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POSICAST VERSUS CONVENTIONAL TYPES OF COMPENSATION IN A CONTROL SYSTEM

by

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B.S., Virginia Polytechnic Institute
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Certified by ___
Thesis Supervisor

Accepted by ____________ __________
Chairman, Departmental Committee on Graduate Students

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Submitted to the Department of Electrical Engineering on May 18, 1962 in partial fulfillment of the requirements for the degree of Master of Science.

ABSTRACT

The purpose of this thesis research was to investigate Posicast, a novel type of compensation for controlling lightly-damped systems proposed by O. J. M. Smith in his book, Feedback Control Systems[3], and to compare a system utilizing Posicast with one utilizing conventional types of compensation.

The research was largely experimental with all testing being performed on a Reeves Analog Computer.

In general the results were quite favorable. The system compensated with Posicast showed a fast response to deterministic inputs and also was well behaved when subjected to a random input. The transient response to a load disturbance was not as good in the system utilizing Posicast as it was in the system utilizing conventional compensation, but the steady-state response was slightly better. The frequency response exhibited no resonance peaking at all, but did exhibit ripples at frequencies above the half-power frequency.

As would be expected, any decision as to whether Posicast or conventional compensation should be used would depend upon the specific application. I do feel that Posicast definitely shows promise and is worthy of consideration where lightly-damped systems must be controlled.

Thesis Supervisor: George C. Newton, Jr.
Title: Associate Professor of Electrical Engineering

ACKNOWLEDGEMENT

I wish to thank Professor G. C. Newton, Jr. for his patience and guidance in supervising this thesis. Thanks also to Mr. G. B. Skelton of the Electronic Systems Laboratory who offered numerous helpful suggestions concerning testing on the analog computer.

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POSICAST VERSUS CONVENTIONAL TYPES OF COMPENSATION IN A CONTROL SYSTEM

CHAPTER 1

INTRODUCTION AND SUMMARY

Introduction

The problem of compensating a control system which is very lightly damped has long confronted control engineers. Numerous solutions to this problem have been proposed and utilized with varying degrees of success. O. J. M. Smith in his book, Feedback Control Systems[3], suggests another solution to this problem. Smith calls his method "Posicast",* and it is the purpose of this thesis to investigate Posicast and compare it to a conventional type of compensation.

Theory of Posicast

Posicast can best be understood by examining an analogous situation. Consider the suspended weight shown in Figure 1. The object is to move the weight from position 1 to position 2 without exciting oscillations. First, the support of the weight is moved one-half the way from position 1^1 to 2^1. The weight swings from position 1 to 2, one-half of a cycle, and stops before beginning its motion back in the direction of position 1.

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* Because of the analogy of his method to casting a fly, Smith first called his method "positive-cast". He then shortened this to the present name, Posicast.

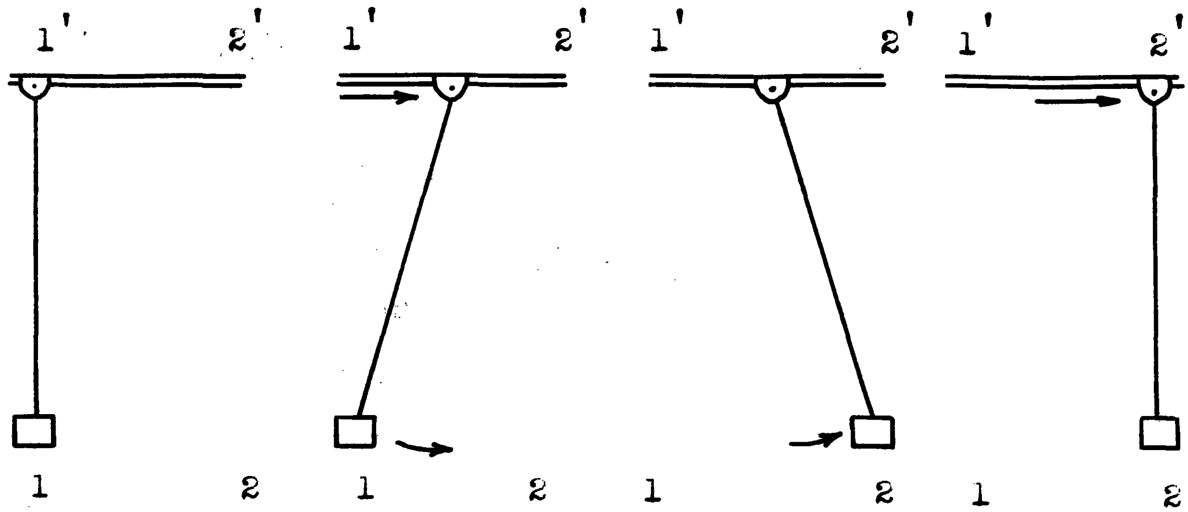


Figure 1.

Pictorial Description of Posicast.

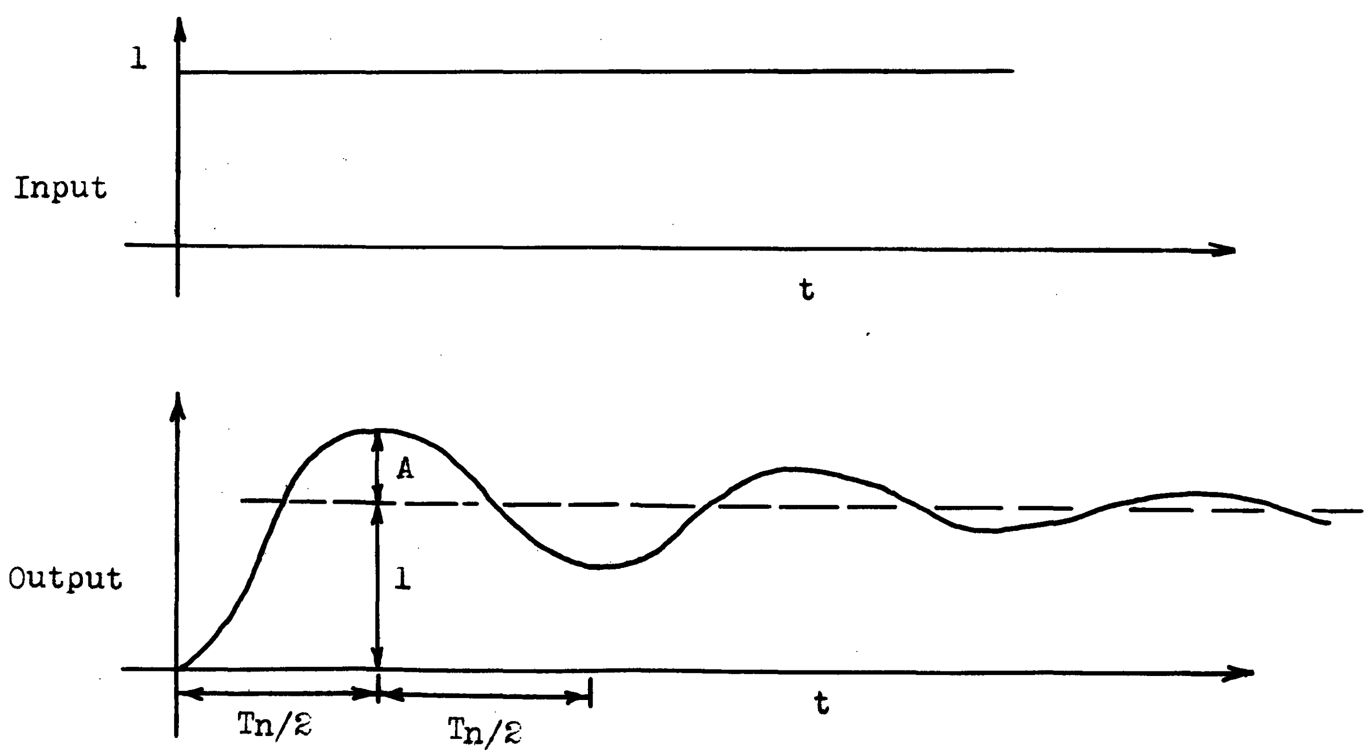


Figure 2.

Step Response of a Typical Lightly-Damped System.

At the instant in which the weight is stopped, the support is quickly moved to position 2^1, relaxing the system or removing all energy from it. The weight now having neither driving force nor momentum remains at position 2, the desired position.

Half-cycle Posicast functions in a similar manner. For example, consider the unit step response of a typical lightly-damped uncompensated system shown in Figure 2 where T_n is the natural period of this system. Posicast acts to subtract a portion of the input signal so that the first output peak instead of having the value 1 + A will have the value 1. This means that Posicast must allow only $\frac{1}{1+A}$ of the input signal to be seen by the system initially. Then at the time $T_n/2$ seconds later when the system output has reached the value 1 and stopped for an instant before swinging back toward a lesser value, Posicast must act to bring the input seen by the system to 1 and thus reduce the actuating error to zero. The system now having neither driving force nor momentum will remain at rest at the desired position. A block diagram of a lightly-damped system, $W_{(s)}$, utilizing Posicast is shown in Figure 3. The input operated upon by Posicast and the corresponding output are shown in Figure 4.

In the frequency domain Posicast can be shown to have an infinite number of zeros.

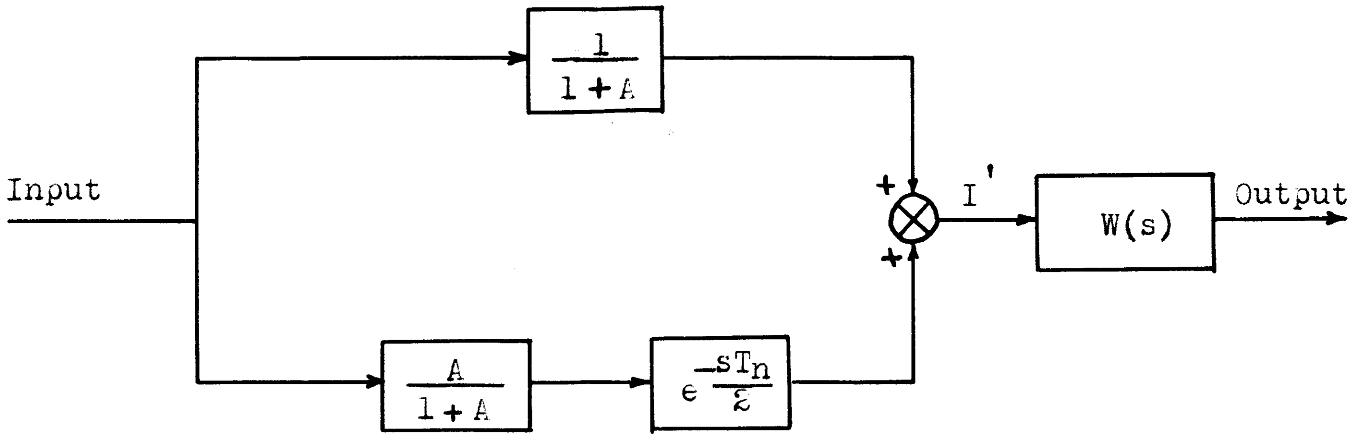


Figure 3.
A System Utilizing Posicast.

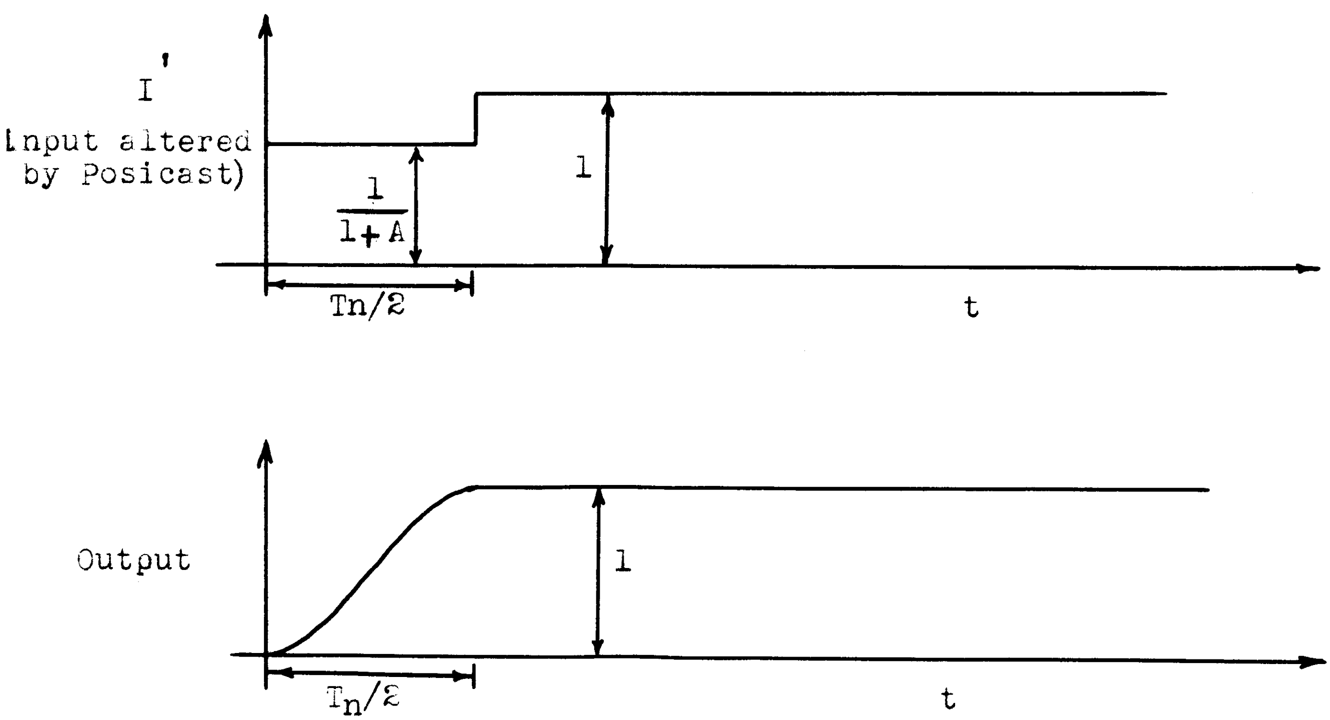


Figure 4.
Step Response of Lightly-Damped System Utilizing Posicast.

$$\frac{1}{1+A} + \frac{A}{1+A} e^{-sT_n/2} = 0$$

now $T_n = \frac{2\pi}{\omega_n}$

$$\therefore \frac{1}{1+A} + \frac{A}{1+A} e^{-s\pi/\omega_n} = 0$$

$$e^{-(\sigma + j\omega)\pi/\omega_n} = -\frac{1}{A}$$

$$e^{-(\sigma + j\omega)\pi/\omega_n} = \frac{1}{A} e^{-j\pi(1+2n)} \qquad n \text{ is any integer.}$$

$$-(\sigma + j\omega)\pi/\omega_n = \ln\frac{1}{A} - j\pi(1+2n)$$

$$\sigma = -\frac{\omega_n}{\pi}\ln\frac{1}{A}$$

$$\omega = \omega_n(1+2n)$$

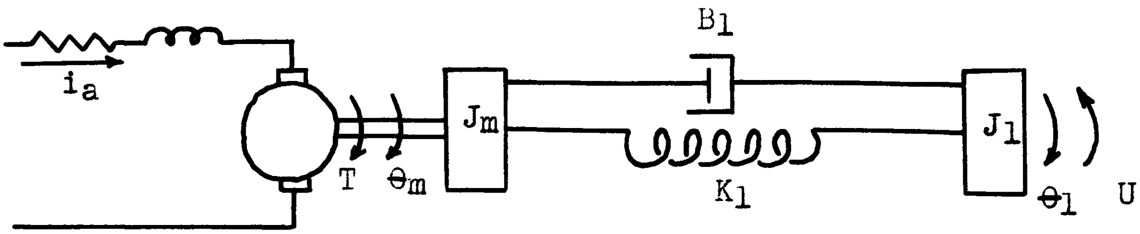
The first pair of zeros at $S = -\frac{\omega_n}{\pi}\ln\frac{1}{A} \pm j\omega_n$ cancels the pair of resonant poles of $W_{(s)}$ and eliminates any resonant peaking. The higher frequency pairs of zeros produce a pronounced ripple in the frequency response. This effect is illustrated in Figures 19 through 24 which show the actual frequency response of the system investigated and is discussed in detail following these figures.

