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Recover a Latent, Long-term Price Series for Oil**

by

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# Using Futures Prices to Filter Short-term Volatility and Recover a Latent, Long-term Price Series for Oil

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*Oil prices are very volatile. But much of this volatility seems to reflect short-term, transitory factors that may have little or no influence on the price in the long run. Many major investment decisions should be guided by a model of the long-term price of oil and its dynamics. Data on futures prices can be used to filter out the short-term volatility and recover a time series of the latent, long-term price of oil. We test a leading model known as the 2-factor or short-term, long-term model. While the generated latent price variable is clearly an improvement over the raw spot oil price series, we also find that (1) the generated long-term price series still contains some of the short-term volatility, and (2) a naïve use of a long-maturity futures price as a proxy for the long-term price successfully filters out a large majority of the short-term volatility and so may be convenient alternative to the more cumbersome model.*

## INTRODUCTION

A striking characteristic of the spot oil price series is the fact that it occasionally exhibits large swings up or down which are then followed by reversals back towards some central tendency. The most dramatic example of such a swing is the sharp price spike occasioned by the first Gulf War in late 1990. Starting from a level below \$18 per barrel in mid-July, the price peaked above \$40 per barrel in October, then falling back below \$18 per barrel by late-February 1991. A less dramatic example is the drop in prices that occurred in 1993 when conflicts within OPEC resulted in a temporary glut of supplies and prices went from over \$18 per barrel in August to below \$14 per barrel in

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December, recovering back to over \$18 per barrel again in June 1994.<sup>1</sup> From December 1998 to November 2000 the price nearly tripled from \$11 to \$35 per barrel. But this dramatic increase was marked by several reversals down by \$10 per barrel before the upward trend recovered. Then within the space of slightly more than a year, the price fell again down to \$18 per barrel and recovered as quickly back above \$30, continuing its rise marked by swings of as much as \$10 per barrel. These dramatic fluctuations can be seen in Figure 1 which shows the graph of the spot price of oil from September 1989 through December 2003.<sup>2</sup>

How much of the daily movements in the oil price is attributable to such transitory, short-term factors? How much volatility is left after the effect of these short-term factors is filtered out? When we observe a sharp spike in the price, how much of that is likely to be reversed, and how quickly? Answering these questions is important for a large number of decisions. Short-term swings in the oil price don't change the value of a major oil-related investment, so it would be useful to be able to filter them out. If we can filter them out and arrive at a cleaner measure of the long-term volatility of oil prices, this would be the right input with which to value oil-related projects with large option features.

The large short-term volatility in the oil spot price is at the center of a current dispute over how oil companies should calculate the reserves they report in their financial

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<sup>1</sup> It was this temporary drop in the spot price that occasioned the brush with bankruptcy by the German company Metallgesellschaft due to its speculation in oil futures.

<sup>2</sup> Consistent with the practice in the finance literature, we refer to the price of the near month futures contract as the spot price, although it is not a price for truly immediate physical delivery as the term 'spot' denotes. Futures prices have a number of useful properties that make it worthwhile to work with the shortest maturity futures contract instead of with the actual spot price series.

statements. “Proven reserves” are those that are likely to be recovered under existing economic conditions, including price. What price should be used in making such a calculation? The Financial Accounting Standards Board (FASB) and the SEC recommend that companies use the end-of-year spot price. Many companies, however, would prefer to use a more stable forecasted price which they claim is used when they actually plan their development of reserves.

Until recently, many oil companies resisted the pressure to report reserves based on the end-of-year spot price. The recent scandal at Shell over misreporting reserves put new pressure on all companies to move to the uniform methodology advocated by the FASB and SEC. When in February 2005 Exxon Mobil made the switch and announced its 2004 results using the end-of-year pricing method, it simultaneously issued a press release to advertise the method’s main flaw: the arbitrary effect of short-term volatility. Exxon Mobil was forced to remove from its reserves approximately 500 million barrels in its Cold Lake field, a heavy oil-bitumen steam project in Canada. Although bitumen prices “were strong for most of 2004,...on the day of December 31, 2004, prices were unusually low due to seasonally depressed asphalt sales and industry upgrader problems in Western Canada. Prices quickly rebounded from December 31, and through January 2005, returned to levels that have restored the reserves to the proved category.”<sup>3</sup> The end-of-year pricing method resulted in a reserve replacement ratio for Exxon of 83%: under Exxon’s former methodology it would have been 112%. Exxon Mobil together with other oil companies, including Anadarko, BP, Chevron, ConocoPhillips, El Paso, Kerr-McGee

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<sup>3</sup> Exxon Mobil press release, February 18, 2005, “Exxon Mobil Corporation Announced 2004 Reserves Replacement”.

and Marathon are lobbying for consideration of alternatives to the end-of-year pricing methodology.<sup>4</sup>

## **2. INFORMATION FROM FUTURES PRICES**

One tool for filtering out the short-term transient volatility is data on futures prices. The price of a futures contract of a given maturity reflects expectations about the future spot price at that horizon. So the dynamics of the futures price at any given maturity is unaffected by short-term factors that dissipate within that maturity horizon. Prices on longer maturity futures contracts should reflect less and less of the short-term factors and should give a cleaner picture of the remaining volatility.

