The Simulation Analysis on the Accuracy and Efficiency of MIT Transaction Index
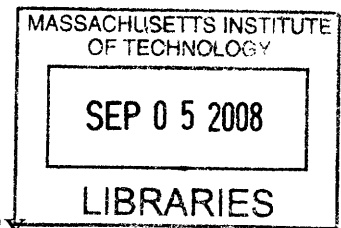
By

Jungsoo Park

Bachelor of Science, Pusan National University, South Korea, 1999
Master of Planning, University of Southern California, 2004

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Author

_____   Jungsoo Park
                                            Department of Urban Studies and Planning
                                            August 6, 2008

Certified by

_____   Professor William C. Wheaton
                                            Professor of Economics
                                            Thesis Advisor

Accepted by

_____   Professor Xavier de Souza Briggs
                                            Chair, MCP Committee
                                            Department of Urban Studies and Planning

Abstract

The paper aims to investigate the improvement that Ridge regression brings to the commercial real estate index estimation. The MIT transaction based index extends the use of the last model by Fisher, Gatzlaff, Geltner and Haurin (2003.) They address the sample selection bias and the noise effect of the transaction prices in small sample. To correct sample selection bias, MIT transaction index use the Heckman two-step. In addition to the correction of sample selection bias the MIT transaction based index applies the Ridge filters into the index creation procedures to dampen the effect of noise in transaction prices on estimations. Among them, this paper primarily focuses on: (1) investigating error structures, defined by the difference between the true market return and the estimated index return; (2) examining the efficiency improvement of the Ridge Regression as estimation method over Ordinary Least Square (OLS); (3) selecting the proper value of weight factor "k."

The findings on the error structures are as follows. First, MIT transaction index does not suffer from smoothing and lagging which is apparent in the appraisal based index. Second, with larger sample size, the estimated market indexes become close to the true market values. In addition, the estimated market index is more biased when it uses small number of sample. Third, the error does not cumulate over time regardless of the sample size.

To correct noise effect on the estimation procedure under the observation poor scenario, this study examines the efficiency of the Ridge regression. First, we examine the qualitative form of relationship between ridge coefficients and the control factor of "k." Generally, ridge trace is not significantly changed with respect to "k." It might be argued that the noise effect is not severe in the estimation procedure, yet the introduction of ridge regression apparently improves the estimation. Indeed, between $k = 4$ and 5 ridge coefficients stabilize. Second, to investigate the improvement of ridge regression, we examine the mean square error. Between $k = 4$ and 5, mean square error, defined by the difference between the true and estimated market index, achieve the lowest value. Taken together, the system stabilizes between $k = 4$ and 5 and achieve the minimum mean squared error. This result leads to the conclusion that we can obtain better estimation using the control factor of 5.

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Introduction

The MIT transaction based index extends the use of the last model by Fisher, Gatzlaff, Geltner and Haurin (2003.) They address the sample selection bias and the noise effect of the transaction prices in small sample. To correct sample selection bias, MIT transaction index use the Heckman two-step. In addition to the correction of sample selection bias the MIT transaction based index applies the Ridge filters into the index creation procedures to dampen the effect of noise in transaction prices on estimations. Among them, this paper primarily focuses on: (1) investigating error structures, defined by the difference between the true market return and the estimated index return; (2) examining the efficiency improvement of the Ridge Regression as estimation method over Ordinary Least Square (OLS); (3) selecting the proper value of weight factor "k."

MIT transaction index improves the FGGH model by employing the Ridge filters to remove the noise of the transaction prices in the quarterly based index construction procedures. Real estate prices deviate from its true value because each transaction is consummated by idiosyncratic buyer and seller, and their negotiation ability is different. Generally, noise in the transaction price has more significant influence on the estimation procedure when sample size is small. Thus, high frequency index with small sample size should be estimated with different estimation method.

Ridge filters dampen the noise in each transaction price by appending the synthetic observations for each time period which is generated from a noise-free index. MIT transaction index benchmarked the annual return index to generate the Ridge observations for each quarter. Because ridge observation is generated for each period (in MIT transaction index, quarter), its time dummy variables has a diagonal square matrix of ones. To control the strength of the ridge observations in each quarter, the ridge filter is weighted by the control factor of "k." That is, by multiplying the ridge observations with different "k", the ridge filter has varying effects on the estimation results: different coefficients for each variable by different "k" and even sometimes different signs on coefficients.

The major improvement of MIT transaction based index in estimating high frequency index is the use of Ridge regression. However, it is not clear what is optimal "k" and what kinds of criteria can be used to select optimal "k." The paper proposes two different selection criteria and examines the efficiency improvement of Ridge regression on MIT transaction based index by using Monte Carlo simulation analysis. First, the study examines the qualitative form of relationship between ridge coefficients and the control factor of "k." Second, to investigate the improvement of ridge regression over OLS regression, the study will examine the characteristic of the mean square error, which is defined by the difference between the true and estimated market index.

The first part of the paper briefly reviews pricing methods developed last decades. The second part of the paper describes the MIT transaction index and the simplified version of the MIT transaction index which is used in the Monte-Carlo simulation analysis. The third part of the paper explains the design of the simulation analysis and the last part of the paper reports the simulation results on error estimates and the Ridge filter

Background

The Construction of commercial real estate index has long been received particular attention in the private real estate investment. There are two main approaches to identify the real estate returns series. One group of studies employs econometric methods (Repeated Sales Regression (RSR) and Hedonic Regression) and they use transaction prices as input data. The other group makes use of appraisal value to measure real estate investment performance. So far, most widely used real estate return series is appraisal value because real estate transactions are scarce. However, it comes at costs--smoothing and lagging. The first and the second moment bias in appraisal based index introduce estimation error.

Considering problems inherent in the use of appraisal values in constructing commercial real estate index, a natural alternative is transaction

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prices. Until recently, transaction prices have not been received as much attention as appraisal value, because transaction prices are relatively scarce comparing to appraisal value and those may contain noise. Still large portion of commercial real estate transactions is privately consummated by buyers and sellers. Information that buyers and sellers possess may be different – asymmetric information – and this infuses noise into transaction prices. Hence, noise filtering technique should be applied in the procedure of commercial real estate index construction to alleviate inefficiency induced by noise. When noise is in presence, RSR reveals unwanted features of estimation such as high, spurious negative auto correlation and standard deviation is two orders of magnitude above the true standard deviation (see Goetzmann [1992]).

Hoerl and Kennar (1970) first introduced the Ridge regression to correct the multi-collinearity problem in small samples. This estimator is belongs to a class of shrinkage estimators, in which the estimator is shrunk toward (or pulled toward) a prior mean (Colin Cameron, PravinTrivedi.) In this paper, annual index is used as prior information because its sample size is larger than quarterly index. In this regard, the ridge regression belongs to the Bayesian approach because it utilizes prior knowledge to gain better estimates.

Ordinary Least Square (OLS) method is Best Linear Unbiased Estimator (BLUE) when Gauss Markov theorem is satisfied. Ridge regression takes different approach in its estimation of unknowns. It minimizes mean squared error,

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not sum of squared residuals, which OLS minimizes. Ridge regression induces

slight bias in its estimation process by attaching synthetic data observations based

on prior information and obtain lower mean squared error than OLS does. Such

bias in its estimation procedure prohibited Ridge regression from being widely

used. However, lower mean squared error is a favorable feature in high frequency

commercial real estate index estimation procedure even though ridge filter

introduces slight bias.

Ridge regression generates biased coefficients with smaller variance.

Mean square error for $\hat{\beta}^*$ (Coefficients of Ridge regression) is decomposed into

two elements.

$$
\begin{aligned}
E\left[L_1^2(k)\right] &= E\left[\left(\hat{\beta}^* - \beta\right)'\left(\hat{\beta}^* - \beta\right)\right] \\
&= \sigma^2 \sum_1^p \frac{\lambda_i}{\left(\lambda_i + k\right)^2} + k^2 \beta'\left(\mathbf{X'X} + k\mathbf{I}\right)^{-2}\beta \\
&= \gamma_1(k) + \gamma_2(k)
\end{aligned}
\tag{2.12}
$$

The first element is the sum of the variance of the parameter estimates and the

second element is the square of bias introduced by using $\hat{\beta}^*$ rather than β. The

second term is 0 if k is 0. The first term is a monotonic decreasing function of k.

