURE

| <u>PROJECT NAME</u> | <u>REDUCED FATALITIES</u> | <u>TOTAL POPULATION</u> | <u>HEALTH ACCESS BENEFIT MEASURE(1)</u> |
|---------------------|---------------------------|-------------------------|---|
| AX                  | 570(2)                    | 3600                    | 158                                     |
| BX                  | 540                       | 2200                    | 245                                     |
| CX                  | 540                       | 4300                    | 126                                     |
| DY                  | 560                       | 3400                    | 165                                     |
| EY                  | 520                       | 3500                    | 149                                     |
| FY                  | 390                       | 1800                    | 217                                     |
| GY                  | 720                       | 2700                    | 267                                     |
| HZ                  | 350                       | 2300                    | 152                                     |
| JZ                  | 330                       | 1100                    | 300                                     |
| KZ                  | 520                       | 3900                    | 133                                     |

(1) This Is The Ratio Of The Reduced Fatalities To Population Expressed As A Scalar

(2) Sample Calculation for Project AX:

| <u>WITHOUT</u>          | <u>WITH</u>           |
|-------------------------|-----------------------|
| 1800 x 30% = 540        | 200 x 30% = 60        |
| 1500 x 10% = <u>150</u> | 600 x 10% = <u>60</u> |
| 690                     | 120 = 570 saved       |

Table 24

STRATEGY DEFINITION FOR HEALTH ACCESS  
WITH A BUDGET CONSTRAINT

A. Maximize Health Access Improvement With Budget

| <u>Project Name</u> | <u>Health Access Score</u> | <u>Project Cost</u> |        |
|---------------------|----------------------------|---------------------|--------|
| JZ                  | 300                        | 900                 |        |
| GY                  | 267                        | 988                 | (ST-6) |
| BX                  | 245                        | 651                 |        |
| FY                  | 217                        | 832                 |        |
| KZ                  | <u>133</u>                 | <u>525</u>          |        |
|                     | 1162                       | 3986                |        |

B. Equal Distribution of Health Access Improvement

| <u>Project Name</u> | <u>Health Access</u> |            | <u>Project Cost</u> |        |
|---------------------|----------------------|------------|---------------------|--------|
| A                   | 158                  | > 403      | 735                 |        |
| B                   | 245                  |            | 651                 |        |
| D                   | 165                  | > 382      | 780                 | (ST-2) |
| F                   | 217                  |            | 832                 |        |
| J                   | 300                  | <u>300</u> | <u>990</u>          |        |
|                     |                      | 1085       | 3988                |        |

### Equal Distribution

Using the equation at 5-3 and a substitute objective yields a regional distribution guide of 314.62 and, on a state weight basis, a regional guide of:

$$X = 283$$

$$Y = 378$$

$$Z = 283$$

Strategy ST-2, shown in Table 24B, appears to satisfy both guidelines.

### Balanced Investment

Strategy ST-2 has been shown previously to satisfy this criterion on both a regional and on a state weighted basis.

### 5.6 The Project Set - Incremental Employment

The road investment will require unskilled construction labor from the area. This is a benefit to the area and to the state. The data confirm there is surplus labor available in each project area hence no farm labor will be diverted to the detriment of the agricultural production capacity. The local government authority will also assume responsibility for maintaining the road for 10 years using special funds earmarked for the purpose. The following rules were established to estimate the employment impact:

- Construction will proceed at 25 kilometers per year, and will require 40 unskilled local workers per year plus 10 unskilled workers per each T 100,000 of value

- Maintenance productivity will be 15 kilometers per worker per year over the 10 years of the project life

Table 25 summarizes the employment impact of each project being considered. As with the other objectives, a preliminary review was performed to identify strategies which maximized employment within the budget constraint, gave the best distribution of new employment, and gave the best distribution of investment while maximizing for employment. The results of this are presented as Table 26.

Maximizing for labor we find that the top 4 projects (J, G, C, H) do not permit a fifth project within the budget constraint. As for the length and arable land objectives, FY substituted for GY to permit KZ and making the maximized strategy ST-1.

The best distribution of employment benefits is calculated as in 5.2.2 and 5.2.3 using Equation 5-3. For the region based guideline, use 277.43, and for the state weighted guideline use:

$$X = 249.69$$

$$Y = 332.92$$

$$Z = 249.69$$

Strategy ST-2 best satisfies this criterion. It also best satisfies the equal investment criterion for both the regional and state-weighted guidelines.

Table 25

LOCAL EMPLOYMENT IMPACT DATA  
(Man-Years)

| <u>Project Name</u> | <u>Project Cost</u> | <u>Labor Need<sup>1</sup></u> | <u>Project Length</u> | <u>Labor Needed<sup>2</sup></u> | <u>Maintenance Labor<sup>3</sup></u> | <u>Total Labor<sup>4</sup></u> |
|---------------------|---------------------|-------------------------------|-----------------------|---------------------------------|--------------------------------------|--------------------------------|
| AX                  | 735                 | 73.5                          | 35                    | 56.0                            | 23.3                                 | 153                            |
| BX                  | 651                 | 65.1                          | 31                    | 49.6                            | 20.7                                 | 135                            |
| CX                  | 882                 | 88.2                          | 42                    | 67.2                            | 28.0                                 | 183                            |
| DY                  | 780                 | 78.0                          | 30                    | 48.0                            | 20.0                                 | 146                            |
| EY                  | 910                 | 91.0                          | 35                    | 56.0                            | 23.3                                 | 170                            |
| FY                  | 832                 | 83.2                          | 32                    | 51.2                            | 21.3                                 | 156                            |
| GY                  | 988                 | 98.8                          | 38                    | 60.8                            | 25.3                                 | 185                            |
| HZ                  | 705                 | 70.5                          | 47                    | 75.2                            | 31.3                                 | 177                            |
| JZ                  | 990                 | 99.0                          | 66                    | 105.6                           | 44.0                                 | 249                            |
| KZ                  | 525                 | 52.5                          | 35                    | 56.0                            | 23.3                                 | 132                            |

---

<sup>1</sup>at 10 man-years per T 100,000.

<sup>2</sup>at 40 man-years per year of construction at 25 kilometers per year constructed.

<sup>3</sup>at 0.67 man-years per kilometer for the full 10 years period.

<sup>4</sup>totalled and rounded.

Table 26

STRATEGY DEFINITION USING THE NEW EMPLOYMENT  
OBJECTIVE WITH A BUDGET CONSTRAINT

A. Maximize For New Employment

| <u>Project Name</u> | <u>Labor Needed</u> | <u>Project Cost</u> |        |
|---------------------|---------------------|---------------------|--------|
| JZ                  | 249                 | 990                 |        |
| FY                  | 156                 | 832                 | (ST-1) |
| CX                  | 183                 | 882                 |        |
| HZ                  | 177                 | 705                 |        |
| KZ                  | <u>132</u>          | <u>525</u>          |        |
|                     | 897                 | 3934                |        |

B. Best Distribution And Even Investment

| <u>Project Name</u> | <u>Labor</u> | <u>Cost</u> |        |
|---------------------|--------------|-------------|--------|
| AX                  | 153          | 735         | (ST-2) |
| BX                  | 135 > 288    | 651 > 1386  |        |
| DY                  | 146          | 780         |        |
| FY                  | 156 > 302    | 832 > 1612  |        |
| JZ                  | <u>249</u>   | <u>990</u>  |        |
|                     | 839          | 3988        |        |

## 5.7 The Project Set - Evaluation

### 5.7.1 Evaluation of Multiple Objectives

The project objectives and data set have been defined and a series of strategies identified. There are four objectives:

- L - Length of the route (kilometers)
- A - New Arable land (hectares)
- H - Improved Health access (persons per 1000)
- E - Incremental employment (man years of labor)

There are two constraints:

- a. - the budget of T4 million
- b. - each region must have at least one project

There are ten projects, one from each state (Table 27). Of the nearly 8000 combinations (5 at a time) which can be made from these 10 projects, there are 1944 combinations which meet constraint b and of these, not more than 108 combinations meet the budget constraint. These combinations were subjected to a preliminary evaluation using three criteria:

- MAX - maximize the objective
- DIST - equal distribution of the objective
- INV - balance investment while achieving equal distribution

A fourth criterion, maximize projects was dropped because no 6 project strategy was possible given the budget

Table 27

SUMMARY OF PROJECT SCORES BY OBJECTIVE

| <u>Proj.<br/>Name</u> | <u>Project<br/>Cost</u> | <u>Proj.<br/>Length</u> | <u>Available<br/>Arable<br/>Land</u> | <u>Health Access<br/>Improvement<br/>Score</u> | <u>Employment<br/>Impact</u> |
|-----------------------|-------------------------|-------------------------|--------------------------------------|--|------------------------------|
| A X                   | 735                     | 35                      | 33                                   | 158  | 153                          |
| B X                   | 651                     | 31                      | 26                                   | 245  | 135                          |
| C X                   | 882                     | 42                      | 45                                   | 126  | 183                          |
| D Y                   | 780                     | 30                      | 18                                   | 165  | 146                          |
| E Y                   | 910                     | 35                      | 26                                   | 149  | 170                          |
| F Y                   | 832                     | 32                      | 21                                   | 217  | 156                          |
| G Y                   | 988                     | 38                      | 31                                   | 267  | 185                          |
| H Z                   | 705                     | 47                      | 59                                   | 152  | 177                          |
| J Z                   | 990                     | 66                      | 95                                   | 300  | 249                          |
| K Z                   | 525                     | 35                      | 36                                   | 133  | 132                          |



constraint. Also, the distribution and investment criteria appeared frequently to be satisfied by the same strategy.

The six strategies which most frequently surfaced are shown in Table 28.

#### 5.7.2 Presentation Of Multiple Objective Analysis

No specific interobjective weighting, cardinal or ordinal, was given the Director. Furthermore many decision makers will be involved in the final resolution. As no consensus seems to exist, nothing can be aggregated further, nor is ranking feasible. The analysis will be presented graphically.

A presentation for the impact on each state by strategy by objective is possible; however, five states must be disappointed with each strategy and several states are only in one or two strategies. There is little to be gained by this presentation.

Similarly, there is nothing to be gained by presenting impact by region by strategy by objective. The bias is clear. Region X observes that ST-2, ST-3, and ST-5 are identical and rejects the other three. Region Y rejects ST-1, ST-2, and ST-5 as inferior; sees ST-4 and ST-6 as identical and has a minor tradeoff of investment against health between them and ST-3. Region Z has no choice but to move for ST-1 as it includes all their projects while

Table 28  
SUMMARY OF STRATEGIES

| STRATEGY | PROJECTS | COST                         | LENGTH                  | AVAILABLE ARABLE LAND (A) | HEALTH ACCESS IMPROVEMENT (H) | EMPLOYMENT (E)            | REMARKS   |
|----------|----------|------------------------------|-------------------------|---------------------------|-------------------------------|---------------------------|---|
|          | (P)      | (C)                          | (L)                     | X Y Z                     | X Y Z                         | X Y Z                     |   |
|          | X Y Z    | X Y Z                        | X Y Z                   | X Y Z                     | X Y Z                         | X Y Z                     |   |
| ST-1     | C F HFK  | <u>882 832 2220</u><br>3934  | <u>42 32 148</u><br>222 | <u>45 21 190</u><br>256   | <u>126 217 585</u><br>928     | <u>183 156 558</u><br>897 | MAX L, A, E   |
| ST-2     | AB DF J  | <u>1386 1612 990</u><br>3988 | <u>66 62 66</u><br>194  | <u>59 39 95</u><br>193    | <u>403 382 300</u><br>1085    | <u>288 302 249</u><br>839 | DIST <sub>R</sub> L,H,E<br>INV <sub>R</sub> C,L,A,H<br>INVS <sub>S</sub> C,L,A,H<br>DIST <sub>S</sub> A,H,E |
| ST-3     | AB EG H  | <u>1386 1898 705</u><br>3989 | <u>66 73 47</u><br>186  | <u>59 57 59</u><br>175    | <u>403 416 152</u><br>971     | <u>288 355 177</u><br>820 | DIST <sub>S</sub> L<br>DIST <sub>R</sub> A  |
| ST-4     | C FG HK  | <u>882 1820 1230</u><br>3982 | <u>42 70 82</u><br>194  | <u>45 52 95</u><br>192    | <u>126 484 285</u><br>895     | <u>183 341 309</u><br>833 | DIST <sub>R</sub> L<br>INV <sub>S</sub> L   |
| ST-5     | AB G JK  | <u>1386 988 1515</u><br>3889 | <u>66 38 101</u><br>205 | <u>59 31 131</u><br>221   | <u>403 267 433</u><br>1103    | <u>288 185 381</u><br>854 | INV <sub>R</sub> C  |
| ST-6     | B FG JK  | <u>651 1820 1515</u><br>3986 | <u>31 70 101</u><br>202 | <u>26 52 131</u><br>029   | <u>245 484 433</u><br>1162    | <u>135 341 381</u><br>857 | MAX H   |

returning the maximum for 3 out of 4 objectives to the nation. In fact only Region Y is faced with any choice at all.

One useful presentation is strategy by objectives. One problem with this presentation could be there are four objectives and only three dimensions. The first step would be to prepare the 4 plots showing 3 objectives at a time:

LAH - LENGTH - ARABLE LAND - HEALTH (Figure 16)

LAE - LENGTH - ARABLE LAND - EMPLOMENT (Figure 17)

LHE - LENGTH - HEALTH - EMPLOYMENT (Figure 18)

AHE - ARABLE LAND - HEALTH - EMPLOYMENT (Figure 19)

Reviewing these four graphs we note that ST-2, ST-3, and ST-4 are always fully interior, i.e. they are never superior to any other strategy. Except for the LAE graph (Figure 17) where ST-1 contains all the other strategies, ST-5 and ST-6 are viable solutions. The explanation for this is that we are only viewing graphs with a national bias. The interior strategies were selected during the preliminary inspection (Chapter 5.2) because they provided a measure of regional distribution for each objective. They also provide a degree of investment balance, the third criterion used in the preliminary inspection. This is not to say that Figures 16 through 19 are useless, but only that they mask the distribution criterion.

Figures 20 through 23 display the regional disaggregation of the six strategies one objective at a time. Figure

Figure 16  
STRATEGY BY OBJECTIVES (LENGTH-ARABLE LAND-HEALTH ACCESS)

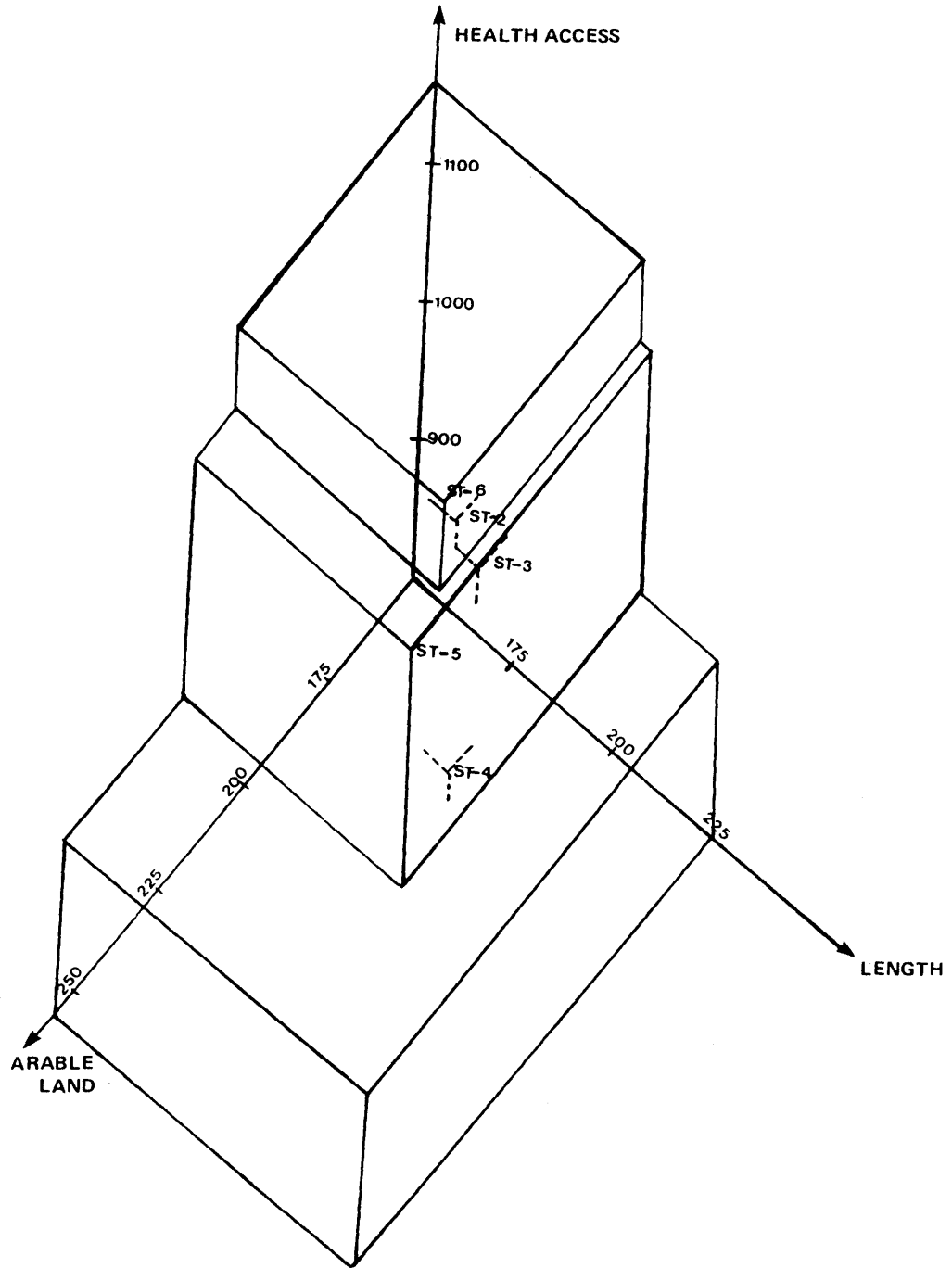


Figure 17  
 STRATEGY BY OBJECTIVES (LENGTH-ARABLE LAND-EMPLOYMENT)