Gibson and Schwartz (1990) and Schwartz (1997) develop a two factor dynamic model of the term structure of oil futures prices which exploits this feature of futures prices. Baker, Mayfield and Parsons (1998) and Smith and Schwartz (2000) show that the two-factors can be represented as a short-term, transient component in the spot oil price and a long-term, lasting component. Each component is subject to shocks. Shocks to the short-term component do not have a lasting effect on the future price of oil. They dissipate gradually. In contrast, shocks to the long-term component are lasting and so cumulate. The observed volatility of the spot price is a function of the volatilities of both factors. Estimation of the model allows us to filter out the portion of the spot price

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<sup>4</sup> These companies sponsored a study of the issue by CERA which was released in February 2005 under the title *In Search of Reasonable Certainty: Oil and Gas Reserves Disclosures*.

volatility due to the short-term transitory factor and determine the volatility due to the long-term factor.<sup>5</sup>

We estimated this short-term, long-term model using the oil futures price data from September 1989 to December 2003 applying the usual Kalman filter technique. We used prices on futures contracts with maturity out to 17-months.<sup>6</sup> Table 1 reports the model parameter estimates as well as the raw volatility on the spot price. Focusing on the estimation made using the 1, 5, 9, 13 and 17 month contracts, the estimated volatility for the long-term component is only 14.9%, or less than half the raw spot price volatility of 37.1%. The mean reversion parameter is estimated at 0.902, which translates to an estimated half-life for short-term swings in price of about 9 months.<sup>7</sup>

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<sup>5</sup> Formally the model is written as  $\ln(P_t) = \xi_t + \chi_t$ , where  $\xi$  is the log of a latent, long-run price of oil and  $\chi$  is the current short-run deviations from the current latent price. The latent, long-run price component is modeled as a simple random-walk with drift:  $d\xi_t = \mu_\xi dt + \sigma_\xi dz_\xi$ , where  $\mu_\xi$  is the instantaneous rate of growth,  $\sigma_\xi$  is the volatility, and  $dz_\xi$  is the standard increment to a Wiener process. The dynamics of the short-run deviations are modeled as a simple mean reverting process:  $d\chi_t = -\kappa\chi_t dt + \sigma_\chi dz_\chi$ , where  $\kappa$  is the rate of mean reversion,  $\sigma_\chi$  is the volatility in the short-run component, and  $dz_\chi$  is the standard increment to a Wiener process. The correlation between the two Wiener processes is written:  $dz_\xi dz_\chi = \rho_{\xi\chi} dt$ . This model is a hybrid of a pure random walk model and a pure mean reverting model. The spot price is mean reverting, but the mean to which it reverts is itself evolving as a random walk.

<sup>6</sup> The key factor in selecting the longest maturity contract to use in the estimation is the availability of the data based on the way futures contracts are denominated. The maturity of a contract shortens through time, so what is today the 5-month contract becomes the 4-month contract. And what becomes the 5-month contract was once the 6-month contract. A continuous series of prices for the constant maturity 5-month contract requires a regular set of contracts for each neighboring month's maturity so that as the maturity of one contract shortens to less than 5-months, it is replaced by a new contract that has shortened to 5-months. While there exist oil futures contracts out several years in maturity, there is not a series of contracts with a maturity for each month at that horizon. When the 2-year contract becomes a 23-month contract, there is no formerly 25-month contract becoming a 2-year contract. These interruptions make it difficult to construct a continuous constant maturity contract dataset at longer maturities. Through time as the oil futures market has evolved, the horizon where this is possible has gotten longer. But then the tradeoff is whether one can go back far enough to construct an adequately long time series. We chose the 17-month contract as a compromise that gives us a meaningfully long time series and an adequately long maturity.

<sup>7</sup> Although the model produces estimates of the drift on each factor and of the price of risk for each factor, as with forecasts for expected returns on other assets, these estimates are not very precise. We report them for completeness.

Implicit in the estimation of the model is the construction of a time series for the latent, long-run price of oil—i.e., the price that would obtain if the effects of the short-run component were removed. Figure 1 shows this series overlaid on the graph of the actual spot oil price. The effect of filtering out the short-term, transitory factor is evident. Many of the largest swings and reversals present in the spot oil price series disappear in this series. The volatility of this series is the 14.9% estimated volatility of the long-term component.<sup>8</sup>

### **3. HAS SHORT-TERM VOLATILITY BEEN SUCCESSFULLY FILTERED?**

How successful is the two-factor model in filtering out all of the short-run, transitory volatility? If it is successful, then the estimated latent, long-term price series should be a random walk, free of any swings and reversals. Is it?

Visual inspection of Figure 1 suggests that the estimated series may fail this test. Although the long-term price series doesn't exhibit price swings and reversals of the same magnitude as the spot price, tempered versions of many of these swings and reversals still seem evident. For example, the long-term series spikes during the Gulf War when the spot price spiked so sharply. The temporary dip in the spot oil price during 1993 is also reflected in the estimated long-term series. The long-term spot price series shows a sharp spike and reversal in October 2000. Are these casual observations truly signs of remaining short-term volatility that was not successfully filtered out, or simply peculiar realizations in an essentially random walk pattern?