System Investigated

The lightly-damped system to which Posicast was applied is shown in Figure 5. This system was one under study by Professor G. C. Newton, Jr., who supervised this thesis. The load is a tracking-radar antenna, and the spring-friction coupling accounts for resilient load members with damping between the drive and the load. An ideal torque source was assumed, and the entire system was assumed linear over the range of operation. A block diagram of the uncompensated system operating in closed-loop fashion is shown in Figure 6. Here K which has the dimensions ft. lb./rad. takes into account the transfer function of the transducer, amplifier, and torque source.

A large portion of the work involved in obtaining the final system compensated by Posicast was preliminary work, that is preparing the system so that Posicast could be used advantageously.

From the system parameters given in Figure 5 one sees that the spring coefficient, K_1, and the friction coefficient, B_1, can each vary by a factor of sixteen. Considering only the extreme values of K_1 and B_1, there are four possible combinations of these parameters and therefore essentially four different systems which must be compensated by a single compensation scheme. One of the points of most interest was the effect varying these parameters would have upon the response of the system utilizing Posicast. From the introductory discussion of Posicast it is seen that the two governing factors of Posicast design are the natural period, T_n, and the overshoot, A, of the system without Posicast. Therefore, if the preliminary design, that is the final design just



Motor inertia, $J_m = 1.6 \times 10^5$ ft.lb.sec.^{2}/rad.
Load inertia, $J_1 = .6 \times 10^5$ ft.lb sec.^{2}/rad.
Friction coefficient, $B_1 = 1.09 \times 10^5$ to 17.4×10^5 ft.lb.sec./rad.
Spring constant, $K_1 = 1.09 \times 10^8$ to 17.4×10^8 ft.lb./rad.

Figure 5.

Lightly-Damped System to Which Posicast Was Applied.

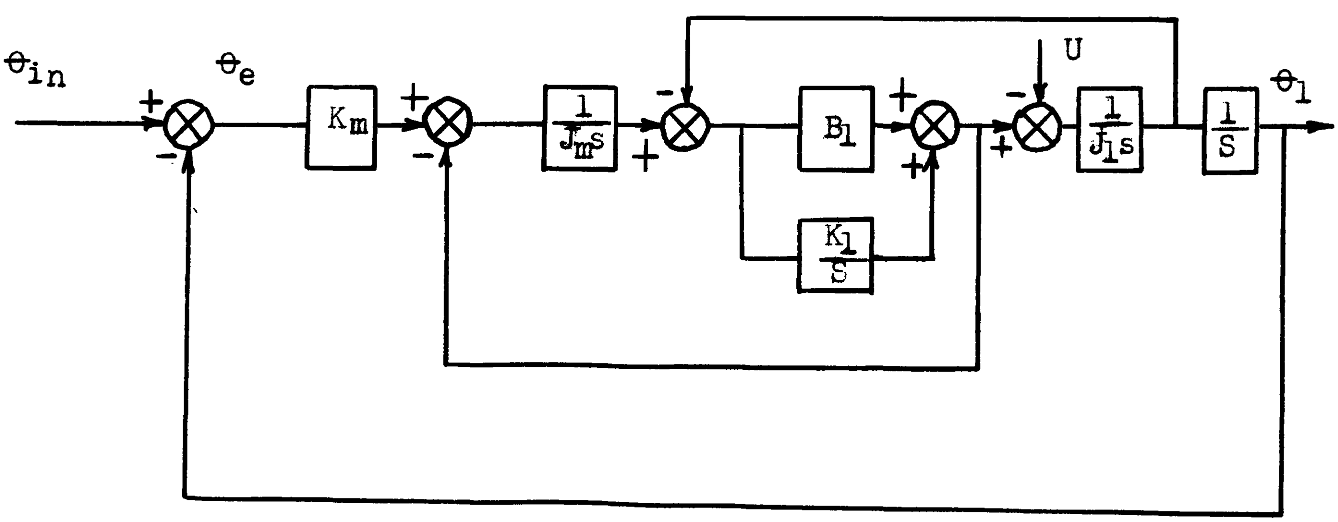
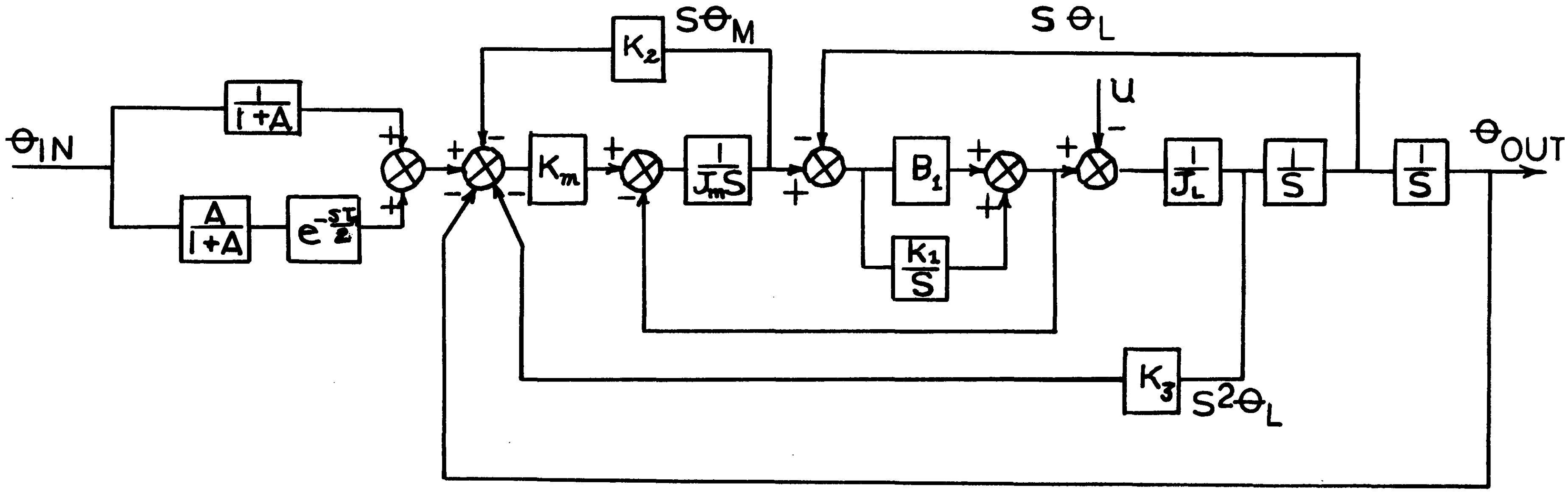


Figure 6.

Uncompensated System of Figure 5. Operating in a Closed-Loop Fashion.

before Posicast has been added, results in a system in which the overshoot and natural period remain constant as K_1 and B_1 are varied, then one Posicast design will fit all four systems and no changes will be exhibited in the system response as K_1 and B_1 are varied. For the preliminary design chosen, the overshoot and natural period did not remain constant in all four systems, but they did remain within ten percent of each other. (The problem of obtaining a preliminary design which results in the natural period and overshoot keeping approximately the same values while the system parameters are varied is discussed in Chapter 2.) Since the natural period and overshoot were not the same for all four systems, the design of the Posicast took the form of an engineering compromise, being designed according to the natural period and overshoot which most closely approximated those of all four systems. The values chosen for the Posicast design were T_n=.205 seconds and A=.786 . A block diagram of the final system compensated by use of Posicast is shown in Figure 7.

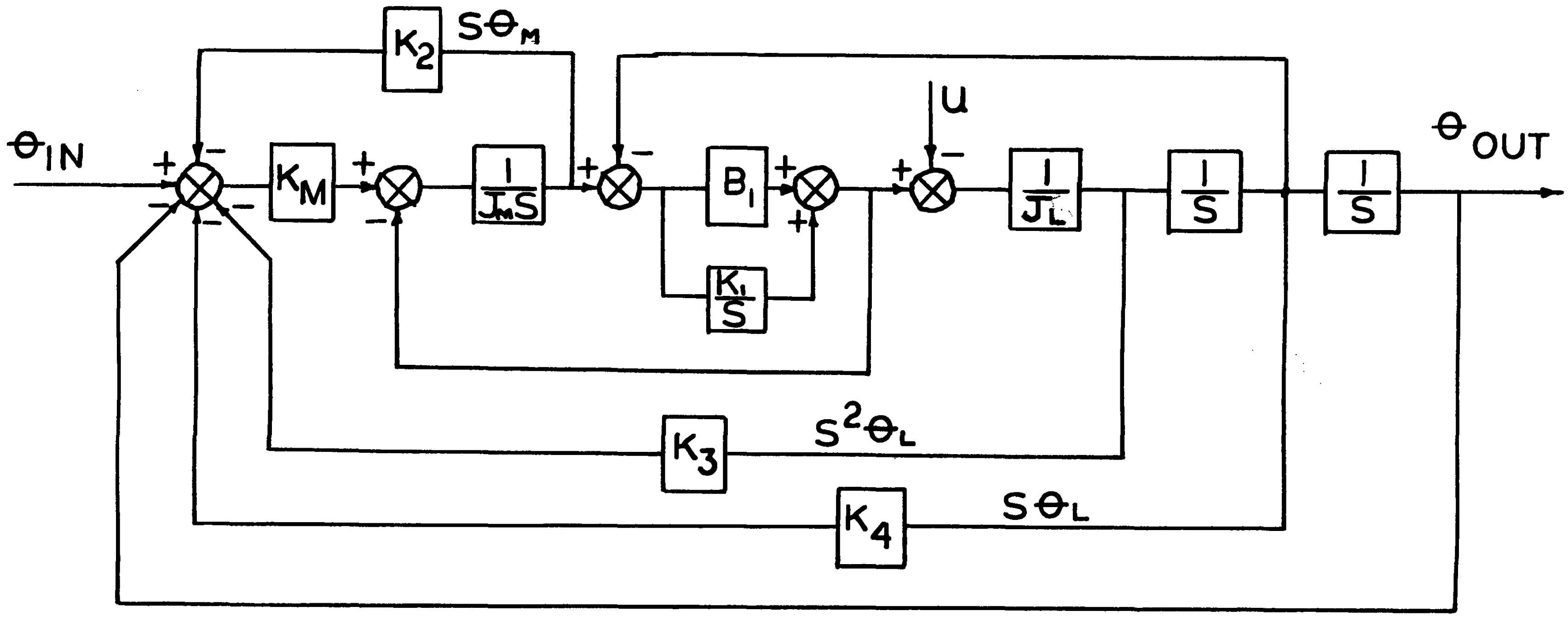
Professor Newton in his study of this system had designed a compensation utilizing synthetic damping. It is this system to which will be compared the system utilizing Posicast. Throughout the remainder of this thesis the system designed by Professor Newton will be referred to as the "conventional" system. The block diagram of the system is shown in Figure 8.



A = .765
T_n = .205 SEC.
$K_m = 10 \times 10^8$ FT. LB./RAD.
$K_2 = 1 \times 10^{-2}$ SEC.
$K_3 = 9.4 \times 10^{-4}$ SEC.^{2}

FIGURE 7.

FINAL SYSTEM UTILIZING POSICAST.



$K_M = 4 \times 10^8$ FT. LB./RAD.
$K_2 = 4.85 \times 10^{-2}$ SEC.
$K_3 = 1.85 \times 10^{-3}$ SEC.^{2}
$K_4 = 2.5 \times 10^{-2}$ SEC.

FIGURE 8.

CONVENTIONAL SYSTEM

Experimental Results

Both the conventional system and the Posicast system were simulated on a Reeves Analog Computer and tested for various inputs. (The delay required for the Posicast system was achieved by use of a tape recorder and is discussed in detail in Chapter 2.) In Figure 9 is shown the response to a step input of the preliminary system before Posicast had been added. The natural period and overshoot can be determined from this figure and have values of which the averages are those stated previously, $T_n = .205$ seconds and $A = .786$. Also Figure 9 contrasted with Figure 10 serves to point up the effect brought about by Posicast. Figure 10 shows the response of the complete Posicast system to a step input. This is the system which was designed as a compromise for the four different combinations of values of K_1 and B_1. The rise times for the different parameter combinations are all approximately 0.1 seconds, the rise time spoken of here being defined as the time required for the system output to rise from zero percent to one-hundred percent of its final value.

Another Posicast compensation was designed, this one specifically for the parameter values $B_1 = 1.09 \times 10^5$ ft. lb. sec./rad. and $K_1 = 1.09 \times 10^8$ ft. lb./rad. Figure 11 shows that the step response for this system with B_1 set at 1.09×10^5 ft. lb. sec./rad. and K_1 set at

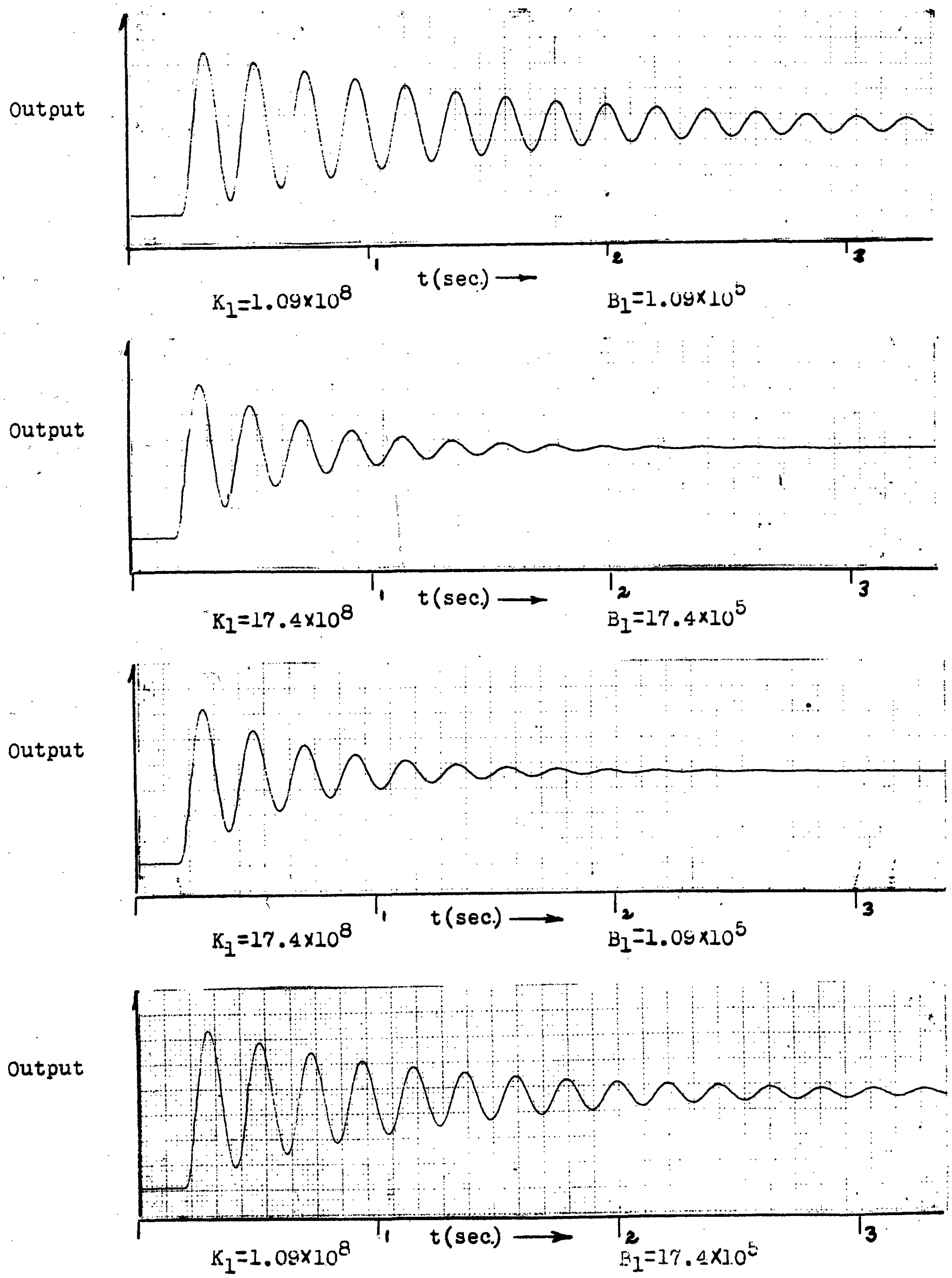


Figure 9.
Step Response of Preliminary-Design System.

