Handout 4 - Hedging Currency Risk

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November 20, 2005

Two agents. A car dealer and a beverage exporter.

A car dealer ordered 10 cars at 10.000 euros each. The cars will be shipped and payed in 6 months. He is selling the cars at 12.000 US$ each. Suppose he knows all the cars will be sold.

The profits of the car dealer will be

$$y_C = 12.000 * 10 - e * 10.000 * 10$$

where e is the euro/dollar exchange rate in 6 months.

The beverage exporter is purchasing 10.000 cases of coke for 10 dollars each and plans to sell them abroad in 6 months for 10 euros each. The profits for the beverage exporter are

$$y_B = e * 10 * 10.000 - 10 * 10.000.$$

The exchange rate today is $e_0 = 1$.

There are two "scenarios" or "states of the world": either the dollar will depreciate against the euro and $e = 1.2$ or the dollar will not depreciate and $e = 1$.

Both agents attach probability π to the event "depreciation".

1 The demand for forward contracts

Their profits in the two states of the world are:

state	1	2
e	1.2	1
y_C	0	20.000
y_B	20.000	0

The preferences of the two agents are given by the *expected utility function*

$$\pi u\left(c_1^j\right) + (1-\pi) u\left(c_2^j\right)$$

with $j = B$ or C. Here, c_1^j is their consumption in state of the world 1, and c_2^j in state of the world 2 (both expressed in US dollars).

Let $u(.)$ be a concave function. Remember that the concavity of $u(.)$ reflects the fact that agents are *risk averse*.

Agents have access to a *forward market*. They can buy or sell 1 units of foreign currency in 6 months, in exchange for f units of domestic currency, also in 6 months.

Suppose they consider the following financial strategy: buy a forward contract for x euros and then sell the x euros they receive in six months. The net gains on this financial strategy are

$$(e - f)\, x$$

US dollars.

Therefore, if they follow this financial strategy their consumption in 6 months will be

$$c_s^j = y_s^j + (e_s - f) * x$$

Notice that e_s depends on the state of the world realized in 6 months, while f and x are chosen today.

Now, we can write the consumer problem

$$\begin{aligned} \max \quad & \pi u\left(c_1^j\right) + (1-\pi)\, u\left(c_2^j\right) \\ s.t. \quad & c_1^j = y_1^j + (e_1 - f) * x \\ & c_2^j = y_2^j + (e_2 - f) * x \end{aligned}$$

Notice that, after some algebra, we can rewrite the budget constraints of the consumer in the following way

$$c_1^j + \frac{e_1 - f}{f - e_2} c_2^j = y_1^j + \frac{e_1 - f}{f - e_2} y_2^j.$$

This is similar to a traditional 2-goods model. The relative price of the goods in the two states of the world is given by $\frac{e_1 - f}{f - e_2}$ and depends on the forward rate f at which you can trade.

Another way of rewriting the problem is

$$\max_x \pi u\left(y_1^j + (e_1 - f)\, x\right) + (1-\pi)\, u\left(y_2^j + (e_2 - f)\, x\right)$$

From this problem we get the first order conditions

$$\pi (e_1 - f)\, u'\left(y_1^j + (e_1 - f)\, x\right) + (1-\pi)(e_2 - f)\, u'\left(y_2^j + (e_2 - f)\, x\right) = 0,$$

and from this, if we knew the utility function u we could derive the demand for forward contracts x.

Exercise 1 *Suppose*

$$u(c) = \log(c)$$

show that the demand for forward contracts is given by

$$x^j = \pi \frac{y_2^j}{f - e_2} - (1 - \pi) \frac{y_1^j}{e_1 - f}$$

this demand is: (1) decreasing in f*, (2) increasing in* π*. Interpret these two properties.*