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<sup>8</sup> Volatility reported is the annualized weekly standard deviation of returns, where  $R_t = \log(P_t/P_{t-1})$  and weekly volatilities are annualized as  $\sigma_a = \sigma_w \sqrt{52}$ .

To address this question more rigorously, we applied a formal statistical test to the series. A random walk is a specific case of a unit process – one for which there is no serial correlation in the first differences. Accordingly, we applied a unit root test and then tested for serial correlation in the first differences of the process. The results are shown in Table 2. Consistent with the random walk specification, the test fails to reject the null hypothesis of a unit root. However, there is strong evidence of negative serial correlation in the first differences. This is inconsistent with the estimated long-term price series being a random walk, and seems to confirm that the two-factor model has not been completely successful in filtering out all of the short-term, transitory contributions to volatility.

Why might the estimated long-term price series still show signs of the short-term volatility that ought to have been filtered out? One obvious problem will arise if there is measurement error in the observed prices on long maturity futures contracts. This might arise, for example, due to illiquidity in long maturity contracts. Another possibility is that the short-term/long-term model does not describe the true dynamics and therefore does not predict the correct term structure, and this misspecification error is translated into a misestimated long-term price series. Alternative models for the term structure have been proposed by Routledge, Seppi, and Spatt (2000), by Casassus, Collin-Dufresne and Routledge (2004) and by Kogan, Livdan, and Yaron (2005).

A particular case of the model misspecification arises if the curvature of the term structure changes with the maturity. A key assumption of the model is that all of the short-term, transient shocks dissipate at a constant rate, so that the curvature of the term structure is fixed. If the curvature of the term structure changes with the maturity, then



the model is mis-specified and the parameter estimates will be affected. This is especially relevant for the estimate of the long-term volatility parameter, since the model is essentially projecting the curvature of the term structure within the range of the maturities used for estimation, i.e., between 1- and 17-months, out to the infinite horizon. This projection of the curvature applies equally to the term structure of volatilities.

We re-estimated the model on two different subsets of contracts within the 17 month horizon: the shorter maturity contracts of 1, 3, 5, 7 and 9 months, and the longer maturity contracts of 9, 11, 13, 15 and 17 months. The parameter estimates are shown in Table 1. The rate of mean reversion is highly sensitive to the choice of contracts used to estimate the model: at the short end the estimated rate of mean reversion is 1.326 or a half-life of approximately 6 months, while at the long end the estimated rate of mean reversion is 0.827 or a half-life of 10 months. The estimated long-term volatility using the shorter maturity contracts is 16.8% and using the longer maturity contracts is 15.7%. This is not a large difference, although in both cases it is higher than the 14.9% arrived at using the full range of contracts.

We also ran the test for serial correlation on the long-term price series generated by each of the alternative estimations. The results are shown in Table 2. In all cases the long-term price series shows evidence of serial correlation.

#### **4. A NAÏVE ESTIMATOR**

By reducing to only two factors all of the dynamics in the term structure of futures prices, the short-run, long-run model is a significant simplification of the possible dynamics driving the oil price. Nevertheless, it is a mathematically demanding model for

industry analysts to employ, and estimation of the parameters is correspondingly difficult. Few people understand the mechanics of the Kalman filter technique, and even fewer of them are capable of sharing that knowledge with less statistically inclined members of a corporate team involved in decision making. Moreover, although we have not touched on the complications here, there are certain subtle choices to be made and a number of issues and caveats that could be raised in favor of alternative approaches to the estimation. Quite often those with the mechanical knowledge of the Kalman filter technique and those who have the familiarity with key market issues relevant to choosing among these approaches are different groups of people and it may not be practical to assume they can be brought together.

So the question arises, how much extra information does one get as a result of deploying this complicated machine? Or put another way, is there a simpler foundation for deriving the same results that practicing analysts would find more appealing? How large of a compromise is made if a simpler estimator is used?

The premise of the short-run, long-run model is that the effect of the variations in the short-run factor is largely felt in the price of short maturity futures contracts. The effect of these variations in the short-run factor die out gradually as one looks to contracts with greater and greater maturity. Therefore, variation in the price of contracts with a relatively long maturity reflects primarily the effect of volatility in the latent, long-term factor. A naïve estimator, then, could be to simply view the price on a long maturity contract as a direct, if noisy, observation of the latent, long-run price. Let us take as our example using the raw 17-month futures price, the longest maturity contract used in our estimation. This naïve inference would contain some error, because even a long-maturity

futures price will reflect some residual amount of the short-run factor that has not yet dissipated. But so long as this residual amount is small, the error is small. How much error is left in this naïve model? How does it compare against the estimated latent, long-run price from the full blown 2-factor model?