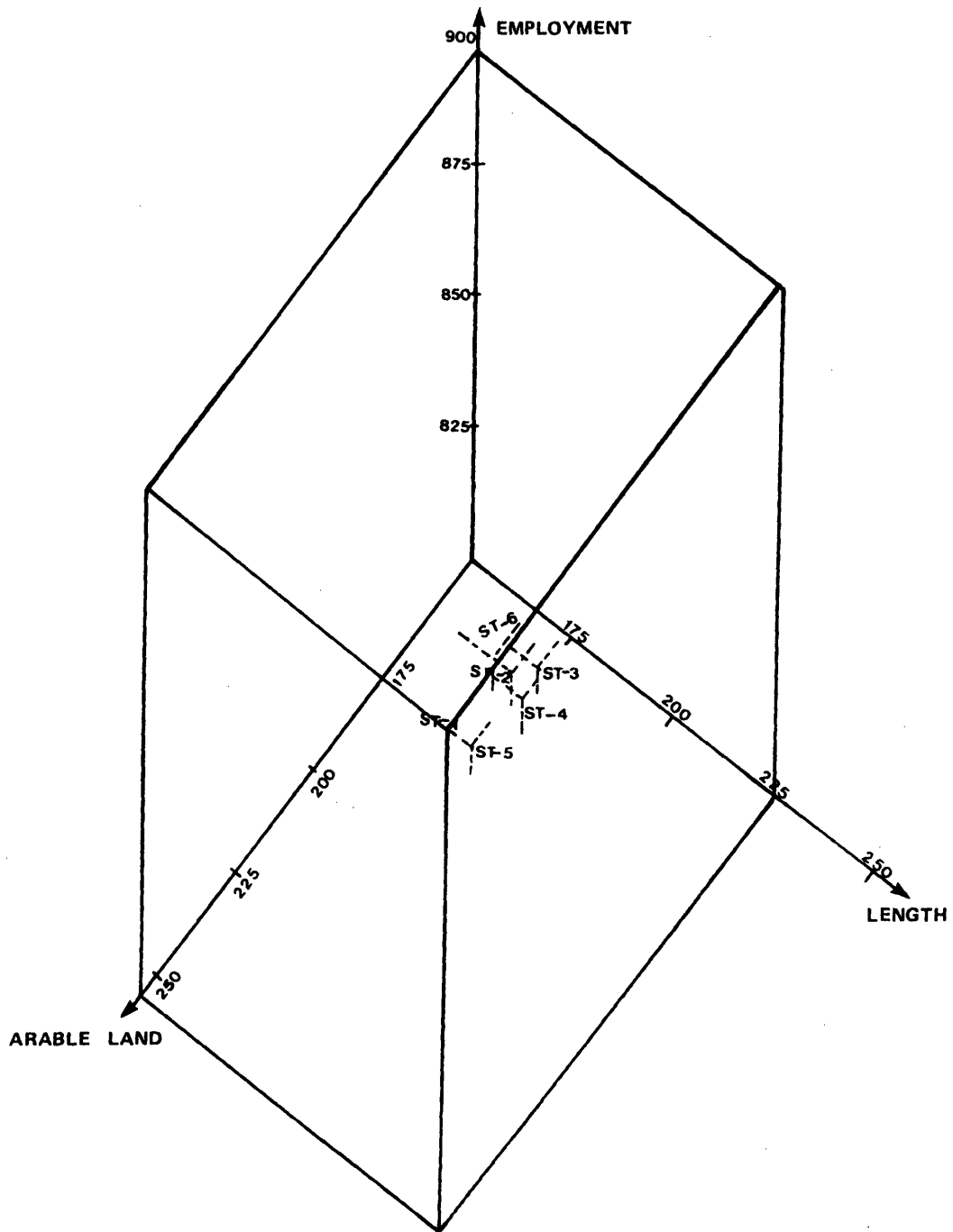


Figure 18  
 STRATEGY BY OBJECTIVES (LENGTH-HEALTH ACCESS-EMPLOYMENT)

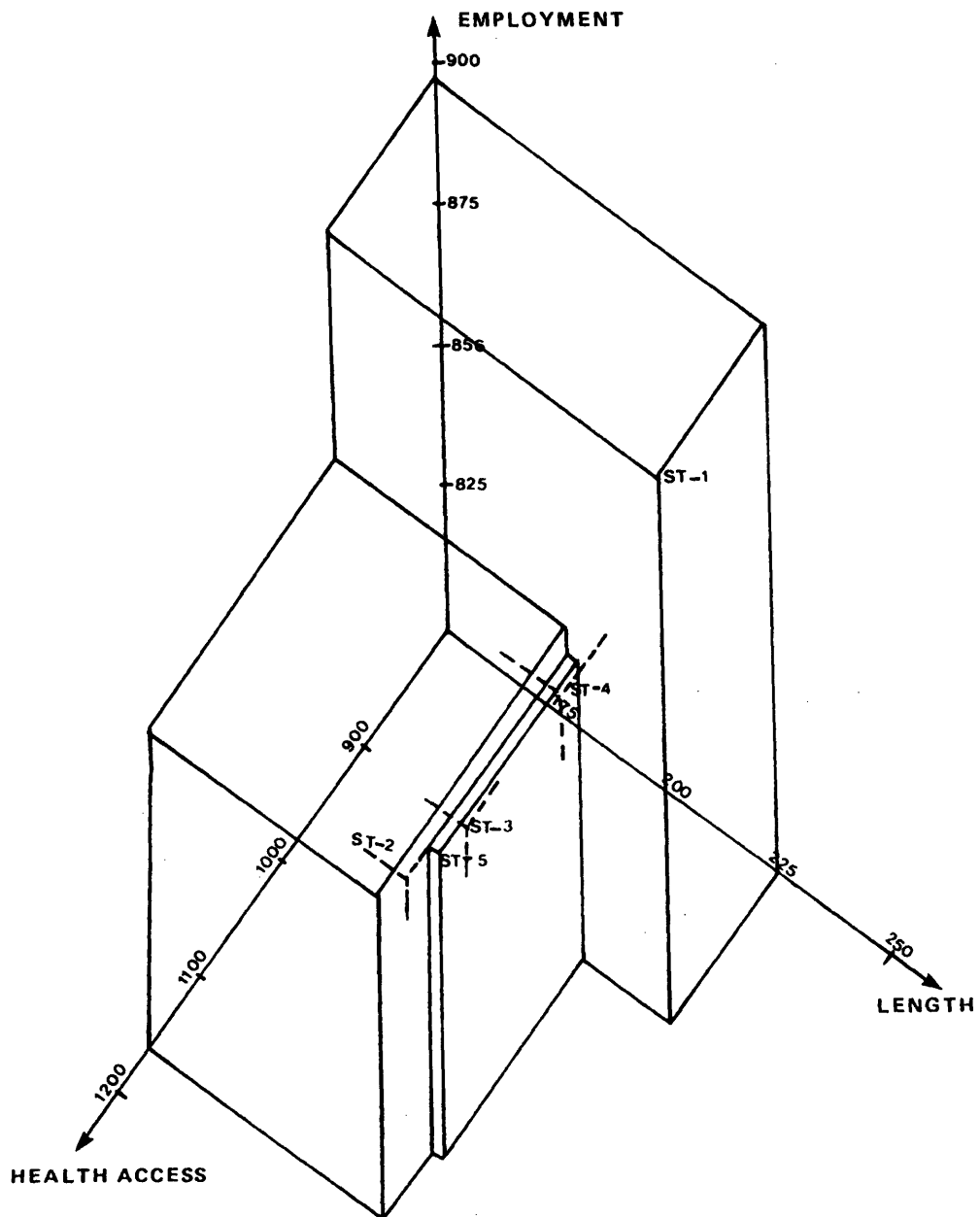


Figure 19  
 STRATEGY BY OBJECTIVES (ARABLE LAND-HEALTH ACCESS-EMPLOYMENT)

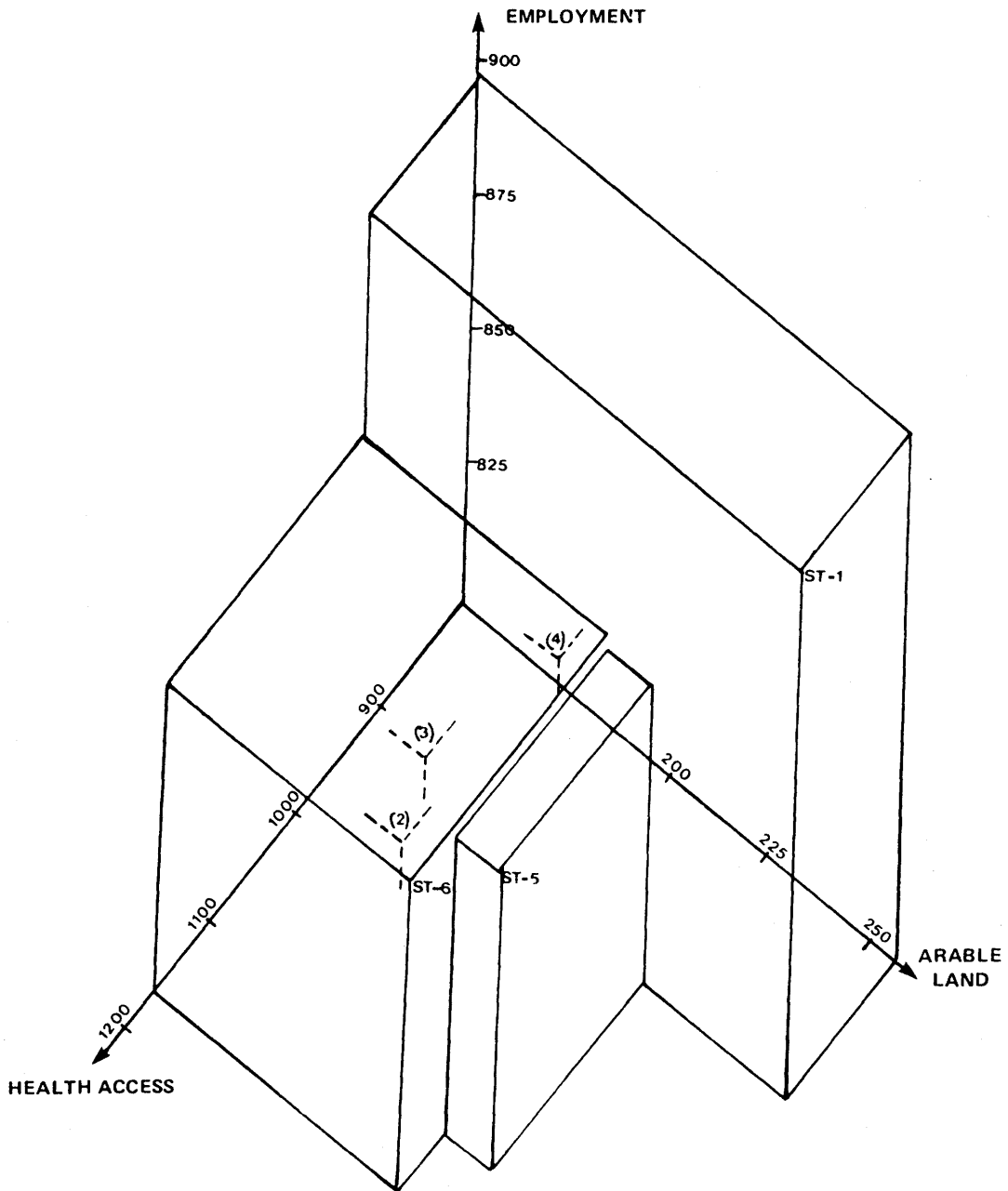
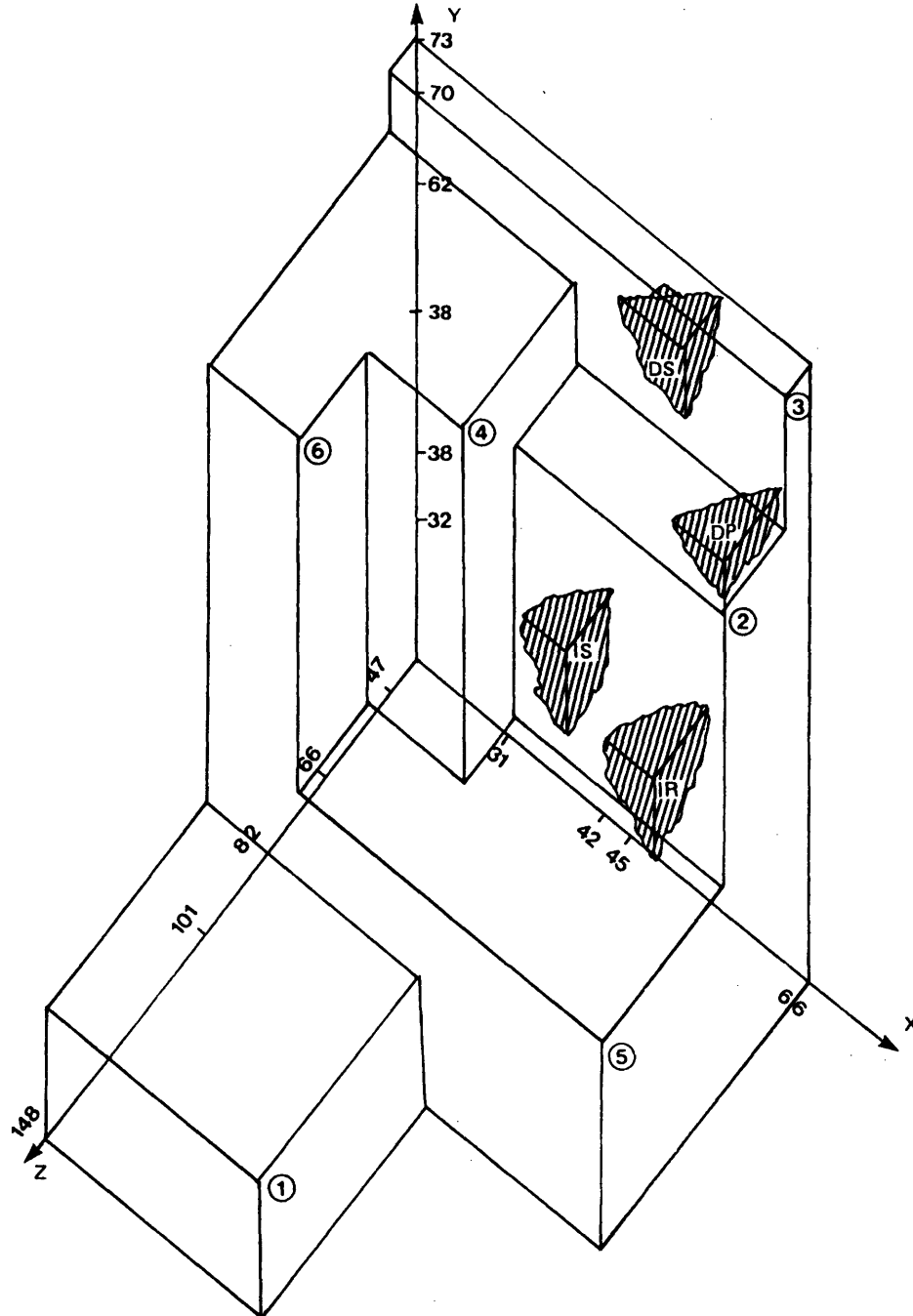


Figure 20

LENGTH OBJECTIVE BY REGION - ALL STRATEGIES WITH GUIDELINE CRITERIA

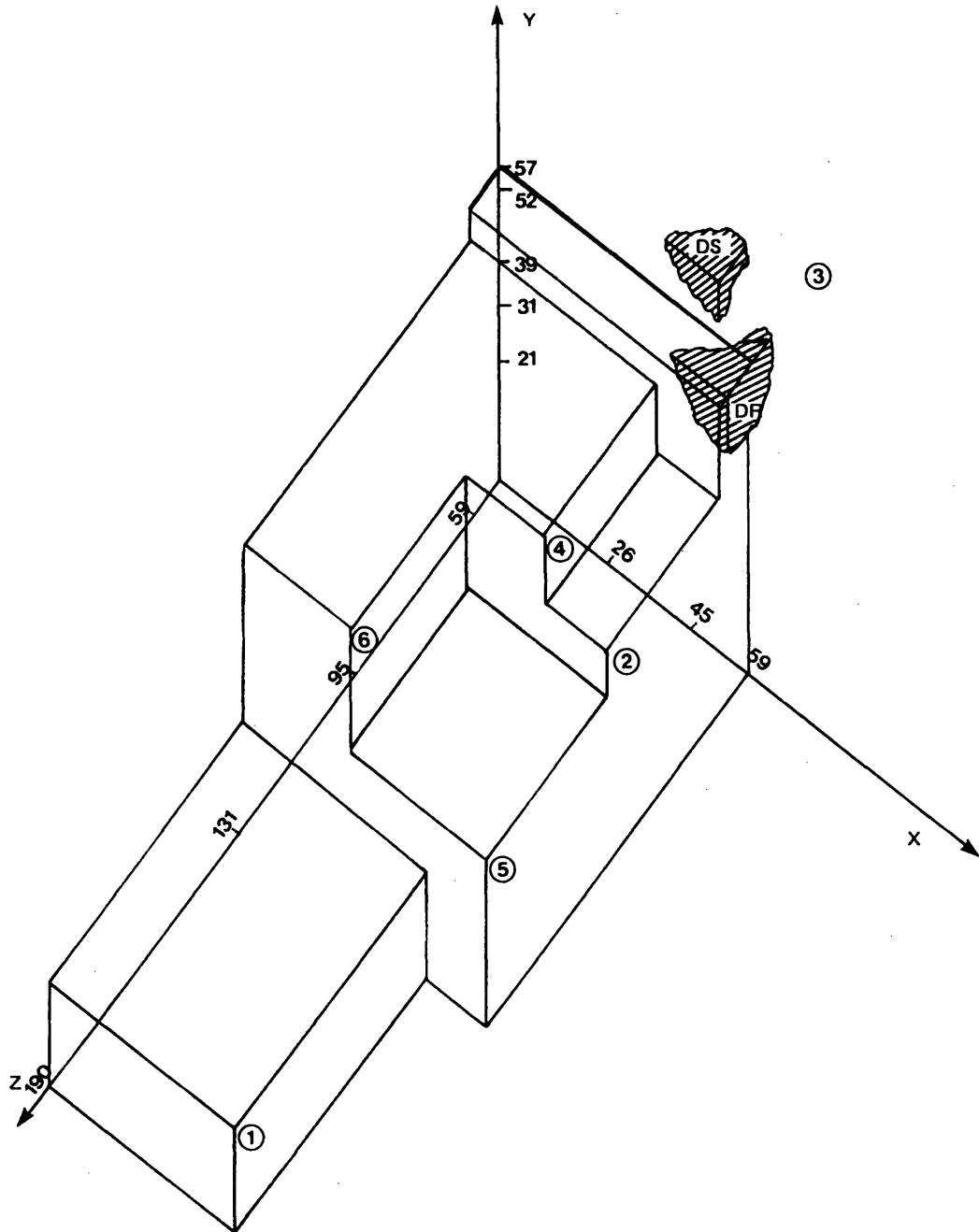


| CRITERIA COORDINATES |    |    |    | 1. STRATEGY NUMBER   |
|----------------------|----|----|----|----------------------|
|                      | X  | Y  | Z  |                      |
| DS=                  | 57 | 76 | 57 | D EVEN DISTRIBUTION  |
| IR=                  | 63 | 51 | 89 | I EVEN INVESTMENT    |
| IS=                  | 57 | 62 | 80 | R REGIONAL WEIGHTING |
|                      |    |    |    | S STATE WEIGHTING    |