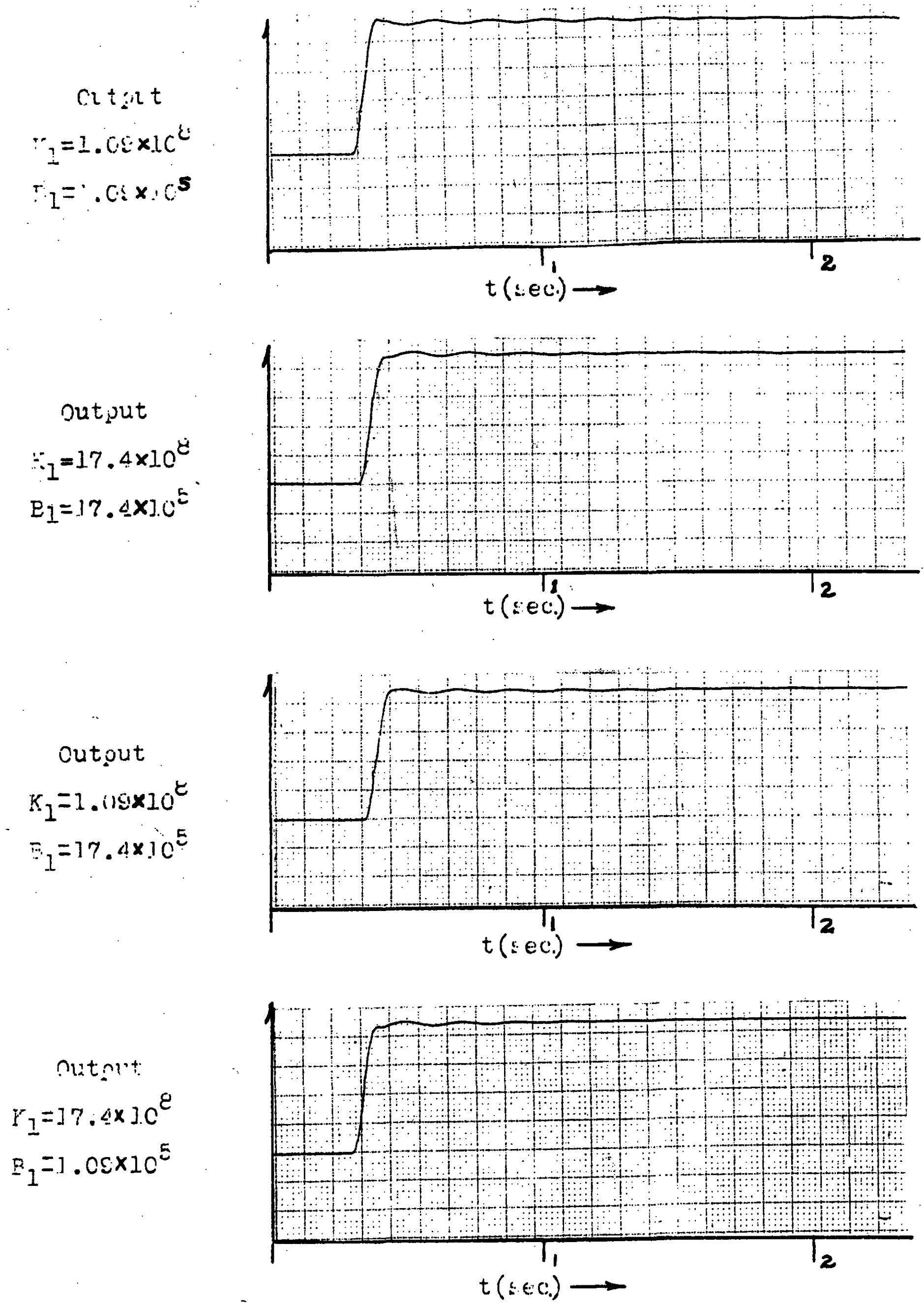


Figure 10.

Step Response of Posicast Compromise Design.

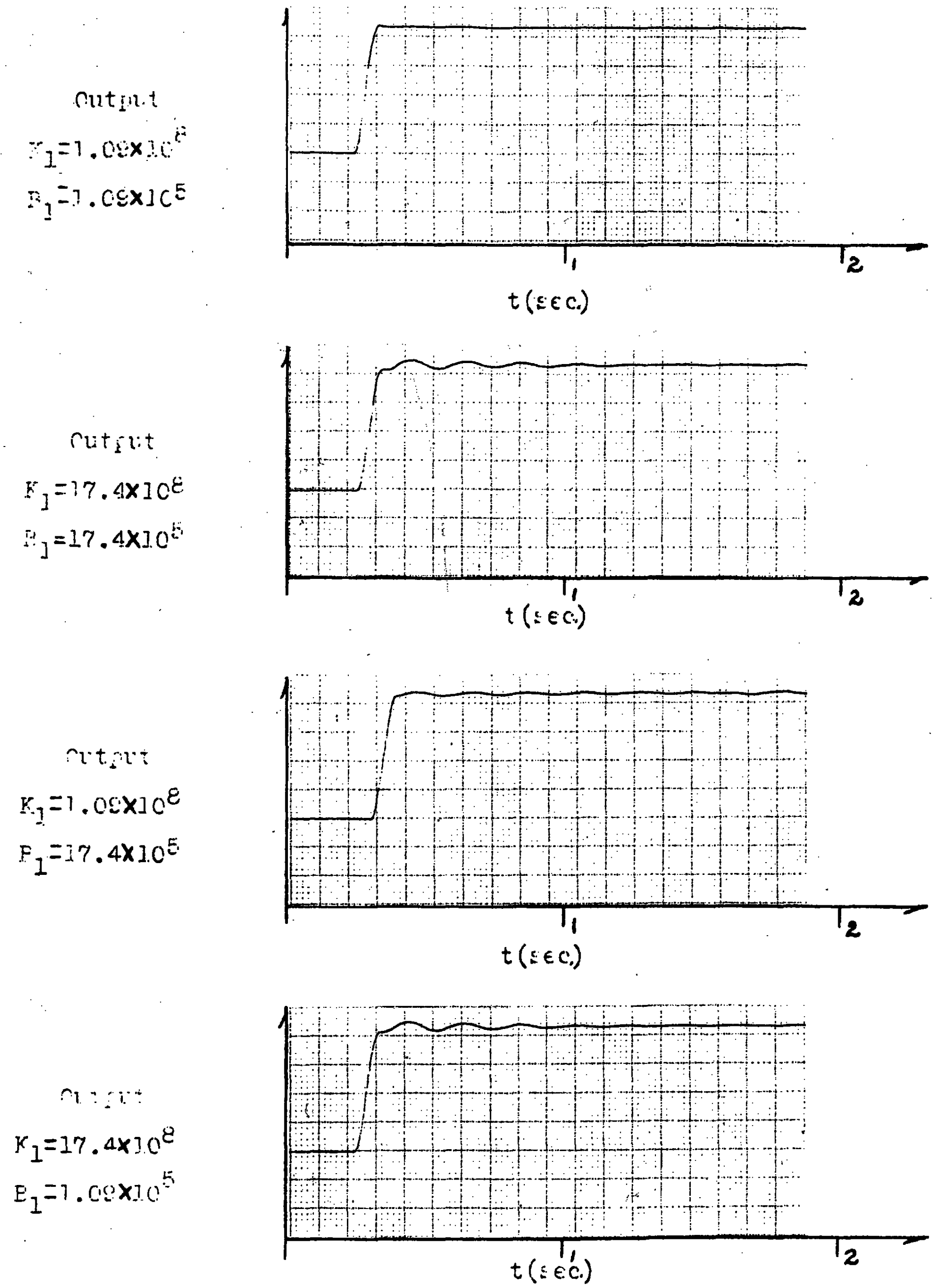


Figure 11.

Step Response of Posicast Designed for $K_1=1.09\times10^8$, $B_1=1.09\times10^5$.

1.09 x 10^8 ft. lb./rad. was quite good with a rise time of .09 seconds. However, when K_1 and B_1 were varied, the results were not as good as they were for the system which was a compromise. The rise times were almost the same, but the slight oscillations observed in Figure 10 are seen to have increased considerably. These oscillations are caused by both inaccurate timing and improper step magnitudes in the Posicast. It should be understood that Posicast does not increase the damping in the system, but rather eliminates oscillations when properly designed by not permitting them to be excited. When mistuned Posicast allows slight oscillations to be excited, their magnitude depending upon the degree to which the Posicast is mistuned.

The step response of the conventional system is shown in Figure 12. This system is seen to be slightly underdamped with a very small overshoot. The rise times varied from .15 seconds to .2 seconds. Note that the time scale in Figure 12 is different from that of Figures 10 and 11.

The ramp responses of the Posicast and conventional systems are shown in Figures 13 and 14 respectively. In each case the output is well behaved. The steady-state errors differ somewhat. The velocity constants, $\lim_{s \to 0} sG(s)$