Figure 2, Panel A shows the estimated model volatilities and the observed volatilities for futures contracts of various maturities when estimated using contracts of 1, 5, 9, 13 and 17 months. The solid curved line displays the modeled volatilities.<sup>9</sup> The solid horizontal line is the long-term volatility to which the solid curve line asymptotes. The markers show the observed volatilities, with the contract maturities used in the estimation shown as solid diamonds and the maturities not used in the estimation marked with crosses. Assuming that the model estimate of 14.9% for the long-term volatility is correct, short-term factors contribute 22.2 percentage points to bring the observed spot price volatility up to 37.1%. The naïve estimator of the long-term volatility is the 16.7% volatility on the 17-month contract. This is only 1.8 percentage points greater than the 14.9% model estimate for the long-run volatility. In this case the naïve estimator would have identified 92% of the short-term contribution to the spot volatility identified by the more complicated model.<sup>10</sup>

Panels B and C of Figure 2 show the same comparison of model and observed volatilities when the model is estimated either using the short maturities of 1, 3, 5, 7 and

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<sup>9</sup> Per the formula in Schwartz and Smith (2000):  $\sigma_T = e^{-2\kappa T} \sigma_\chi^2 + \sigma_\xi^2 + 2e^{-\kappa T} \rho_{\chi\xi} \sigma_\chi \sigma_\xi$ .

<sup>10</sup> We chose the 17-month contract price as the naïve estimator because it was the longest maturity used in our estimation. Others may prefer a contract maturity that is more of a focal point such as the 1-year futures price. This, too, does reasonably well. The 1-year contract has a volatility of 18.7%. This is only 3.8 percentage points greater than the 14.9% model estimate for the long-run volatility. In this case the naïve estimator would have identified 83% of the short-term contribution to the spot volatility identified by the more complicated model.

9 months or using the long maturities of 9, 11, 13, 15 and 17 months. Since both of these estimations produce a higher long-term volatility, the naïve estimator looks even better in comparison.

Another way to evaluate the naïve estimator is to visually compare the 17-month future price series against the estimated long-term price series as is done in Figure 3. The dynamics of the two series are remarkably similar. Many of the transitory price spikes that seem apparent in the spot price series, and that appear to have some residual influence in the 17-month futures price series, show themselves also in the long-term price series. And while for most apparently transitory, short-term price movements the long-term price is constructed to show a dampened response as compared against the 17-month futures price series, there are certain cases where the long-term price series spikes more sharply than does the 17-month futures price series—for example, in May 1990 and October 2000.

## **5. CONCLUSIONS**

The short-term, long-term model of oil prices clearly helps us understand the distinction between the daily volatility observed in the spot price and the long-run volatility in forecasted prices. Estimation of the model provides a markedly improved measure of the correct long-term volatility that is useful for most major investment decisions linked to the oil price. We have shown that although this improvement is significant, there remains some amount of the transient volatility still contained in the model's estimate of the long-term volatility. We have also shown that a naïve estimator using the raw volatility on the longest maturity futures contract succeeds in filtering out

most of the transient volatility caught by the full blown model. This naïve estimator is significantly easier to work with. It requires no complicated statistical knowledge and no implementation of difficult estimation procedures. The formula for estimation can be simply written in a single cell of an Excel spreadsheet.

The success of the naïve estimator also provides an opening for the resolution of the conflict over the right price to use in calculating proven reserves for financial reporting purposes. While many of the oil companies have properly complained about the volatility in the spot price and the curious results that follow from that, an alternative that is sometimes proposed is to leave them discretion to utilize whatever price forecast they deem best. Regulators are obviously cautious about granting that kind of discretion. This paper shows, however, that longer maturity futures contracts contain very little of the short-term transitory volatility that is at the heart of the objection to end-of-year spot pricing. They provide a viable alternative that is both simple and effective.

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**Table 1. Parameter Estimates for the Short-Term, Long-Term Model**

Parameter	Contract Months Used in Estimation		
	1, 5, 9, 13, 17	1, 3, 5, 7, 9	9, 11, 13, 15, 17
Volatility of Long-Term Price (estimated)	14.9% (33.218)	16.8% (37.835)	15.7% (37.274)
Volatility of Short-Term Factor (estimated)	31.0% (32.967)	33.5% (36.143)	31.9% (37.515)
Correlation of Volatilities (constrained)	0.0	0.0	0.0
Volatility of Spot Price, Model (calculated)	32.4%	34.4%	33.7%
Volatility of Spot Price (raw data)	37.1%	37.1%	37.1%
Speed of Mean Reversion (estimated)	0.902 (53.480)	1.326 (50.865)	0.827 (51.648)
Half-Life of Short-Term Factor	0.77	0.52	0.84
Growth Rate of the Long-Term Price (constrained)	0.0%	0.0%	0.0%
Instantaneous Drift of the Long-Term Price (calculated)	-1.1%	-1.4%	-1.2%
Risk-Neutral Instantaneous Drift of the Long-Term Price (estimated)	1.4% (6.042)	-2.2% (-6.565)	1.7% (8.109)
Long-Term Risk Premium (calculated)	-2.5%	0.8%	-3.0%
Short-Term Risk Premium (estimated)	17.4% (1.998)	14.8% (1.558)	17.7% (2.363)