Figure 21

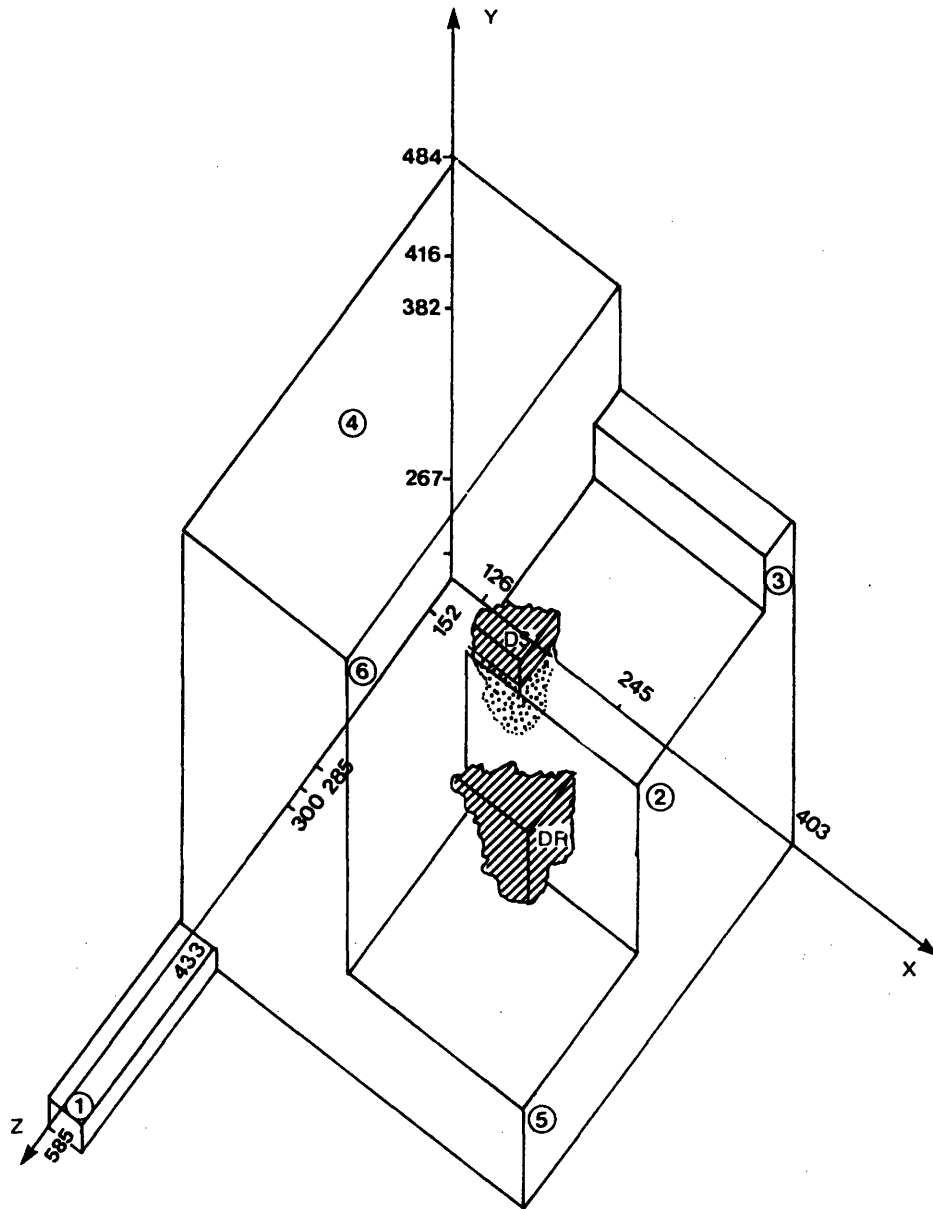
NEW ARABLE LAND OBJECTIVE BY REGION - ALL STRATEGIES WITH GUIDELINE CRITERIA



| CRITERIA COORDINATES |    |    |    |                      |
|----------------------|----|----|----|----------------------|
|                      | X  | Y  | Z  |                      |
| DR:                  | 64 | 64 | 64 | 1 STRATEGY NUMBER    |
| DS:                  | 58 | 77 | 58 | D EVEN DISTRIBUTION  |
|                      |    |    |    | R REGIONAL WEIGHTING |
|                      |    |    |    | S STATE WEIGHTING    |

Figure 22

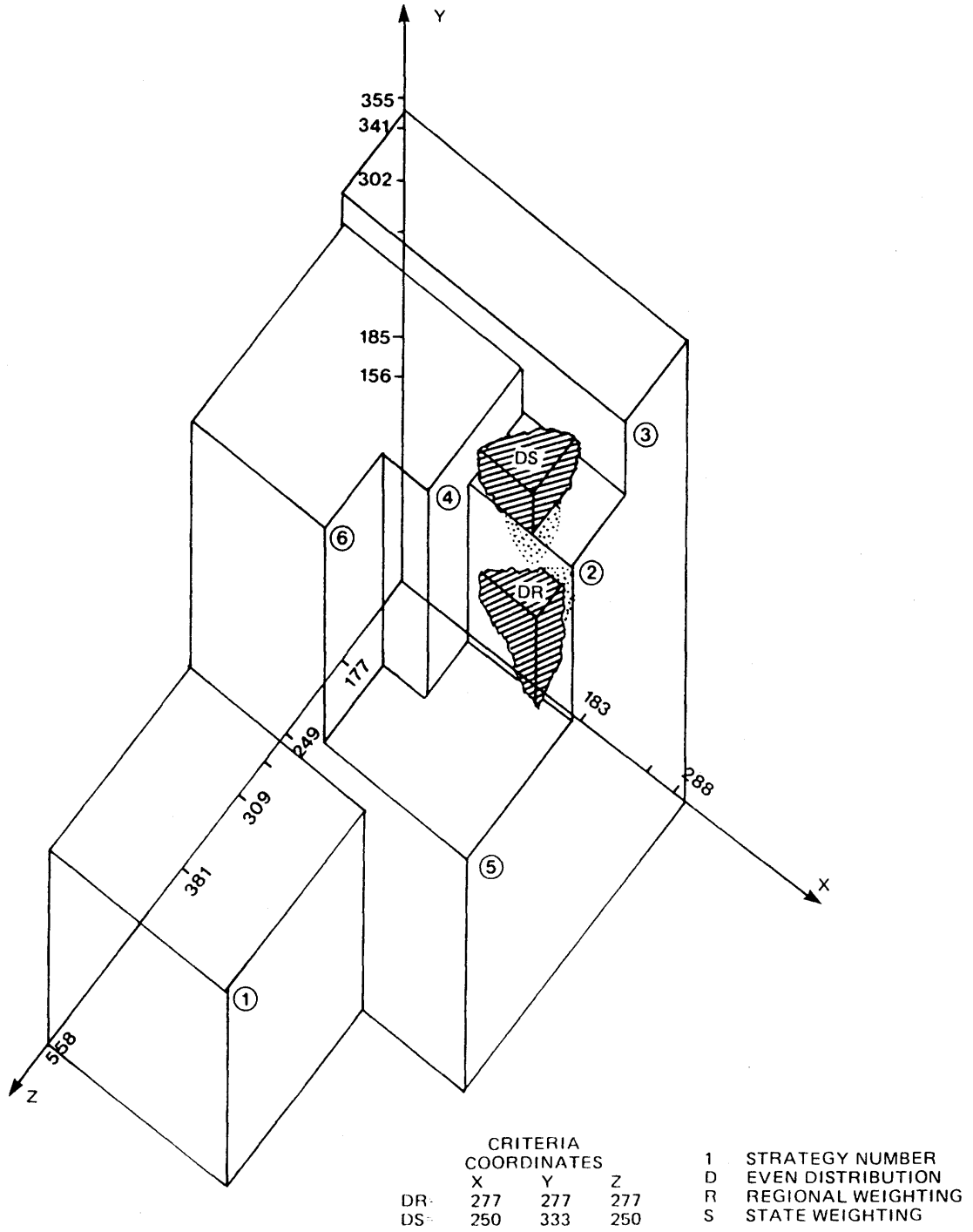
HEALTH ACCESS OBJECTIVE BY REGION - ALL STRATEGIES WITH GUIDELINE CRITERIA



|     | CRITERIA COORDINATES |     |     |                      |
|-----|----------------------|-----|-----|----------------------|
|     | X                    | Y   | Z   |                      |
| DS= | 315                  | 315 | 315 | 1 STRATEGY NUMBER    |
| IR= | 283                  | 378 | 283 | D EVEN DISTRIBUTION  |
|     |                      |     |     | R REGIONAL WEIGHTING |
|     |                      |     |     | S STATE WEIGHTING    |

Figure 23

EMPLOYMENT IMPACT OBJECTIVE BY REGION - ALL STRATEGIES WITH GUIDELINE CRITERIA



20 shows the location in the three dimension space of each of the even distribution and even investment criteria using both regional and state weighting. The remaining figures only show the two even distribution criteria as the even investment criteria is within a few percent and does not change the location materially.

The first interesting observation regarding the regionally disaggregated figures is the contrast with the aggregated ones. Whereas in the latter a minimum of 3 strategies were always interior, in this case none is interior. In fact only in the case of the health objective (Figure 22) is there an instance of one strategy lying on the face of another. Clearly strategies ST-2, ST-3, and ST-4 are quite relevant to the issue of regional balance.

This conclusion is supported by the observation that these three strategies seem normally to lie more proximate to the even distribution guideline criteria corners. Of the three strategies, ST-2 and ST-3 seem to more closely satisfy the distribution criteria.

#### 5.8 Final Recommendation to the Decision Makers

There is no reason to expect any conclusion, ranking, or final recommendation to result from this analyses. The purpose is solely to simplify the presentation of voluminous information to decision makers where no consensus exists. The best guidance the Director of Planning could give the Minister of Works would be:

- If maximization is important, select ST-1, unless health access improvement is more significant than the other objectives. If so, then chose ST-5 or ST-6 depending upon the extent it is more significant.
- If regional balance is more important than total national maximization then select ST-2 or ST-3 depending upon the trade-offs desired. If the intention is to favor Region Y, then select ST-3, unless health is very much more important, in which case ST-6 to favor Region Z, ST-4 for X.
- The best for Region Z is ST-1. Region X is indifferent to ST-2, ST-3, or ST-5. ST-3 favors Y, ST-5 favors Z, and ST-2 is intermediate but slightly towards Y.

## 5.9 Summary

Chapter Five presented a hypothetical case study requiring analysis of ten rural road projects in a multiple-objective analysis space where no decision rules were disclosed and no decision maker consensus existed. It was shown that without rules, consensus, and a common metric for aggregation, it was not possible to rank the projects definitively. However, using a series of policy objectives, it was possible to preprocess the information for presentation in tables or graphs to the decision makers for discussion. As preferences or perspectives became clearer, the presentation could be modified to emphasize or delete certain

aspects. On this iterative basis, a set of tables and graphs which supported various positions was produced to aid the decision makers.

Moreover, it was concluded that in this situation attempts to rank or compel consensus were inappropriate.

## Chapter Six

### SUMMARY AND RECOMMENDATIONS

In the preceding chapters the idea of multi-objective analysis was introduced in the context of rural road development. The need resulted from the inability of traditional economic efficiency criteria to handle the problem adequately.

In Chapter Two the range of possible objectives was discussed non-exhaustively. These included national and regional income, agricultural surplus, health, labor, and education. It was shown that no reasonable metric existed which could permit aggregation. Without aggregation a single objective utility function could not be defined, therefore the analysis required multiple utility functions. The techniques for multi-objective evaluation were presented in Chapter Three.

The issue of techniques for multi-objective evaluation was broken into two general classes. These two techniques were not redundant. One was appropriate for multiple objectives of independent metrics where the interrelation was known, i.e. there was consensus among the decision makers. Techniques were presented for defining the consensus (equal weights, cardinal, ordinal and partial ordinal) and producing a ranking. If any measure can be

developed to interrelate the various objectives, then this technique should be investigated. Chapter Four presented a hypothetical case study of 36 projects and 5 criteria. Each procedure was applied systematically to show that ranking could be achieved so long as a cardinal or ordinal relationship could be defined that represented the decision maker consensus.

The second technique presented in Chapter Three was appropriate to the context of no consensus or undisclosed decision rules. This is often the case where the decision maker/technician dialogue is poor, e.g. where decisions are on highly political, non-technical, policy objectives. In the event no consensus exists, then a general technique for data organization, manipulation, and presentation is offered. This technique does not attempt to reach conclusions; for to do so presumes consensus can be compelled. It is strongly suggested that more extensive use of "no consensus" techniques be encouraged in difficult program evaluation. It will either be helpful directly to the decision makers, or force them to some degree of consensus.

The no consensus techniques were then applied to the hypothetical case study in Chapter Five. This case study, comprising 10 projects and 5 objectives, sought to achieve an overall strategy of investment which would address a series of policy objectives. These objectives could not be



translated into discrete utility functions. A system of graphs and tables was developed which could aid the decision makers in reviewing the implications of various decisions without compelling a consensus or proposing a definitive ranking.

Optimum rural road investment decisions are normally quite difficult to achieve. Use of multiple objective techniques by technicians as part of an iterative evaluation process with decision makers can be productive in defining objectives and developing strategies for investment. Use of the techniques presented above is encouraged where appropriate.

Additional research into rural road investment and the definition of key variables and indicators will be helpful; however, political considerations will always require the analysis to be performed in a multiple objective context.

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APPENDIX A. LINEAR PROGRAMMING ORDINAL RULES

A.1 Pure Ordinal Preference

To begin, let us state the following:

- a) The number of independent criteria under consideration total  $n$ .
- b) The weights,  $w_i$ , corresponding to each criteria are not specified, but the order of their preference is specified. They are ranked as follows:

$$w_1 \geq w_2 \geq \dots \geq w_n$$

- c) The weights are normalized and non-negative:

$$\sum_{i=1}^n w_i = 1.0$$

$$w_i \geq 0 \quad (i = 1, 2, \dots, n)$$

For each project, let us determine the maximum and minimum total scores given any set of cardinal weights which obeys the stated ordinal preferences of the decision maker; we call them  $S_j^{\max}$  and  $S_j^{\min}$ , respectively, for the  $j^{\text{th}}$  project.

One way of formalizing this is to treat it as two linear programming problems, as follows:

$$\text{Maximize (Minimize) } S_j = \sum_{i=1}^n w_i u_i(x_{i,j}) \quad (\text{A-1})$$

$$\text{Subject to: } \sum_{i=1}^n w_i = 1 \quad (\text{A-2})$$

$$w_i - w_{i+1} \leq 0 \quad (i = 1, 2, \dots, n-1) \quad (\text{A-3})$$

$$w_i \geq 0 \quad (i = 1, 2, \dots, n) \quad (A-4)$$

Equation (A-3) reflects the ordering of the criteria, while Equations (A-2) and (A-4) ensure that the weights are non-negative and normalized.