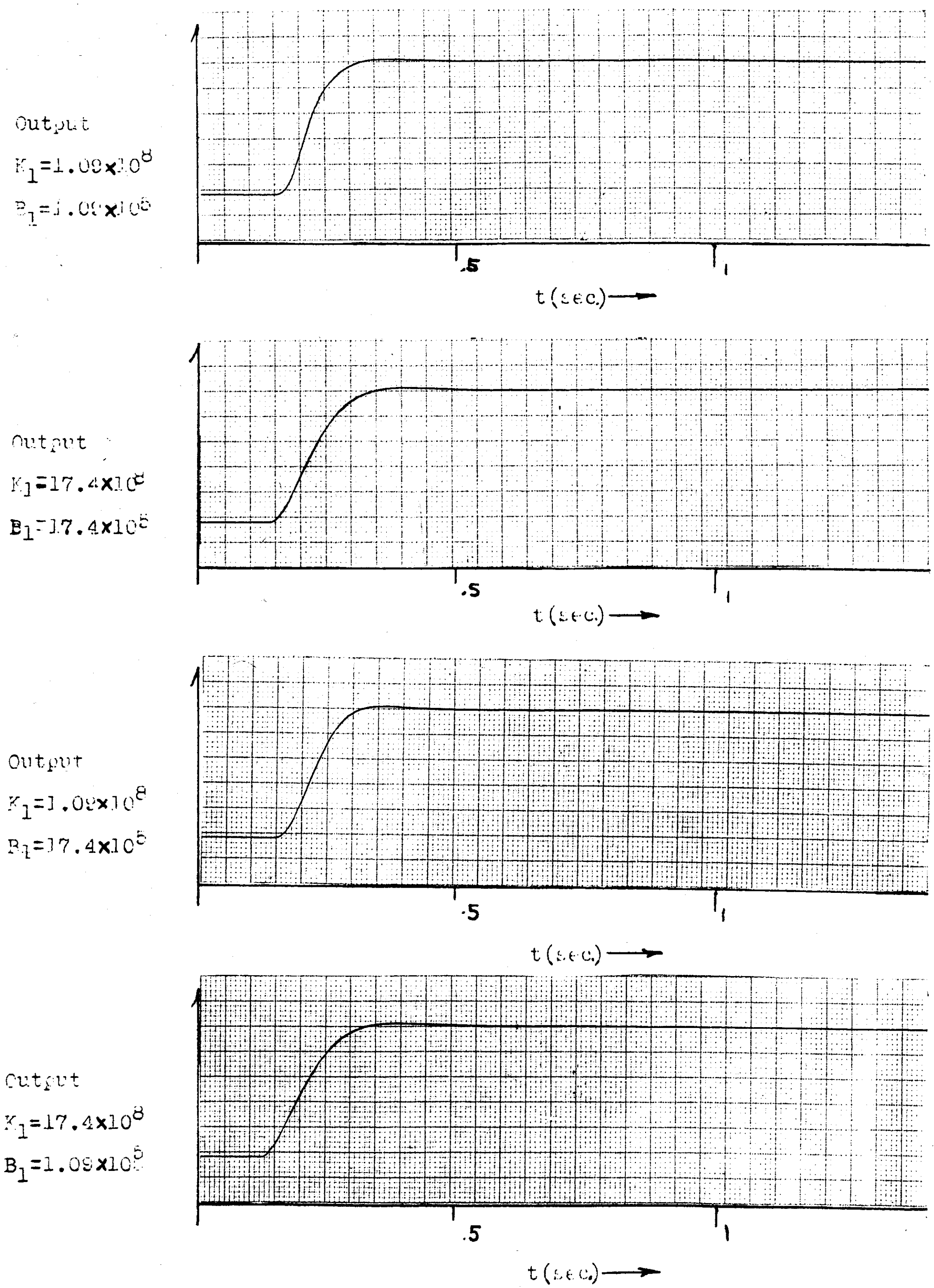


Figure 12.
Step Response of Conventional Design System

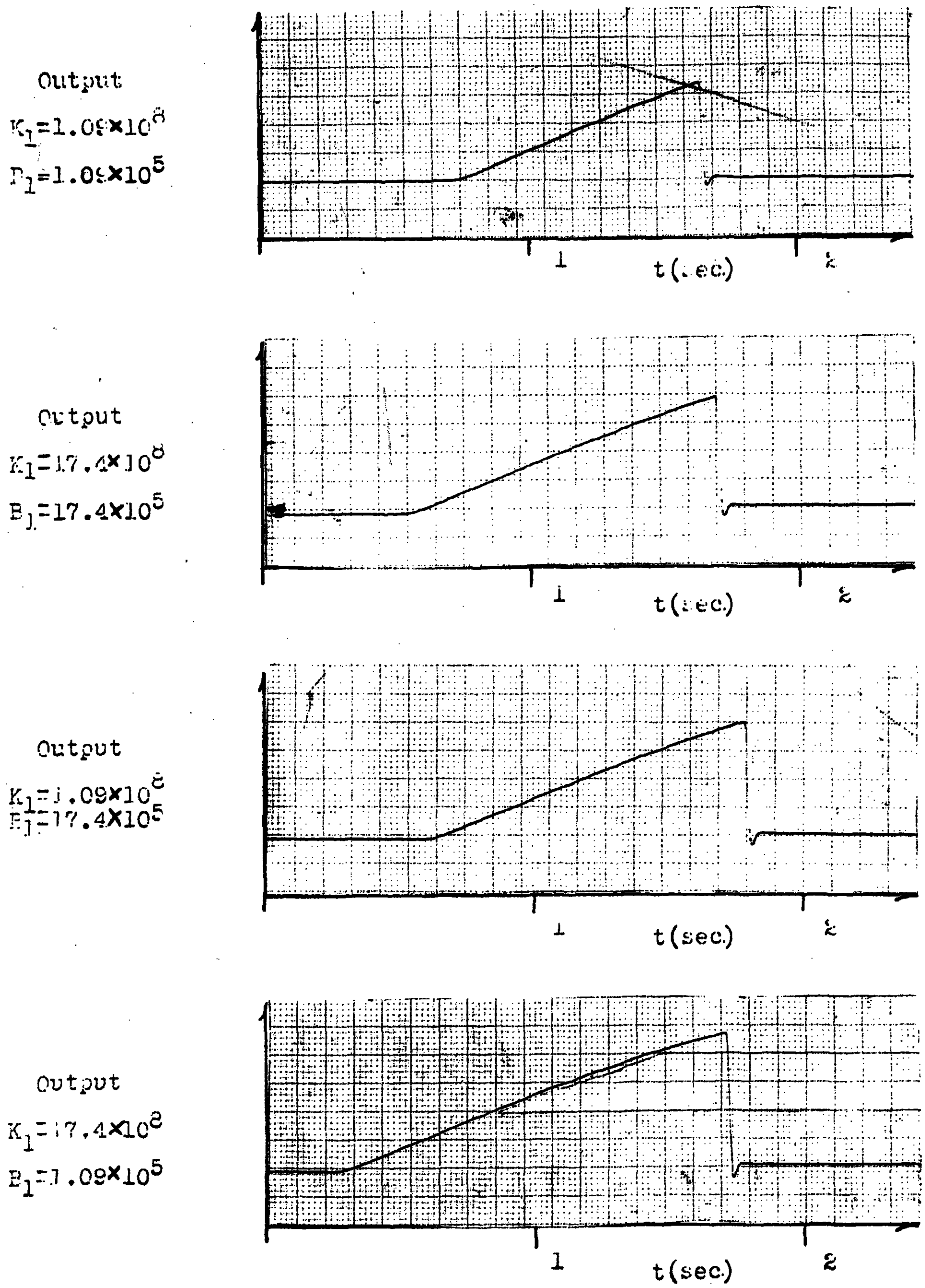


Figure 13.
Ramp Response of Posicast Compromise Design.

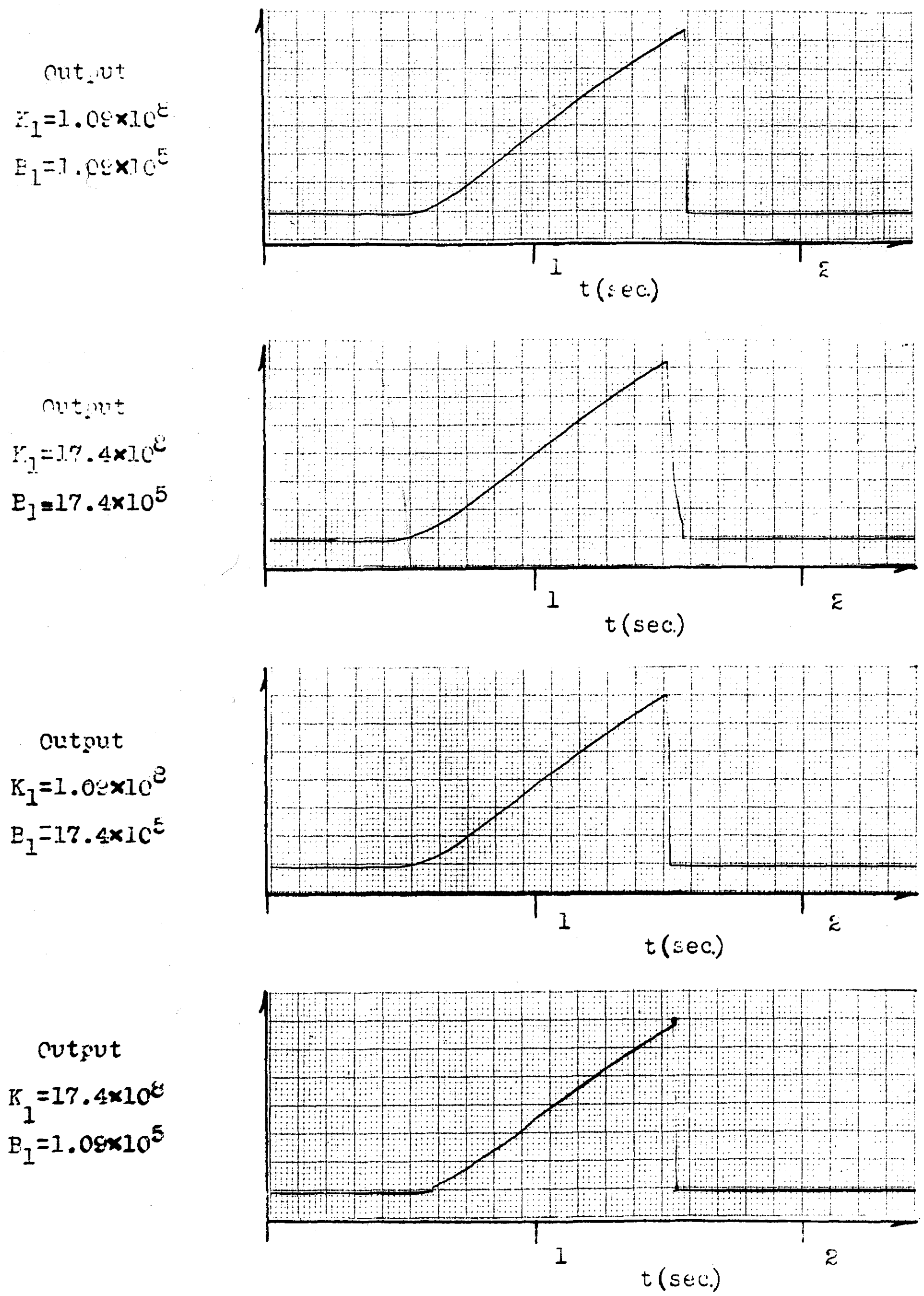


Figure 14.
Ramp Response of Conventional System.

are $K_2^{-1} = 100$ sec.$^{-1}$ for the Posicast system and $(K_2 + K_4)^{-1} = 13.6$ sec^{-1} for the conventional system. For a ramp input of ω radions/second, the steady-state error for the conventional system is $.074\,\omega$ radians. For the Posicast system the steady-state error between I^1 of figure 3 and the output is $.01\,\omega$ radians. However, an additional error is incurred between the input and I^1 because of the delay brought about by Posicast. This error is of the amount $\omega \times T_n/2 \times \frac{A}{1+A} = .045\omega$ radians and the total error becomes $.055\,\omega$ radians.

Figures 15 and 16 show the responses of the systems to step load disturbances. The Posicast system response is seen to be highly oscillatory. As far as load disturbances are concerned, the Posicast system is no different than the preliminary design system since Posicast as used here does not modify load disturbances before permitting the system to see them. Posicast can be utilized in such a manner as to eliminate oscillations caused by load disturbances, and the reader is referred to Feedback Control Systems[3], page 341 for further discussion of this.

The torque constants for the systems are 10×10^8 ft. lb./rad. for the Posicast system and 4×10^8 ft. lb./rad. for the conventional system. Therefore, the steady-state error of the Posicast system $U \times 10^{-9}$ radians is less than that of the conventional system, $U \times 2.5 \times 10^{-9}$ radians, but the transient response of the Posicast system is much less desirable than that of the conventional system.

Figures 17 and 18 show the response of the two systems to a "random" input. The input signal was achieved by manually varying the amplitude and frequency of a triangle-wave generator. The frequency was varied from zero to ten cycles per second, approximately twice the bandwidth of the systems.

No attempt was made to derive quantitative results from the random input test, but speaking qualitatively, it is seen that both systems are well behaved, and this was the main point of interest, especially concerning the Posicast system.

The frequency response curves for the systems are shown in figures 19 through 24. As was mentioned in the introductory discussion on Posicast, the resonance peaking has been entirely eliminated by Posicast. The most striking feature of the Posicast response curves is the ripple exhibited at frequencies above the half-power frequency. However, this phenomenon is directly related to the infinite column of zeros of Posicast in the S plane demonstrated earlier. As the frequency of the system is varied along the $j\omega$ axis, the $\frac{1}{1+A} + \frac{A}{1+A} e^{-j\omega T_n/2}$ term varies in magnitude from $\frac{1-A}{1+A}$ to $\frac{1+A}{1+A} = 1$. The value $\frac{1-A}{1+A}$ occurs at $\omega T_n/2 = \pi(2n+1)$ or $\omega = \omega_n(2n+1)$ where $\omega_n = 2\pi/T_n$ and n is any integer. The value 1 occurs at $\omega T_n/2 = 2n\pi$ or $\omega = \omega_n(2n)$.

Since the remaining factor of the transfer function,

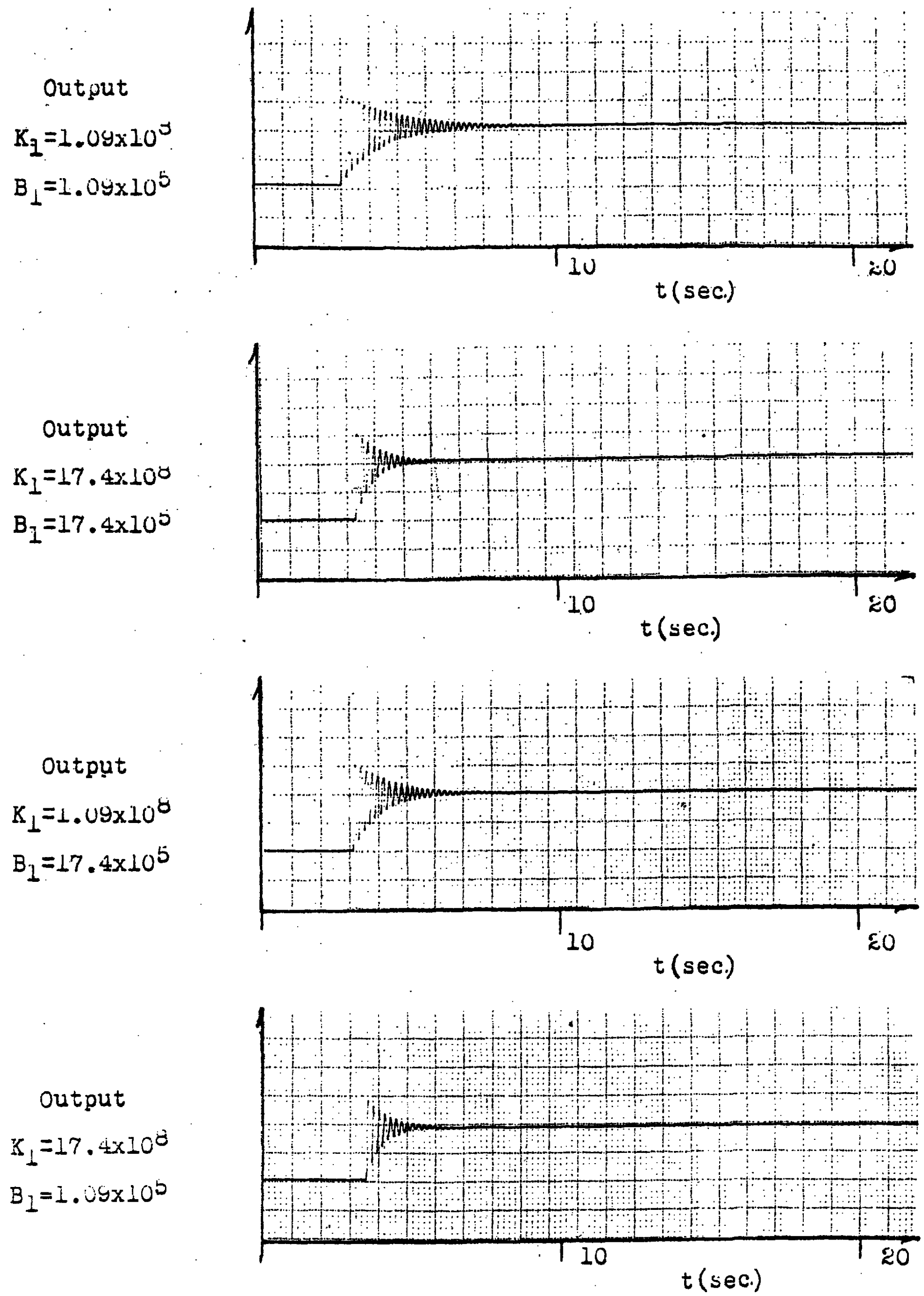


Figure 15.

Load Disturbance Response of Posicast Compromise Design.

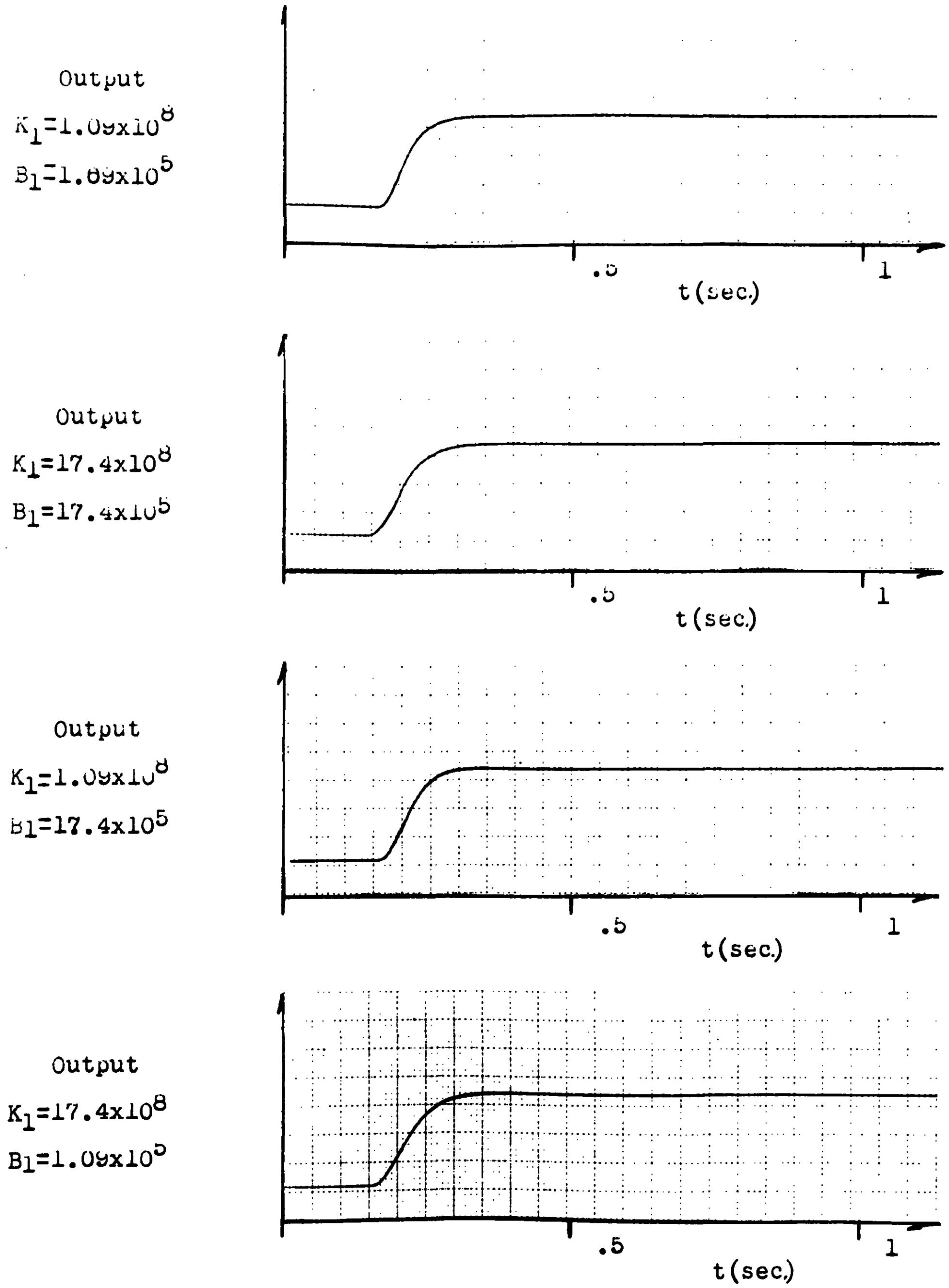


Figure 16.