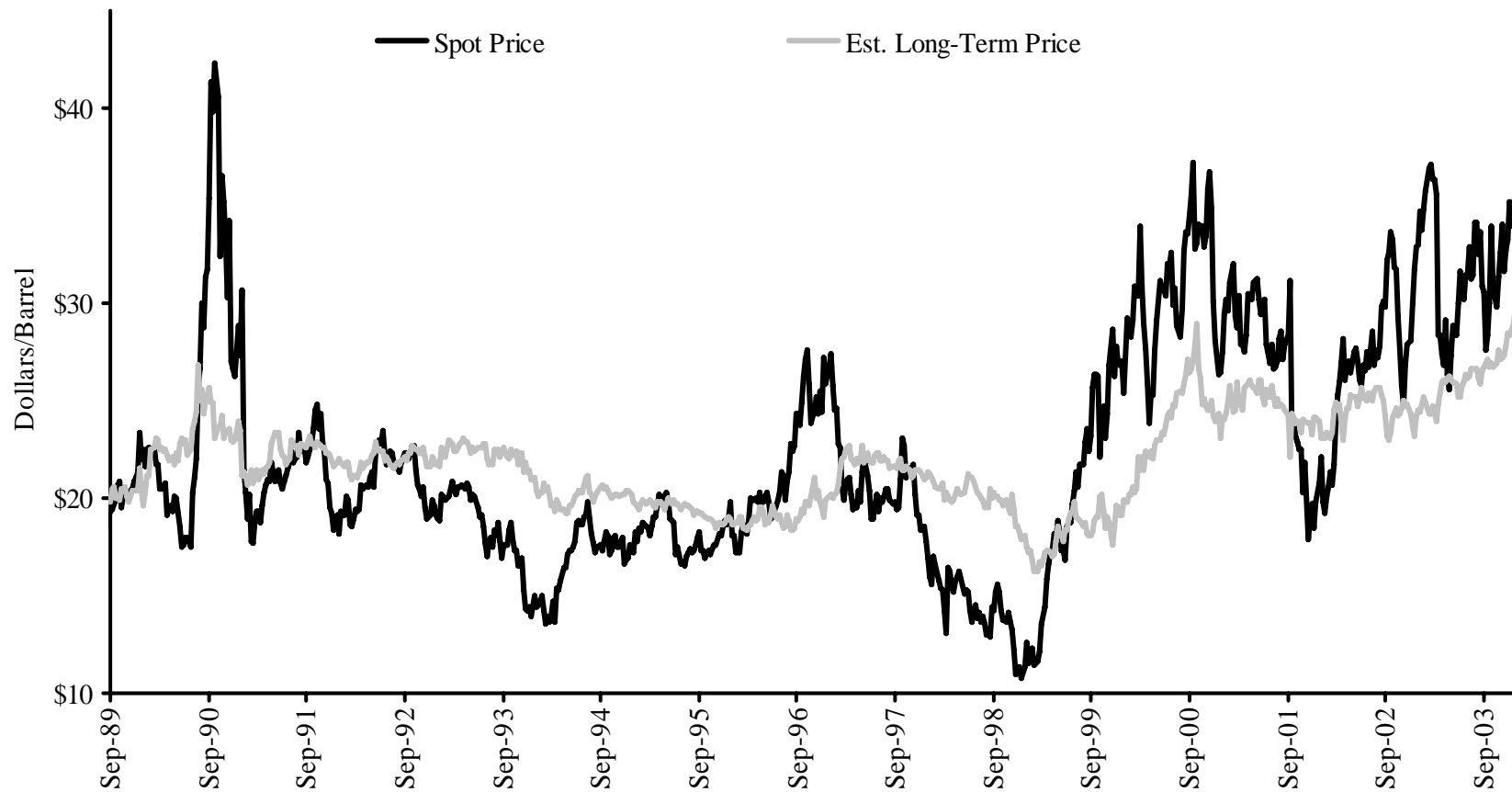
Parameter estimates are for the short-term, long-term model in Schwartz and Smith (2000) using weekly futures price data from September 1989 to December 2003. In each estimation five futures contracts are used with the months of the contracts shown at the top of the column. T-statistics are shown in parentheses. We estimate the model constraining the correlation coefficient between the short- and long-term shocks to be zero and the long-term spot price drift to be zero. Since the model spot price volatility is a function of the volatility of the long- and short-term factors and the correlation coefficient, this is not independently estimated but merely calculated and displayed for convenience. Similarly, the half-life of short-term price movements is a restatement of the speed of mean reversion and so is not estimated and has no t-stat shown. The instantaneous drift is a function of the long-term price drift and the estimated long-term volatility. The long-term risk premium is a function of the two instantaneous drift parameters.