By the application of two transformations:

$$Z_i = w_i - w_{i+1} \quad (i = 1, 2, \dots, n-1) \quad (A-5)$$

$$M_{i,j} = \sum_{k=1}^i u_k (x_{k,j}) \quad (i = 1, 2, \dots, n) \quad (A-6)$$

the above formulation can be simplified to:

$$\text{Maximize (Minimize) } S_j = \sum_{i=1}^n M_{i,j} Z_i \quad (A-7)$$

$$\text{subject to: } \sum_{i=1}^n i Z_i = 1 \quad (A-8)$$

$$Z_i \geq 0 \quad (i = 1, 2, \dots, n-1) \quad (A-9)$$

The problem has now been transformed to a form whereby there is only one functional constraint. The optimal solution to a linear programming problem of this form will have only one of the functional variables  $Z_i$  positive with all other functional variables equal to zero. Utilizing this it can be observed that Equation (4-5) becomes:

$$Z_i = \frac{1}{i} \quad (A-10)$$

for some  $i$ , since only one variable  $Z_i$  will be non-zero. The important result that follows is that  $S_j$  will be maximized

(minimized) according to whether  $\frac{1}{i} M_{i,j}$  is maximized (minimized). The process is thus reduced to the identification of the maximum (minimum) value from all  $\frac{1}{i} M_{i,j}$  from Equation (A-6):

$$\frac{1}{i} M_{i,j} = \sum_{k=1}^i \frac{1}{i} u_k(x_{k,j}) \quad (i = 1, 2, \dots, n) \quad (A-11)$$

From the above, we can see that the linear programming formulation produces n sets of cardinal weights, as follows:

|             |       |       |     |     |       |     |                              |        |
|-------------|-------|-------|-----|-----|-------|-----|------------------------------|--------|
| $w_{i,k} =$ | k \ i | 1     | 2   | 3   | ..... | n   |                              |        |
|             | 1     | 1     | 0   | 0   | ..... | 0   |                              |        |
|             | 2     | 1/2   | 1/2 | 0   | ..... | 0   |                              |        |
|             | 3     | 1/3   | 1/3 | 1/3 | ..... | 0   | $\frac{1}{k}$ for $i \leq k$ |        |
|             | .     | ..... |     |     |       |     |                              | (A-12) |
|             | .     | ..... |     |     |       |     | 0 for $i \geq k$             |        |
|             | n     | 1/n   | 1/n | 1/n | ..... | 1/n |                              |        |

one of which will produce the maximum total score for any given project and one which will produce the minimum. The set of weights maximizing (minimizing) one project will, however, not be the same as that maximizing (minimizing) another.

The max-ordinal rule is then found from Equations (A-1) and (A-12) as:

$$S_j^{\max} = \max_k [S_{j,k}] = \sum_{i=1}^n w_{i,k} u_i(x_{i,j}) \quad (A-13)$$

and the min-ordinal rule as:

$$S_j^{\min} = \min_k [S_{j,k}] = \sum_{i=1}^n w_{i,k} u_i(x_{i,j}) \quad (A-14)$$

The projects are then ranked according to the value of  $S_j^{\max}$  or  $S_j^{\min}$ , respectively, for the two decision rules.

## A.2 Partial Ordinal Preference

Similar to the pure ordinal preference case above, we define the following conditions:

- a) The number of independent criteria under consideration is  $n$ .
- b) The weights,  $w_i$ , are not specified, but the partial order of their preference is specified. They are ranked as follows:

$$(w_i = w_2 = \dots = w_p) \geq (w_q = w_r = \dots = w_s) \geq \dots \\ \geq (w_t = w_u \dots = w_n)$$

That is, a preferential ordering is given between subsets of criteria, with criteria within a subset being of equal weight.

- c) The total number of subsets,  $T_k$ , is  $m$ .
- d) The number of criteria in each  $T_k$  subset is given by  $n_k$ .
- e) The weights are normalized:

$$\sum_{i=1}^n w_i = 1.$$

We now define the following linear programming problems:

$$\text{Maximize (minimize) } S_j = \sum_{i=1}^n w_i u_i(x_{i,j}) \quad (\text{A-15})$$

$$\text{subject to: } \sum_{i=1}^n w_i = 1$$

$$w_i - w_{i+1} \geq 0 \quad \text{if } i \in T_k \text{ and } i+1 \notin T_k \text{ for some } k$$

$$w_i - w_{i+1} = 0 \quad \text{if } i, i+1 \in T_k \text{ for some } k \quad (\text{A-16})$$

$$w_i \geq 0 \quad (i = 1, 2, \dots, n) \quad (\text{A-17})$$

Equation (A-16) states that weights in the same subset are equal, while those in different subsets are ordered.

Applying our same two transformations (A-5) and (A-6), we have:

$$\text{Maximize (Minimize) } S_j = \sum_{i=1}^n M_{i,j} Z_i \quad (\text{A-18})$$

$$\text{subject to: } \sum_{i=1}^n i Z_i = 1 \quad (\text{A-19})$$

$$Z_i \geq 0 \quad \text{if } i \in T_k \text{ and } i+1 \notin T_k \text{ for some } k$$

$$Z_i = 0 \quad \text{if } i, i+1 \in T_k \text{ for some } k \quad (\text{A-20})$$

This is the same formulation as (A-10) above, except for the restriction that  $Z_i$  can be non-zero only for  $i$ 's representing the last criterion<sup>i</sup> in a subset. Thus, for example, if  $T_1$  includes the first three criteria, then  $Z_1 = Z_2 = 0$  and  $Z_3 = 1/3$ , since the 3rd and 4th criteria are in different subsets.

We have the following set of  $k$  cardinal weights, as follows:



| k \ i | 1   | 2   | 3     | ..... | n   |                              |
|-------|---|---|-------|-------|-----|------------------------------|
| 1     | 1/n <sub>1</sub>                                    |   |       | ..... | 0   |                              |
| 2     | 1/(n <sub>1</sub> +n <sub>2</sub> )                 | 1/(n <sub>1</sub> +n <sub>2</sub> )                 |       | ..... | 0   | 1                            |
| 3     | 1/(n <sub>1</sub> +n <sub>2</sub> +n <sub>3</sub> ) | 1/(n <sub>1</sub> +n <sub>2</sub> +n <sub>3</sub> ) | ..... |       | 0   | $\frac{1}{\sum_{j=1}^k n_k}$ |
| m     | 1/n   | 1/n   | 1/n   | ..... | 1/n | $\frac{1}{\sum_{j=1}^k n_k}$ |

$i < \sum_{j=1}^k n_k$   
 $i > \sum_{j=1}^k n_k$

and Equations (A-13) and (A-14) provide the max- and min-ordinal decision rules.

### A.3 Mixed Cardinal and Ordinal Preference

If the decision maker specifies a mixture of cardinal and ordinal preferences, providing the ordinal information in (A-3) or the partial ordinal information in (A-16) plus cardinal weights for a subset C of the criteria containing m elements, i.e.,

$$w_i = c_i \quad \text{if } i \in C \quad (A-22)$$

then we may use the following linear programming problems to compute the upper and lower bound project scores:

$$\text{Maximize (Minimize) } S_j = \sum_{i=1}^n w_i u_i(x_{i,j}) \quad (A-23)$$

$$\text{subject to: } \sum_{i=1}^n w_i = 1 \quad (A-24)$$

$$w_i - w_{i+1} \geq 0 \quad (i = 1, 2, \dots, n-1) \quad (A-25)$$

$$w_i \geq 0 \quad (A-26)$$

$$w_i = c_i \quad \text{if } i \in C \quad (A-27)$$

for the pure ordinal mixed case, with constraint (A-25) replaced by:

$$w_i - w_{i+1} \geq 0 \quad \text{if } i \in T_k \text{ and } i+1 \in T_k \text{ for some } k$$

$$w_i - w_{i+1} = 0 \quad \text{if } i, i+1 \in T_k \text{ for some } k \quad (\text{A-28})$$

for the partial ordinal mixed case. While these problems can be reduced to closed form solutions, the algebra is tedious and adds little to the understanding of the min-ordinal and max-ordinal decision rules.

## APPENDIX B. THE MODE-ORDINAL RULE

### B.1 Pure Ordinal Preference

The mode-ordinal decision rule is based upon determining the "most-likely" set of cardinal weights obeying a specified ordinal preference. We define the following:

- a) The number of independent criteria under consideration is  $n$ ;
- b) The weights,  $W_i$ , corresponding to each criteria are not specified, but the order of their preference is as follows:

$$W_1 \geq W_2 \geq \dots \geq W_n$$

- c) The weights are non-negative and normalized

$$W_i \geq 0 \quad (i = 1, 2, \dots, n)$$

$$\sum_{i=1}^n W_i = 1$$

The "most likely" set of weights is that set which has the greatest probability of occurrence. In an equivalent formulation, the most likely first weight, is that which provides maximum flexibility for the values of the remaining weights. We will use the latter line of reasoning in this development.

We pose the question, give  $n$  ordered criteria, what is the most-likely value of the weight on the first criteria. Clearly if  $n=2$ , the question has no answer, since all weights of criteria one greater than or equal to one-half are equally likely. What if  $n=3$ ? The most-likely value for  $W_1$  is determined from the following integrals:

$$N_3(x \geq 1/2) = N_3^+(x) = \int_{\frac{1-x}{2}}^{1-x} \int_0^y \int_0^{1-x-y} dz \quad (B-1)$$

$$N_3(x \leq 1/2) = N_3^-(x) = \int_0^x \int_0^y \int_0^{1-x-y} dz \quad (B-2)$$

where  $x$  is the value of  $W_1$

$y$  is the value of  $W_2$

$z$  is the value of  $W_3$

and  $N_n$  is the "number" of sets of weights having  $W_1 = x$  and  $n$  criteria

If  $x \geq 1/2$  the second criterion,  $y$ , can range between  $1-x$  (all the remaining weight) or  $(1-x)/2$  (half the remaining weight, since it cannot be less than the weight on criterion 3). Criterion 3 has a weight between that of criterion 2 and the remaining unassigned weight,  $1-x-y$ . These integrals evaluate as follows:

$$N_3^+(x) = \frac{1}{2} \left( \frac{1-x}{2} \right)^2 \quad (B-3)$$

$$N_3^-(x) = \frac{1}{2} \left( \frac{3x-1}{2} \right)^2$$

Similar integrals for greater numbers of weights are evaluated as follows:

$$N_4^+(x) = 3 \left( \frac{1-x}{3} \right)^3 \quad (B-4)$$

$$N_4^-(x) = \frac{3}{8} \left( \frac{4x-1}{3} \right)^3$$

$$N_5^-(x) = 3^2 \cdot 2 \left(\frac{1-x}{4}\right)^4 \tag{B-5}$$

$$N_5^-(x) = \frac{2}{9} \left(\frac{5x-1}{4}\right)^4$$

$$N_{n+1}^+(x) = \left[ \left( \int N_n^-(y) dy \right) * \{1-x\} \right] \left| \begin{array}{l} 1-x \\ \frac{1-x}{n} \end{array} \right. \tag{B-6}$$

$$N_{n+1}^-(x) = \left[ \left( \int N_n^-(y) dy \right) * \{1-x\} \right] \left| \begin{array}{l} x \\ \frac{1-x}{n} \end{array} \right.$$

where \* denotes multiplication of each power,  $y^k$ , resulting from the integration by its binomial counterpart power of  $(1-x)$ ; i.e.,  $y^k \cdot (1-x)^{n-k}$ . That is, for  $N_4^+$  as an example:

$$N_3^-(y) = \frac{1}{8} (1-6y+9y^2)$$

$$\int N_3^-(y) dy = \frac{1}{8} (y-3y^2+3y^3) \tag{B-7}$$

$$\left( \int N_3^-(y) dy \right) * \{1-x\} = \frac{1}{8} [y(1-x)^2 - 3y^2(1-x) + 3y^3] = A$$

$$\text{so that } N_4^+(x) = A \left| \begin{array}{l} 1-x \\ \frac{1-x}{3} \end{array} \right.$$

$$= \frac{1}{8} ((1-x)^3 - 3(1-x)^3 + 3(1-x)^3) - \frac{1}{8} \left( \frac{1}{3}(1-x)^3 - \frac{3}{9}(1-x)^3 + \frac{3}{27}(1-x)^3 \right)$$

$$= \frac{1}{8}(1-x)^3 - \frac{1}{8} \left( \frac{1}{9}(1-x)^3 \right)$$

(B-8)

$$= \frac{1}{8} \left( \frac{8}{9}(1-x)^3 \right) = \frac{1}{9}(1-x)^3$$

Similarly for the other values of N.

From Equation (B-6), we can determine the values of  $N_n$  for any value of n, as follows:

$$N_n^+(x) = \frac{(n-2)^{n-1} (1-x)^{n-1}}{2 [(n-1)!]^2} \quad (B-9)$$

$$N_n^-(x) = \frac{(nx-1)^{n-1}}{2 [(n-1)!]^2}$$

From this, it is quite easy to see that the mode, i.e., the largest value of  $N_n$  indicating the greatest number of distributions having a weight of x on the first criterion, occurs for  $x = \frac{1}{2}$  for all  $n > 2$ . That is, the most likely weight on the first criterion is

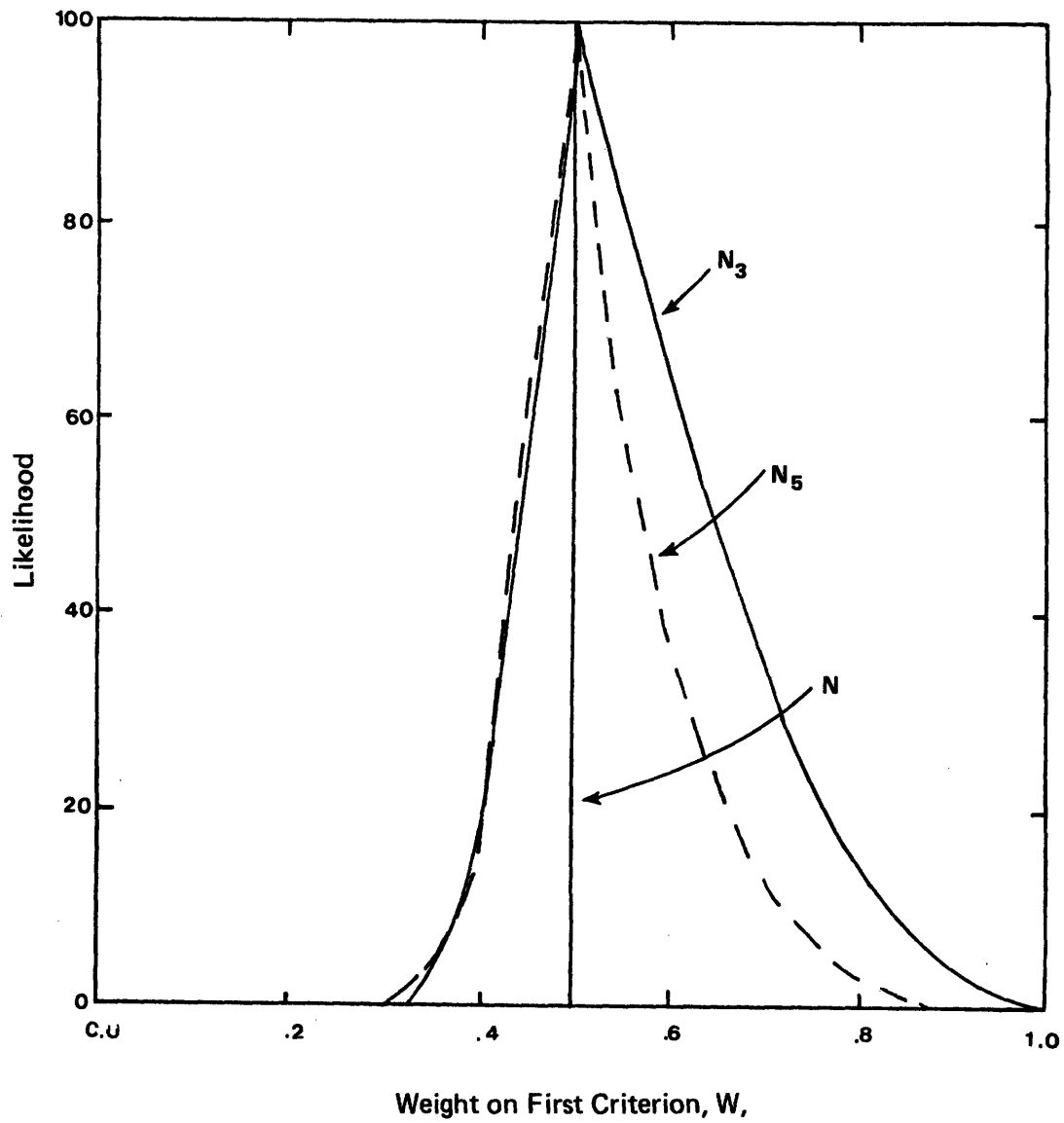
$$W_1^{\text{mode}} = \frac{1}{2} \quad (B-10)$$

The values of  $N_n^+$  and  $N_n^-$  are shown in Figure B-1 for  $n = 3, 5,$  and  $\infty$ .

This result should not be surprising. To grasp it intuitively, let us investigate the effects of  $W_1$  on the range of values left for the second criterion only. The range of  $W_1$  is constrained between 1 and  $1/n$  if there are n criterion, due to the ordering. Similarly, the second criterion is constrained between  $1/2$  and 0 independently of the value of  $W_1$ . Thus, if  $W_1 > 1/2$ , the range of values for  $W_2$  has been further constrained; since  $W_2 \leq 1 - W_1 \leq 1/2$ . Similarly if  $W_1 < 1/2$ , since then  $W_2 \leq W_1 \leq 1/2$ . Thus, any value for  $W_1$  not equal to  $1/2$  constrains the choices of the second criterion. This effect "trickles down" through all subsequent criteria, the extent of the effects being determined by how much  $W_1$  deviates from  $1/2$  and the number of criteria, as can be seen from Figure B-1.

Now that we have determined the most likely weight for the first criterion, we can remove it from our problem. We now have  $n-1$  ordered criteria whose weights must sum to  $1/2$ . Clearly, this will produce a value of  $1/4$  for the weight on the second criteria, and so on.

Figure B - 1  
THE LIKELIHOOD OF VARIOUS WEIGHTS ON THE FIRST CRITERION,  $W_1$ ,  
WITH THE MODE ASSIGNED A LIKELIHOOD OF 100,  
FOR VARIOUS NUMBERS OF CRITERIA



We now have the following cardinal weights as the most-likely given the ordinal preference:

$$W_i = 2^{-i} \quad (B-11)$$

These would be normalized if we had an infinite number of criteria; however, given a finite number of criteria  $n$ , the last two weights,  $W_{n-1}$  and  $W_n$  are undefined. We have the following choices to ensure normality:

$$W_i = 2^{-i} / \sum_{i=1}^n 2^{-i} \quad (i = 1, 2, \dots, n) \quad (B-12)$$

or

$$W_i = 2^{-i} \quad (i = 1, 2, \dots, n-1) \quad (B-13)$$

$$W_n = W_{n-1}$$

For large number of criteria the two become equivalent; for the 5-criteria case used as a case study, the two sets of weights would be:

$$\text{from (B-12) } W_i \Big|_{n=5} = (.5161, .2581, .1290, .0645, .0323) \quad (B-14)$$

$$\text{from (B-13) } W_i \Big|_{n=5} = (.5000, .2500, .1250, .0625, .0625)$$

We have opted for the latter (B-13) interpretation due to its simplicity.



