

# Handout 5 - A Monetary Model

Guido Lorenzoni

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## 1 One country

A model in continuous time.

Money demand

$$M^D = PYL(i)$$

$M^D$  is money demand,  $P$  is the price level,  $i$  is the nominal interest rate in domestic currency (pesos).

Output is constant  $\bar{Y}$  (full employment level of output), the real interest rate is constant  $\bar{r}$  (equal to the marginal productivity of capital).

Then we have:

$$\frac{M^D}{P} = \bar{Y}L(i)$$

Suppose money supply grows at a constant growth rate in country 1.

$$\frac{\dot{M}}{M} = \mu.$$

The Fisher equation (is really a definition of the real interest rate) is

$$i = \pi^e + r$$

where  $\pi^e$  is expected inflation.

Equilibrium in money market

$$M^D = M.$$

Rational expectations

$$\pi^e = \pi = \frac{\dot{P}}{P}.$$

Then

$$\frac{M}{P} = \bar{Y}L(\bar{r} + \pi).$$

Inflation constant and

$$\frac{\dot{M}}{M} = \frac{\dot{P}}{P} = \mu.$$

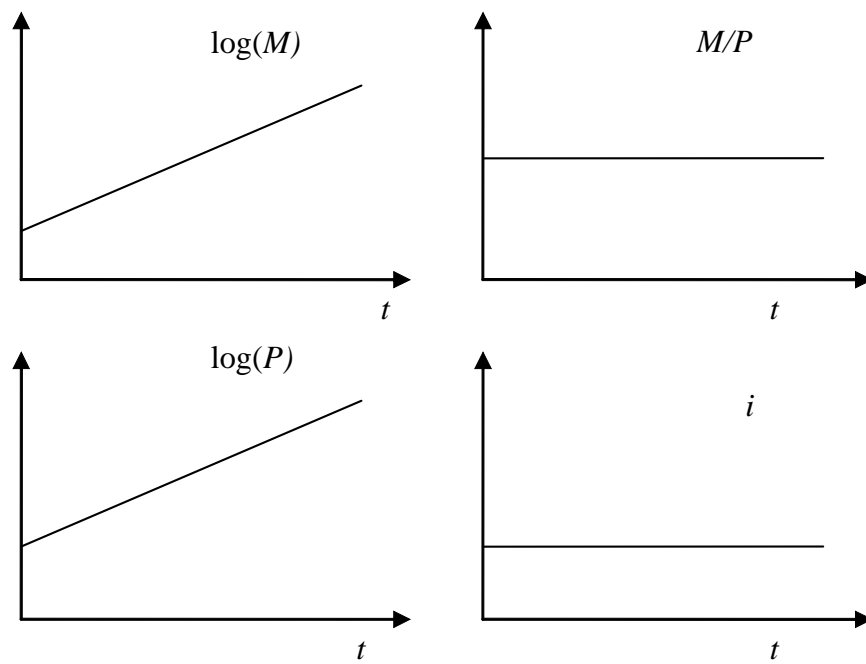


Figure 1: Time path of money, prices, real balances, nominal interest rates

## 2 Two countries

Two countries with different monetary policy

$$\frac{\dot{M}}{M} = \mu > \frac{\dot{M}^*}{M^*} = \mu^*.$$

Suppose there is only one good, so PPP holds:

$$P = EP^*$$

$E$  is nominal exchange rate, the real exchange rate always 1.

With open goods markets, we have PPP and we have

$$\frac{\dot{E}}{E} = \pi - \pi^* \tag{1}$$

The one country analysis still applies and we have

$$\frac{\dot{E}}{E} = \mu - \mu^*.$$

What is the effect of having open capital markets?

The UIP has to hold too:

$$i = i^* + \frac{\dot{E}}{E}$$

this together with (1) gives

$$i - \pi = i^* - \pi^*$$

that is we need

$$\bar{r} = \bar{r}^* = \bar{r}_w$$

same real interest rate in the two countries (remember the model with one good in Handout 3).

Here the nominal exchange rate only depends on the monetary side.