**Table 2. Test Results for Unit Root and Serial Correlation in First Differences**

	Model Estimated Long-Term Price Series			Raw Futures Price Series			
	Contract Months Used in Estimation			Spot, 1-Mos	9-Mos	1-Year	17-Mos
	1, 5, 9, 13, 17	1, 3, 5, 7, 9	9, 11, 13, 15, 17				
Unit Root Test							
Test Statistic (6 lags)	-1.68	-1.93	-1.84	-2.85	-1.93	-1.94	-1.74
Test Statistic (20 lags)	-1.82	-2.13	-1.95	-2.91	-2.13	-2.11	-1.97
Serial Correlation Test (1st Difference, 1 lag)							
Coefficient	-0.145	-0.11	-0.201	-0.155	-0.11	-0.140	-0.179
Q-Stat	15.021	8.722	28.957	17.237	8.724	14.181	23.122
P-Value	0.0000	0.0030	0.0000	0.0000	0.0003	0.0000	0.0000



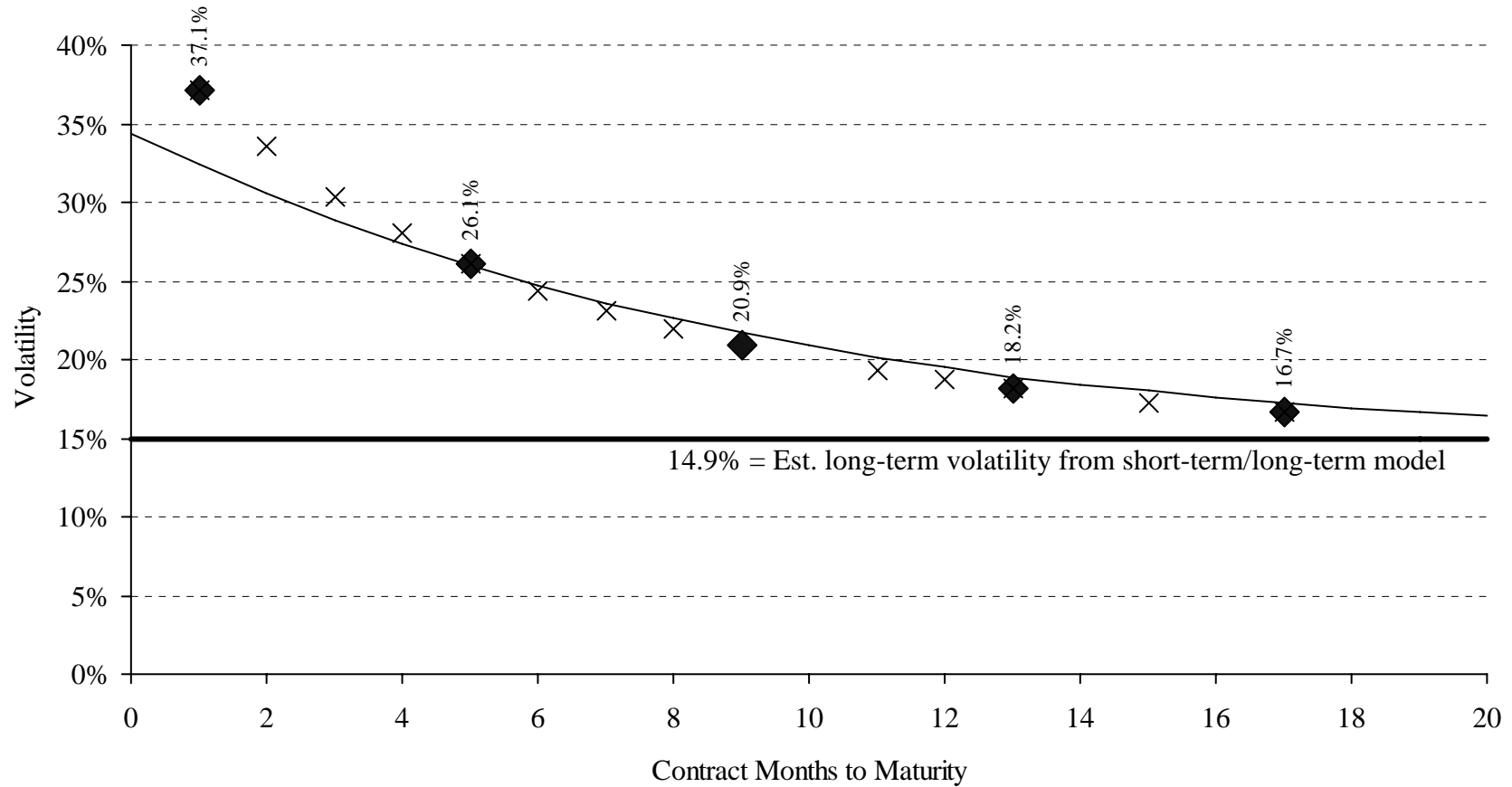
**Figure 1. Spot Oil Price Overlaid with an Estimated Long-Term Price Series**



The spot oil price weekly from September 1989 to December 2003 where the 1-month NYMEX futures price is used as the spot price. The estimated long-term price series is calculated using the short-term, long-term model in Schwartz and Smith (2000). Five futures contracts are used—the 1-month, 5-month, 9-month, 13-month and 17-month contracts. We estimate the model constraining the correlation coefficient between the short- and long-term shocks to be zero and the long-term spot price drift to be zero. Model parameter estimates are shown in Table 1.