In contrast, the second term is a monotonic increasing function of k. Thus, as k

increases, the first term (sum of variance of the parameter) will decrease, but the

second term (the square of bias) will increase. However, it is not clear how we

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can decide the optimal "k" that introduces minimal bias and at the same time minimizes the sum of variance of parameter.

This study employs simulation analysis to examine such tradeoff between bias and efficiency improvement. Because true market value is not observable, this study utilizes Monte Carlo Simulation analysis. By using simulation analysis, we generate true market values based on the data generating process assumptions of how commercial real estate market values respond to news arriving in markets. Once we generate true market values under assumptions, the next step is to develop price indexes with true market value we generated[1]. We are mainly interested in the bias and precision of the estimation. In order to examine the different explanatory power of commercial real estate price indexes, we need to test statistics that demonstrate distinctive features of estimation. Because no one index construction method dominates other methods, each has its own weakness and strength. That is, under a certain condition, one method might be able to estimate better than others but under different circumstances, it may not work efficiently comparing to other methods. Considering unknown and unobservable characteristics of true value, simulations analysis is expected to shed light on the efficiency and accuracy of the various index construction methodologies.

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[1] Detailed simulation procedure will be discussed in the methodology section.

Literature Review

Literatures on the commercial real estate price index construction have focused on the questions pertinent to the disturbance or error terms: What is the smoothing and how does smoothing affect on the estimation of commercial price index at both individual and aggregated level? How can we adequately address various error terms in the model specification and which model specification is better? Does a transaction based index work better than appraisal based index and if so, what criteria should be used to measure the performance. Among this diversified body of research, this study will particularly focus on the review of the literatures on: the error terms pertinent to the smoothing, the development of the commercial real estate price (return) index and simulation analysis as an evaluation method.

Assumptions on disturbance or noise terms make each methodology unique. The most widely applied but strong assumption on the noise is that it is randomly distributed with mean zero and a constant variance. That is, each commercial property is homogeneous. It is rare to see these randomly distributed samples in the real world, however, because each individual property has its own idiosyncratic characteristics. To shed light on the unique behaviors of error terms in the construction of the commercial real estate value indexes, Clayton (2001) examines the lagging with particular emphasis on the variations at the level of the individual property appraisals. He argues that appraisers tend to weight more on

the previous appraisal values when the markets are more volatile. Along with this variation at the individual level, another main source of lagging is due to the temporal aggregation of transaction prices. Geltner (1993) asserts that temporal aggregation of the time series tends to underestimate the observed real estate index. He further states that empirical based investment analysis may produce wrong estimation because of the smoothing from temporal aggregation regardless of the usage of transaction based index.

At the earlier stage of the index development, appraised values are the most widely analyzed database because transactions of commercial real estate properties were not frequent enough to provide sufficient number of samples. This lack of data availability made researchers focus on the effect of appraised values on the commercial real estate index estimation (Geltner 1993, Giaccotto and Clapp 1992). NCREIF property index (NPI) estimates the change of prices and returns based on the appraisal values. Geltner and Goetzmann (2000) examine the revised version of NPI index using the repeated-sales method with the aim of eliminating "stale appraisal[2]" and seasonality problems.

The enhancement of econometric methodologies and the applications of those new methods improve the precision of the commercial real estate price index construction. Most widely used regression methods are "Hedonic" and

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[2] Stale valuations resulted from the appraisal schedule that serious reappraisals are conducted only yearly basis, but quarterly appraisal values are compiled into the NPI index. Hence, property values reported each quarter staggered throughout the year.

"Repeat-Sale (RS)" regression[3]. Bailey, Muth, and Nourse (1963) first proposed repeat-sale regression in order to control for the widely varying quality across properties; nonetheless, repeat-sale regression has a weakness in gathering a sufficient number of samples because it is restricted to the repeated transactions for the same properties. Furthermore, if properties change dramatically through renovation or remodeling, it is hard to eliminate these transactions which might bias estimates. Clapp and Giaccotto (1992) compare repeat-sale regression with assessed value regressions and point out that small size of sub-sample makes RS inefficient. In spite of the remaining flaws of RS method, it is still most widely used method because it provides good estimates under plausible assumptions.

Geltner (1999) employs simulation analysis to examine the bias and precision of housing investment risk with small sample. He focuses on the relationship between smoothing and noise under two different assumptions: informationally efficient or ineifficient market. He makes a comparison between three various specifications and points out that repeated sales regression may work efficiently at smaller geographical analysis units. Other study employing simulation analysis was done by Geltner (1997) to answer the two questions: Are appraisal procedure for individual property optimal for the new aggregate valuation applications? Can the share price information of publicly traded property companies help in the valuation of properties? He sets forth the two

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[3] Repeat sale regression is also called longitudinal and panel data regression because it is based on the database which compiles sale price at a different time point for the same property.

major implications from the simulations analysis. First, the share price

information will be favorable for appraisers to perform valuations in the future

and second, "repeat-sale" and "hedonic" method will dominate appraisal based

construction in measuring of the commercial property investment performance

Yet, few studies have been done to compare and contrast repeat-sale and

hedonic methodologies in the construction of the commercial property price

indexes. Even though earlier two studies done by Geltner employed simulation

analysis, they did not cover four property types—office, industrial, multi-family,

and retail. Furthermore, the simulation analysis on the recently developed MIT

transaction index may provide valuable insights on the performance of the new

index construction methods.

MIT Transaction Index

MIT transaction index based on the model developed by Fisher, Gatzlaff, Geltner,

and Haurin (2003), referred hereafter as FGGH. The model estimates demand

and supply side indexes separately by taking account heterogeneous

characteristics of the buyers and sellers with search model. Heterogeneity of the

buyers and sellers are expressed as different reservation price distribution with

respect to the various market conditions: "up" and "down" market. When and

only when the buyer's reservation price exceeds the seller's reservation price, we

can observe the transaction prices and therefore, the stochastic error term of the

model is non-orthogonal. That is, the model has a nonzero mean of error term

and if OLS is used to estimate the coefficients, we end up with biased coefficients

because the model violates the fundamental assumptions of OLS method, the

Gauss-Markov theorem. To address this non-orthogonality problem in the

estimation procedure, FGGH employs the Heckman procedure. MIT transaction

index is an extension of the FGGH in the sense that while it applies Heckman two

step methods to remove the estimation bias resulted from sample selection; it uses

appraisal or assessed value capturing all the hedonic characteristics of the

properties instead of using a vector of property specific variables. Moreover, it

utilizes the Ridge filter to dampen the noise of the each transaction which is

especially prevalent in small sample.

FGGH Model

To make the procedure of MIT transaction index clearer, we briefly

review the FGGH model and point out an extension of the FGGH of model. In

the FGGH model, the demand side reservation price is modeled by the equation

(1)

$$RP_{it}^b = \sum \alpha_j^b X_{ijt} + \sum \beta_t^b Z_t + \varepsilon_{it}^b \qquad (2.1)$$

The supply side reservation price is modeled by the equation (2)

$$RP_{it}^s = \sum \alpha_j^s X_{ijt} + \sum \beta_t^s Z_t + \varepsilon_{it}^s \qquad (2.2)$$

In equation (1) and (2), the variables are described below:

RP_{it}^b, RP_{it}^s = the natural logarithm of a buyer's (seller's) reservation price for asset i as of time t;

X_{ijt} = a vector of j asset-specific characteristics of the properties relevant to valuation;

Z_t = a vector of zero/one time-dummy variables;

ε_{it}^b, ε_{it}^s = normally distributed mean zero random errors;

We can observe the transaction prices if and only if the reservation price of potential buyers (demand side) exceeds the reservation price of the potential sellers (supply side) : $RP_{it}^b > RP_{it}^s$. The specification used for the MIT transaction index creation is indicated in equation ( )

$$P_{it} = \alpha A_{it} + \sum \beta_t Z_t + \sigma_{\varepsilon\eta} \lambda_{it} + \nu_{it} \qquad (2.3)$$

However, correction for this sample selection bias is beyond this study because the simulation model draws the observations from normal random distribution and thus in our simulated world, the transaction prices are not censored. The simplified model specification is provided in equation ()

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$$P_{it} = \alpha A_{it} + \sum \beta_t Z_t + \varepsilon_{it} \tag{2.4}$$

This model specification does not include the inverse Mills ratio because the error term in the simplified model for the simulation analysis is orthogonal, which means the error term has a zero mean.