## B.2 Partial Ordinal Preference

If a partial ordinal preference is specified, in that an order of preference is given for  $k$  subsets  $\{S_k\}$  of the criteria within each of which the  $n_k$  criteria are of equal importance, we would produce the most-likely set of weights as follows. Consider first the first subset  $\{S_1\}$  containing  $n_1$  criteria. From Equations (B-3), we would have that the most-likely weight on these criteria would be

$$x_1 = \text{Max} \left[ \frac{1}{2n_1}, \frac{1}{n} \right] \quad (\text{B-15})$$

This leaves  $1-n_1x_1$  as the total weight on the remaining criteria. For the second subset of criteria, we would have:

$$x_2 = \text{Max} \left[ \frac{1-n_1x_1}{2n_2}, \frac{1-n_1x_1}{n-n_1} \right] \quad (\text{B-16})$$

subject to  $x_2 \leq x_1$

That is,

$$x_2 = \text{Min} \left[ x_1, \text{Max} \left[ \frac{1-n_1x_1}{2n_2}, \frac{1-n_1x_1}{n-n_1} \right] \right] \quad (\text{B-17})$$

For example, if the first subset contains three criteria and the second only one,  $x_1 = 1/6$  and  $x_2 = 1/4 > x_1$  so the first four weights would all be  $1/6$ , if there were a very large number of criteria.

This leaves  $1-n_1x_1 - n_2x_2$  as the total weight on the remaining criteria. For the third subset, we would have:

$$x_3 = \text{Min} \left[ x_2, \text{Max} \left[ \frac{1-n_1x_1 - n_2x_2}{2n_3}, \frac{1-n_1x_1 - n_2x_2}{n-n_1-n_2} \right] \right] \quad (\text{B-18})$$

and so on. Thus, we would have the following set of most-likely weights for the partial-ordinal decision rule:

$$\begin{aligned}
 W_1 &= \text{Max} \left[ \frac{1}{2n_1}, \frac{1}{n} \right] \\
 W_i &= \text{Min} \left[ W_{i-1}, \text{Max} \left[ \frac{1 - \sum_{j=1}^{i-1} n_j W_j}{2n_i}, \frac{1 - \sum_{j=1}^{i-1} n_j W_j}{n - \sum_{j=1}^{i-1} n_j} \right] \right] \quad (i=2, 3, \dots, k-1)
 \end{aligned}
 \tag{B-19}$$

$$W_k = W_{k-1}$$

and  $W_i = W_j$  if  $i \in \{S_j\}$

It should be noted that this rule is much "weaker" than the mode-ordinal decision rule, in that it is closer to the equal weights case. It is not recommended as an evaluation tool.

### B.3 Mixed Cardinal and Ordinal Preferences

While this rule could be algebraically developed, it adds little to the understanding of the approach and is not recommended as a decision rule. The linear programming approaches are to be preferred in such cases.

### B.4 Mean-Ordinal Rule

A final rule which could be used, but which was not in the present study, would be the expected cardinal weights given the ordinal preference. Based upon relations (B-9), this could be formulated as follows for  $n$  criteria:

$$W_1 = \frac{n^3 + n - 2}{2n(n+1)(n-1)}$$

$$W_i = \left(1 - \sum_{j=1}^{i-1} W_j\right) \frac{(n+1-i)^3 + (n-1-i)}{2(n+1-i)(n+2-i)(n-i)} \quad (i=2, 3, \dots, n-1) \quad (\text{B-20})$$

$$W_n = 1 - \sum_{j=1}^{n-1} W_j$$

For the five criteria case, this would yield:

$$W_i = (.533, .2567, .1225, .0583, .0292)$$

which is sufficiently close to the mode-ordinal weights that it was deemed redundant. For very large  $n$ , the two become identical.

## APPENDIX C. DATA FOR THE CASE STUDY\*

### C.1 A Detailed Description of One Project

In this section a detailed description of one of the projects, the Makir-Aros feeder road project (Figure C-1), is presented. A brief description of the overall set of projects to be ranked and some of the differences among the projects is given in Section C.2.

The proposed 20 kilometer feeder road project extends out from a small provincial market town of Makir. The community of Aros served by the road has suitable conditions (physical, ecological, demographic, etc.) for agricultural development. At present a trail (earth road not passable by motor vehicles) exists which is mainly used for walking or transport by pack animals to the market town where the peasants periodically come to sell some of their agricultural surplus or to buy some consumer goods (cloth, spices, etc.).

As part of a regional development effort, this project has been identified and a proposed package of investment projects has been advocated by the design team, consisting of a gravel road, extension services and help to grow some

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\*This is a hypothetical case study initiated by Chew in 1977, published in 1979, and further developed by Brademeyer in an unpublished paper (1980).

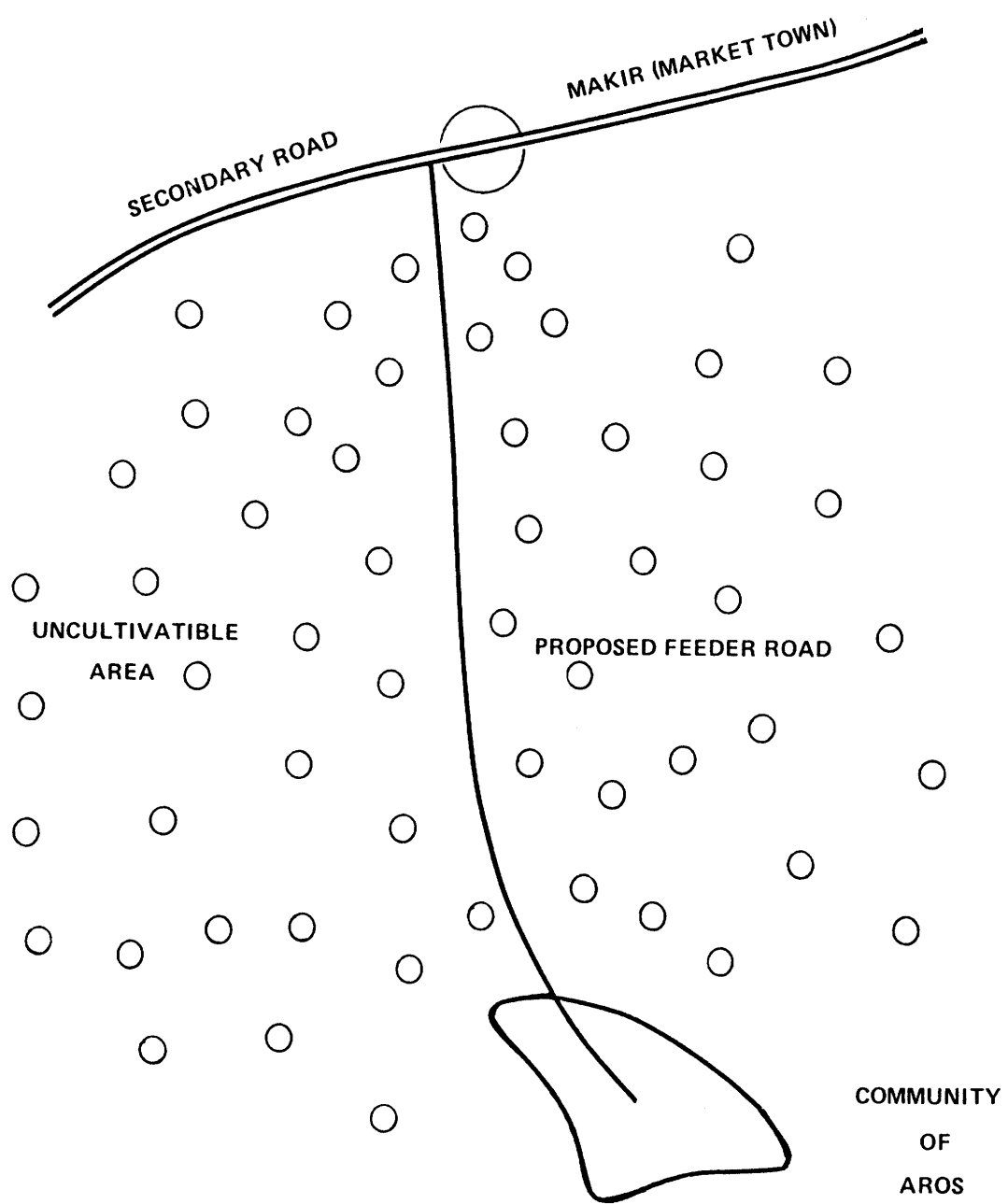


FIGURE C-1: ILLUSTRATION OF THE PROJECT AREA

new crops, a new health clinic in the community of Aros, and provision of some general education for both adults and children.

#### Land Ownership Pattern

The community of Aros has 109 families, and the population totals some 600 persons. In a survey of the area, the population was divided into three different groups using land ownership characteristics. There are 5 relatively rich families who own between 45 to 50 hectares of land per family. A second group, consisting of 34 families, is characterized by the ownership of 2 to 10 hectares of land per family. The third group, totaling some 70 families, is characterized by being landless. They can be further subdivided into two groups. One group, numbering 50 families, rents a total of about 100 hectares from the relatively richer families for subsistence farming. They pay the landlords with some of their agricultural produce and occasionally with some services. The other 20 families are directly under the "umbrella" of the relatively richer group and do not farm any land of their own. They work for the relatively richer families and in return receive some subsistence wage, mostly in kind.

#### Agricultural Activities Before the Project

Presently the agricultural production consists mostly of cassava, rice and maize, with a bit of livestock. The relatively richer families and some of the small land owners

have some agricultural surplus which they carry by pack animals or even on their heads to the market town of Makir, which is served by a good secondary road.

Table C-1 shows the quantity of land under the 3 crops, the average yields and the market value of the present agricultural production. Figure C-2 summarizes the present distribution and type of agricultural activity.

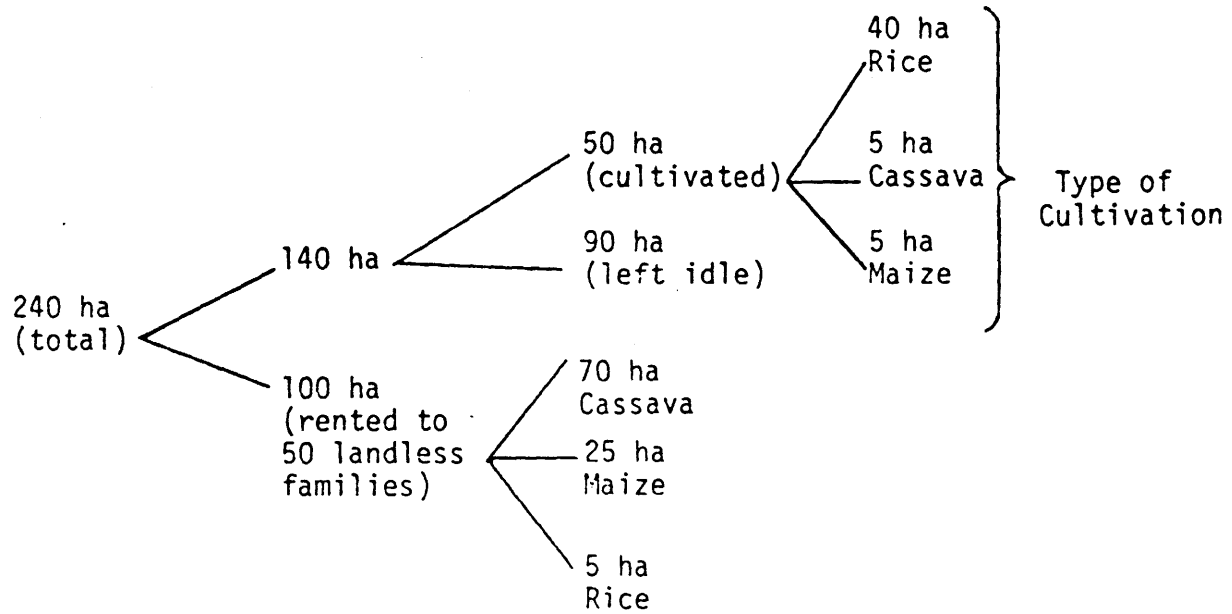
Table C-1  
PRESENT AGRICULTURAL ACTIVITIES

| <u>Crop</u> | <u>Hectares</u> | <u>Yield/<br/>Hectare</u> | <u>Market<br/>Price</u> | <u>Value</u>     |
|-------------|-----------------|---------------------------|-------------------------|------------------|
| Cassava     | 250             | 2,000 lb.                 | \$ 0.25/lb.             | \$125,000        |
| Rice        | 80              | 3,250 lb.                 | \$ 0.50/lb.             | \$130,000        |
| Maize       | <u>75</u>       | 2,020 lb.                 | \$ 0.70/lb.             | <u>\$106,050</u> |
| Total       | 405             |                           |                         | \$361,050        |

Agricultural Activities Planned for the Project

It is planned to introduce new varieties of seeds and the use of fertilizers for the crops of cassava, maize and rice. The proper cultivation of the new varieties with the essential assistance and direction of the proposed extension staff will result in a substantial increase in yield.

GROUP 1  
(families owning  
between 50 to 60  
hectares each)



GROUP 2  
(families owning  
between 2 to 10  
hectares each)

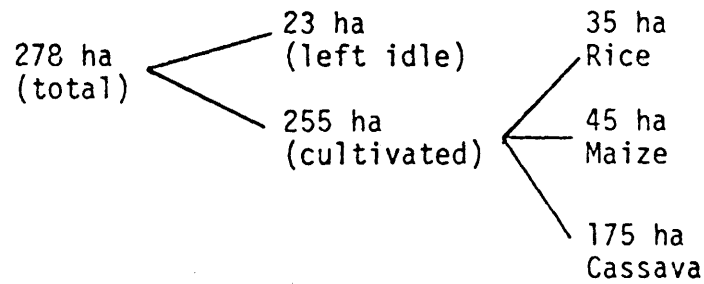


FIGURE C-2: SUMMARY OF PRESENT DISTRIBUTION AND TYPE OF AGRICULTURAL ACTIVITY



There is at present a total of 113 hectares of cultivable land which is presently uncultivated and idle. The proposed plan is to induce the owners of this land to plant cash crops of cocoa which is highly suitable to the local climatic and soil conditions, highly profitable, and commands a good price with high demand as an export commodity.

Adjacent to the community is a parcel of government land which is covered with secondary forests, but which has suitable soil for the cultivation of cocoa. It is planned to clear this parcel to enable 70 landless families to cultivate one hectare each, with the necessary credit (payable in easy installments) provided to them.

#### The Appraisal Report

The benefits accruing to the criteria under consideration are discussed under the stipulation that the contributions are as a result of the provision of the feeder road. Each of them will be elaborated in turn.

#### Economic Benefits Measure

The measure of the economic benefits is the difference in the present value of the expected value of the agricultural activity with implementation of the project with that of the no-project alternative. The technique that will be used to predict the agricultural activity is the assessment of the project by an interdisciplinary appraisal team.

Table C-2 shows the results of the assessment of the appraisal team of the probability that new cultivation will

TABLE C-2  
ASSESSMENT OF THE APPRAISAL TEAM ON THE  
PROBABILITY OF NEW CULTIVATION

| Type of Crop | Rice |     |     | Cocoa |     |     |     | Cassava |     |     | Maize |     |     |
|--------------|------|-----|-----|-------|-----|-----|-----|---------|-----|-----|-------|-----|-----|
| Group        | 1    | 2   | 3a  | 1     | 2   | 3a  | 3b  | 1       | 2   | 3a  | 1     | 2   | 3a  |
| Year         |      |     |     |       |     |     |     |         |     |     |       |     |     |
| 0            | 0    | 0   | 0   | 0     | 0   | 0   | 0   | 0       | 0   | 0   | 0     | 0   | 0   |
| 1            | 0.8  | 0.6 | 0.5 | .9    | .6  | .7  | .4  | 1.0     | .3  | .2  | 1.0   | .3  | .2  |
| 2            | 0.9  | 0.6 | 0.5 | .9    | .6  | .7  | .4  | 1.0     | .3  | .2  | 1.0   | .3  | .3  |
| 3            | 0.9  | 0.7 | 0.5 | 1.0   | .6  | .8  | .5  | 1.0     | .4  | .3  | 1.0   | .4  | .3  |
| 4            | 0.9  | 0.7 | 0.6 | 1.0   | .8  | .8  | .5  | 1.0     | .4  | .3  | 1.0   | .4  | .4  |
| 5            | 1.0  | 0.8 | 0.6 | 1.0   | .9  | .9  | .5  | 1.0     | .5  | .4  | 1.0   | .5  | .5  |
| 6            | 1.0  | 0.8 | 0.6 | 1.0   | 1.0 | 1.0 | .6  | 1.0     | .8  | .5  | 1.0   | .8  | .7  |
| 7            | 1.0  | 0.8 | 0.8 | 1.0   | 1.0 | 1.0 | .7  | 1.0     | .9  | .7  | 1.0   | .9  | .8  |
| 8            | 1.0  | 0.9 | 0.9 | 1.0   | 1.0 | 1.0 | .8  | 1.0     | 1.0 | .8  | 1.0   | 1.0 | 1.0 |
| 9            | 1.0  | 1.0 | 1.0 | 1.0   | 1.0 | 1.0 | .9  | 1.0     | 1.0 | 1.0 | 1.0   | 1.0 | 1.0 |
| 10           | 1.0  | 1.0 | 1.0 | 1.0   | 1.0 | 1.0 | 1.0 | 1.0     | 1.0 | 1.0 | 1.0   | 1.0 | 1.0 |
| 11           | 1.0  | 1.0 | 1.0 | 1.0   | 1.0 | 1.0 | 1.0 | 1.0     | 1.0 | 1.0 | 1.0   | 1.0 | 1.0 |
| 12           | 1.0  | 1.0 | 1.0 | 1.0   | 1.0 | 1.0 | 1.0 | 1.0     | 1.0 | 1.0 | 1.0   | 1.0 | 1.0 |

be carried out by the various groups for the four crops over the project life. The expected average yields per hectare for each of the crops are detailed in Table C-3. The expected value of agricultural production with the implementation of the project is detailed in Table C-4. The figures are obtained from calculations that are illustrated in Tables C-5, C-6, C-7, and C-8 for each particular crop and year. Table C-9 details the calculation of the expected value of agricultural production without the project. The measure that comes out from this analysis is the net expected present value of agricultural activity:

$$\begin{aligned}
 \text{NEPV} &= \text{Present value of agriculture production with the project} & - & \text{Present value of agriculture production without the project} \\
 &= \$ 7,603,868 - \$ 3,258,190^* \\
 &= \$ 4,345,678
 \end{aligned}$$

#### Economic Costs Measure

Table C-10 shows a summary of the stream of expenditures that make up the project costs. Briefly, the items of expenditure include all items related to the construction and maintenance of the road, the costs of building and staffing the health clinic, the costs of the provision of

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\*These are discounted at 8 percent.