Load Disturbance Response of Conventional Design.

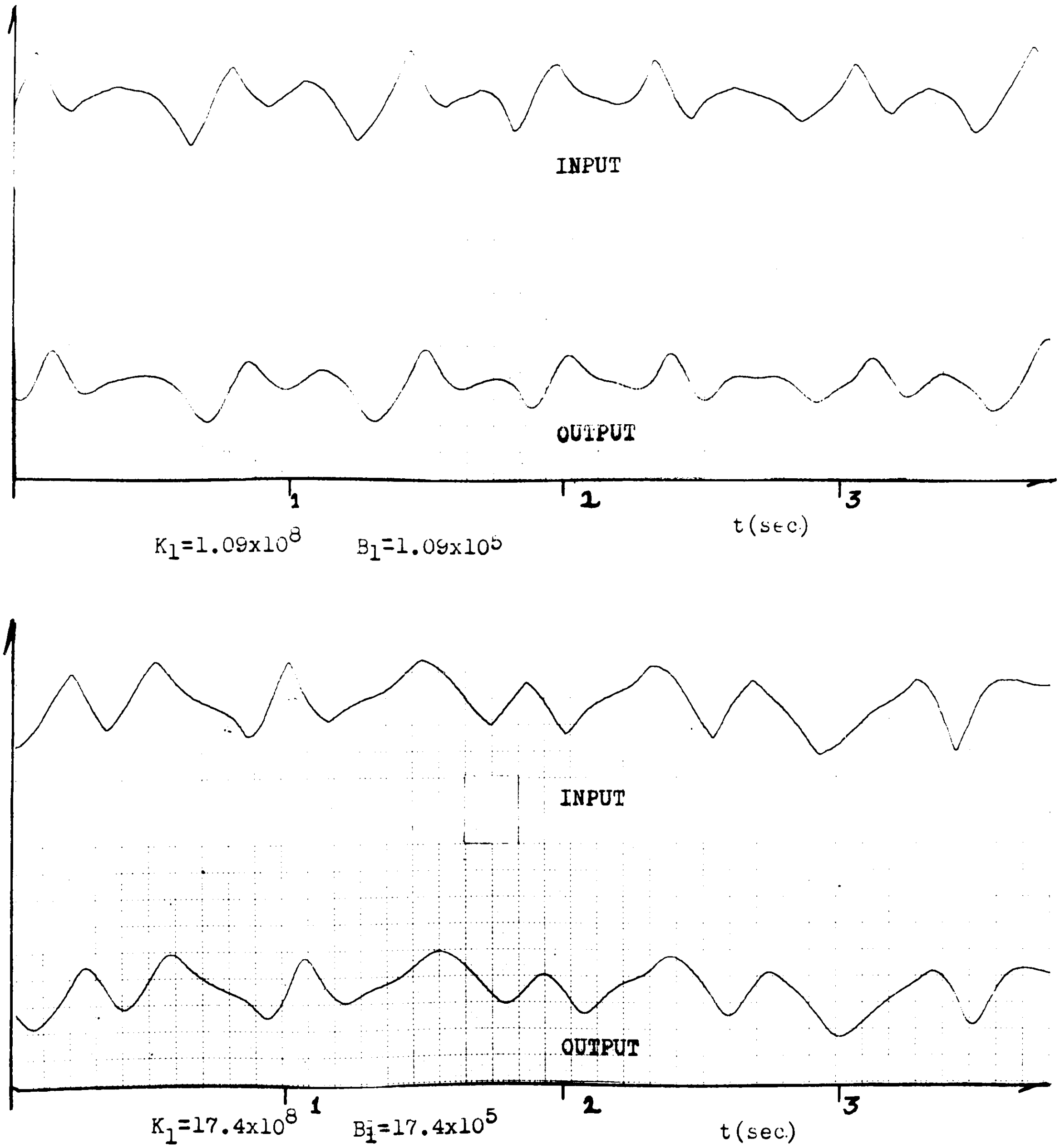


Figure 17.

Random-Input Response of Posicast Compromise Design.

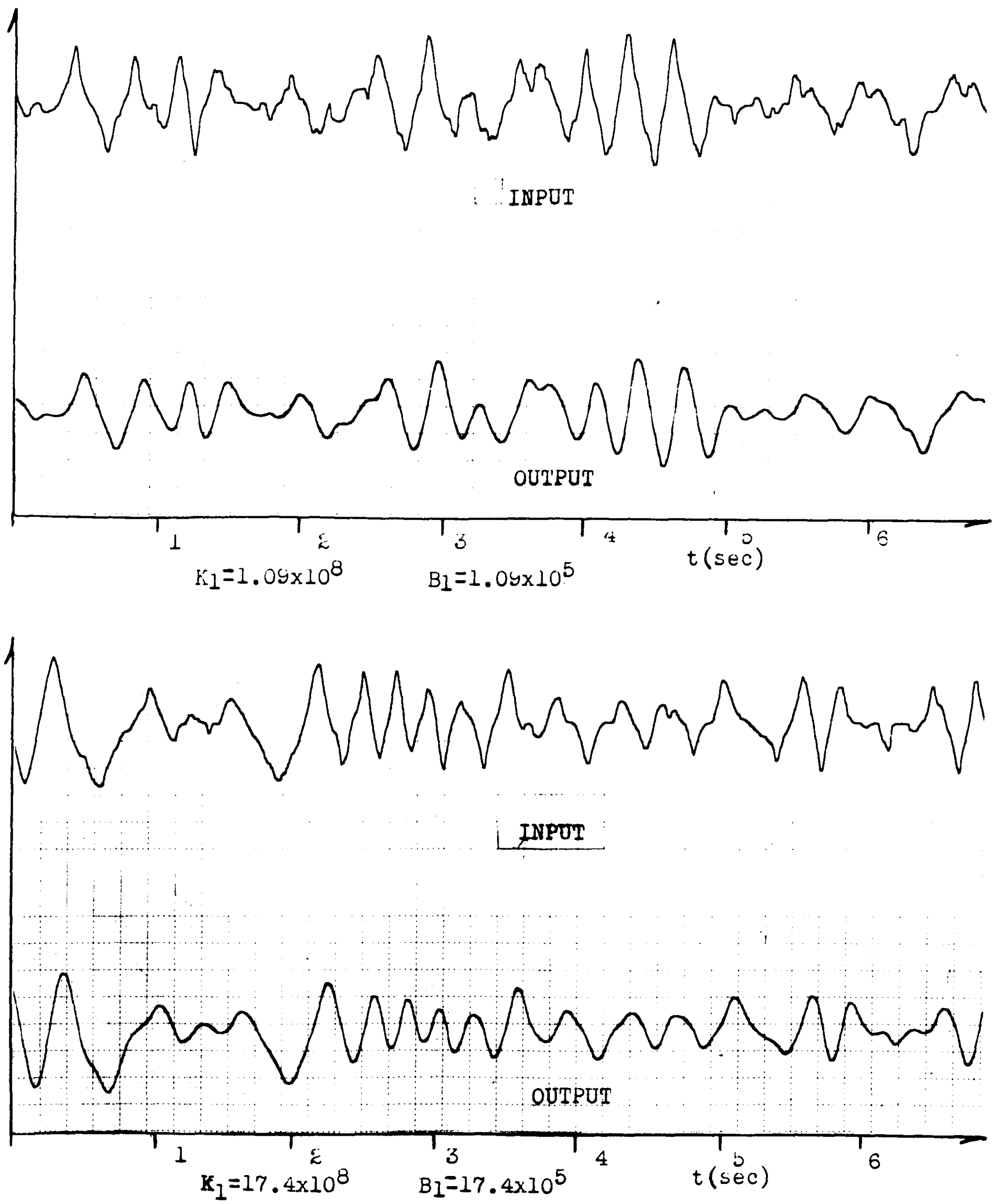


Figure 18.

Random-Input Response of Conventional Design.

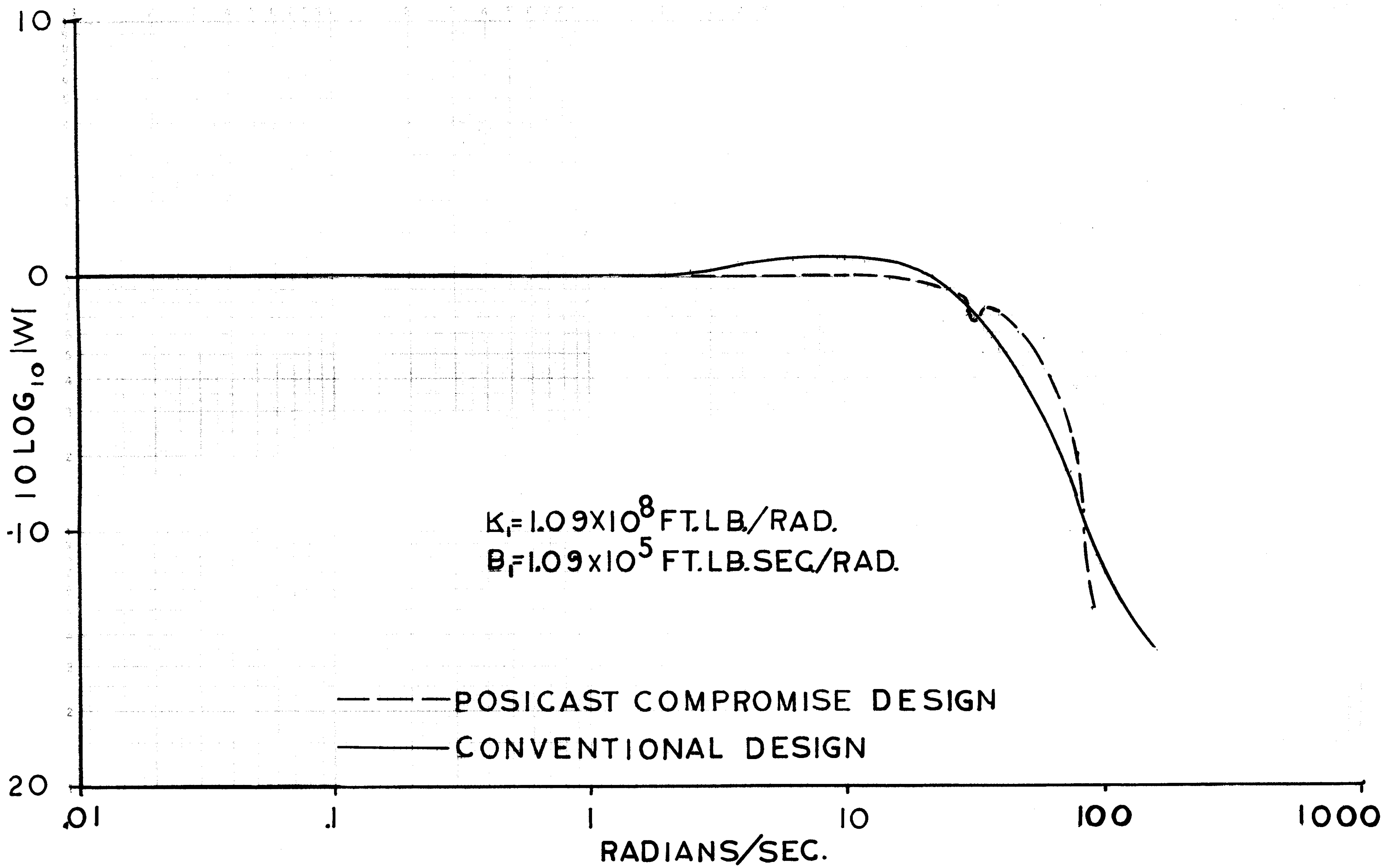

10
0
-10
20
10 LOG$_{10}$ |W|
K_1= 1.09X10^8 FT. LB./RAD.
B_1=1.09x10^5 FT. LB. SEC./RAD.
POSICAST COMPROMISE DESIGN
CONVENTIONAL DESIGN
.01
.1
1
10
100
1000
RADIANS/SEC.

FIGURE 19.