The expanded version of FGGH model: MIT transaction Index

MIT transaction index expand the FGGH model by employing Clapp & Giacotto (1992) "assessed value method", which proves that the assessed value method estimates the residential price index more efficiently than Repeated Sales Regression does because assessed value method is able to utilize relatively large sample. The X_{ijt}, a vector of a cross-sectional variations across the properties, can be substituted by a scalar of A_{it}, which represents the assessed value of the properties. The original FGGH model is simplified by the following equation.

$$RP_{it}^b = \sum \alpha_j^b A_{it} + \sum \beta_t^b Z_t + \varepsilon_{it}^b \tag{2.5}$$

All other variables in the model specification is same with the FGGH model except for the appraised value, A_{it}. The equation for the seller is following the same substitution procedure.

Ridge Regression

MIT transaction index further extend the FGGH model by employing the

Ridge filters to remove the noise of the transaction prices in the quarterly based

index construction procedures. Real estate prices deviate from its true value

because each transaction is consummated by idiosyncratic buyer and seller, and

their negotiation ability is different. Generally, the noise in the transaction price

is more obvious with small samples.

Ridge filters dampen the noise in each transaction price by appending the

synthetic observations for each time period which is generated from a noise-free

index. MIT transaction index benchmarked the annual return index to generate

the Ridge observations for each quarter. Because ridge observation is generated

for each period (in MIT transaction index, quarter), its time dummy variables has

a diagonal square matrix of ones. To control the strength of the ridge

observations in each quarter, the ridge filter is weighted by the control factor of

"k." That is, by multiplying the ridge observations with different "k", the ridge

filter has varying effects on the estimation results: different coefficients for each

variable by different "k" and even sometimes different signs on coefficients.

Hoerl and Kennar (1970) first introduced the Ridge regression to solve the

multicollinearity in small samples. This estimator is belongs to a class of

shrinkage estimators, in which the estimator is shrunk toward (or pulled toward) a

prior mean in this paper, noise-free annual index (Colin Cameron, PravinTrivedi.)

The ridge regression belongs to the Bayesian approach because it utilizes prior

knowledge to gain better estimates.

Ordinary Least Square (OLS) method estimates unknown parameter β on

the basis of the Gauss-Markov theorem (GM theorem.) To be more specific, let

$\mathbf{Y} = \mathbf{X}\beta + \varepsilon$ as a standard model for the multiple linear regression and $\mathbf{X}$ is

$(n \times p)$ matrix and β is $(p \times 1)$ matrix. To satisfy GM, $\mathbf{X}$ is a full rank matrix,

$E[\varepsilon] = 0, and E[\varepsilon\varepsilon'] = \sigma^2 \mathbf{I}$. The unknown parameter β is estimated by following:

$$\beta = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y} \tag{2.6}$$

This is the result of the minimization of the sum of squared residuals, which can

be expressed by the following:

$$\varepsilon'\varepsilon = \left(y - X\hat{b}\right)'\left(y - X\hat{b}\right) \tag{2.7}$$

And the distance between the estimated coefficient and the true coefficient and its

expected value are following (λ_i denotes the eigenvalues of $\mathbf{X}'\mathbf{X}$):

$$L_1^2 = \left(\hat{\beta} - \beta\right)'\left(\hat{\beta} - \beta\right) \tag{2.8}$$

$$E\left[L_1^2\right] = \sigma^2 \text{Trace}(\mathbf{X}'\mathbf{X}) \tag{2.9}$$

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$$E\left[L_1^2\right] = \sigma^2 \sum_{i=1}^{p} (1/\lambda_i)$$ (2.10)

In contrast, the coefficients in ridge regression are estimated by following:

$$\hat{\beta}^* = [\mathbf{X}'\mathbf{X} + k\mathbf{I}]^{-1}\mathbf{X}'\mathbf{Y}$$ (2.11)

By design, it generates biased coefficients with smaller variance. Hoerl and

Kennard (1970) shows that mean square error for $\hat{\beta}^*$ (Coefficients of Ridge

regression) is decomposed into two elements. The mean squared error for $\hat{\beta}^*$ is

modeled by following equation:

$$
\begin{aligned}
E\left[L_1^2(k)\right] &= E\left[\left(\hat{\beta}^* - \beta\right)'\left(\hat{\beta}^* - \beta\right)\right] \\
&= \sigma^2 \sum_{1}^{p} \frac{\lambda_i}{\left(\lambda_i + k\right)^2} + k^2 \beta'\left(\mathbf{X}'\mathbf{X} + k\mathbf{I}\right)^{-2} \beta \\
&= \gamma_1(k) + \gamma_2(k)
\end{aligned}
$$ (2.12)

The first element is the sum of the variance of the parameter estimates and the

second element is the square of bias introduced by using $\hat{\beta}^*$ rather than β. The

second term is 0 if k is 0. The first term is a monotonic decreasing function of k.

In contrast, the second term is a monotonic increasing function of k.

Simulation Model and Analysis

A Monte-Carlo simulation analysis has been designed to investigate the error structures of commercial real estate sample second moments based on the MIT transaction index construction procedures. The length of the return sample time is taken to be 100 quarterly frequencies to reflect the publication length of the MIT transaction index. We use three different sample availability scenarios. Among them, an average observation of 12 and 25 per quarter is used to test small sample situation and we use average observation of 100 per quarter to examine relatively large sample. Considering the fact that in the thin market, transaction price observations of commercial real estate properties are dry up, our small sample scenario can be useful to examine metropolitan level subindices for specific property types. For the simulation, we use 100 independently identical draws, which means we have 100 different underlying histories based on which our news series will be generated.

To test the bias and precision of the simplified version of the MIT transaction index construction procedure, this study made four assumption – news arrival, market assumption, appraisal behavior, and the noise in transaction prices. The news series relevant to the value of the commercial real estate in each history is generated by the iid normal distribution. Innovation or news arrive in quarter "q" and reflect into the value of the commercial real estate properties. For each of

the hundred histories, the values of I_q are generated for 100 quarters as iid

drawings from normal (0, 0.05) random variable. After generating news series for

each quarter, we accumulate the news series by adding the current news with the

sum of the previous news series.

In each of the hundred underling histories, the true market returns are

derived by a moving average process which would be consistent with the

"sluggish" commercial real estate market. To be more specific, let I_q be the news

series arrived in quarter "q" and the let r_q be the true return. We assume that the

news series accumulate in the true return with different weights. For this

simulation analysis, we give weight on the recently arrived information 60% and

the the information arrived the previous quarter 40%, respectively.

$$r_q = 0.6 \times I_q + 0.4 \times I_{q-1}$$ (3.1)

The true market return is the accumulated values of the news series, and the

weight on the previous and current news depends on the perception of the market

on the innovation. Under sluggish market assumption, the market's perception on

the arrival of the news follows the above formula. In each simulated 100-quarter

history, realized true market value (level) is generated by accumulating true

market returns throughout the hundred quarters.

These realized true market value is used to generate the appraisal values

which are assumed in the MIT transaction procedure to capture the whole hedonic characteristics of the commercial real estate properties. Appraisal values are calculated by averaging the current and previous four quarters true market value. To be more specific, let A_q be the appraisal values of quarter "q" and r_q be the true market return in quarter "q". Appraisal values are derived by the following equation.

$$A_q = 0.2 \times r_q + 0.2 \times r_{q-1} + 0.2 \times r_{q-2} + 0.2 \times r_{q-3} + 0.2 \times r_{q-4} \qquad (3.2)$$

In generating the individual commercial real estate price observations which are the left hand side variable in the estimation model, a number, N, of individual properties values are generated based on the assumption that a true value for each property equals to the current market value. More specifically, a true market value for each quarter is calculated by the following equations:

$$V_q = V_{q-1} + r_q \qquad (3.3)$$

While the realized true value of individual properties equal to this market value, the observed transaction price of individual properties has random transaction price "noise":

$$V_q^{j*} = V_q + e_q^j \tag{3.4}$$

V_q^{j*} : The observed individual transaction price

V_q : True market value in quarter "q"

$e_q^j \sim N(0, 0.1)$: Noise or error in each transactions

The random error is drawn from the iid normal distribution with mean zero and 10% of variance. It is further assumed that each transaction occurs every five years for each property and therefore each property transacts five times for 25years (100quarters) in this simulation analysis. The transaction price observations, however, vary by the sample size scenario: large sample with 100 transaction price observations and small sample with 12 or 25 transaction price observations.