Table C-3

EXPECTED AVERAGE YIELDS PER HECTARE

| <u>Year</u> | <u>Cassava (lb)</u> | <u>Rice (lb)</u> | <u>Cocoa (lb)</u> | <u>Maize (lb)</u> |
|-------------|---------------------|------------------|-------------------|-------------------|
| 0           | 2,000               | 3,250            | 0                 | 2,020             |
| 1           | 2,200               | 3,500            | 0                 | 2,200             |
| 2           | 2,400               | 4,000            | 0                 | 2,400             |
| 3           | 2,600               | 5,000            | 0                 | 2,600             |
| 4           | 2,800               | 5,500            | 2,800             | 2,600             |
| 5           | 3,000               | 5,750            | 2,800             | 2,600             |
| 6           | 3,300               | 5,750            | 3,000             | 2,800             |
| 7           | 3,500               | 6,000            | 3,000             | 3,000             |
| 8           | 3,500               | 6,000            | 3,000             | 3,200             |
| 9           | 3,500               | 6,000            | 3,000             | 3,200             |
| 10          | 3,500               | 6,000            | 3,000             | 3,200             |
| 11          | 3,500               | 6,000            | 3,000             | 3,200             |
| 12          | 3,500               | 6,000            | 3,000             | 3,200             |

Table C-4

EXPECTED VALUE OF AGRICULTURAL PRODUCTION WITH THE PROJECT

| <u>Year</u> | <u>Rice</u> | <u>Cocoa</u> | <u>Cassava</u> | <u>Maize</u> |
|-------------|-------------|--------------|----------------|--------------|
| 0           | 130,000     | 0            | 125,000        | 106,050      |
| 1           | 136,938     | 0            | 128,575        | 109,011      |
| 2           | 152,313     | 0            | 132,150        | 112,966      |
| 3           | 185,125     | 0            | 139,400        | 118,433      |
| 4           | 201,438     | 559,468      | 144,200        | 119,448      |
| 5           | 218,750     | 559,468      | 155,125        | 122,290      |
| 6           | 218,750     | 669,030      | 183,500        | 137,991      |
| 7           | 229,000     | 689,040      | 204,313        | 150,983      |
| 8           | 234,500     | 720,795      | 213,500        | 168,000      |
| 9           | 240,000     | 761,250      | 218,750        | 168,000      |
| 10          | 240,000     | 769,950      | 218,750        | 168,000      |
| 11          | 240,000     | 778,650      | 218,750        | 168,000      |
| 12          | 240,000     | 787,350      | 218,750        | 168,000      |

Present value at 8 percent = \$7,603,868

TABLE C-5

ASSESSMENT OF AGRICULTURAL PRODUCTION

TYPE OF CROP: RICE

YEAR: 4

| Type of Group | Cultivable Areas<br>A | "Old"* Average Yield<br>Y <sub>0</sub> | "New"** Average Yield<br>Y <sub>1</sub> | Predicted Average Unit<br>Market Price<br>P <sub>1</sub> | FROR*** to Farmer | Demand Conditions | Other Relevant<br>Parameters   | Subjective**** Assess-<br>ment of PR(cultivation)<br>Pr | Expected Value<br>[Pr x P <sub>1</sub> x Y <sub>1</sub> x A +<br>(1 - Pr) x P <sub>1</sub> x Y <sub>0</sub> x A] |
|---------------|-----------------------|--|---|--|-------------------|-------------------|--|---|--|
| 1             | 40                    | 3250<br>lb/ha.                         | 5500<br>lb/ha.                          | \$ 0.5   | 50%               | Excellent         | Survey indi-<br>cates good<br>response   | 0.9   | \$ 105,500   |
| 2             | 35                    | 3250<br>lb/ha.                         | 5500<br>lb/ha.                          | \$ 0.5   | 50%               | Excellent         | From survey it<br>seems some<br>Farmers exhib-<br>it risk aver-<br>sion to the<br>changeover | 0.7   | \$ 84,438  |
| 3a            | 5                     | 3250<br>lb/ha.                         | 5500<br>lb/ha.                          | \$ 0.5   | 50%               | Excellent         | High risk<br>aversion to<br>change in new<br>seed varieties                                  | 0.6   | \$ 11,500  |

\*From Table C-1    \*\*From Table C-3    \*\*\*Financial rate of return    \*\*\*\*From Table C-2

TABLE C-6

ASSESSMENT OF AGRICULTURAL PRODUCTION

TYPE OF CROP: COCOA

YEAR: 4

| Type of Group | Cultivable Areas<br>A | "Old" Average Yield<br>Y <sub>0</sub> | "New" Average Yield<br>Y <sub>1</sub> | Predicted Average Market Price<br>P <sub>1</sub> | FROR*** to Farmer | Demand Conditions  | Other Relevant Parameters  | Subjective*** Assessment of Pr(cultivation) | Expected Value<br>[Pr x P <sub>1</sub> x Y <sub>1</sub> x A + (1 - Pr) x P <sub>0</sub> x Y <sub>0</sub> x A] |
|---------------|-----------------------|---------------------------------------|---------------------------------------|--|-------------------|--|--|---|---|
| 1             | 90                    |                                       | 2800 lb/ha.                           | \$ 1.45  | 60%               | Excellent, the Cocoa can be sold to buyers in Makir and there is an excellent demand for it as an export commodity | There is a gestation period of four years before benefits are realized. This is a new crop | 0.9   | \$ 328,860  |
| 2             | 23                    |                                       | 2800 lb/ha.                           | \$ 1.45  | 60%               |  |  | 0.6   | \$ 56,028   |
| 3a            | 50                    |                                       | 2800 lb/ha.                           | \$ 1.45  | 60%               |  |  | 0.7   | \$ 142,100  |
| 3b            | 20                    |                                       | 2800 lb/ha.                           | \$ 1.45  | 60%               |  |  | 0.4   | \$ 32,480   |

\*From Table C-1    \*\*From Table C-3    \*\*\*Financial rate of return    \*\*\*\*From Table C-2

TABLE C-7

ASSESSMENT OF AGRICULTURAL PRODUCTION

TYPE OF CROP: CASSAVA

YEAR: 4

| Type of Group | Cultivable Areas<br>A | "Old"* Average Yield<br>Y <sub>0</sub> | "New"** Average Yield<br>Y <sub>1</sub> | Predicted Average Market Price<br>P <sub>1</sub> | FROR*** to Farmer | Demand Conditions | Other Relevant Parameters   | Subjective**** Assessment of PR(cultivation)<br>Pr | Expected Value<br>[Pr x P <sub>1</sub> x Y <sub>1</sub> x A + (1 - Pr) x P <sub>1</sub> x Y <sub>0</sub> x A] |
|---------------|-----------------------|--|---|--|-------------------|-------------------|---|--|---|
| 1             | 5                     | 2000 lb/ha.                            | 3500 lb/ha.                             | \$ 0.25  | 45%               | Excellent         | By now the farmers have realized that the introduction of the new inputs are very profitable as demonstrated by the entrepreneurship of the richer farmers. | 1.0  | \$ 4,375  |
| 2             | 175                   | 2000 lb/ha.                            | 3500 lb/ha.                             | \$ 0.25  | 45%               | Excellent         |   | 0.9  | \$ 146,563  |
| 3a            | 70                    | 2000 lb/ha.                            | 3500 lb/ha.                             | \$ 0.25  | 45%               | Excellent         |   | 0.7  | \$ 53,375   |

\*From Table C-1

\*\*From Table C-3

\*\*\*Financial rate of return

\*\*\*\*From Table C-2



TABLE C-8

ASSESSMENT OF AGRICULTURAL PRODUCTION

TYPE OF CROP: MAIZE

YEAR: 1

58T

| Type of Group | Cultivable Areas<br>A | "Old"* Average Yield<br>Y <sub>0</sub> | "New"** Average Yield<br>Y <sub>1</sub> | Predicted Average Unit<br>Market Price<br>P <sub>1</sub> | FROR*** to Farmer | Demand Conditions   | Other Relevant<br>Parameters | Subjective**** Assess-<br>ment of PR(cultivation)<br>Pr | Expected Value<br>[Pr x P <sub>1</sub> x Y <sub>1</sub> x A ÷<br>(1 - Pr) x P <sub>1</sub> x Y <sub>0</sub> x A] |
|---------------|-----------------------|--|---|--|-------------------|---|------------------------------|---|--|
| 1             | 5                     | 2020<br>lb/ha.                         | 2200<br>lb/ha.                          | \$ 0.7   | 32%               | Good price exists for maize and demand for it is good. Only the Group 1 Farmer will "gamble" on the new cultivation while the majority of the other 2 groups will not dare take the risk. |                              | 1.0   | \$ 7,700   |
| 2             | 45                    | 2020<br>lb/ha.                         | 2200<br>lb/ha.                          | \$ 0.7   | 32%               |   |                              | 0.3   | \$ 68,331  |
| 3a            | 25                    | 2020<br>lb/ha.                         | 2200<br>lb/ha.                          | \$ 0.7   | 32%               |   |                              | 0.2   | \$ 35,980  |

\*From Table C-1    \*\*From Table C-3    \*\*\*Financial rate of return    \*\*\*\*From Table C-2

Table C-9

EXPECTED VALUE OF AGRICULTURAL PRODUCTION  
WITHOUT THE PROJECT

| <u>Year</u> | <u>Rice</u> | <u>Cassava</u> | <u>Maize</u> |
|-------------|-------------|----------------|--------------|
| 0           | 130,000     | 125,000        | 106,050      |
| 1           | 130,000     | 125,000        | 106,050      |
| 2           | 130,000     | 125,000        | 106,050      |
| 3           | 130,000     | 125,000        | 106,050      |
| 4           | 130,000     | 125,000        | 106,050      |
| 5           | 143,000     | 125,000        | 106,050      |
| 6           | 143,000     | 125,000        | 106,050      |
| 7           | 143,000     | 125,000        | 106,050      |
| 8           | 143,000     | 125,000        | 106,050      |
| 9           | 156,000     | 125,000        | 121,958      |
| 10          | 156,000     | 125,000        | 121,958      |
| 11          | 156,000     | 125,000        | 121,958      |
| 12          | 156,000     | 125,000        | 121,958      |

Present value at 8 percent = \$3,258,190

Table C-10

STREAM OF PROJECT EXPENDITURES

| <u>Year</u> | <u>Project Expenditures</u> |
|-------------|-----------------------------|
| 0           | 170,000                     |
| 1           | 310,000                     |
| 2           | 300,000                     |
| 3           | 350,000                     |
| 4           | 100,000                     |
| 5           | 90,000                      |
| 6           | 60,000                      |
| 7           | 60,000                      |
| 8           | 60,000                      |
| 9           | 50,000                      |
| 10          | 40,000                      |
| 11          | 20,000                      |
| 12          | 20,000                      |

Present value at 8 percent = \$1,292,130

general education and the costs related to the introduction of better inputs for the agricultural activities (extension workers, seeds, etc.).

#### Distribution Measure

In this particular project, the number of families belonging to the target group of interest number some 104 families. In fact, all families in the project area except for the five relatively richer families qualify for consideration in this case. The projected area of cultivatable land that will be under the ownership of these 104 families totals 348 hectares. These include the 278 hectares owned by families classified as group 2 and the 70 hectares of government land (one hectare per family) that will be provided to the 70 landless families (groups 3a and 3b).

#### Accessibility to Social Service Measure

At the present moment, the community of Aros enjoys the health services of local midwives. As a consequence of the implementation of the project, the people will have the services of a health clinic which will be built in the community. Thus the accessibility to health service will have a measure of:

$$\begin{aligned} & \text{score of the change*} \\ & \text{in health services} \quad \times \quad \text{population served} \\ & = 10 \times 600 \\ & = 6,000 \end{aligned}$$

---

\*See Table 23

In terms of accessibility to educational facilities, the community will enjoy an initial change from nothing to some general education which will be provided to the younger population of the community. Thus the accessibility to education services (from Table C-11) will have a measure of:

score of the change in  
educational facilities      x      population served

$$= 3 \times 162$$

$$= 486$$

Finally, the measure of the accessibility to social services is the sum of the two measures of accessibility to health services and accessibility to educational facilities. For this project this measure equals  $6,000 + 486 = 6,486$  ATSS units.

#### Employment Measure

The employment that is generated or is a consequence of the implementation of the project arises from that of the construction period of 30 months and the increased agricultural activity over the project life. The employment of the extension workers and other government employees will not be counted because if they were not used in this project they would presumably be employed elsewhere. Thus the employment measure is primarily concerned with accounting for the mobilization of the local factors of production, the rural peasant labor.

Table C-11

ASSESSMENT OF THE UTILITY OF CHANGE  
IN EDUCATIONAL FACILITIES

| <u>Type of Change*</u> | <u>Subjective Assessment<br/>of Educational Experts</u> |
|------------------------|---|
| No change              | 0   |
| E 1 to E 2             | 3   |
| E 1 to E 3             | 6   |
| E 1 to E 3 + E 4       | 9   |
| E 1 to E 2 + E 4       | 7   |
| E 2 to E 3             | 4   |
| E 2 to E 3 + E 4       | 7   |

\*The entire set of possible combination of changes has not been detailed. The listing represents the likely implemented changes.

The project documents indicate a total of 83,050 man-days of employment is generated as a direct consequence of the construction and maintenance activities.

For the cocoa cultivation, the agriculturalist indicated that an average of 180 man-days per year of employment will be generated per hectare of cocoa cultivation. From the information generated by the appraisal team (Tables C-1, C-2), it can be calculated that the expected total annual-area of cocoa cultivation over the project life amounts to 2,014.5 hectares. Correspondingly, the employment generated by cocoa cultivation totals 362,610 man-days.

For the rice cultivation, the agriculturalist indicated that the introduction of the new inputs and cultivation methods will yield an increment of 55 man-days per year for each hectare of transformed rice cultivation. The expected total annual-area of transformed cultivation equals 855 hectares. Consequently, the employment generated by cocoa cultivation totals 47,025 man-days.

For the cassava cultivation, the increment in employment as a consequence of the changeover in agricultural technology equals 39 man-days per year per hectare. The expected total annual-area of transformed cultivation equals 2,083 hectares. Thus the employment generated from cassava cultivation amounts to 81,237 man-days.

The increment in employment for the transformed maize cultivation is 27 man-days per year per hectare. As 652

hectares is the expected total annual-area of transformed cultivation, the resulting employment generated is 17,604 man-days.

Finally, the total employment generated as a direct consequence of the project is:

|   |  |   |
|---|--|---|
| Construction-gene-<br>rated employment        | + Rice cultivation-<br>generated<br>employment | + Cocoa-cultivation-<br>generated<br>employment |
| + Cassava cultivation<br>generated employment | + Maize cultivation<br>generated employment    |   |
| = 83,050 + 47,025 + 362,610 + 81,237 + 17,604 |  |   |
| = 591,526 man-days                            |  |   |

## C.2 A General Description of all Projects

In the previous section a detailed description of a single project was presented. A total of 36 projects have been identified and prepared; their appraisal measures are presented in Table C-12. Each project has five measures and each of these measures is denominated in its own units.