## Figure 2. Volatility of Oil Futures Prices by Contract Maturity

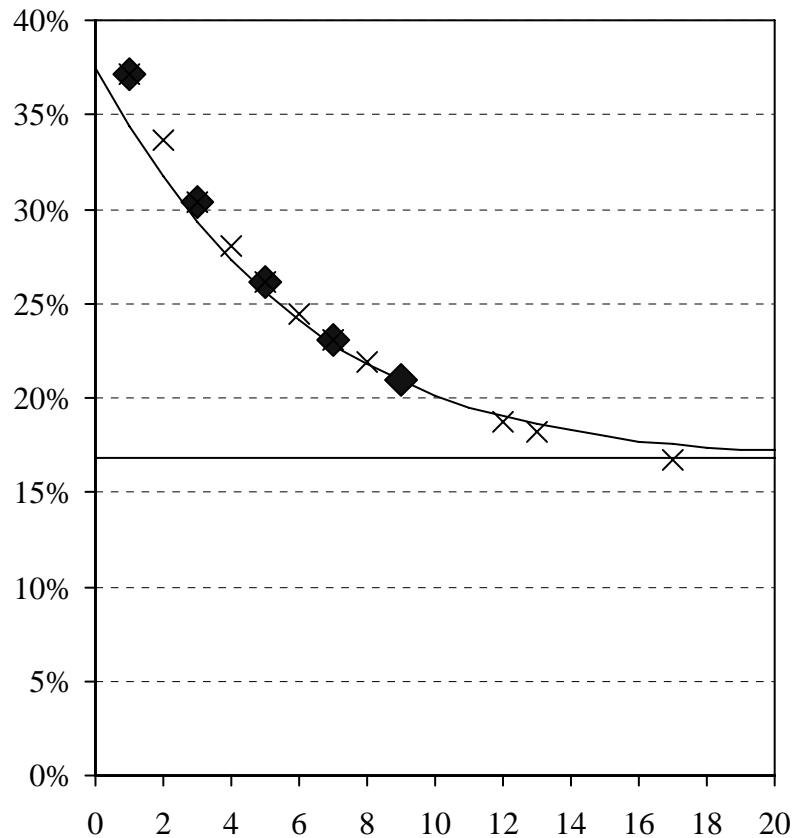
### Panel A: Estimation Using the 1, 5, 9, 13, and 17-Month Contracts



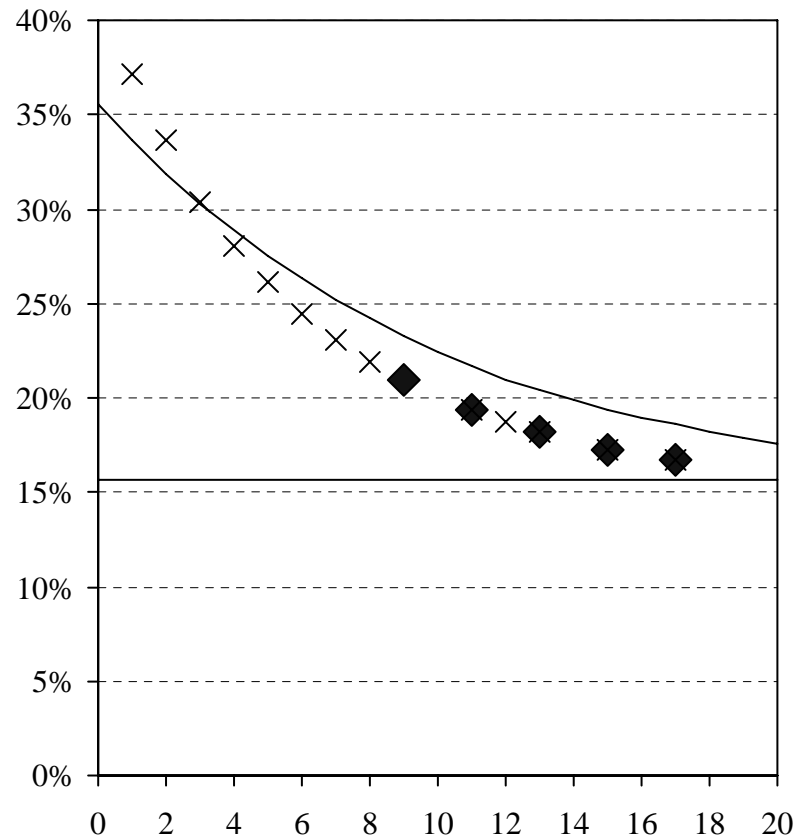
The horizontal line is the estimated long-term volatility as reported in Table 1. The curved line shows model volatilities for futures by maturity per the formula in Schwartz and Smith (2000):  $\sigma_T = e^{-2\kappa T} \sigma_\chi^2 + \sigma_\xi^2 + 2e^{-\kappa T} \rho_{\chi\xi} \sigma_\chi \sigma_\xi$ . Points marked individually are the observed volatilities for futures prices of the corresponding maturities; diamonds mark the maturities used in the estimation, and crosses mark other maturities. Reported values are annualized weekly standard deviation of returns, where  $R_t = \log(P_t/P_{t-1})$  and weekly volatilities are annualized as  $\sigma_a = \sigma_w \sqrt{52}$ .