Within each 100 history, the market return index is estimated by first regressing transaction prices onto appraisal values representing hedonic characteristics and the time dummies which have value "1" when each property in the index database transacts and otherwise value "0." To calculate market index, we plug the representative appraisal value in a quarter into the above specification.

$$\hat{V}_{m,t} = \hat{\alpha} \times A_{m,t} + \sum_{t=1}^{100} \hat{\beta}_t \times D_t \tag{3.5}$$

$\hat{V}_{m,t}$: Estimated Market Index

$\hat{\alpha}$: Coefficients of Appraisals

$A_{m,t}$: Appraisal value of Representative property

$\hat{\beta}$: Coefficients of time dummy variables

D_t : Time dummy variables for each quarter

We use OLS regression methods to estimate coefficients for each variable.

Estimated market index-return is calculated by subtracting the current estimated

market index from the previous quarter's estimated market index.

This study use the same sample second moments which Geltner (1997)

employed in his study to examine the risk characteristics of the commercial real

estate. We investigate the quarterly-frequency time-series sample moments of the

25 year simulated histories: first, the quarterly return volatility; second, the beta;

third, the first order auto correlation coefficient:

$$\text{STD}[r_q] = \sqrt{\text{VAR}[r_q]} = \sqrt{\frac{1}{99}\sum_{q=1}^{100}(r_q - \bar{r}_q)^2} \qquad (3.6)$$

$$\text{BETA}[r_q, I_q] = \frac{\sum_{q=1}^{100}(r_q - \bar{r}_q)(I_q - \bar{I}_q)}{\sum_{q=1}^{100}(I_q - \bar{I}_q)^2} \qquad (3.7)$$

$$\text{AUTO}[r_q] = \frac{\sum_{q=2}^{100}(r_q - \bar{r}_q)(r_{q-1} - \bar{r}_{q-1})}{\sum_{q=2}^{100}(r_q - \bar{r}_q)^2} \qquad (3.8)$$

Simulation Results on errors

This section reports the results of the simulation analysis described above. Various statistics on errors, calculated by subtracting the estimated market index from true market return, is summarized in table 1. Through the simulation analysis, usually unobservable true market value becomes available and with the true market value, this study is able to report the behavior of the errors. Table 1 shows mean, mean absolute error, RMSE, standard deviation, and 1st order auto-correlation across the 100 repetitions.

The bias of the estimation is examined as the deviation of estimated return from its true return and the precision of the estimation is reported as the standard deviation of error. In this study, we test the precision and the bias of the estimation with three different sample numbers. For the smallest sample size scenario, we generate 250 properties and assume that the transaction consummated for every five years. Thus, the price observations under the smallest sample size scenario are 12.5, which is calculated by dividing 250 underlying properties by 20 quarters (5years.) The price observations with 500 and 1000 properties are 25 and 50 for each quarter, respectively.

Across the three scenarios, the index generated by MIT transaction index method shows very small bias, and the precision of the estimation is increased by adding more observations in the index creation procedure. The mean of the error

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almost converges to ideal value of 0. Standard deviation, RMSE and the mean

absolute error shows a similar pattern of increasing precision with bigger sample

size. In addition to the precision and the bias of the estimation, the table reports

the 1st order auto-correlation of the error. Unlike the previous four statistics, the

1st order auto-correlation does not decrease as more observation become

available. That is, regardless of the sample size, the error does not cumulate over

time. This is shown in figure 1. The right axis represents the error in the each

quarter, and the each bar represents the magnitude of the error in each quarter.

The left axis represents the cumulative error across quarters and the dark line

shows the vanishing pattern of error across quarters.

Table 1 Errors of three observation scenarios

	Sim250	Sim500	Sim1000
Mean Error:			
Mean	0.00001	-0.00002	0.00000
Standard Deviation	0.00027	0.00021	0.00013
RMSE:			
Mean	0.03679	0.02574	0.01683
Standard Deviation	0.00314	0.00239	0.01852
Mean Absolute Error:			
Mean	0.03025	0.02061	0.01474
Standard Deviation	0.02920	0.00187	0.00137
Standard Error:			
Mean	0.03697	0.02587	0.01849
Standard Deviation	0.00315	0.00240	0.00161
1st-order Autocorrelation of Errors:			
Mean	-0.50295	-0.49592	-0.49933
Standard Deviation	0.07696	0.07327	0.06848
2nd-order Autocorrelation of Erros:			
Mean	0.01239	0.01011	-0.00369
Standard Deviation	0.13131	0.12221	0.11482

Error = True Market Return - Estimated Market Return (Market Index)
RMSE = (Average of Mean Sqared Error) ^ (1/2)

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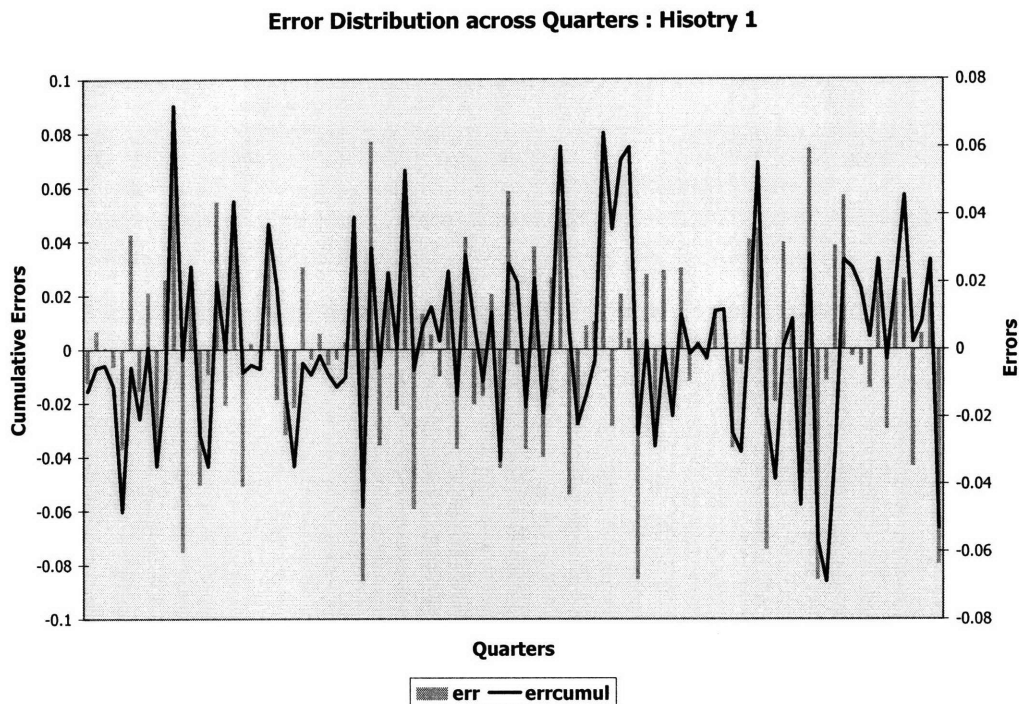**Error Distribution across Quarters : Hisotry 1**

Figure 1 Error Distribution across Quarters

Table 2 reports the Comparison Statistics for estimated index vs. true second-moments a measure of the contemporaneous correlation between true and estimated return. The VOL and BETA is the ratio of the estimated index to the true market volatility. The AUTO is the arithmetic difference between true market values and estimated index of the sample moment. Because the VOL and the BETA measure the ratio of estimated index to the true market value, the ideal number of these sample second moment is "1." The ideal number of the AUTO, however, is 0 because it measures the difference from sample second moment of the true market value. Moreover, the table reports the simple correlation between

estimated return index and the true market return. Because CORR measures the temporal fit of the estimated index, the bigger CORR implies that estimated index move closer with the true market value.

The simulation results are consistent with a qualitative understanding of the effect that the smoothing and the noise have on the estimated index. More specifically, the VOL biased most with the smallest sample because of the strong effect of noise on the estimated index. The VOL diminishes as the sample size increases and the VOL with average 50 observations per quarter is about 1.13, which is quite small comparing to the 1.44 in the simulation with average 12.5 observations per quarter. The BETA converges to the ideal number "1" regardless of the sample size, the result which reinvigorates our understanding that transaction based index is not subject to the smoothing. Furthermore, larger sample size obtains better temporal fit which is measured by the CORR.