The economic benefits measure is represented by the difference in the expected present value of agricultural activity with the project with the expected present value of agricultural activity without the project. The economic costs criteria is represented by the present value of all



TABLE C-12 SUMMARY OF THE CRITERIA MEASURES FOR ALL THE PROJECTS

| Project Number | Economic Benefits | Economic Costs | Distribution (Hectares) | Employment (Man-Days) | Accessibility (ATSS Units) |
|----------------|-------------------|----------------|-------------------------|-----------------------|----------------------------|
| 1              | 4345678           | 1292130        | 348                     | 591526                | 6846                       |
| 2              | 3943230           | 2864390        | 623                     | 700105#               | 8324                       |
| 3              | 2432100           | 2032000        | 326                     | 200010                | 5324                       |
| 4              | 9008000#          | 2324430        | 20*                     | 356320                | 3234                       |
| 5              | 8432432           | 1101368        | 35                      | 253436                | 1634                       |
| 6              | 7436562           | 1343520        | 39                      | 275343                | 2398                       |
| 7              | 5100000           | 1400362        | 421                     | 623431                | 5002                       |
| 8              | 4400320           | 968400         | 521                     | 532432                | 4326                       |
| 9              | 2960860           | 940500         | 631                     | 361432                | 2498                       |
| 10             | 1960000           | 932632         | 720                     | 213432                | 5142                       |
| 11             | 3943200           | 1010060        | 203                     | 349946                | 3924                       |
| 12             | 4900632           | 1326360        | 196                     | 436248                | 3062                       |
| 13             | 5100656           | 2010612        | 800                     | 632160                | 11432                      |
| 14             | 4562020           | 2306542        | 953                     | 639196                | 12530                      |
| 15             | 3862062           | 1743680        | 732                     | 432960                | 8410                       |
| 16             | 7632952           | 1560000        | 621                     | 395432                | 6458                       |
| 17             | 7800432           | 3400738        | 432                     | 296050                | 4322                       |
| 18             | 2860432           | 708680         | 205                     | 543200                | 1900                       |
| 19             | 1143620           | 693620         | 103                     | 132000*               | 820                        |
| 20             | 800632*           | 620632*        | 190                     | 140000                | 0*                         |
| 21             | 5432620           | 900632         | 25                      | 290000                | 2320                       |
| 22             | 3960462           | 1152120        | 1000#                   | 360000                | 4324                       |
| 23             | 6832420           | 3500000#       | 612                     | 496000                | 7234                       |
| 24             | 5823400           | 3010000        | 528                     | 430000                | 13000#                     |
| 25             | 3000620           | 1500060        | 700                     | 180000                | 2848                       |
| 26             | 3400621           | 1200362        | 420                     | 362620                | 3620                       |
| 27             | 6236520           | 2003620        | 600                     | 490000                | 6200                       |
| 28             | 3900000           | 2400000        | 800                     | 450000                | 8200                       |
| 29             | 1500000           | 1200000        | 862                     | 290000                | 4320                       |
| 30             | 2190000           | 1500000        | 622                     | 320000                | 7000                       |
| 31             | 6620000           | 1600000        | 523                     | 420000                | 7200                       |
| 32             | 4702620           | 2010320        | 502                     | 580000                | 8206                       |
| 33             | 4230060           | 3206820        | 202                     | 362000                | 9200                       |
| 34             | 3630000           | 1100000        | 392                     | 392000                | 1020                       |
| 35             | 4000000           | 2163000        | 252                     | 432000                | 7260                       |
| 36             | 4526387           | 2456700        | 159                     | 362140                | 6588                       |

\* Lowest  
# Highest

the relevant project expenditure items. The distribution measure is captured by the ownership of land in hectares by the target group of interest. The employment is represented by the amount of employment in the construction and agricultural activities generated by the project. Finally, the accessibility to social services measure is represented by the score in ATSS units resulting from the change in accessibility to the services of health and education as a consequence of the project.

There is a great variation in the contributions to the five criteria for the 36 projects. No pattern can be used to point out these differences as they depend on the specific circumstances of each particular project. As an extreme example, any project may have the best contribution with respect to one criterion, but at the same time may have the worst contribution with respect to another criterion. Project 4 has the characteristics just described. It has the best contribution with respect to the economic benefits measure (\$9,008,000), but on the other hand, it has the worst contribution with respect to the distribution criteria (20 hectares). This is an example of a situation where the land, in the project area of influence, is owned by rich landlords who will be able to take advantage of the transport improvement, resulting in high economic benefits in the form of induced agricultural production but low

distribution effects in the sense that the poorer farmers will only reap an insignificant amount of the economic benefits.

The other variations in the criteria among all the projects can also be interpreted in an analogous fashion. However, the interpretation will have to be made with respect to the different contexts involved and the resulting different implications.

### C.3 Determination of the Individual Criterion Preference Function

The various techniques that can be used to assess the individual criterion preference (or utility) function have been discussed in Chapter Two. The technique to be used depends on the type of multicriteria problem encountered and the preferences of the decision maker. Nevertheless, the proper education and familiarization of the decision maker with the technique used cannot be overemphasized.

In the present case, the direct method has been chosen as the assessment technique. For each criteria, the lowest to the highest attainment from all the projects under consideration are put on the horizontal axis. Along the vertical axis is a common scale of from 0 to 100. The lowest attainment is assigned a value of 0, while the highest attainment is assigned the value of 100. The exception is the economic costs criteria, whereby the lowest attainment is assigned a value of 0. The decision maker is asked to

articulate an approximate value of the measure which corresponds to 50 (mid-way) on the scale. This is repeated for the two mid-points: one between 0 and 50 (25) and the other between 50 and 100 (75).

The articulation of these three points by the decision maker enables the analyst to arrive at approximate but appropriate preference functions for each of the five criteria. If more refined functions are required, more points may be elicited in a similar manner. Often the elicitation of just these approximate points will suffice for a good representation of the preference function. The preferences functions used in the case study are given in Figures C-3 to C-7.

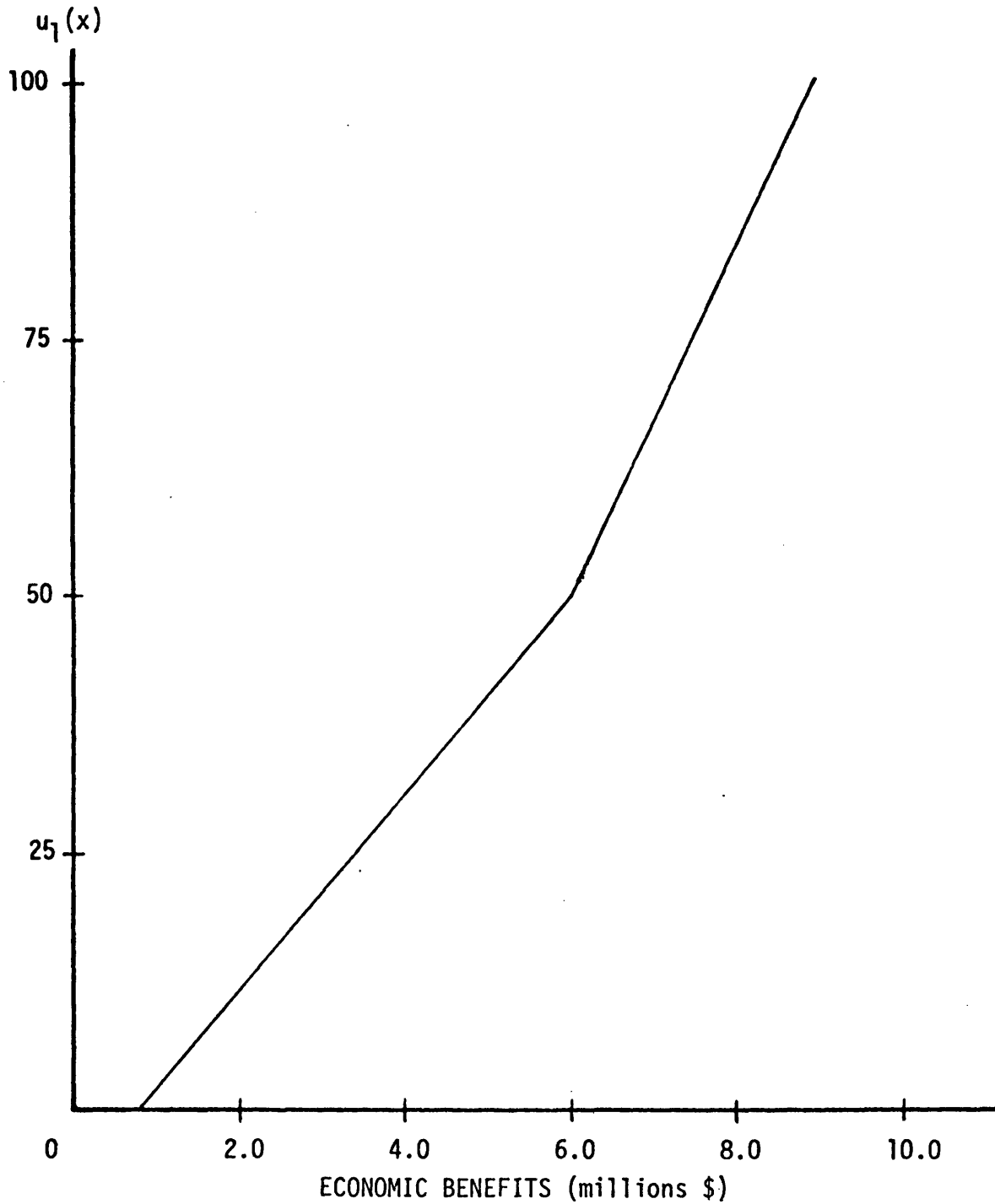


FIGURE C-3 : ECONOMIC BENEFITS PREFERENCE (UTILITY) FUNCTION

$$u_1(x) = \begin{cases} 0.000009617 x - 7.699 & 800,632 \leq x \leq 6,000,000 \\ 0.000015 x - 40 & 6,000,000 < x \leq 9,008,000 \end{cases}$$