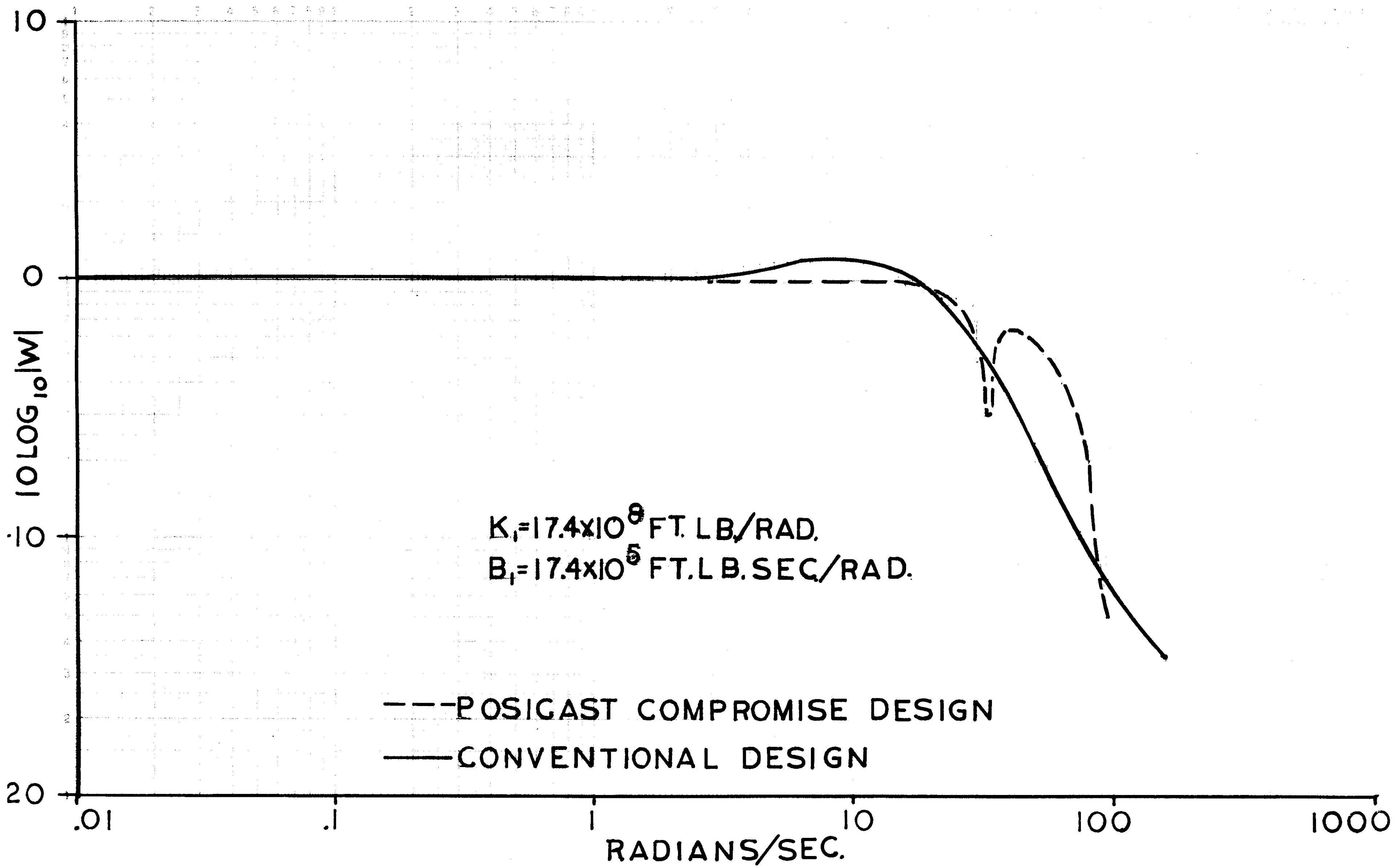

10
0
-10
20
$10 LOG_{10}|W|$
$K_1 = 17.4 \times 10^8$ FT. LB./RAD.
$B_1 = 17.4 \times 10^5$ FT. LB. SEC./RAD.
--- POSIGAST COMPROMISE DESIGN
— CONVENTIONAL DESIGN
.01
.1
1
10
100
1000
RADIANS/SEC.

FIGURE 20.

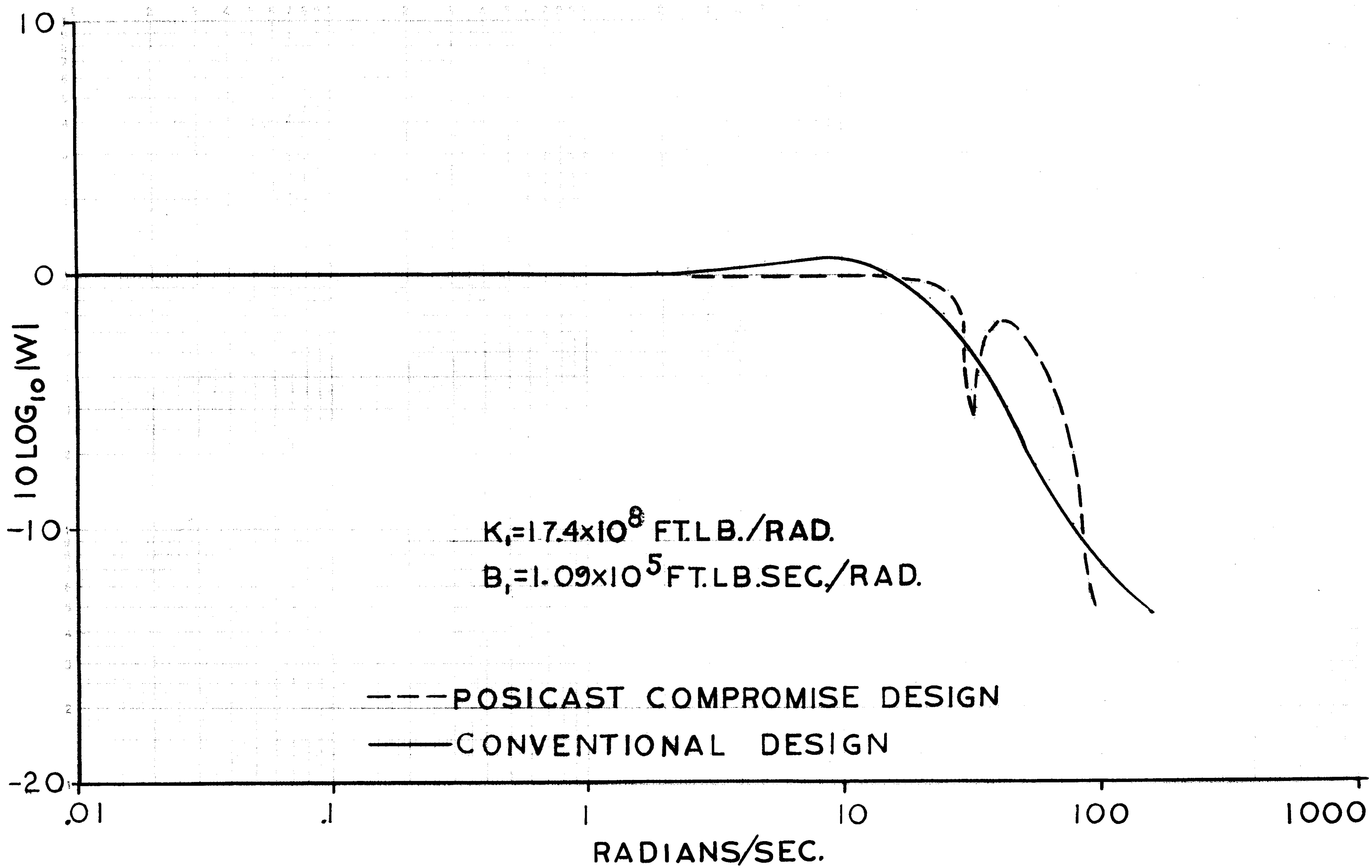
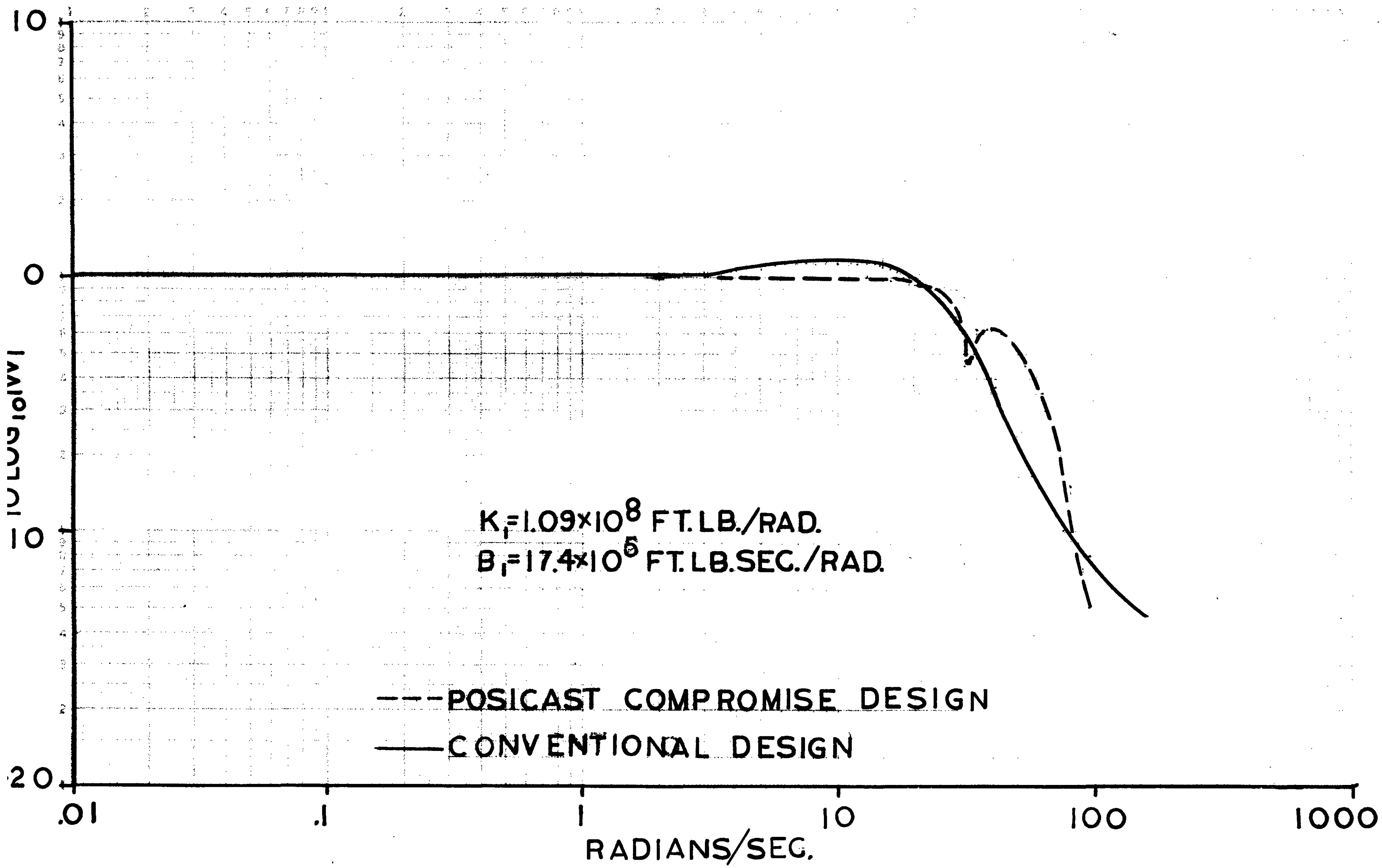
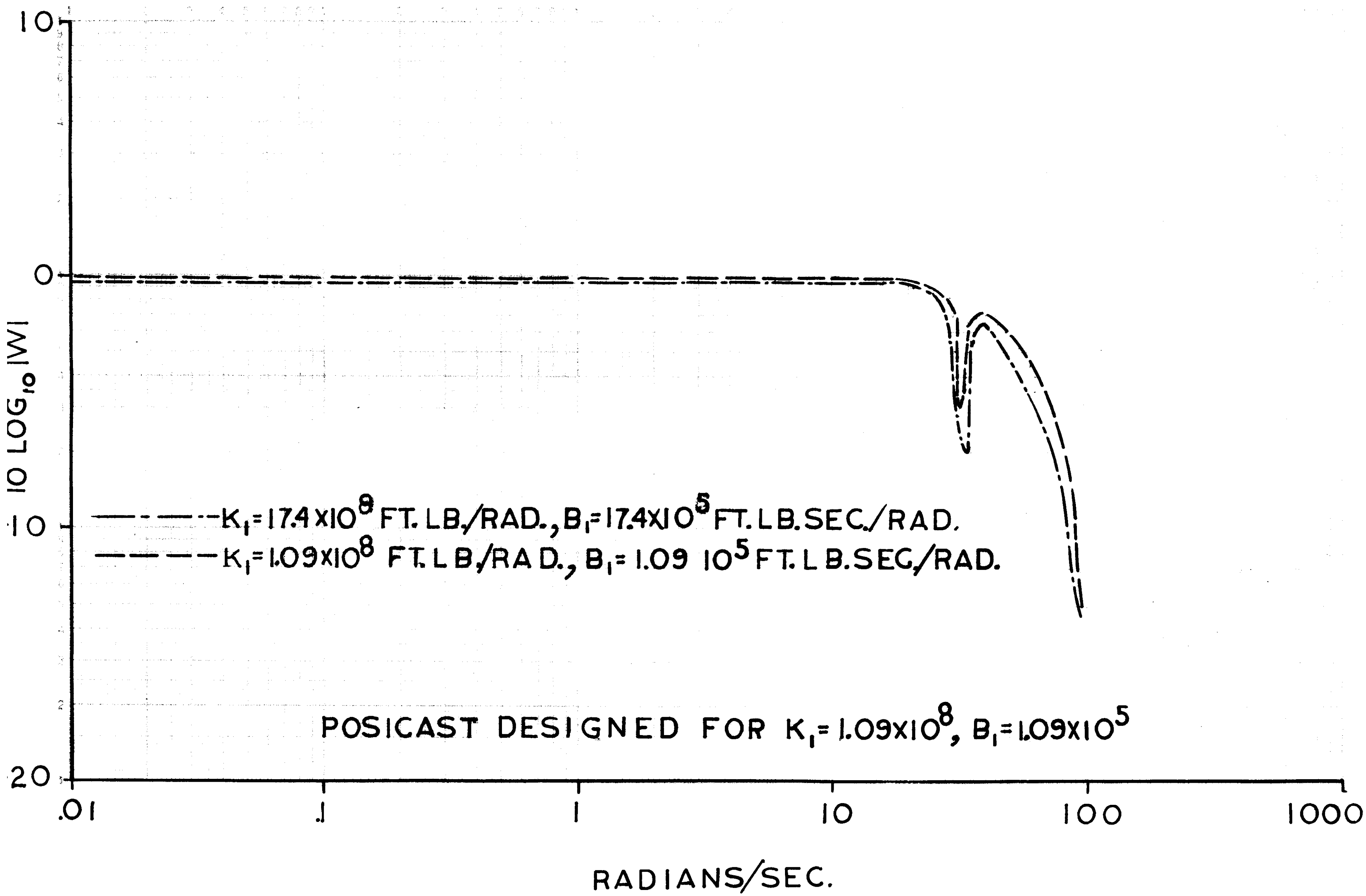

10
0
-10
-20
10 LOG$_{10}$ |W|
K_1=17.4x10^8 FT.LB./RAD.
B_1=1.09x10^5 FT.LB.SEC./RAD.
---POSICAST COMPROMISE DESIGN
——CONVENTIONAL DESIGN
.01
.1
1
10
100
1000
RADIANS/SEC.

FIGURE 21.



10
0
10
20
10 LOG₁₀|M|
.01
.1
1
10
100
1000
RADIANS/SEC.
$K_1 = 1.09 \times 10^8$ FT.LB./RAD.
$B_1 = 17.4 \times 10^6$ FT.LB.SEC./RAD.
---POSICAST COMPROMISE DESIGN
——CONVENTIONAL DESIGN

FIGURE 22.



10
0
-10
20
10 LOG10 |W|
K1=17.4X10^8 FT.LB./RAD., B1=17.4X10^5 FT.LB.SEC./RAD.
K1=1.09X10^8 FT.LB./RAD., B1=1.09 10^5 FT.LB.SEC./RAD.
POSICAST DESIGNED FOR K1=1.09X10^8, B1=1.09X10^5
.01
.1
1
10
100
1000
RADIANS/SEC.

FIGURE 23.