Table 2 Comparison Statistics for estimated index vs. true second-moments

	Sim250	Sim500	Sim1000
VOL:			
Mean	1.444899	1.240197	1.128566
Standard Deviation	0.106734	0.069561	0.047744
BETA:			
Mean	0.994836	1.01695	1.00333
Standard Deviation	0.122307	0.085971	0.062233
AUTO:			
Mean	-0.48382	-0.31623	-0.20086
Standard Deviation	0.095554	0.076764	0.057184
CORR:			
Mean	0.697689	0.811188	0.888419
Standard Deviation	0.055581	0.039807	0.023871

VOL = STD(estimated return) / STD(True Return)

BETA = COV(est. return, Innovations) / COV(True return, Innovations)

AUTO = Auto_CORR(est.return @ t-1, est. return @ t) - Auto_CORR(true return @ t-1, true return @ t)

CORR = CORR(est. return, true return)

Figure 2 and 3 displays the index traced out for a single example 25 year simulated history. The true underlying market value is marked as a solid line and the estimated index, the appraisal value and the mean price is shown as a dashed line. The smoothing and lagging is apparent in the representative appraisal value and the estimated index leads the appraisal value. It seems the mean price and the estimated index move closely and the deviation from the true value is not significant.

The bigger bias in simulation analysis with small sample supports the

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application of the ridge filter in the MIT transaction index. In the observation

poor scenario, the MIT transaction index without Ridge filter displays greater

volatility with the sample size of average 12.5 observations per quarter.

Nonetheless, MIT transaction index with greater sample size of average 50

observations per quarter does not seem to pose a great problem though it is still

sensitive to the noise. For instance, the overestimation of volatility is just over

10%.

History1: True, Price, Appraisals and Market Index with 250 samples
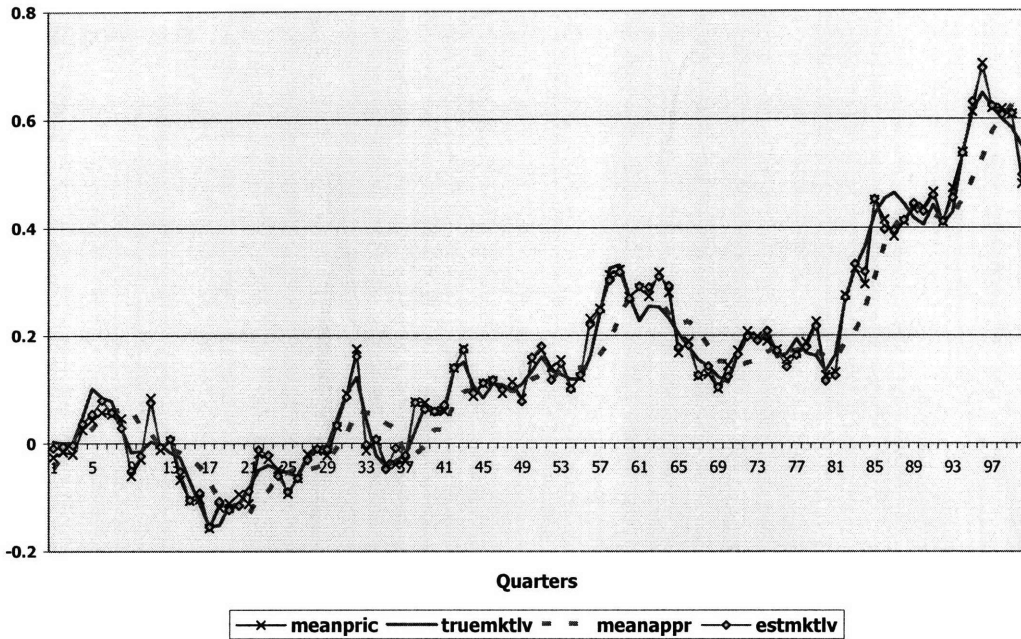
Figure 2 Example simulated history, showing true(solid line), estimated MIT transaction index level, mean price, and appraisal value under observation poor scenario

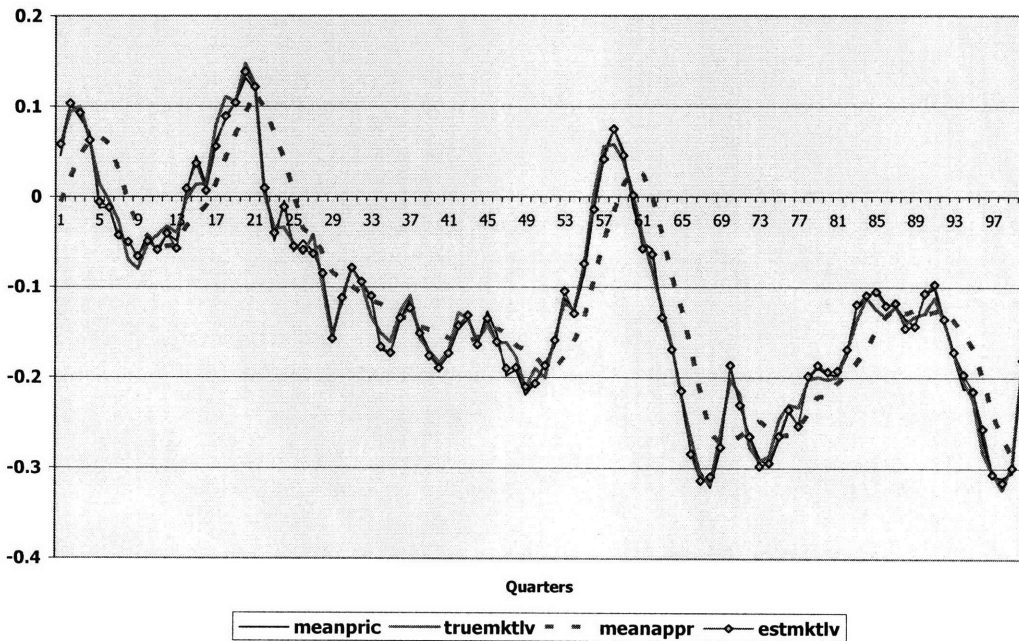**History1: True, Price, Appraisals and Market Index with 1000 observations**

Figure 3 Figure 2 Example simulated history, showing true(solid line), estimated MIT transaction index level, mean price, and appraisal value under observation rich scenario

The simulation Result with Ridge Regression

In this section, we report the simulation analysis results with Ridge filters. The simulation analysis with ridge filter is conducted to examine the selection process of a better estimate of β. Hoerl and Kennard (1970) pointed out that the best method for obtaining a better estimate of β is to use $k_i = k$ for all i and use the Ridge Trace to select a k and a unique $\hat{\beta}^*$. More specifically, they proposed four guidelines: (1) At a certain value k the system will stabilize; (2) Coefficients will not have unreasonable absolute value; (3) Incorrect sign of coefficients at k=0 will change to the proper sign; (4) The residual sum of squares will not increased to an unreasonable value. This paper borrows these guidelines to find a better estimate of β.

Table () reports the simulation results of the second moment of sample. VOL and BETA decreases monotonically as k increases. For instance, we obtain the ideal value of "1" for the VOL between k=3 and 4 and after this range, it increases. A possible explanation on the diminishing pattern of the VOL could be the small variance in the estimated market index. The coefficients of Ridge regression bias away from those of OLS and shrink toward the moment restriction, in this study the annual noise-free index. Accordingly, the estimated market indexes, calculated by the model specification with better coefficients, obtain smaller standard error.

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Table2 reports the same sample second moments with different sample numbers. General movement of the statistics shows similar patterns, but the numbers are slightly different from the previous one because of the more number of samples. VOL and BETA under 25 observations has smaller bias with OLS method (k=0) comparing to those under 12 observations scenario. Nevertheless, VOL and BETA under 12 observations scenario shrink much faster than those under 25 observations scenario. This might imply that when we use ridge filter with small sample, its influence on the estimation is stronger than when ridge filter is used in the estimation with larger sample. In our model, the VOL and BETA might shrink faster under small sample scenario because of the relatively larger effect of noise on the coefficients. This seems consistent with the guidelines which were proposed by Hoerl and Kennard (1970) because they argue that wrong coefficients tend to shrink quickly when ridge filter is applied to the estimation.

VOL with 25 observations per quarter obtains its ideal value "1" between k=4 and 5, which is actually used to build the MIT transaction index. The CORR representing the temporal fit of the estimated index to true market value has the greatest value at k=3.