2 Equilibrium

Now we want to find the forward rate f that gives an equilibrium in the market for forward contracts. This means that if C wants to buy euros forward, $x^C > 0$, then B needs to sell euros forward, $x^B < 0$ and

$$x^C = -x^B.$$

I want to show that an equilibrium of the forward market implies that we have *full insurance*, that is, both agents have stable consumption, and consume the same amount in the two states

$$\begin{aligned} c_1^C &= c_2^C \\ c_1^B &= c_2^B \end{aligned}$$

Let's start with this guess and plug it in the consumer first order conditions:

$$\begin{aligned} \pi (e_1 - f) u'\left(c_1^C\right) + (1 - \pi)(e_2 - f) u'\left(c_2^C\right) &= 0, \\ \pi (e_1 - f) u'\left(c_1^B\right) + (1 - \pi)(e_2 - f) u'\left(c_2^B\right) &= 0 \end{aligned}$$

If consumption is equal in the two states so are the marginal utilities

$$\begin{aligned} u'\left(c_1^C\right) &= u'\left(c_2^C\right) \\ u'\left(c_1^B\right) &= u'\left(c_2^B\right) \end{aligned}$$

so the first order conditions are satisfied as long as

$$\pi (e_1 - f) + (1 - \pi)(e_2 - f) = 0$$

or

$$f = \pi e_1 + (1 - \pi) e_2.$$

We still need to check that the budget constraints of the two agents are satisfied. This requires that

$$c_1^j = c_2^j = \frac{y_1^j + \frac{e_1 - f}{f - e_2} y_2^j}{1 + \frac{e_1 - f}{f - e_2}}.$$

So the consumption levels for the two consumers are in general different, but *for each consumer* they are equal across states of the world.

Exercise 2 *Using the result of the previous exercise plot the aggregate demand for forward contracts, $x^C + x^B$, in as a function of the forward rate f. Show that $x^C + x^B = 0$ when $f = \pi e_1 + (1 - \pi) e_2$. (Use $\pi = 1/2$ and the values of y and e given above for your plots).*

Therefore, the forward exchange rate in this economy is identical to the mathematical expectation of the spot exchange rate e.

$$f = \pi e_1 + (1 - \pi) e_2.$$

The forward market essentially allows the consumers to share optimally the exchange-rate risk they are exposed to.

Example 3 *Suppose $\pi = 1/2$, then we have the following forward rate*

$$f = \frac{1}{2} 1.2 + \frac{1}{2} 1 = 1.1$$

and the following consumption profiles (in thousands of dollars)

$$\begin{aligned} c^C &= 10 \\ 10 &= 0 + 10\% * x^C \\ 10 &= 20 - 10\% * x^C \end{aligned}$$

and

$$\begin{aligned} c^B &= 10 \\ 10 &= 20 + 10\% * x^B \\ 10 &= 0 - 10\% * x^B. \end{aligned}$$

Notice that this requires

$$\begin{aligned} x^C &= 100 \\ x^B &= -100 \end{aligned}$$

the car dealer buys 100.000 euros forward from the beverage exporter.

Example 4 *Now suppose we have $\pi = 1/4$, then we have the following forward rate*

$$f = \frac{1}{4}1.2 + \frac{3}{4}1 = 1.05$$

and consumption

$$\begin{aligned} c^C &= 15 \\ 15 &= 0 + 15\% * x^C \\ 15 &= 200 - 5\% * x^C \end{aligned}$$

and

$$\begin{aligned} c^B &= 5 \\ 5 &= 20 + 15\% * x^B \\ 5 &= 0 - 5\% * x^B \end{aligned}$$

Notice that this requires

$$\begin{aligned} x^C &= 100 \\ x^B &= -100 \end{aligned}$$

the car dealer buys 100.000 euros forward from the beverage exporter.

Interpretation: in both examples after six months the beverage exporter takes his euros and gives them to the car dealer, so he can pay for his imports. In both examples this transacation completely *eliminates the exchange rate risk*. In practice, neither of the two agents has to trade on the spot market, when one needs euros he simply gets euros from the other agent, so they are fully insulated from movements in the spot market.

The one thing that needs to be determined is the price at which they will do this transaction. In the first example a depreciation of the dollar is very likely so the car dealer has to pay dearly for the forward euros he is buying. In this case his consumption is 10.000 dollars. In the second example a depreciation is less likely and the euros forward are cheaper. In this case is consumption is 15.000 dollars.