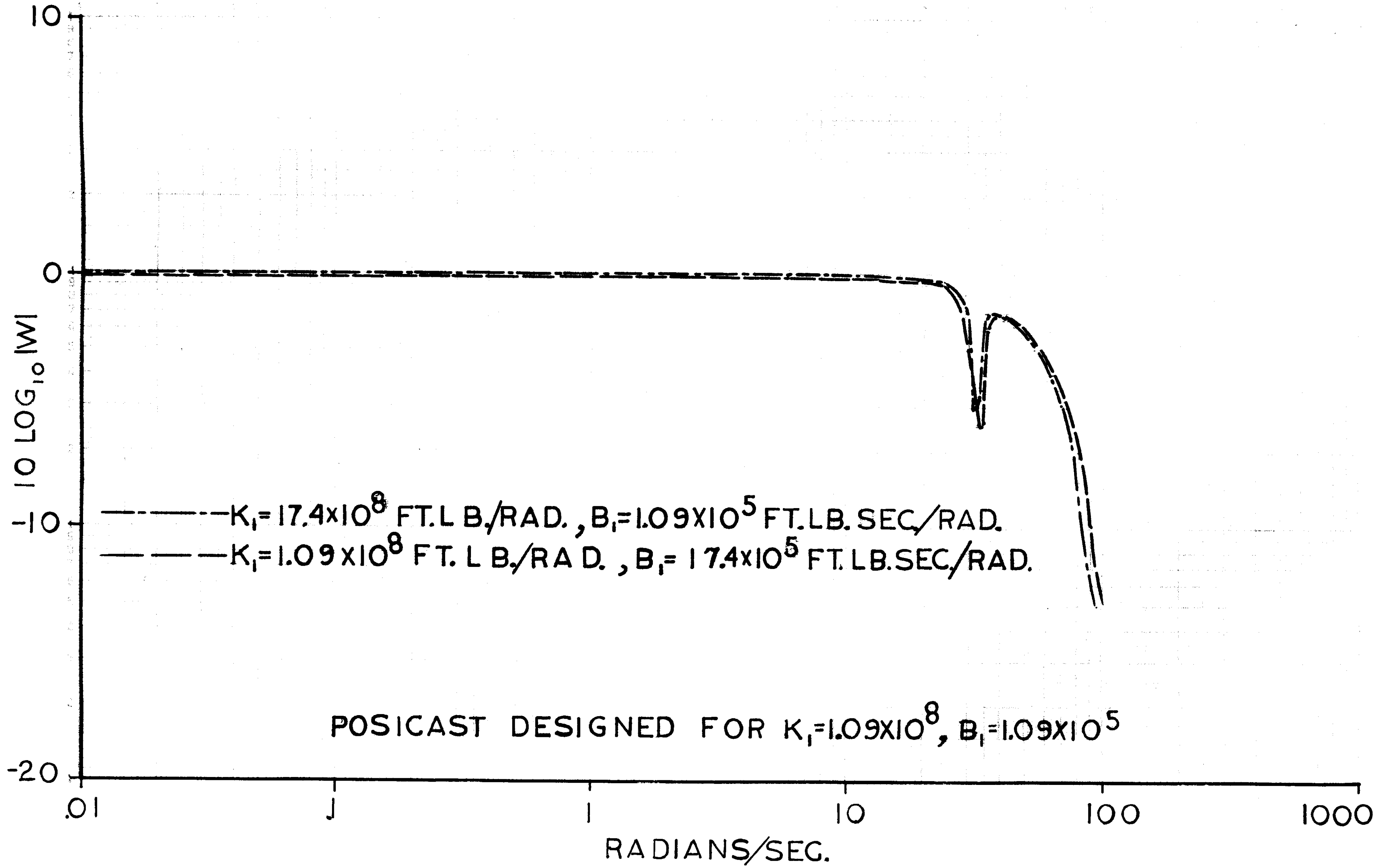

10
0
-10
-20
10 LOG10 |W|
K1= 17.4X10^8 FT. LB./RAD., B1=1.09X10^5 FT. LB. SEC./RAD.
K1= 1.09X10^8 FT. LB./RAD., B1= 17.4X10^5 FT. LB. SEC./RAD.
POSICAST DESIGNED FOR K1=1.09X10^8, B1=1.09X10^5
.01
.1
1
10
100
1000
RADIANS/SEC.

FIGURE 24.

$W_{(s)}$ as shown in Figure 3, is generally a monotonically decreasing function at frequencies above ω_n, it is seen that depending upon the rate at which $W_{(s)}$ drops off with increasing frequency and the ratio of the extreme values which the $\frac{1}{1+A} + \frac{A}{1+A} e^{-j\omega T_n/2}$ term may take on, various degrees of rippling may be exhibited at the higher frequencies.

The important results of the investigation are listed in Table 1.

<u>TABLE 1</u>

	Conventional System	Posicast Compromise System	Posicast Designed For: $K_1 = 1.09 \times 10^8$ $B_1 = 1.09 \times 10^5$
Average rise time for step input.	.17 sec.	.1 sec.	.08 sec.
Average overshoot for step input of magnitude N radians.	.012 N rad.	.03 N rad.	.033 N rad.
Average steady-state error for velocity input of magnitude ω rad/sec.	.074 ω rad.	.055 ω rad.	.056 ω rad.
Average error overshoot for step load disturbance of magnitude U ft.lbs.	0	.5 U rad.	.5 U rad.
Average settling time for step load disturbance (time to settle within 5% of final value)	.11 sec.	3 sec.	3 sec
Average steady-state error to step load disturbance of Magnitude U.	2.5×10^{-9} U rad.	1×10^{-9} U rad.	1×10^{-9} U rad.

TABLE 1 CONTINUED

	Conventional System	Posicast Compromise System	Posicast Designed For $K_1 = 1.09 \times 10^8$ $B_1 = 1.09 \times 10^5$
Average half-power frequency (frequency where $\|W\| = .707$).	27.25 rad./sec.	29.5 rad./sec.	29 rad./sec.
Average Resonant Peak (arithmetic scale).	1.17	1.0	1.0

Conclusions

From Table 1, we see which factors have been improved and which ones have suffered adverse effects through the use of Posicast. Perhaps the most important improvement is the reduction of the step response rise time. The bandwidth has been increased but only slightly. The resonant peaking has been eliminated entirely, but in the conventional system the peaking was not enough to create any serious problems. The steady-state error for velocity inputs favors Posicast. By far the factor which suffered most through the use of Posicast was the transient error response to a load disturbance. However, even with its fifty percent overshoot the error of the Posicast system is at no time as large as the steady-state error of the conventional system. Because Posicast eliminates resonant peaking and overshoot in the response to input signals, higher gain and less damping can be used in a system, thus bringing about a reduction in steady-state errors.

One requirement imposed by the use of Posicast is that the input signal must be explicit. In most control systems the only signal which is of importance as far as the system is concerned is the error signal, and this is the signal measured by the system when in operation. For the use of Posicast, the input can be made explicit by measuring the output signal and adding this to the error signal through a summing amplifier. However, these measurements as well as the Posicast network must be very accurate since the output of the system can be no more accurate than the input regardless of how good the system is. Also the input in the case of the tracking-radar antenna may vary from zero to 360 degrees.

The net requirement then becomes an accurate measurement of angles up to 360 degrees, an accurate summing device to add the error and output signals, and a Posicast network capable of handling a wide range of inputs. If a magnetic tape or similar device is used to achieve the time delay, the range of the input signal may be quite significant. One possible means of alleviating the situation would be to shift the output reference periodically to the operating point at that time so that the magnitude of the output signal measured from the reference could be maintained reasonably small. More work on this problem would be required in any system in which the input is not explicit.

An analysis of how Posicast compares to the conventional system depends upon which characteristics of system behavior are most important in the specific application. If the problem just mentioned can be overcome, I believe that Posicast should be considered for compensating lightly-damped systems since it has demonstrated several favorable characteristics. The remainder of this thesis is concerned with the problem of achieving the preliminary design and with simulating and testing the systems.

CHAPTER 2

PRELIMINARY DESIGN AND TESTING

The Preliminary Design

This section is concerned with the preliminary design, that is the design required in modifying the original system shown in Figure 6, to a form to which Posicast could be applied advantageously.

The main objective of the preliminary design was to obtain a stable system with a short natural period, a large velocity constant, and a large torque constant. No effort was made to limit the overshoot since theoretically this could be eliminated by Posicast. It was desired that the overshoot and also the natural period vary only slightly as the system parameters K_1 and B_1 were varied since the design of the Posicast depended upon these two factors, natural period and overshoot, and was to remain unchanged for all values of K_1 and B_1.

An examination of the closed loop transfer function of the original system shown in Figure 6 reveals that it is unstable for all real values of K_m.

$$\frac{\theta_L}{\theta_{error}} = \frac{K_m(B_1 S + K_1)}{S^2\left[J_m J_L S^2 + B_1(J_m + J_L)S + K_1(J_m + J_L)\right]}$$

$$\frac{\theta_L}{\theta_{in}} = \frac{K_m(B_1 S + K_1)}{J_m J_L S^4 + B_1(J_m+J_L)S^3 + K_1(J_m+J_L)S^2 + K_m B_1 S + K_m K_1}$$

The corresponding array for the Routh Criterion is:

$J_m J_L$

$B_1(J_m + J_L)$

$K_1 B_1 (J_m + J_L)^2 - J_m J_L K_m B_1$

$$\frac{K_m B_1 \left[K_1 B_1 (J_m + J_L)^2 - J_m J_L K_m B_1\right] - B_1^2 (J_m + J_L)^2 K_m K_1}{B_1(J_m + J_L)}$$

$K_1(J_m + J_L)$ $\qquad$ $K_m K_1$

$K_m B_1$

$K_m K_1$

By restricting all the terms in the first group to being greater than zero, we arrive at the condition $K_m^2 < 0$ which cannot be satisfied by a real number. For $K_m = 0$, the system is marginally stable.

By examining the open-loop transfer function of the system and noting the second order pole at the origin, it is realized that phase lead is very desirable. Based upon this observation it was decided that a lead network in the forward loop should be given some consideration.

Examining the pole zero sketch of this transfer function shown in Figure 25 and realizing the desirability of moving the poles to the left and away from the imaginary axis, one recognizes the need of increased damping. From this observation it was decided to consider synthetic damping through the use of rate and acceleration feedback subtracted at the input summing point.

Once the types of compensation to be considered had been selected, the design was largely trial and error since the objectives of the preliminary design did not correspond to those of any known optimizing techniques. Much of the work was performed on the analog computer.

First a second order lead network of the form $\left(\frac{\alpha T_c S+1}{T_c S+1}\right)^2$ was designed for the system. The resonant frequency of the term $J_m J_L S^2 + B_1(J_m+J_L)S + K_1(J_m+J_L)$ of the open loop transfer function was found to be fifty radians per second. The lead network was designed to give its maximum lead effect at this frequency. α was chosen to be 20 since this is generally the largest practical value it can be given. T_c was .0045 seconds, calculated from the relation $T_c = \frac{1}{\omega_m\sqrt{\alpha}}$ where ω_m is the frequency where maximum lead is desired. The lead network achieved a phase lead of 130° at ω_m and stabilized the system for a limited range of values of K_m. However, when this system was simulated on the analog computer and tested for a step input, the shortest natural period obtainable was two seconds. It was not known how short a natural period it would be possible to achieve, but it was known that the average rise time of the conventional system was .17 seconds. Since in the Posicast system the rise time

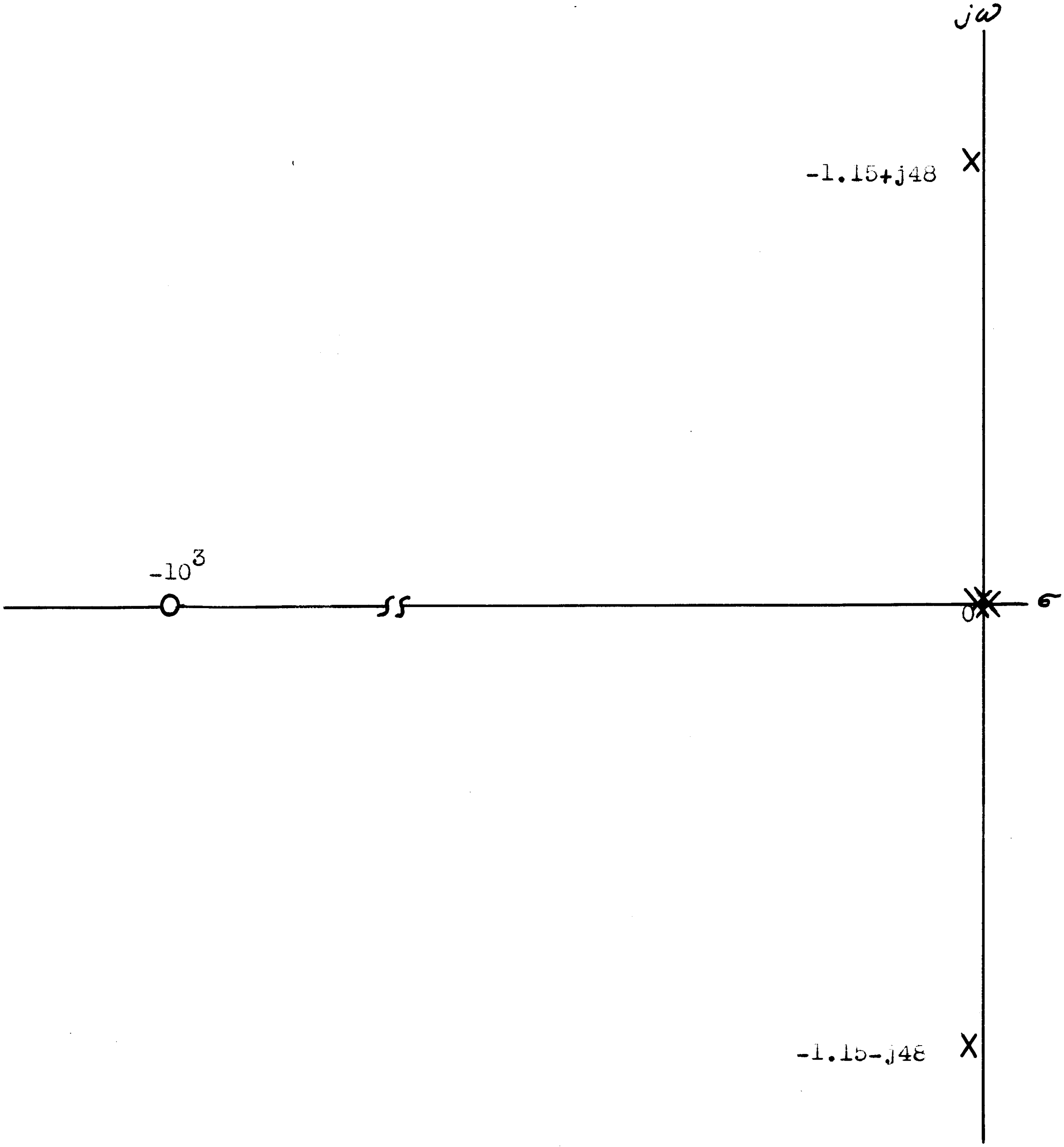


Figure 25.

Pole-Zero Plot of Uncompensated System.

is one half the natural period, it was felt that a natural period considerably less than two seconds should be striven for.

Next rate feedback from the load was attempted. This compensation added damping and stabilized the system for a limited range of values of K_m. The system was simulated on the computer and tested for a step input, varying the amount of rate feedback and in each case using the largest value of K_m permitted by stability considerations. Still the shortest natural period obtainable was approximately two seconds. At this point load-acceleration feedback was introduced, but no significant improvement resulted.

Rate feedback from the torque source was then attempted and the natural period was decreased to .26 seconds. The relatively short natural period was encouraging, but neither it nor the overshoot remained even close to constant when K_1 and B_1 were varied. However, since this scheme had given the most favorable results up to that time, it was decided to give it further consideration and to attempt to improve its performance by modifications. Load-rate feedback in conjunction with the torque-source-rate feedback was then attempted but with no improvement.