Figure 1-4 displays the mean squared error and RMSE with 12 and 25 observations. Ridge regression is designed to minimize the mean squared error, not the sum of squares of residuals which is the least square method tends to

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minimize. That is, with small bias Ridge regression achieve smaller mean square error. Mean square error with 12 and 25 observations show similar decreasing pattern until k approximately reaches 4 or 5 and after it hits the low bound, mean square error increases with larger k. The concaveness of the mean square error function of k might be due to the different magnitude of the monotonic increasing squared of bias and the monotonic decreasing variance caused by the ridge filter[4]. This results in the improvement of the mean square error of estimation and predictions with substantially low variance at the cost of little bias. Indeed, these properties of ridge regression dampen the noise effect on the quarterly MIT transaction index. Furthermore, it might be able to improve the efficiency of the sub-indices construction.

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[4] Hoerl and Kennard (1970) provide the qualitative form of relationship between the variances, squared bias and the parameter k, and as k increases the mean square error of ridge coefficients initially decrease and then increase. This pattern is consistent with our simulated mean square error.

Table 3Comparison statistics for estimated MIT transaction index vs. true second-moments with 12 observations per quarter (major noise)

	k=0	K=1	k=2	k=3	k=4	k=5	k=6	k=7	K=8	k=9	k=10
VOL:											
Mean	1.513	1.429	1.251	1.082	0.963	0.890	0.848	0.824	0.810	0.802	0.797
Standard Deviation	0.091	0.084	0.073	0.070	0.073	0.079	0.083	0.086	0.089	0.090	0.091
BETA:											
Mean	1.125	1.073	0.956	0.834	0.735	0.661	0.609	0.571	0.543	0.522	0.507
Standard Deviation	0.139	0.132	0.116	0.101	0.092	0.089	0.088	0.089	0.090	0.091	0.092
CORR:											
Mean	0.718	0.725	0.740	0.749	0.744	0.727	0.705	0.682	0.661	0.644	0.629
Standard Deviation	0.053	0.052	0.050	0.048	0.047	0.049	0.053	0.059	0.064	0.068	0.071

Table 4 Comparison statistics for estimated MIT transaction index vs. true second-moments with 25 observations per quarter (minor noise)

|	k=0	k=1	k=2	k=3	k=4	k=5	k=6	k=7	K=8	k=9	k=10
VOL:											
Mean	1.343	1.307	1.217	1.108	1.009	0.930	0.873	0.834	0.807	0.789	0.776
Standard Deviation	0.064	0.061	0.057	0.058	0.064	0.072	0.080	0.086	0.091	0.095	0.098
BETA:											
Mean	1.142	1.115	1.043	0.951	0.861	0.782	0.717	0.665	0.624	0.592	0.566
Standard Deviation	0.089	0.086	0.080	0.076	0.075	0.077	0.080	0.084	0.087	0.090	0.092
CORR:											
Mean	0.817	0.819	0.824	0.828	0.826	0.816	0.800	0.779	0.758	0.736	0.717
Standard Deviation	0.036	0.036	0.034	0.033	0.032	0.031	0.032	0.034	0.038	0.041	0.045

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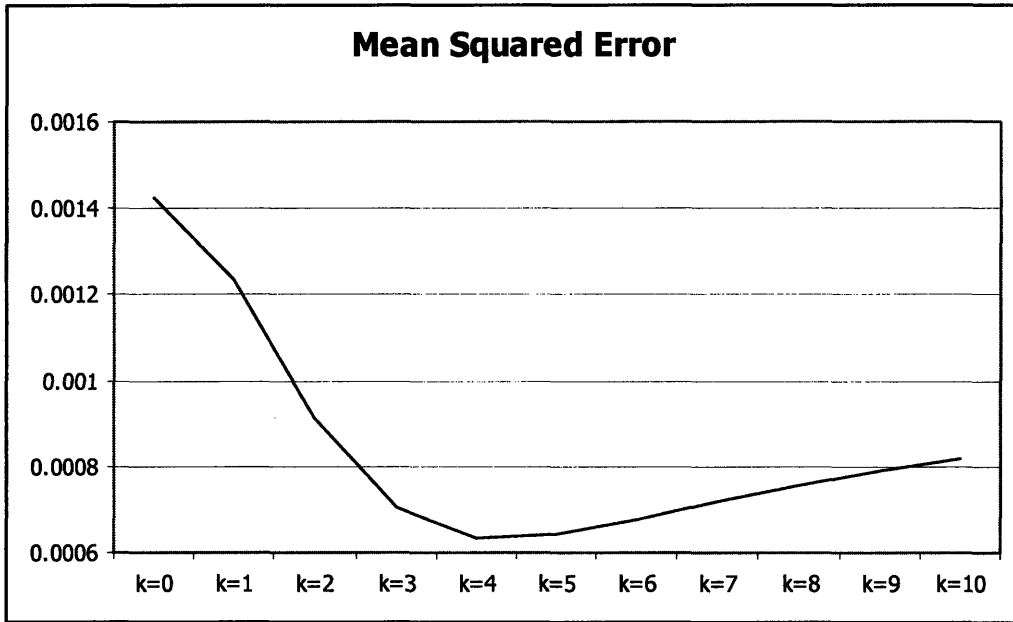
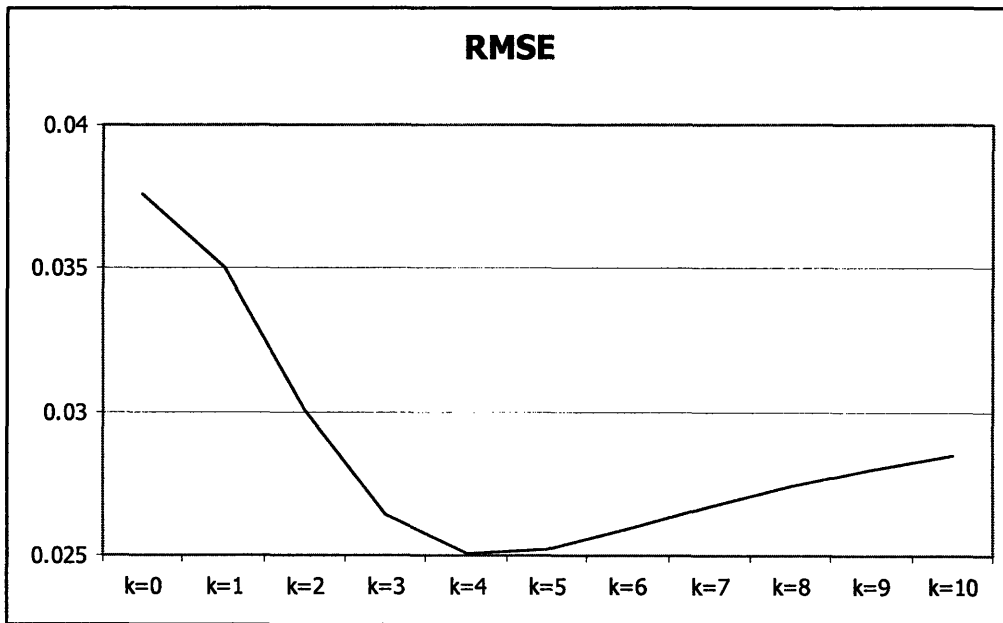**Figure 4 Mean Squared Error with 12 observations per quarter**

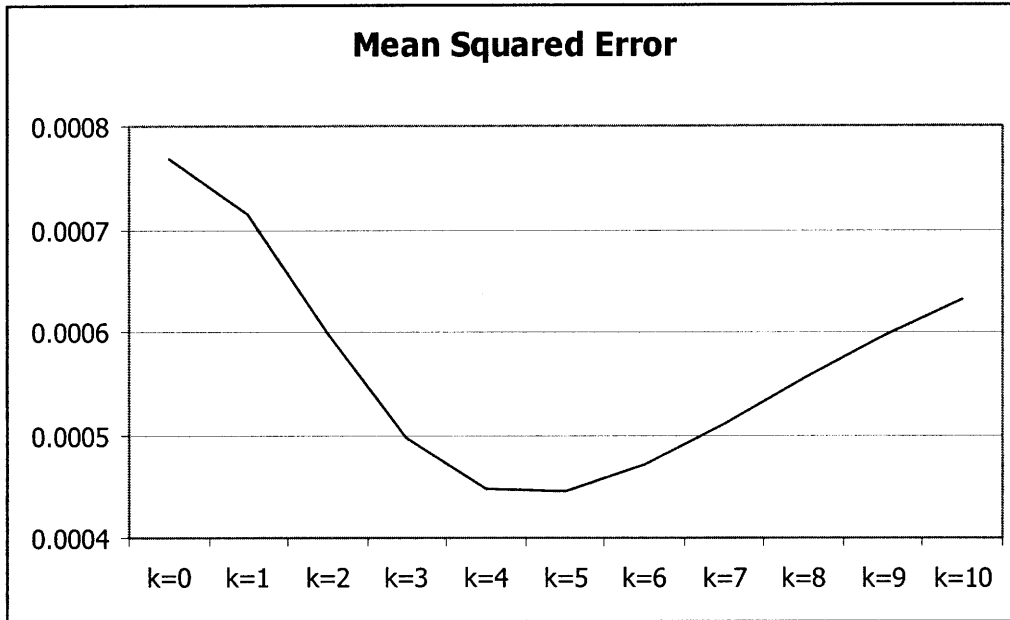**Figure 5 RMSE with 12 observations per quarter**