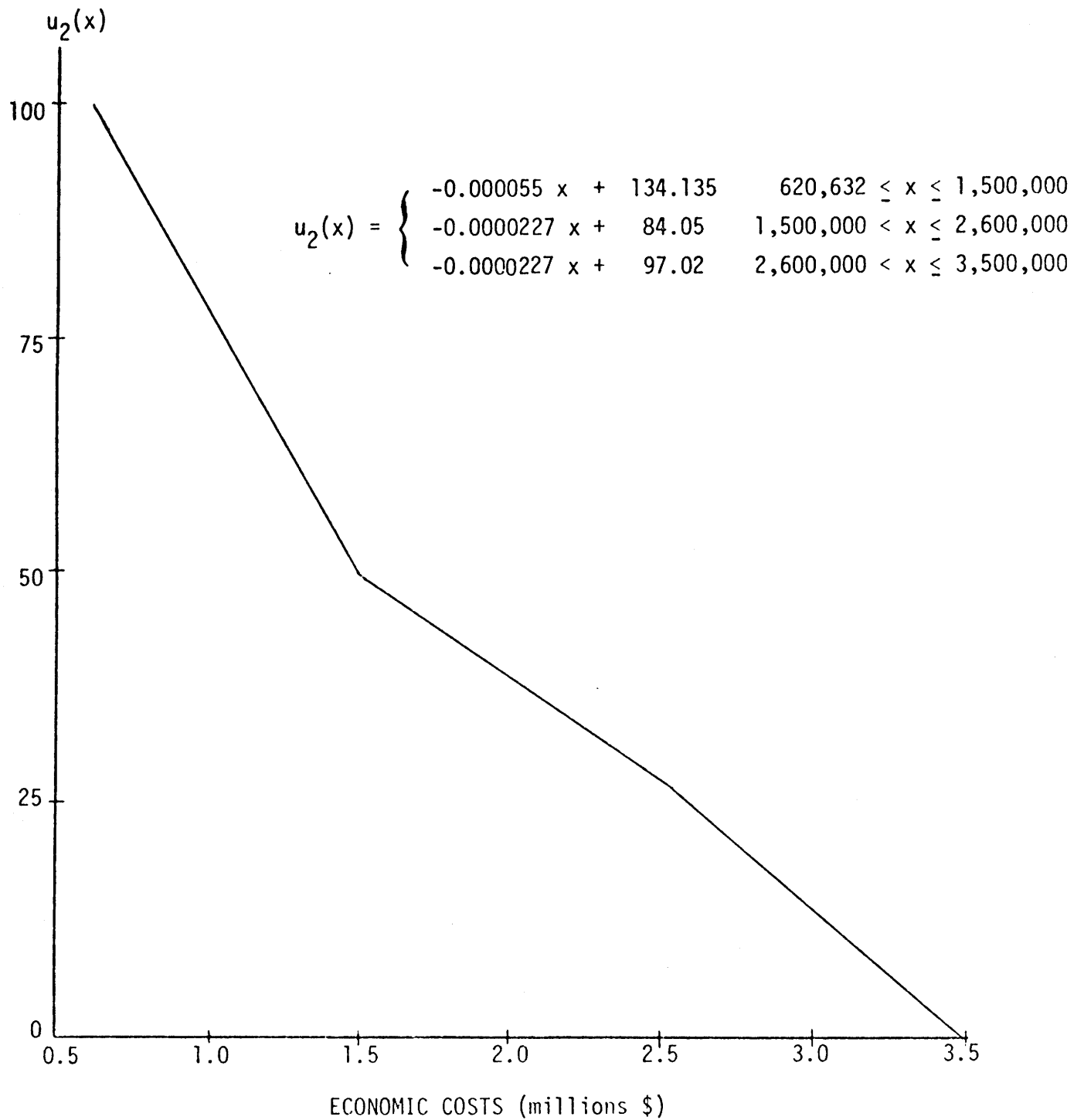


FIGURE C-4. ECONOMIC COSTS PREFERENCE (UTILITY) FUNCTION

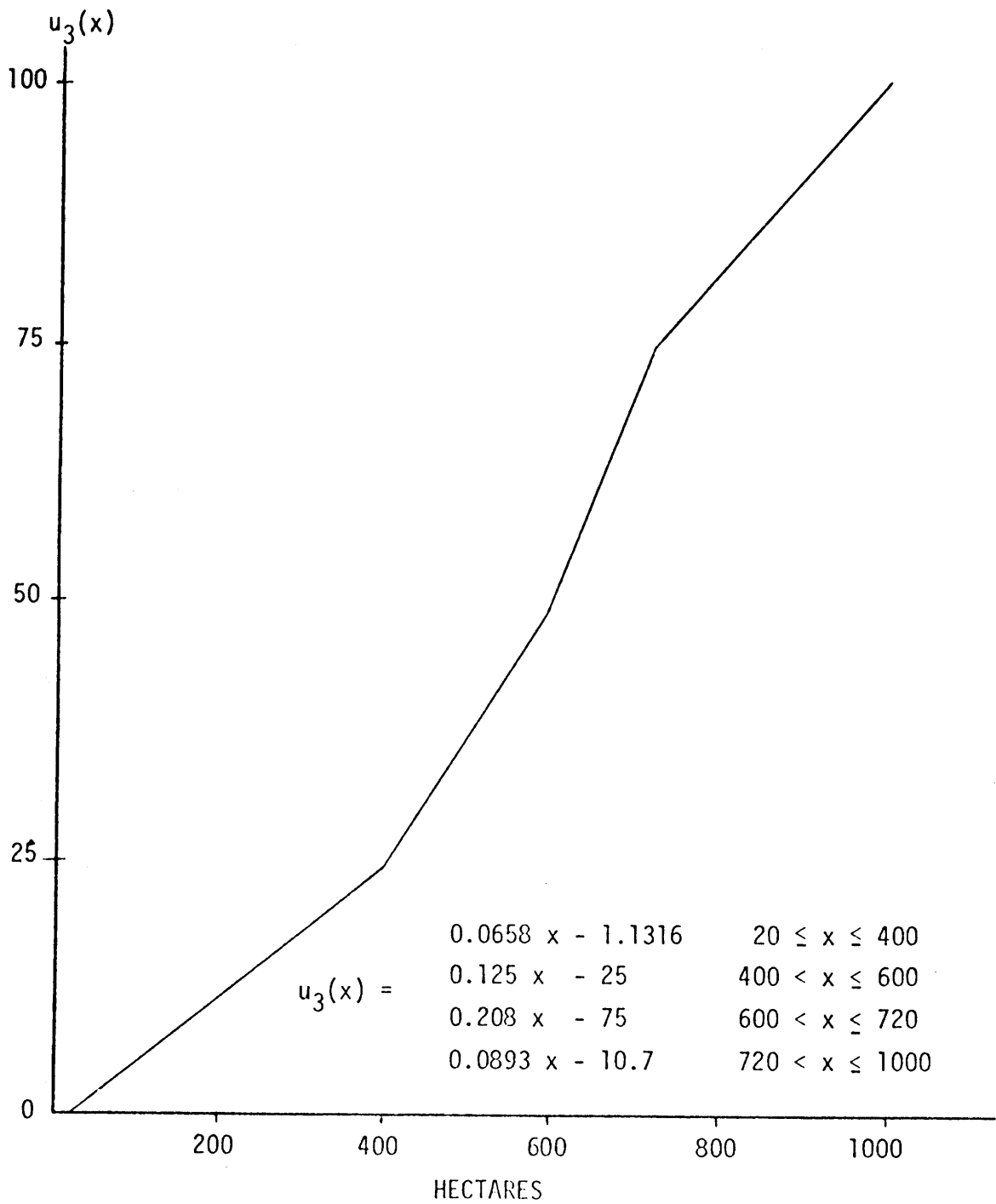


FIGURE C-5: DISTRIBUTION PREFERENCE (UTILITY) FUNCTION

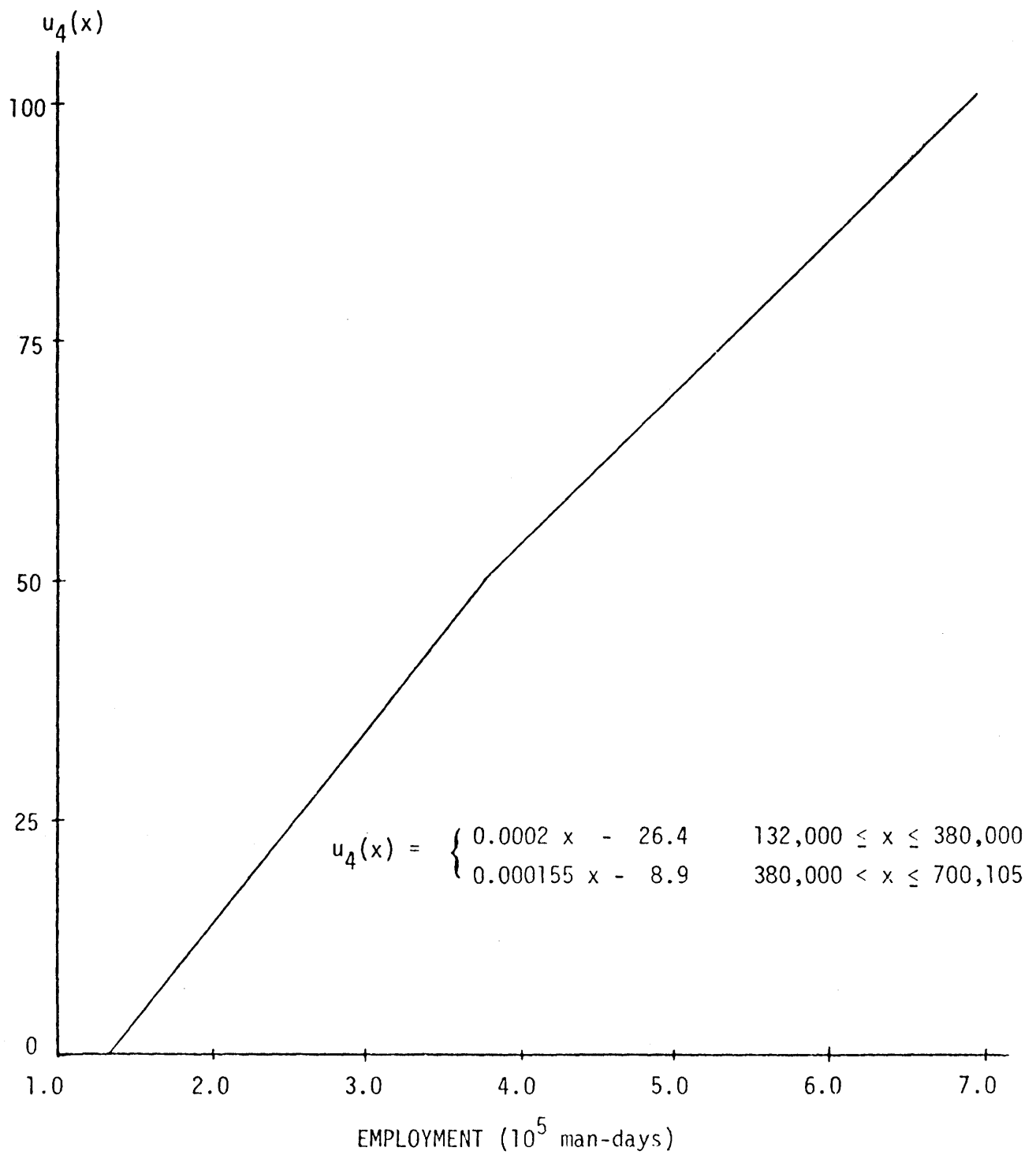
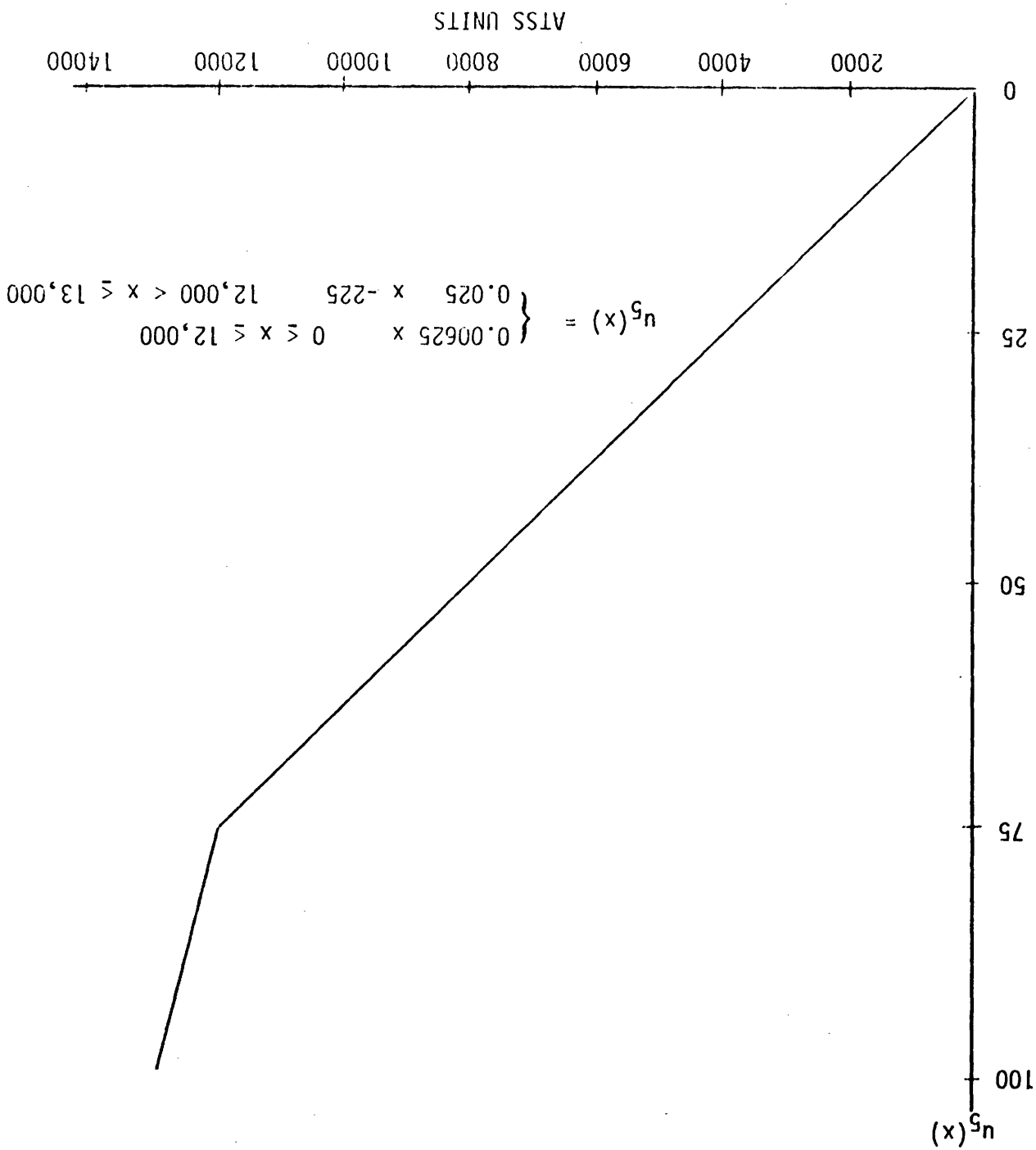


FIGURE C-6: EMPLOYMENT PREFERENCE (UTILITY) FUNCTION



FIGURE C-7: ACCESSIBILITY TO SOCIAL SERVICES PREFERENCE (UTILITY) FUNCTION



APPENDIX D. THE "RECOMMENDED" RANKING RULE

The "recommended" rankings produced in Chapter Four were produced by a list-intersection, or Venn diagram, rule. Let us assume the following:

- a) Three decision-rule rankings have been produced (the actual number is irrelevant, but more than three causes diagrammatic problems).
- b) The rank of project  $j$  under the  $i^{\text{th}}$  decision rule is denoted  $R_{i,j}$ .
- c)  $\{V_k\}$  denotes the Venn intersection of the three rankings considering only the top projects from each list, such that projects in  $\{V_k\}$  appear in all but one (or 2, in this case) of the partial lists.  $\{V_k\}$  is shown in Figure D-1.
- d) The best project has the lowest numerical rank, which is one, and there are  $N$  projects.

The recommended or "compromise" ranking is then produced as follows:

1. Determine  $\{V_k\}$ ,  $K = 1, 2, \dots, N$ .
2. Assign rank  $k$  to project  $j$  if the following are true:

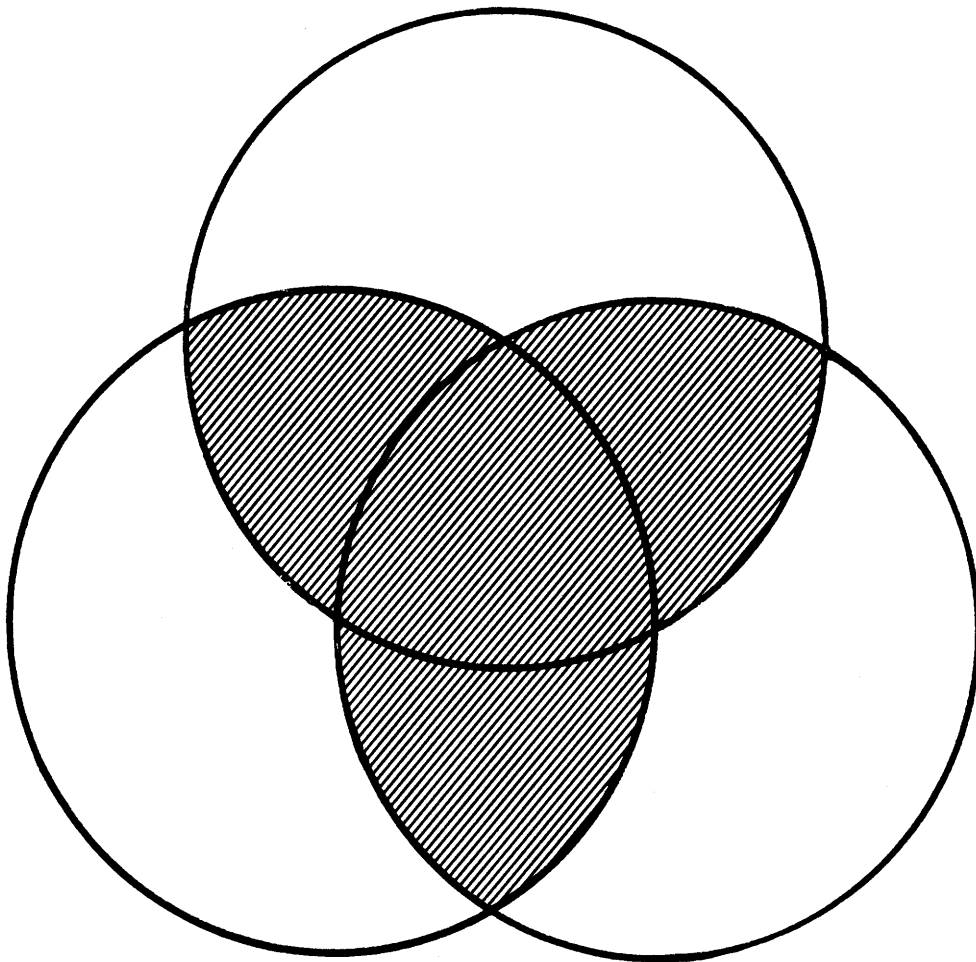
$$j \in \{V_k\}$$

$$j \notin \{V_m\} \text{ for all } m < k$$

i.e., assign rank  $k$  to all projects entering the Venn intersection of iteration  $k$ .

3. Rank the projects by the value of  $k$ , with lowest  $k$  being ranked one.

Figure D - 1  
VENN INTERSECTION FOR "RECOMMENDED" RULE  
WITH RULES OF EQUAL IMPORTANCE



4. Break any ties between project rankings by the highest total of each project's ranks on all but one (omit the lowest ranking for each project) decision rule.
5. Break any remaining ties by the lowest rank achieved by each project.

From Table 10 , omitting the mode-ordinal decision rule, we have the project rankings given in Table D-1. The  $V_k$  are as given in Table D-2.

Table D-1

TOP ELEVEN PROJECTS UNDER ORDINAL DECISION RULES

| <u>Rank</u> | <u>Equal<br/>Weights</u> | <u>Ordinal Preference<br/>Rules</u> |            |
|-------------|--------------------------|-------------------------------------|------------|
|             |                          | <u>Max</u>                          | <u>Min</u> |
| 1           | 14                       | 4                                   | 16         |
| 2           | 13                       | 5                                   | 31         |
| 3           | 22                       | 17                                  | 27         |
| 4           | 16                       | 16                                  | 7          |
| 5           | 15                       | 6                                   | 13         |
| 6           | 24                       | 14                                  | 5          |
| 7           | 8                        | 22                                  | 4          |
| 8           | 2                        | 13                                  | 32         |
| 9           | 28                       | 21                                  | 12         |
| 10          | 31                       | 23                                  | 6          |
| 11          | 27                       | 31                                  | 21         |

Table D-2

VENN INTERSECTIONS,  $\{V_k\}$ , AND ENTRANCES  $\{V_k^*\}$

| k  | $\{V_k\}$                       | $\{V_k^*\}$ |
|----|---------------------------------|-------------|
| 1  | --                              | --          |
| 2  | --                              | --          |
| 3  | --                              | --          |
| 4  | 16                              | 16          |
| 5  | 16, 13                          | 13          |
| 6  | 16, 13, 14, 5                   | 14, 5       |
| 7  | 16, 13, 14, 5, 4, 22            | 22          |
| 8  | 16, 13, 14, 5, 4, 22            | --          |
| 9  | 16, 13, 14, 5, 4, 22            | --          |
| 10 | 16, 13, 14, 5, 4, 22, 31, 6     | 31, 6       |
| 11 | 16, 13, 14, 5, 4, 22, 31, 6, 27 | 27          |

The rank under  $\{V_k^*\}$  gives us the recommended ranking. The ties of projects 14 and 5 are broken by the sum of their top two rankings:

$$\text{Project 14} = 1 + 6 = 7$$

$$\text{Project 5} = 2 + 6 = 8$$

So Project 14 is ranked above Project 5. In a similar fashion, Project 31 is ranked above Project 6.

The formal mathematical statement of the rule is as follows:

Let:  $R_{i,j}$  denote the  $i^{\text{th}}$  highest rank of the  $j^{\text{th}}$  project, considering all  $k$  decision rules.

Then: The projects are ranked according to their second-lowest ranking:

$$\text{RANK}(P_j) = R_{k-1,j}$$

If:  $\text{RANK}(P_q) = \text{RANK}(P_r)$  for some  $q, r$

then:  $\text{RANK}(P_q) > \text{RANK}(P_r)$  if  $\sum_{i=1}^{k-2} R_{i,q} < \sum_{i=1}^{k-2} R_{i,r}$

If the projects are still ranked equally after this operation, their worst performances are taken into account:

$\text{RANK}(P_q) > \text{RANK}(P_r)$  if  $R_{k,q} < R_{k,r}$

If the projects are still tied, they are practically identical and may be ranked in either order.

For the "cardinal recommended" rule, or the case where one ranking is considered to be of greater significance than the others, the definition of  $\{V_k\}$  becomes:

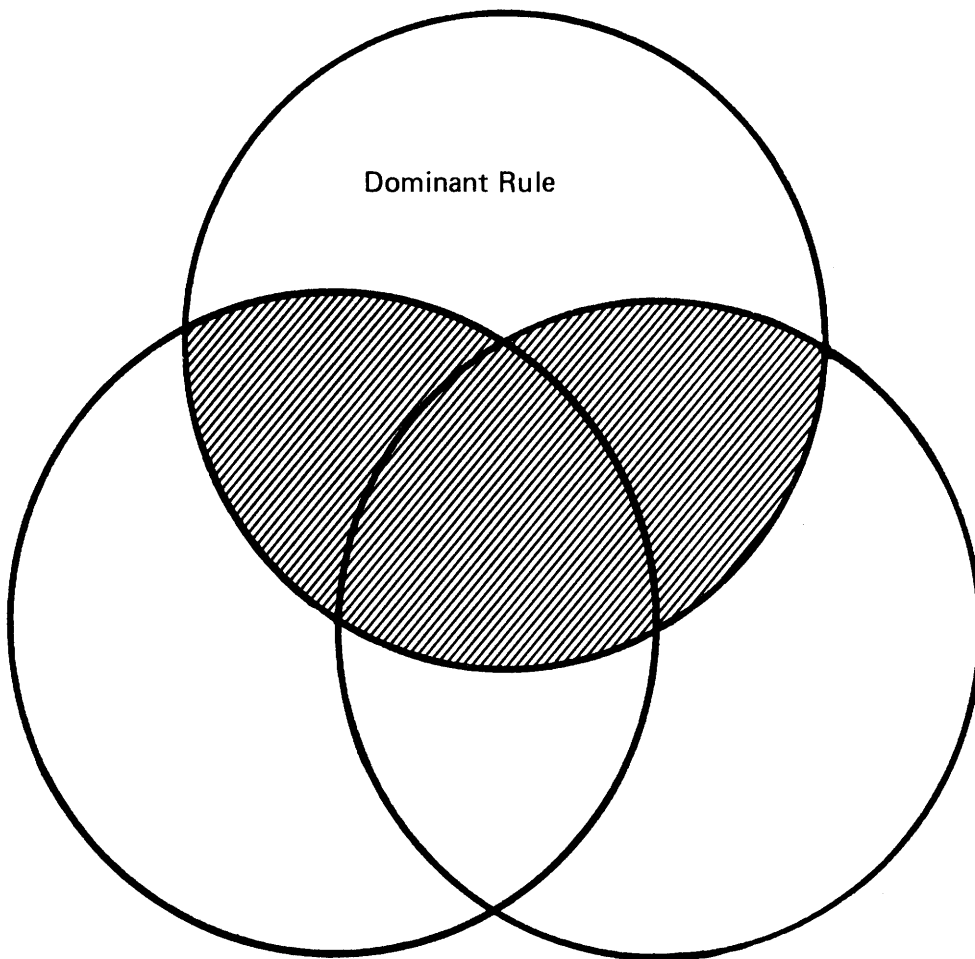
- c)  $\{V_k\}$  denotes the Venn intersection of the rankings considering only the top  $k$  projects from each list, such that the projects in  $\{V_k\}$  are in the top  $k$  projects in the dominant list. This  $\{V_k\}$  is shown in Figure D-2.

The first three steps of our rule are the same; the fourth becomes:

4. Break any ties between project rankings by their rank under the dominant decision rules.

The fifth step can never occur. Given our above notation, and defining  $R_{*,j}$  to be the rank of the  $j^{\text{th}}$  project under the dominant decision rule, we may formalize this as follows:

Figure D - 2  
VENN INTERSECTION FOR "RECOMMENDED" RULE  
WITH ONE RULE DOMINANT



$$\text{RANK } (P_j) = R_{*,j} \text{ if } R_{*,j} > R_{1,j}$$

$$\text{RANK } (P_j) = R_{2,j} \text{ if } R_{*,j} = R_{1,j}$$

if  $\text{RANK } (P_q) = \text{RANK } (P_r)$  for some  $q, r$

then

$$\text{RANK } (P_q) > \text{RANK } (P_r) \text{ if } R_{*,q} < R_{*,p}$$

Again, RANK denotes position in the list, with  $l > 2$ , and  $R_{i,j}$  denotes numerical value of the rank position, with  $l < 2$ .