Finally it was found that best results could be achieved by using load-acceleration feedback along with torque-source-rate feedback. The natural periods for the four combinations of values of K_1 and B_1 averaged .205

seconds and the overshoots averaged .786. All values were within five percent of these average values. The step responses are shown in Figure 9. The torque constant was 10 x 10^8 ft. lb./rad. and the velocity constant was 100/second. The reason for desiring large torque and velocity constants obviously was to decrease the steady-state errors to load disturbances and velocity inputs respectively.

One may wonder at the preliminary design being terminated at this point since better results could conceivably be achievable. In a trial and error design such as this in which there is no analytical method which can be used to establish guide figures, there is no way of knowing when an optimum or near optimum system has been achieved. Therefore, recognizing that the torque constant, velocity constant and rise time compared favorable with those of the conventional system, and realizing that a limited amount of time was available for this phase of the project, it was decided that this preliminary design would be used. A block diagram of this design is shown in Figure 7 where the Posicast has been included also.

The open loop transfer function of the preliminary design system chosen is given below.

$$\frac{\theta_L}{\theta_{error}} = \frac{K_m(B_1 S + K_1)}{J_m J_L S^4 + \left[B_1(J_m - J_L) - K_m(K_3 B_1 - K_2 J_L)\right] S^3 + \left[K_1(J_m + J_L) + K_m(K_1 K_3 + B_1 K_2)\right] S^2 + K_m K_2 K_1 S}$$

Figure 26 shows a pole zero sketch of this system and exhibits the increased damping achieved through the use of the rate and acceleration feedback. The damping could be increased further by increasing the quantities of rate and acceleration feedback or decreasing K_m, but increasing the quantity of rate feedback decreases the velocity constant and decreasing K_m decreases the torque constant. Doing either of the above increases the natural period. Damping was required to stabilize the system and permit use of a sufficiently large value of K_m to make the system reasonably fast. Beyond this it was felt that damping was more detrimental than helpful toward the system response since Posicast prevents oscillations due to input signals from being excited. Therefore, higher damping was not sought.

Realizing The Time Delay

After the preliminary design was obtained, the next problem confronted was that of achieving a time delay for the Posicast network. Smith[3] had suggested the use of a transmission line. An effort was made to approximate this by means of lumped elements, but when simulated on the analog computer this scheme gave very poor results. The possibility of approximating the Taylor expansion for $e^{-ST_n/2}$ with a ratio of two polynomials and simulating this on the analog computer was also considered, but

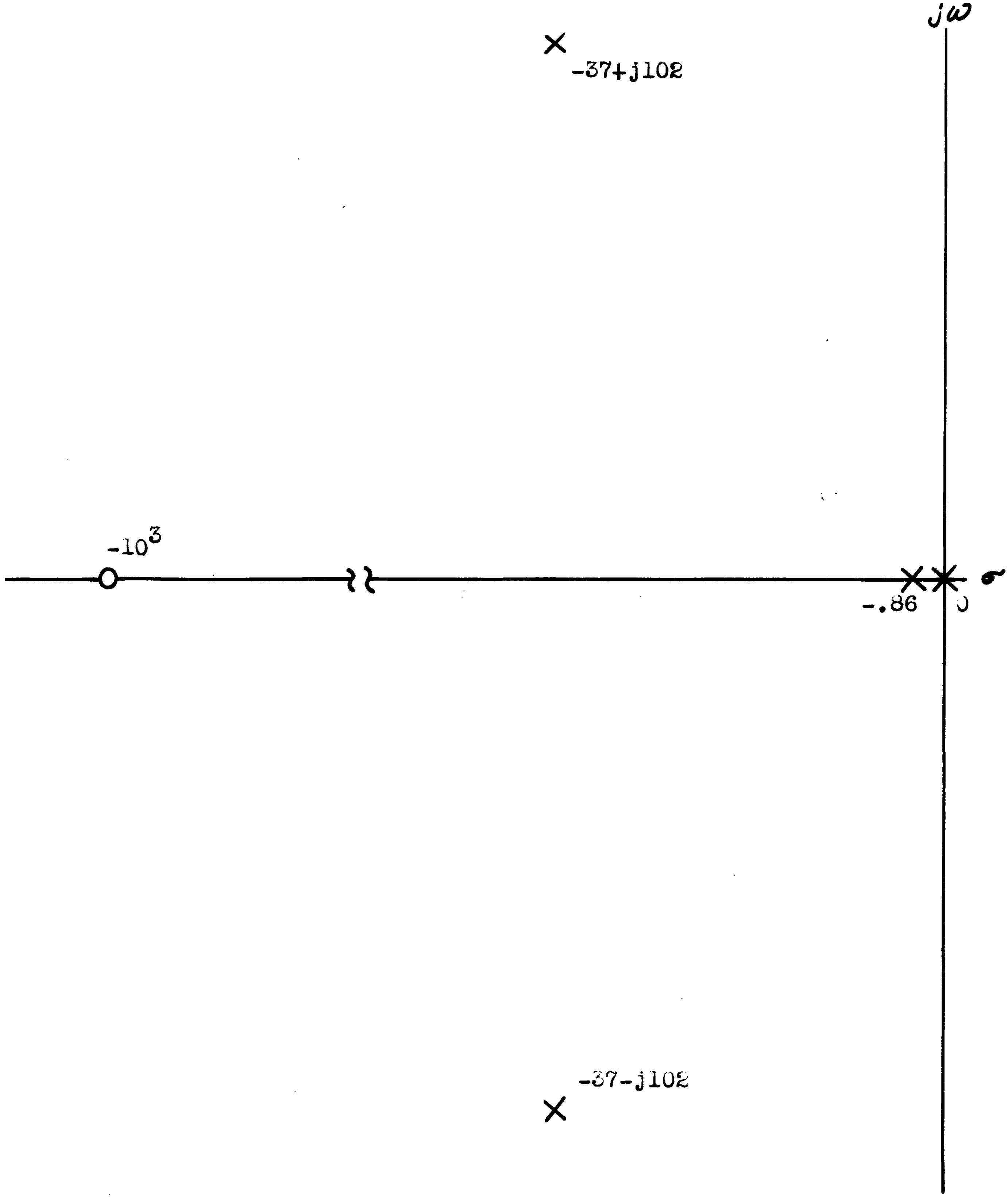


Figure 26.

Pole-Zero Plot of Preliminary System.

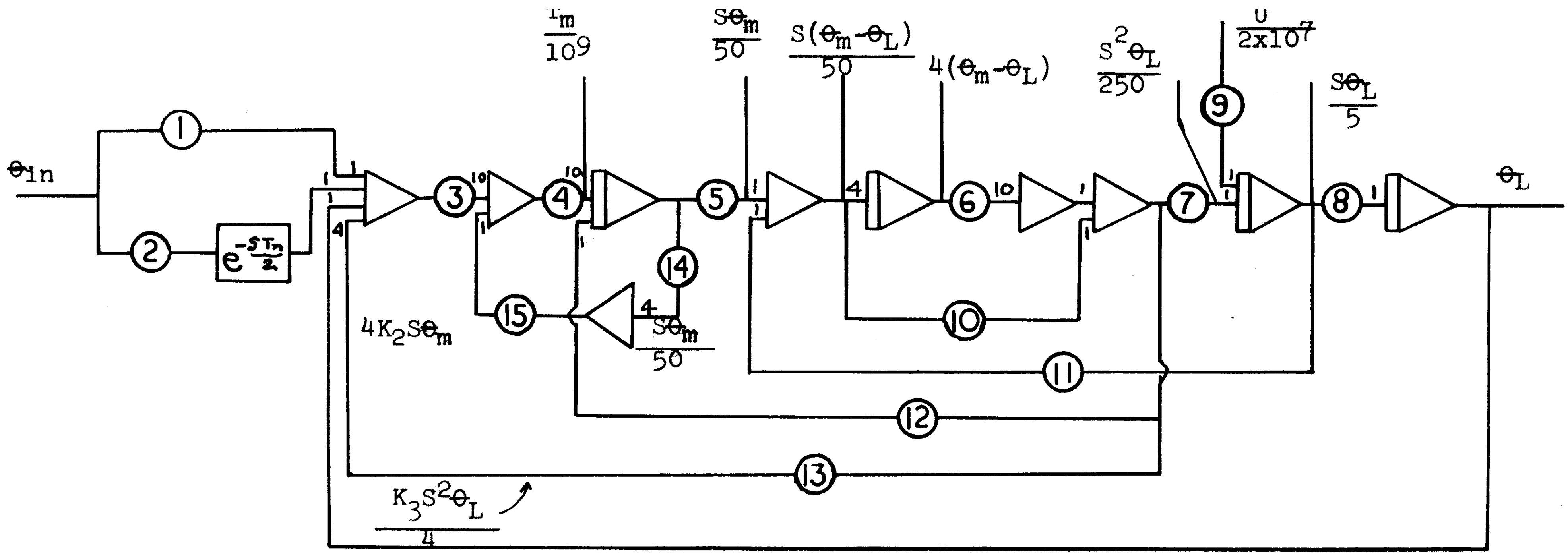
published literature on simulation techniques reported this method to be stable only for frequencies from zero to fifty cycles per second which would rule out step inputs.

Finally an Ampex F. M. tape recorder with separate record and play-back heads was obtained. The spacing of the heads was 1.5 inches and the only tape speed at which the recorder would both record and play back was thirty inches per second. This meant that a delay of .05 seconds could be achieved through the use of this tape recorder.

Since a delay of $T_{n/2}$ or .103 seconds was required, time scaling had to be introduced into the computer simulation to make the system operate at .103/.05 or 2.06 times real speed. This was achieved without difficulty by increasing all integrator gains in the simulation by a factor of 2.06. The system was now ready to be tested. Simulations of the Posicast system and the conventional system appear in Figures 27 and 28 respectively.

Testing of the Two Systems

Throughout the testing of the Posicast and conventional systems and the interpretation of the results, an attempt was made to maintain an objective approach. The tests used were those which it was believed would give the most information about the behavior of the systems. The same tests were performed several times

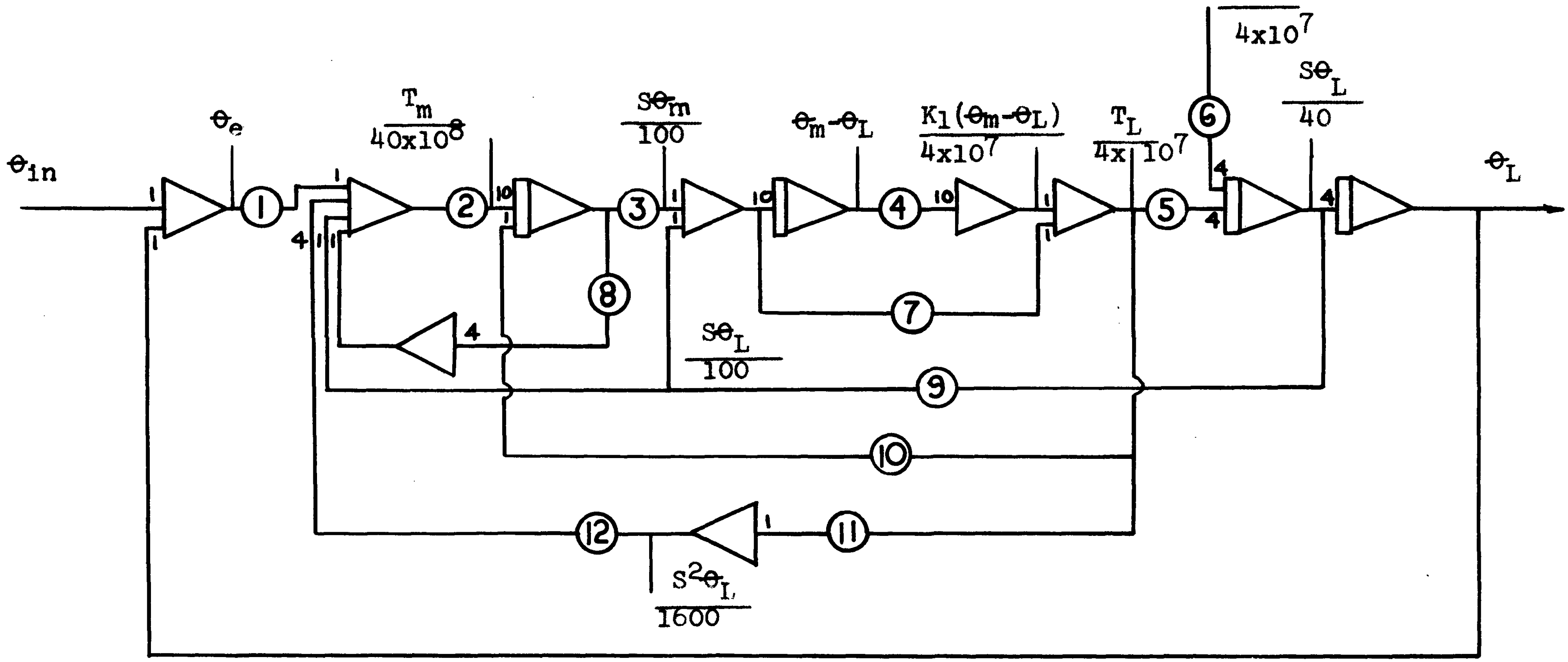


① $= 1/1+A$	④ $= K_m/40\times10^8$	⑦ $= 8\times10^4/J_L$	⑩ $= B_1/4\times10^5$	⑬ $= K_3\times10^7/2J_L$
② $= A/1+A$	⑤ $= .4\times10^5/J_m$	⑧ $= .1$	⑪ $= .1$	⑭ $= .4\times10^5/J_m$
③ $= .4$	⑥ $= K_1/8\times10^8$	⑨ $= 8\times10^4/J_L$	⑫ $= .2$	⑮ $= 50K_2$

Figure 27

Analog Computer Simulation of Posicast System at 1/50th Real Speed

(Computer was switched to 100 times real speed to give a final speed of 2 times real speed)



① = .4
② = $K_m / 40 \times 10^7$
③ = $10^5 / J_m$
④ = $K_1 / 4 \times 10^8$
⑤ = $10^5 / 4 J_L$
⑥ = $10^5 / 4 J_L$
⑦ = $B_1 / 4 \times 10^5$
⑧ = $K_2 \times 10^6 / J_m$
⑨ = .4
⑩ = .4
⑪ = $10^5 / 4 J_L$
⑫ = $160 K_3$

FIGURE 28

Analog Computer Simulation of Conventional System at 1/10th Real Speed on the systems to make sure that the results were consistent. Also, each test was performed with various magnitudes of input to ensure that the response was independent of magnitude.

The step input and the step load disturbance were achieved simply by actuating a contactor connected to a D. C. voltage source. The velocity input was obtained by passing a step function through an integrator on the analog computer. A sine-wave generator was used for determining the frequency response. For the random input, a triangle-wave generator was utilized with the frequency and magnitude of the signal being varied manually. It is recognized that this type of input is not truly random, but it is believed that it is somewhat representative of the type input to which one might expect a tracking-radar antenna to be subjected. It was hoped that the systems could be tested with a noise generator in cascade with a low-pass filter, but no noise generator was readily available of which the lower half-power frequency was less than the upper half-power frequency of the two systems to be tested.

All important results and conclusions of this investigation have been listed and discussed at the end of Chapter 1.

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