The difference between the two countries is the level of real balances:

$$\begin{aligned} \frac{M}{P} &= \bar{Y}L(\bar{r}_w + \mu), \\ \frac{M}{P} &= \bar{Y}^*L(\bar{r}_w + \mu^*). \end{aligned}$$

## 3 A surprise change in monetary policy

Surprise change in  $\mu$  from  $\mu$  to  $\mu'$  at date  $T$ .

What happens to the price level?

The inflation rate goes from  $\mu$  to  $\mu'$ .

The level of real balances has to *jump*.

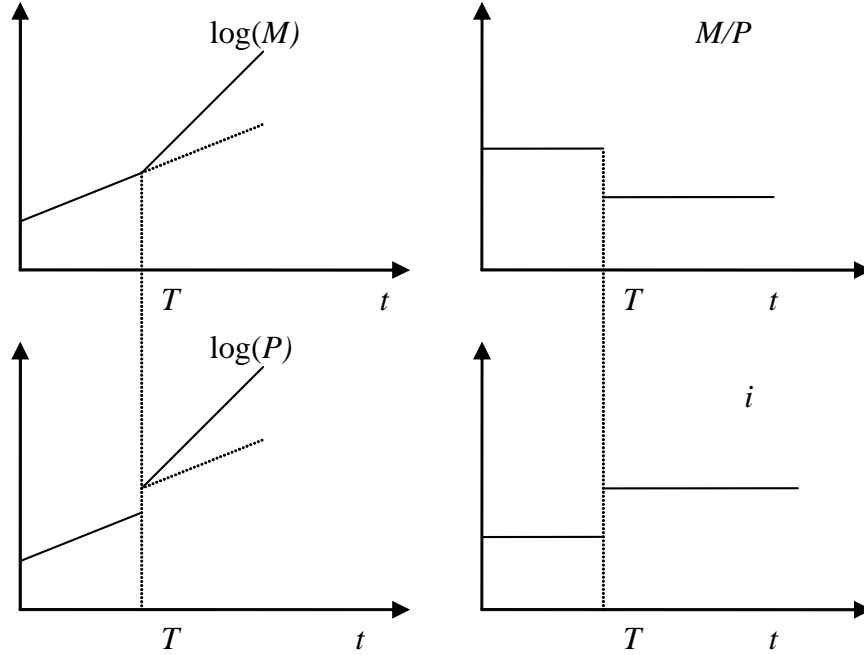


Figure 2: A change in money growth

$$\frac{M(t)}{P(t)} = \bar{Y}L(\bar{r}_w + \mu) \text{ for } t < T$$

$$\frac{M(t)}{P(t)} = \bar{Y}L(\bar{r}_w + \mu') \text{ for } t > T$$

$$\lim_{\epsilon \rightarrow 0^+} \frac{P(T + \epsilon)}{P(T - \epsilon)} = \frac{\frac{M_T}{\bar{Y}L(\bar{r}_w + \mu')}}{\frac{\bar{Y}M_T}{L(\bar{r}_w + \mu)}} = \frac{L(\bar{r}_w + \mu)}{L(\bar{r}_w + \mu')} > 1$$

This together with PPP means that the nominal exchange rate  $E(t)$  has to jump at time  $T$ .

## 4 The overshooting model

Suppose constant money supply in both countries at  $M$ . Zero inflation in both countries.

Then money supply jumps at  $M'$  in country home. Stays the same abroad.

Now no effect on growth rate of money.  
 Only effect is that  $P$  and  $E$  change.

Now suppose that  $P(t)$  adjusts gradually.  
 The price level follows the adjustment rule

$$\dot{p} = \delta(e - p)$$

where  $p = \ln P$  and  $e = \ln E$  (notice that  $\dot{p}$  is the same as  $\pi$ , inflatio  
 Now suppose

$$\begin{aligned} \bar{Y} &= 1 \\ L(i) &= e^{-\eta i} \end{aligned}$$

and let  $m = \ln M$ . The money demand becomes

$$m - p = -\eta i.$$

Now capital movements are crucial UIP

$$i = i^* + \dot{e}.$$

Putting together these equations we obtain a system of two differential equa-  
 tions

$$\begin{aligned} m - p &= -\eta(i^* + \dot{e}) \\ \dot{p} &= \delta(e + p^* - p) \end{aligned}$$