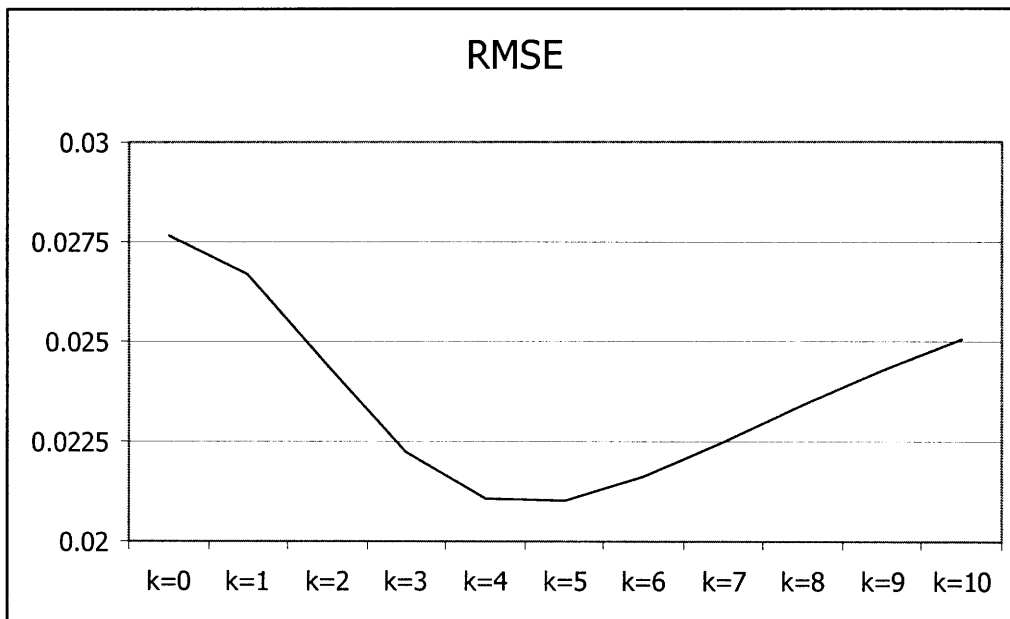**Figure 6 Mean Squared Error with 25 observations per quarter**

Figure 7 RMSE with 25 observations per quarter

Figure ( ) displays the qualitative form of the relationship between the ridge coefficients and k. The ridge trace of appraisal and the time dummies are separately reported because the value of appraisal and time dummies varies widely. The ridge trace is generated by calculating the 11 regressions using $\hat{\beta}^* = [\mathbf{X'X} + k\mathbf{I}]^{-1}\mathbf{X'Y}$ and 11 different k ranging from 0 to 10. The ridge trace enables graphical assessments by collapsing the dimensions related to dependent variables into a two-dimensional portrayal. Indeed, ridge trace makes it possible to observe sensitivity of the coefficients with respect to k. That is, we can examine the stability of the coefficients and we can track the change of coefficients graphically. For example, some wrongfully estimated coefficients with OLS method tend to shrink toward the moment restrictions specified in the Ridge regression, and the ridge trace highlight relative shrinkage speed and sometimes even the change of the sign of coefficients.

The ridge trace for appraisal values behaves in a similar way under 12 and 25 observations per quarter. The coefficient of the appraisal values increase from its origin, which is the coefficient estimated by OLS, and after it passes around the value 5 of k, it stabilizes. Ridge trace for time dummies is selected from the 100 overall periods to clarify graphical implications of the ridge trace. The ridge traces of time dummies with 12 and 25 observations per quarter share the similarity with the ridge traces of appraisal values in the light that the ridge traces stabilize after passing around the value 4 or 5 of k. A group of ridge traces do not

deviate much from its OLS origin, yet another group of the ridge traces shrink

toward some points, the points which is the moment restrictions specified in the

ridge regression. Taken together with the previous result from the graphical

examination of RMSE and mean squared error, between 4 and 5 the system seems
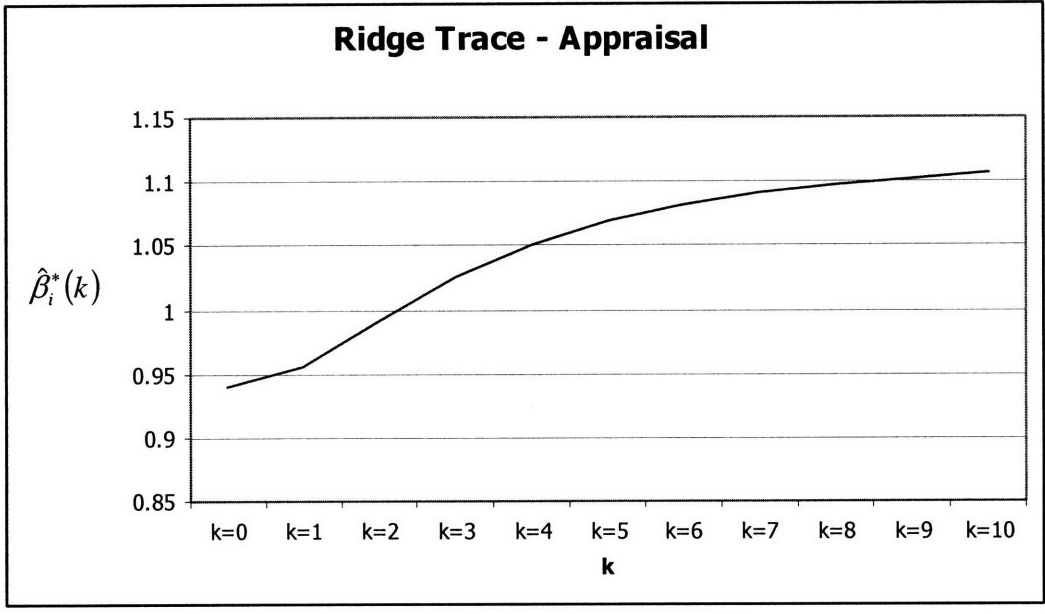
stabilized and achieve minimum mean squared error.