**Figure 2. Observed and Modeled Volatility of Oil Futures Prices by Contract Maturity**

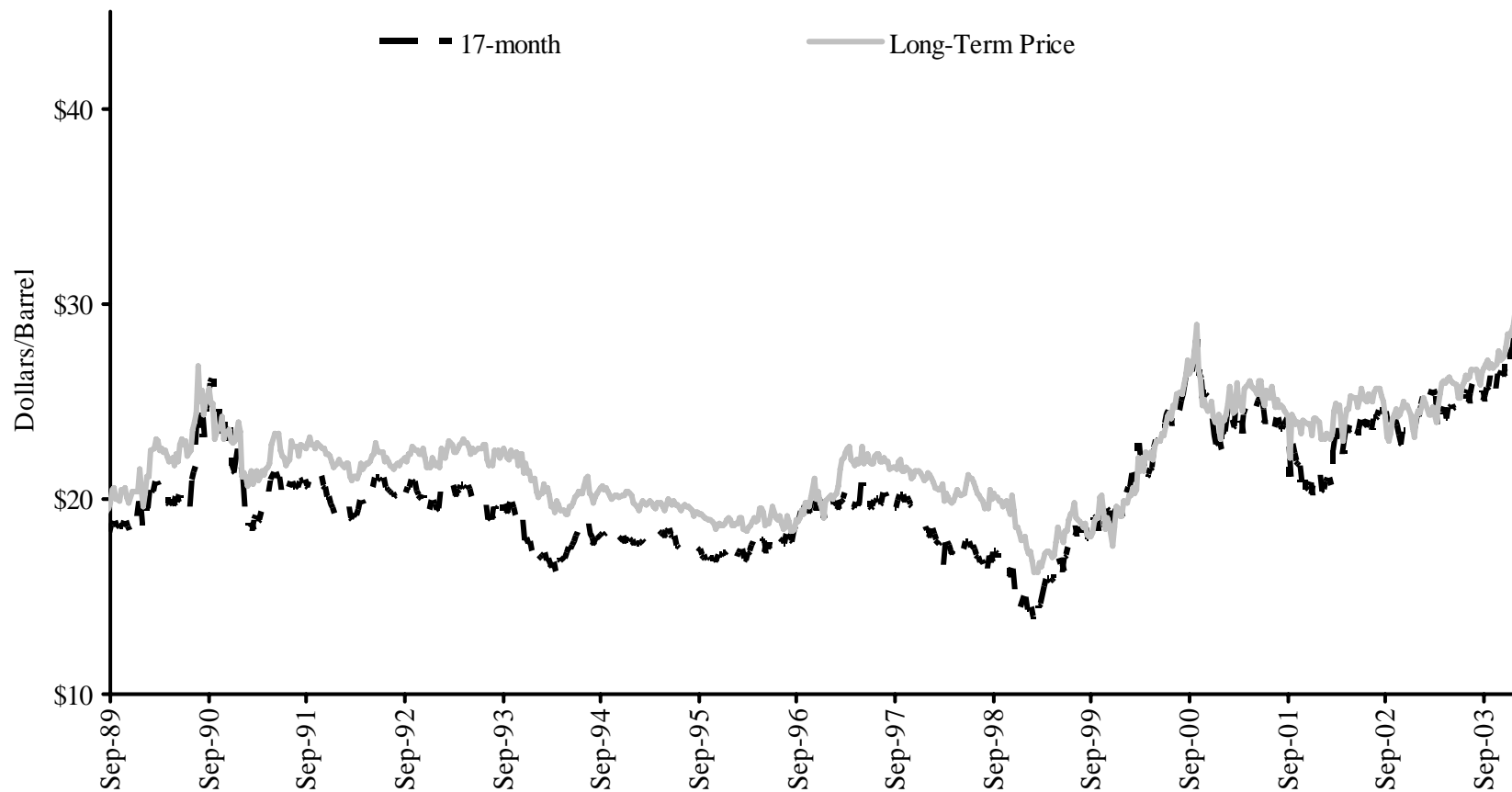
**Panel B: Estimation Using the 1, 3, 5, 7, and 9-Month Contracts**



**Panel C: Estimation Using the 9, 11, 13, 15 and 17-Month Contracts**



**Figure 3. The 17-Month Futures Price Versus the Estimated Long-Term Price**



The 17-month oil futures price weekly from September 1989 to December 2003. The estimated long-term price series is calculated using the short-term, long-term model in Schwartz and Smith (2000). Five futures contracts are used—the 1-month, 5-month, 9-month, 13-month and 17-month contracts. We estimate the model constraining the correlation coefficient between the short- and long-term shocks to be zero and the long-term spot price drift to be zero. Model parameter estimates are shown in Table 1.