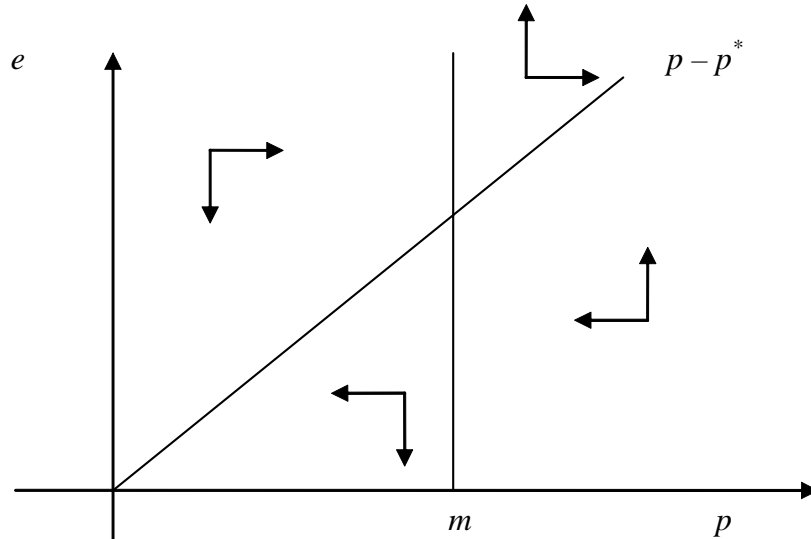
The long-run equilibrium is

$$\begin{aligned} p &= m - \eta i^* \\ e &= p - p^* \end{aligned}$$

Notice that when  $m > p$  there is excess money supply in the country, this  
 pushes down the nominal interest rate,  $i < i^*$ , and this can only be consistent  
 with an expected *appreciation* of our exchange rate,  $\dot{e} < 0$ .

On the other hand, when  $e + p^* > p$  PPP fails to hold, domestic goods are  
 cheaper than foreign goods, the demand for domestic goods increases and there  
 is a pressure for the domestic prices to increase.

With this two observations we can describe the phase diagram of the system.



Phase diagram

Suppose the money supply goes from  $m$  to  $m'$ .

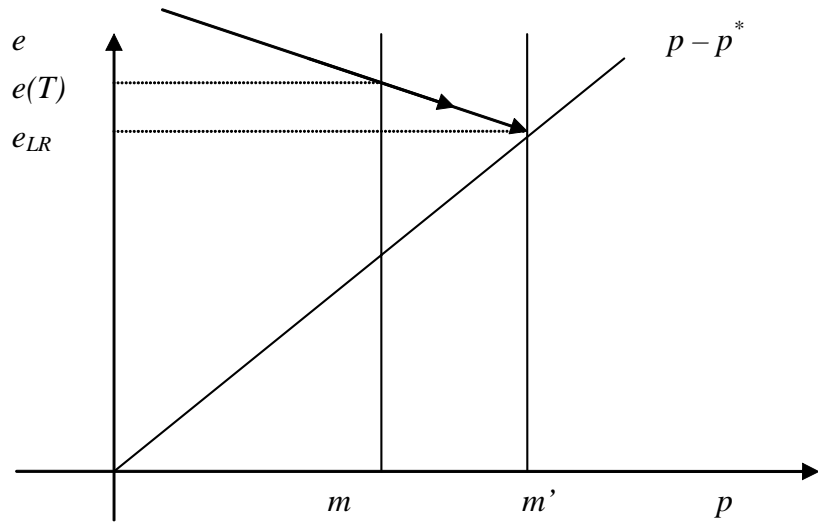
Now the price level cannot adjust instantaneously. We have  $m > p$ ,  $i < i^*$  and  $\dot{e} < 0$ .

The price level has to converge to  $m'$ . Therefore the exchange rate has to converge to  $e_{LR} = m' - p^*$  in the long run. In the short run, given that  $\dot{e} < 0$  the exchange rate has to be higher than  $m' - p^*$ .

The jump of the exchange rate at date  $T$  is

$$\Delta e > m' - m$$

larger than the money supply shock.



Overshooting