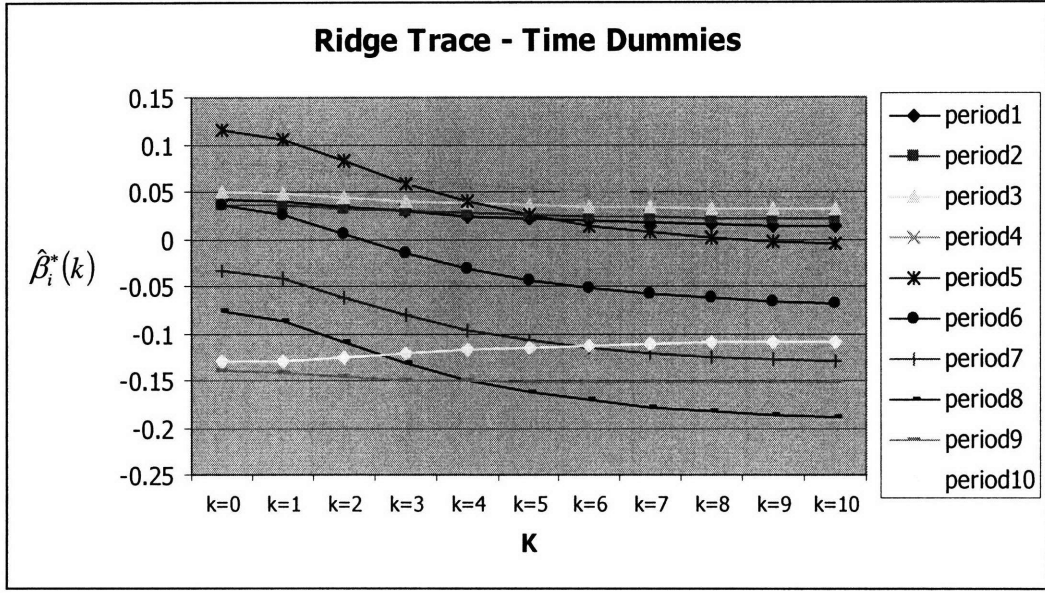**Figure 8 Ridge Trace of Appraisal Value (n=12)**
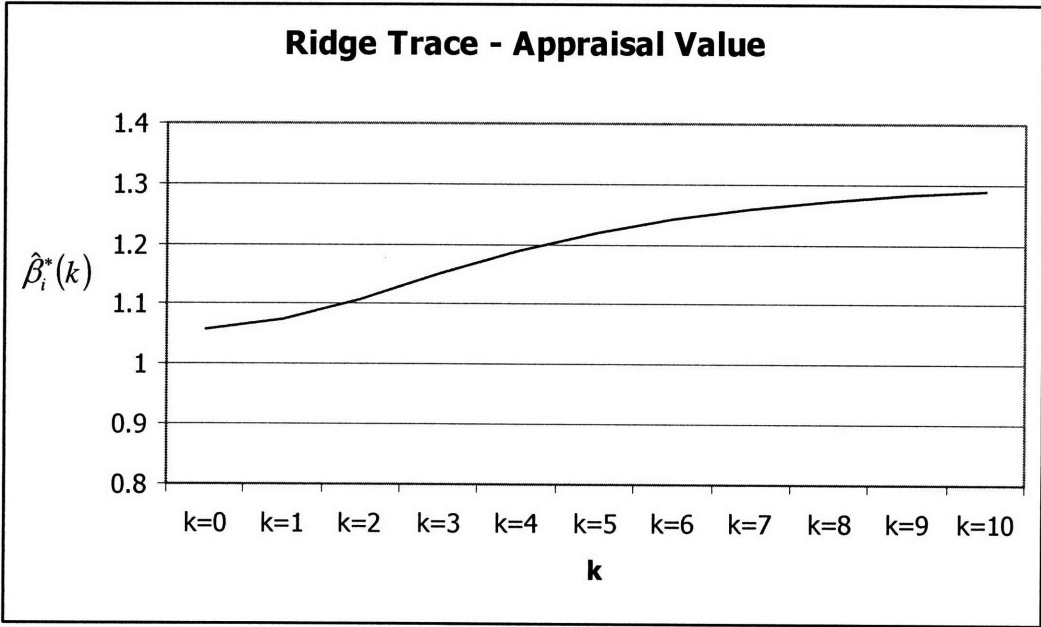

Figure 9 Ridge Trace of Time Dummies (n=12)

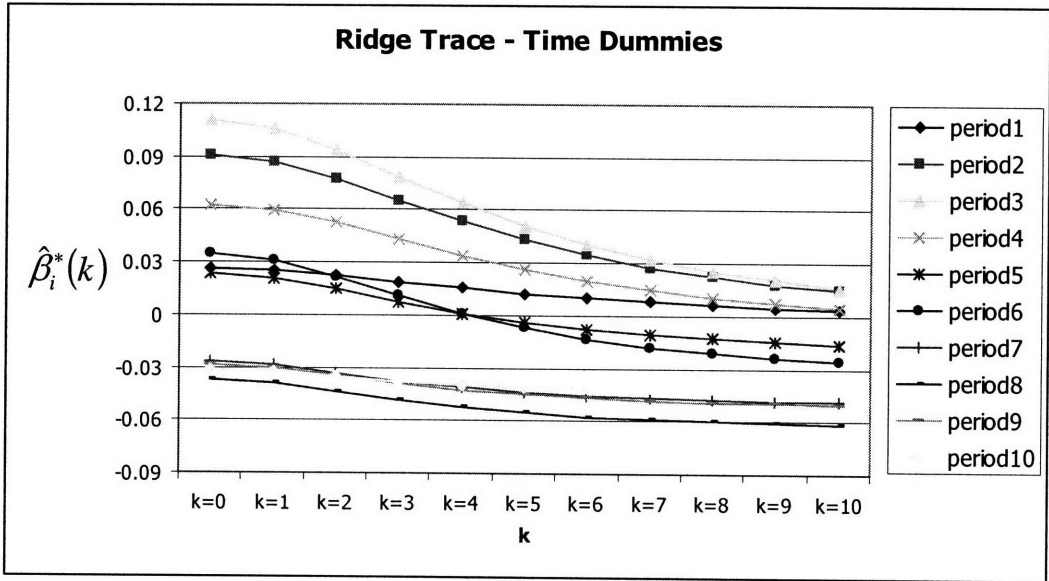**Figure 10 Ridge Trace of Appraisal (n=25)**

Figure 11 Ridge Trace of Time Dummies (n=25)

Conclusion

This paper was designed to examine the bias and the precision of the MIT

transaction index. To test the sample second moment, this study employs the

Monte-Carlo simulation analysis through which we generate true market values,

transaction prices and the appraisal values. The MIT transaction index extends

the use of the last model by Fisher, Gatzlaff, Geltner and Haurin (2003), which

addresses the sample selection bias and the noise effect of the transaction prices in

small sample. To correct sample selection bias, MIT transaction index use the

Heckman two-step and to dampen the effect of noise in transaction prices on

estimations, it applies the Ridge filters into the index creation procedures. Among

them, the primary focus of this study is on: (1) investigating error structures,

defined by the difference between the true market return and the estimated index

return; (2) examining the efficiency of the ridge filter as noise filter; (3) selecting

the proper value of weight factor "k."

The findings on the error structures are as follows. First, MIT transaction

index does not suffer from smoothing and lagging which is apparent in the

appraisal based index. Second, with larger sample size, the estimated market

indexes become close to the true market values. In addition, the estimated market

index is more biased when it uses small number of sample. Third, the error does

not cumulate over time regardless of the sample size.

Through the comparison of sample second moments between the true and

the estimated market index, we found that under rich observations (50 price observations per quarter) scenario VOL and BETA approaches ideal value of "1." In addition, the temporal fit, which is examined by CORR, shows better number under the rich observation scenario. Like the findings from the examination of the error structure, the estimation results improve by using more observations in the index construction procedure.

The graphical presentation of the estimated market index, true value, the mean of price and appraisal value provides the clear sense of different characteristics of three values. The MIT transaction index leads the mean of appraisal, which is calculated by the similar procedure of NPI, and move closely with the mean of price. Nonetheless, under the observation poor scenario (12.5 price observations per quarter) the mean of the price deviates farther away from the true values than under the observation rich scenario. Taken together, under the observation poor scenario the noise effect on the index is more apparent than under the observation rich scenario because the MIT transaction index seems to have a tendency to follow the mean price.

To correct noise effect on the estimation procedure under the observation poor scenario, this study examines the efficiency of the Ridge regression. First, we examine the qualitative form of relationship between ridge coefficients and the control factor of "k." Generally, ridge trace is not significantly changed with respect to "k." It might be argued that the noise effect is not severe in the

estimation procedure, yet the introduction of ridge regression apparently improves the estimation. Indeed, between $k = 4$ and 5 ridge coefficients stabilize. Second, to investigate the improvement of ridge regression, we examine the mean square error. Between $k = 4$ and 5, mean square error, defined by the difference between the true and estimated market index, achieve the lowest value. Taken together, the system stabilizes between $k = 4$ and 5 and achieve the minimum mean squared error. This result leads to the conclusion that we can obtain better estimation using the control factor of 5.

Finally, here are some thoughts about further studies. The MIT transaction index employs the Ridge filters, which is based on the Bayesian methods, to dampen the noise in each transaction prices without causing lagging and smoothing. While the MIT transaction index makes use of a well behaved noise-free annual index as prior knowledge, it seems worthwhile to compare the ridge regression results with Generalized Lease Square (GLS) which roots on the classical econometrics and gains its popularity in recent days. In addition, comparative study with RSR (Repeat Sales Regression) is also helpful to understand the different approaches of index construction. Also, a study on the effect of spatial auto-correlation on the index creation procedures expects to improve our understanding on the price change in the real world. Nevertheless, the simulation results of MIT transaction index assure the application of the index creation methods into the real world.

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